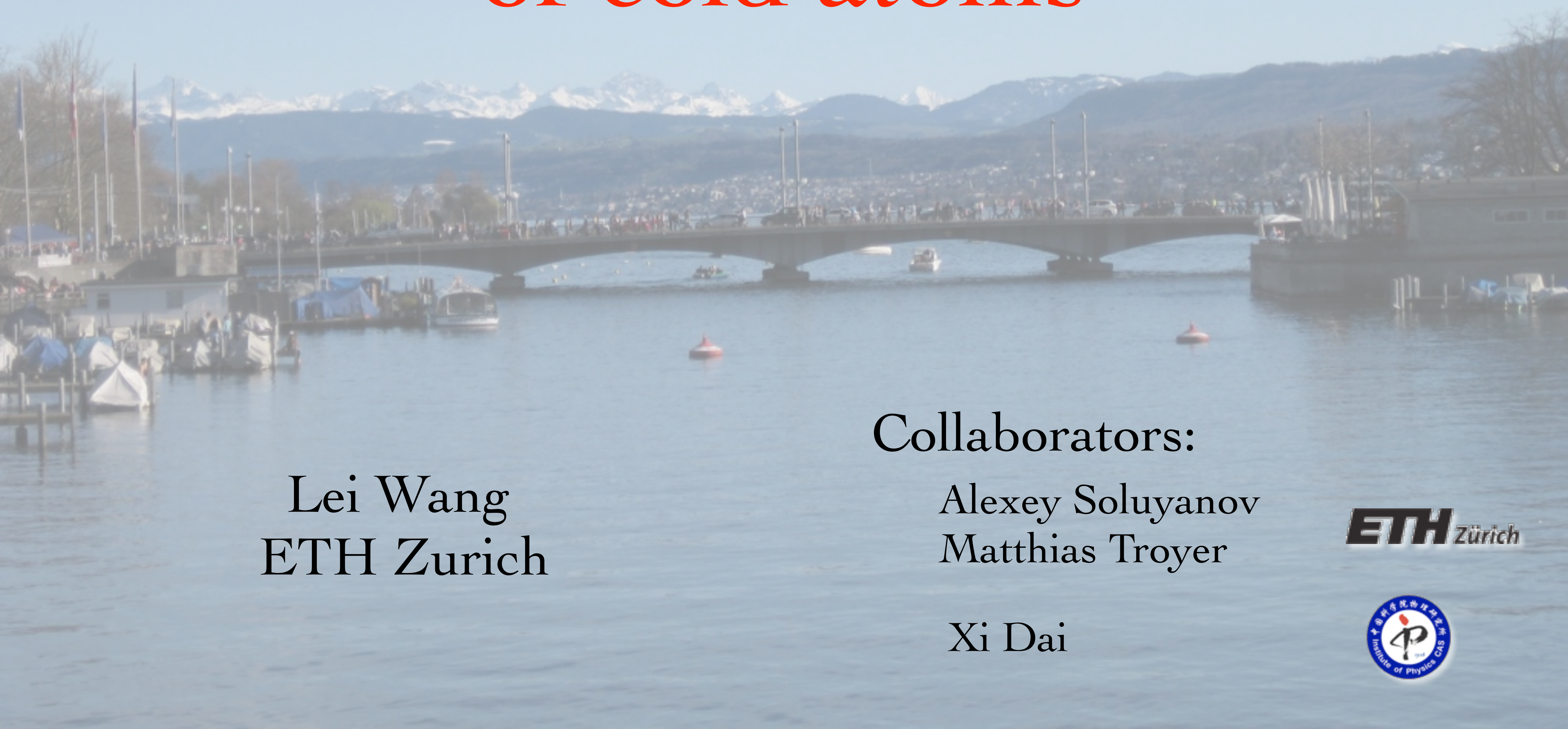
arXiv:1301.7435
PRL 110, 166802

Topological charge pumping of cold atoms



Lei Wang
ETH Zurich

Collaborators:

Alexey Soluyanov
Matthias Troyer

ETH Zürich

Xi Dai

Plan

Topological charge pumping
in a 1D optical lattice

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Topological charge pumping in a 1D optical lattice

$\downarrow$

Measure topological index of 2D optical lattices

Pumps

A **pump** is a device that moves fluids, or sometimes slurries, by mechanical action.

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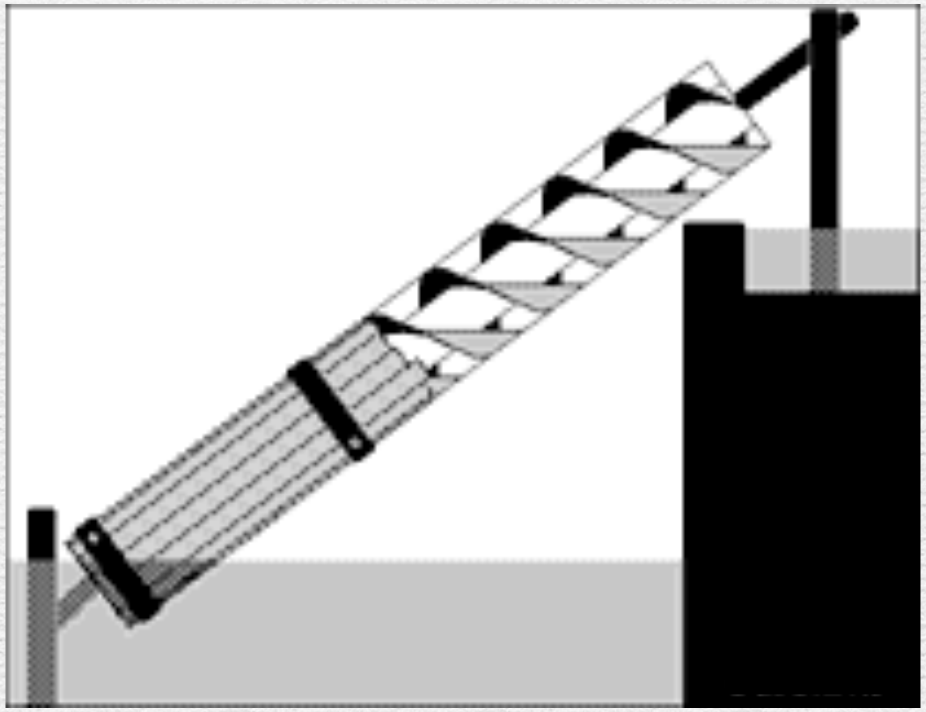

Archimedes' screw ~250 BC

Pumps

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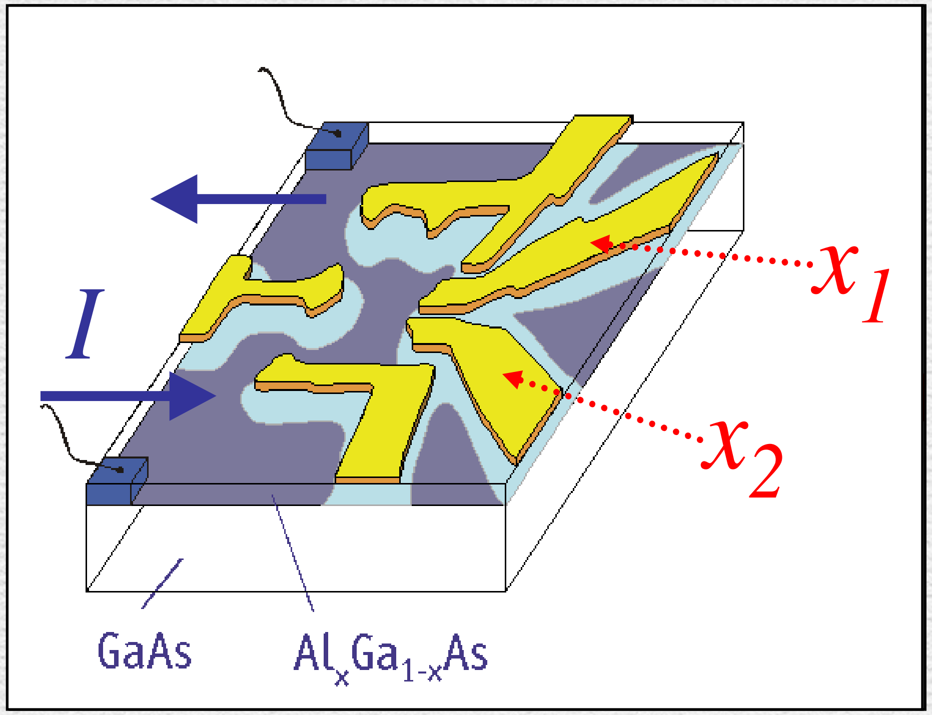
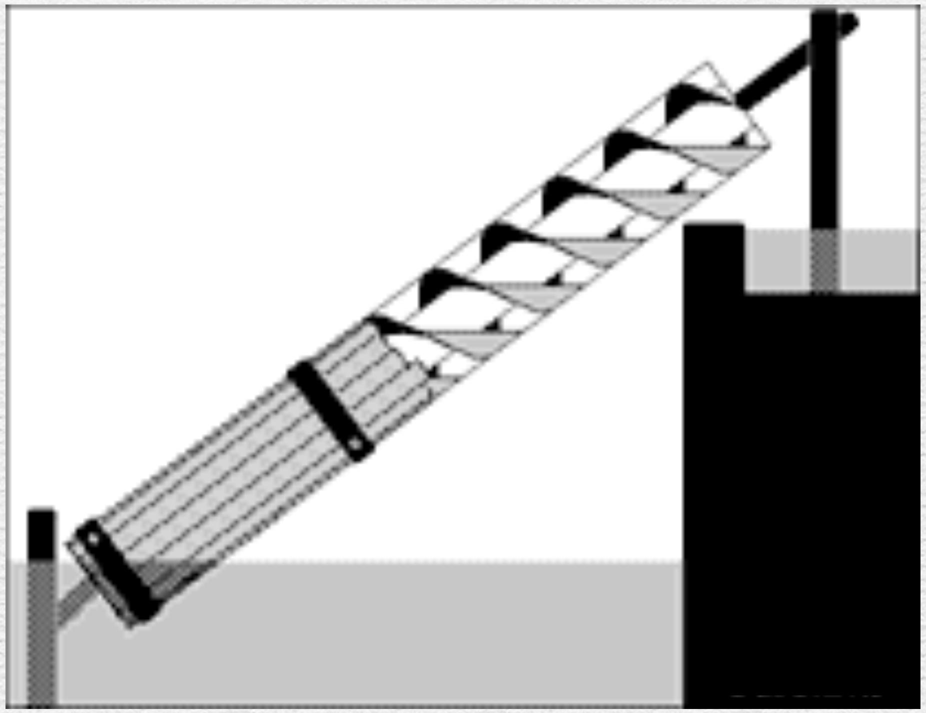
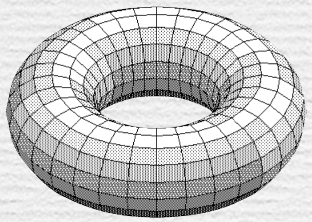


Switkes *et al* 1999
Buttiker, Brouwer, Zhou, Spivak, Altshuler ...



Archimedes' screw ~250 BC

Topological pump

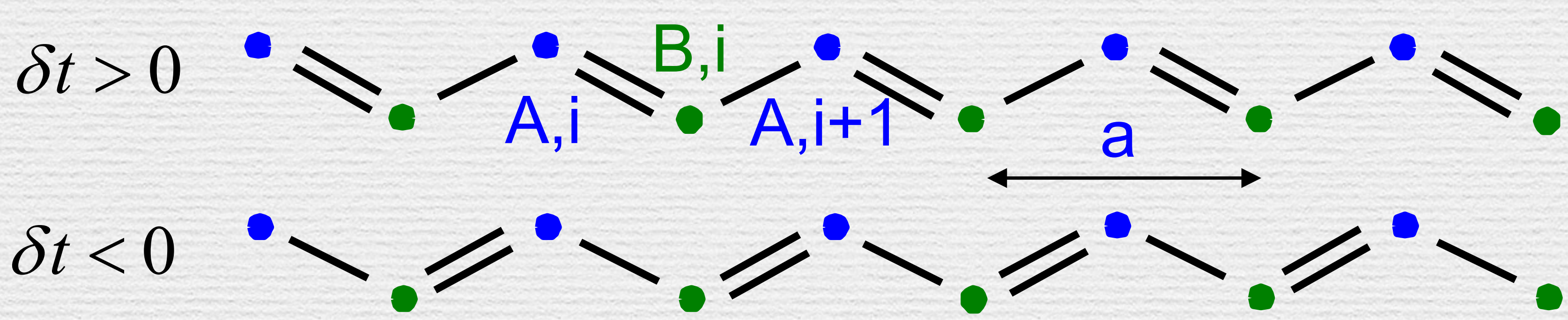


A device transfers **quantized charge** in each pumping cycle.

Thouless 1983

$$H = \sum_i (t+\delta t) c^\dagger_{Ai} c_{Bi} + (t-\delta t) c^\dagger_{Ai+1} c_{Bi} + H.c.$$

Su, Schrieffer, Heeger, 1979

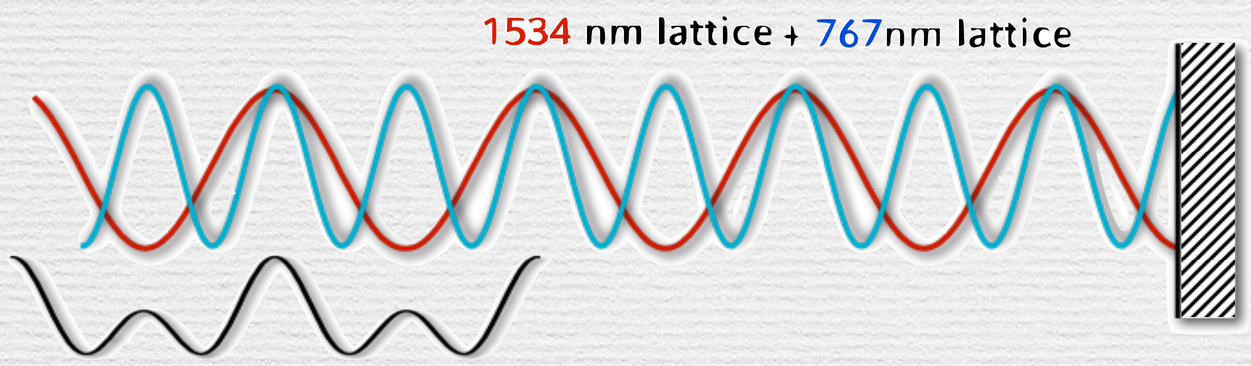


- Current flows in an insulating state
- No dissipation!
- Dynamical analog of quantum Hall effect

Experimental progresses

Optical Superlattice

Fölling *et al*, Atala *et al*

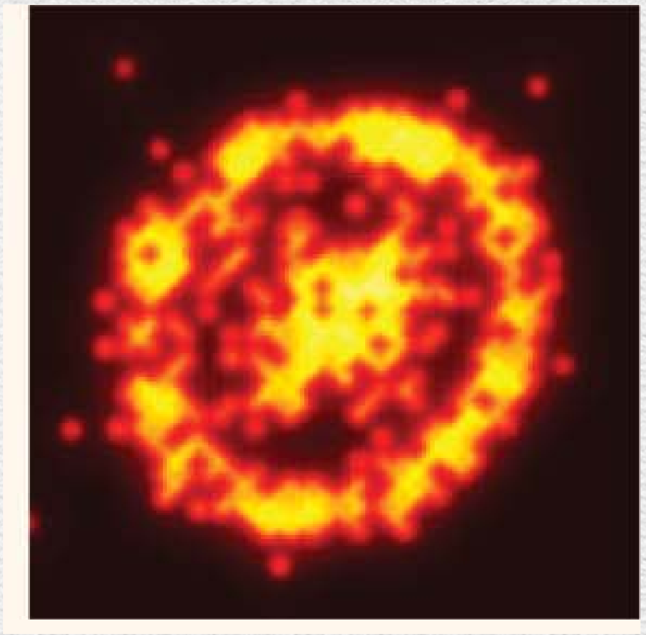


$$V_{\mathrm{OL}}(x) = V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \varphi\right)$$

Full (independent) dynamical control over V_1, V_2 and φ

in-situ imaging

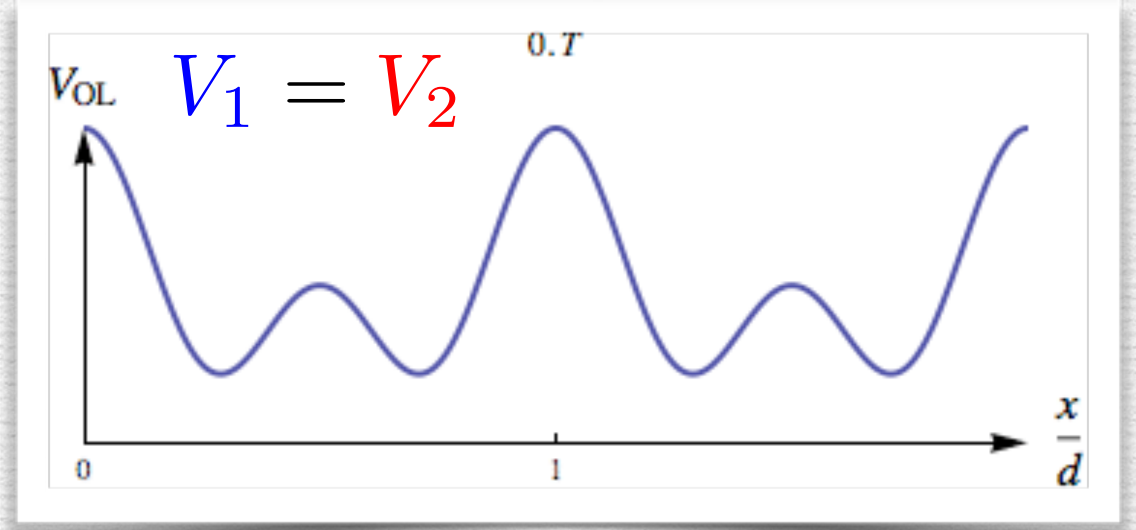
Gemelke, *et al*, Sherson *et al*, Bakr *et al*



Allows to measure exact quantization of pumped charge

1D pumping lattices

$$V_{\mathrm{OL}}(x,t) = V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \frac{\pi t}{T}\right)$$



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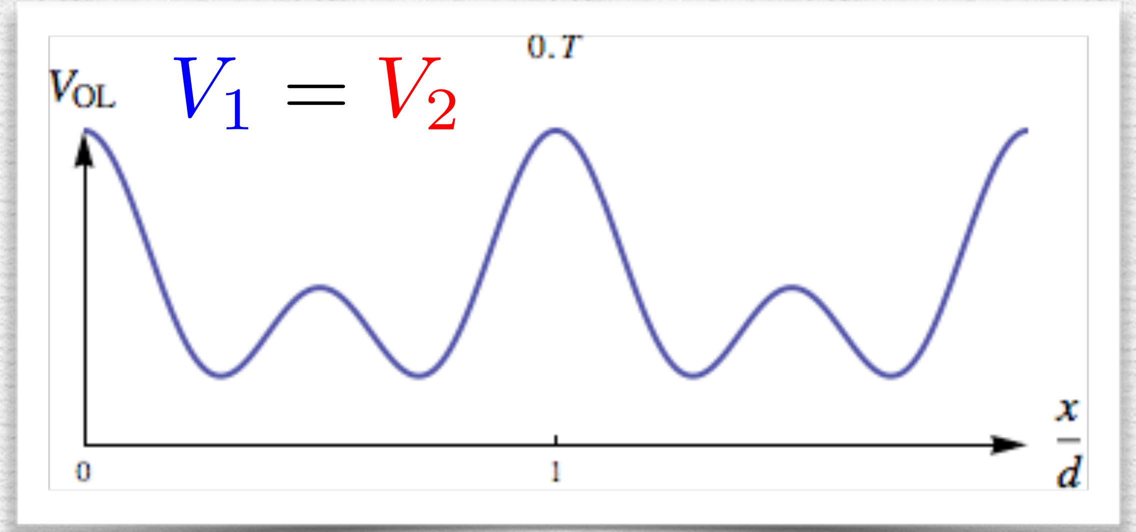

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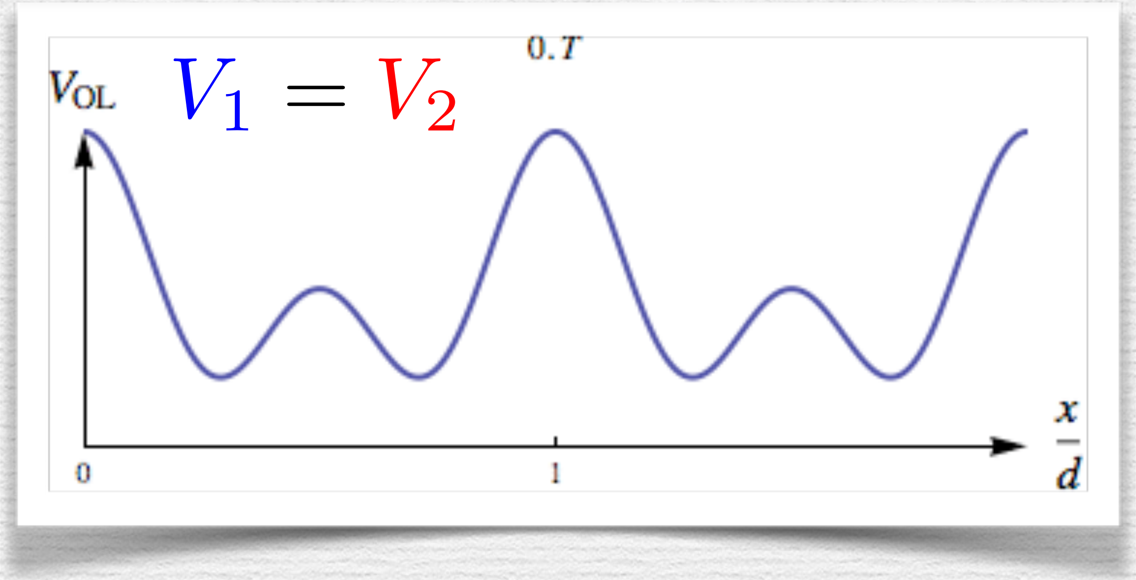

0 A = B — A = B

Su, Schrieffer, Heeger, 1979

T/2 A — B = A — B

1D pumping lattices

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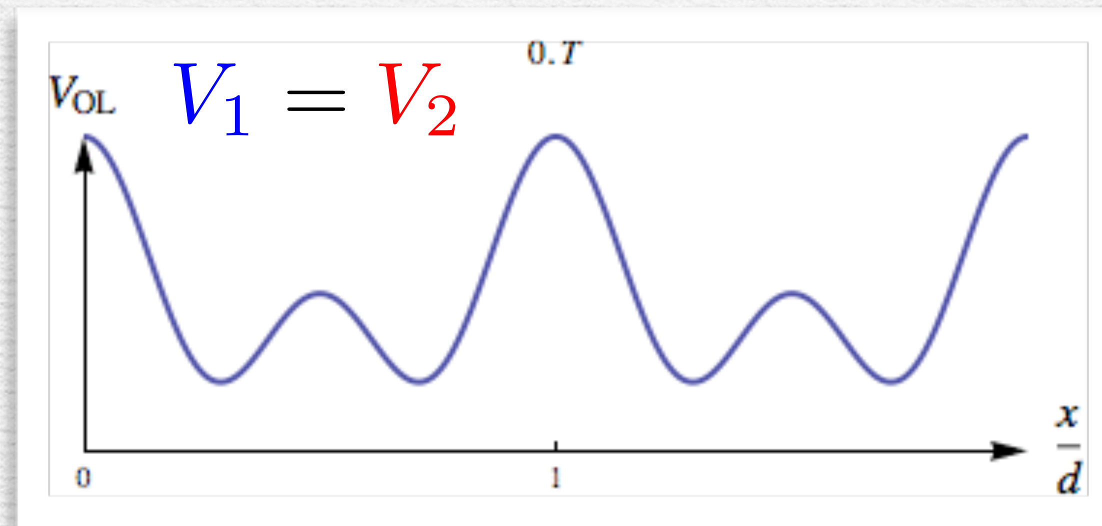


0 A ═══ B ─── A ═══ B

T/4 A ----- B ----- A ----- B

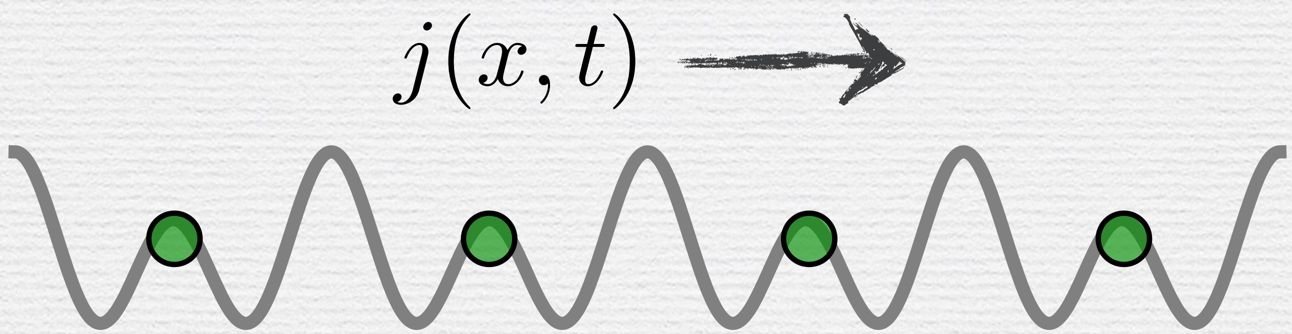
T/2 A ─── B ═══ A ─── B

3T/4 A ----- B ----- A ----- B

Su, Schrieffer, Heeger, 1979

Rice, Mele, 1982

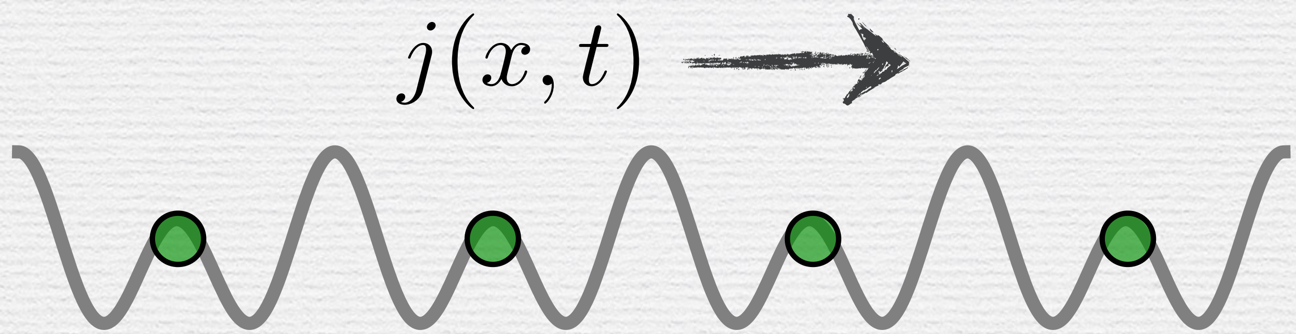
Pumping dynamics

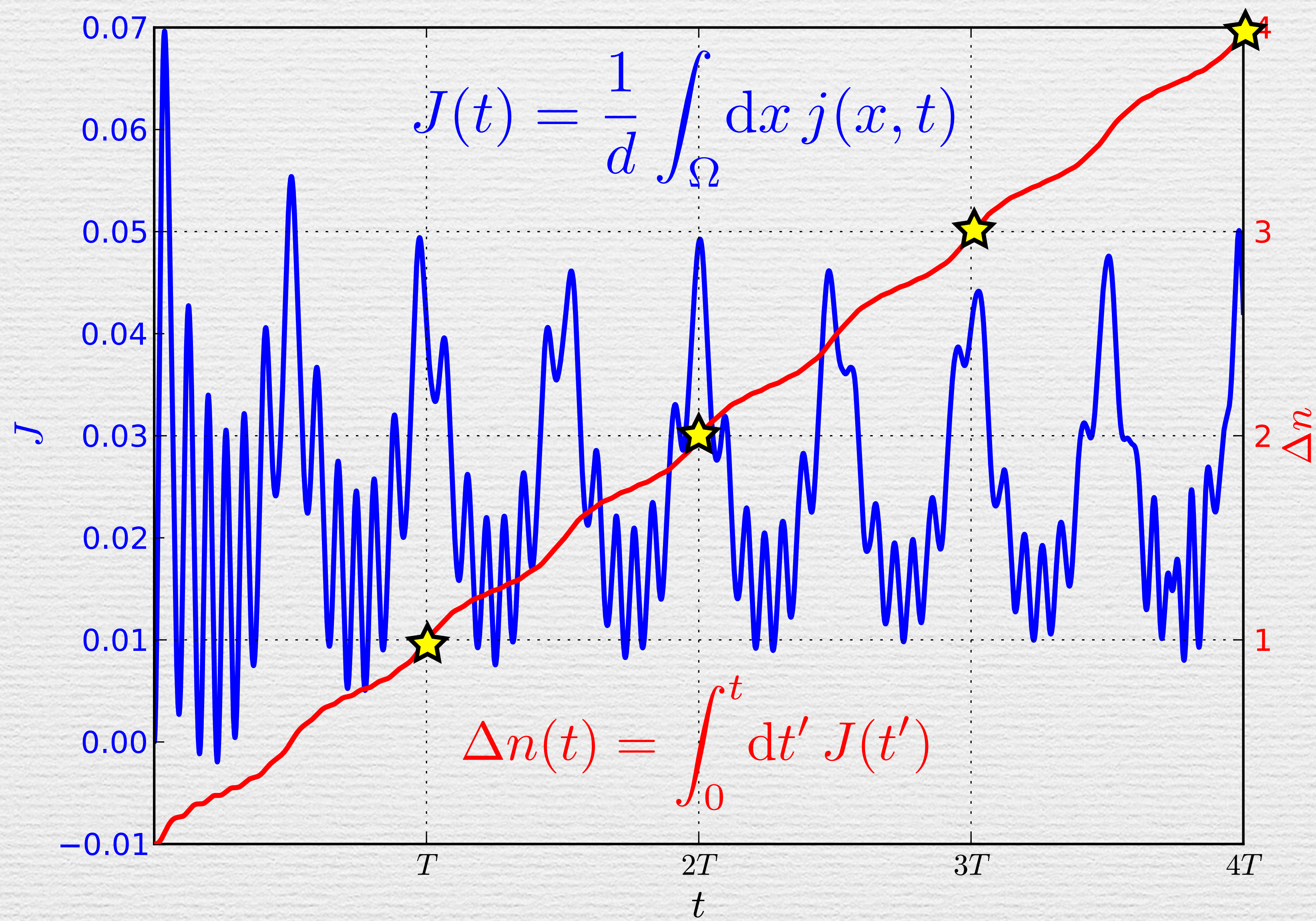


$$H(x,t) = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{OL}}(x,t)$$

$$i\frac{\partial}{\partial t}|\Psi\rangle = H(x,t)|\Psi\rangle$$

Pumping dynamics

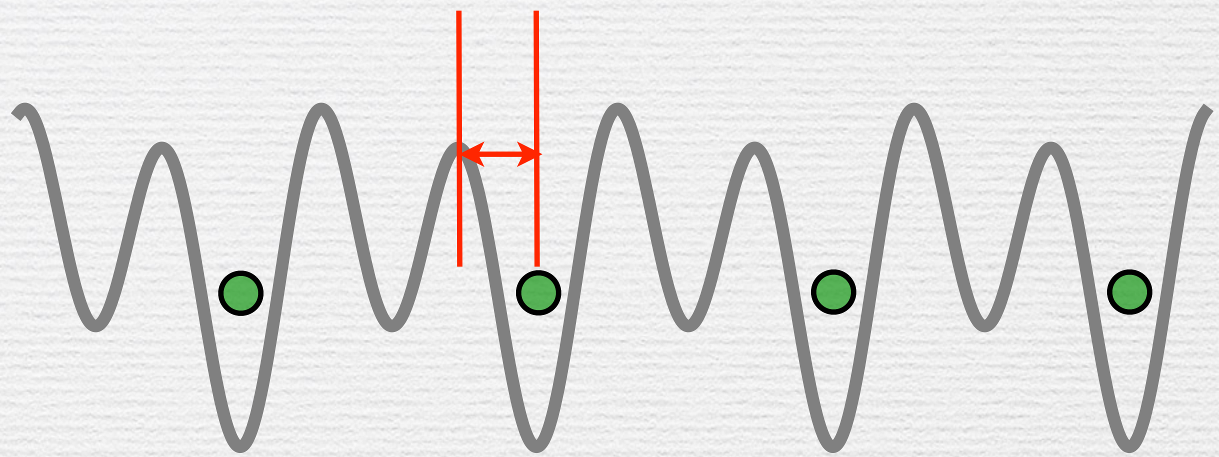




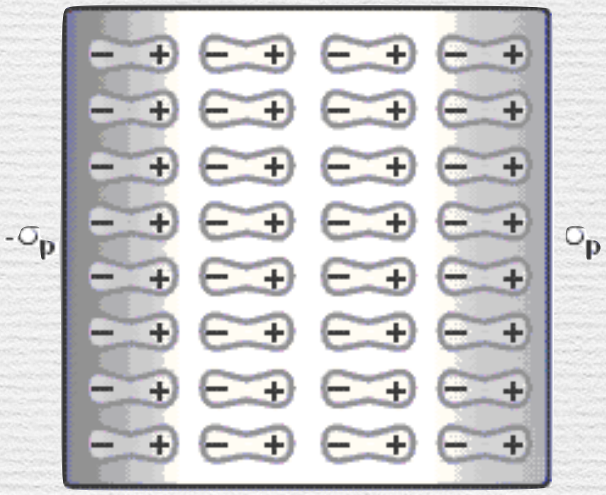
Polarization and Berry phase

Resta, King-Smith, Vanderbilt, ...

