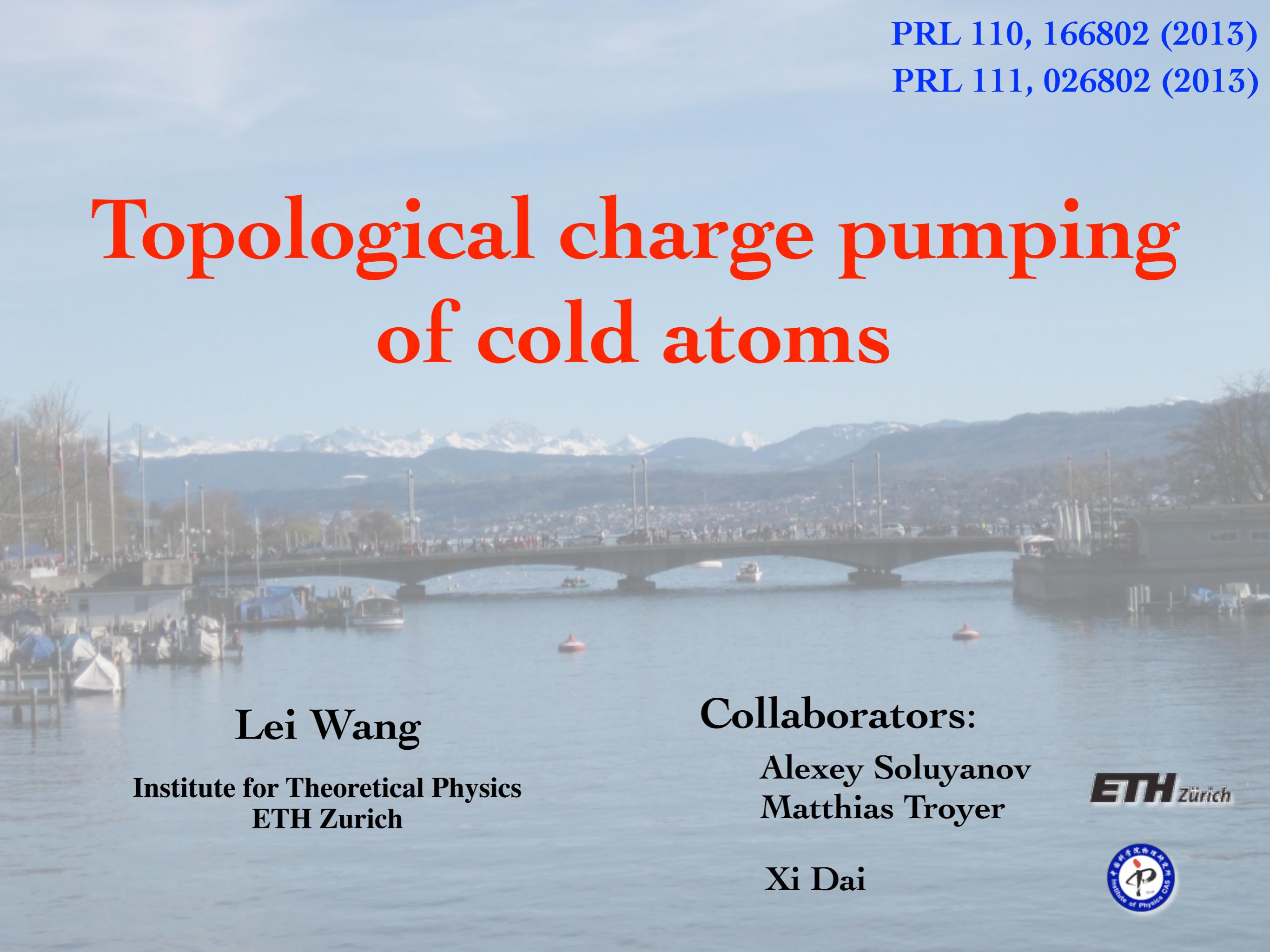


PRL 110, 166802 (2013)
PRL 111, 026802 (2013)

Topological charge pumping of cold atoms



A scenic view of a lake with many sailboats and a bridge in the background, with snow-capped mountains under a clear blue sky.

Lei Wang

Institute for Theoretical Physics
ETH Zurich

Collaborators:

Alexey Soluyanov
Matthias Troyer

Xi Dai



Pumps



Pump is a device that moves fluids, or sometimes slurries, by mechanical action.

Pumps



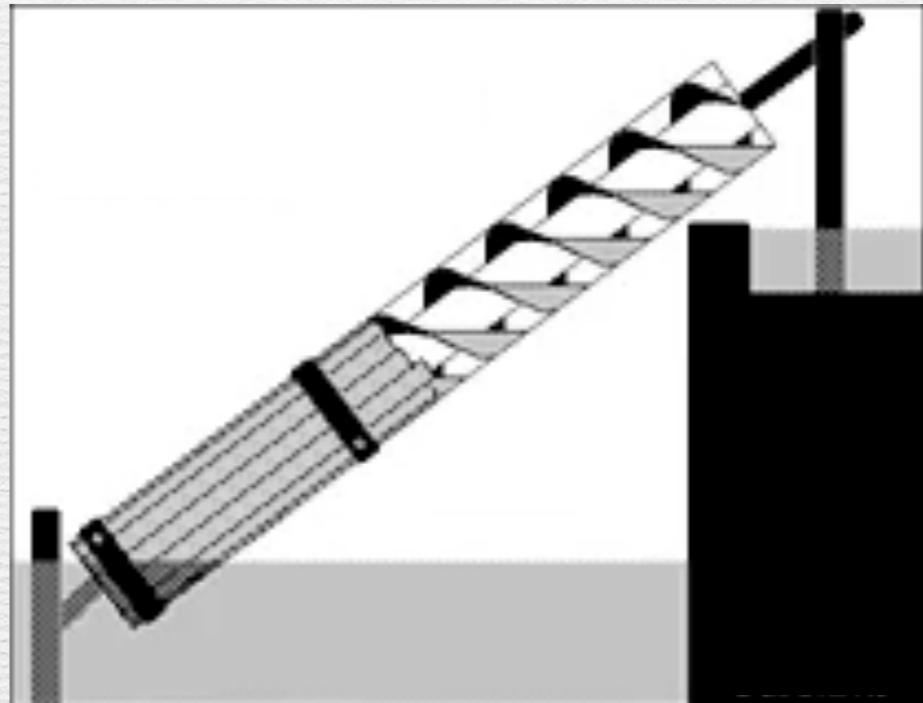
Pump is a device that moves fluids, or sometimes slurries, by mechanical action.



Pumps



Pump is a device that moves fluids, or sometimes slurries, by mechanical action.

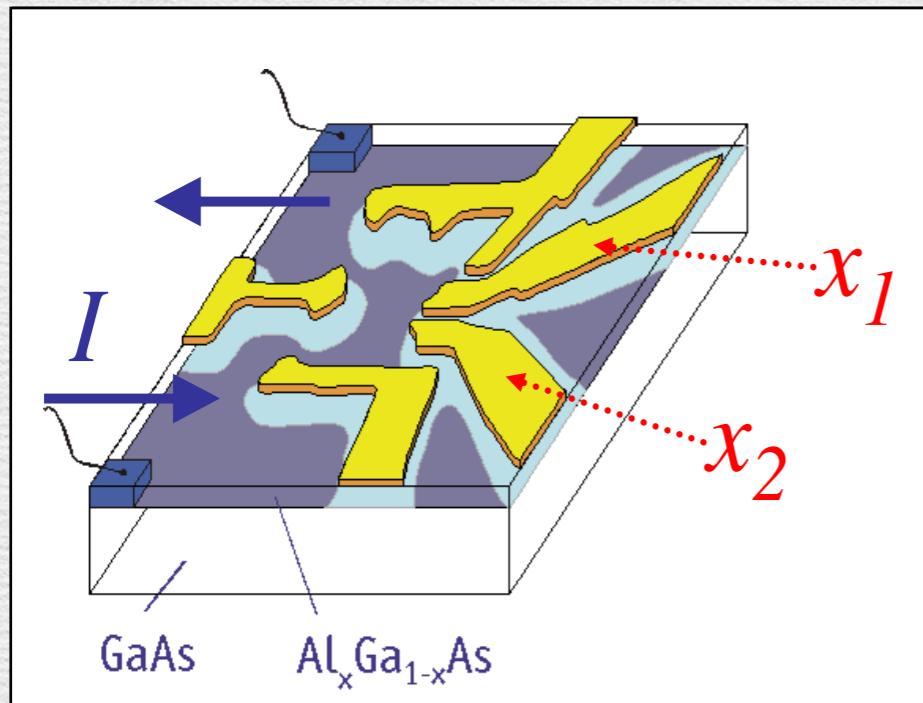


Archimedes' screw ~250 BC

Pumps

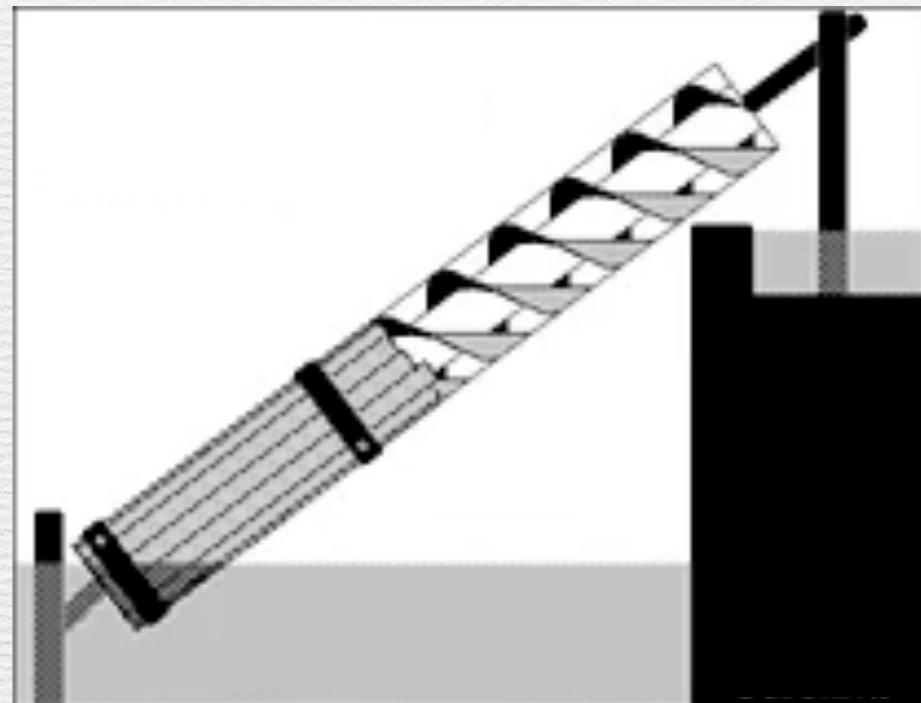


Pump is a device that moves fluids, or sometimes slurries, by mechanical action.



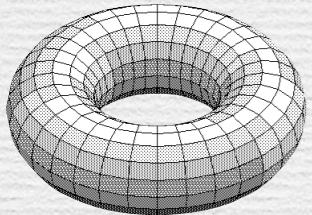
Switkes *et al* 1999

Buttiker, Brouwer, Zhou, Spivak, Altshuler ...



Archimedes' screw ~250 BC

Topological pump

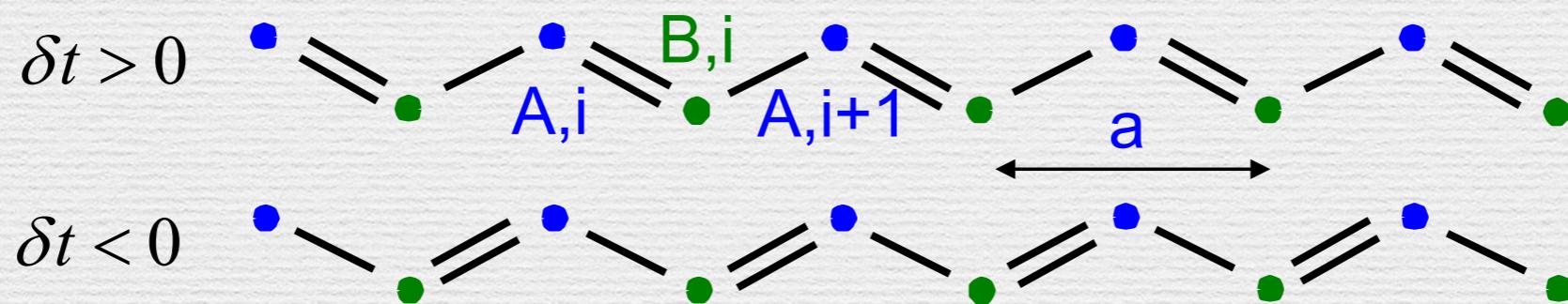


A device transfers **quantized charge** in each pumping cycle.

Thouless 1983

$$H = \sum_i (t + \delta t) c_{Ai}^\dagger c_{Bi} + (t - \delta t) c_{Ai+1}^\dagger c_{Bi} + H.c.$$

Su, Schrieffer, Heeger, 1979

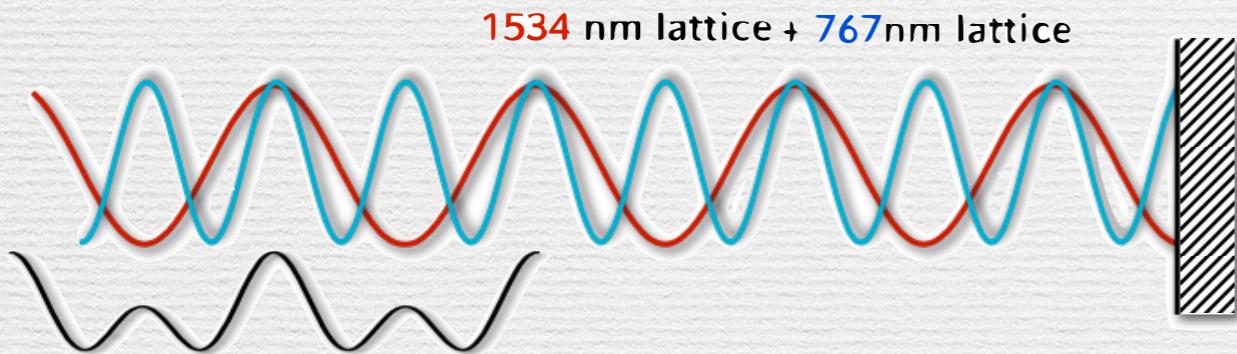


- ~ Current flows in an insulating state
- ~ No dissipation!
- ~ Dynamical analog of quantum Hall effect

Experimental progresses

Optical Superlattice

Fölling *et al*, Atala *et al*

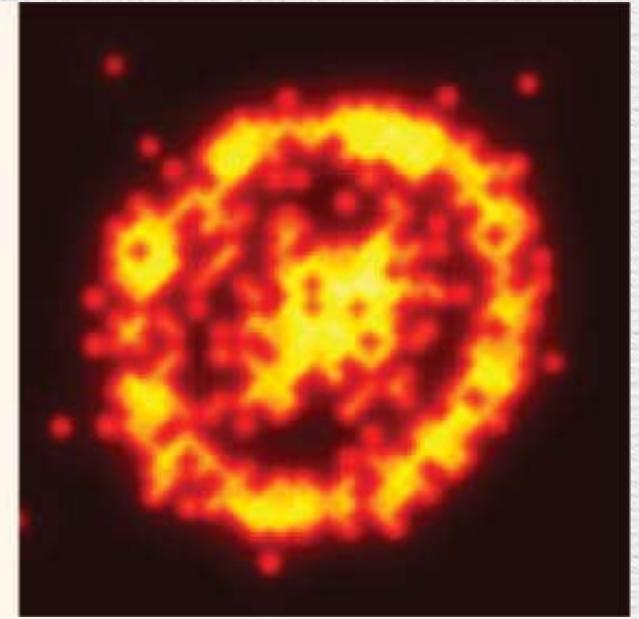


$$V_{\text{OL}}(x) = V_1 \cos^2 \left(\frac{2\pi x}{d} \right) + V_2 \cos^2 \left(\frac{\pi x}{d} - \varphi \right)$$

Full (independent) dynamical control over V_1 , V_2 and φ

in-situ imaging

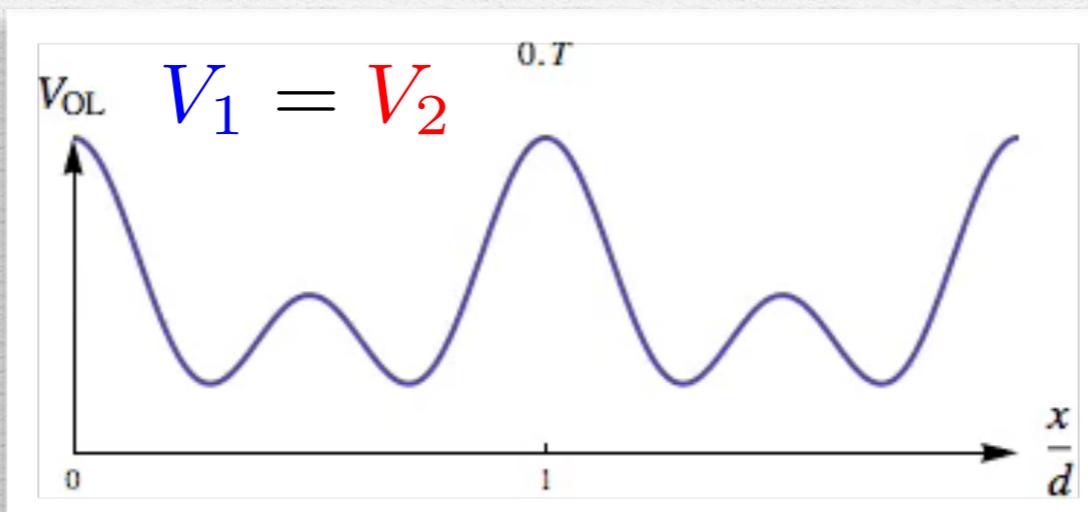
Gemelke *et al*, Sherson *et al*, Bakr *et al*



Allows to measure exact quantization of pumped charge

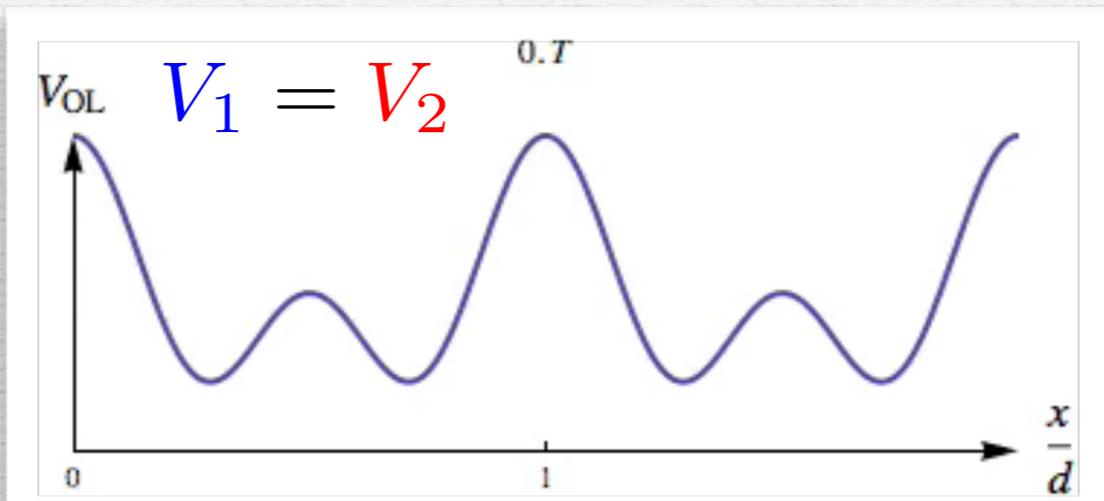
1D pumping lattices

$$V_{\text{OL}}(x, t) = V_1 \cos^2 \left(\frac{2\pi x}{d} \right) + V_2 \cos^2 \left(\frac{\pi x}{d} - \frac{\pi t}{T} \right)$$



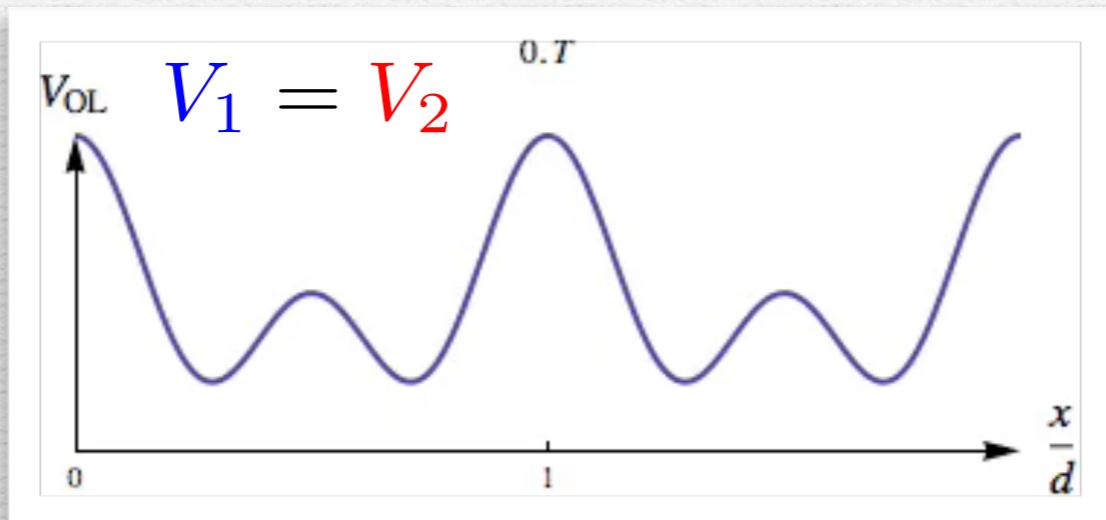
1D pumping lattices

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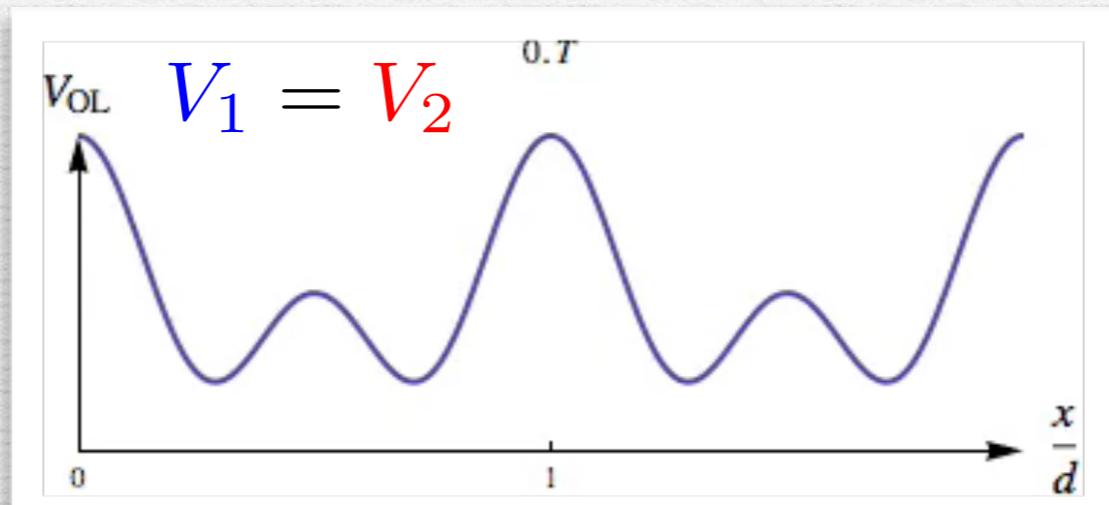
0 A $\overline{\quad}$ B $\overline{\quad}$ A $\overline{\quad}$ B

Su, Schrieffer, Heeger, 1979

$T/2$ A $\overline{\quad}$ B $\overline{\quad}$ A $\overline{\quad}$ B

1D pumping lattices

$$V_{\text{OL}}(x, t) = V_1 \cos^2 \left(\frac{2\pi x}{d} \right) + V_2 \cos^2 \left(\frac{\pi x}{d} - \frac{\pi t}{T} \right)$$



