Random Errors Compensation method of Autoregressive Adaptive Kalman Filter based on MEMS Gyroscope

Yingmin Yi , Haiming Sun* , Bojun Liu , Yiwei Yuan , Yankai Li , Xinyu Zhang , Zhexing Li
Faculty of Automation and Information Engineering, Xi'an University of Technology, Xi'an 710048, Shaanxi , China

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***Abstract*—For the low precision problem of MEMS gyroscopes, an adaptive Kalman filter method based on the autoregressive AR model is applied to compensate MEMS gyroscopes in time series modeling. The Allan variance analysis principle is used to obtain various random performance indicators of the gyroscope. The collected non-stationary data are preprocessed and converted into stationary, zero-mean random drift data. Then the model and parameters of the time series are selected for Autoregressive modeling using the AIC criterion. Finally, the adaptive Kalman filtering method is used to compensate for random drift data. Comparing the performance indicators before and after compensation, this method is indeed more effective than Kalman filter in suppressing the random error of gyroscope**

Keywords—MEMS gyroscope, random error, time series modeling, Allan variance analysis, adaptive Kalman filter

I. INTRODUCTION

Due to the continuous development of aerospace technology, the demand for global positioning system(GPS) and Inertial measurement unit is increasing(IMU). However, due to the influence of climate, troposphere, electromagnetic waves, ionosphere and other factors, there will be deviations when determining the position of the global positioning system. In addition, the continuous progress of sensor and semiconductor technology in recent years, and because of its advantages of small size, low price, and easy installation, MEMS inertial navigation has great development potential and value [1]. The gyroscope is the most basic and important unit of the MEMS inertial measurement unit, so the research on the gyroscope is very necessary. Due to the level of manufacturing and design, MEMS gyroscopes have disadvantages such as large random errors and poor stability compared with other high-precision gyroscopes, which will lead to the measurement accuracy of the system. Therefore, in order to reduce the error of gyroscope data in MEMS inertial measurement unit, it is urgent to analyze its own random error and compensate for the error.

In recent years, with the widespread application of MEMS inertial measurement units in autonomous driving, there are more and more methods for compensating for random errors of MEMS gyroscopes. For example, Some scholars proposed a method based on wavelet threshold to remove the error, and applied the improved wavelet denoising algorithm to remove the random error of the gyroscope in the inertial measurement unit.[2-3] , and some scholars use Empirical Mode Decomposition (EMD) combined with time series modeling to compensate for random errors of gyroscopes [4-5]. For most papers, time series modeling is generally used to obtain the error model of gyroscope, and Kalman filter is configured according to the model parameters to compensate for its random error [6-8]. However, Kalman filter has some limitations in solving this problem. The process noise matrix and measurement noise matrix of the system play a significant role. In the process of filtering the measurement data, the adaptive Kalman filter dynamically judges the filtering process according to the output value of the filter. If the model parameters change, or the statistical characteristics of square noise change, the filter can be corrected, and the parameters of the filter can be modified to reduce the error. Therefore, the adaptive Kalman filter is a better choice than other methods to compensate the random error of gyroscope. So, this experiment adopts this method to reduce the random error of gyroscope. The experimental steps are as follows: Firstly, the measured data is analyzed using the Allen's variance principle, and the random error coefficients of various gyroscopes such as quantization noise, angle random walk, bias instability, rate random walk, and rate ramp are obtained[9]. Secondly, the collected non-stationary data are preprocessed, tested, and trend items extracted, and converted into stable, zero-mean random drift data. Then the model and parameters of the time series are selected for AR modeling using the AIC criterion. Finally, the adaptive Kalman filtering method is used to compensate for random drift data. Comparing the performance indicators before and after compensation can be determined, this method can effectively suppress the random error of MEMS gyroscope.