$$\text{“}x = i\partial_{k_x}\text{”}$$



$$P(t) = \int_0^{2\pi} \frac{\mathrm{d}k_x}{2\pi i} \langle u(k_x,t)|\partial_{k_x}|u(k_x,t)\rangle$$



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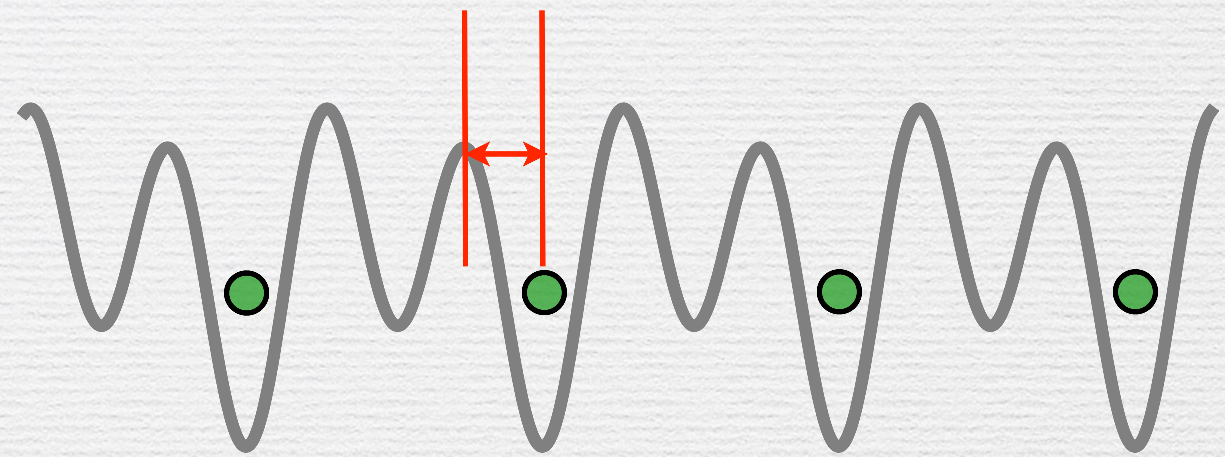

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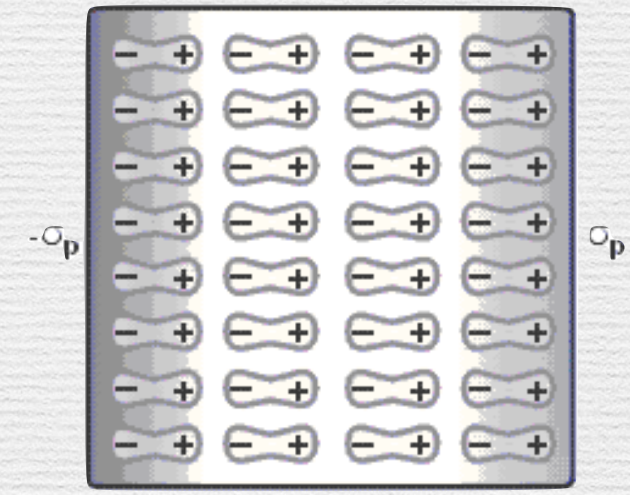


Change of Polarization

$$\Delta P = \int_0^T \mathrm{d}P$$

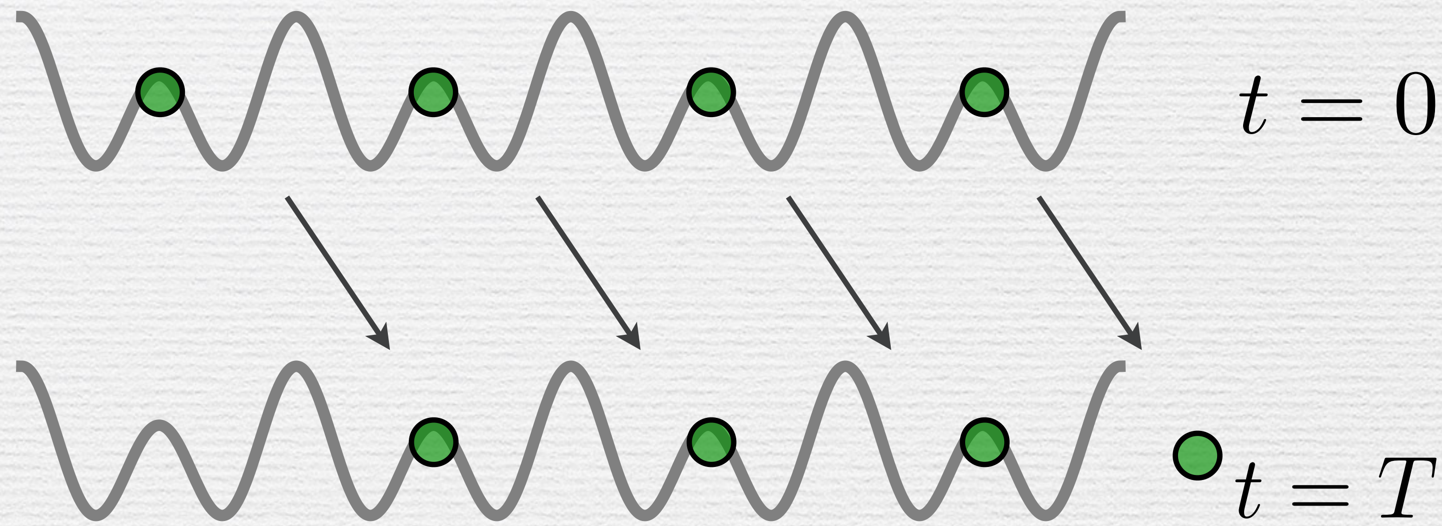
Berry Curvature

$$= \frac{1}{2\pi i}\int_0^T \int_0^{2\pi} \mathrm{d}t\,\mathrm{d}k_x \boxed{\left(\langle \partial_t u|\partial_{k_x} u\rangle - h.c.\right)}$$

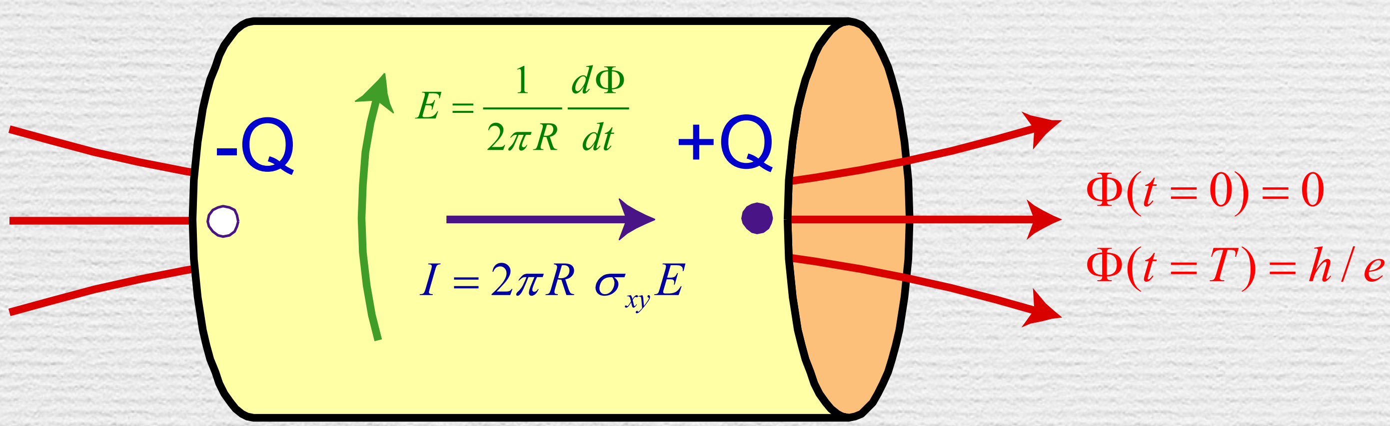
$= $ Chern number

Connection to IQHE

$$H(k_x, t) = H(k_x, t + T)$$

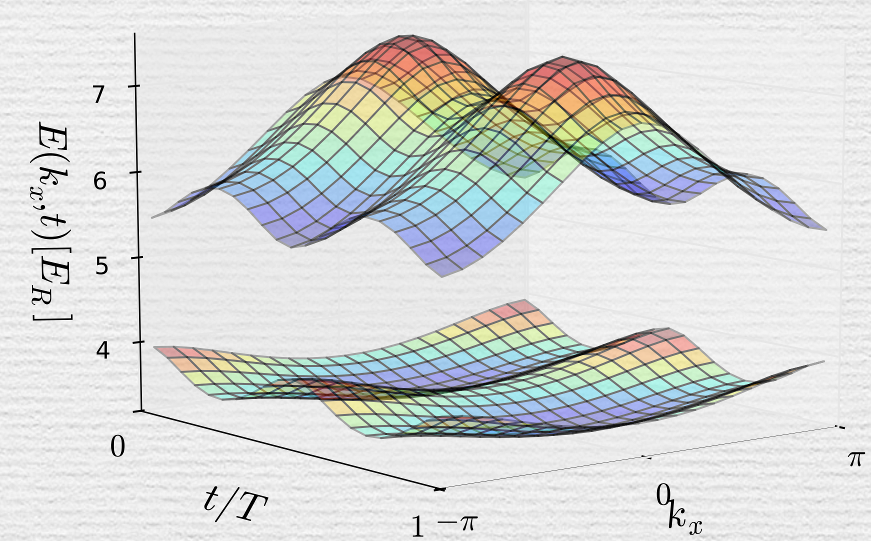


Adiabatically thread a quantum of magnetic flux through cylinder.



Laughlin, 1981

$V_1 = 4E_R \quad V_2 = 4E_R$



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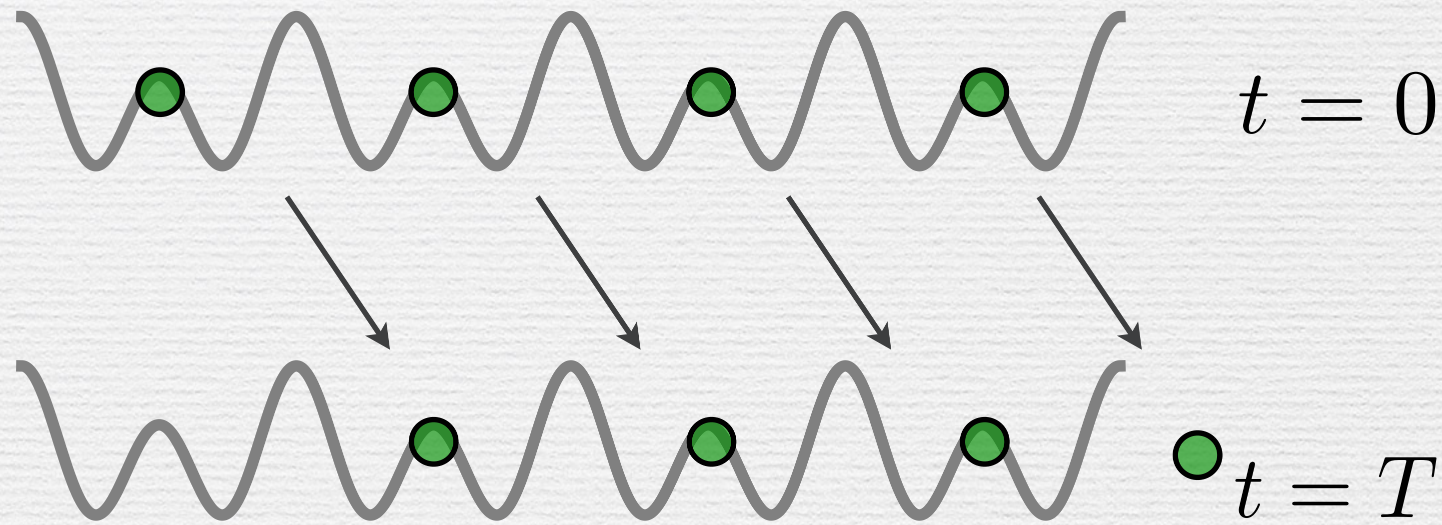

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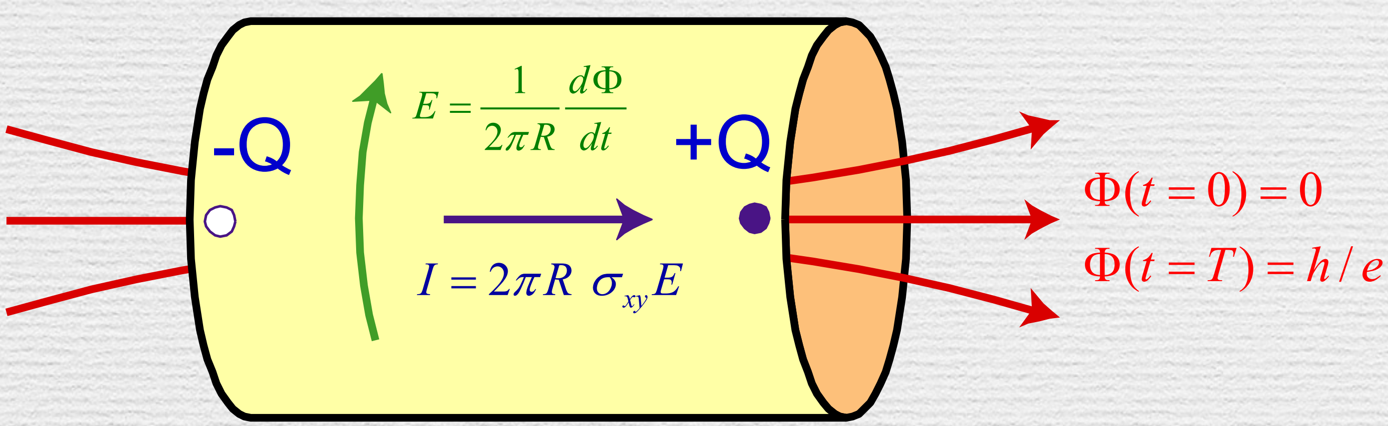

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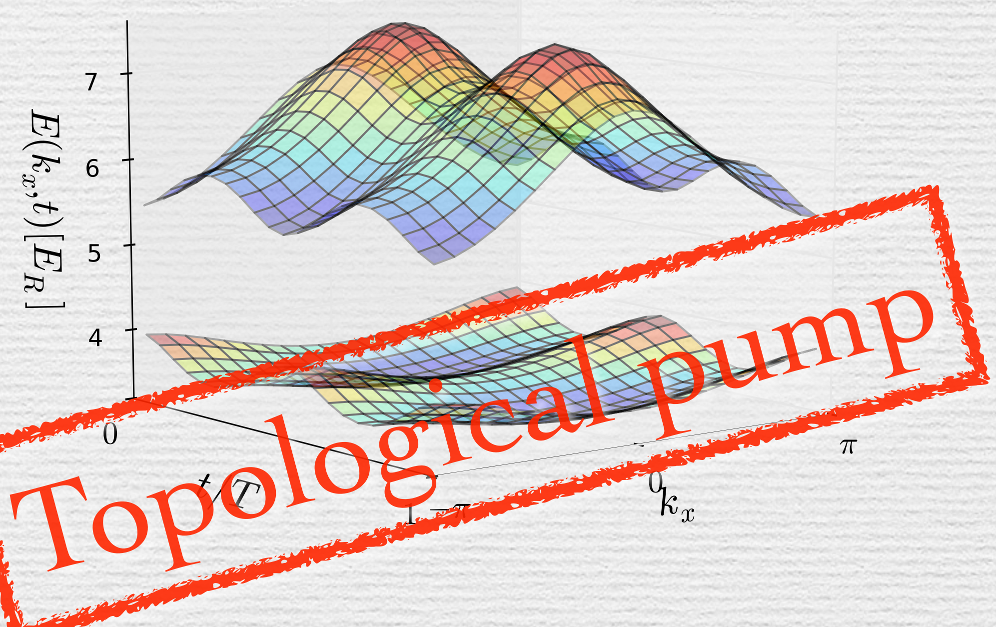

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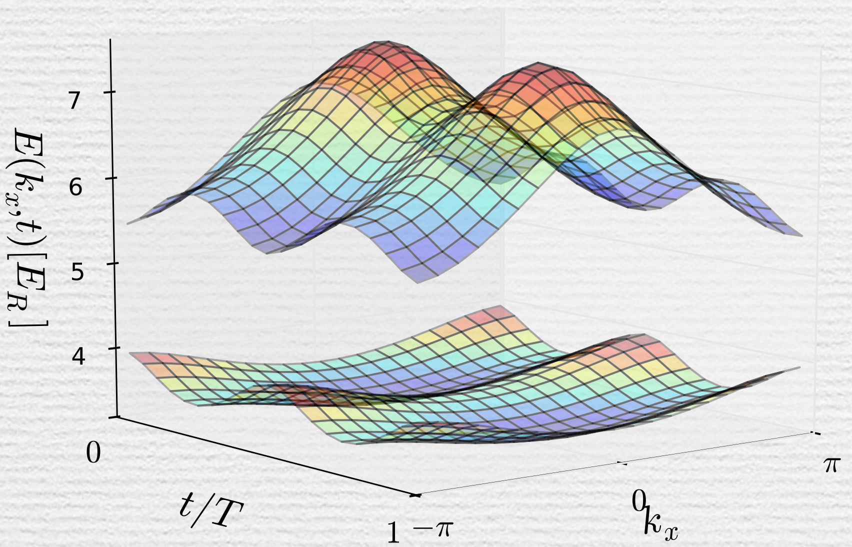

$$V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \frac{\pi t}{T}\right)$$

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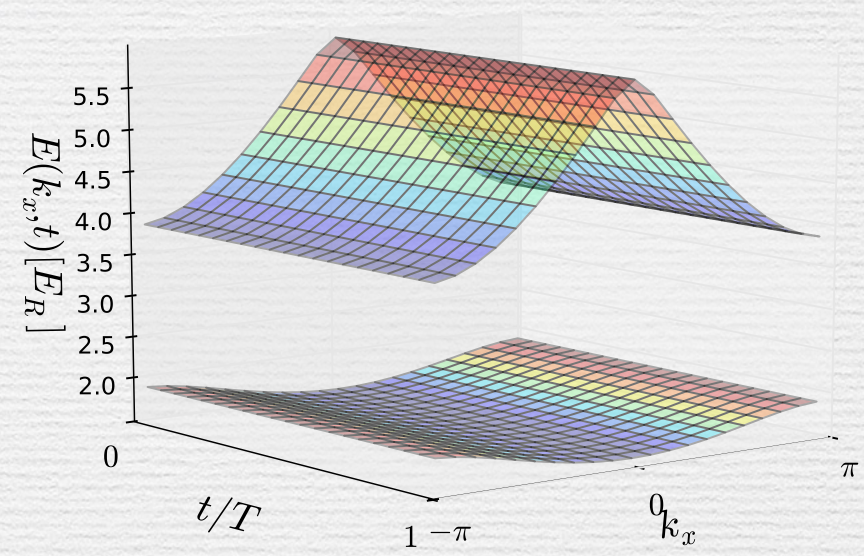

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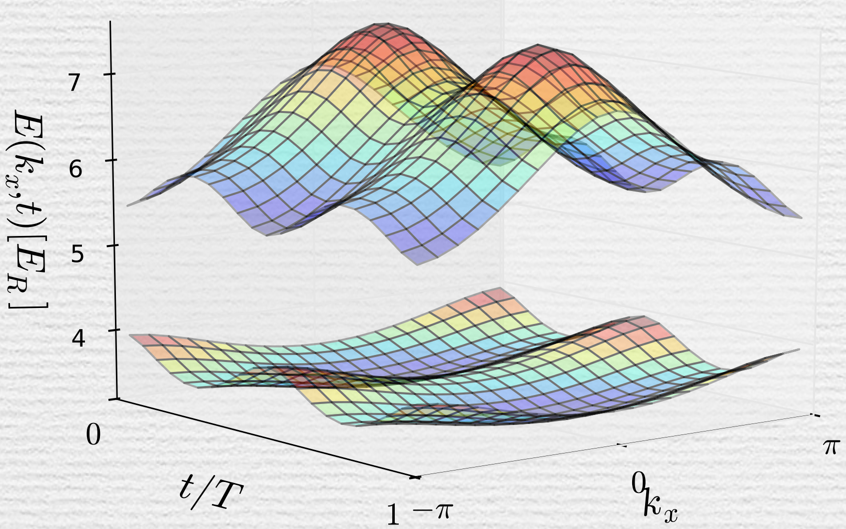

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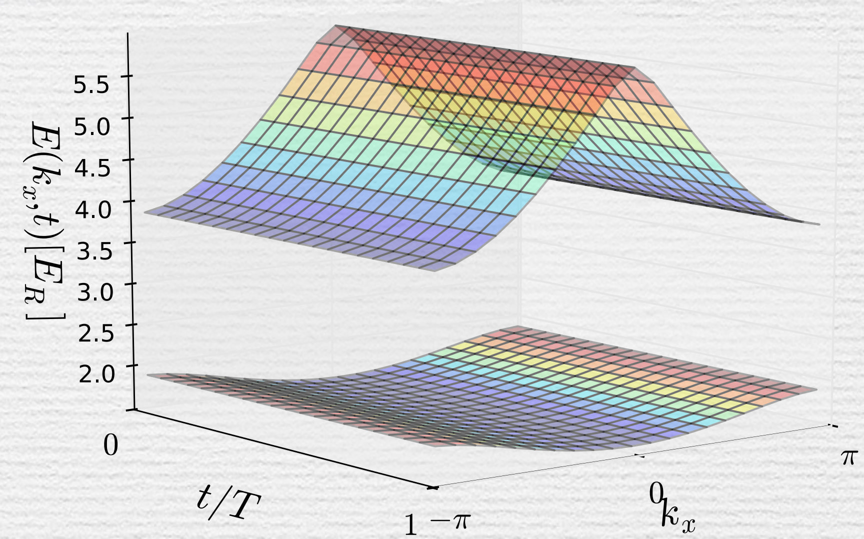

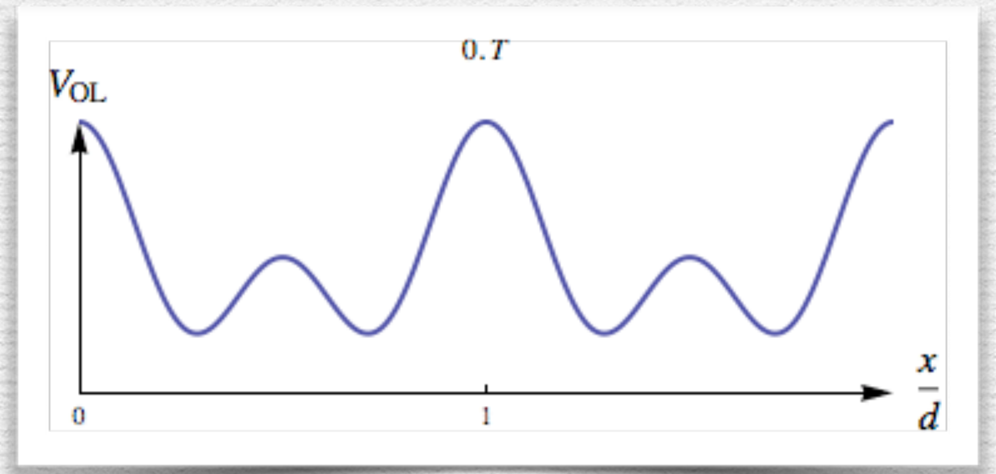

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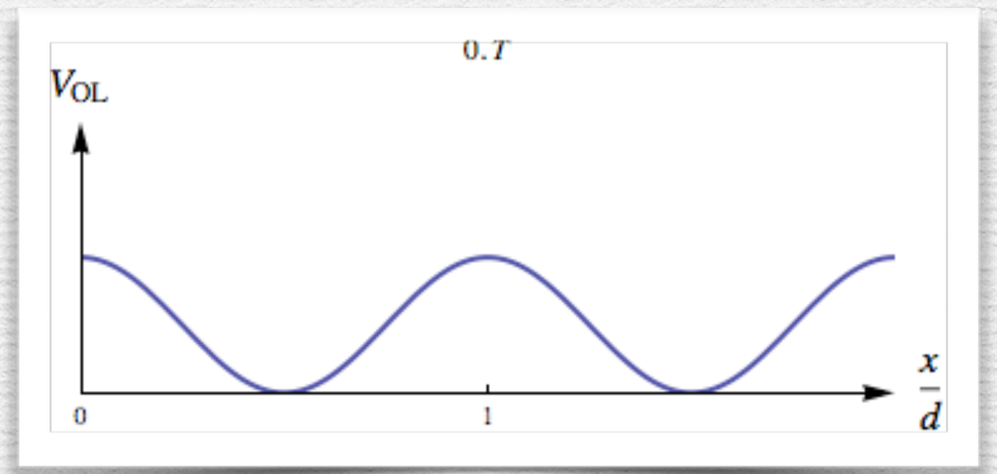
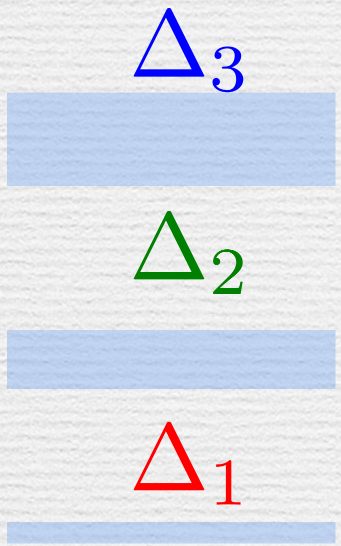
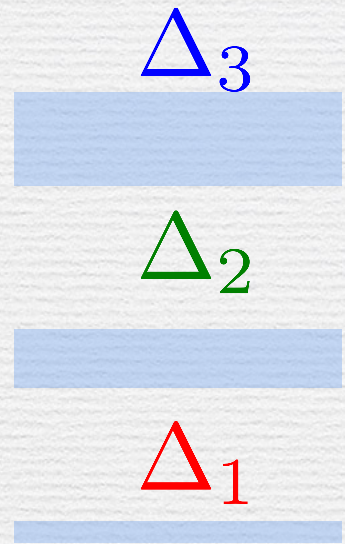