0 A \equiv B \equiv A \equiv B

Su, Schrieffer, Heeger, 1979

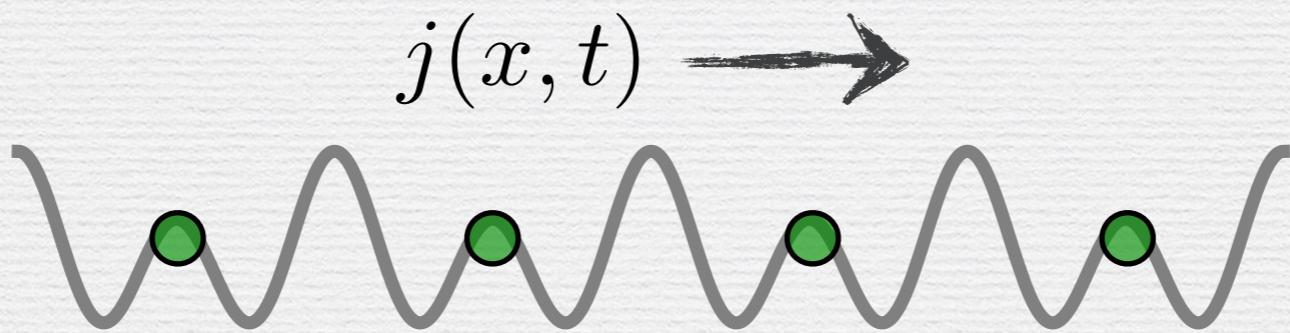
$T/4$ A \cdots B \cdots A \cdots B

Rice, Mele, 1982

$T/2$ A \equiv B \equiv A \equiv B

$3T/4$ A \cdots B \cdots A \cdots B

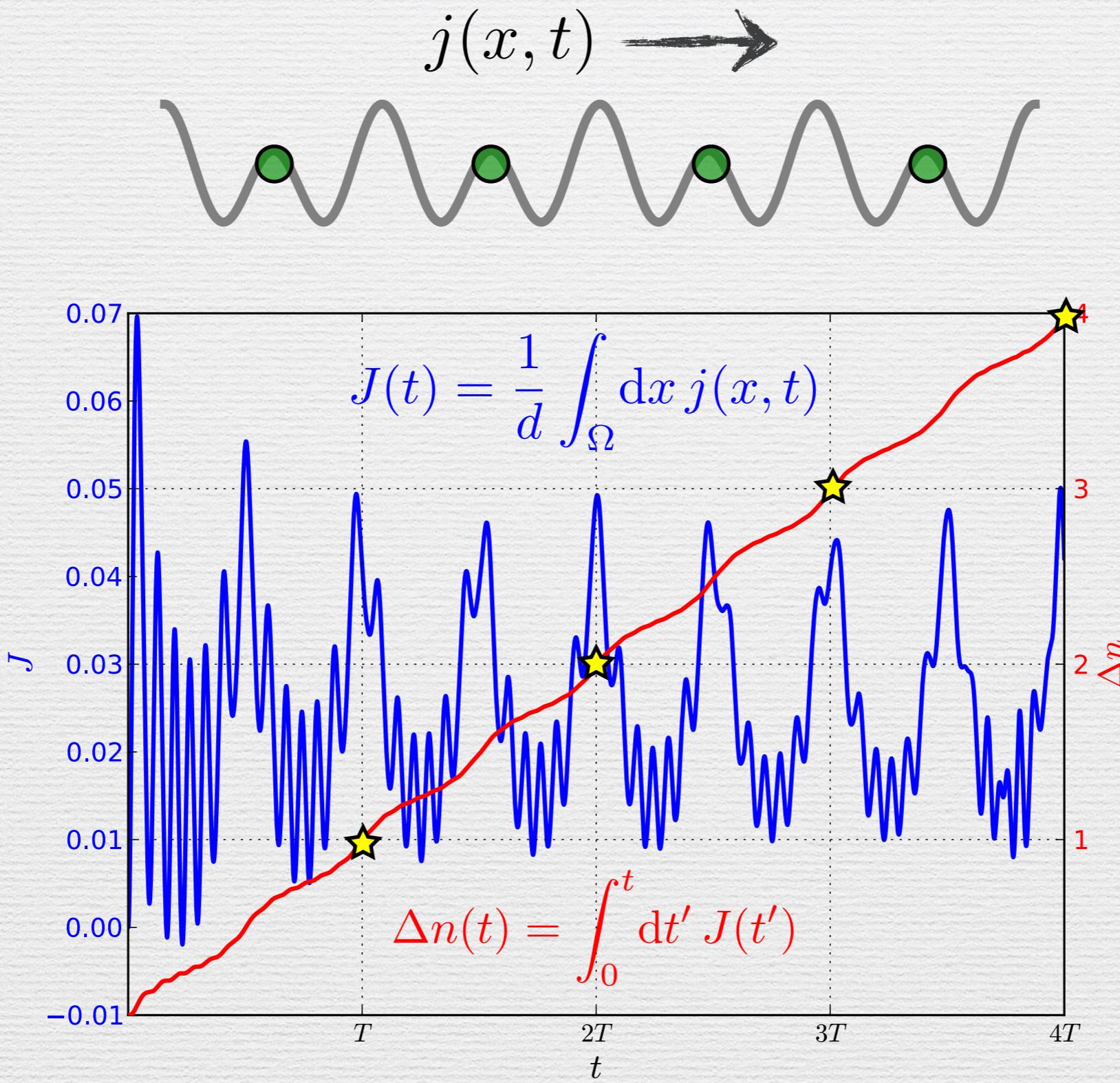
Quantization of pumped charge



$$H(x, t) = -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{OL}}(x, t)$$

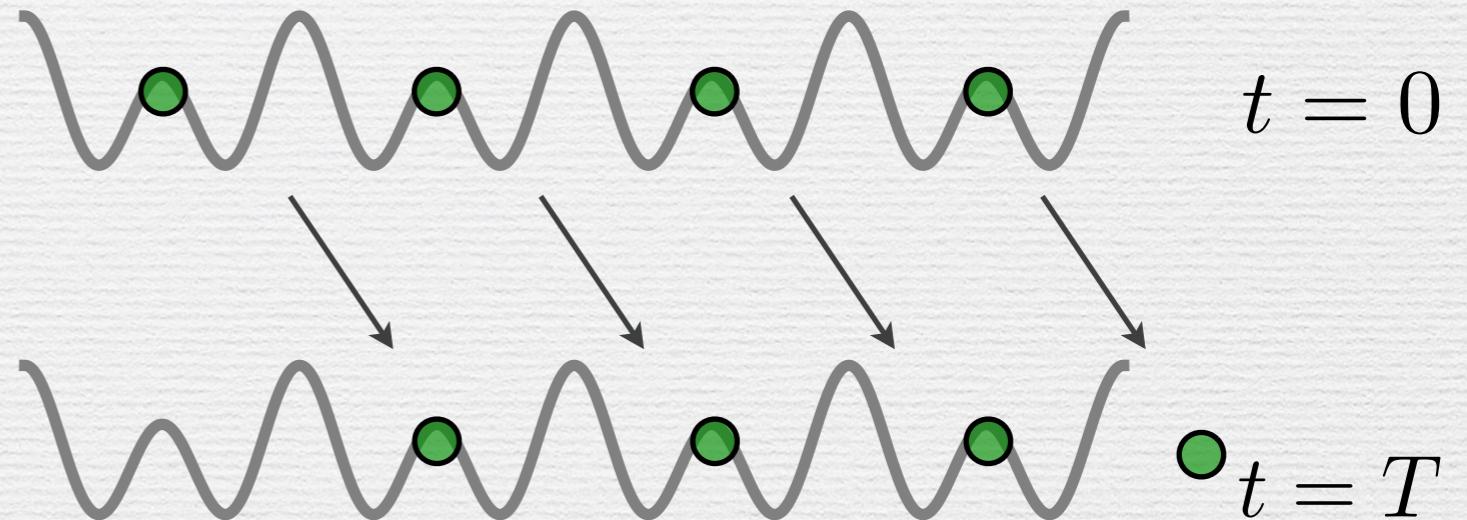
$$i \frac{\partial}{\partial t} |\Psi\rangle = H(x, t) |\Psi\rangle$$

Quantization of pumped charge



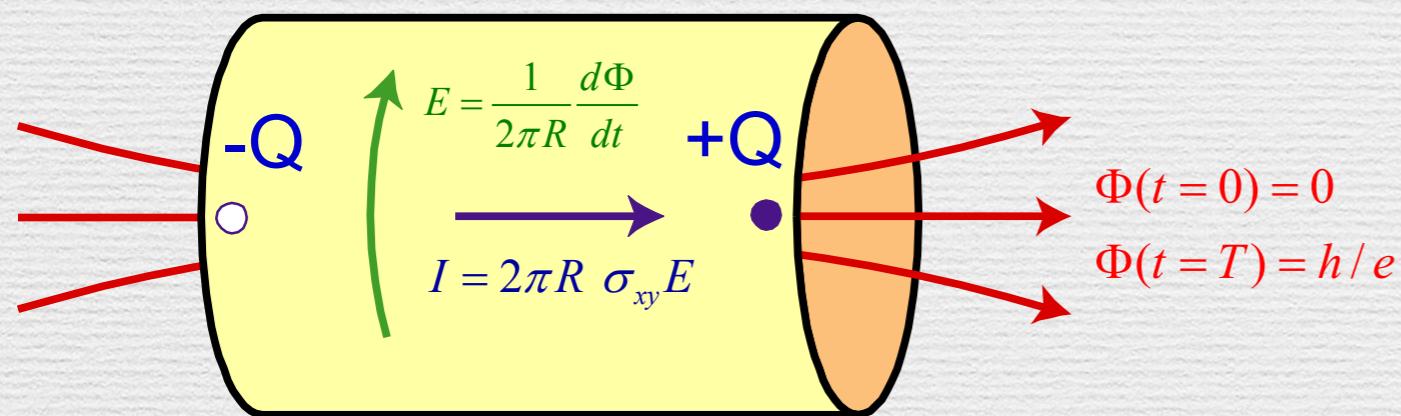
Connection to IQHE

$$H(k_x, t) = H(k_x, t + T)$$

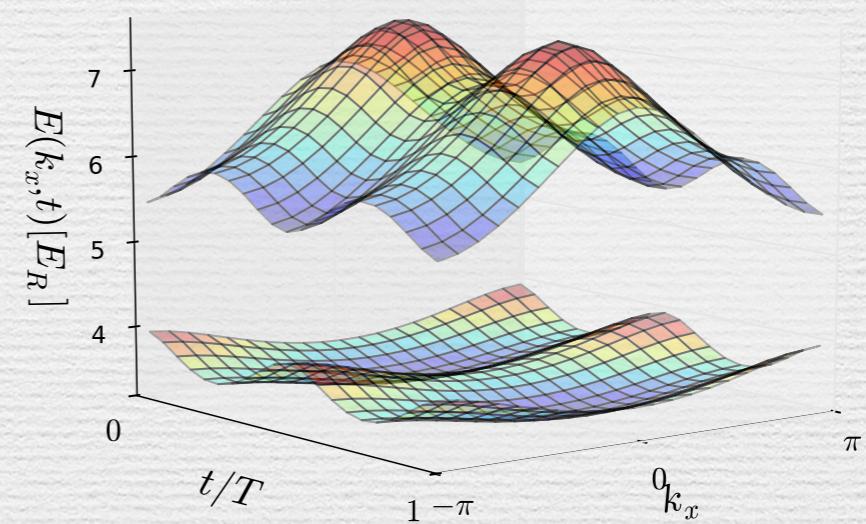


$$V_1 = 4E_R \quad V_2 = 4E_R$$

Adiabatically thread a quantum of magnetic flux through cylinder.

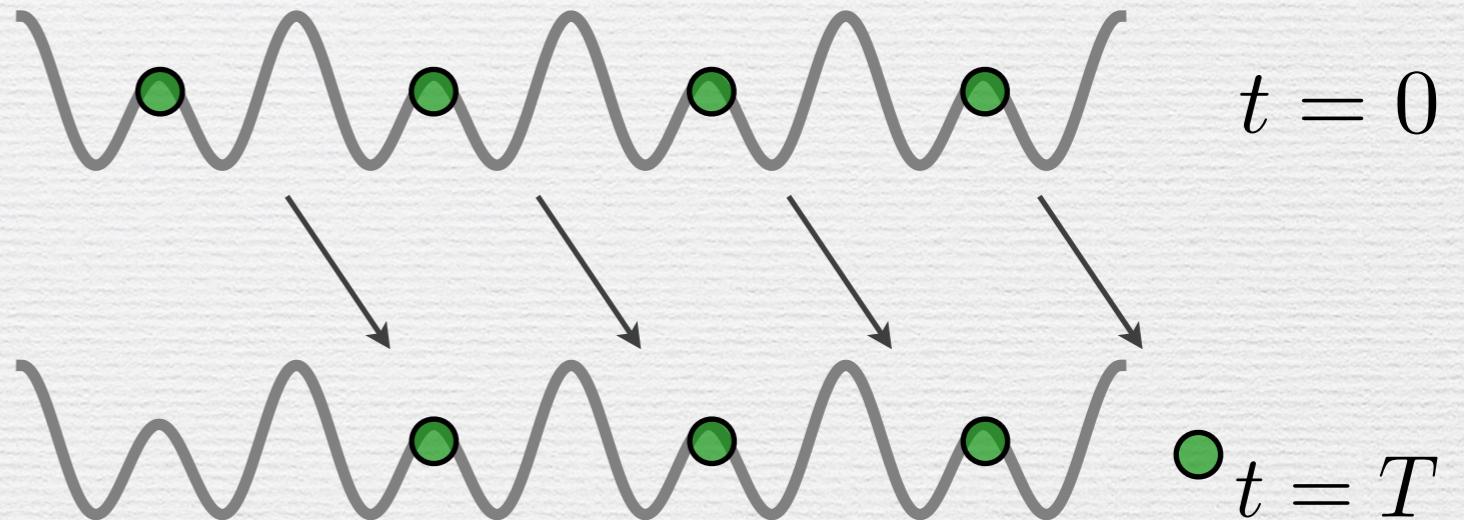


Laughlin, 1981

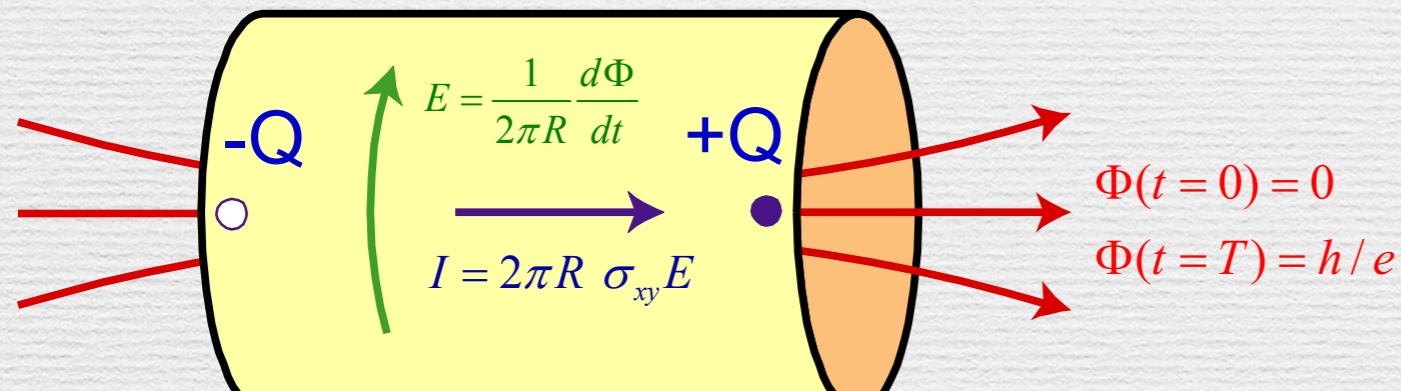


Connection to IQHE

$$H(k_x, t) = H(k_x, t + T)$$

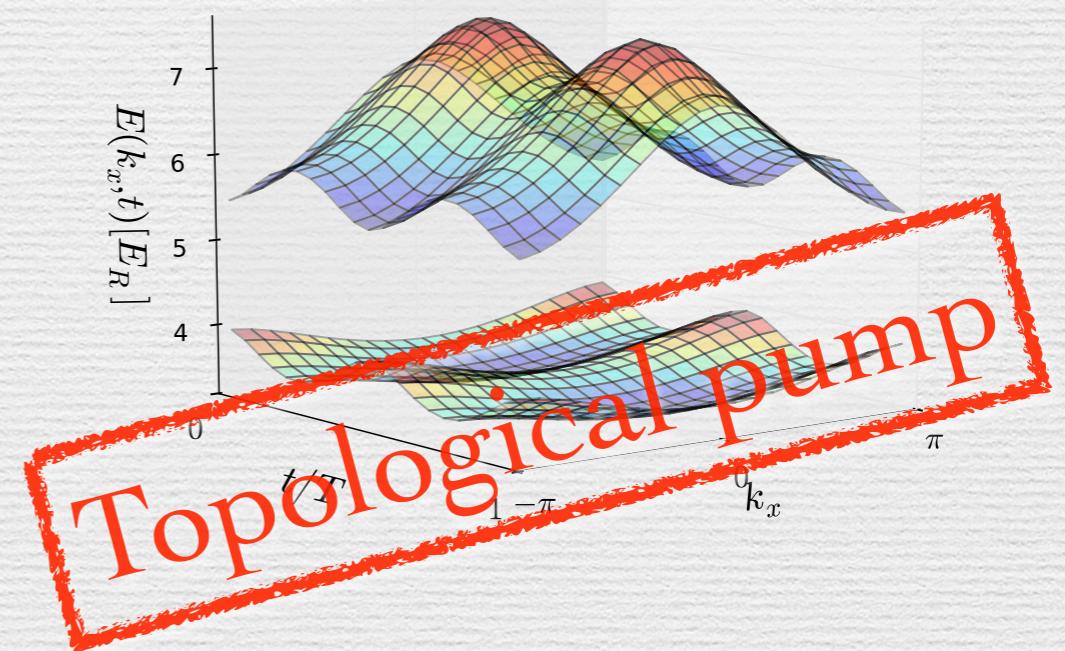


Adiabatically thread a quantum of magnetic flux through cylinder.



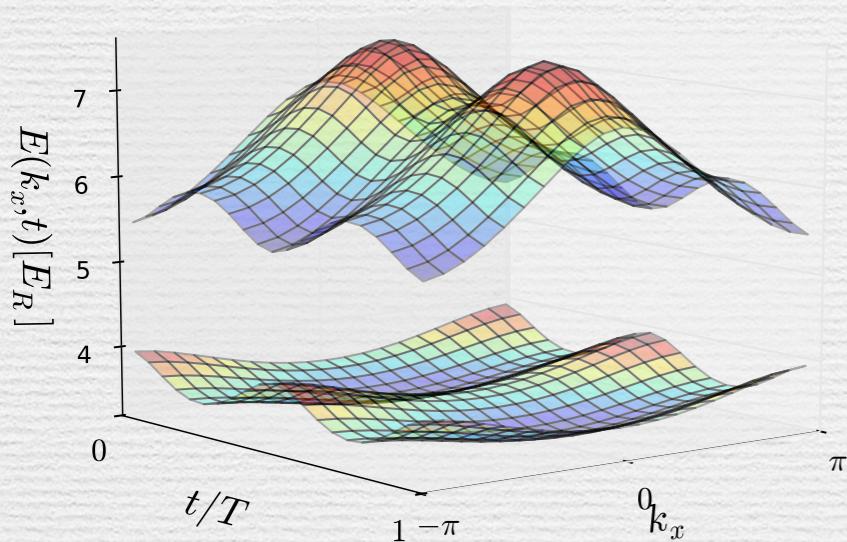
Laughlin, 1981

$$V_1 = 4E_R \quad V_2 = 4E_R$$



Adiabatic Connection

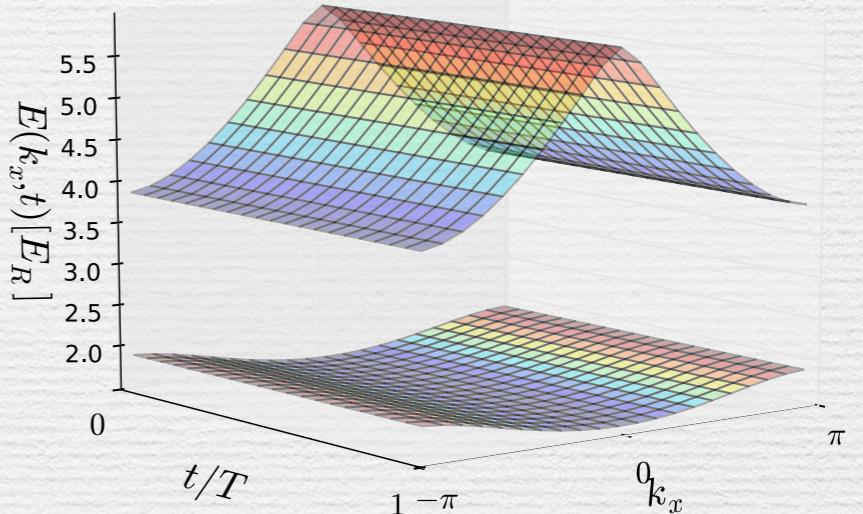
$$V_1 = 4E_R \quad V_2 = 4E_R$$



$$V_1 \cos^2 \left(\frac{2\pi x}{d} \right) + V_2 \cos^2 \left(\frac{\pi x}{d} - \frac{\pi t}{T} \right)$$

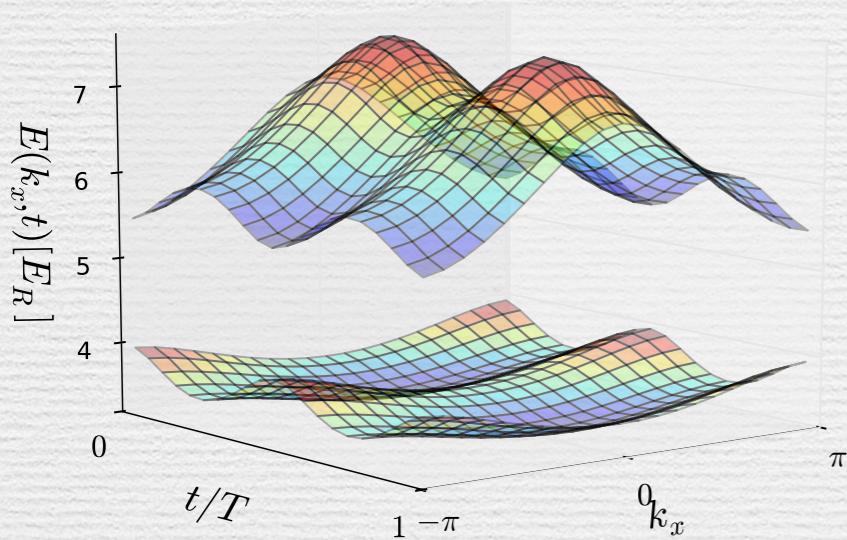


$$V_1 = 0 \quad V_2 = 4E_R$$



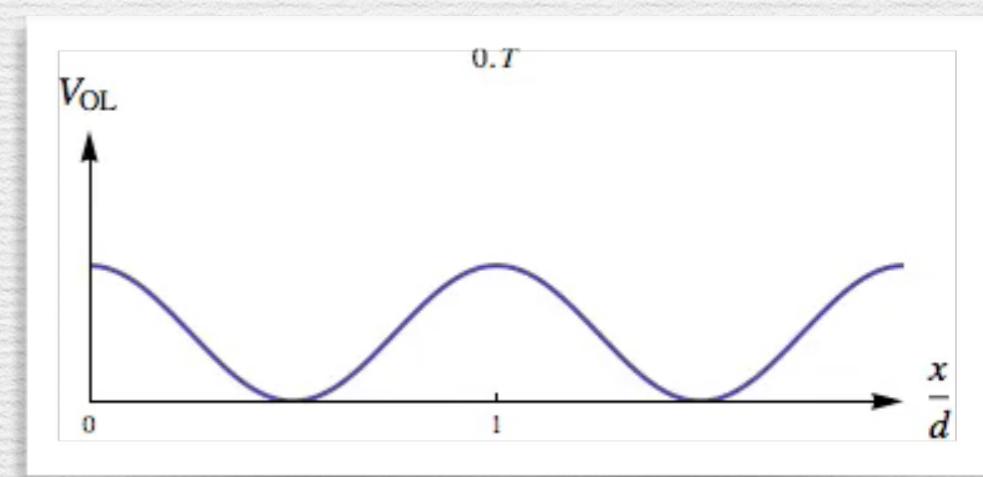
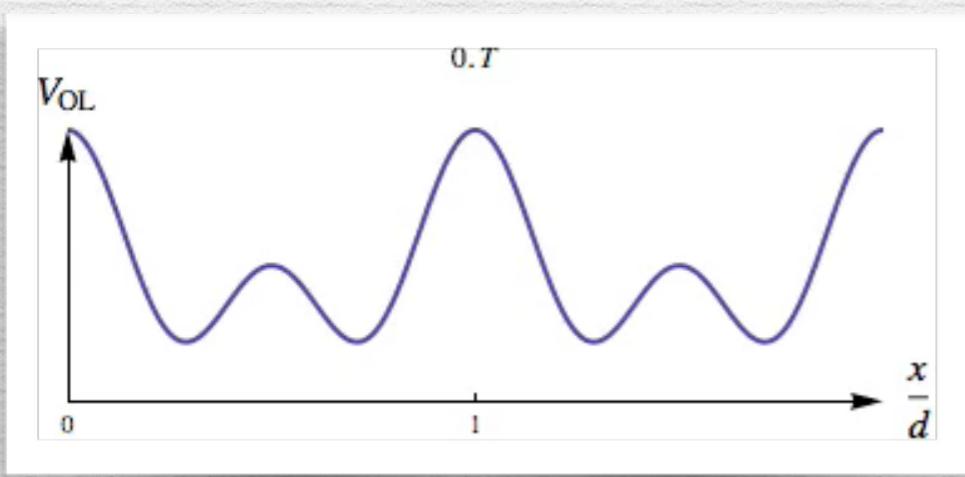
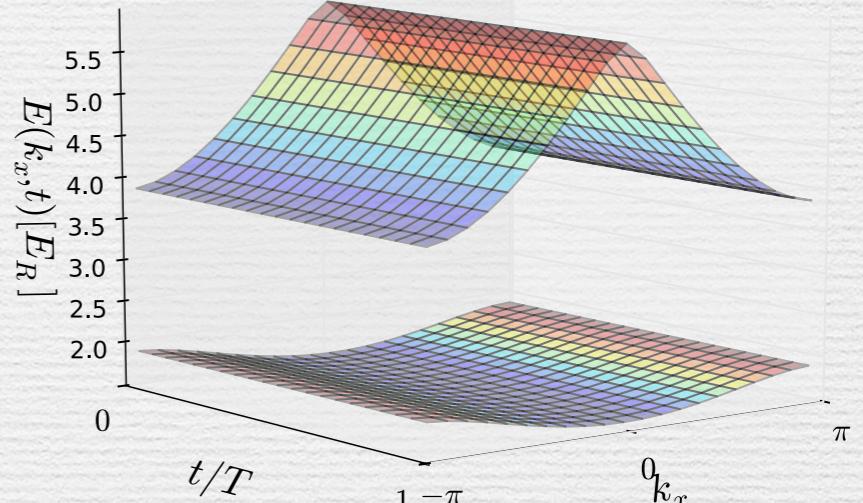
Adiabatic Connection