II. RANDOM ERROR ANALYSIS OF MEMS GYROSCOPE

A. Allan variance analysis

The physical meaning of the Allan variance of an essentially random signal arises from its relationship to the power spectrum:

$$\sigma^2 = 4\int_0^\infty S_w(f)\frac{\sin(\pi f\tau)}{(\pi f\tau)^2}df \tag{1}$$

$\sigma^2(\tau)$ in formula (1) represents the Allan variance of the random error, τ is the average time, $S_w(f)$ represents the rate power spectral density of the gyroscope random error, f is the frequency. It can be seen from the above formula (1) that all the collected MEMS inertial measurement unit gyroscope data is put into a filter with a transfer function of $\sin^4 x / x^2$, and the size of the Allen variance increases with the increase of the random error noise of the gyroscope. Therefore, it can be obtained: The types of deviations can be obtained, and the total deviation amplitude of the gyroscope can also be obtained from the time domain.

Allan variance facilitates the separation of noise statistics in random signals. It quantifies the different noise terms in the signal with a formula. Since the random signal output by the inertial measurement unit has similar statistical characteristics to the frequency fluctuation of the atomic

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clock, the Allan variance analysis can be used as the random error characteristic of the MEMS gyroscope, which is recognized by IEEE. Standard method for gyroscope parameter analysis [10].

B. Allan variance identification of gyroscope random noise terms

The gyroscope in the MEMS inertial measurement unit mainly contains five random error terms, which are described in detail as follows:

1) Quantization noise

In the process of integrating continuous time signals into discrete signals through AD acquisition, the accuracy will be reduced. The size of the accuracy loss is proportional to the step size of AD conversion. The smaller the step size, the smaller the quantization noise. The Allan variance of the quantization noise is:

$$\sigma_{QN}^2(\tau) = \frac{3Q^2}{\tau^2} \tag{2}$$

2) The Angle random walk

The output of the angular velocity is not accurate angular velocity data, and usually contains white noise, and the process of calculating the attitude angle by integrating the angular rate will inevitably integrate the noise, and the integration of white noise is not White noise is a Markov process, and the Markov error contained in the angle error is called angle random walk. The Allan variance of an angle random walk is:

$$\sigma_{ARW}^2(\tau) = \frac{N^2}{\tau} \tag{3}$$

3) Zero-bias instability

The zero-bias instability of a gyroscope is generally not a fixed parameter, but a slow random drift within a certain range. It is necessary to assume a probability interval to describe how likely it is to fall within this interval. The Allan variance of the zero-bias instability is:

$$\sigma_{BI}^2(\tau) = (\frac{B}{0.6648})^2 \tag{4}$$

4) Angular rate random walk

This indicator is similar to the Angle random walk above, and the error contained in the angular rate error is the same as the angle random walk, which is the error of Markov nature, which is called the angular rate random walk. The Allan variance of an angular rate random walk is:

$$\sigma_{RRW}^2(\tau) = \frac{K^2\tau}{3} \tag{5}$$

5) Rate ramp

This indicator of gyroscope is actually a trend error, which is mainly caused by external factors. For example, the most common reason in the gyroscope is the zero position change caused by temperature. The Allan variance of the rate ramp is:

$$\sigma_{RR}^2(\tau) = \frac{R^2\tau^2}{2} \tag{6}$$

The statistical properties of the random errors shown by the above formulas are independent, and the sum of various errors can be regarded as the Allen variance, so the Allen variance can be expressed as:

$$\begin{aligned}&\sigma^2(\tau) = \sigma_{QN}^2(\tau) + \sigma_{ARW}^2(\tau) + \sigma_{BI}^2(\tau) \\ &+\sigma_{RRW}^2(\tau) + \sigma_{RR}^2(\tau) = \frac{3Q^2}{\tau^2} + \frac{N^2}{\tau} + \\ &(\frac{B}{0.6648})^2 + \frac{K^2\tau}{3} + \frac{R^2\tau^2}{2}\end{aligned} \tag{7}$$

It is observed that the above formula can be used $A_n^2\tau^2$ and since the value of the standard deviation is larger than the variance, the fitting standard deviation can be more accurately obtained. The above formula can be approximated as:

$$\begin{aligned}&\sigma^2(\tau) = \frac{3Q^2}{\tau^2} + \frac{N^2}{\tau} + (\frac{B}{0.6648})^2 \\ &+\frac{K^2\tau}{3} + \frac{R^2\tau^2}{2} = \sum_{n=-2}^{2} A_n^2\tau^2\end{aligned} \tag{8}$$