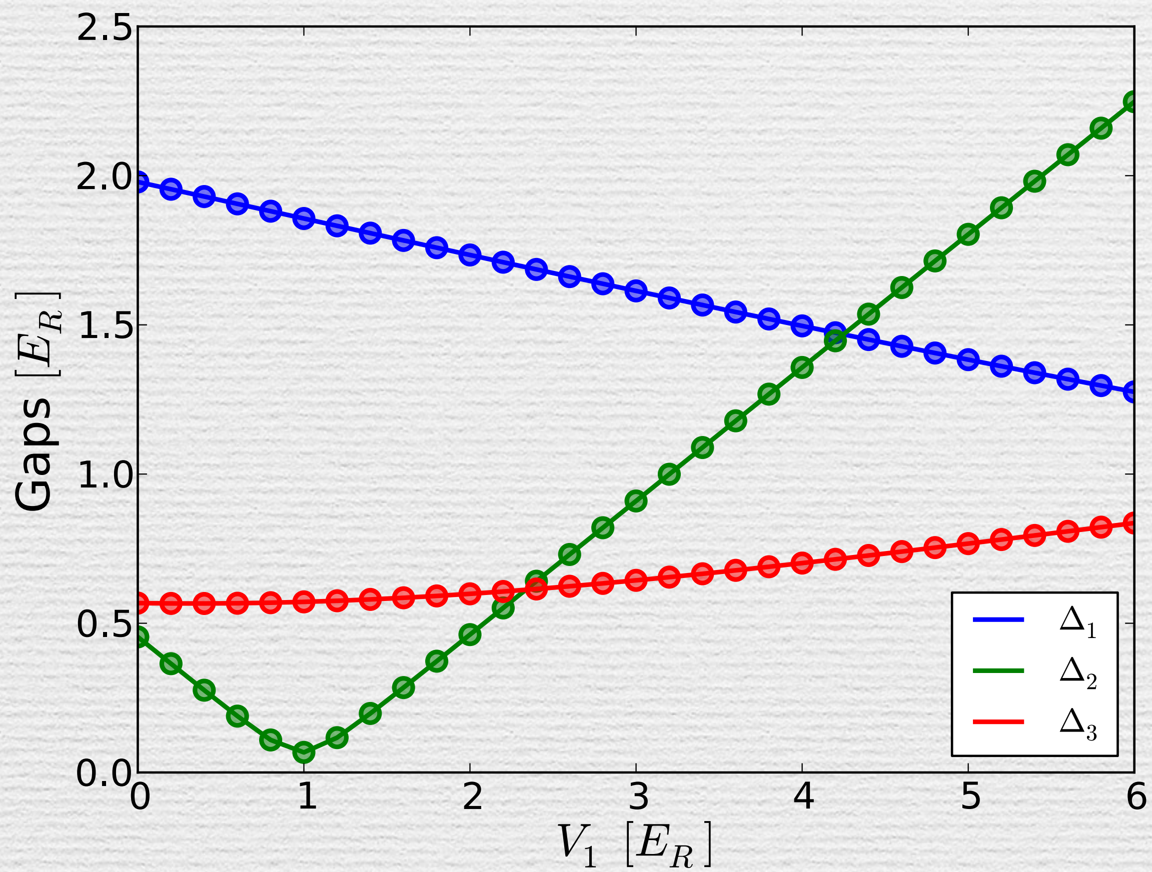
Higher bands



Sliding



Double-well



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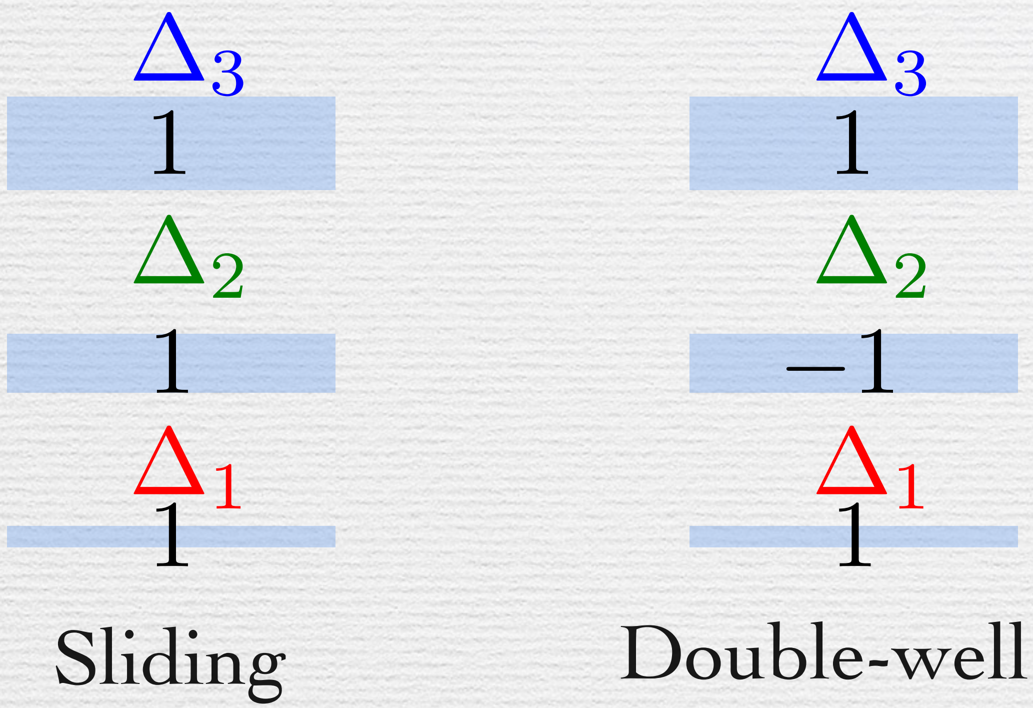

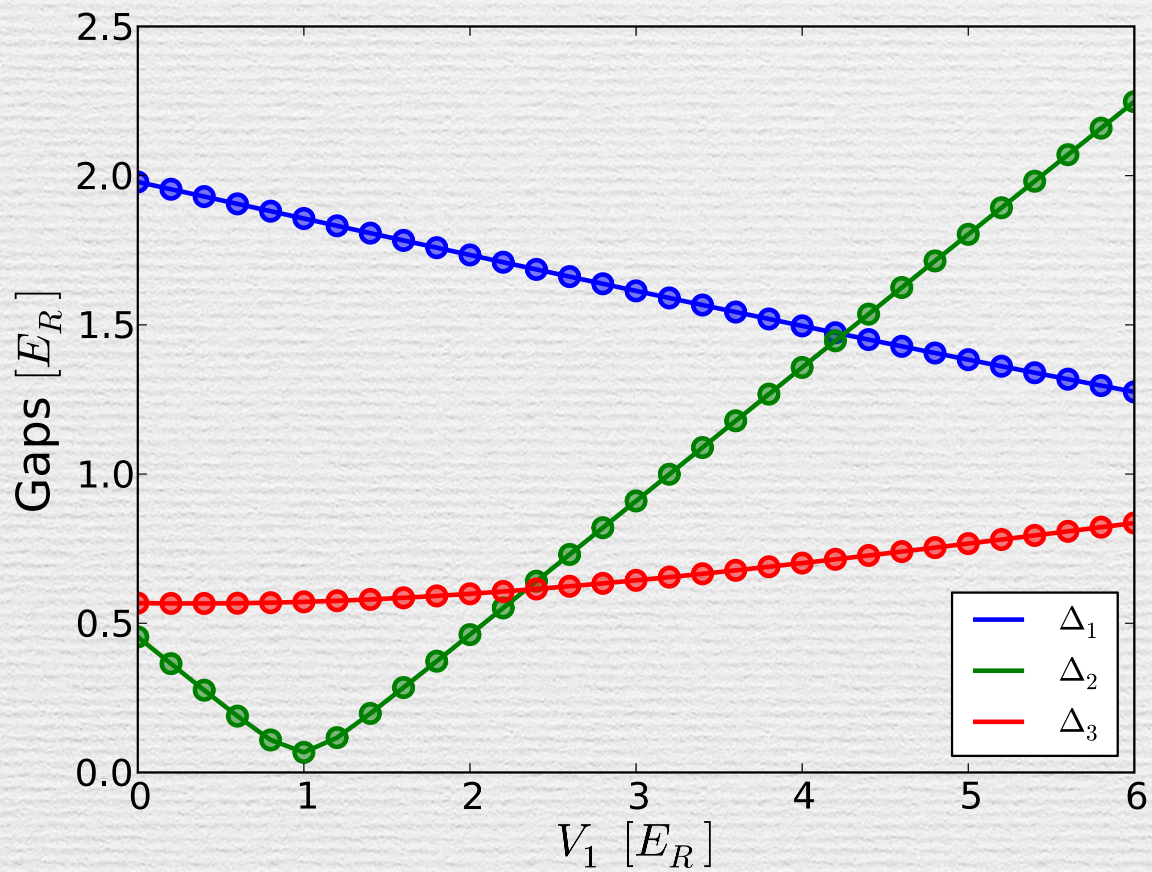

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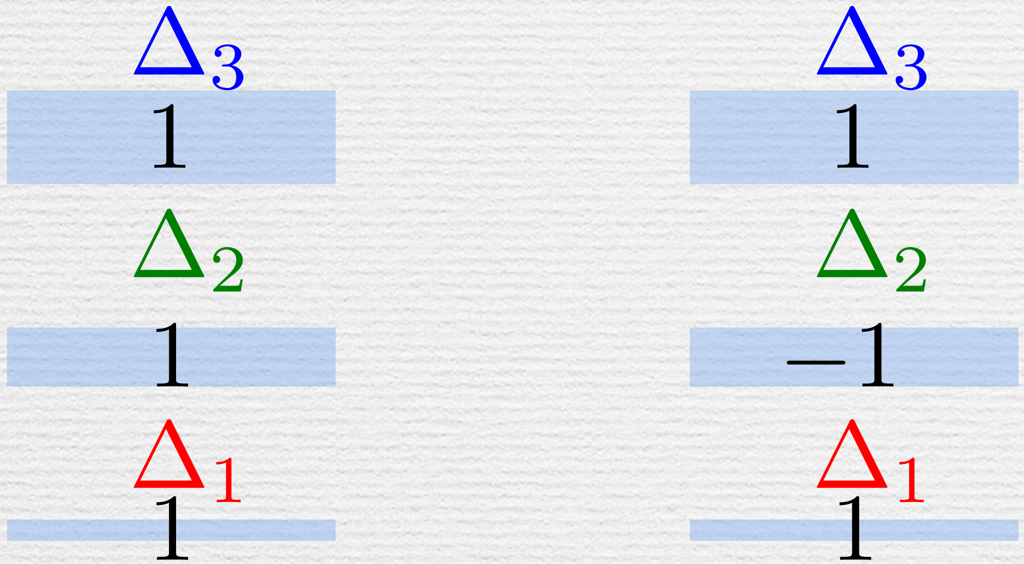

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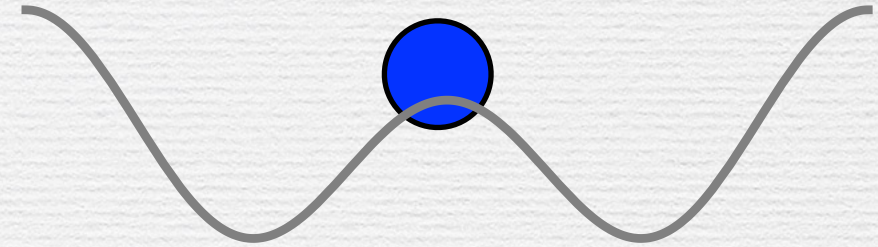

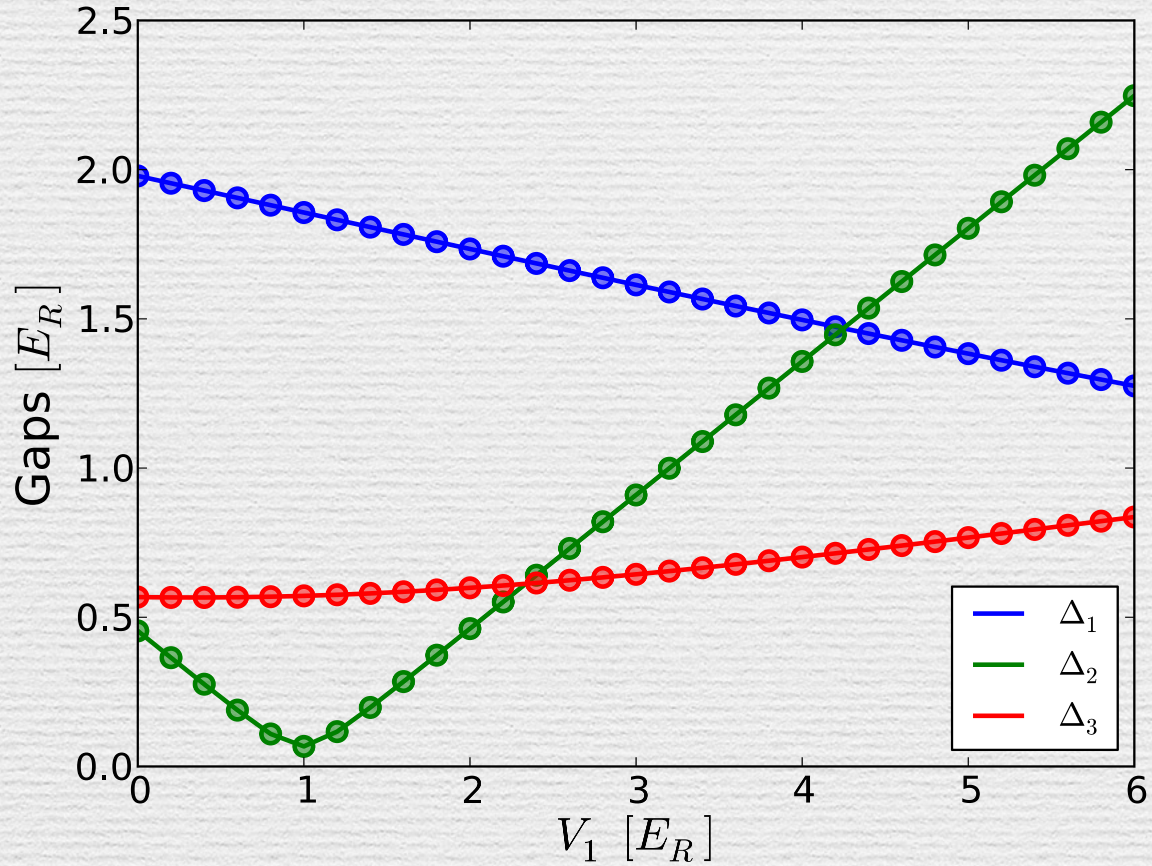

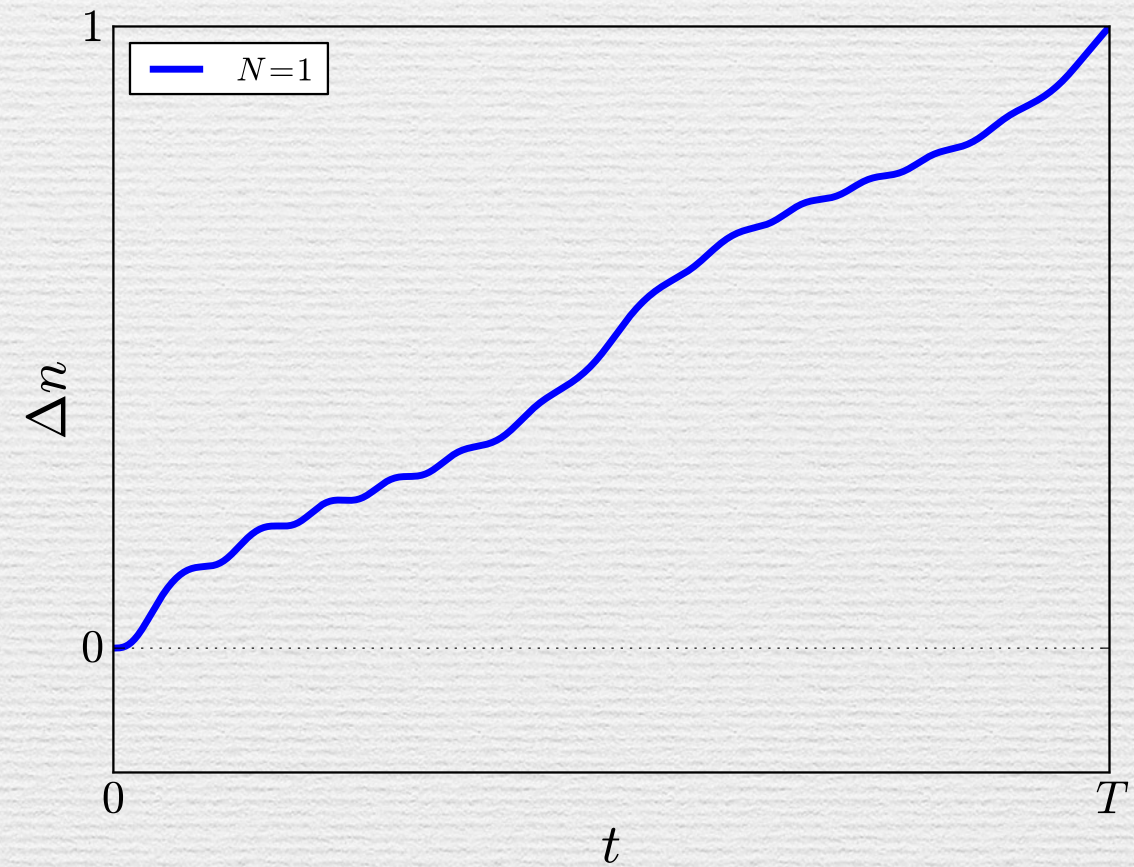

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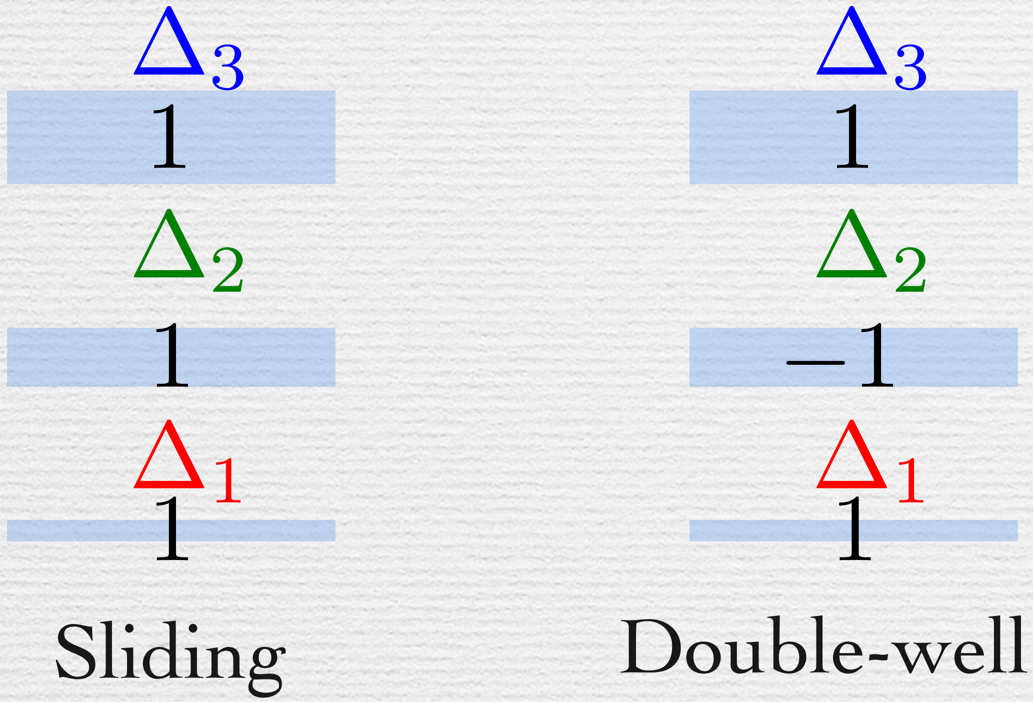

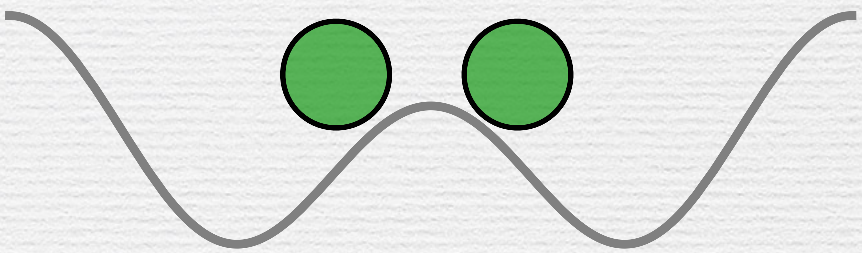

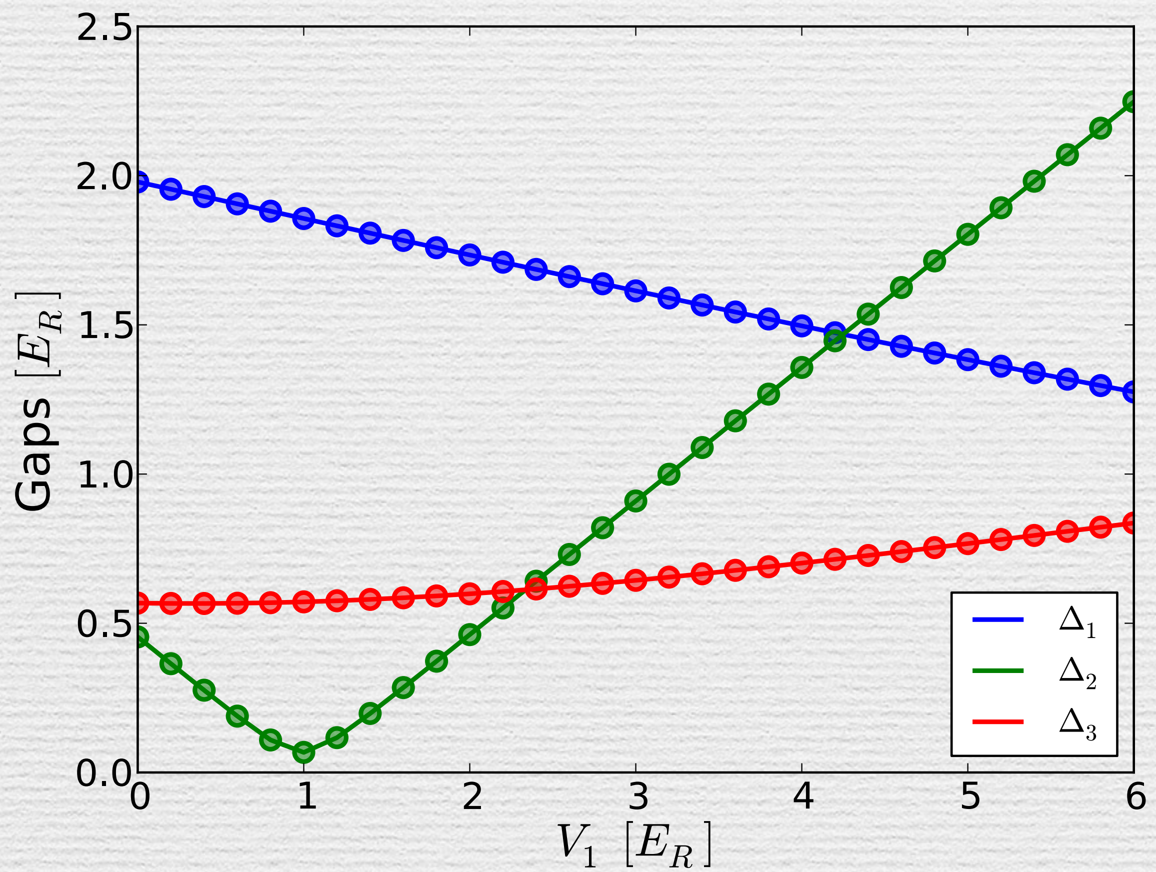

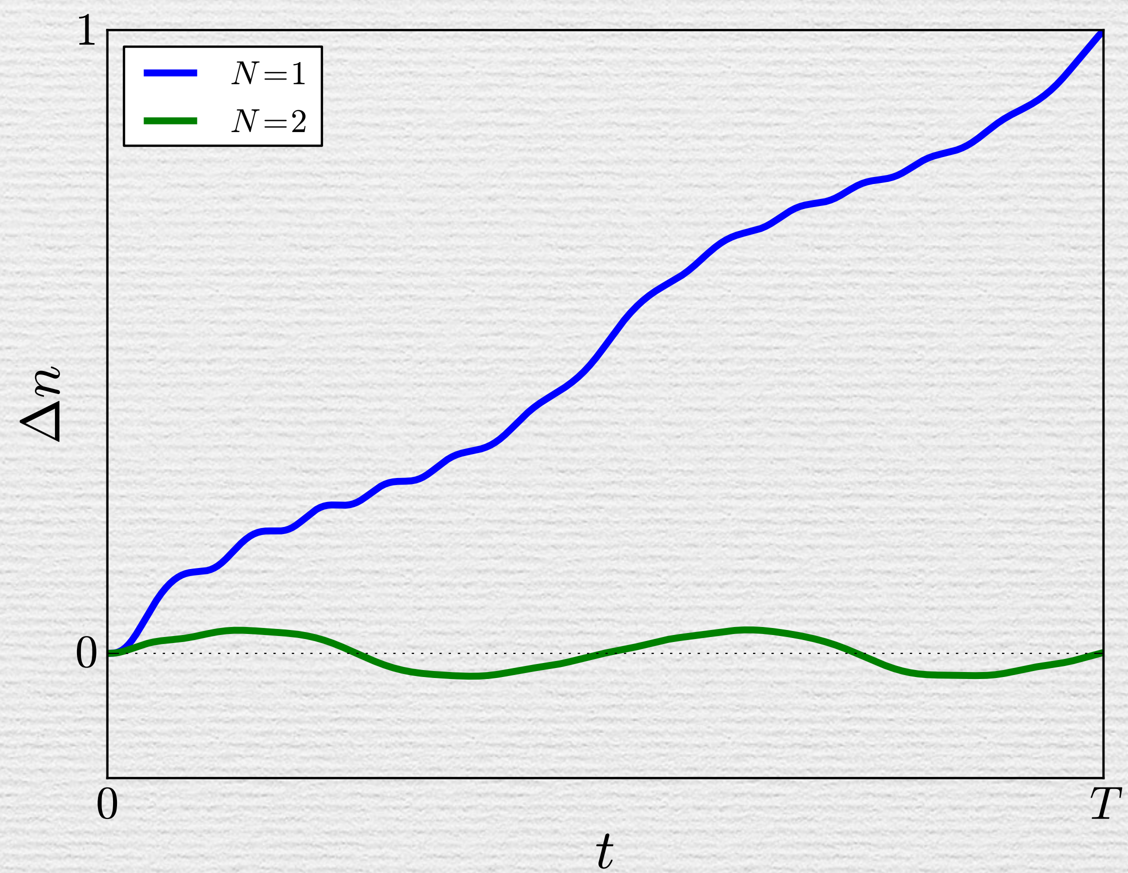

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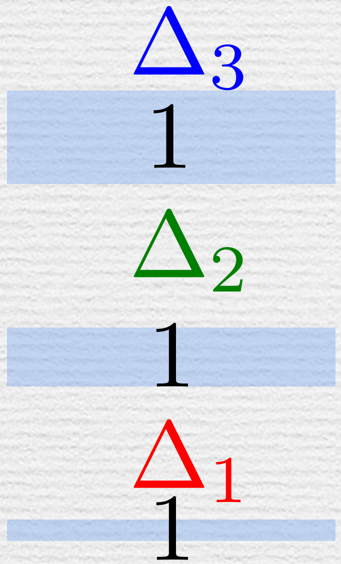

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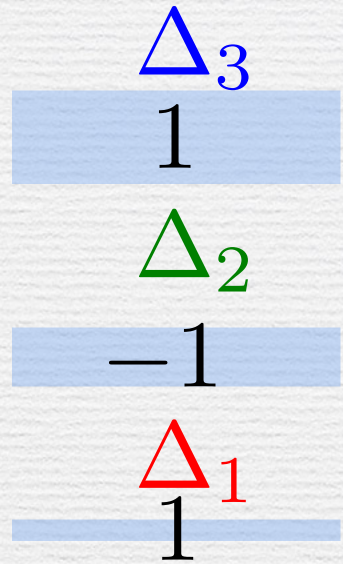

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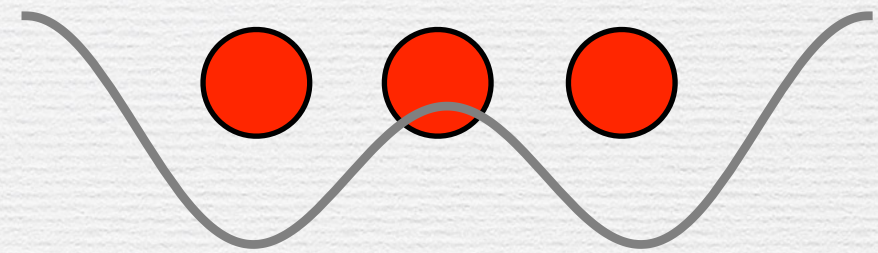
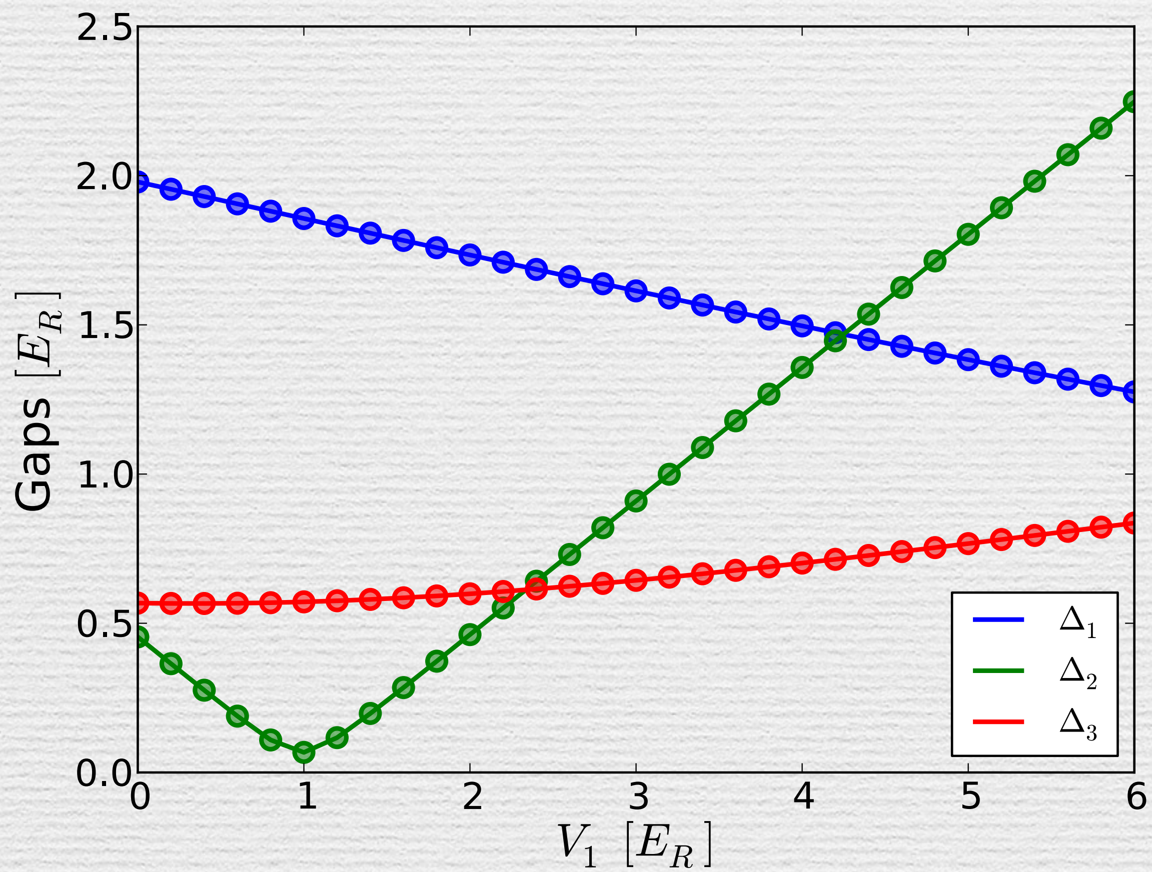
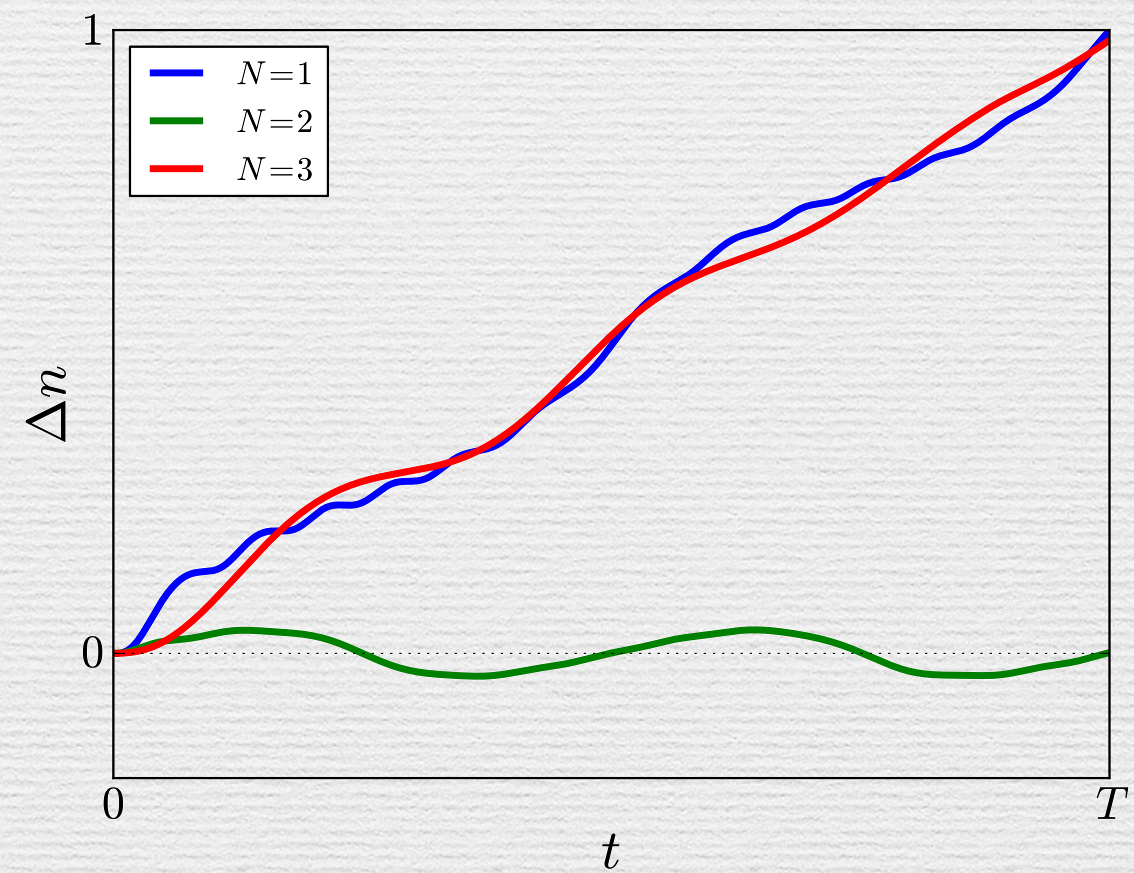
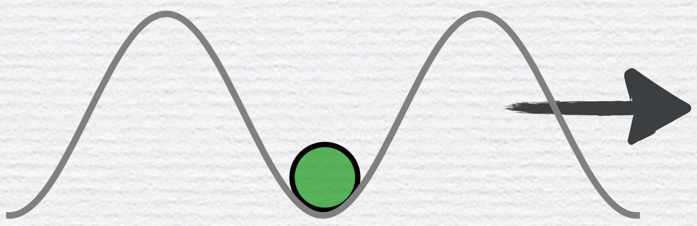