$$V_1 = 4E_R \quad V_2 = 4E_R$$

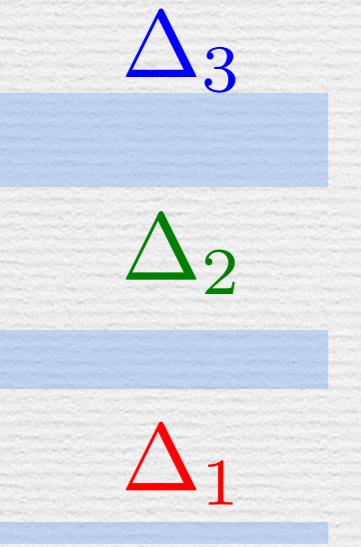


$$V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \frac{\pi t}{T}\right)$$

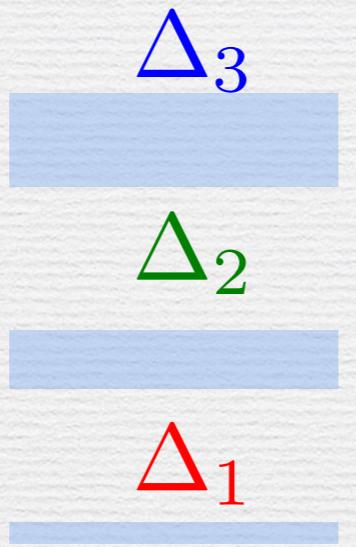
$$V_1 = 0 \quad V_2 = 4E_R$$



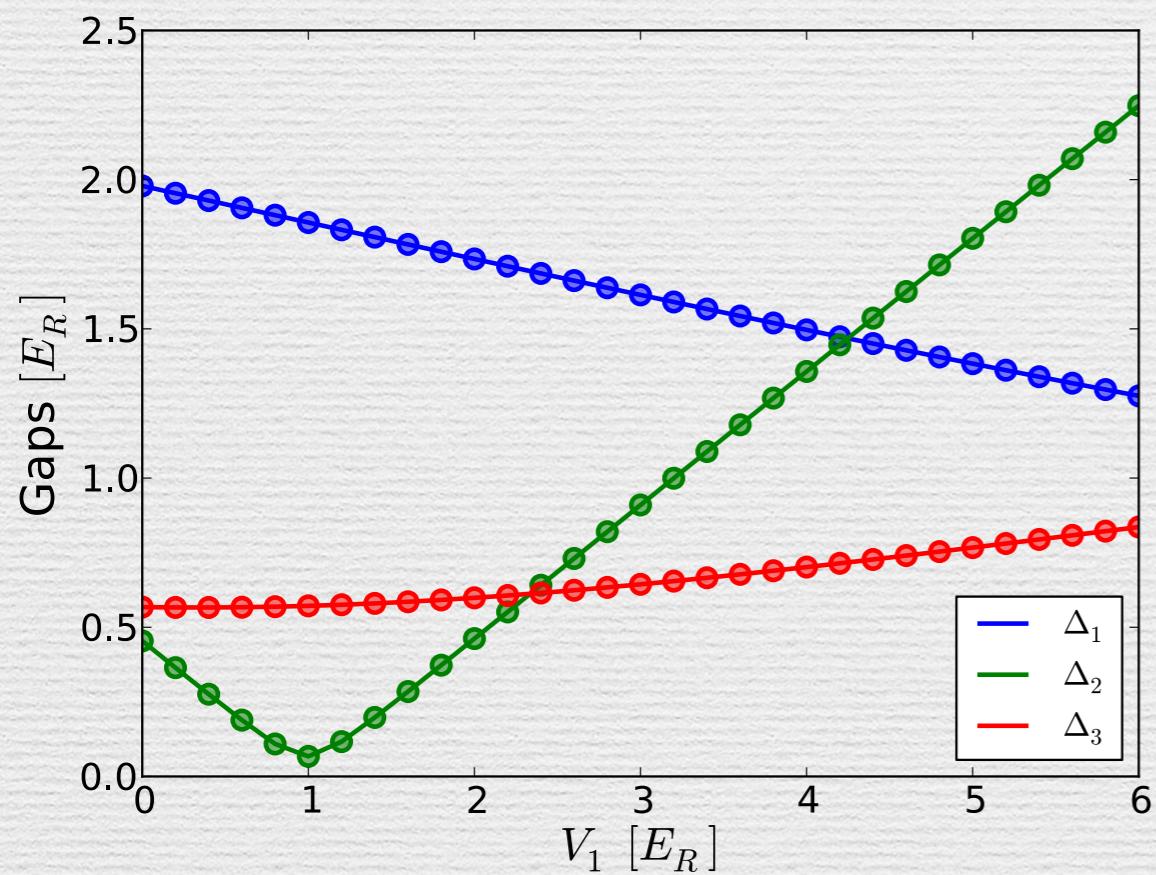
Higher bands



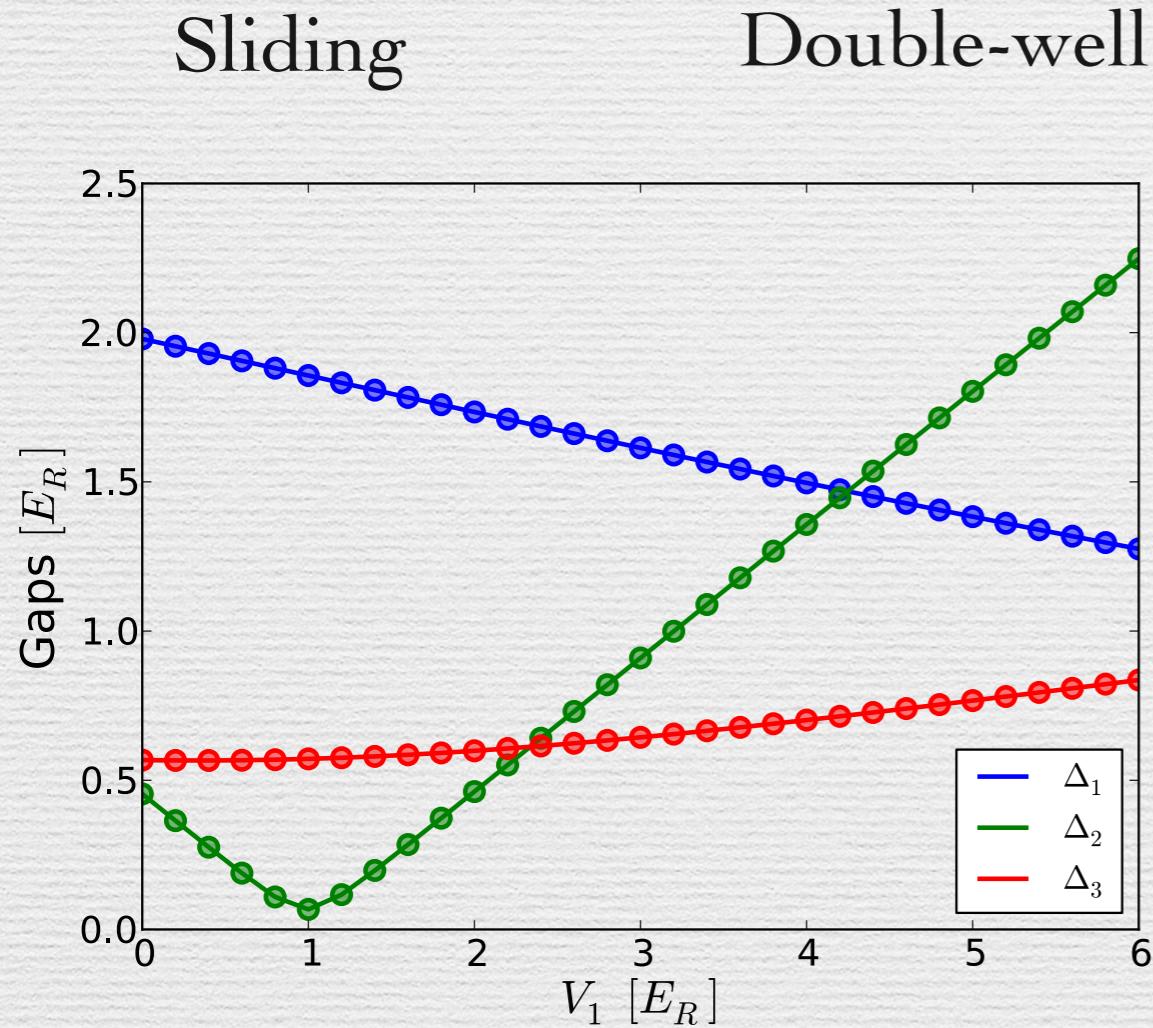
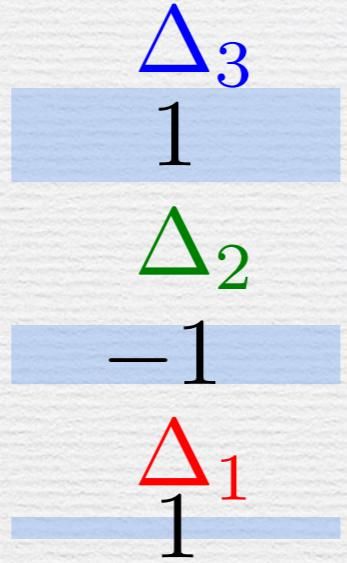
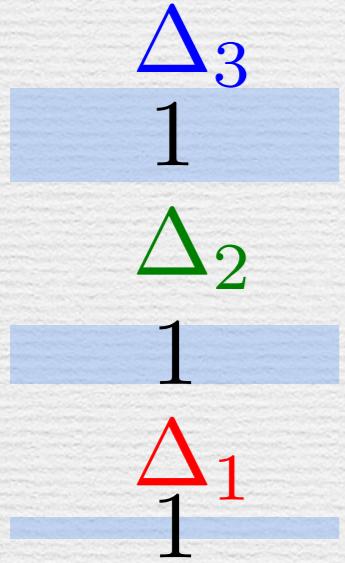
Sliding



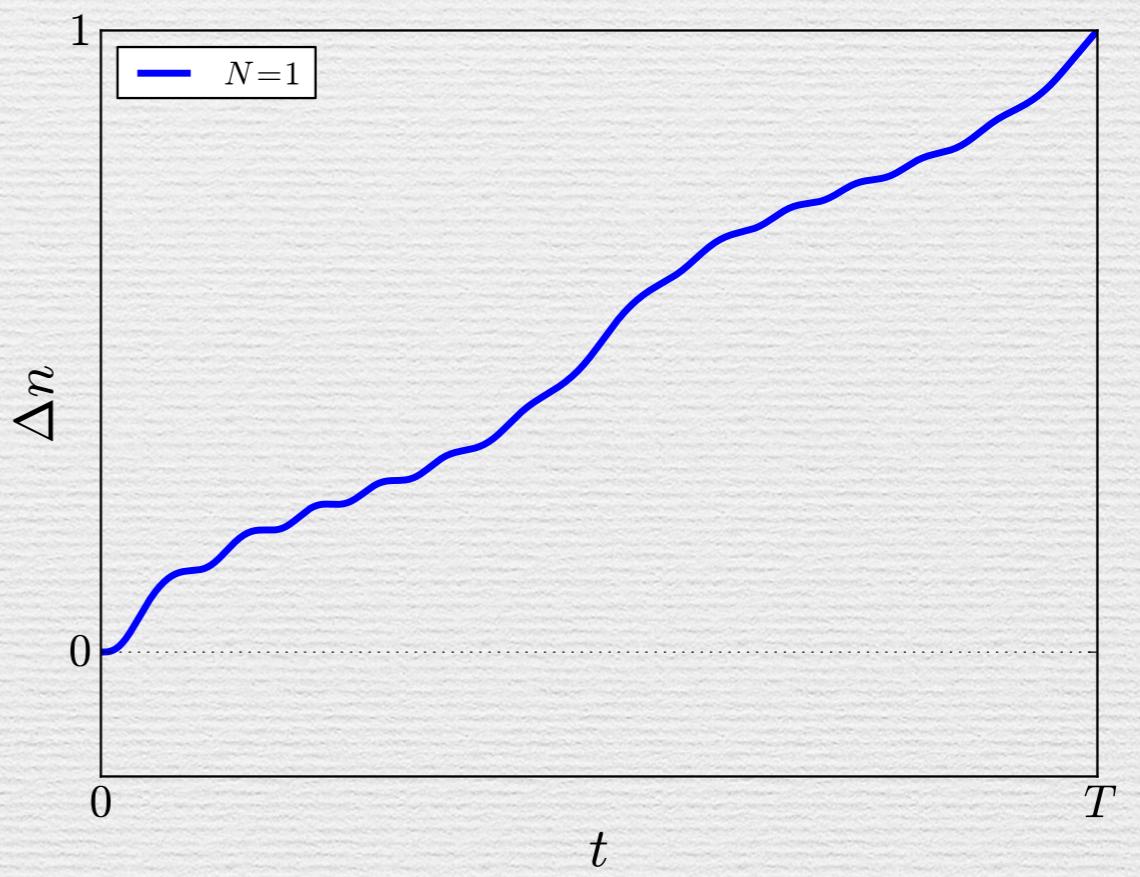
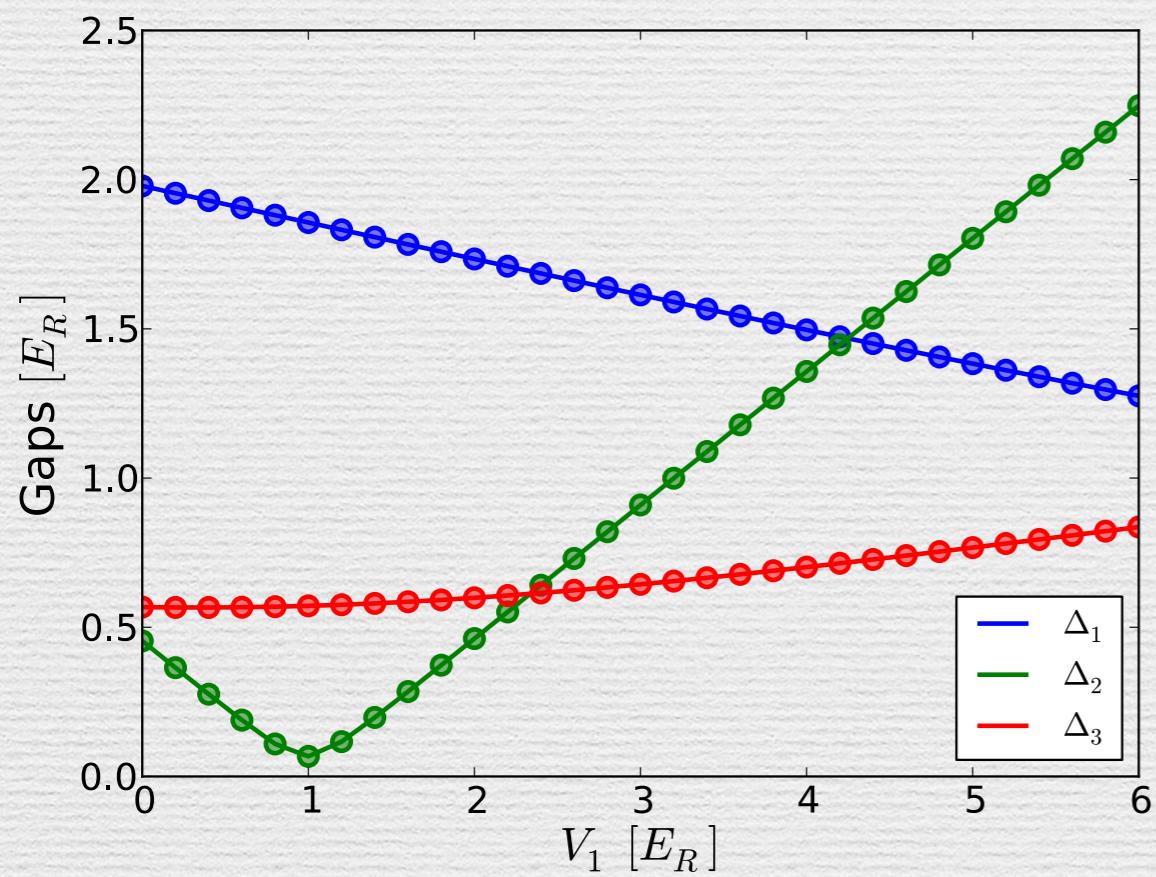
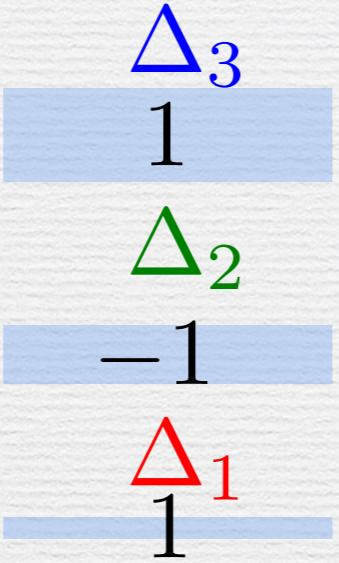
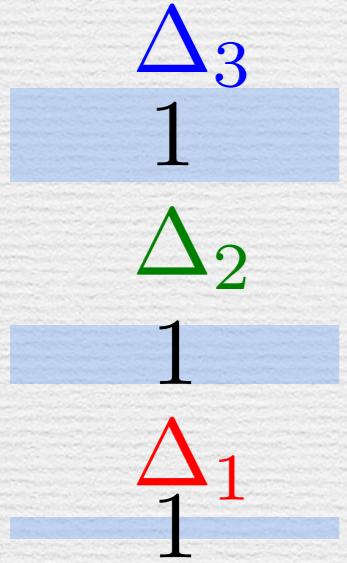
Double-well



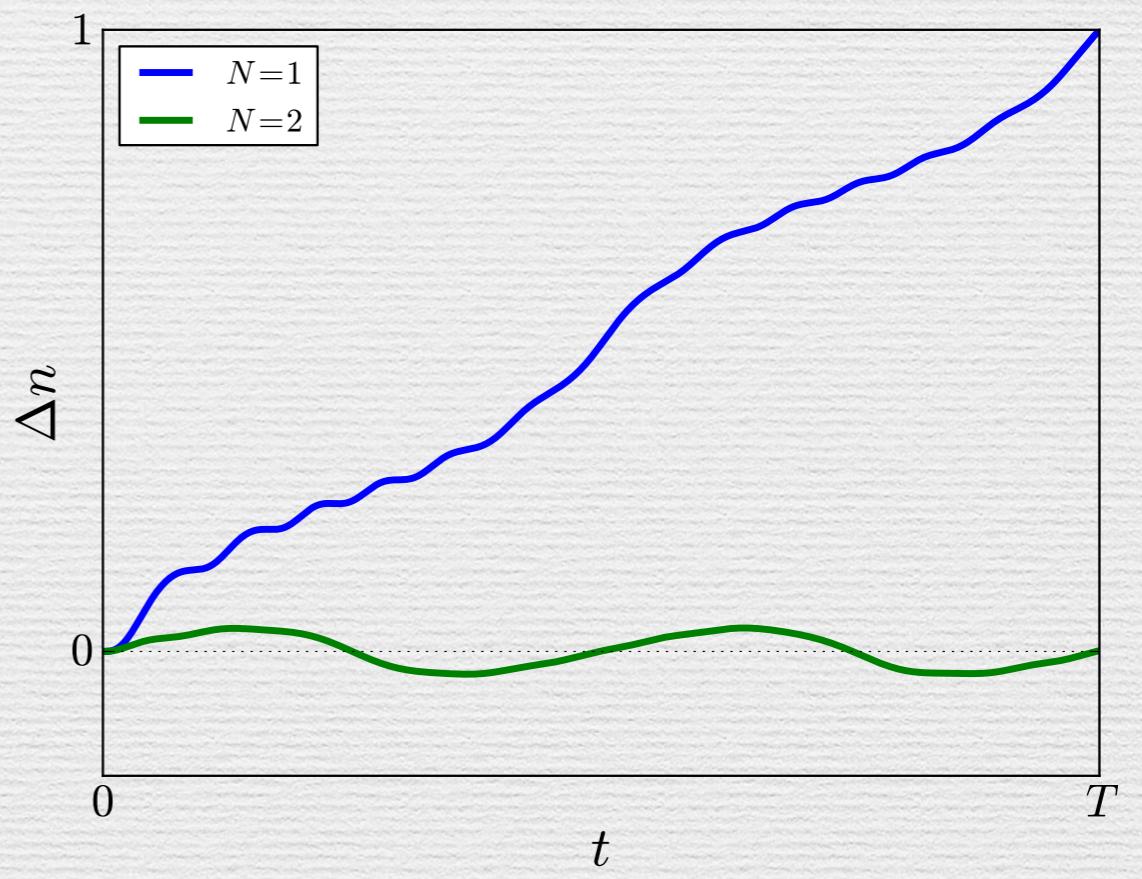
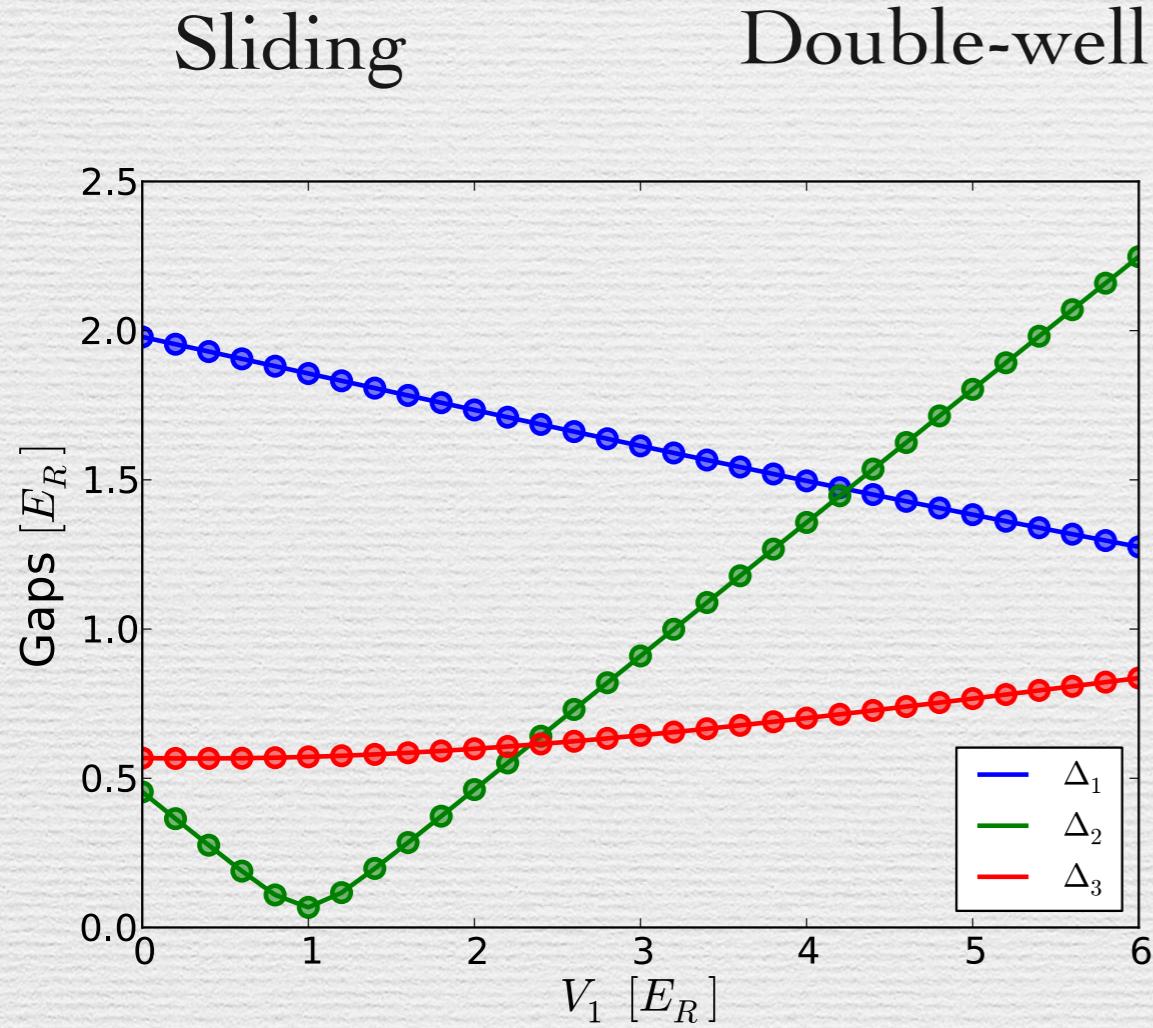
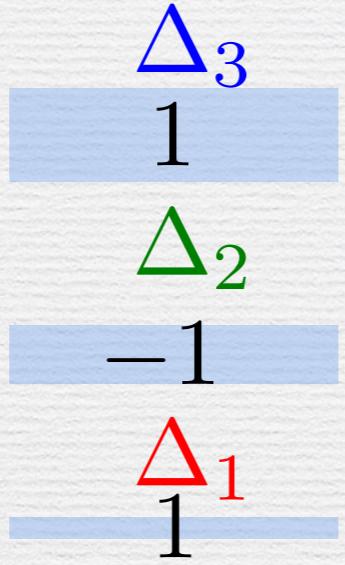
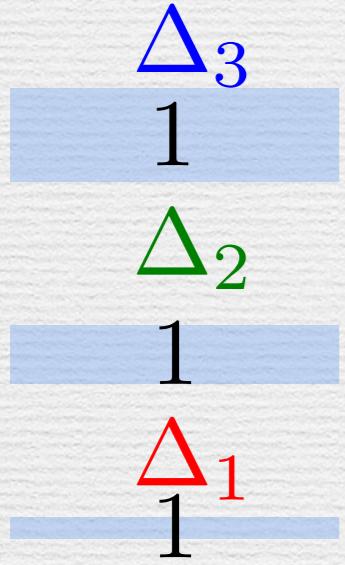
Higher bands



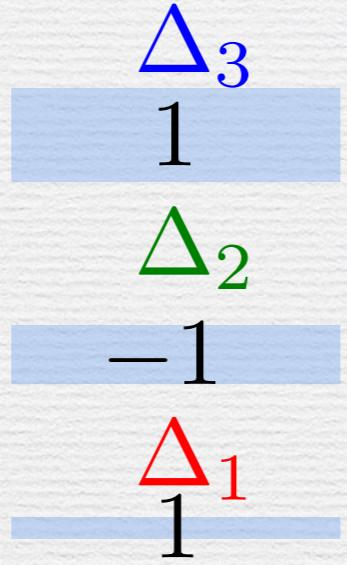
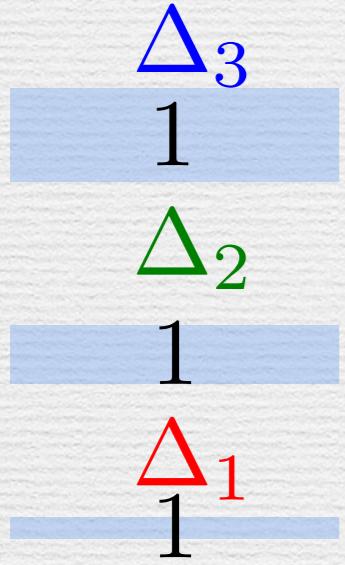
Higher bands