According to formula (8), we can obtain the random error coefficients in the gyroscope as shown in formula (9):

$$\begin{cases} Q = \frac{A_{-2}}{\sqrt{3}}\mu\text{rad} \\ N = \frac{A_{-1}}{60} \circ / \sqrt{h} \\ B = \frac{A_0}{0.6643} \circ / h \\ K = 60\sqrt{3}A_1 \circ / h^{3/2} \\ R = 3600\sqrt{2}A_2 \circ / h^2 \end{cases} \tag{9}$$

III. MEMS Gyroscope Data Acquisition and Processing

A. MEMS Gyroscope Data Acquisition System

The inertial measurement unit used in this experiment is RION16488, with a built-in 3-axis gyroscope and the microcontroller is STM32F103ZET6. The microcontroller first uses the SPI interface to collect the data information of the gyroscope, and transmits it to the host computer through the RS232 serial port for real-time communication. The sampling frequency is set to 5Hz. It is advisable to collect and sample the gyroscope data for 40 minutes after standing for 20 minutes. The physical map of the gyroscope data acquisition system as shown in Fig.1.

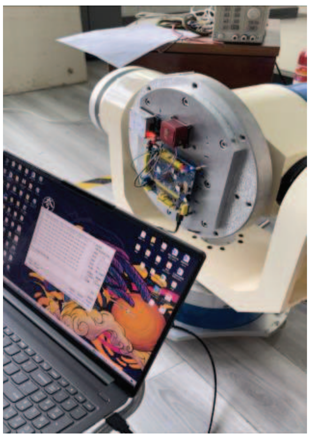

Fig. 1. Gyroscope data acquisition diagram

The gyroscope is relatively horizontally fixed on the inner axis of the turntable by screws. The raw data collected by RION16488 is shown in Fig. 2

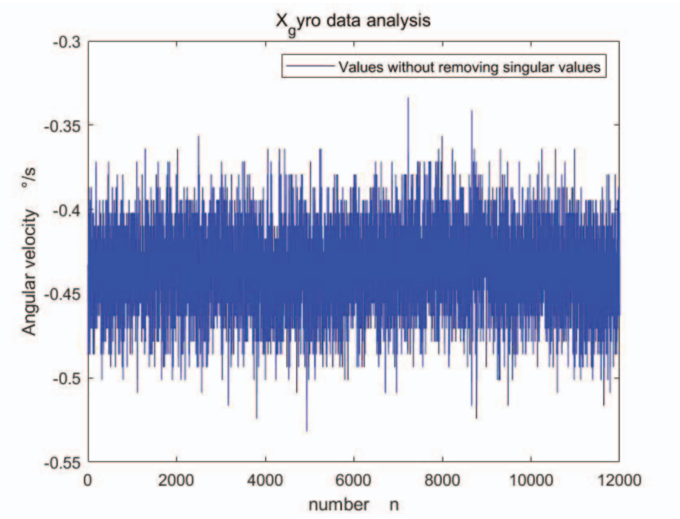

Fig. 2. RION16488 gyroscope x-axis raw data

B. MEMS Gyroscope Data Preprocessing

Observing the collected data, it can be seen that the amplitude of the gyroscope has obvious singularities, and with the increase of time, the amplitude of the gyroscope is not a stable process, it can be clearly seen that it first declines, then rises, and finally again declines. Therefore, before performing time series modeling on this set of data, preprocessing such as singularity removal and trend item extraction should be performed first.

For this set of data, use pauta criterion to remove singularities, This method proposes to calculate the difference between the value x_t of the signal at time t and the mean value $\overline{x_t}$ of all samples sampled to obtain the residual error V_t of each item, When the absolute value of the residual error V_t at a certain moment is greater than triple standard deviation σ of the sampling point, the sampling point x_t at this moment is considered to be a singular point and should be deleted [12].