Classical dynamics

$$\boxed{m\ddot{x} = -\frac{\partial V_{\mathrm{OL}}(x,t)}{\partial x}}$$



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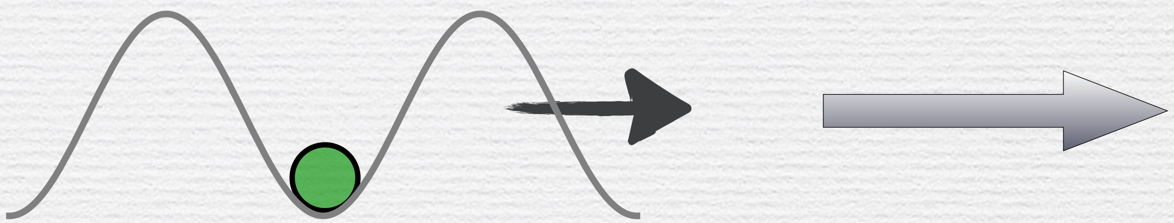

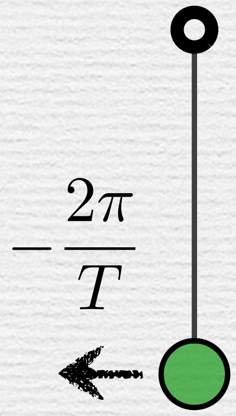

- Without V_1 term, pumping maps to a simple pendulum

Classical dynamics

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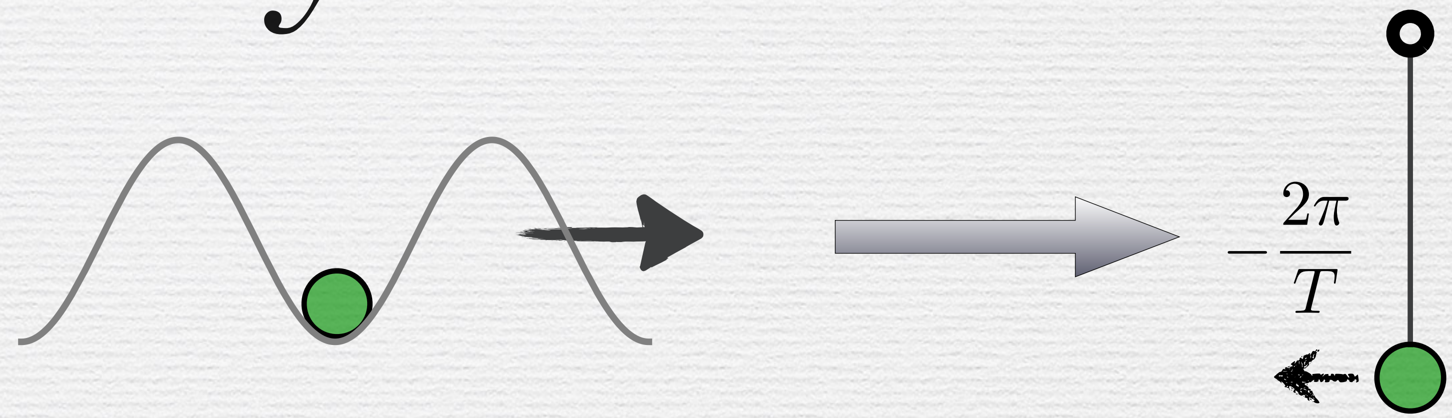
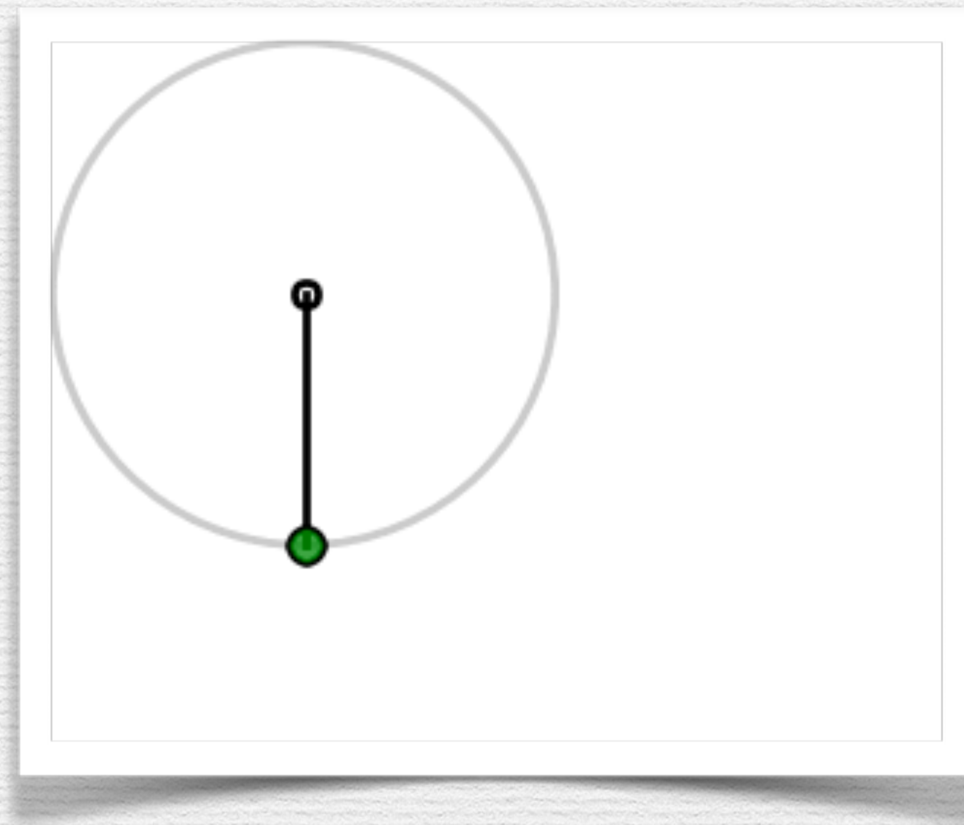


- Without V_1 term, pumping maps to a simple pendulum
  - Slow pumping-> small oscillations-> follows pumping



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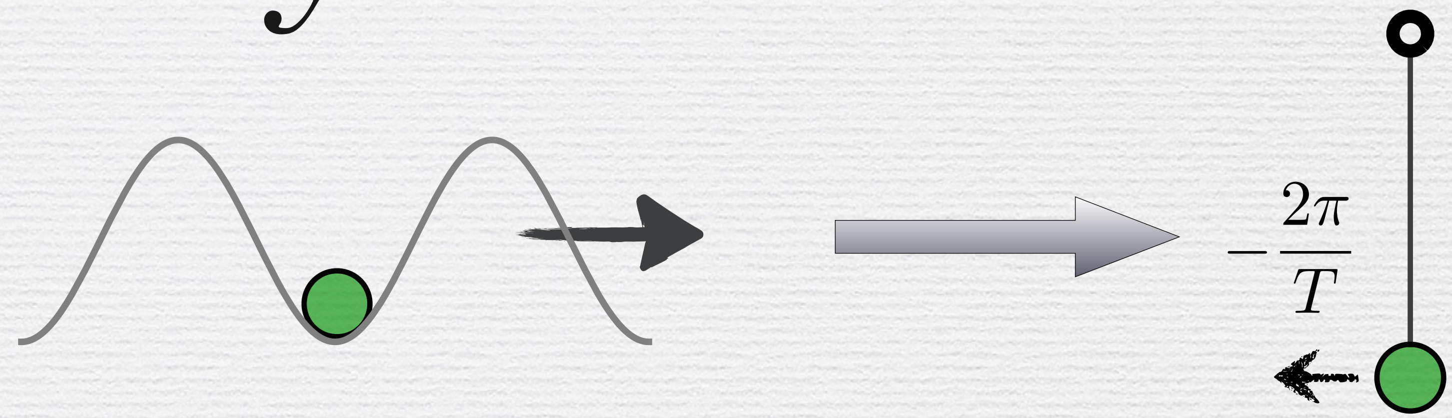

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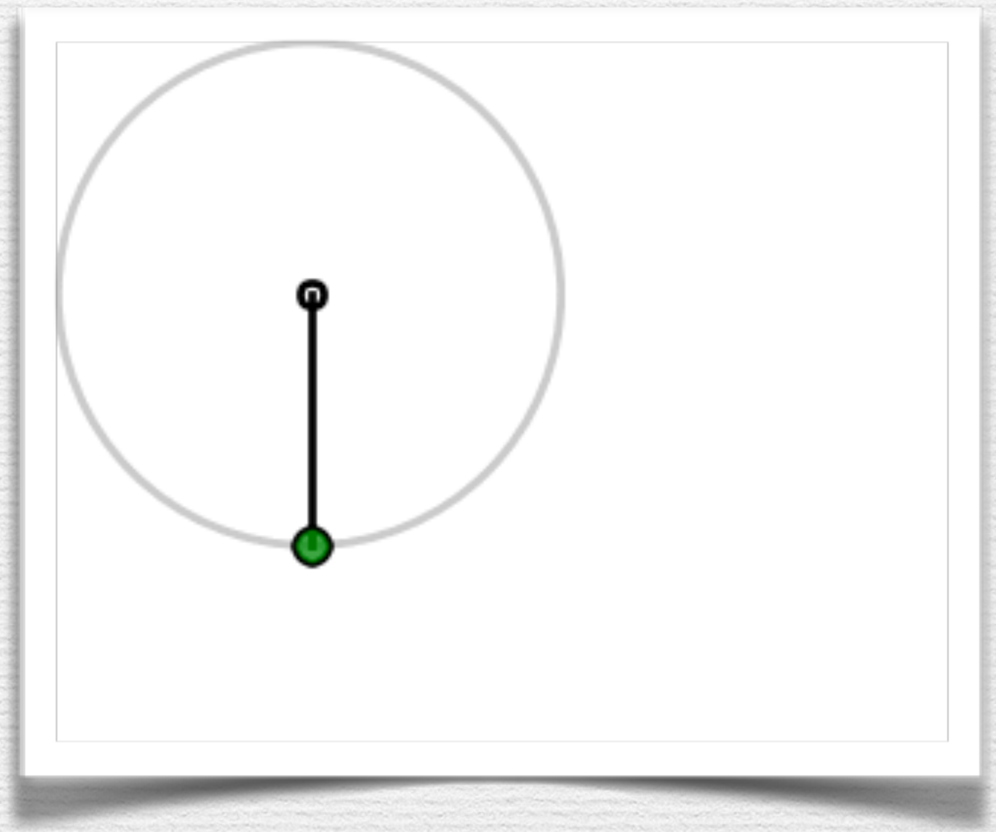
  - Fast pumping-> swings around pivot-> can not follow pumping



Classical dynamics

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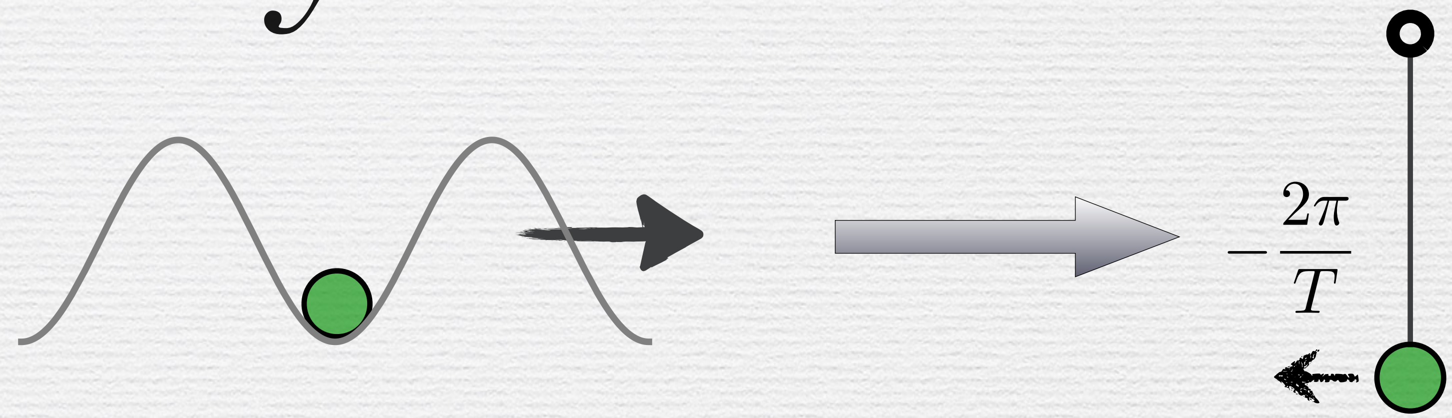
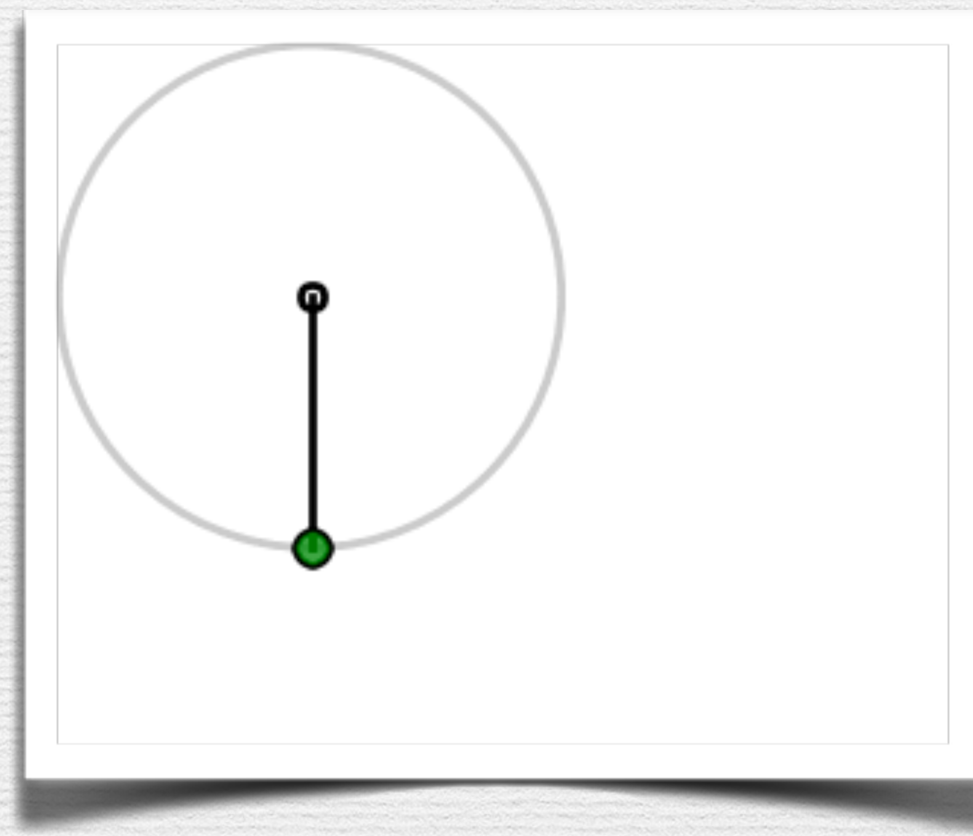


- Without V_1 term, pumping maps to a simple pendulum
  - Slow pumping-> small oscillations-> follows pumping
  - Fast pumping-> swings around pivot-> can not follow pumping
- V_1 term: Driven pendulum shows chaotic behavior.



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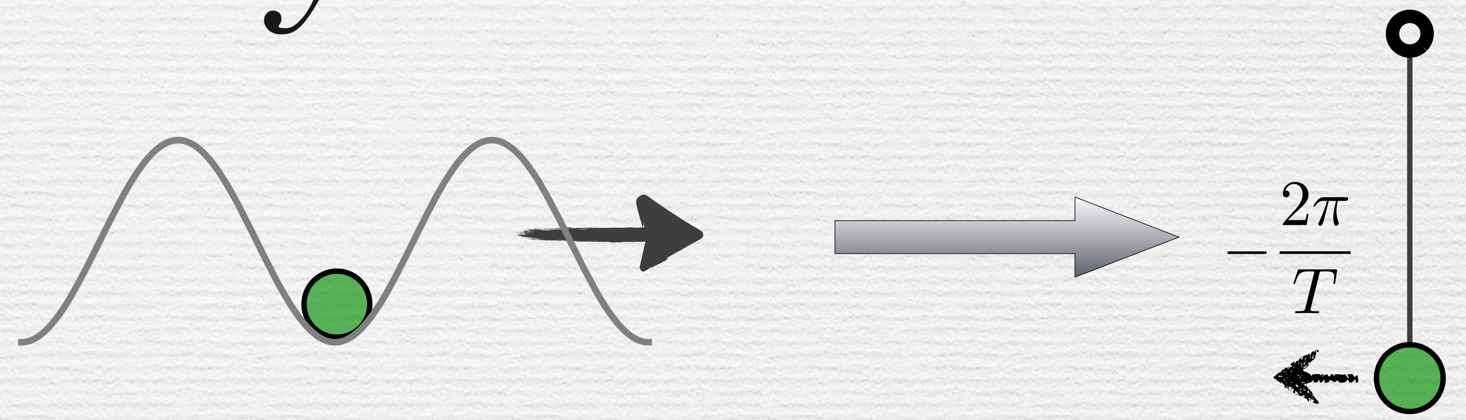

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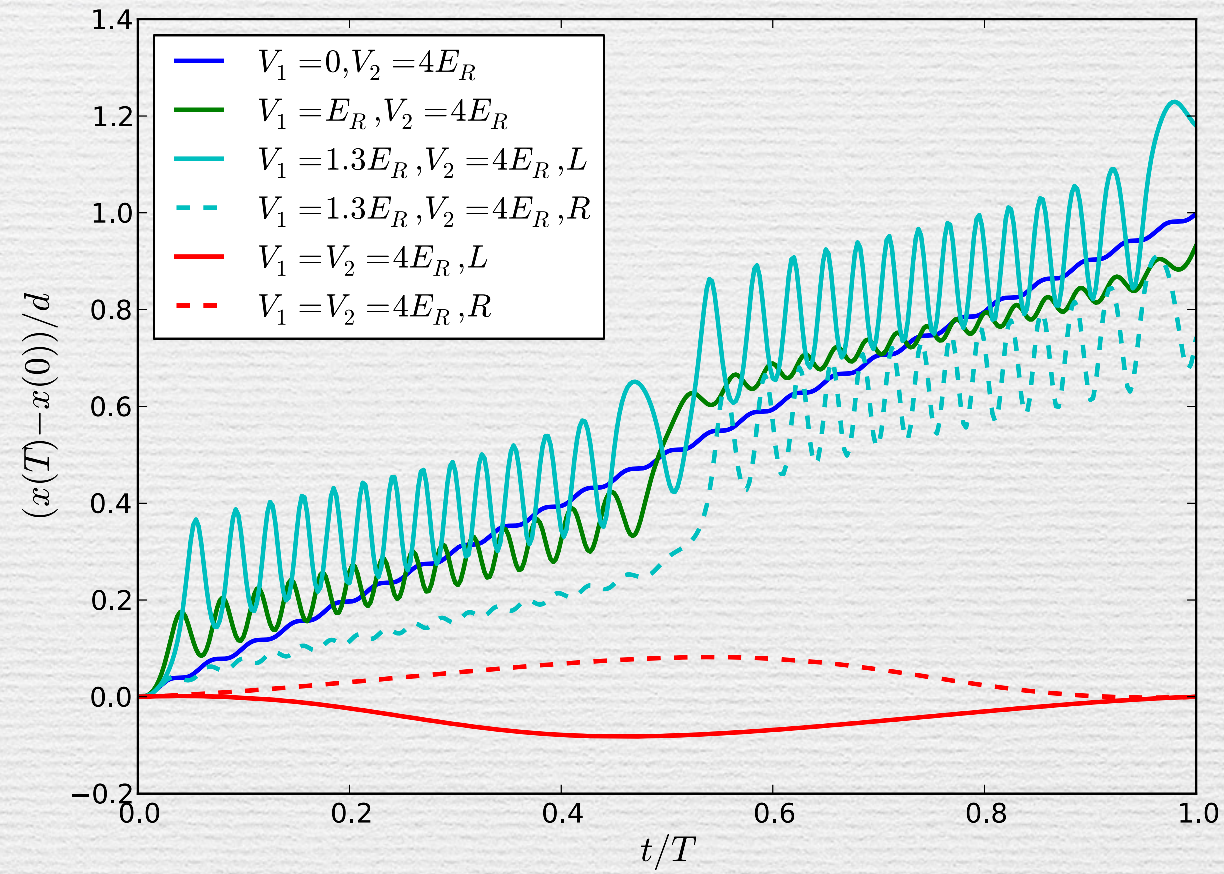
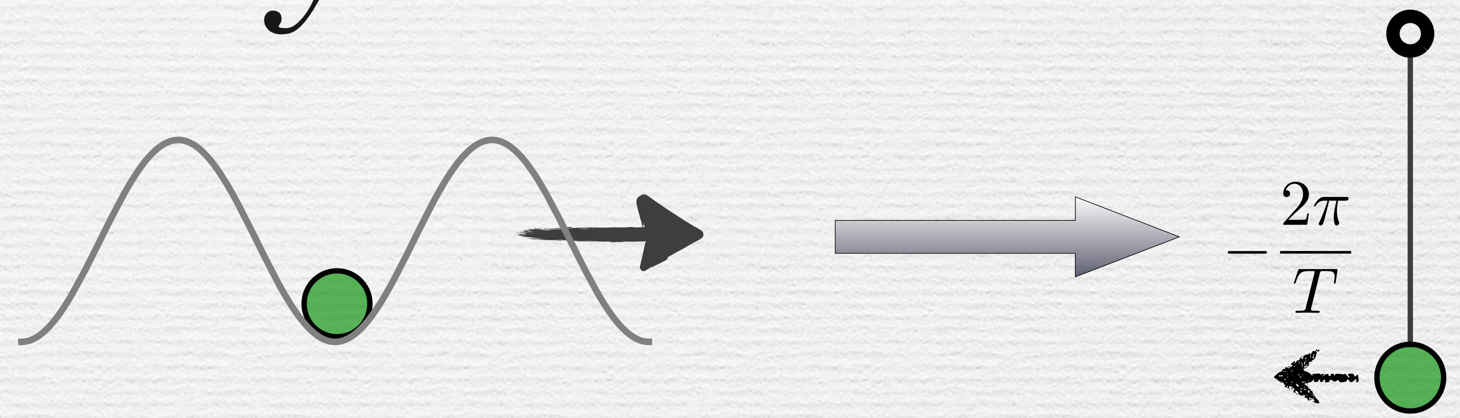
- V_1 term: Driven pendulum shows chaotic behavior.



Classical dynamics

$$m\ddot{x} = -\frac{\partial V_{\mathrm{OL}}(x,t)}{\partial x}$$



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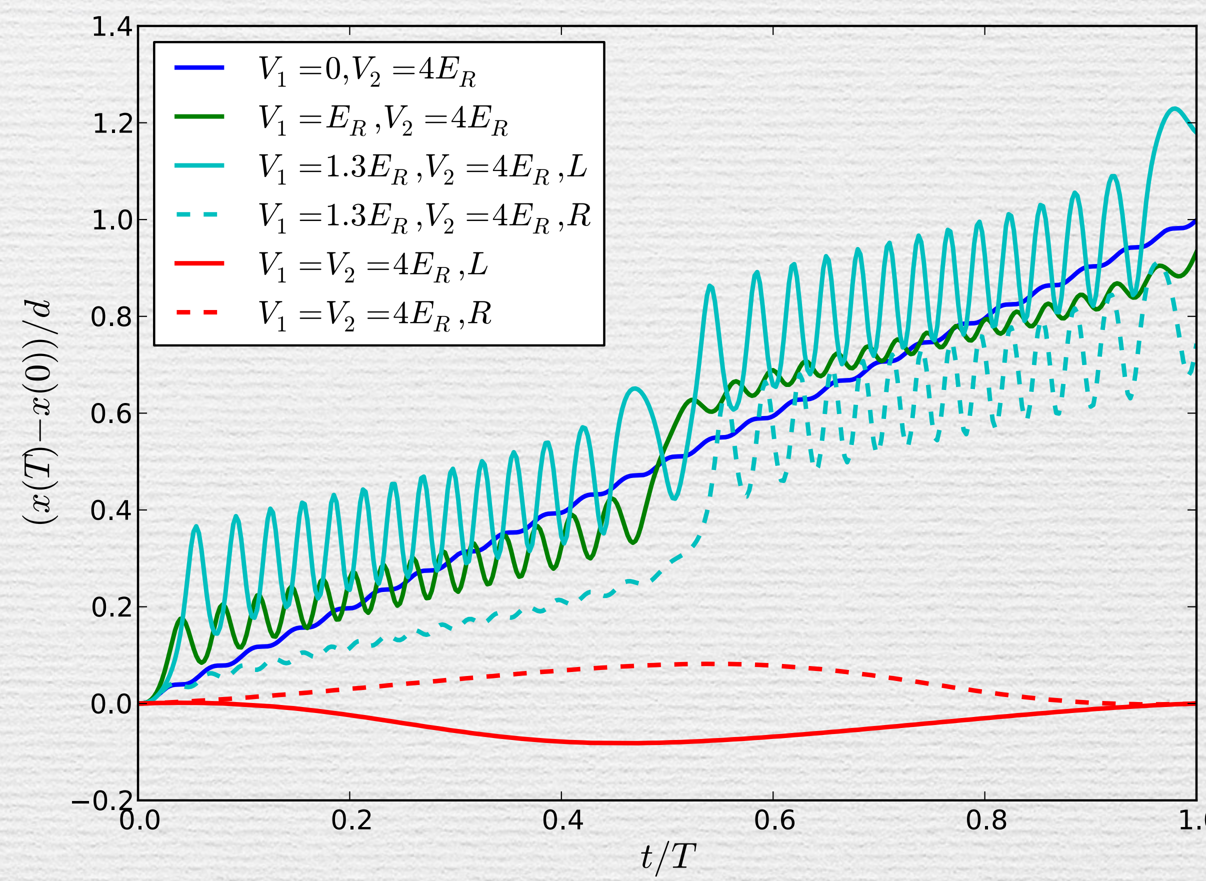