Higher bands

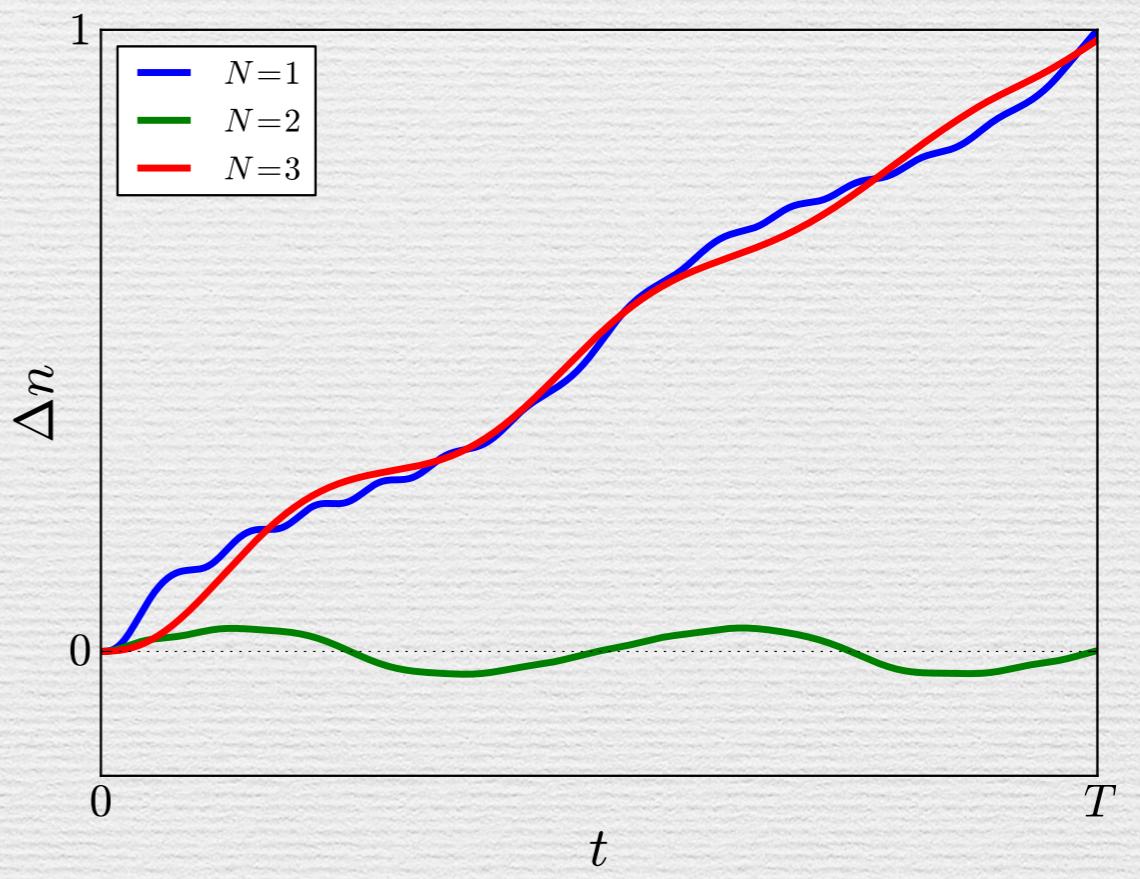
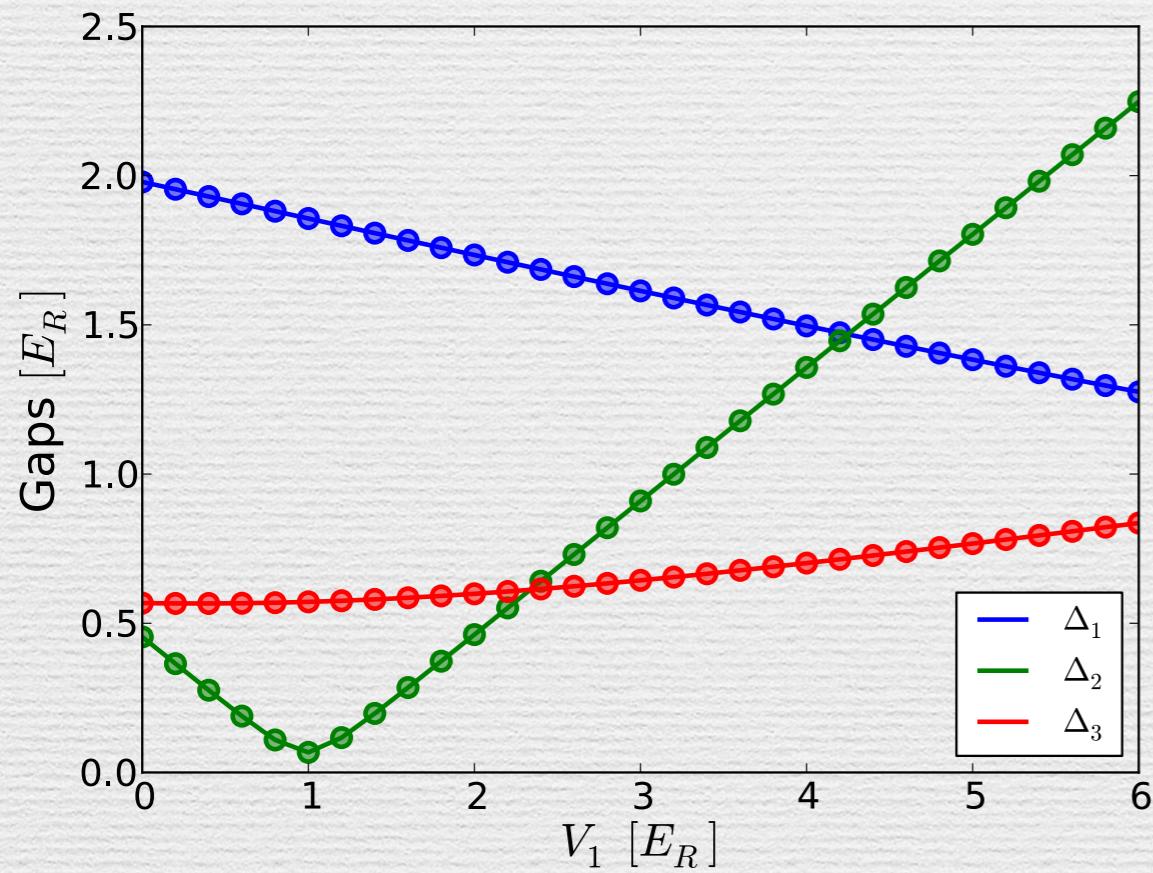


Higher bands



Sliding

Double-well

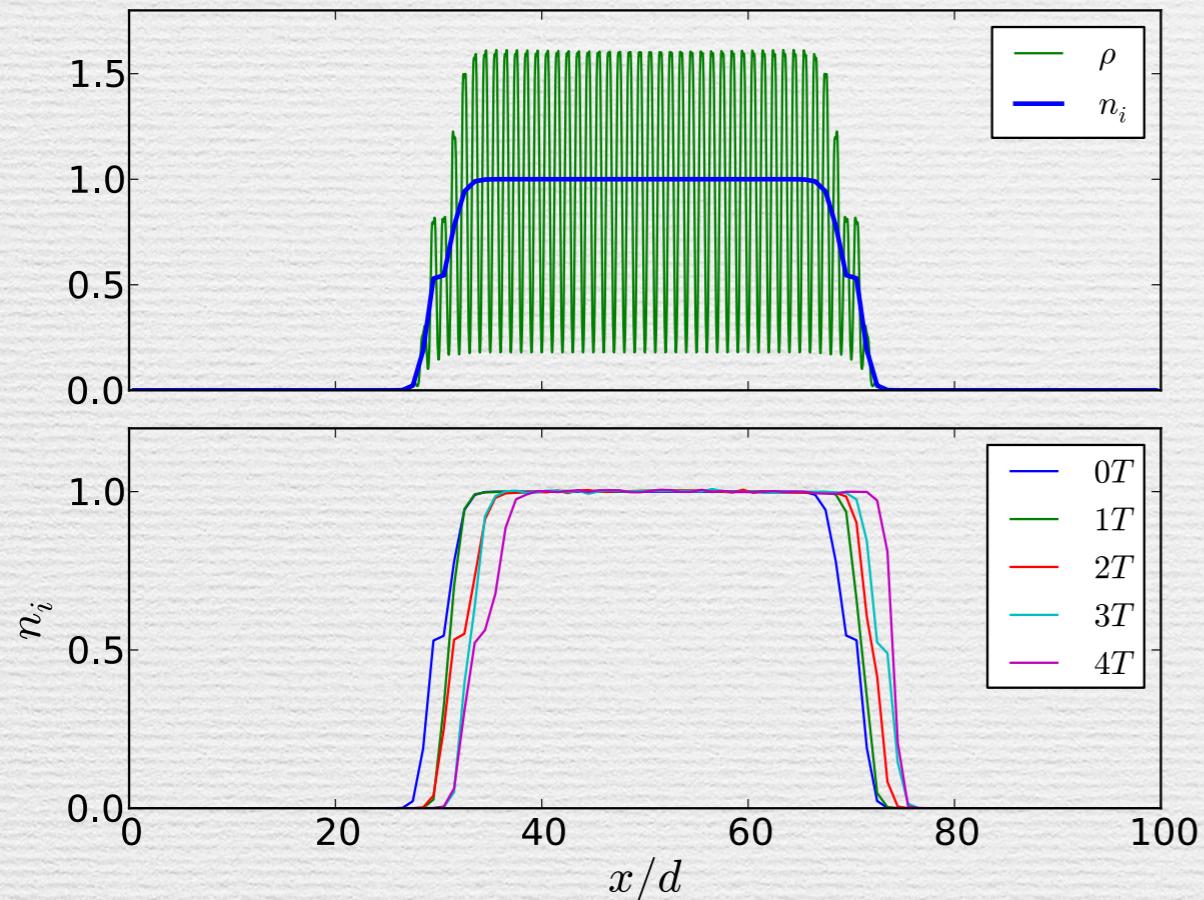


Practical issues

- ❖ Detection
- ❖ External trap
- ❖ Temperature effect
- ❖ Non-adiabatic effect

Trapping & Detection

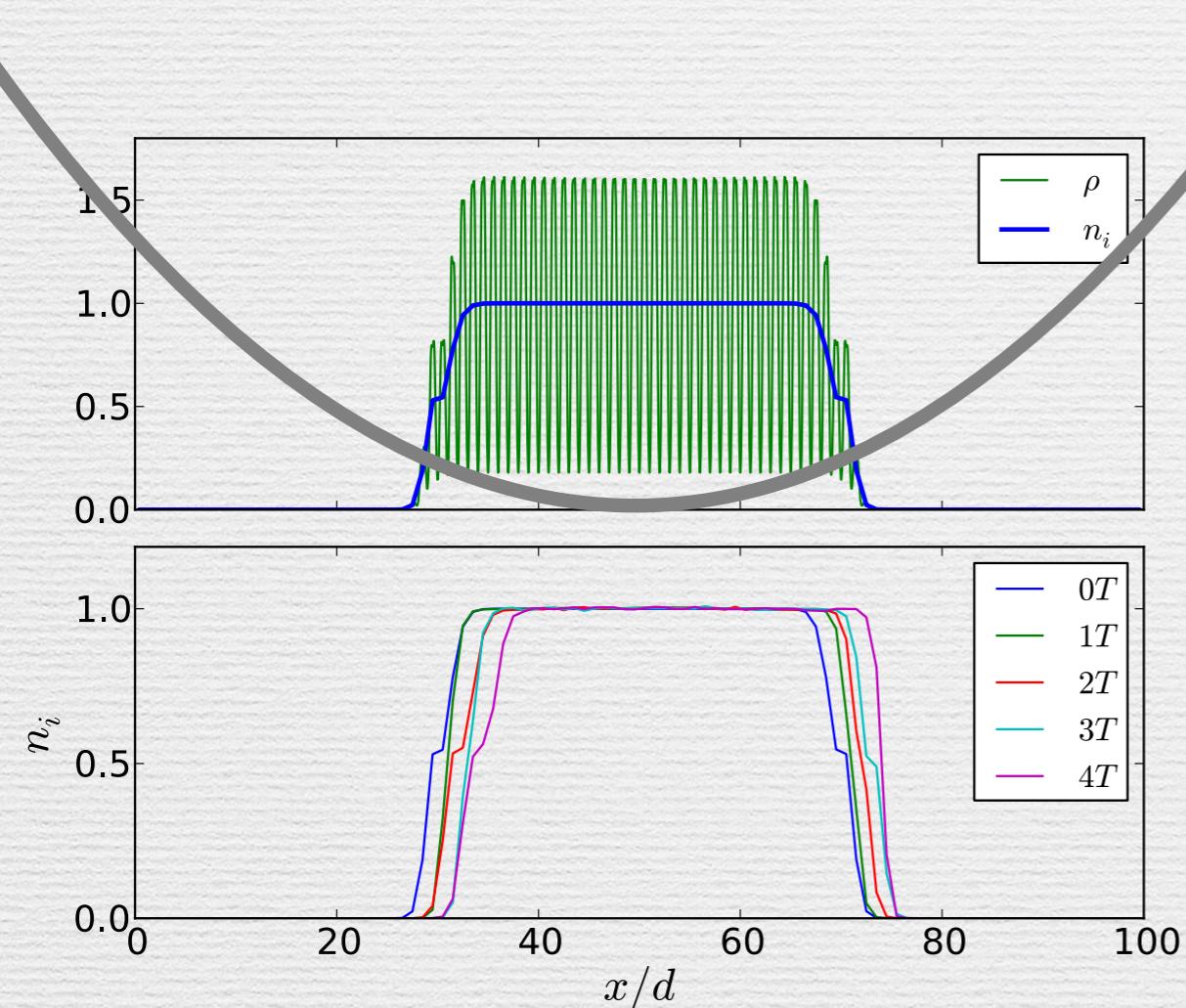
LW, Troyer and Dai, PRL 111, 026802 (2013)



$$\Delta n = \langle x \rangle / d$$

Trapping & Detection

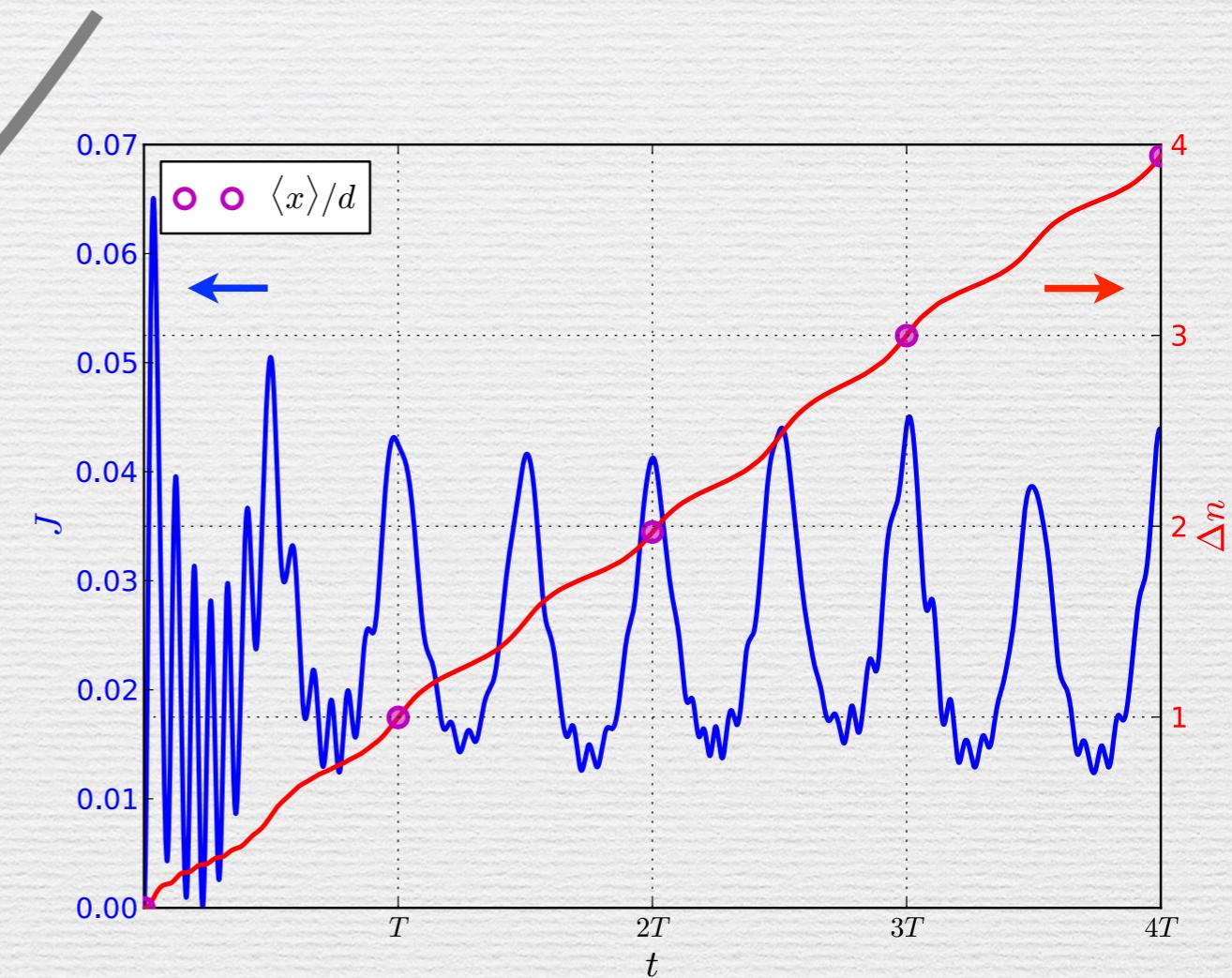
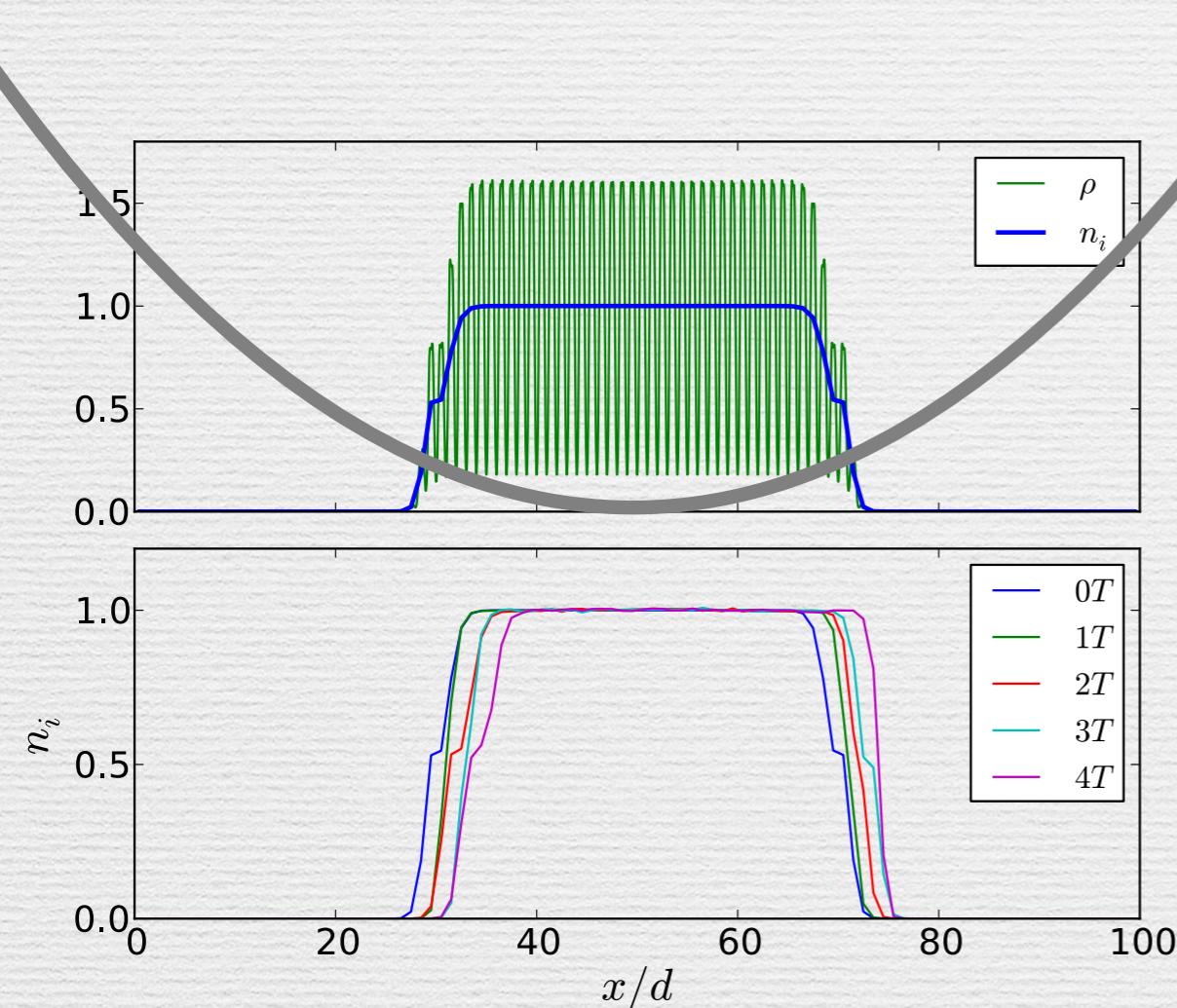
LW, Troyer and Dai, PRL 111, 026802 (2013)



$$\Delta n = \langle x \rangle / d$$

Trapping & Detection

LW, Troyer and Dai, PRL 111, 026802 (2013)



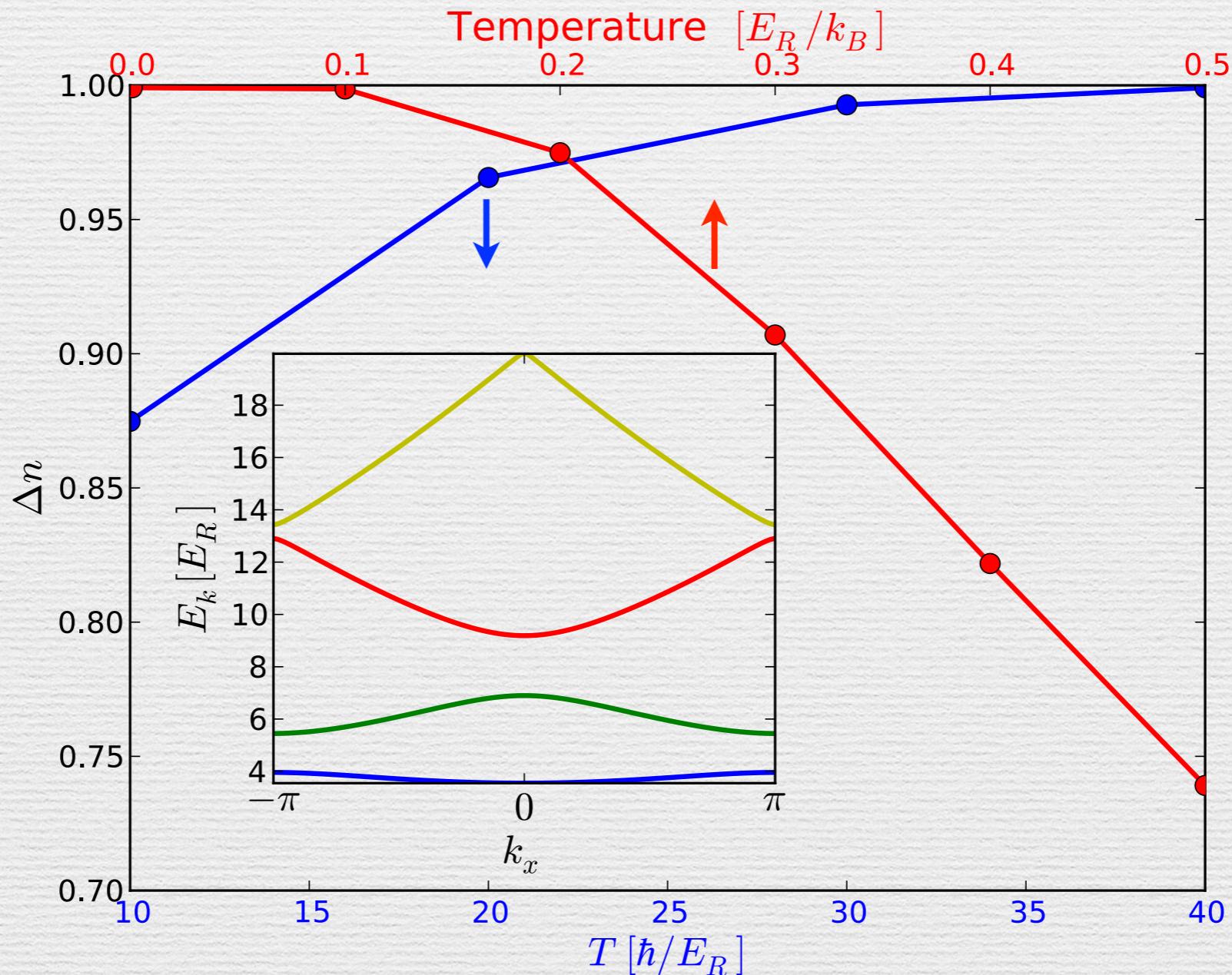
$$\Delta n = \langle x \rangle / d$$

Temperature & Non-adiabatic effect

$$\text{Temperature} \ll \frac{\Delta}{k_B} \qquad \qquad \textcolor{blue}{T} \gg \frac{\hbar}{\Delta}$$

Temperature & Non-adiabatic effect

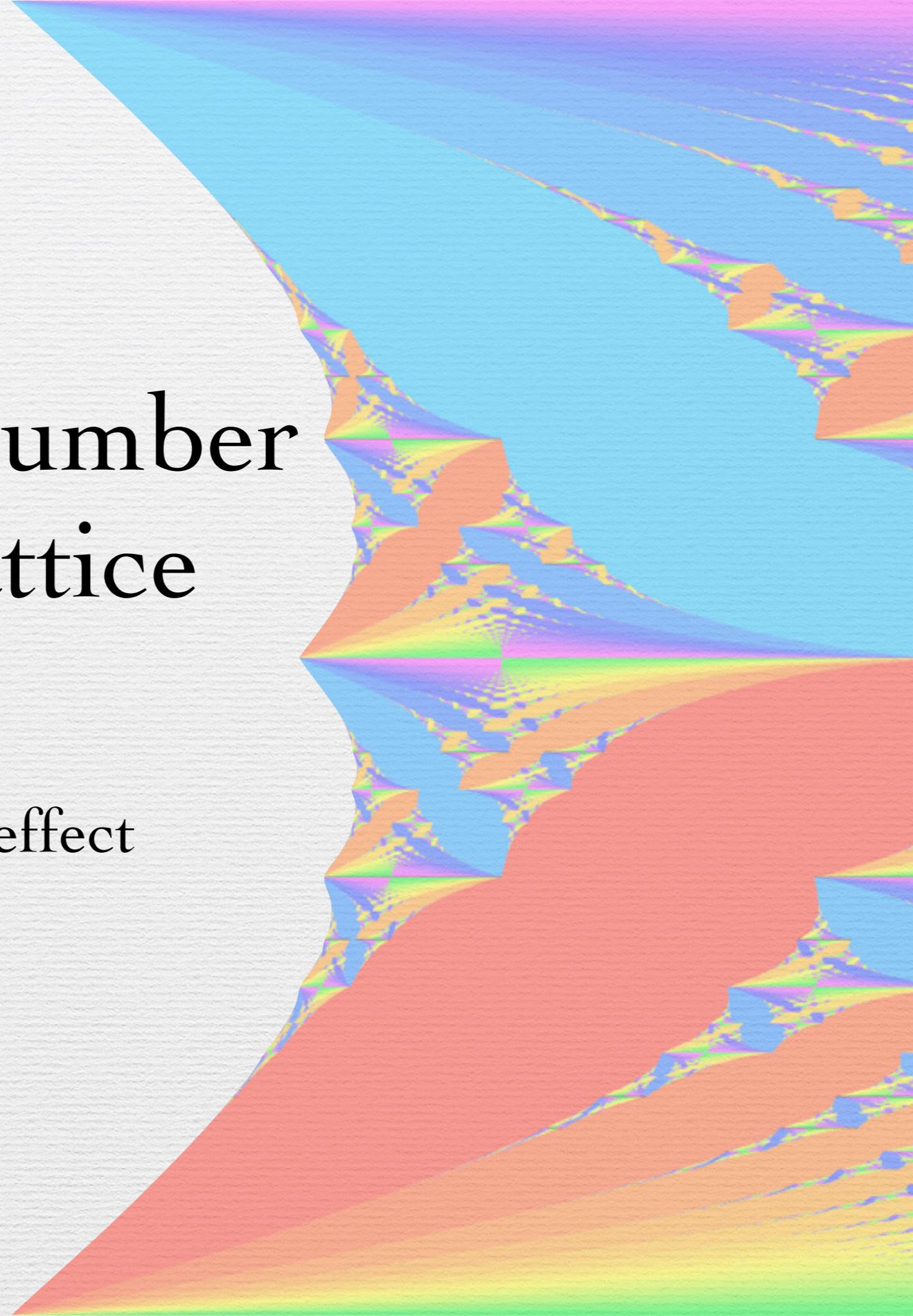
Temperature $\ll \frac{\Delta}{k_B}$ $T \gg \frac{\hbar}{\Delta}$