When the data contains a deterministic trend, the trend is clearly expressed by mathematical methods, usually using the gray prediction method of exponential trend, polynomial fitting method, etc. In this paper, the polynomial fitting method is used, and the fitting function of the trend is obtained as :

$$f(t) = -4.71\times10^{-14}x^3 + 7.22\times 10^{-10}x^2 - 2.51\times10^{-6}x - 0.4353 \tag{10}$$

Figure 3 shows the image of the gyroscope x-axis data with singular points removed and the extracted trend items. From the figure, the blue line represents the value of the original data after removing the singular point according to pauta criterion, and the red line is the graph of the function obtained by the polynomial fit trend. After removing the singular point and extracting its trend item, the x-axis gyroscope data we have collected is a set of data that is stable, random, and has a normal distribution:

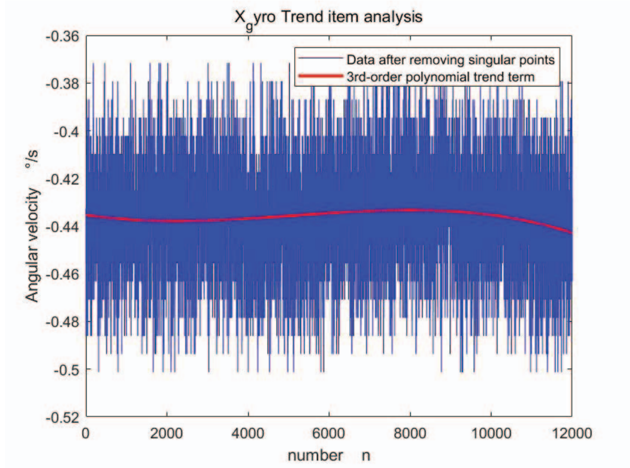

Fig. 3. Removing singular points and extracting trend item data

C. Data verification

After obtaining the preprocessed gyroscope data sampling value, it is necessary to perform statistical test on the group of data samples to determine whether it is a stable and normal random process, and the time series modeling can only be carried out after reaching the standard.

- Stationarity Test

ADF test (Augmented Dickey-Fuller Testing) is one of the most commonly used unit root test methods. It is used to judge whether the sequence is stationary by testing whether the sequence has a unit root. When an autoregressive process:

$$y_t = by_{t-1} + a + \varepsilon_t \tag{11}$$

If the value of b in Equation (11) is 1, it is the root of unit. If a series is stationary, it does not produce a unit root, but when the series is not stationary, it does. And the significant statistical characteristics of a set of sequences are less than 10%, 5%, 1% confidence level, then there must be 90%, 95%, 99% probability to reject the original hypothesis, that is, the data is not stationary, otherwise the data is stationary.

- Normality test

The normality test often adopts the skewness-kurtosis normality test method, but this paper adopts a more intuitive method to draw a histogram of the probability density of each quantity of the sequence, which can simply and intuitively determine that the set of data obeys the normal distribution , as shown in Fig. 4.

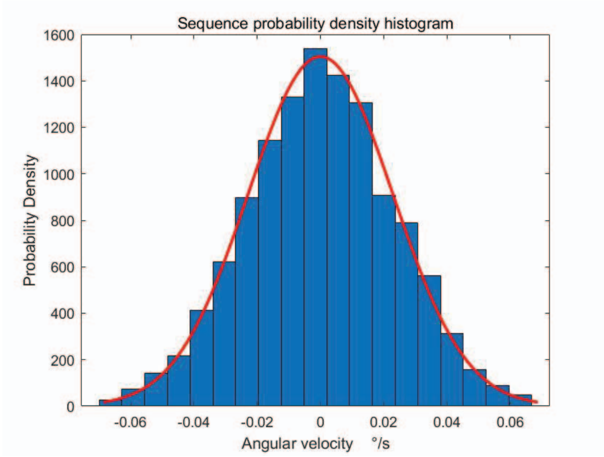

Fig. 4. Histogram of sequence probability density

IV. Time Series Models of Random Errors

After the above-mentioned preprocessing operations such as removing singular points and extracting trend items, and after testing for normality, periodicity, and stationarity, it can be seen that this group of data series is already a group of random and stable time series, so the time series can be used to construct The mathematical model of this group of sequences is established in a modular manner.