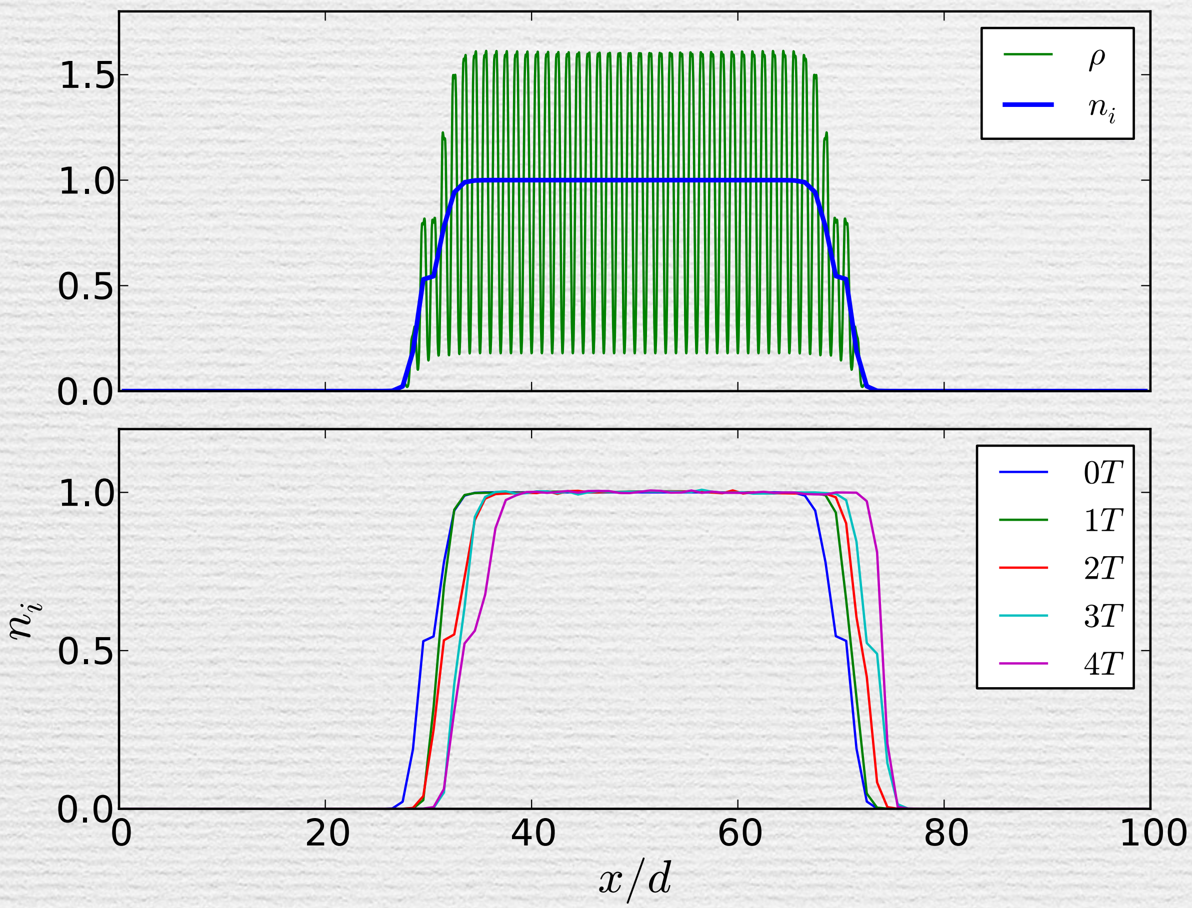
In general, classical pumped charge is not quantized

Practical issues

- Detection
- External trap
- Temperature effect
- Non-adiabatic effect

Trapping & Detection

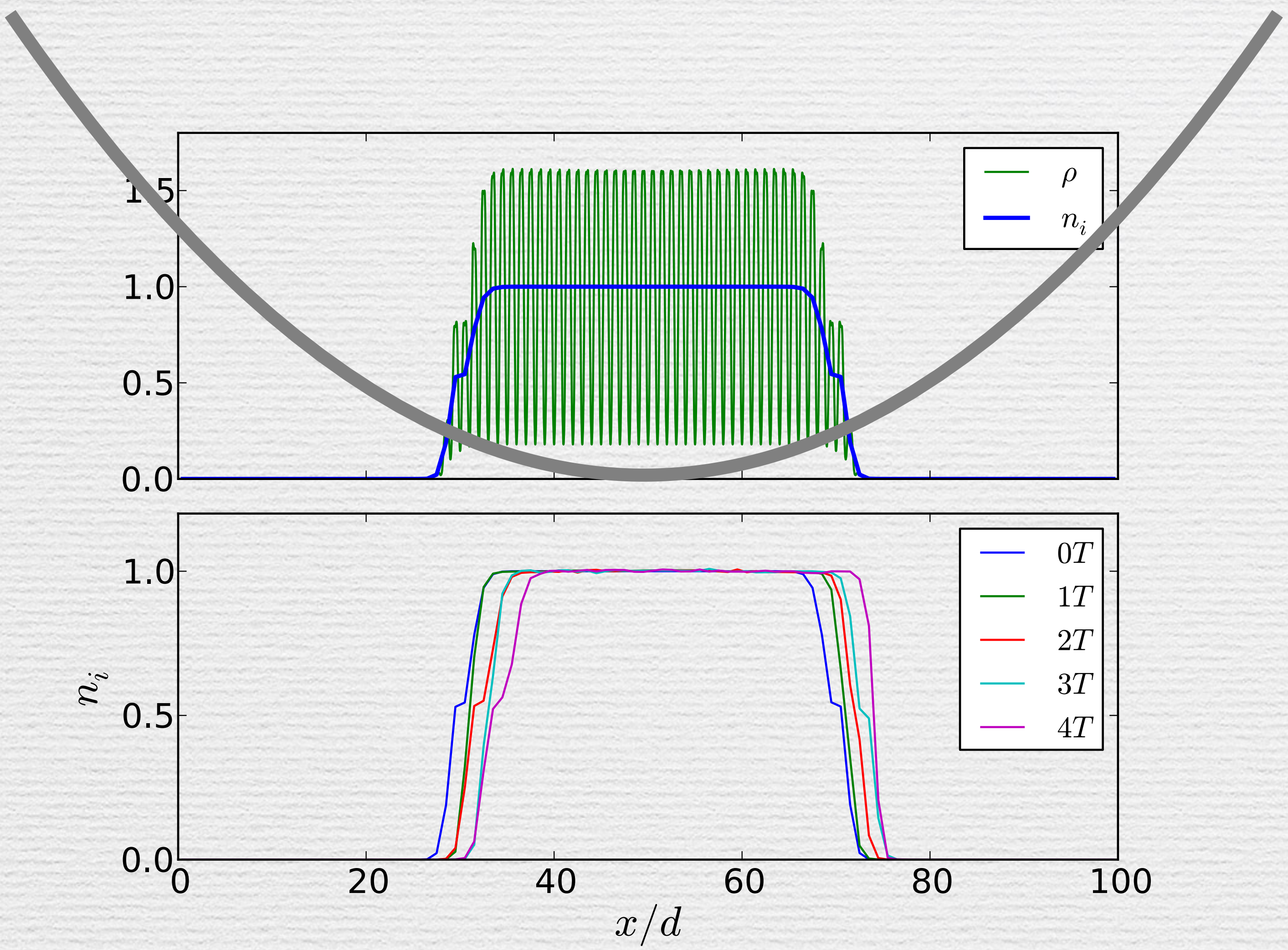
LW, Troyer and Dai, 1301.7435



$$\langle x \rangle / d = \Delta n$$

Trapping & Detection

LW, Troyer and Dai, 1301.7435



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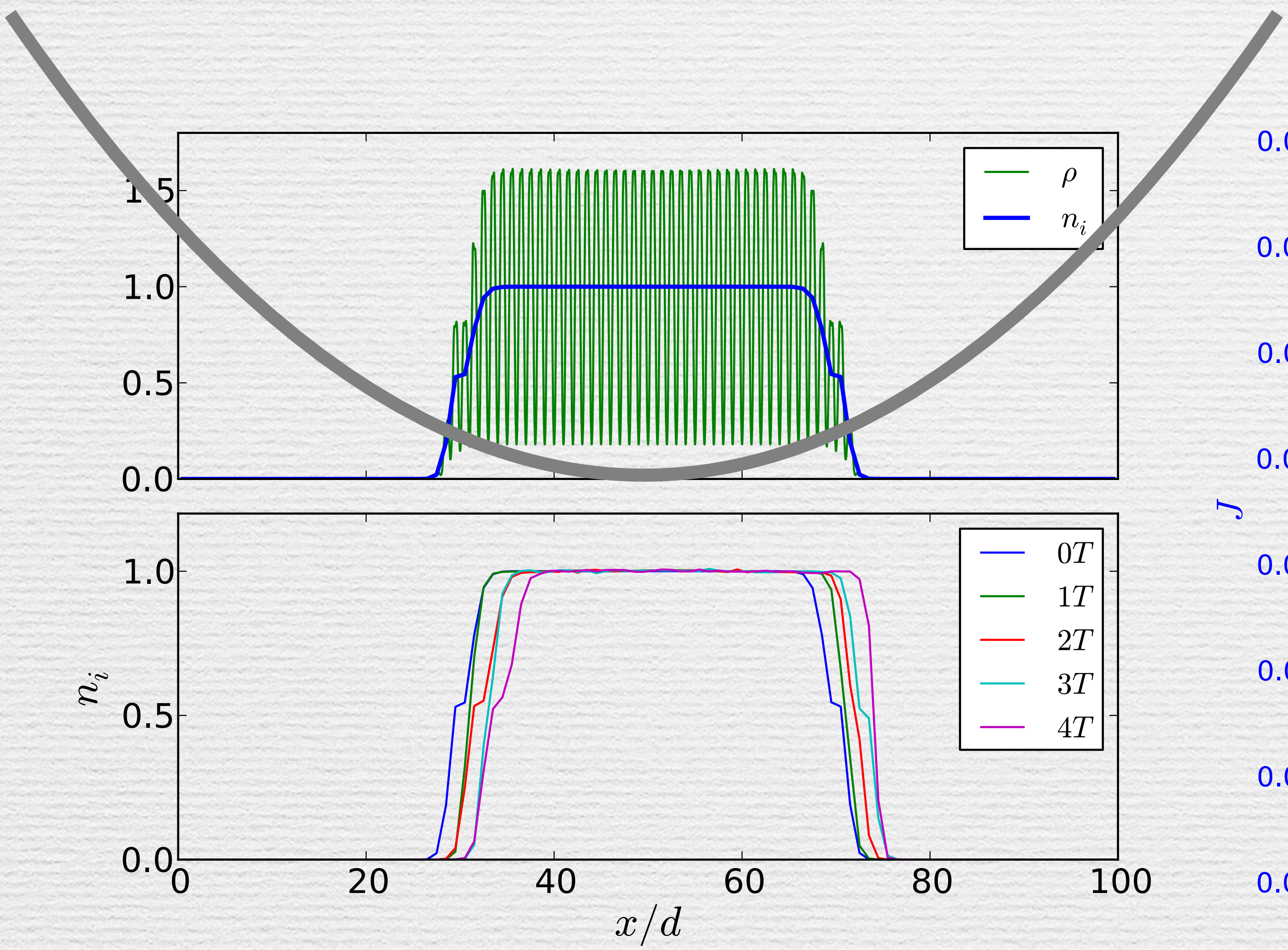

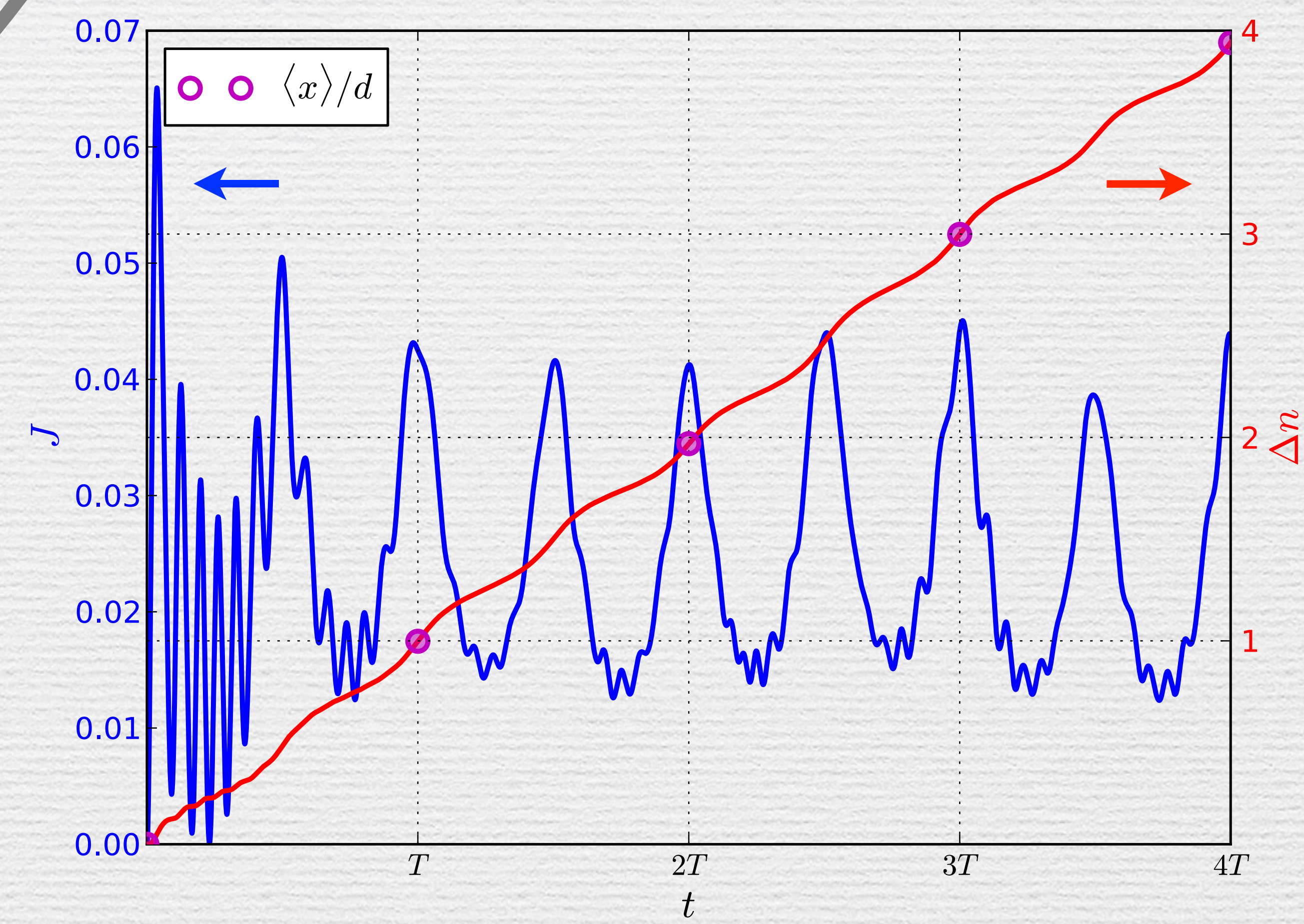

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Temperature & Non-adiabatic effect

$$\text{Temperature} \ll \frac{\Delta}{k_B} \qquad\qquad T \gg \frac{\hbar}{\Delta}$$

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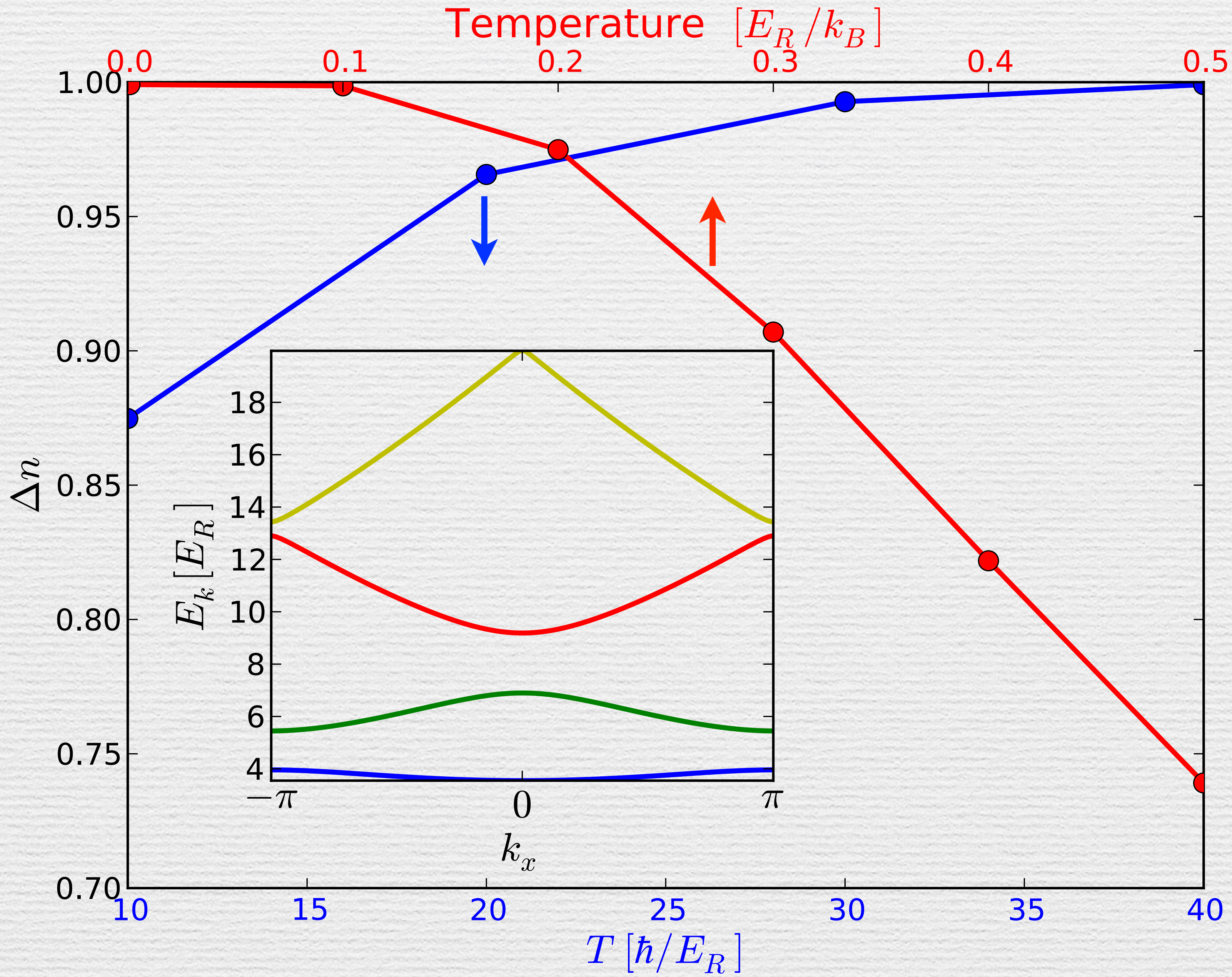


Measure Chern number of 2D optical lattice

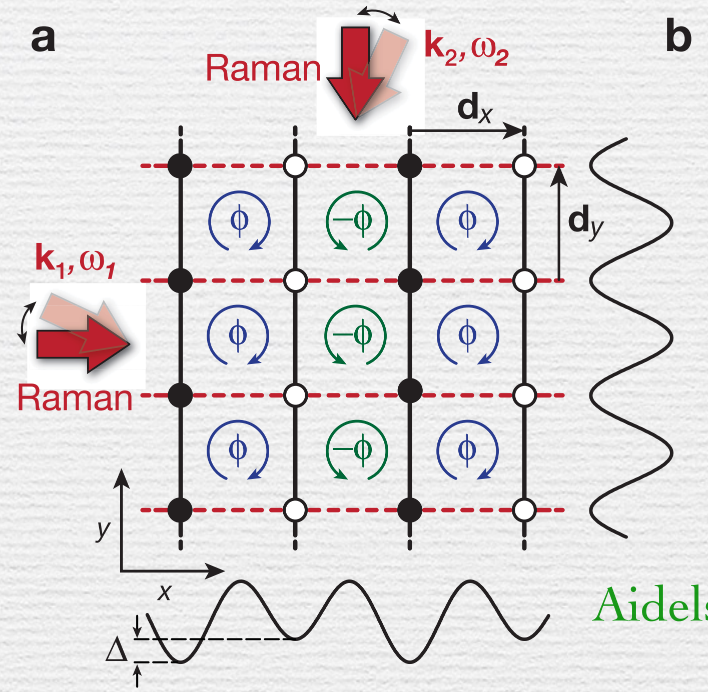
with

Topological charge pumping

Synthetic gauge-field in optical lattices

- Imprint complex phases to the hopping amplitude

- Staggered flux lattice Munich



Aidelsburger *et al*

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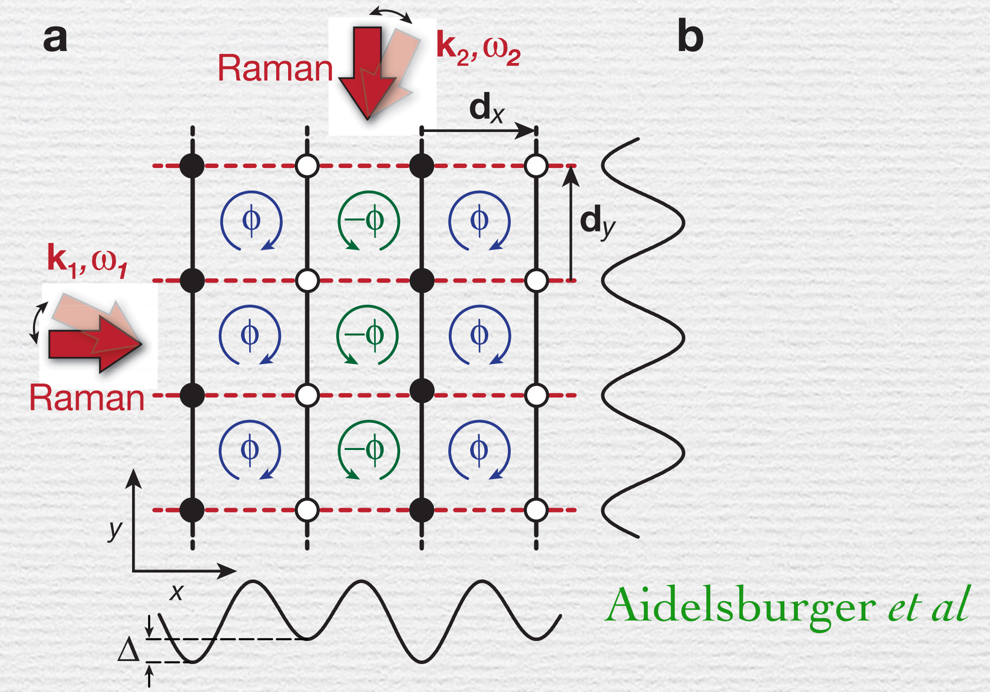
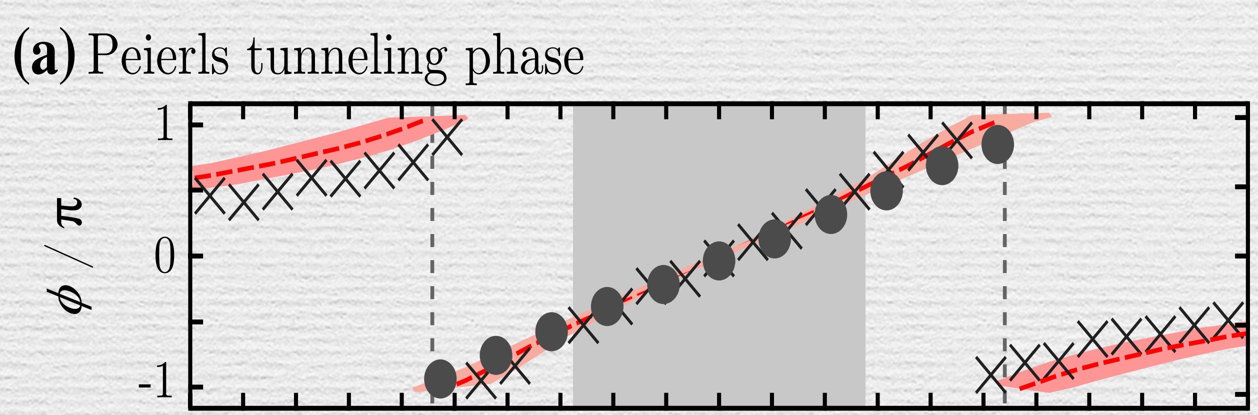


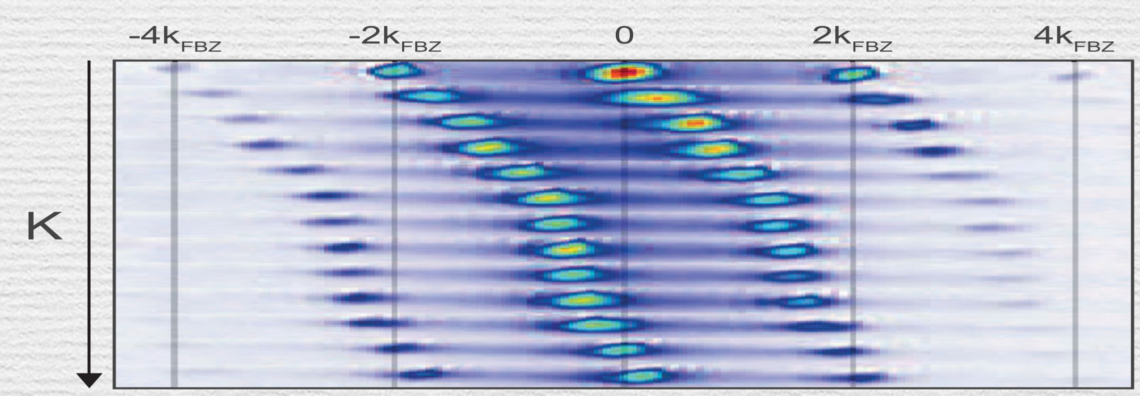
Aidelsburger *et al*

- 1D Peierls lattice NIST, Hamburg

$$H = -J \sum_m e^{i2\pi\Phi} c^\dagger_{m+1} c_m + H.c.$$



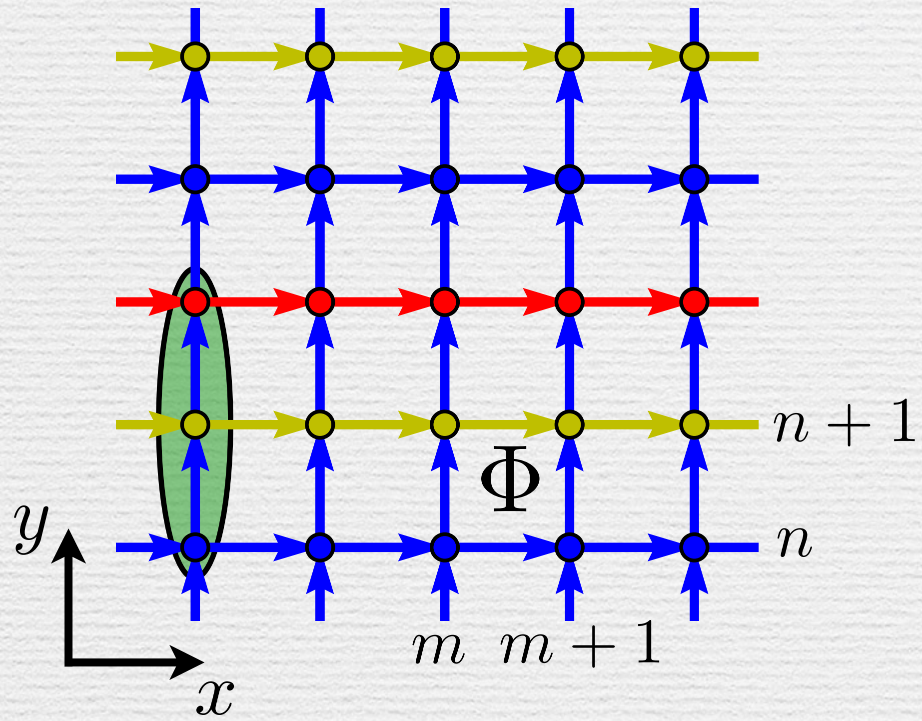
Jimenez-Garcıa *et al*



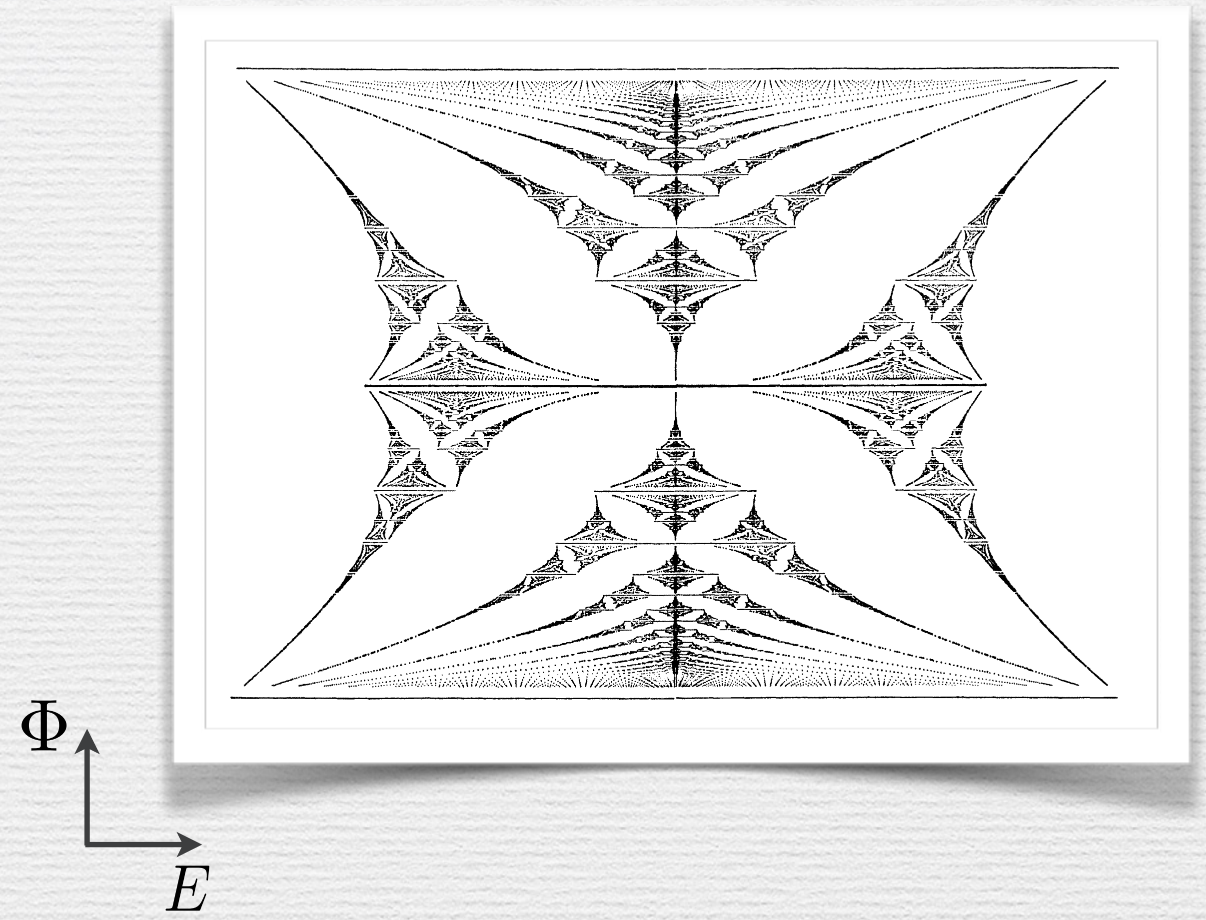
Struck *et al*

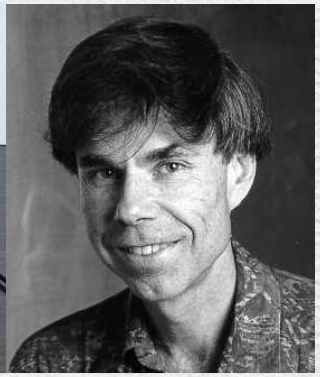
Hofstadter optical lattice

$$H = -J \sum_{m,n} e^{i2\pi n\Phi} c^\dagger_{m+1,n} c_{m,n} + c^\dagger_{m,n+1} c_{m,n} + H.c. \qquad \Phi = p/q$$