Measure Chern number
of 2D optical lattice

with

Topological pumping effect



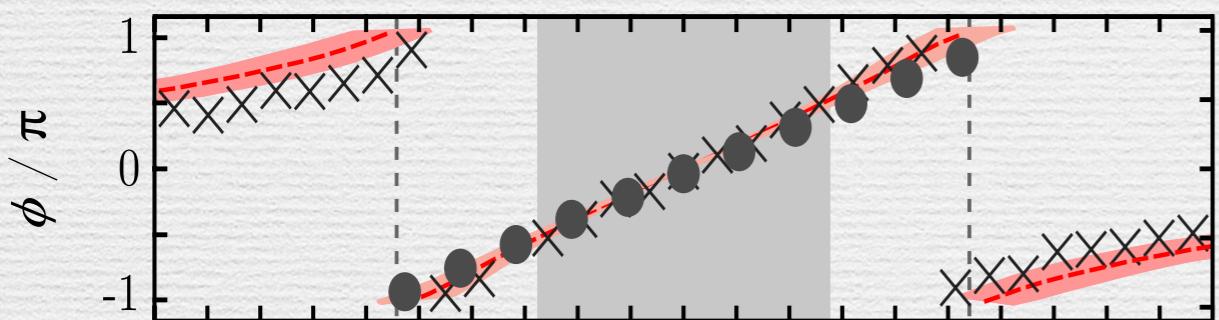
Synthetic gauge-field in optical lattices

- ❖ Imprint **complex phases** to the hopping amplitude

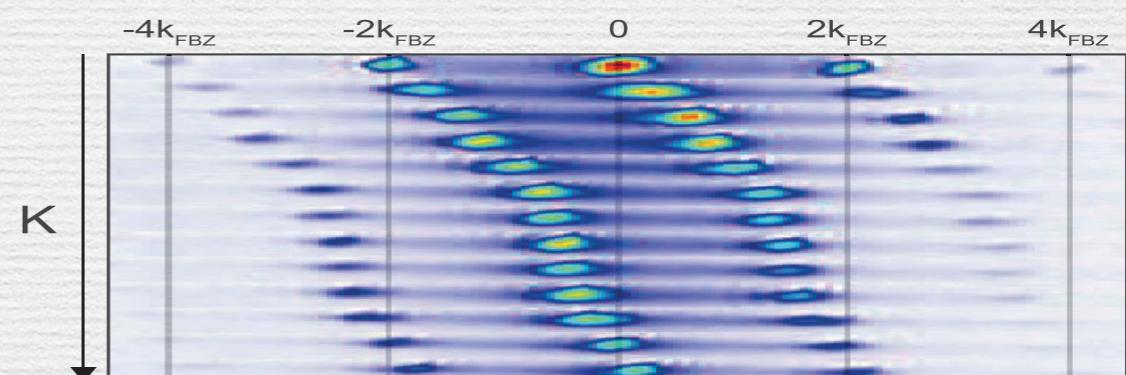
- ❖ 1D Peierls lattice **NIST, Hamburg**

$$H = -J \sum_m e^{i2\pi\Phi} c_{m+1}^\dagger c_m + H.c.$$

(a) Peierls tunneling phase



Jimenez-Garcia *et al*



Struck *et al*

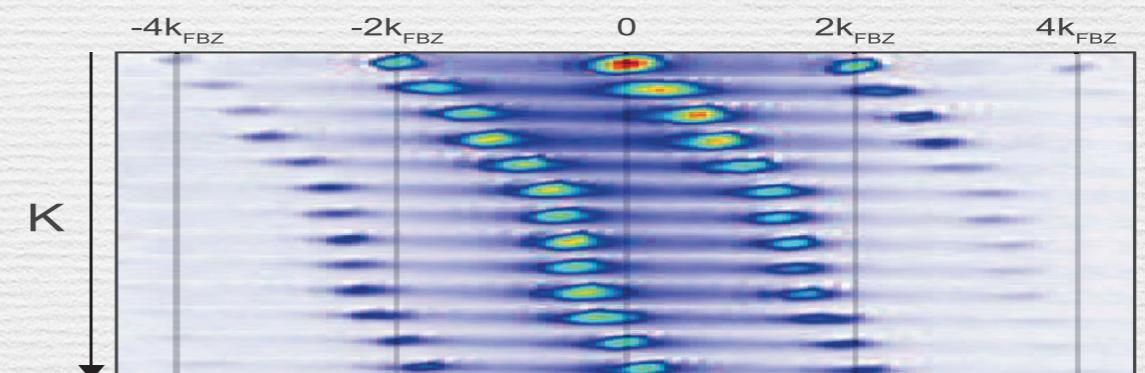
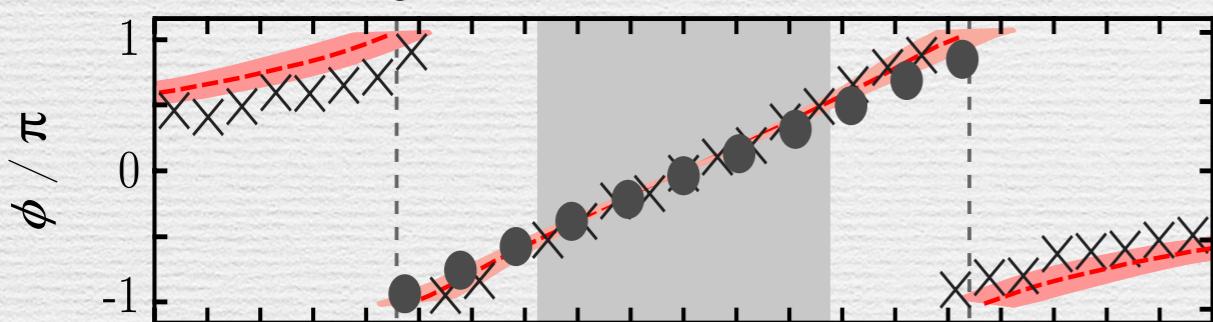
Synthetic gauge-field in optical lattices

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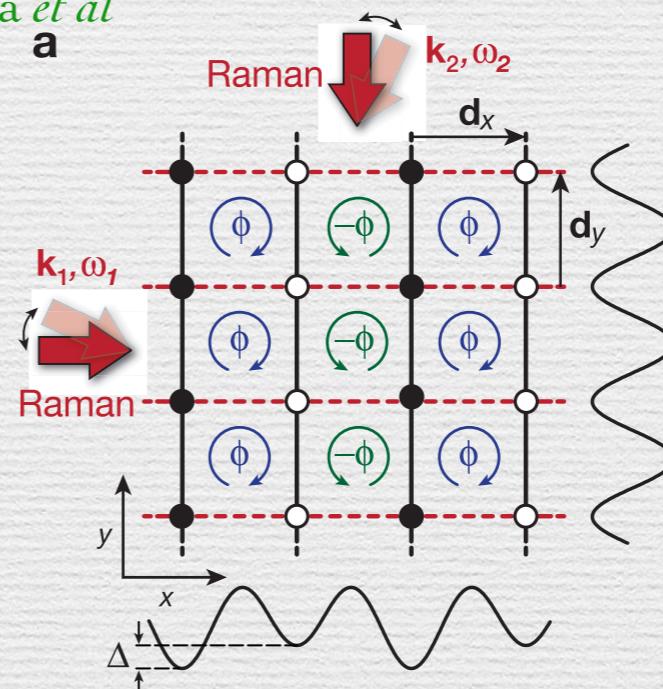
(a) Peierls tunneling phase



Jimenez-Garcia *et al*

Struck *et al*

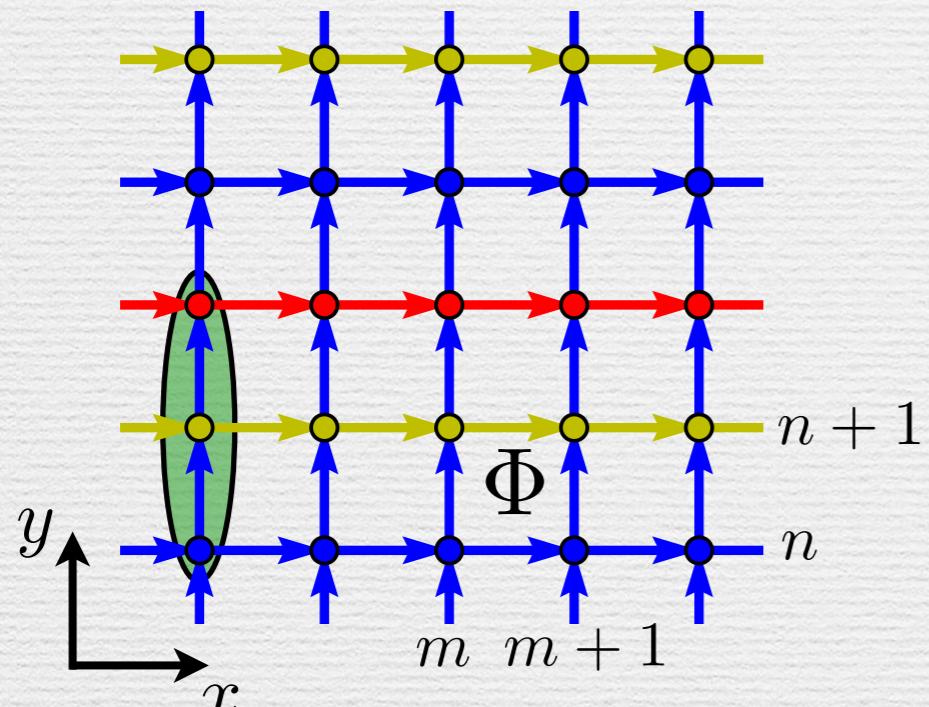
- ✿ Staggered flux lattice **Munich**



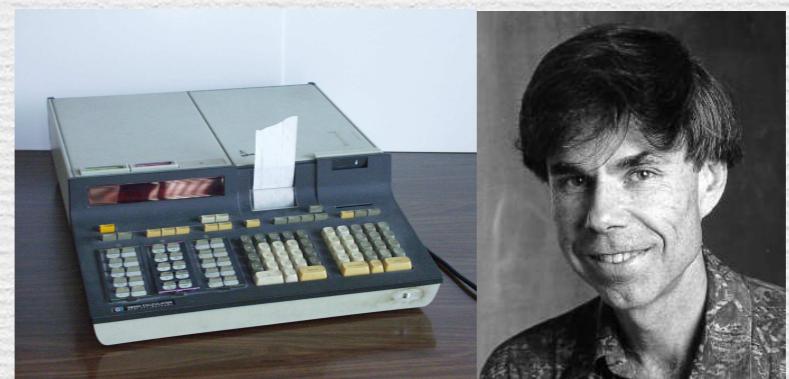
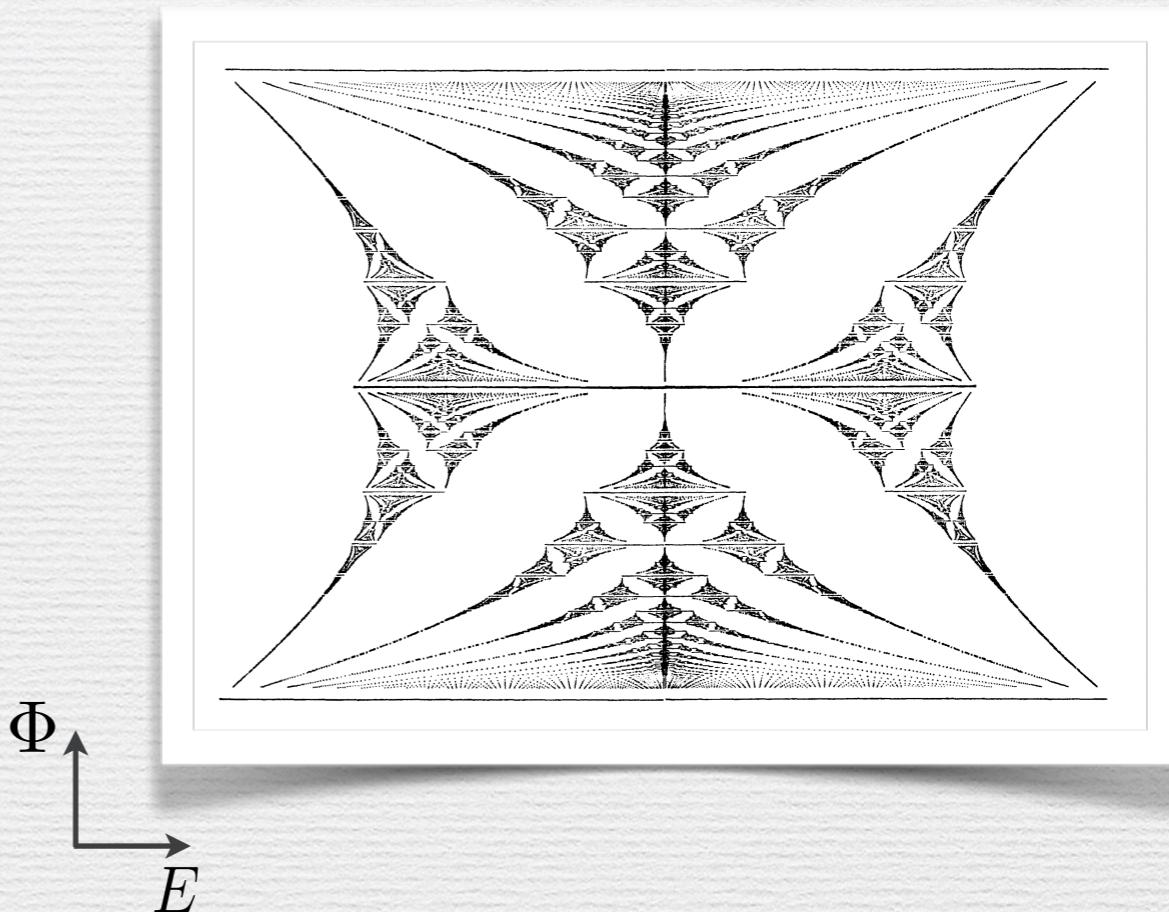
Aidelsburger *et al*

Hofstadter optical lattice

$$H = -J \sum_{m,n} e^{i2\pi n\Phi} c_{m+1,n}^\dagger c_{m,n} + c_{m,n+1}^\dagger c_{m,n} + H.c. \quad \Phi = p/q$$

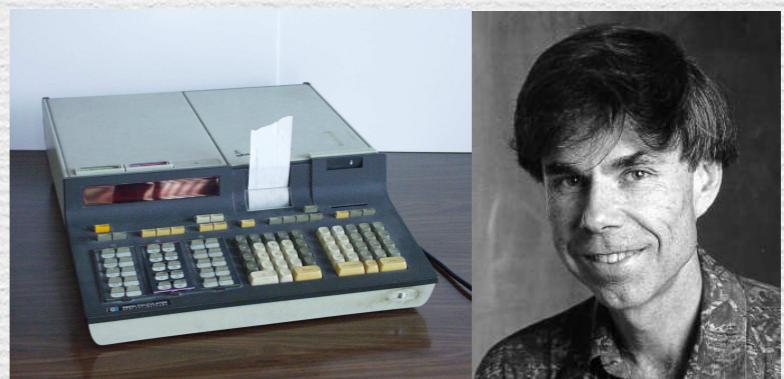
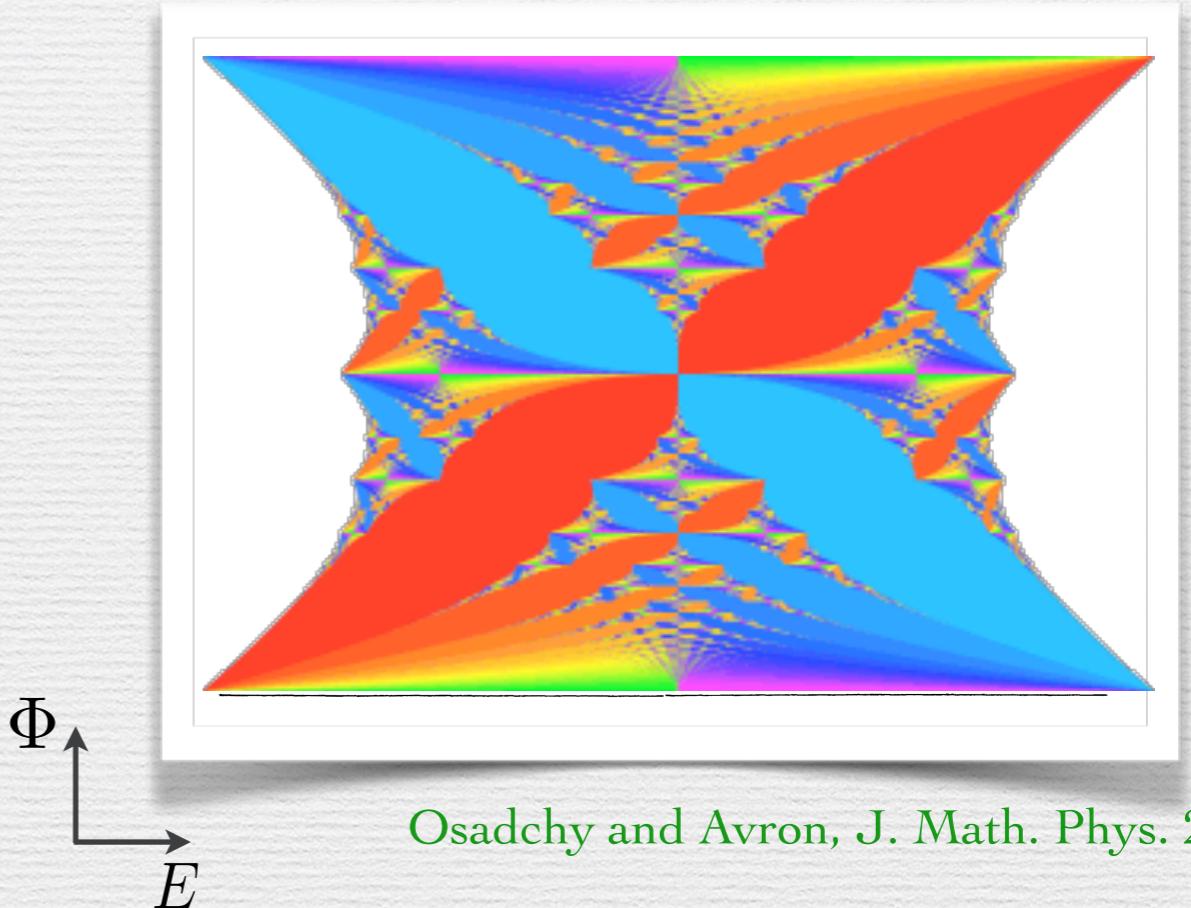
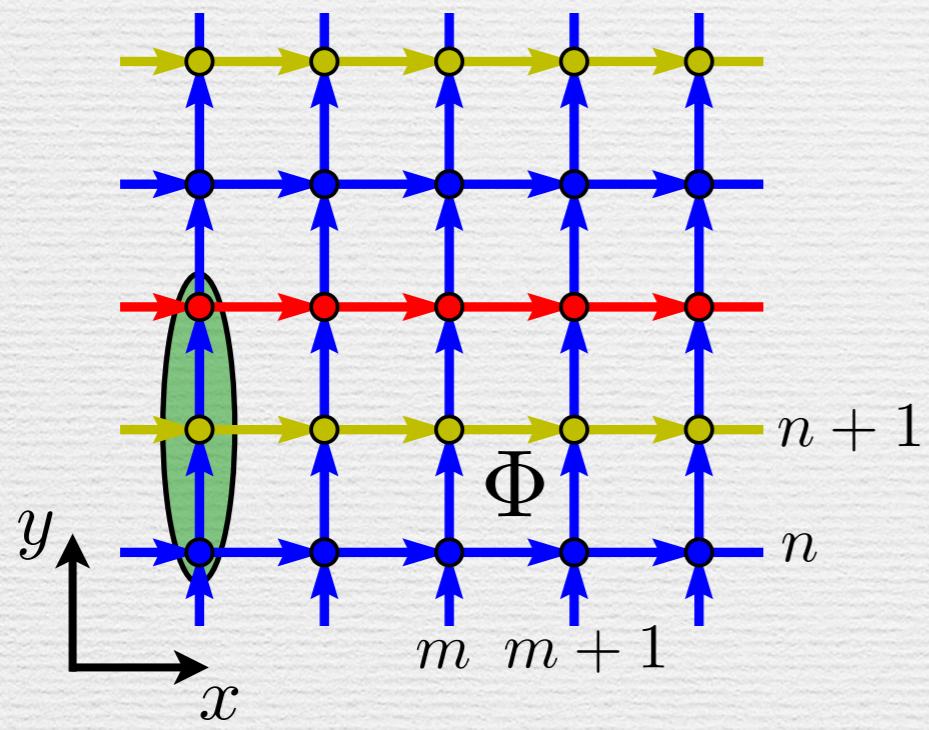


Hofstadter, 1976



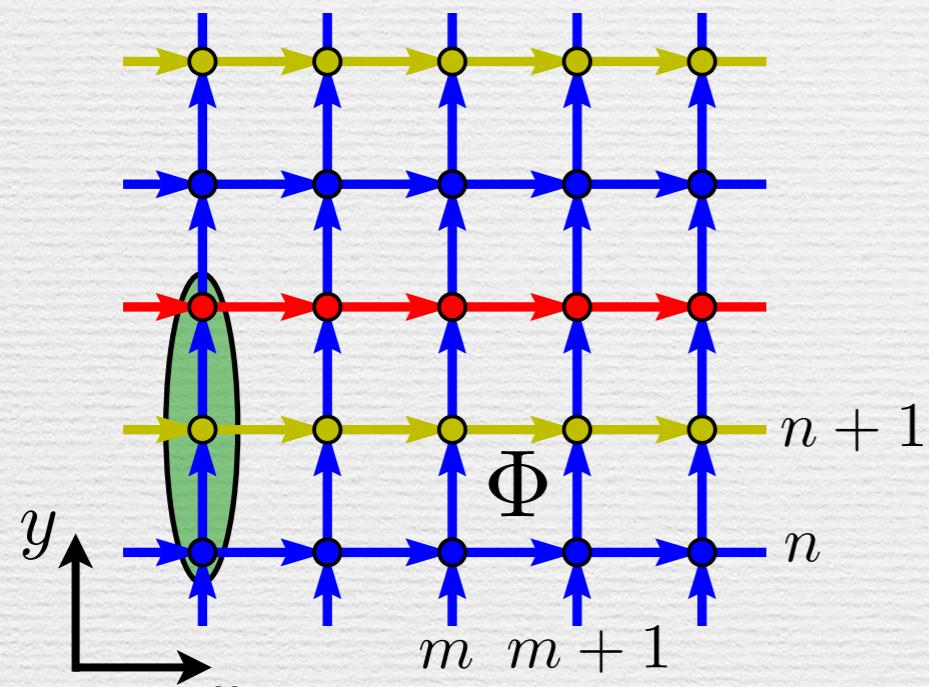
Hofstadter optical lattice

$$H = -J \sum_{m,n} e^{i2\pi n\Phi} c_{m+1,n}^\dagger c_{m,n} + c_{m,n+1}^\dagger c_{m,n} + H.c. \quad \Phi = p/q$$

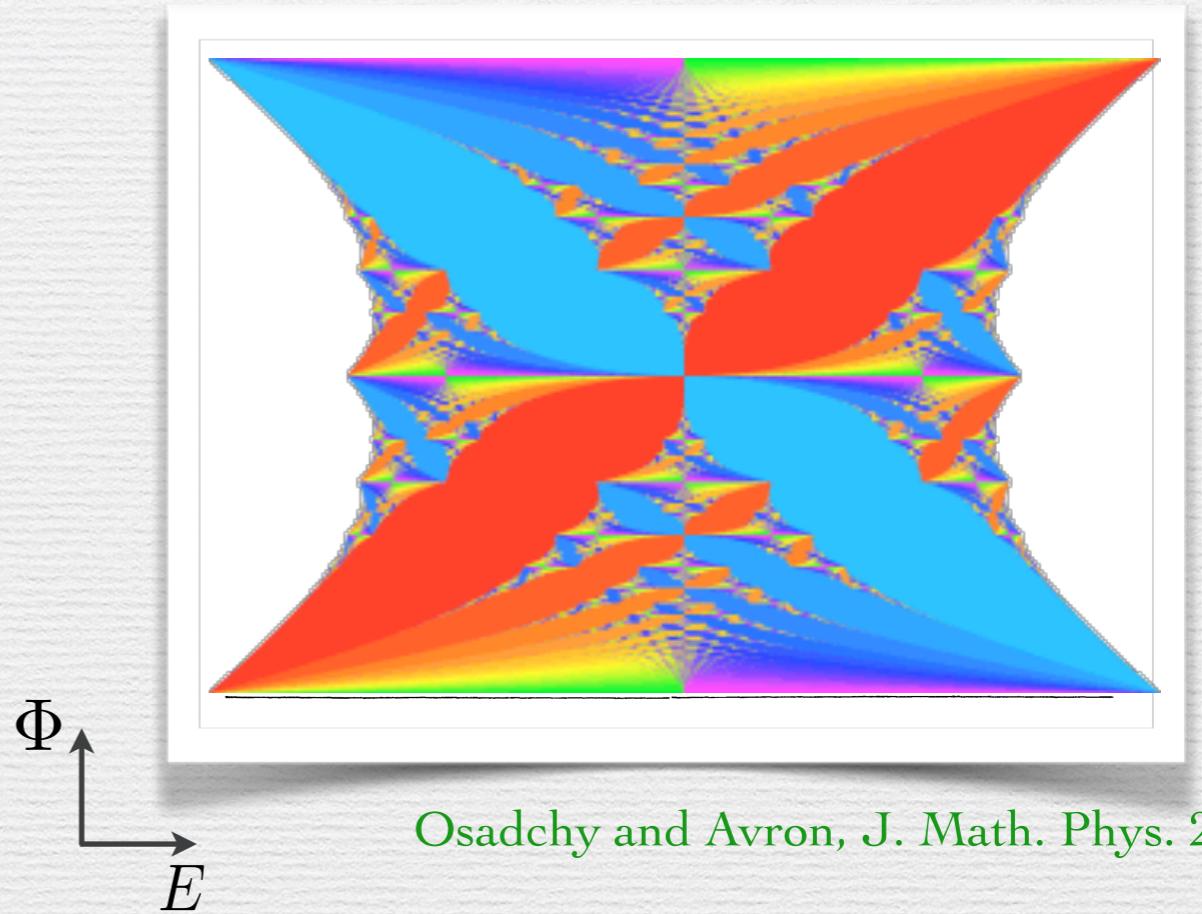


Hofstadter optical lattice

$$H = -J \sum_{m,n} e^{i2\pi n\Phi} c_{m+1,n}^\dagger c_{m,n} + c_{m,n+1}^\dagger c_{m,n} + H.c. \quad \Phi = p/q$$



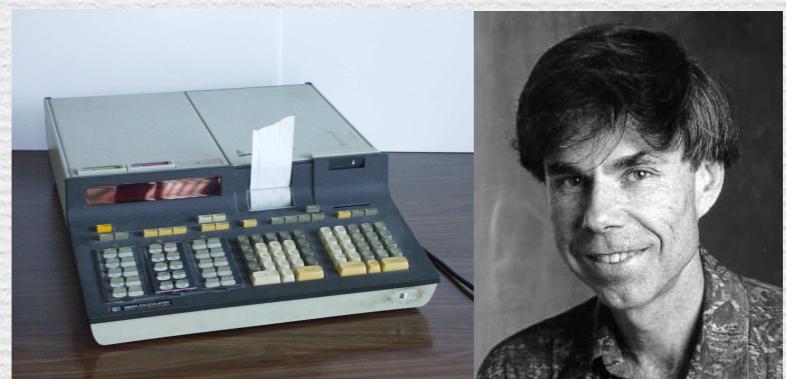
Hofstadter, 1976



Osadchy and Avron, J. Math. Phys. 2001

NO sharp edge states in
harmonic trapping potential

Buchhold *et al*

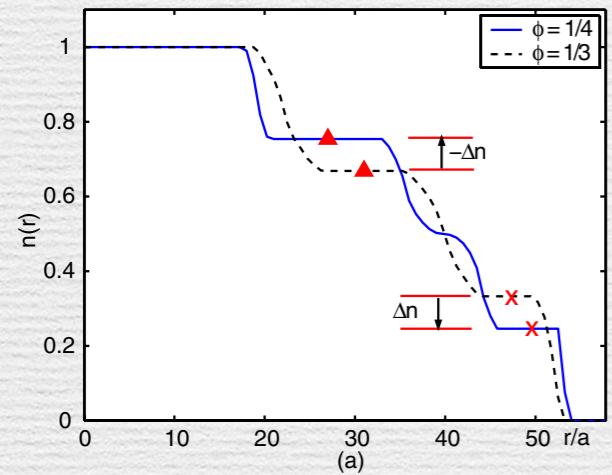


How to measure Chern # ?