The autoregressive model AR(p), which points out that the measured value at any time in the sequence is a combination of linear regression of the measured values at p times before time t, and the expression is:

$$x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + ... + \varphi_p x_{t-p} + a_k \tag{12}$$

where φ_t is the autoregressive coefficient at time t, p is the order of the autoregressive model, and a_k is white noise sequence.

When the likelihood function in a model is larger, its model is more accurate, but the complexity of the model will also increase according to the increase of fitting accuracy, which will lead to overfitting. In order to avoid this situation, this paper adopts the Akaike Information Criterion, which is a weighting function of the fitting accuracy and the number of parameters. The expression is:

$$AIC = 2p - 2\ln(L) \tag{13}$$

In the above formula, L is the maximum likelihood function of the model.

As the order of the model increases, the likelihood function will also increase. When p is a certain value, the smallest AIC value will be obtained, and the p value will be the most suitable model order. Table I below shows the AIC values of each model. It can be found that AIC value of AR (1) is less than that of AR (2) and AR (3), so AR (1) model is the best.

TABLE I. AIC VALUE COMPARISON TABLE CORRESPONDING TO DIFFERENT MODELS

model	AR(1)	AR(2)	AR(3)
AIC	-4.8930	-4.8929	-4.8912

In summary, the model for establishing AR(1) is:

$$x_t = 0.409975 x_{t-1} + a_k \tag{14}$$

V. Adaptive Kalman Filter Compensation for Random Errors

The process noise matrix and measurement noise matrix of the system play a significant role. However, Kalman filter has some limitations in solving this problem. Although the system model has been obtained before, for the gyroscope in the MEMS inertial measurement unit, with the working environment and measurement If conditions and other factors change, the parameters of the Kalman filter will also change. Therefore, the adaptive Kalman filter is used in this experiment, and the filtering process can be dynamically adjusted according to the output value of the filter. Changes in statistical characteristics can be modified to achieve higher filtering accuracy.

The actual system input includes process noise and measurement white Gaussian noise, so the system state equation and observation equation of the gyroscope are:

$$X_k = \phi_{k,k-1} X_{k-1} + G_{k,k-1} W_{k-1} \tag{15}$$

$$Y_k = H_k X_k + V_k \tag{16}$$

In the above formula, X_k is the state variable at time k, Y_k is the observed value at time k, W (k) is the process noise, and V (k) is the measurement Gaussian white noise, both of which are independent random variables, and the relationship between them should be:

$$\begin{cases} E(W(k)) = q_k \\ E(W^2(k)) = Q_k \delta_{kj} \\ E(V(k)) = r_k \\ E(V^2(k)) = R_k \delta_{kj} \\ COV(W(k)V(k)) = 0 \end{cases} \tag{17}$$

If the mean parameters q_k and r_k of Gaussian white noise and the parameters Q_k and R_k of the variance are all known, the formula of the adaptive Kalman filter is:

$$\begin{cases} x_{k,k-1} = \phi_{k,k-1} x_{k-1} + q_{k-1} \\ y_{k,k-1} = H_k x_{k,k-1} + r_k \\ Y_k = Y_k - H_k x_{k,k-1} - \hat{r}_{k-1} \\ K_k = P_{k,k-1} - H_k^T [H_k P_{k,k-1} H_k^T + R_k]^{-1} \\ x_k = x_{k,k-1} + K_k (y_k - y_{k,k-1}) \\ P_k = P_{k,k-1} - K_k P_{k,k-1} K_k^T \\ P_{k,k-1} = \phi_{k,k-1} P_{k-1} \phi_{k,k-1}^T + Q_{k-1} \end{cases} \tag{18}$$

If the mean parameters q_k and r_k of Gaussian white noise and the parameters Q_k and R_k of the variance are all unknown, these parameters can be estimated from noise statistics:

$$q_k = (1 - d_{k-1}) q_{k-1} + d_{k-1} [x_k - \phi_{k,k-1} x_{k-1}] \tag{19}$$

$$Q_k = (1 - d_{k-1}) Q_{k-1} + d_{k-1} [K_k Y_k Y_k^T K_k^T + P_k - \phi_{k,k-1} \phi_k^T] \tag{20}$$

$$\hat{r}_k = (1 - d_{k-1}) \hat{r}_{k-1} + d_{k-1} [Y_k - H_k x_{k,k-1}] \tag{21}$$

$$R_k = (1 - d_{k-1}) R_{k-1} + d_{k-1} [Y_k Y_k^T - H_k P_k H_k^T] \tag{22}$$

Among them, $d_k = (1-b)/(1-b^{k+1})$ b is the forgetting factor, which can limit the memory length of the filter[13]. The general value range is between 0.95 and 0.99, which can make the new data play a dominant role in the estimation process.