Hofstadter, 1976





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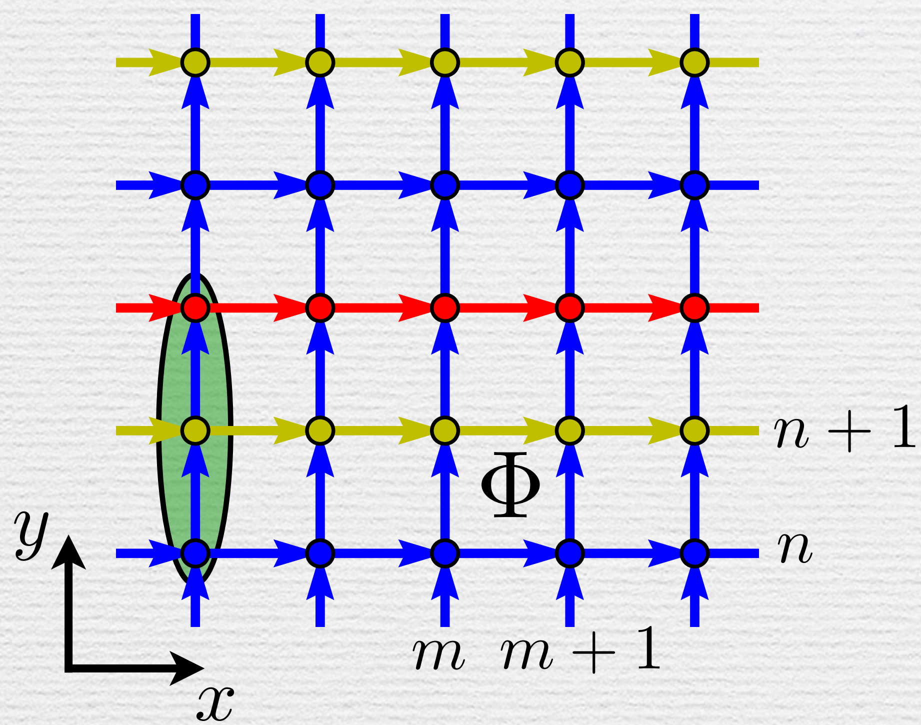
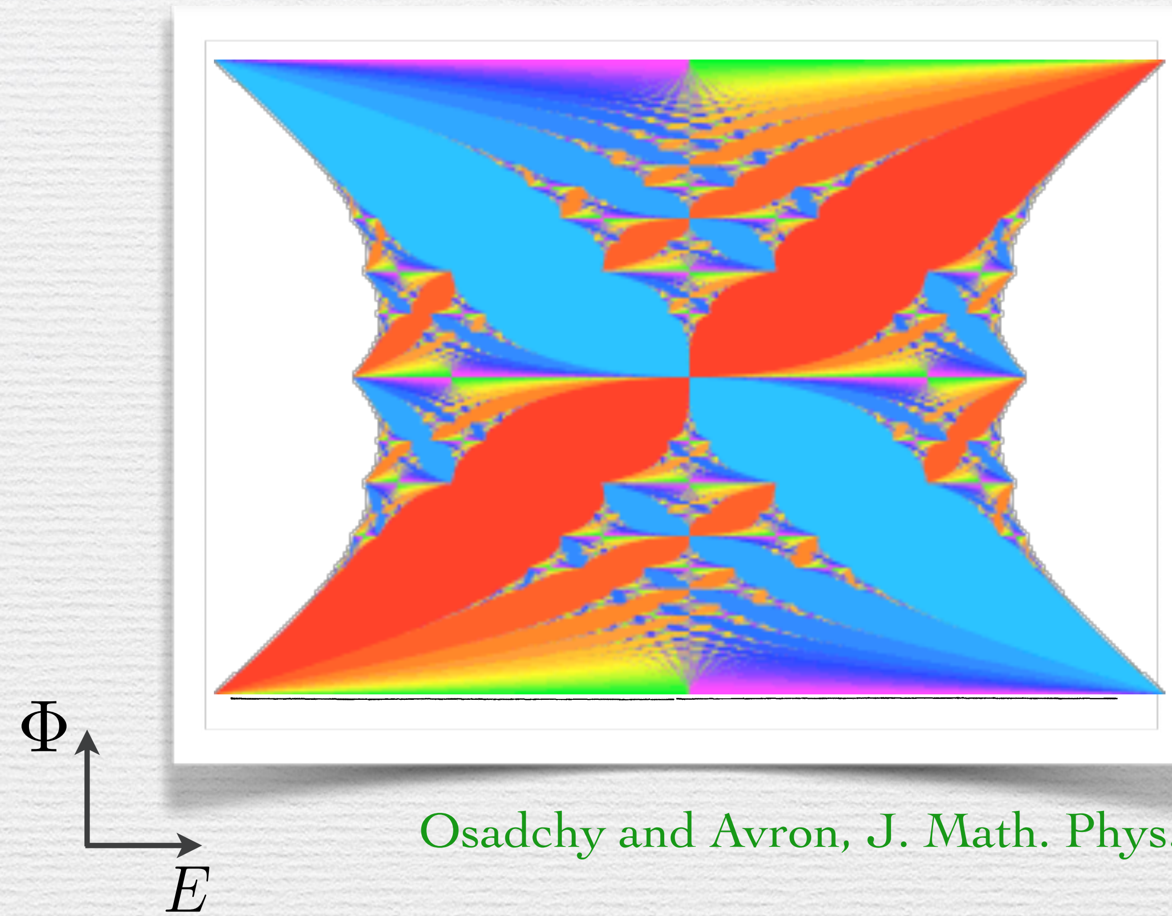
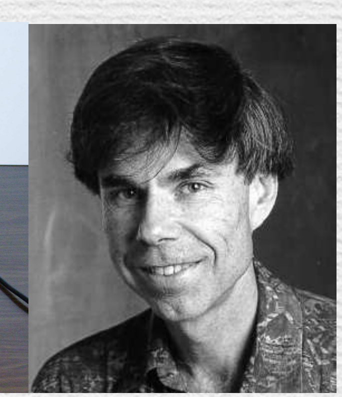


Hofstadter, 1976



Osadchy and Avron, J. Math. Phys. 2001



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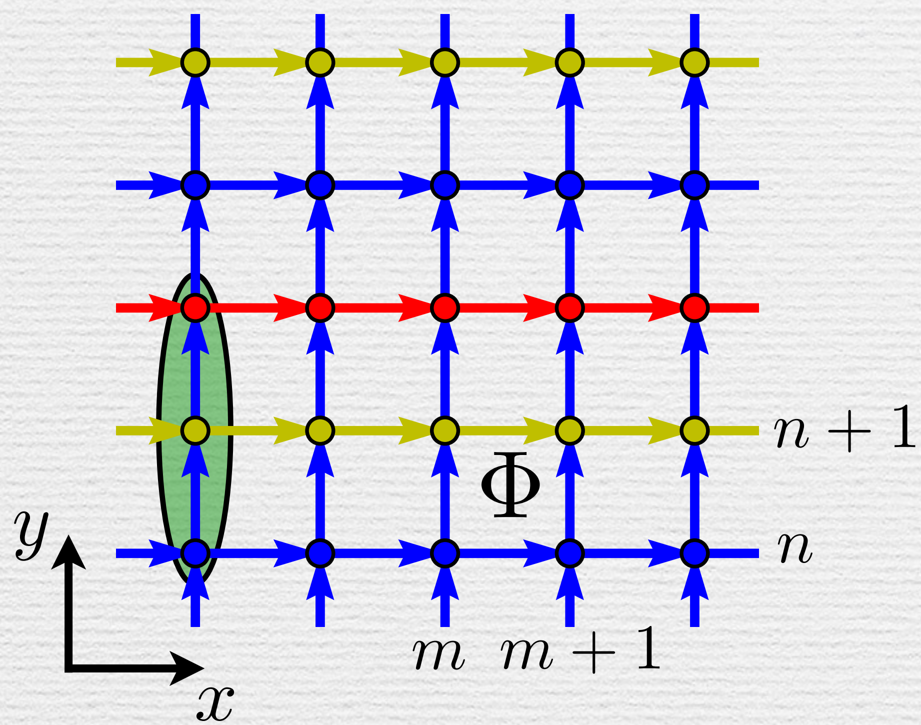

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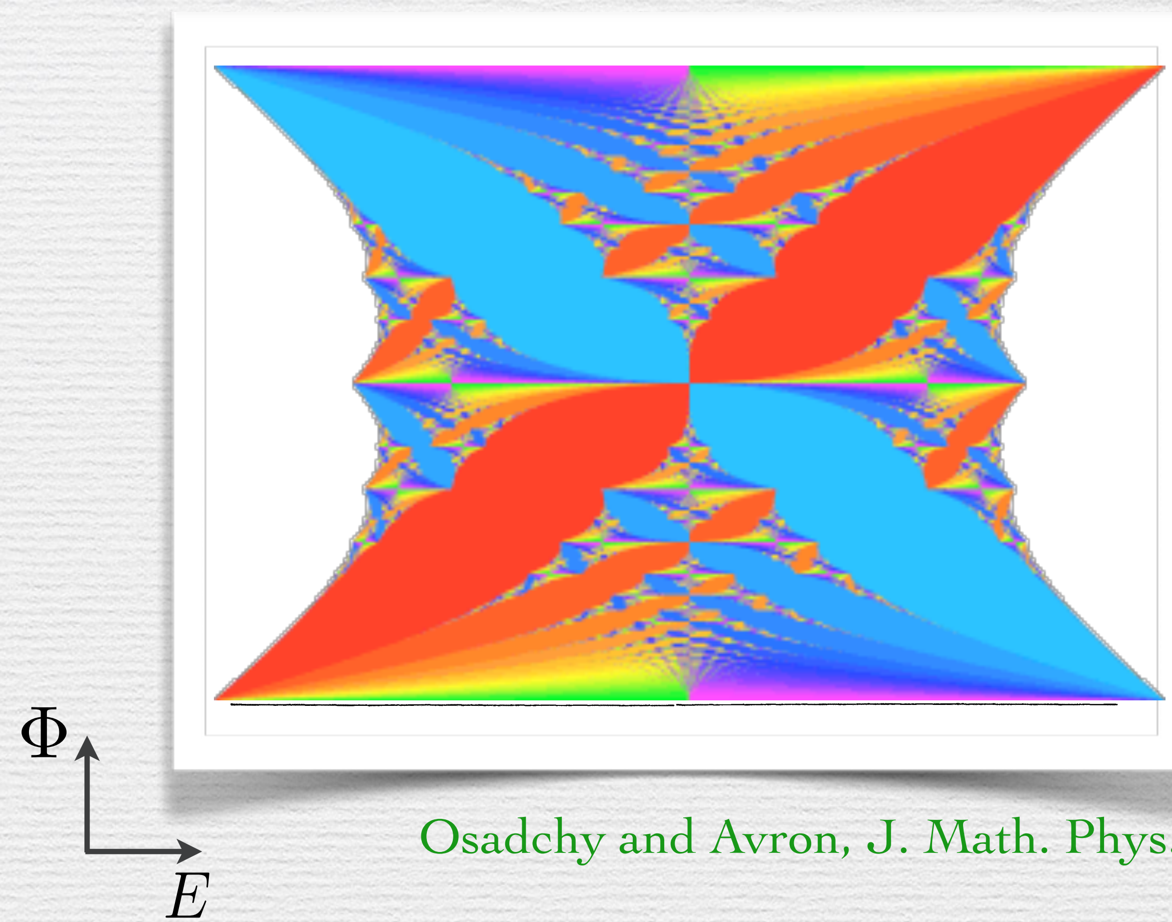
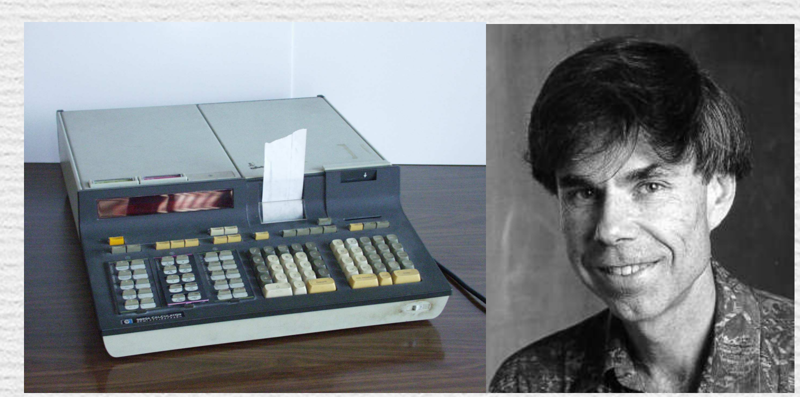


Osadchy and Avron, J. Math. Phys. 2001

NO sharp edge states in harmonic trapping potential

Buchhold *et al*



How to measure Chern # ?

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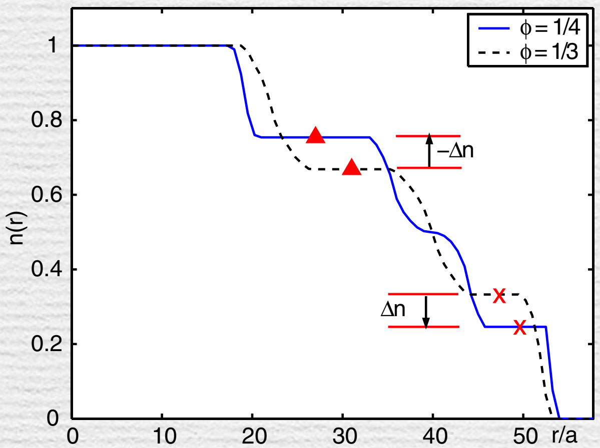
Density profile

Umucalilar *et al*

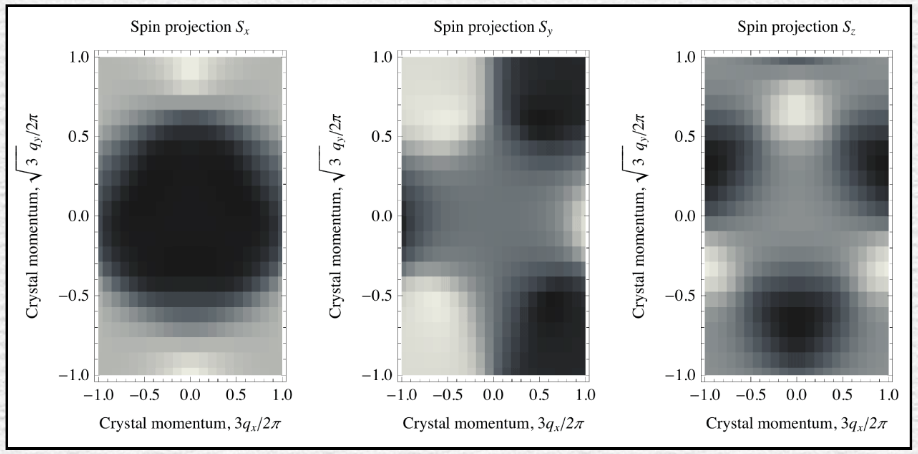


(a)

How to measure Chern # ?

Time-of-flight

Alba *et al*, Zhao *et al*



Density profile

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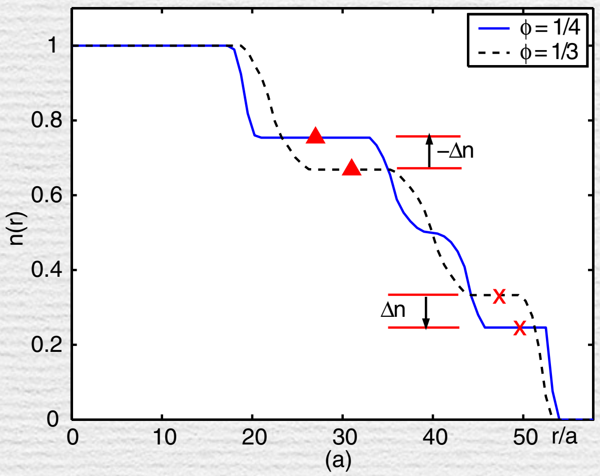

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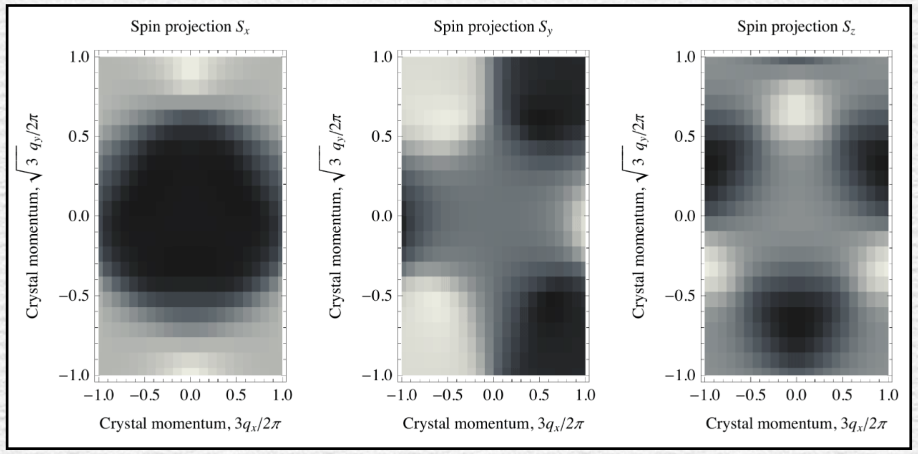

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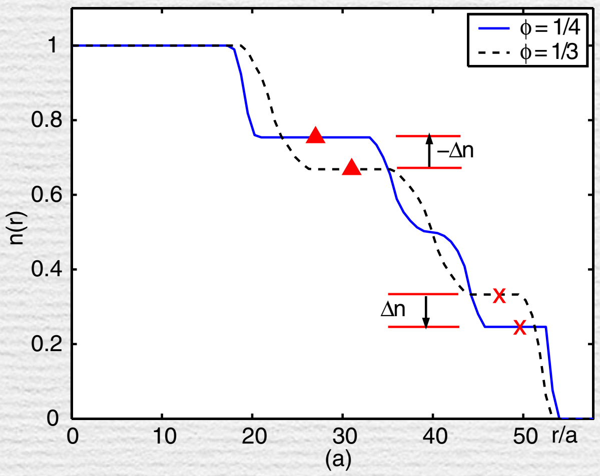
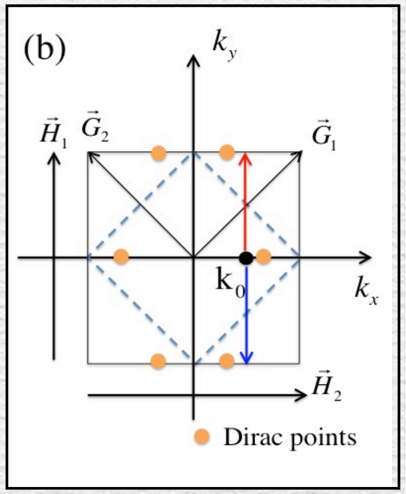


Zak phases

Abanin *et al*



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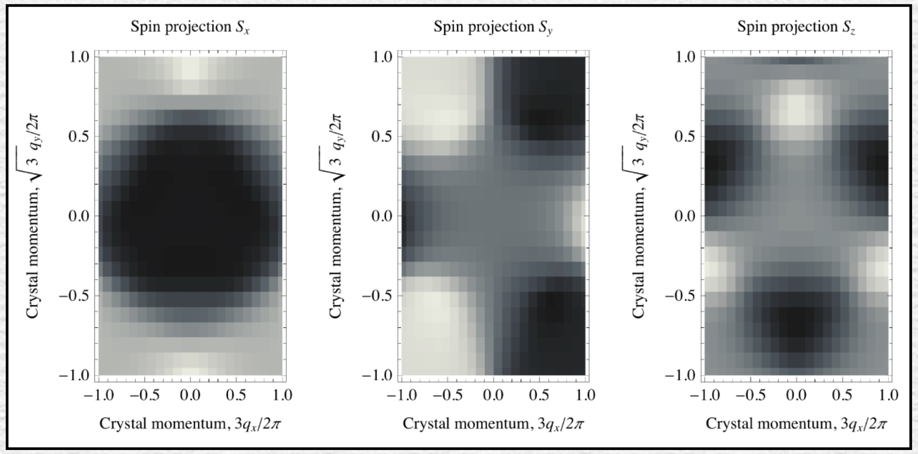
Alba *et al*, Zhao *et al*



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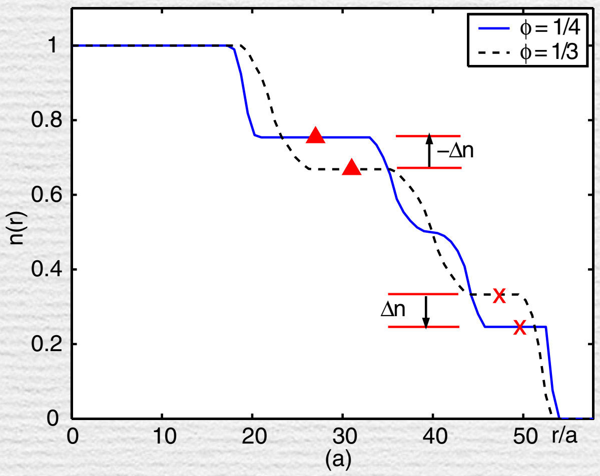

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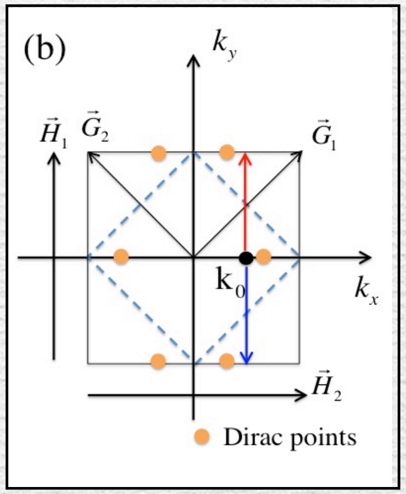


Semi-classical dynamics

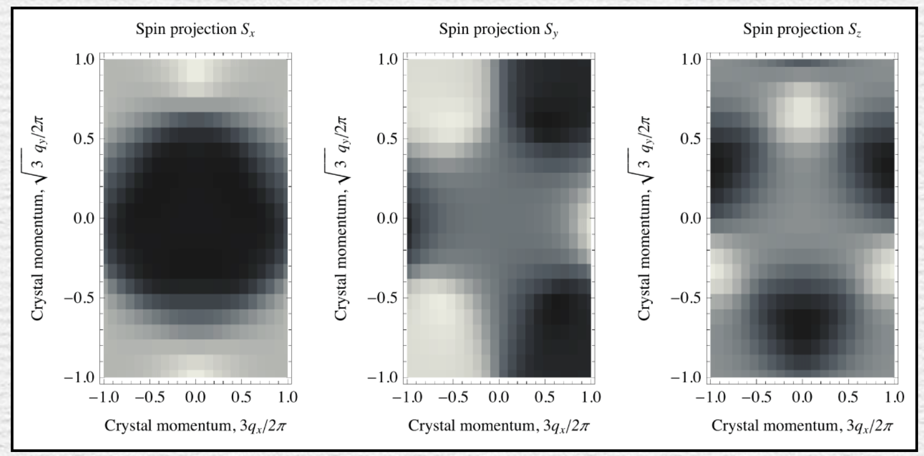
Price *et al*

$$\dot{\mathbf{r}} = \frac{1}{\hbar}\frac{\partial \varepsilon(\mathbf{k})}{\partial \mathbf{k}} - \frac{\mathbf{F}}{\hbar} \times \hat{\mathbf{z}}\Omega(\mathbf{k})$$

How to measure Chern # ?

Time-of-flight

Alba *et al*, Zhao *et al*



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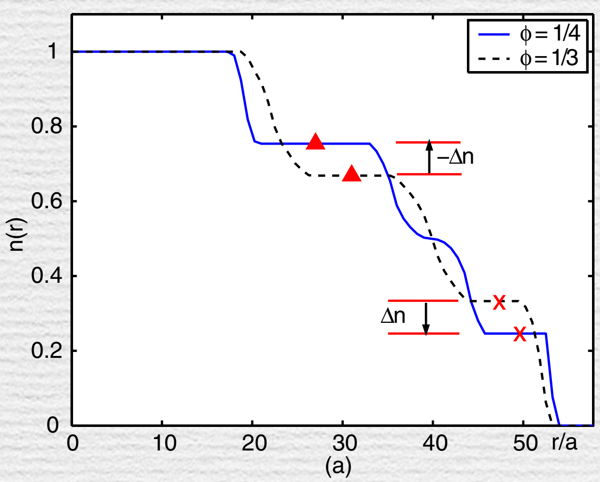

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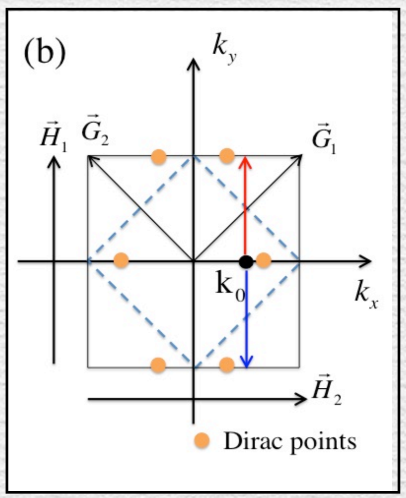

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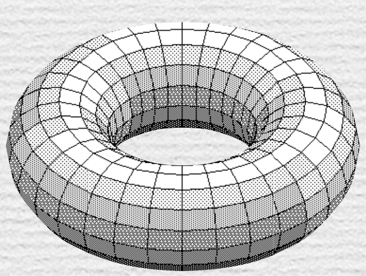


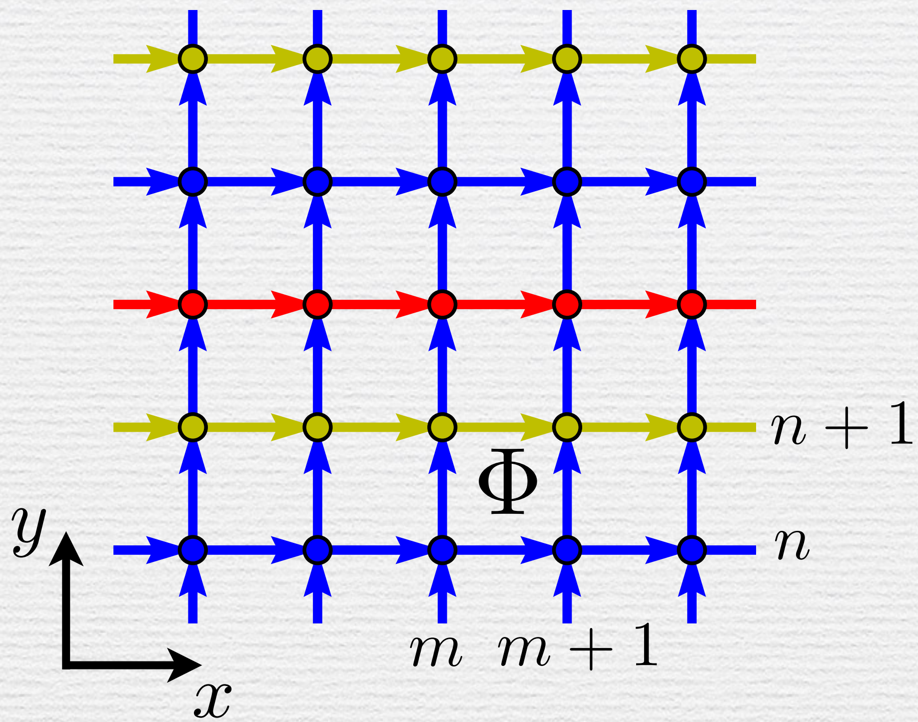
We propose a new probe based on

Topological Pumping Effect

$$\rho(k_x, y)$$

Hybrid time-of-flight

LW, Soluyanov and Troyer, PRL 110, 166802



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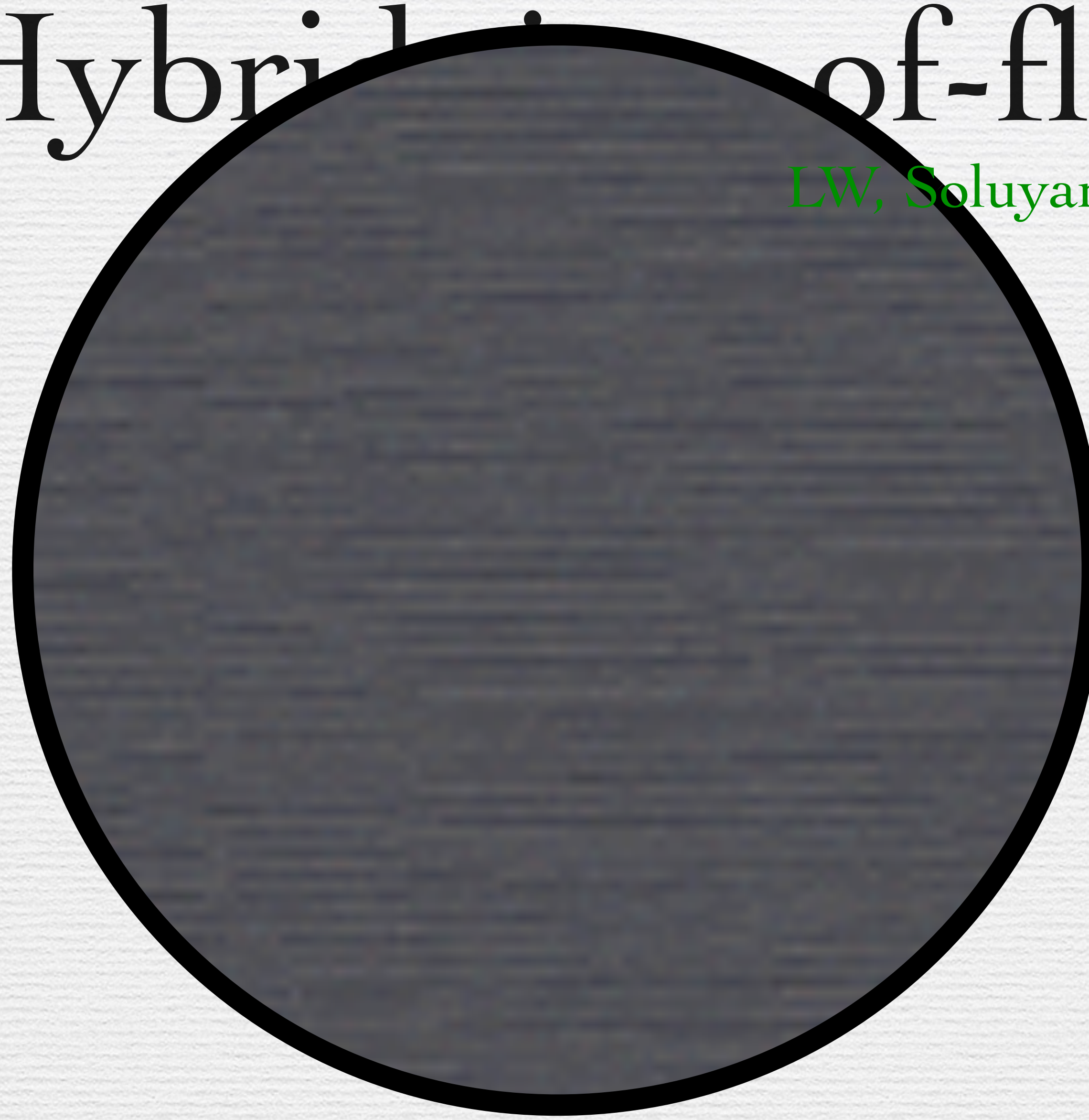
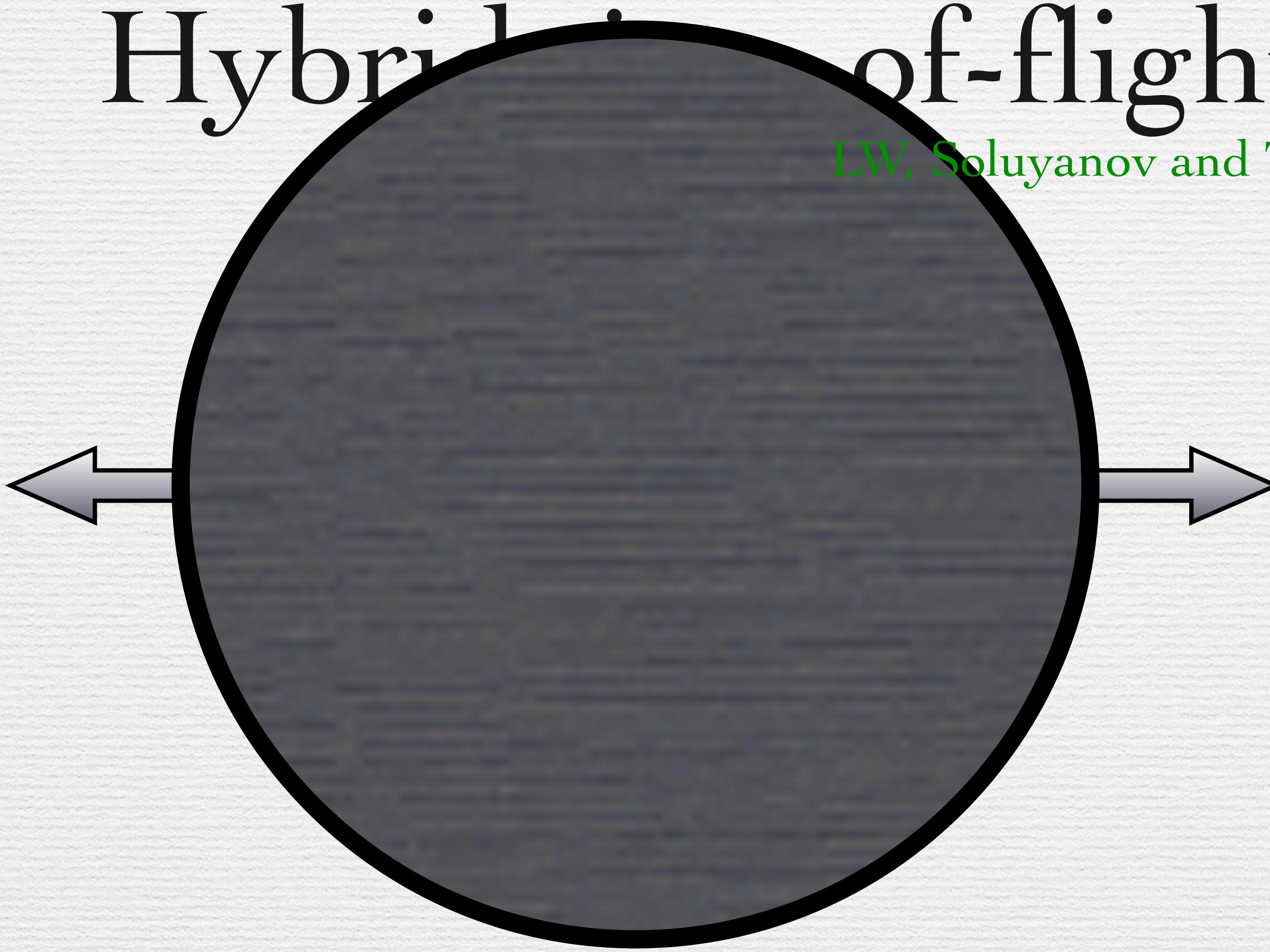