How to measure Chern # ?

Density profile

Umucalilar *et al*



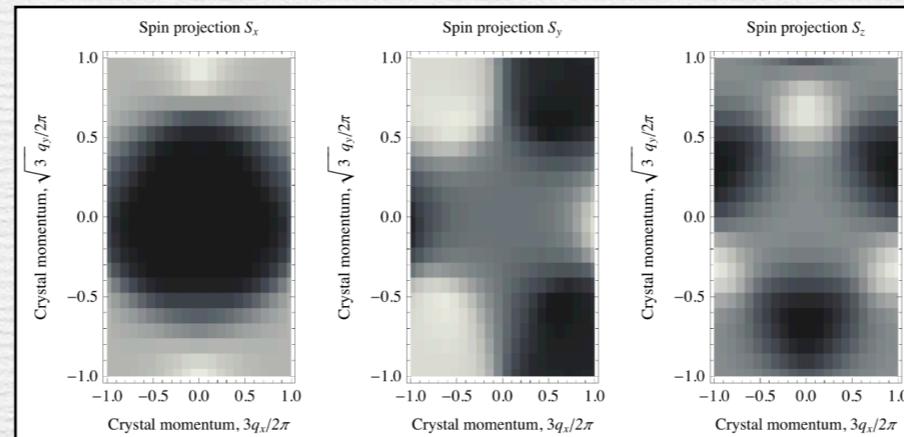
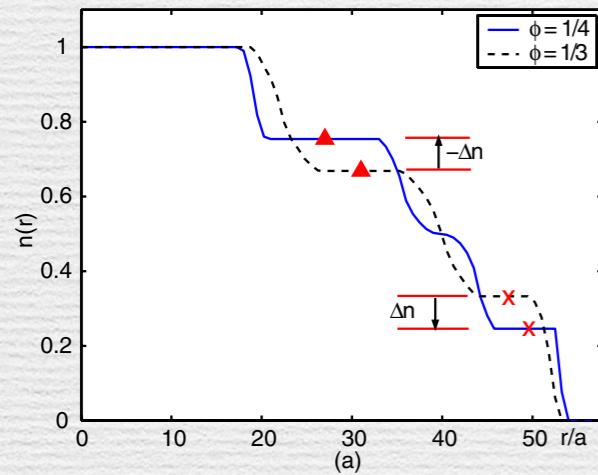
How to measure Chern # ?

Time-of-flight

Alba *et al*, Zhao *et al*

Density profile

Umucalilar *et al*



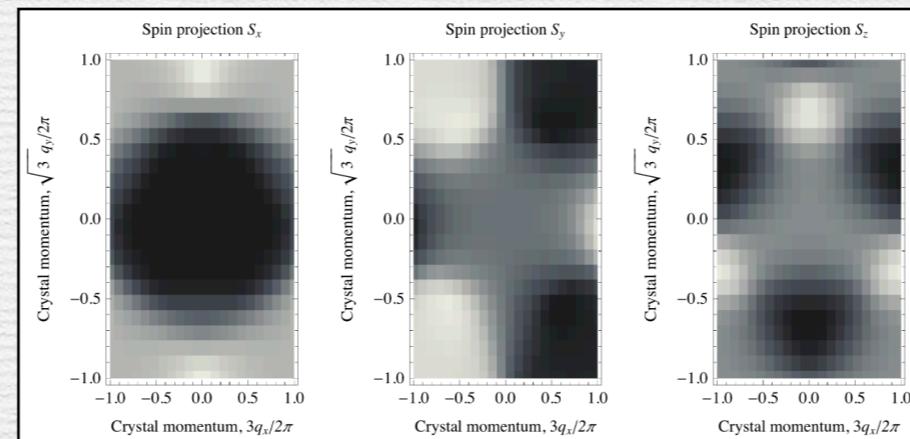
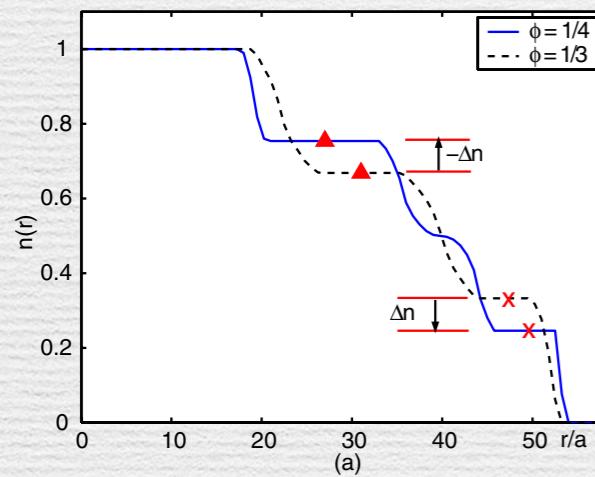
How to measure Chern # ?

Time-of-flight

Alba *et al*, Zhao *et al*

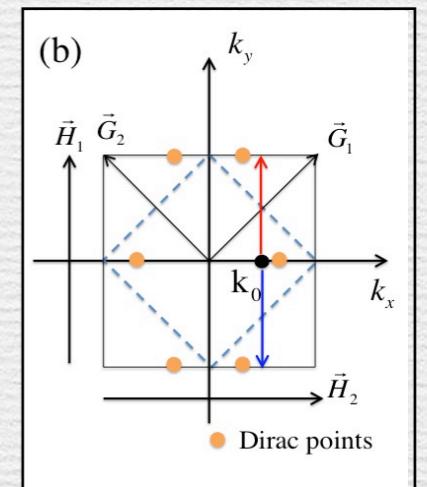
Density profile

Umucalilar *et al*



Zak phases

Abanin *et al*



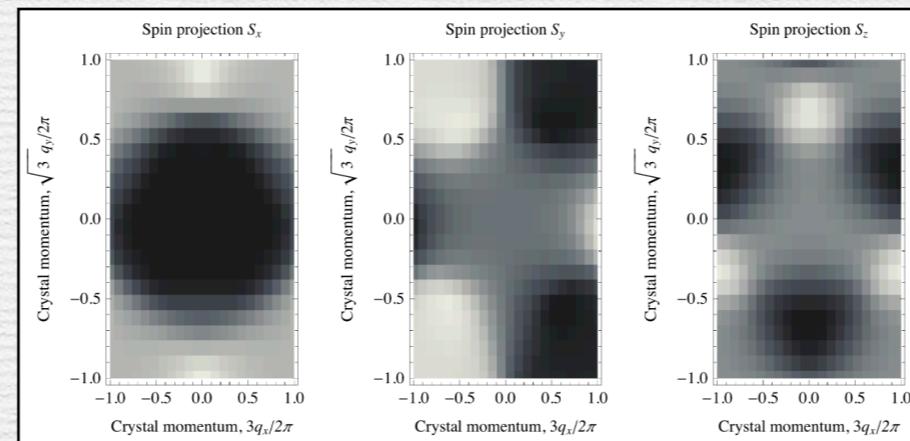
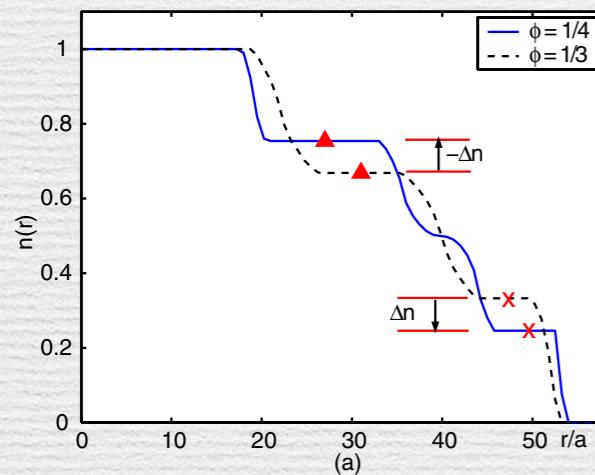
How to measure Chern # ?

Time-of-flight

Alba *et al*, Zhao *et al*

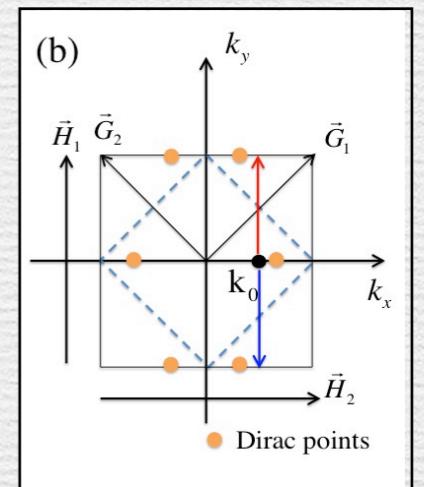
Density profile

Umucalilar *et al*



Zak phases

Abanin *et al*



Semi-classical dynamics

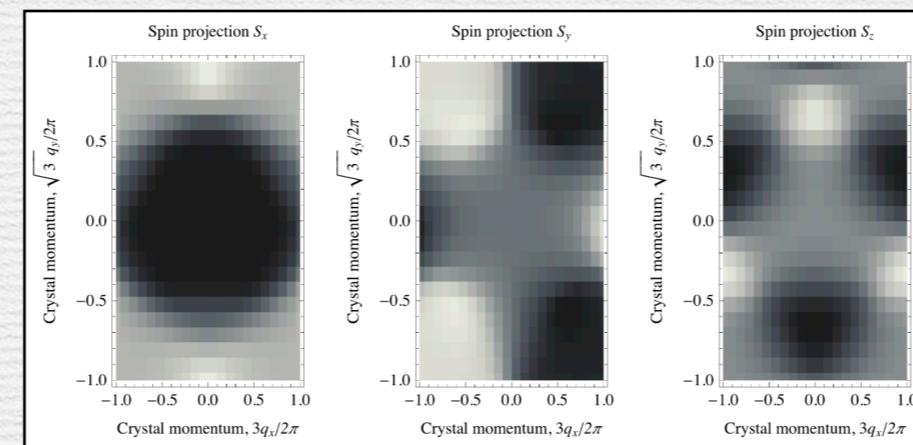
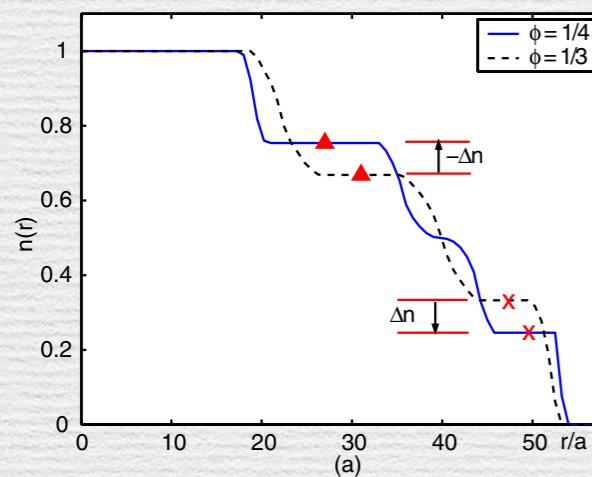
Price *et al*

$$\dot{\mathbf{r}} = \frac{1}{\hbar} \frac{\partial \varepsilon(\mathbf{k})}{\partial \mathbf{k}} - \frac{\mathbf{F}}{\hbar} \times \hat{\mathbf{z}} \Omega(\mathbf{k})$$

How to measure Chern # ?

Density profile

Umucalilar et al



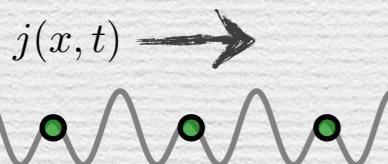
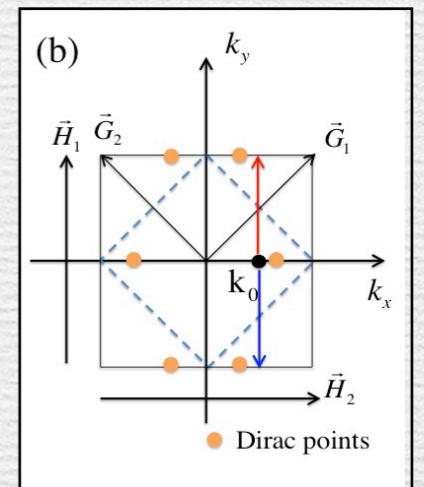
Semi-classical dynamics

Price et al

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Zak phases

Abanin et al

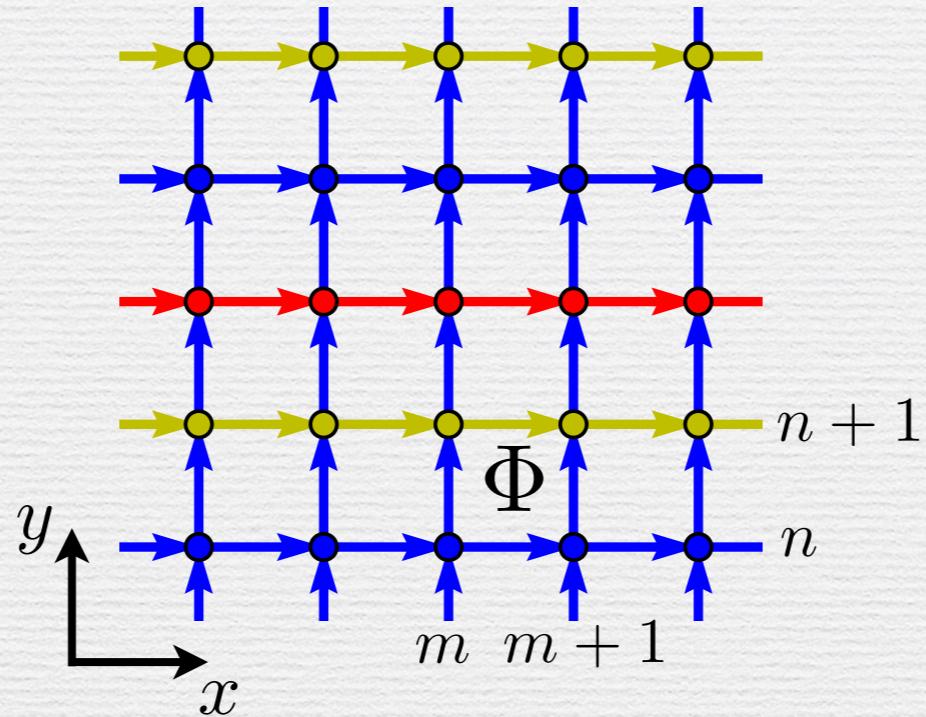


We propose a **new** probe based on
Topological Pumping Effect

$\rho(\mathbf{k}_x, y)$

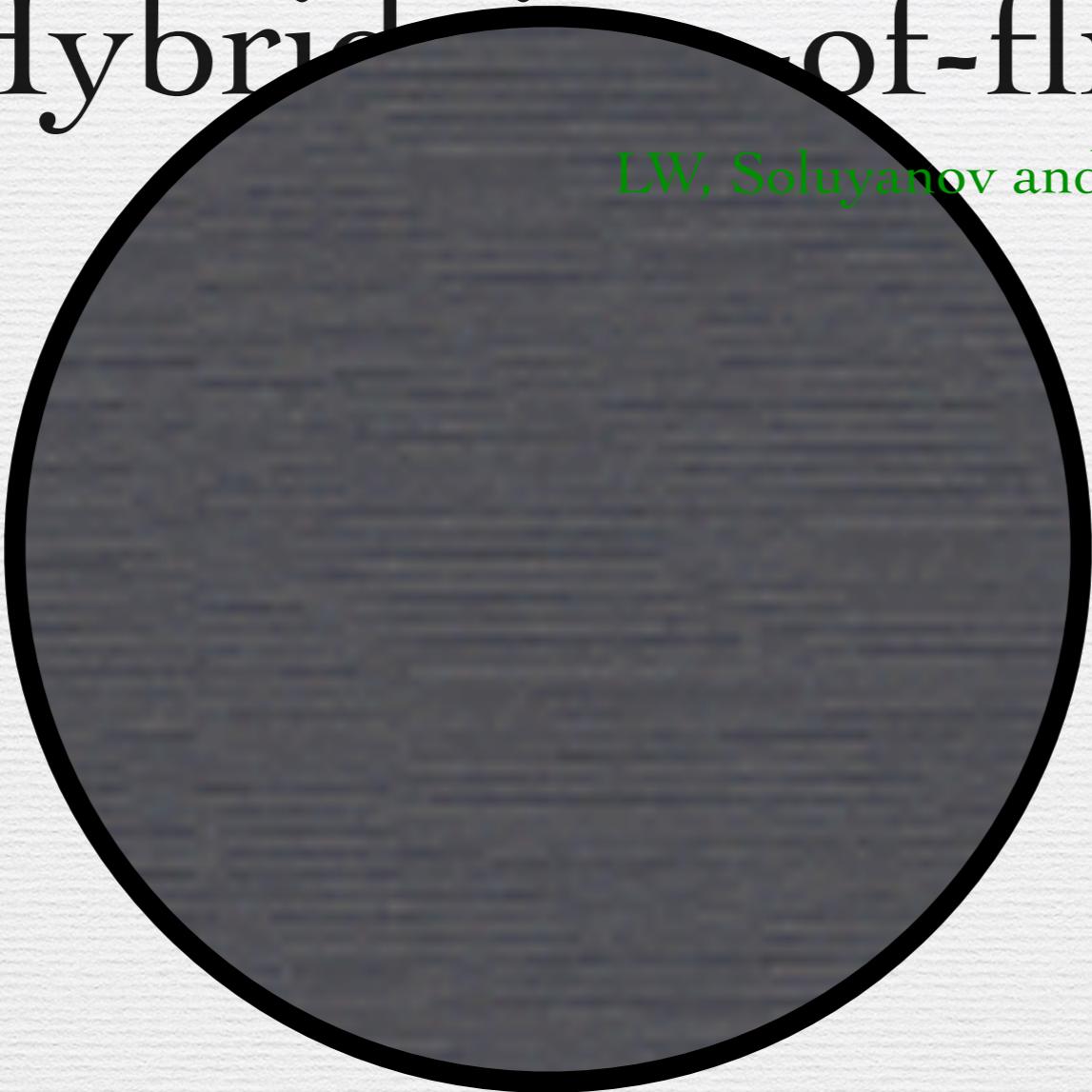
Hybrid time-of-flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



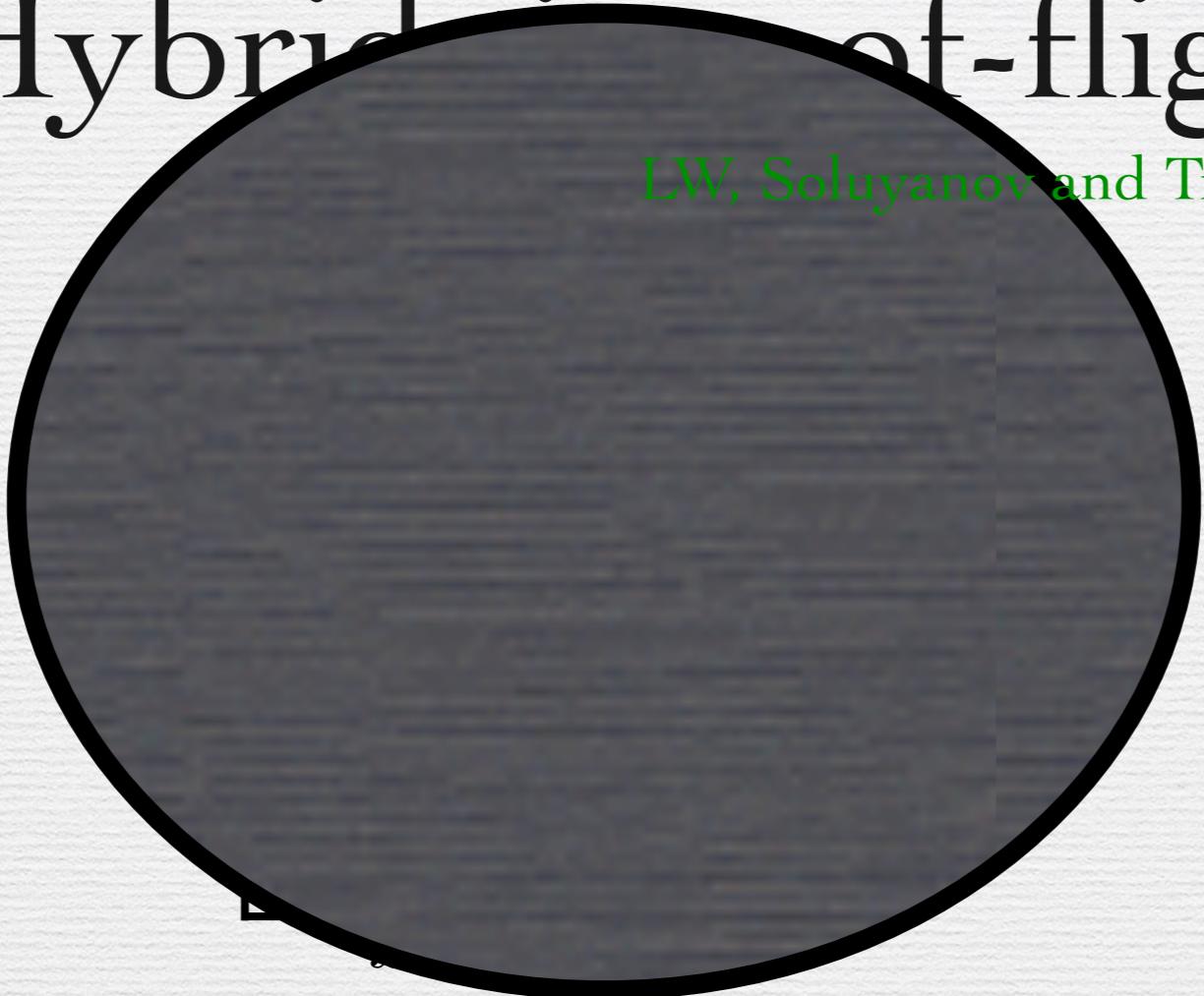
Hybrid ion-of-flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



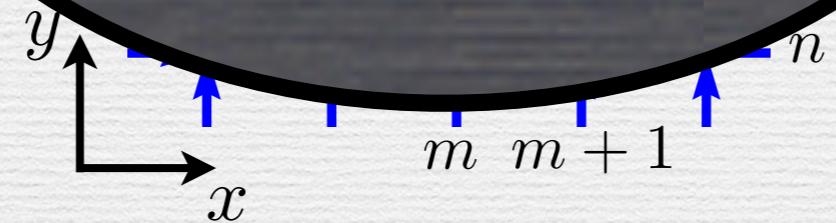
Hybrid ion-of-flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



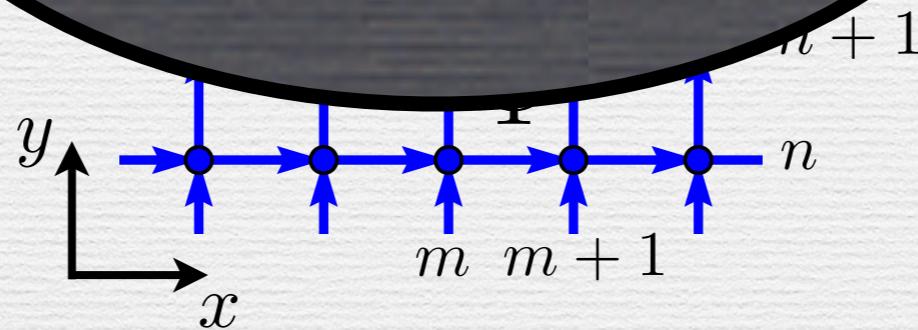
Hybrid self-flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