According to the formula(18) of the adaptive Kalman filter, the corresponding adaptive Kalman filter can be designed, and random error compensation is performed on the output data of this group of MEMS gyroscopes. The compensation result is shown in Fig. 5.

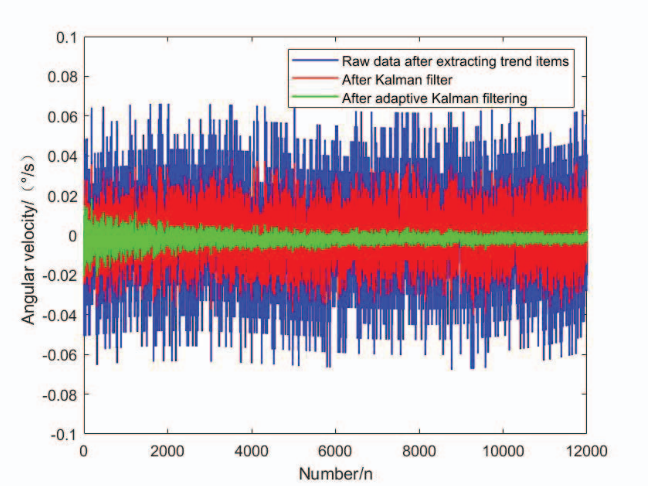

Fig. 5. Data output before and after random error compensation

The blue line in Figure 5 represents the raw observation data that has not been processed, the red line is the output value after Kalman filter processing random errors, and the green line means the result of the data after adaptive Kalman filtering, It can be concluded that the adaptive Kalman filter is much better than the Kalman filter in compensating the random error of gyroscope. The specific quantitative comparison of the above three cases is shown in Table II.

TABLE II. DATA PROCESSING RESULT TABLE OF DIFFERENT ALGORITHMS

method	mean	variance
Raw data	2.74×10^{-5}	5.24×10^{-4}
KF	8.9×10^{-7}	1.48×10^{-4}
AKF	-1.77×10^{-8}	6.82×10^{-6}

According to this set of data, the time series modeling is carried out, the parameters are derived after the model is obtained, and the adaptive Kalman filter is constructed to reduce the random error. Finally, it can be seen from Table III that the adaptive Kalman filter reduces the quantization noise of the gyroscope by 86.15%, the angle random walk by 87.67%, the bias instability by 79.15%, and the random angular rate by 64.93%, and reducing the rate ramp by 70.83%.

TABLE III. COMPARISON TABLE OF ERROR TERMS

random error coefficient	Raw data	After AKF	percent reduction
Quantization noise $\circ/h$	0.007359	0.001019	86.15%
Angle random walk $\circ/\sqrt{h}$	0.000365	0.000045	87.67%
Zero Bias Instability $\circ/h$	0.003277	0.000683	79.15%
Angular rate random walk $\circ/h^{3/2}$	0.006234	0.002186	64.93%
rate ramp $\circ/h^2$	0.007542	0.002200	70.83%

VI. CONCLUSION

In this experiment, the adaptive Kalman filter compensation for the random error of the gyroscope in the MEMS inertial measurement unit is studied. First, the Allan variance analysis principle is used to do experiments with the actual measured data, and various random performance indicators of the gyroscope are obtained. Then, the collected unstable data are preprocessed and transformed into relatively stable, zero-mean random drift values. Then, the mode and parameters of the time series are selected by AIC criteria, and AR modeling is carried out. Finally, the adaptive Kalman filtering method is used to compensate the random drift data, and the random error after compensation is significantly improved compared with the original data. It is proved that the method has practical value and significance for compensating the random error of gyroscope.

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