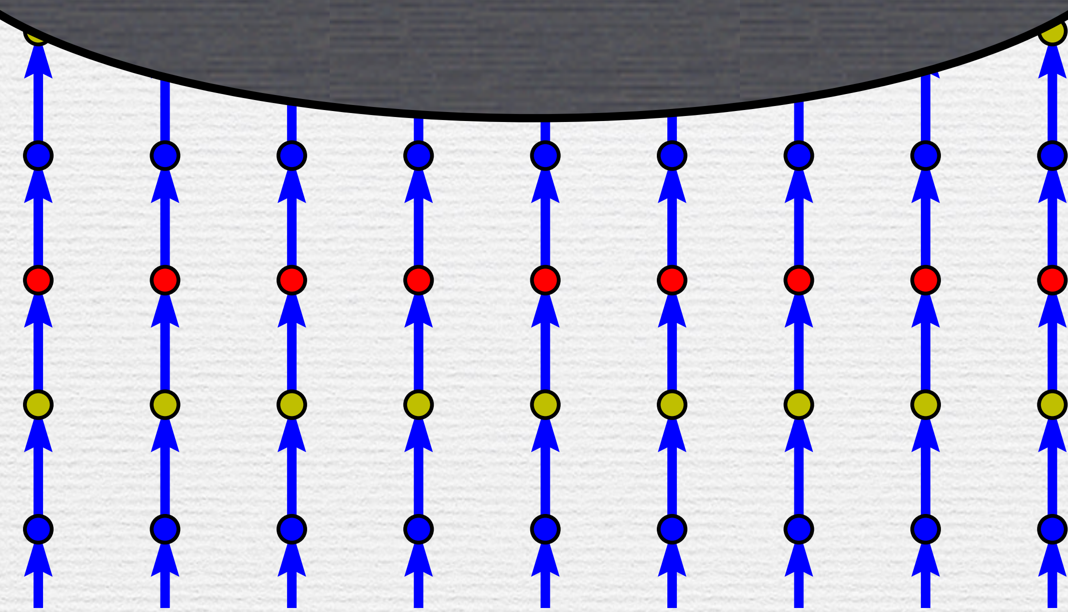
Hybri[illegible]of-flight

LW, Soluyanov and Troyer, PRL 110, 166802



Hyb[redacted]f flight

LW, Soluyanov and Troyer, PRL 110, 166802



Hy[redacted]f flight

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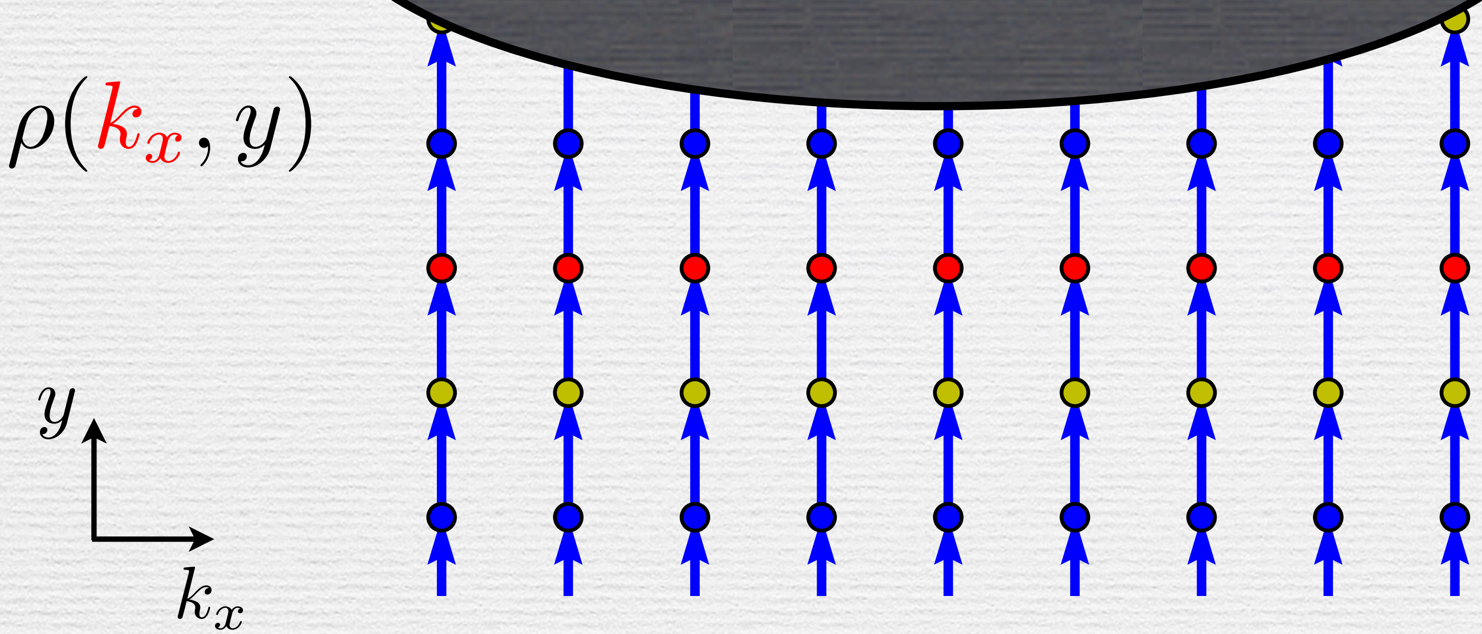

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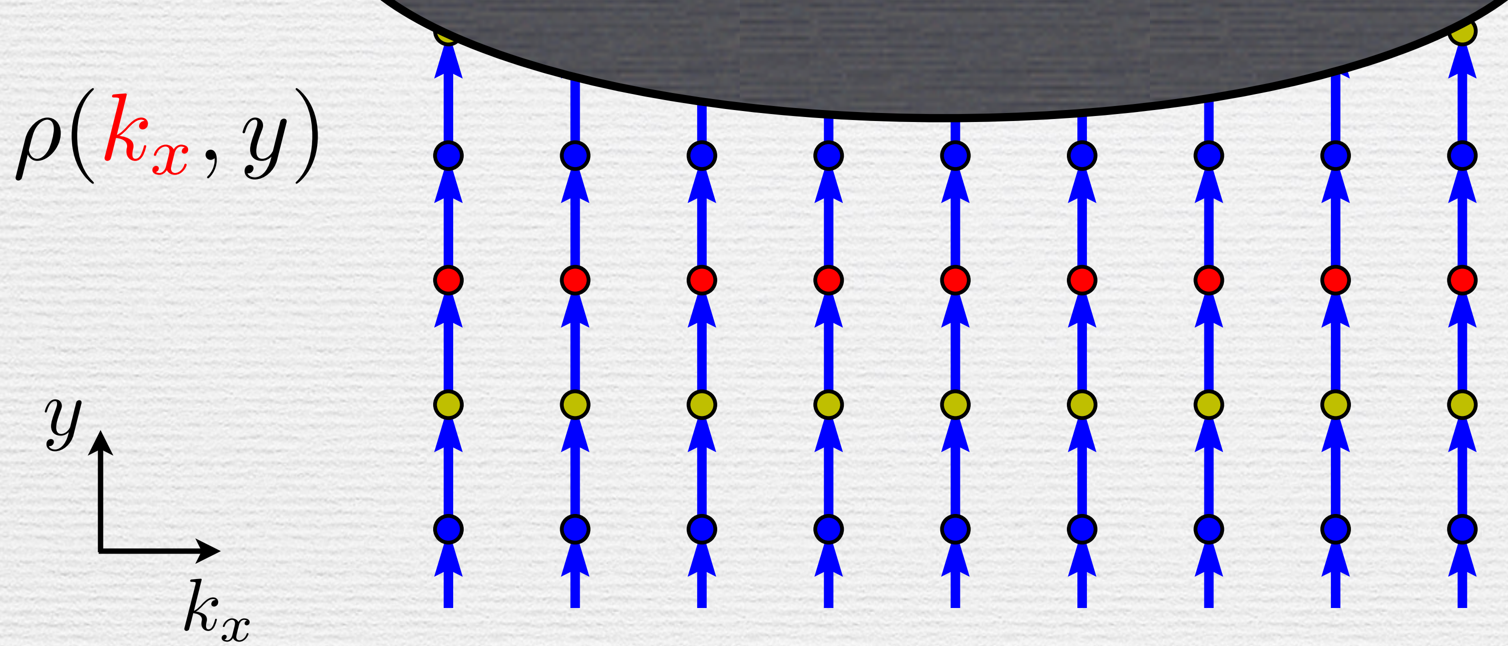
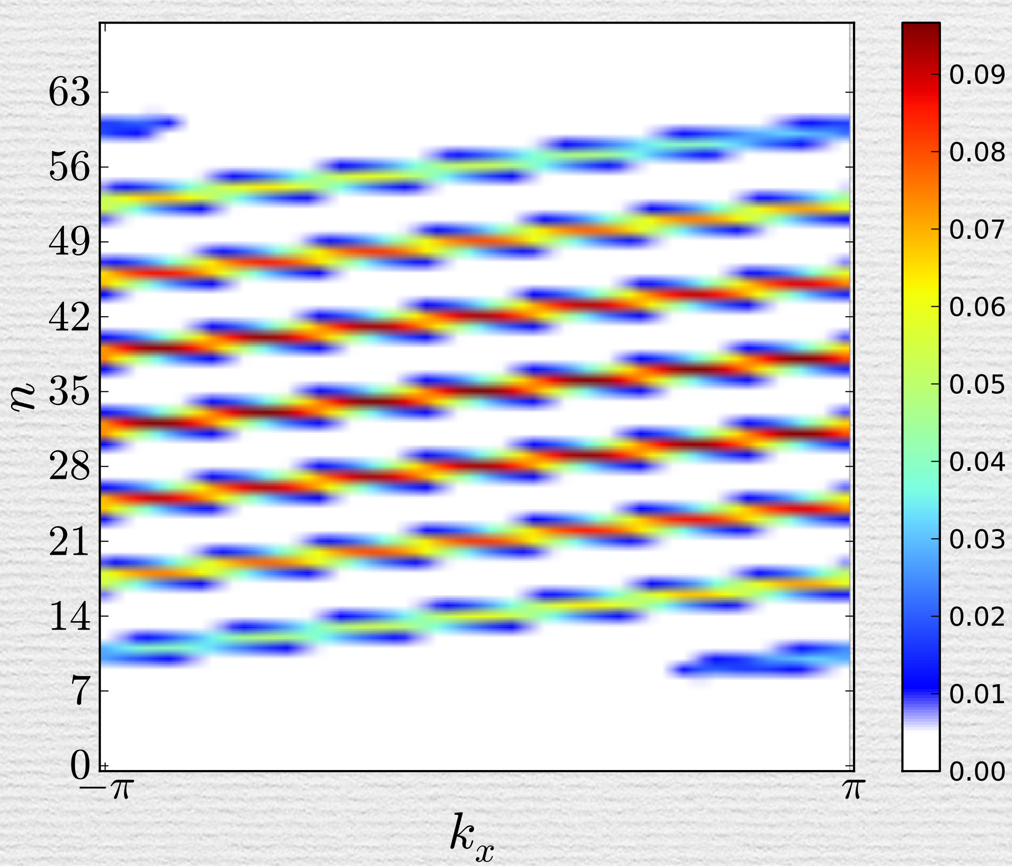


$\Phi = 1/7 \quad C = 1$



Hy [redacted] f flight

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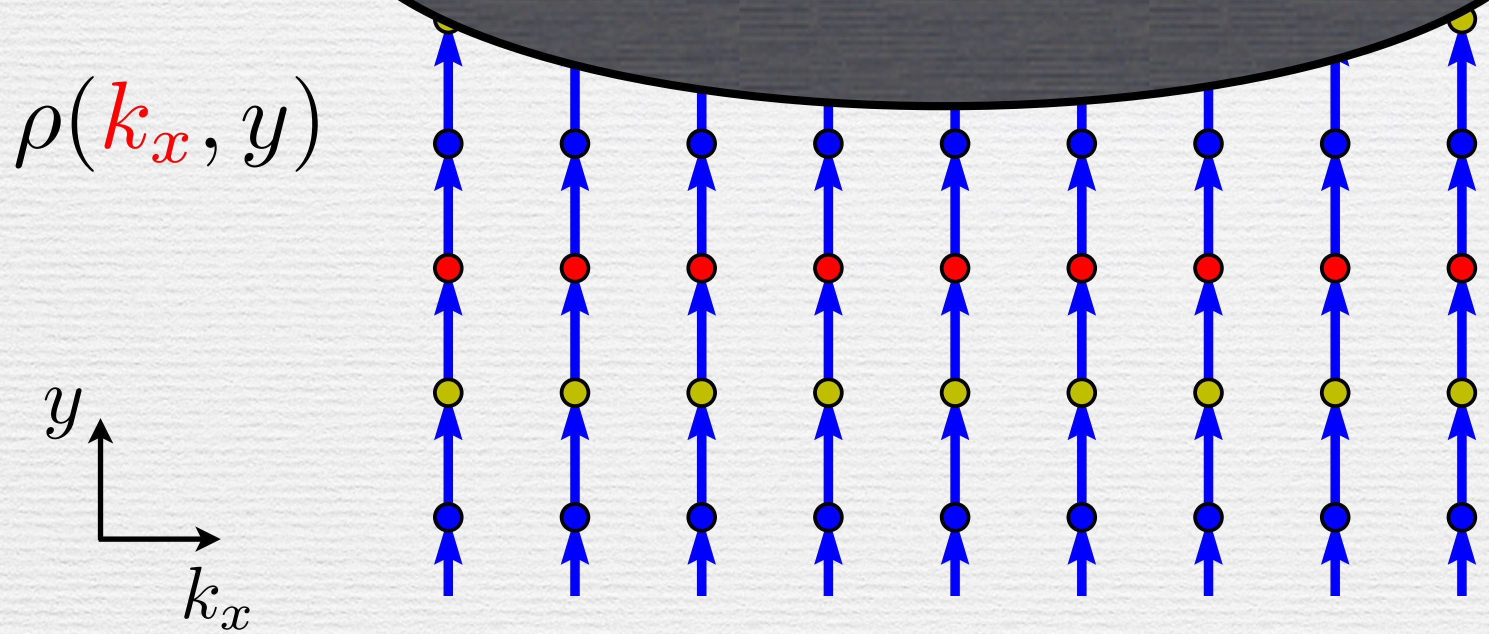

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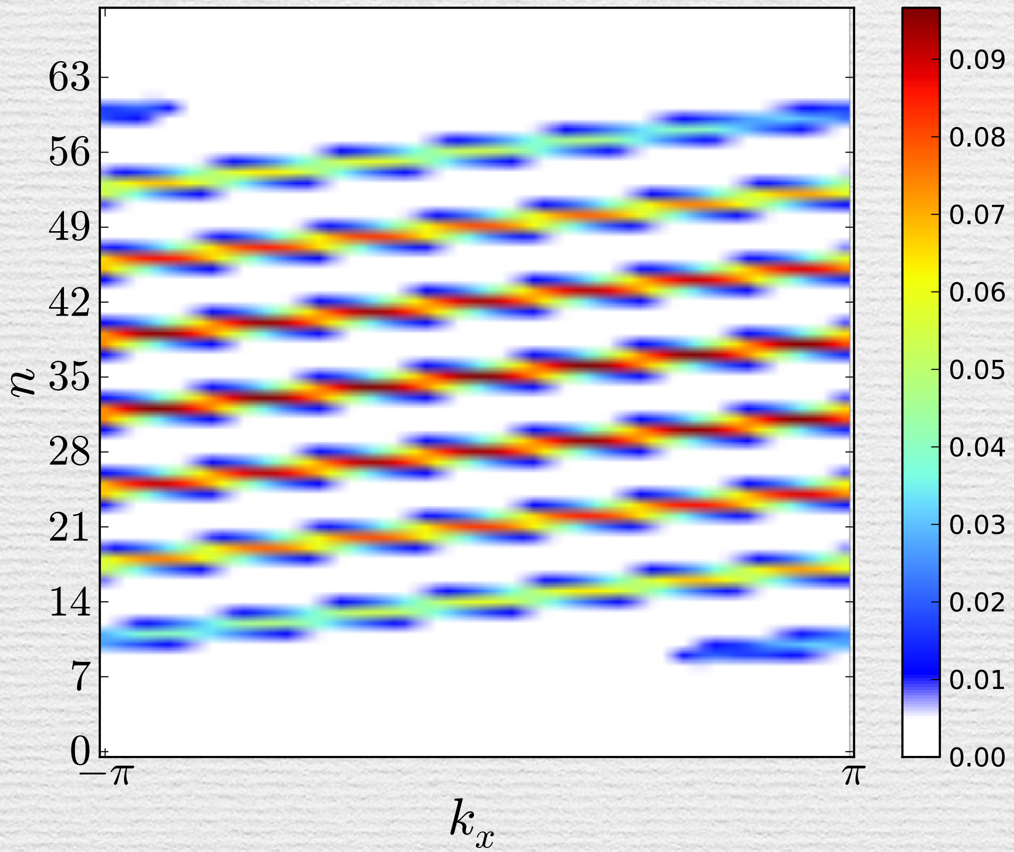
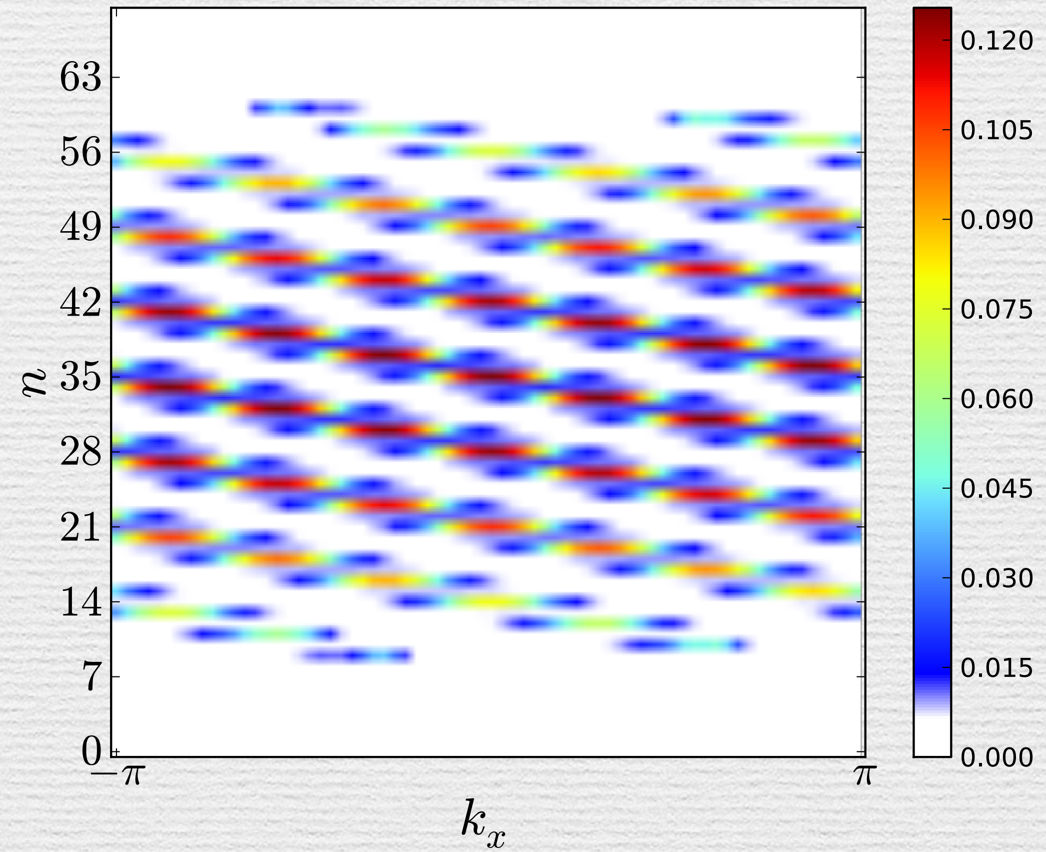
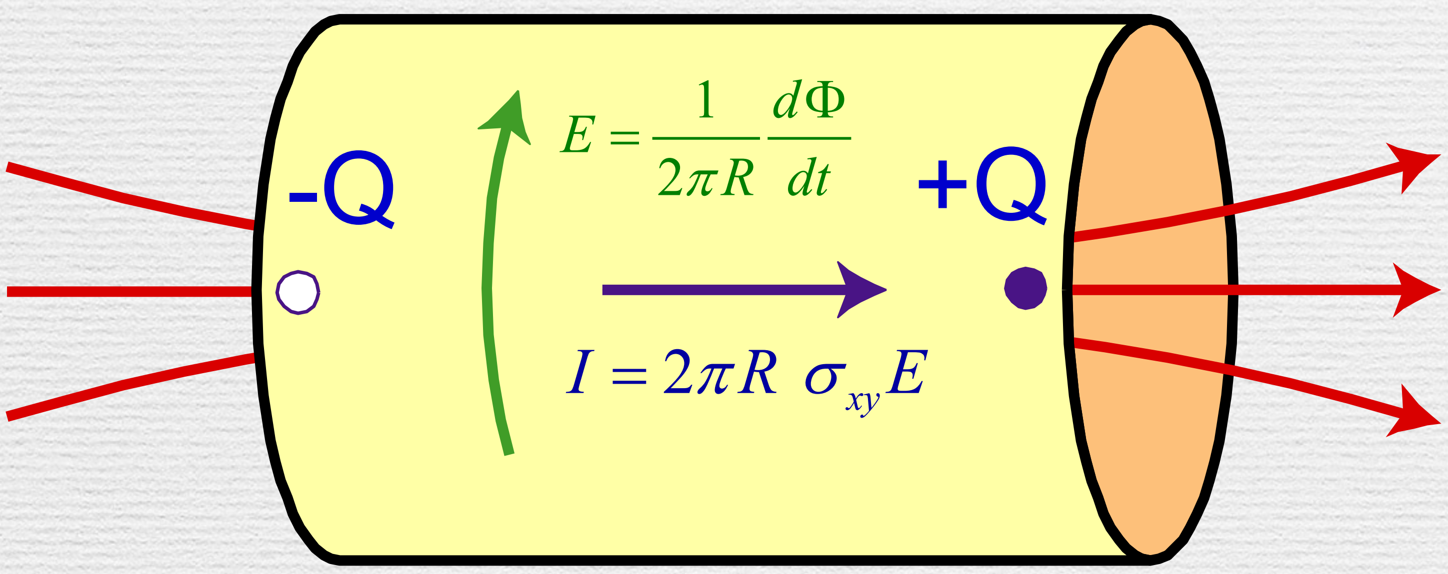


$\Phi = 3/7 \quad C = -2$



Why it works?