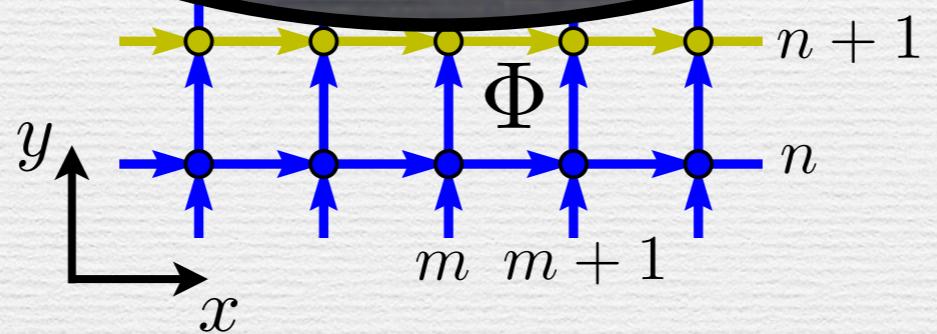
Hybrid $\text{e}^+ \text{e}^-$ -flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



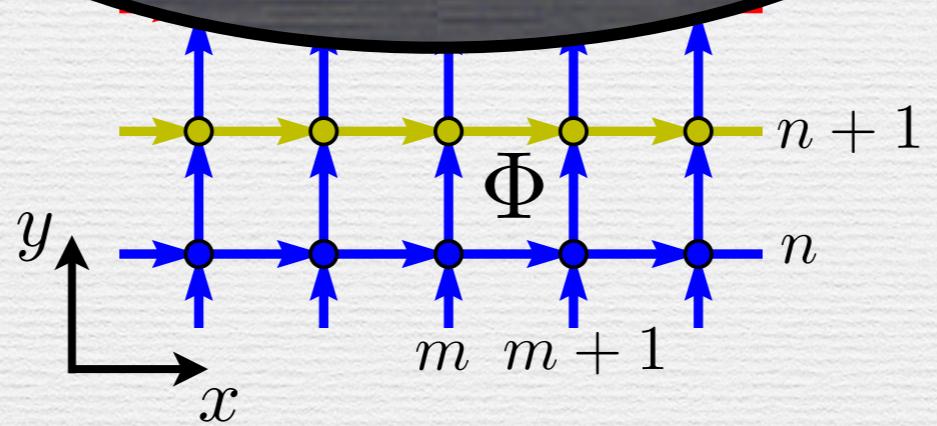
Hybrid f-f f-flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



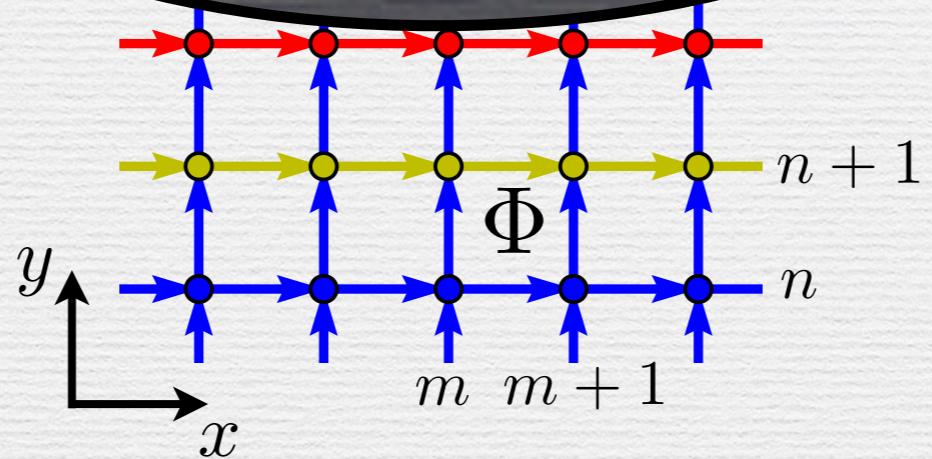
Hybrid f-f flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



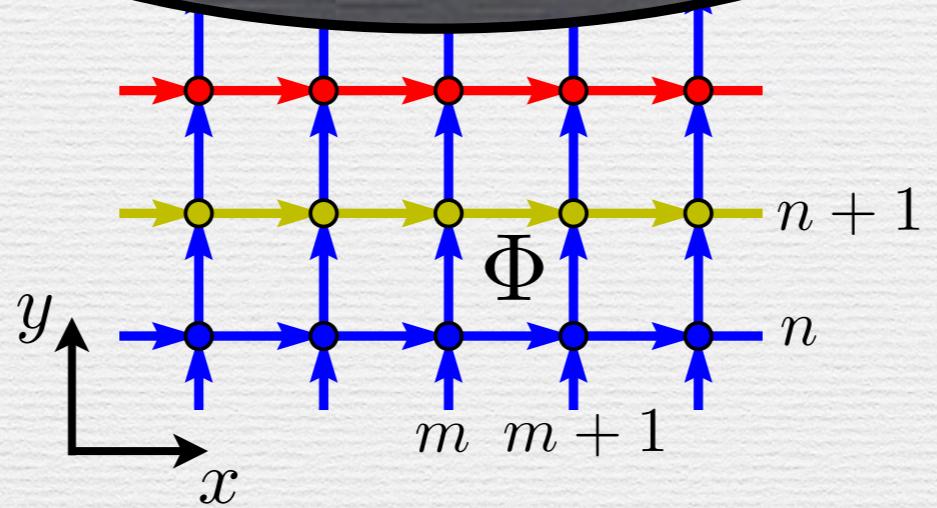
Hybrid topological flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



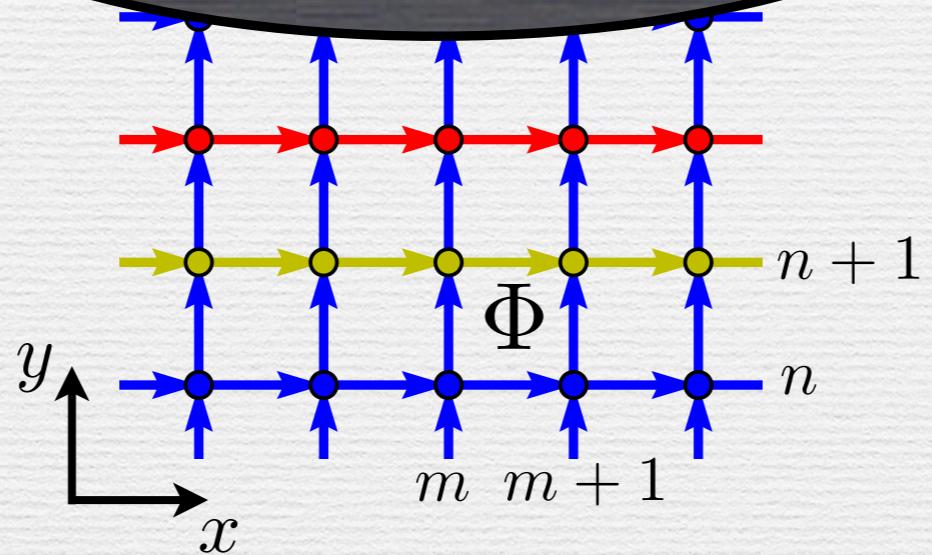
Hybridization flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



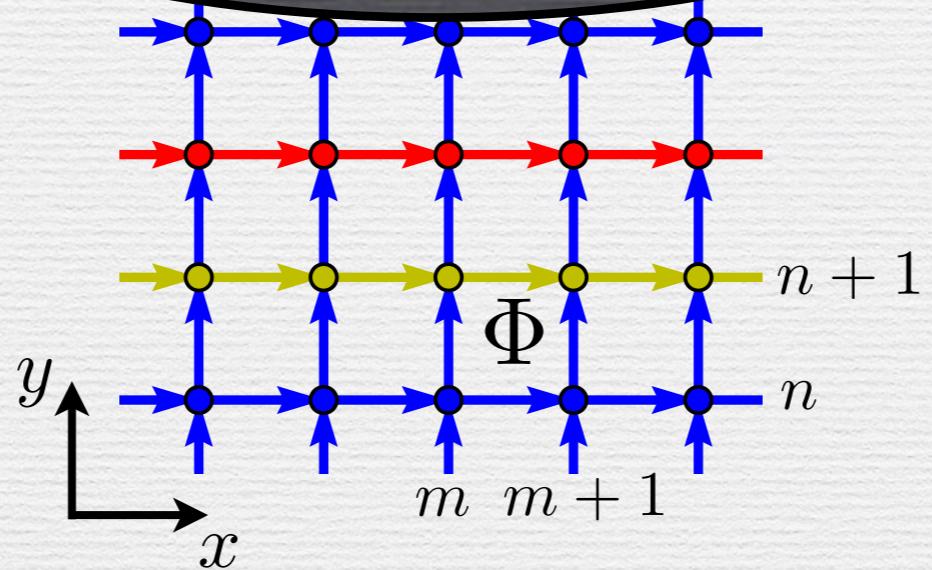
Hybridization flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



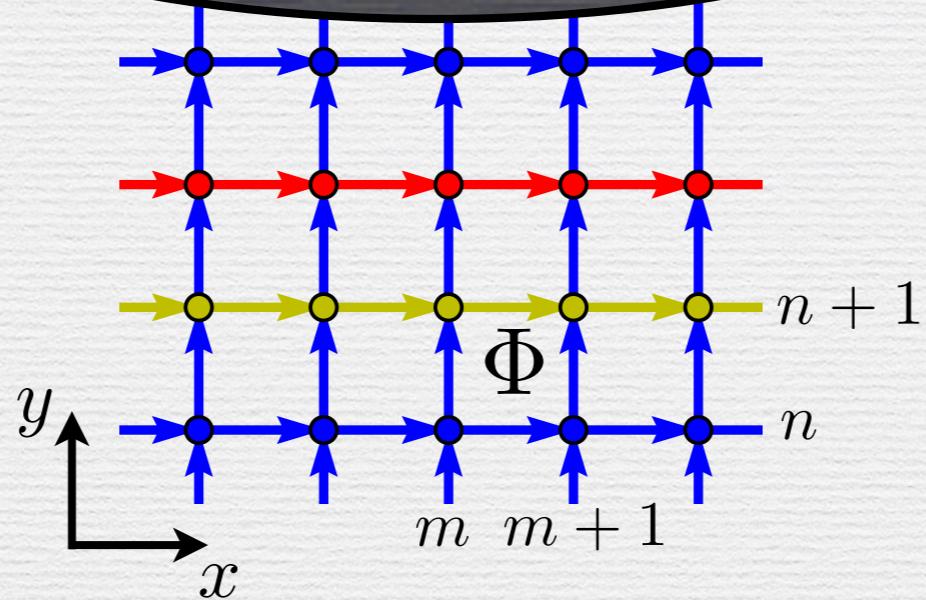
Hybrid state flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



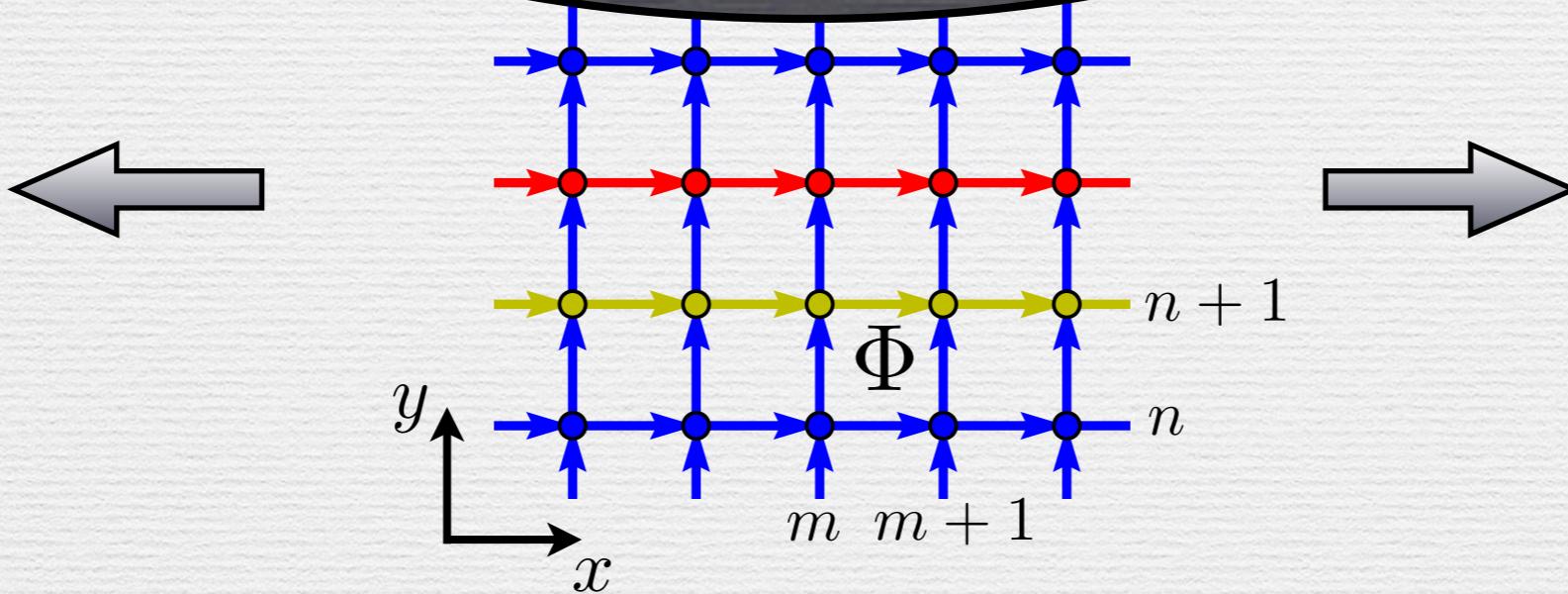
Hybridization flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



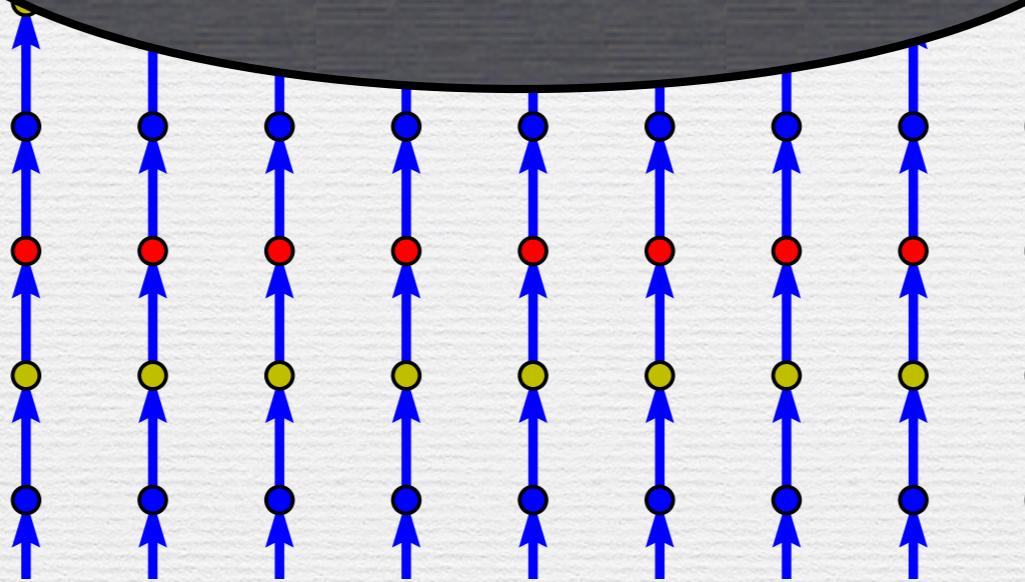
Hybridization flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



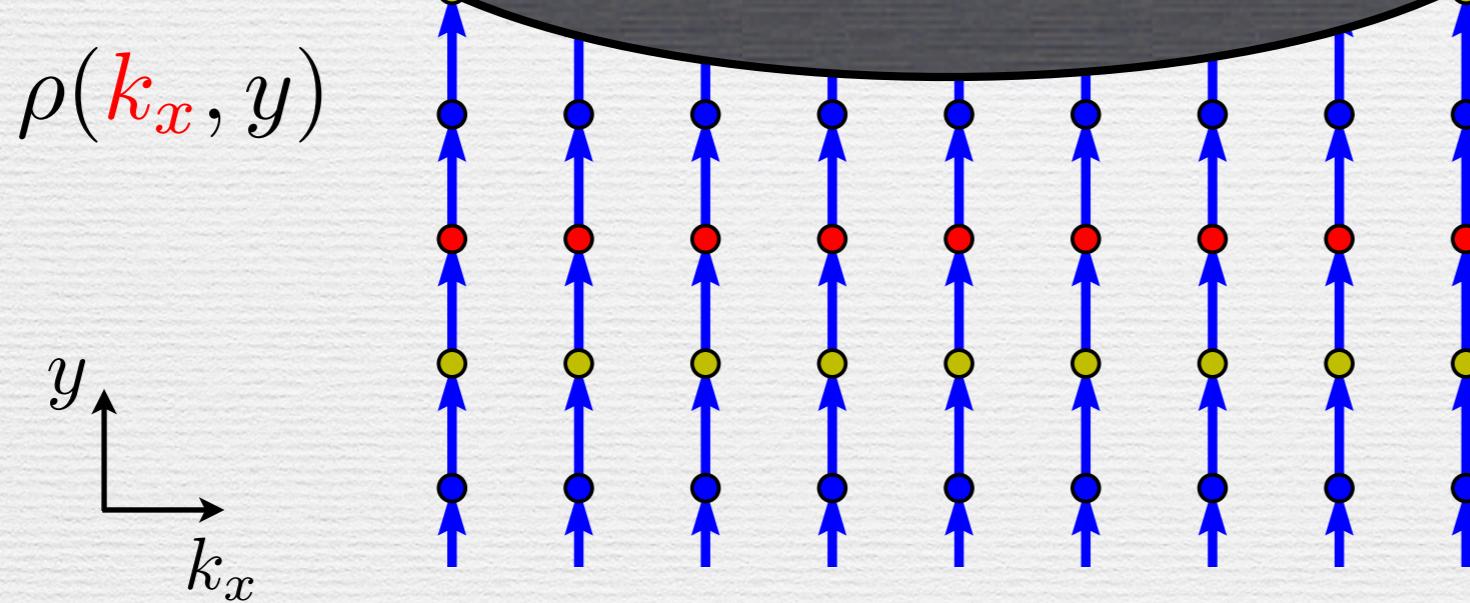
Hybrid state of flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



Hybrid state of flight

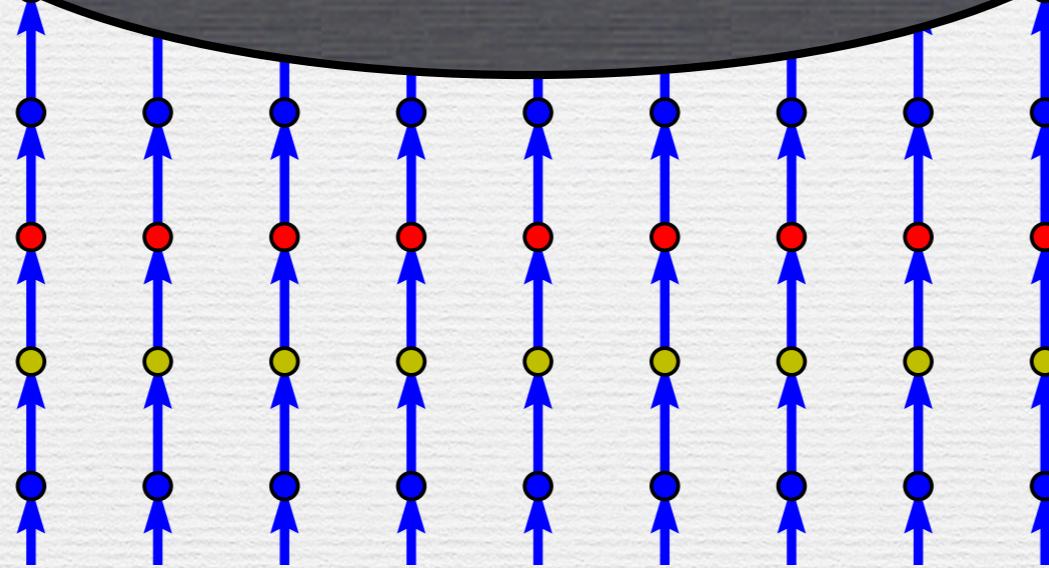
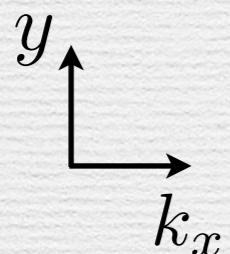
LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



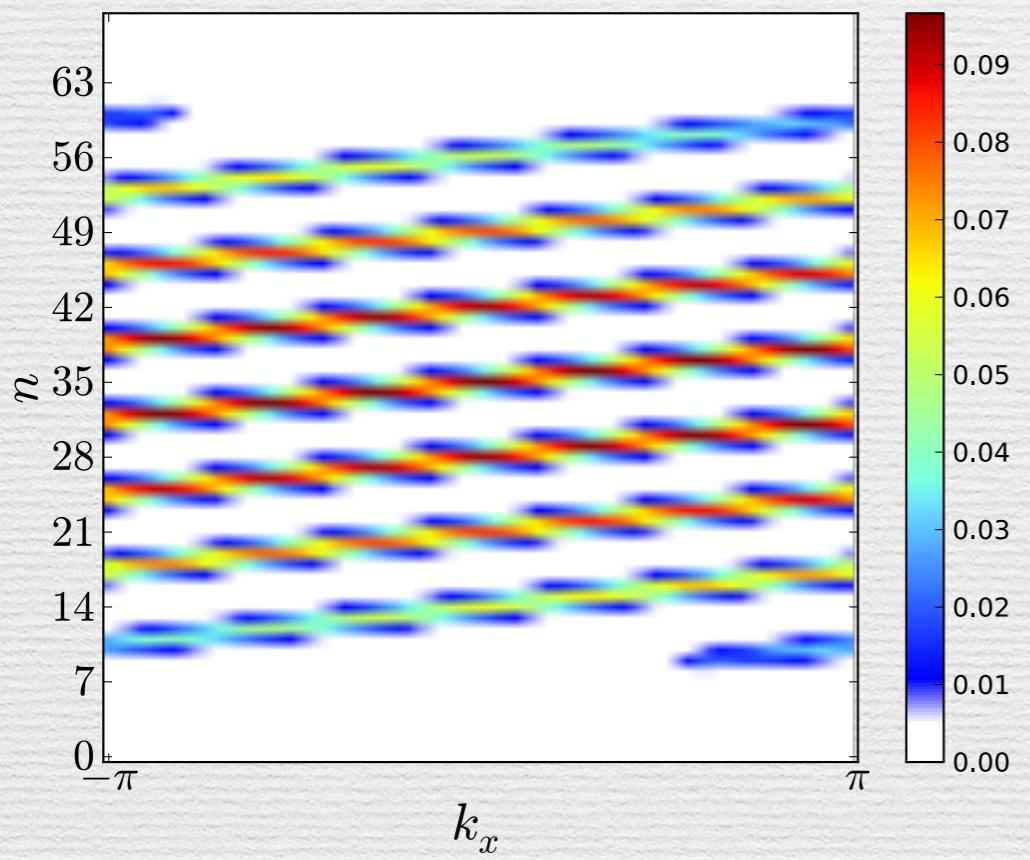
Hybrid state of flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)

$$\rho(k_x, y)$$



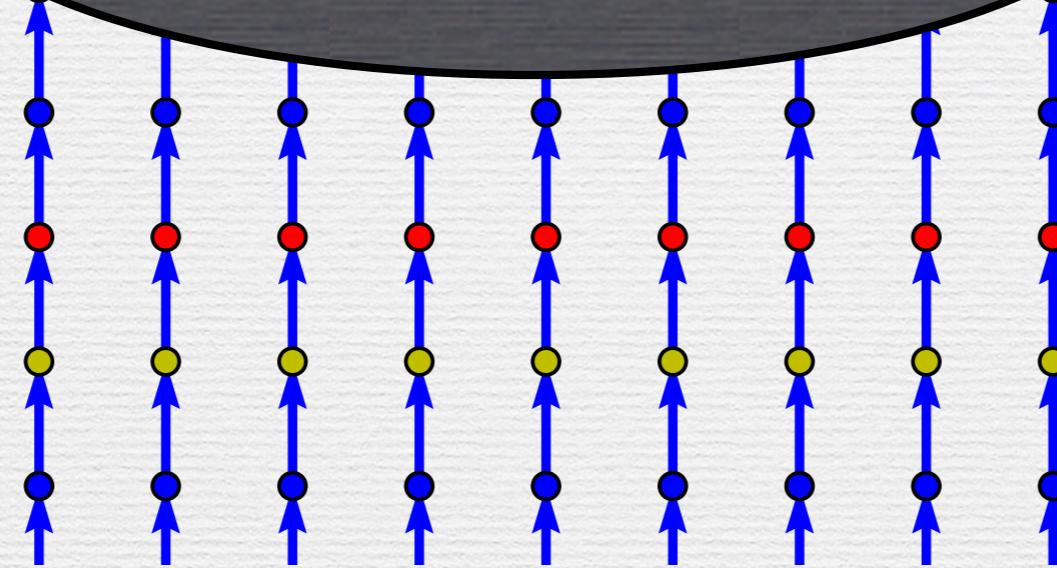
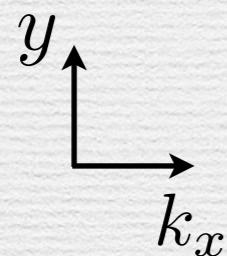
$$\Phi = 1/7 \quad C = 1$$



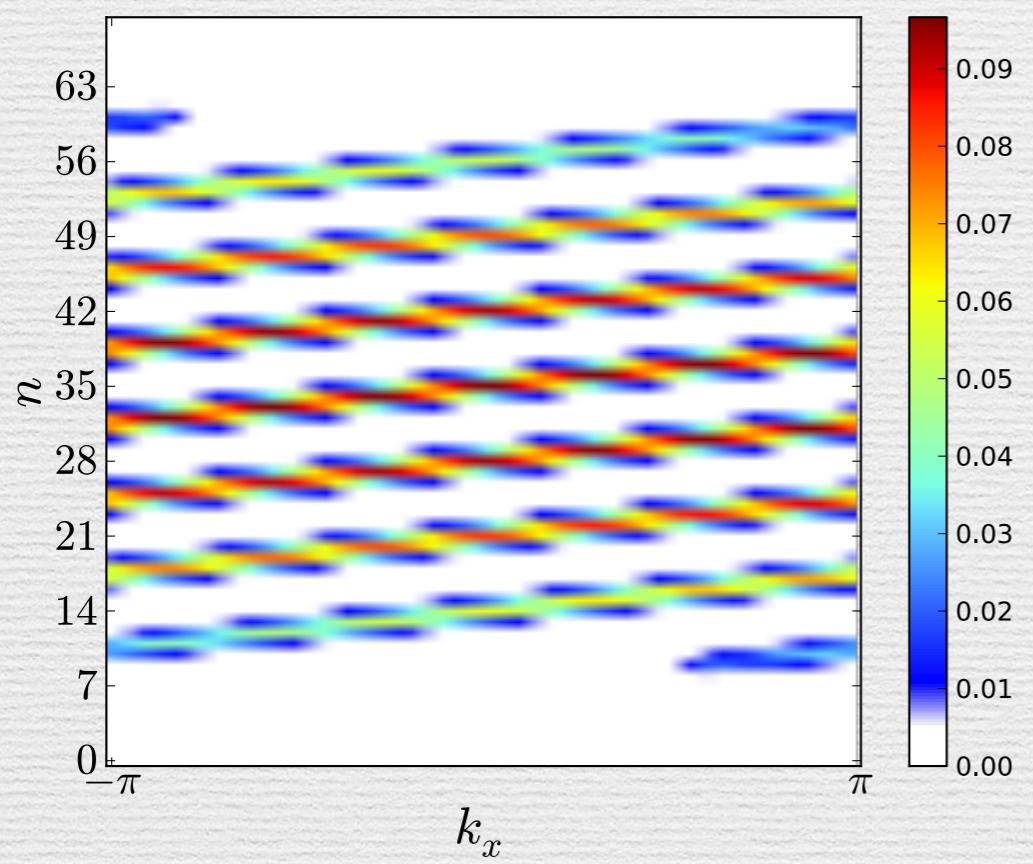
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LW, Soluyanov and Troyer, PRL 110, 166802 (2013)

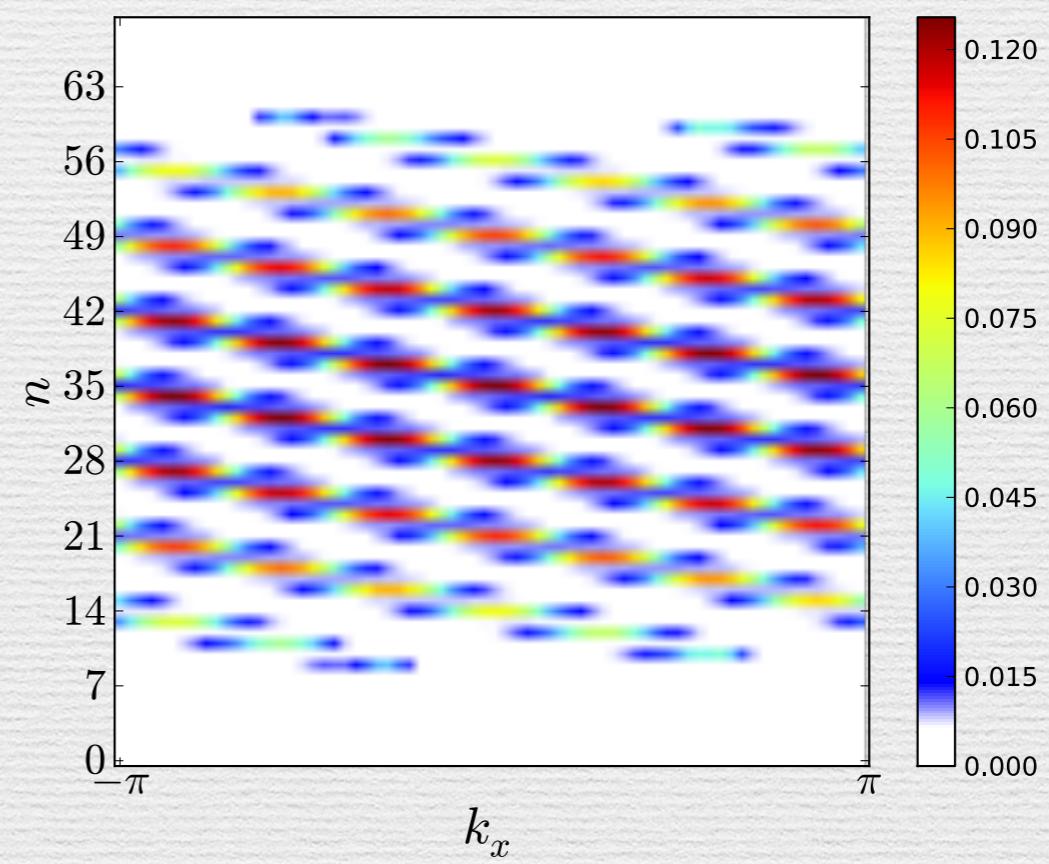
$$\rho(k_x, y)$$



$$\Phi = 1/7 \quad C = 1$$



$$\Phi = 3/7 \quad C = -2$$



Hybrid state of flight

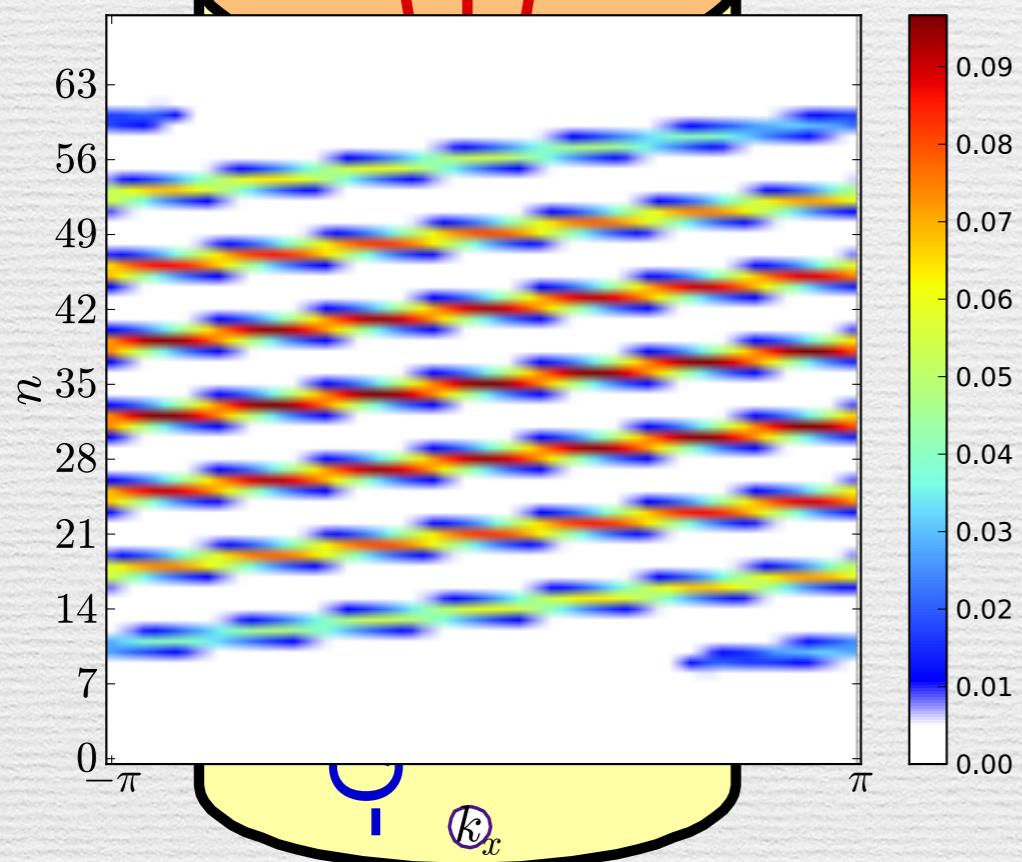
LW, Soluyanov and Troyer, PRL 110, 166802 (2013)

$$\rho(k_x, y)$$

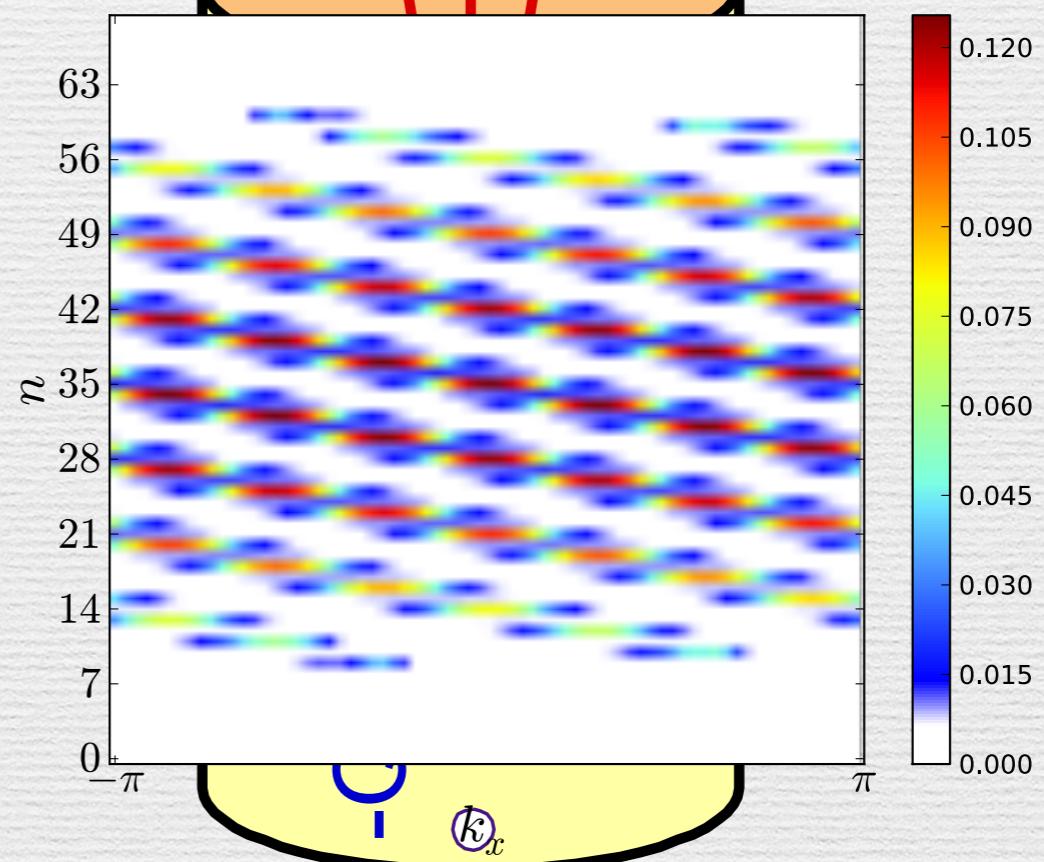
y

$$k_x$$

$$\Phi = 1/7 \quad C = 1$$

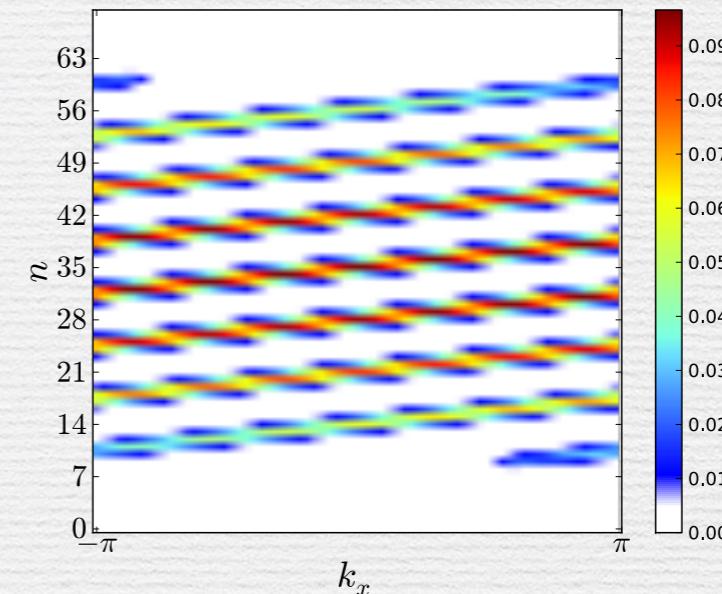


$$\Phi = 3/7 \quad C = -2$$



Quantitative Characterizations

- ❖ Slope
- ❖ # of cuts (edge modes)
- ❖ COM along y-direction
- ❖ Bipartition particle number (trace index)



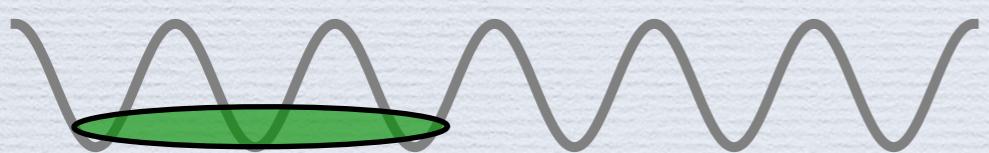
Alexandradinata *et al*

Salient features

- ❖ Bulk detection, does not require edge states
- ❖ $\rho(k_x, y)$ is almost impossible to measure in solids, but is natural to cold atom toolbox
- ❖ Can be extended to interacting case

Fractional charge pumping

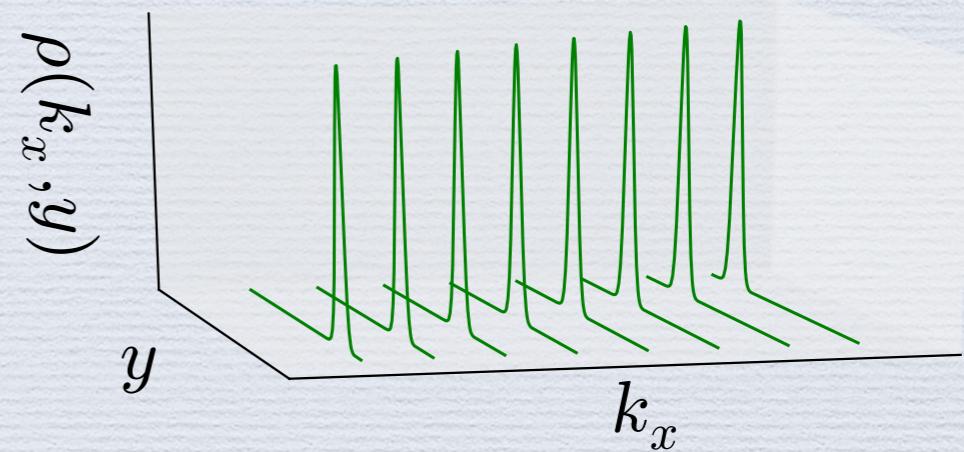
1D lattice



Interaction is crucial
for opening an energy gap

2D Laughlin state

$$\rho(k_x, y) = \frac{\nu}{\sqrt{\pi}} e^{-(y - k_x)^2}$$



Use hybrid ToF to detect FQHE and fractional Chern insulators realized in optical lattices

Fractional charge pumping

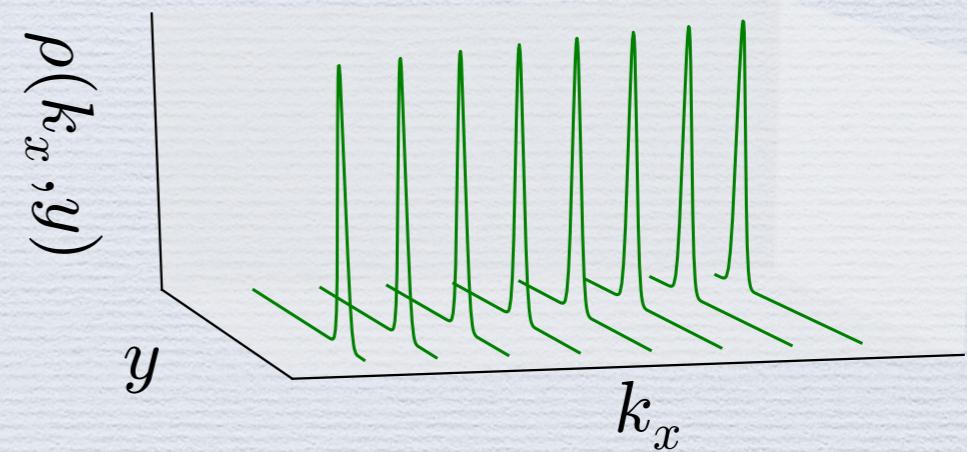
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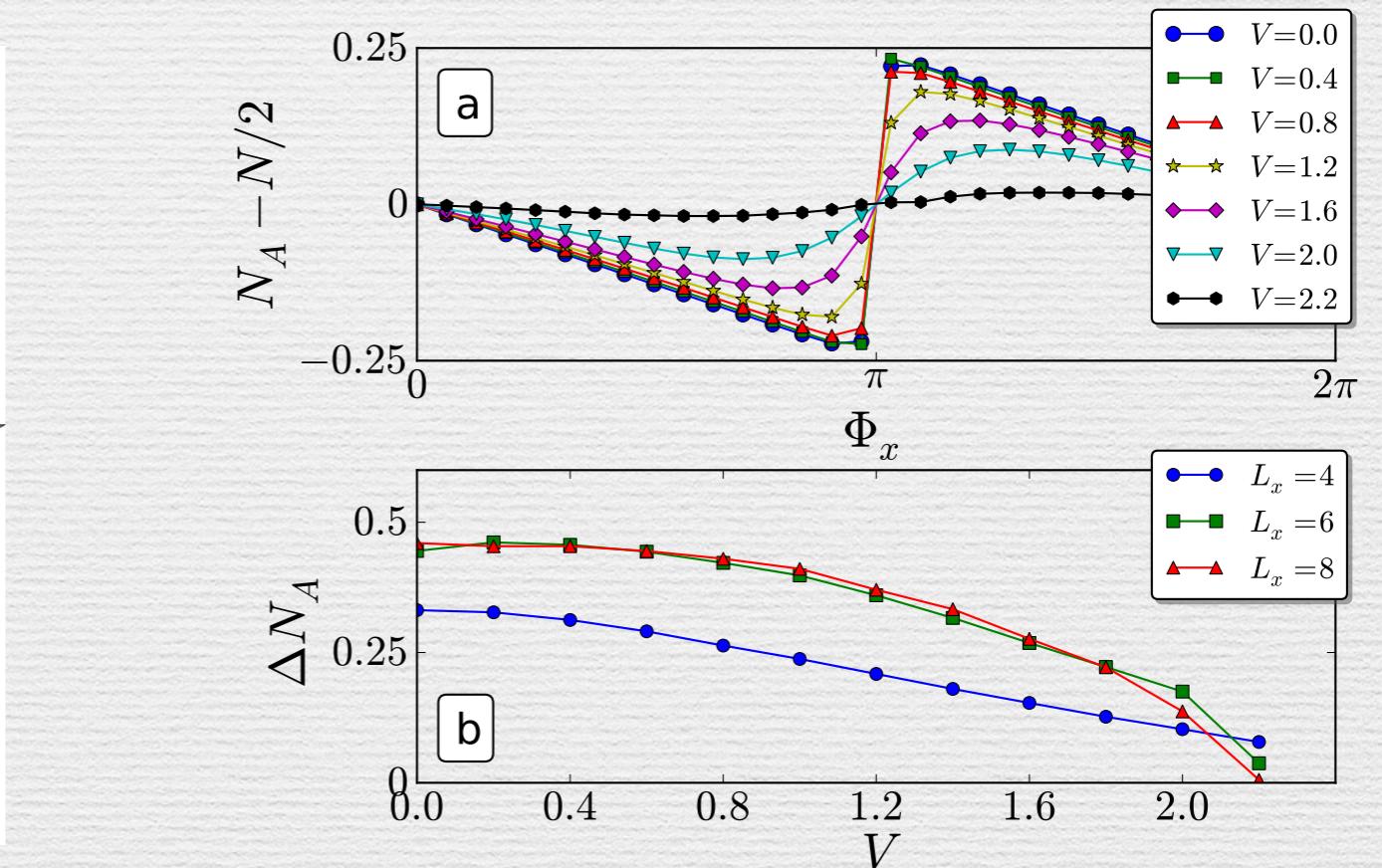
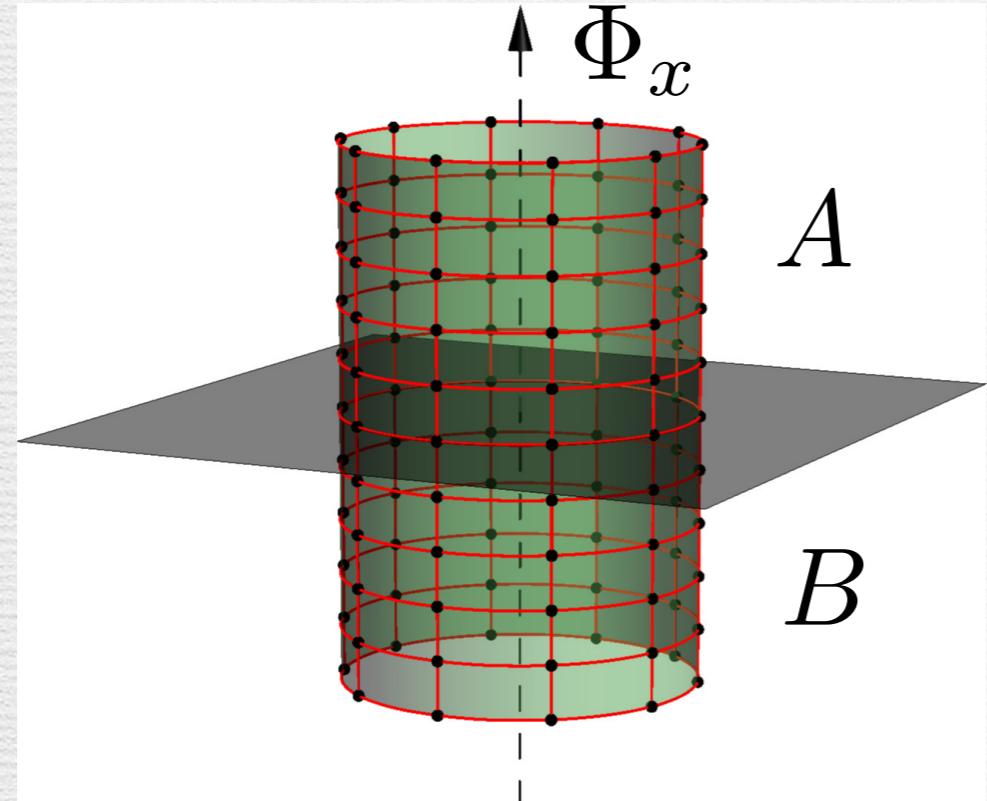
$$\rho(k_x, y) = \frac{\nu}{\sqrt{\pi}} e^{-(y - k_x)^2}$$



Use hybrid ToF to detect FQHE and fractional Chern insulators realized in optical lattices

Epilog: Numerical diagnosis of fractional Hall conductance

$$N_A(\Phi_x) = \sum_{\mathbf{r} \in A} n_{\mathbf{r}}(\Phi_x)$$

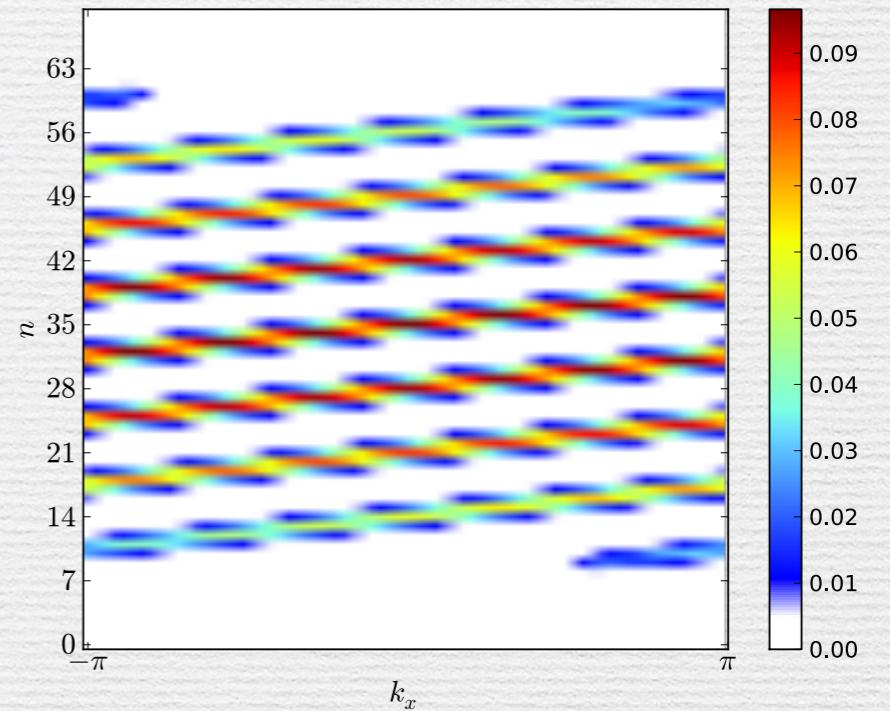
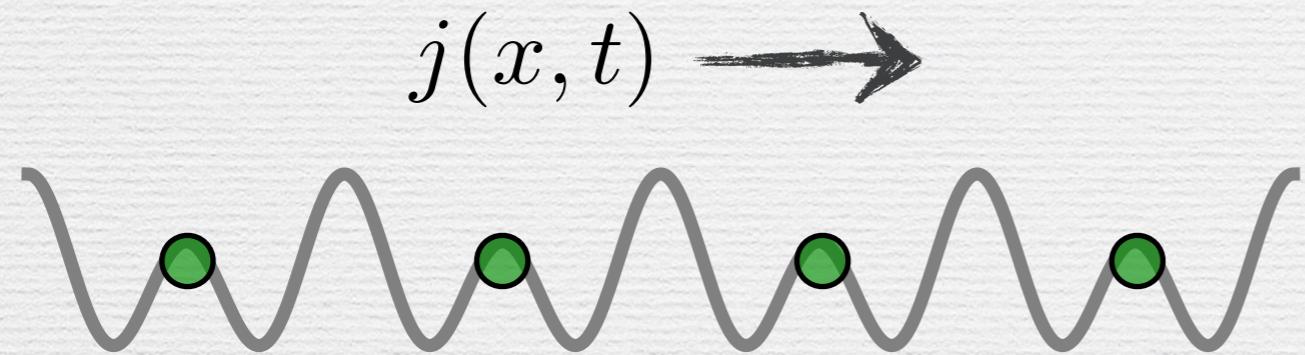


LW, Soluyanov and Troyer, to appear

Avoid calculating overlap between wavefunctions
 Thouless, Kohmoto, Nightingale,
 den Nijs

Summary

PRL 111, 026802 (2013)
PRL 110, 166802 (2013)



Topological charge pumping is a common thread
unifies many features of topological states

Guideline for design and detection of topological
phases in cold atom systems

Thank you!