Topological charge pumping



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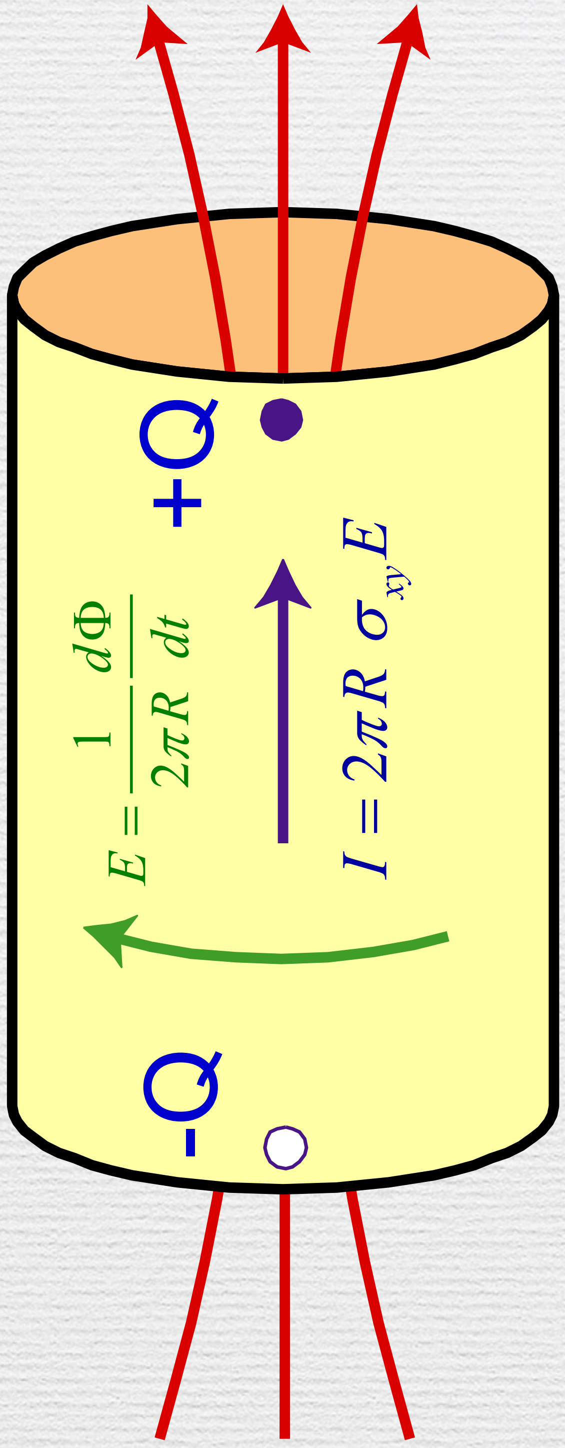

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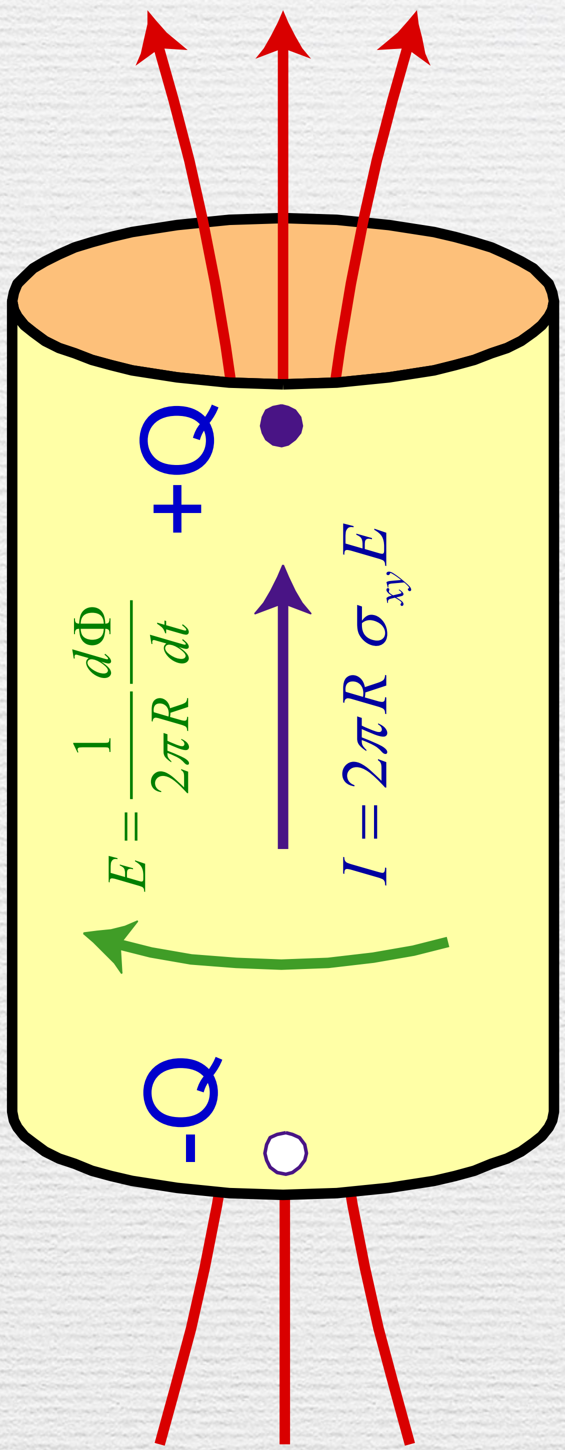

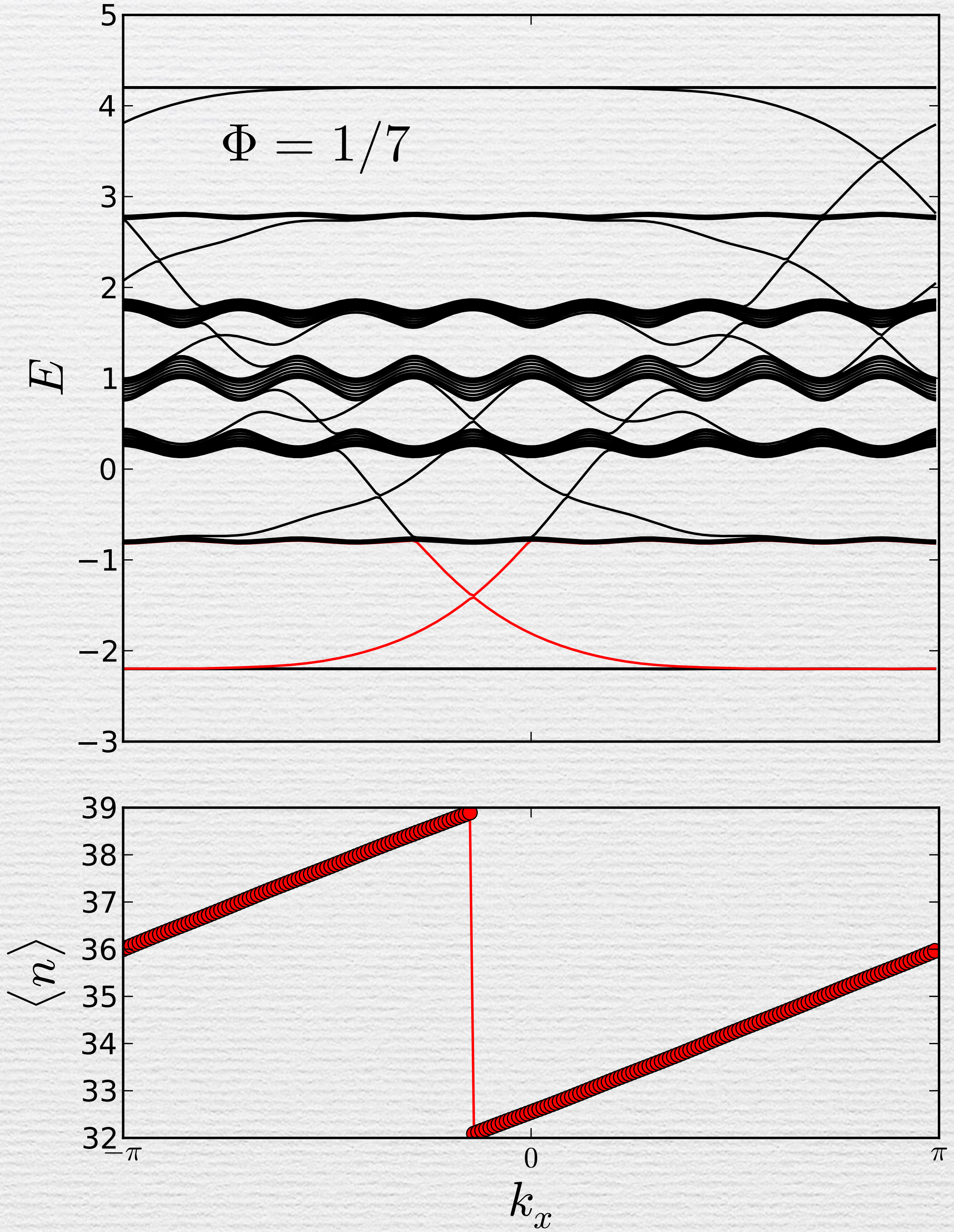

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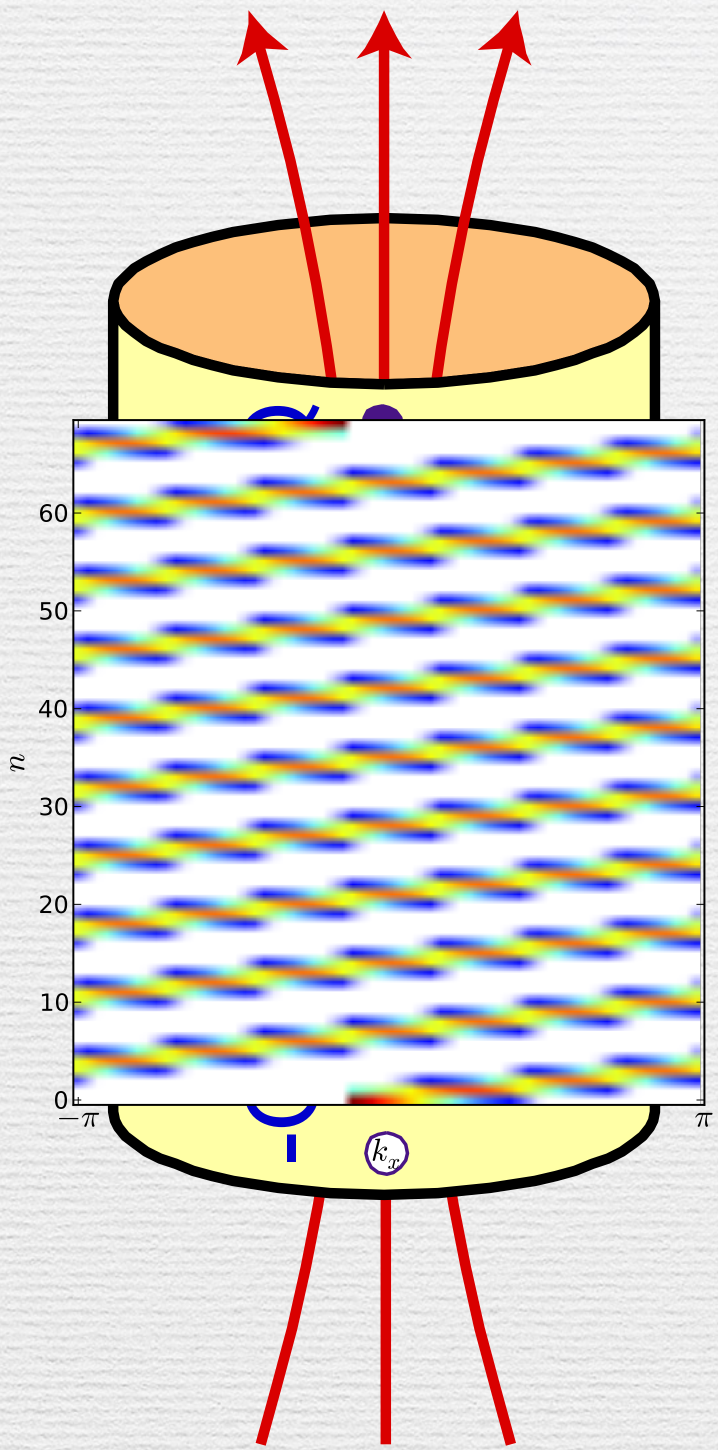
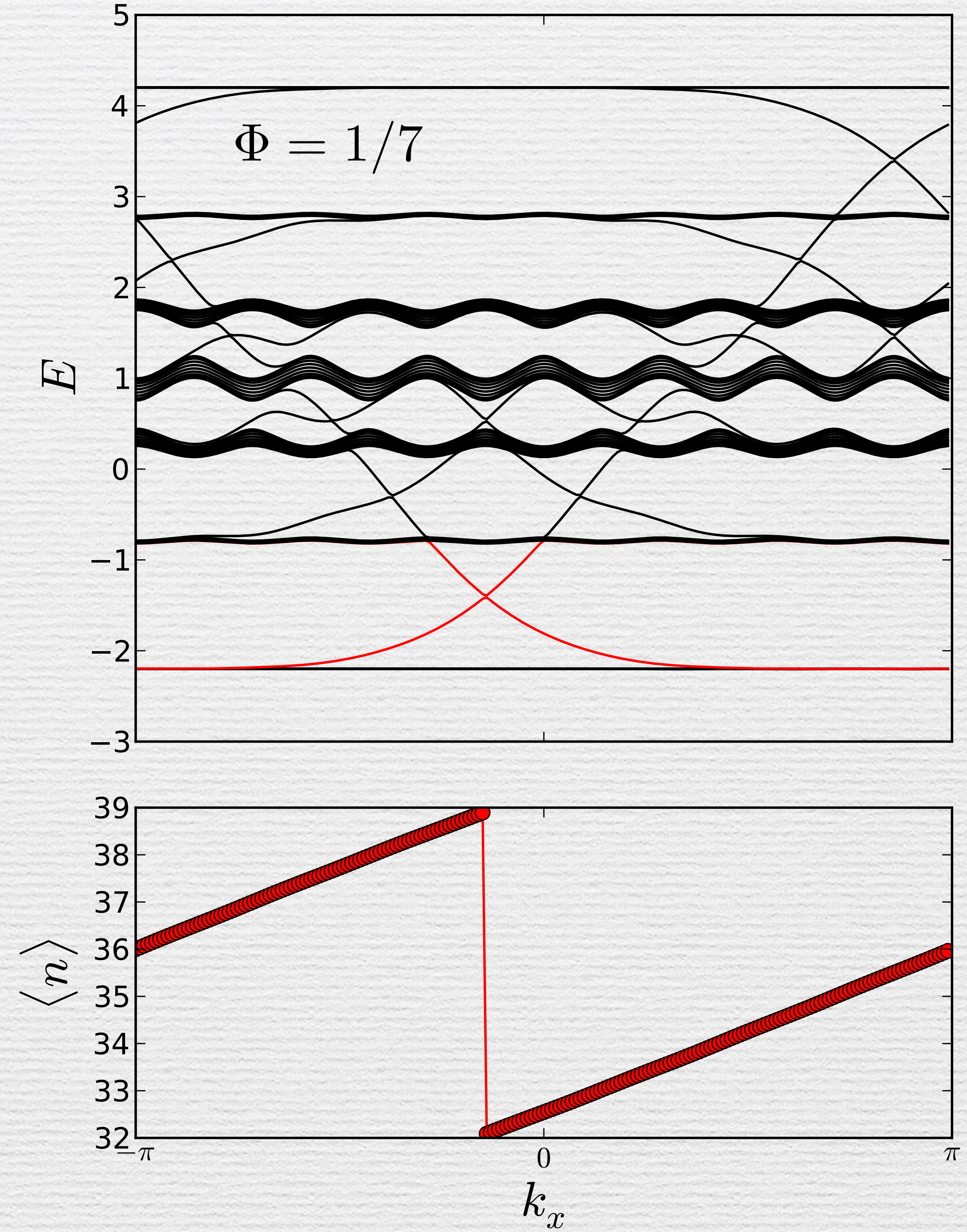
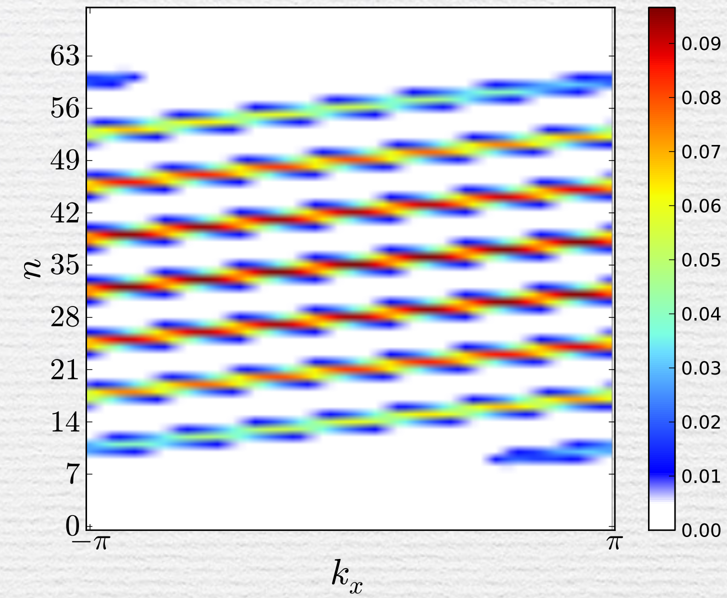




Quantitative Characterizations



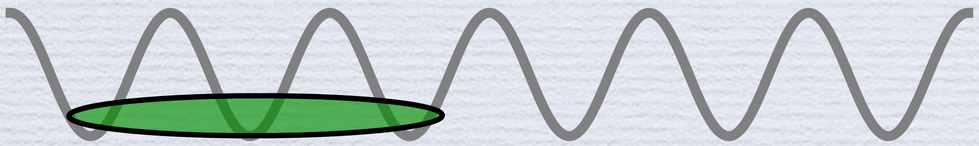
- Slope
- # of cuts (edge modes)
- COM along y-direction
- Bi-partition number of particle (trace index)

Salient features

- Bulk detection, does not require edge states
- $\rho(k_x, y)$ is nearly impossible to measure in solids, but accessible to cold atom toolbox
- Can be extended to interacting case

Fractional charge pumping

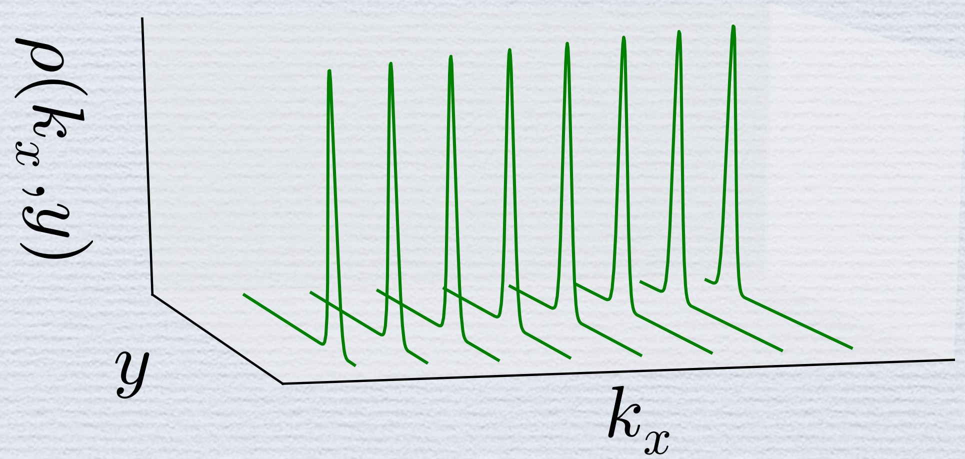
1D lattice



Interaction is crucial
for opening an energy gap

2D Laughlin state

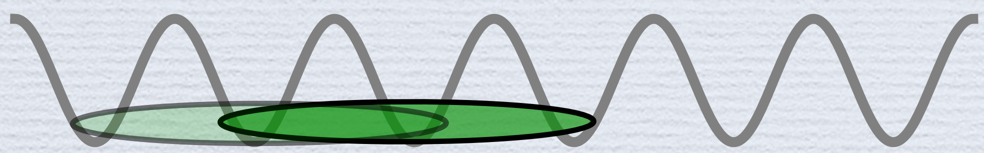
$$\rho(k_x, y) = \frac{\nu}{\sqrt{\pi}} e^{-(y-k_x)^2}$$



Can be used to detect FQHE and fractional Chern insulators realized in optical lattice Cooper *et al*, Yao *et al*, Nielsen *et al*

Fractional charge pumping

1D lattice



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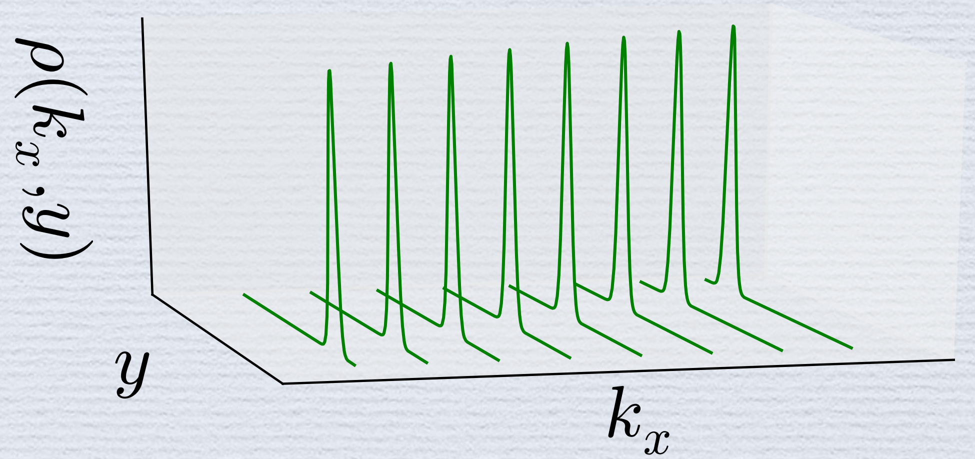

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Laughlin states on lattice

$$\Phi_{\text{Laughlin}} = \Phi_{\text{Hofstadter}}^{1/\nu}$$

Laughlin states on lattice

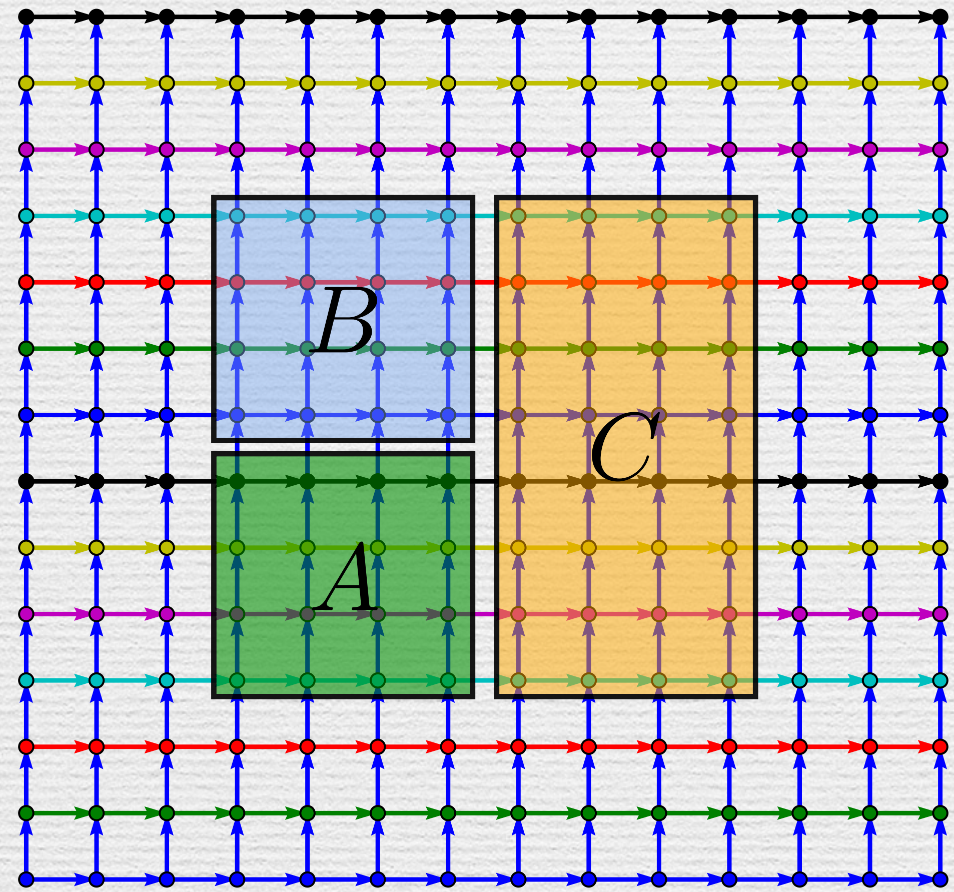
$$\boxed{\Phi_{\text{Laughlin}} = \Phi_{\text{Hofstadter}}^{1/\nu}}$$

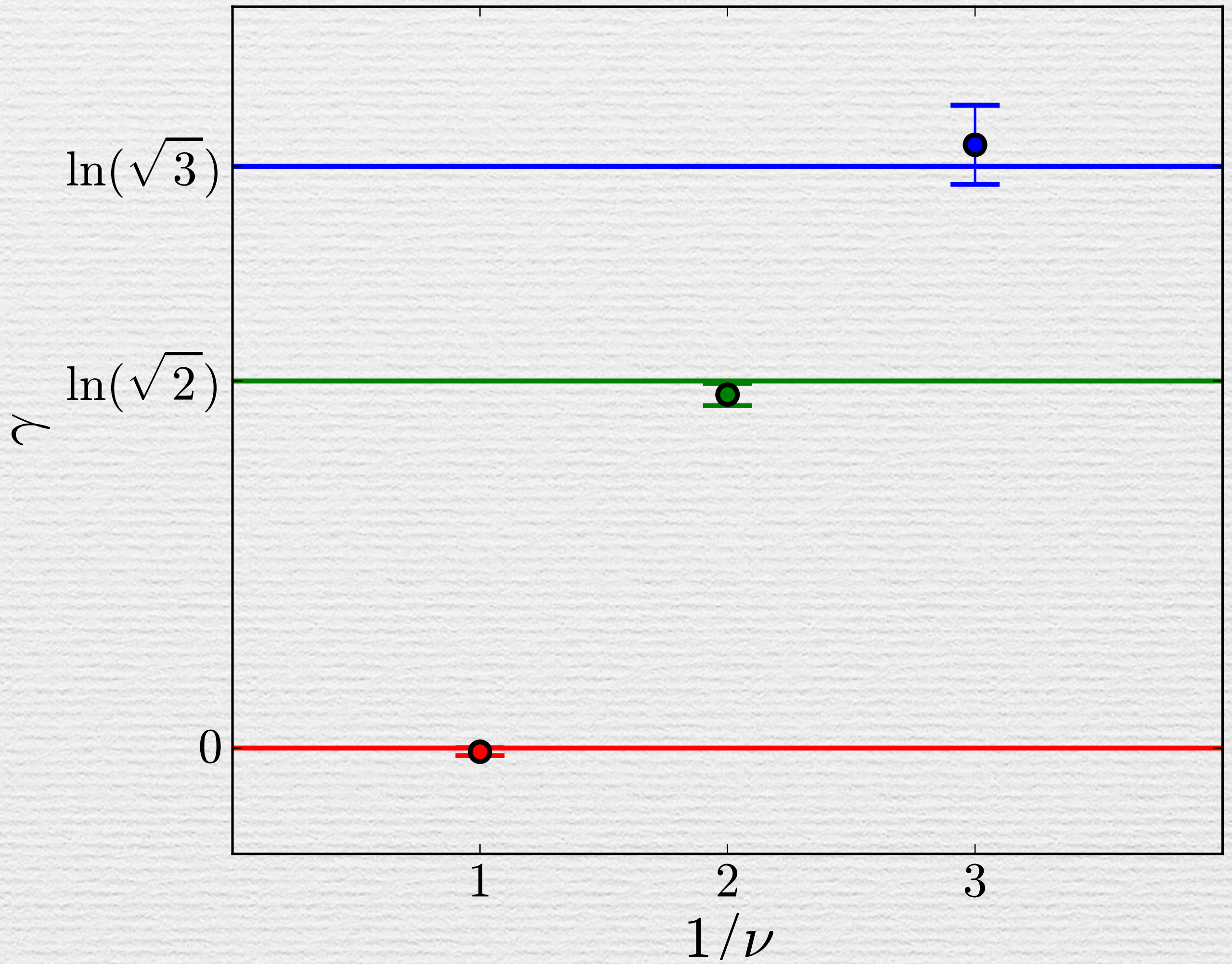
Topological entanglement entropy

Preskill, Kitaev, Levin, Wen

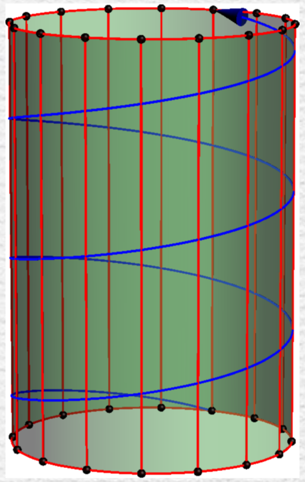
Zhang *et al*

$$-\gamma = S_A + S_B + S_C - S_{AB} - S_{BC} - S_{AC} + S_{ABC}$$

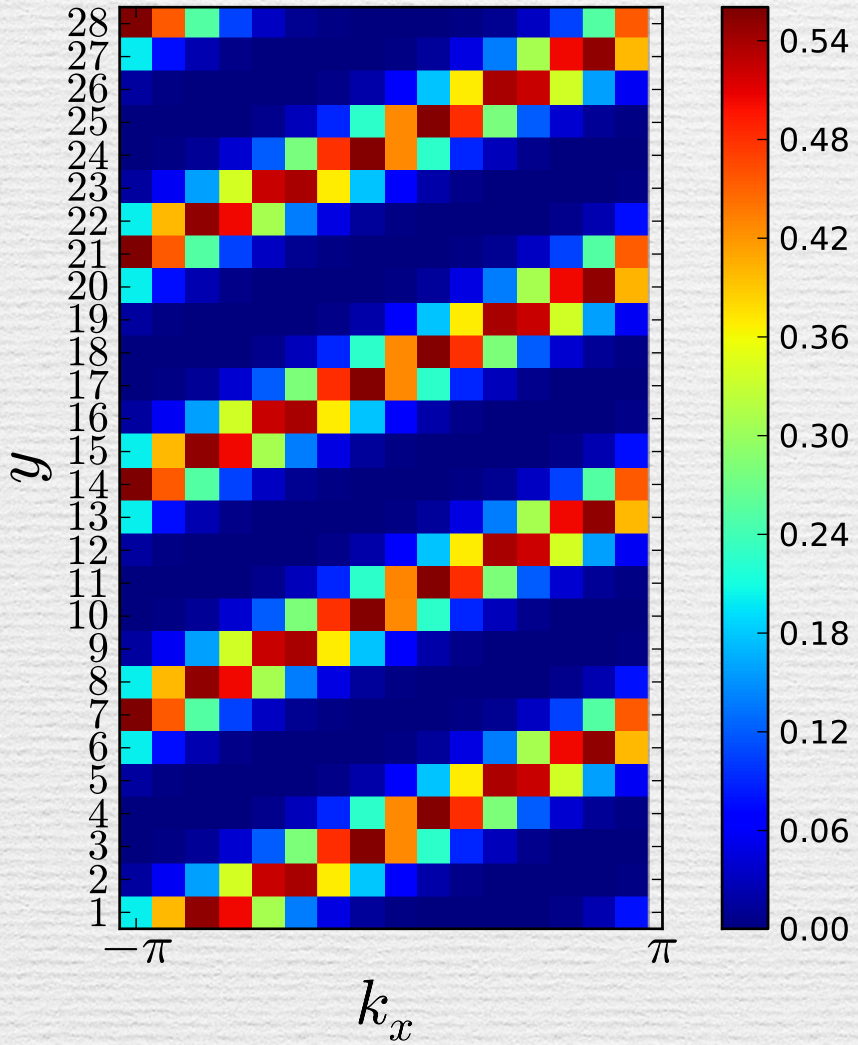




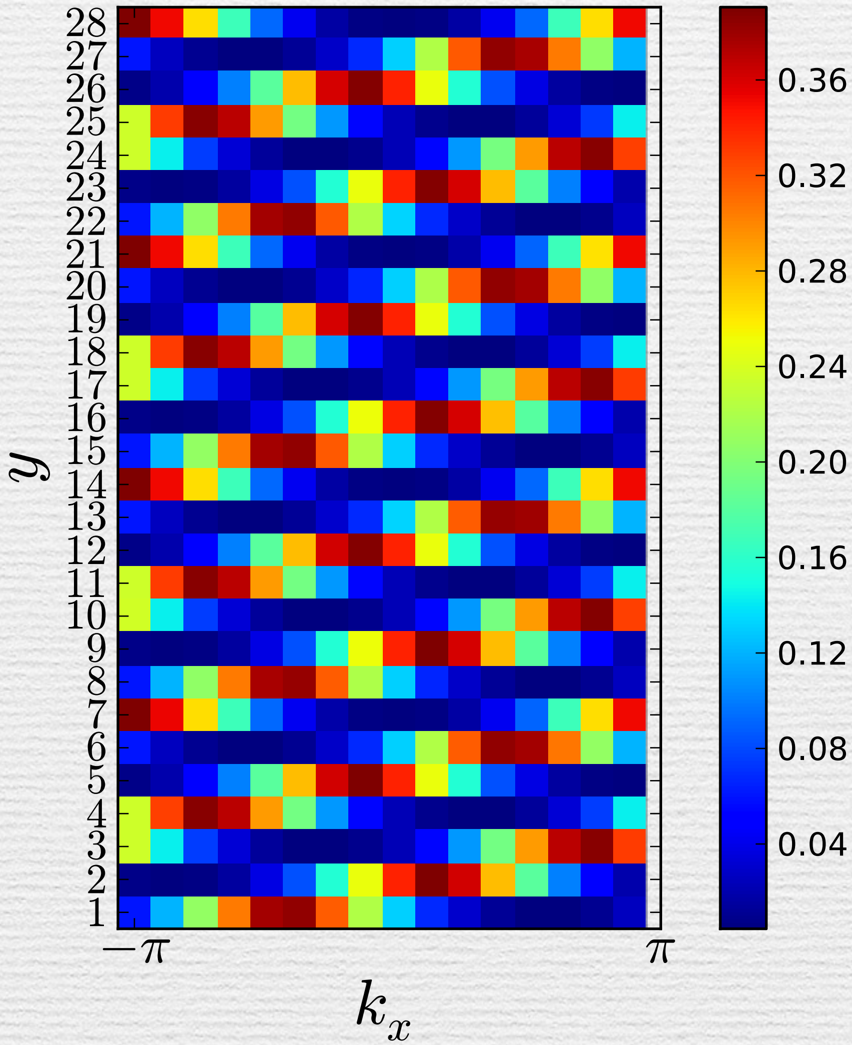
Hybrid densities



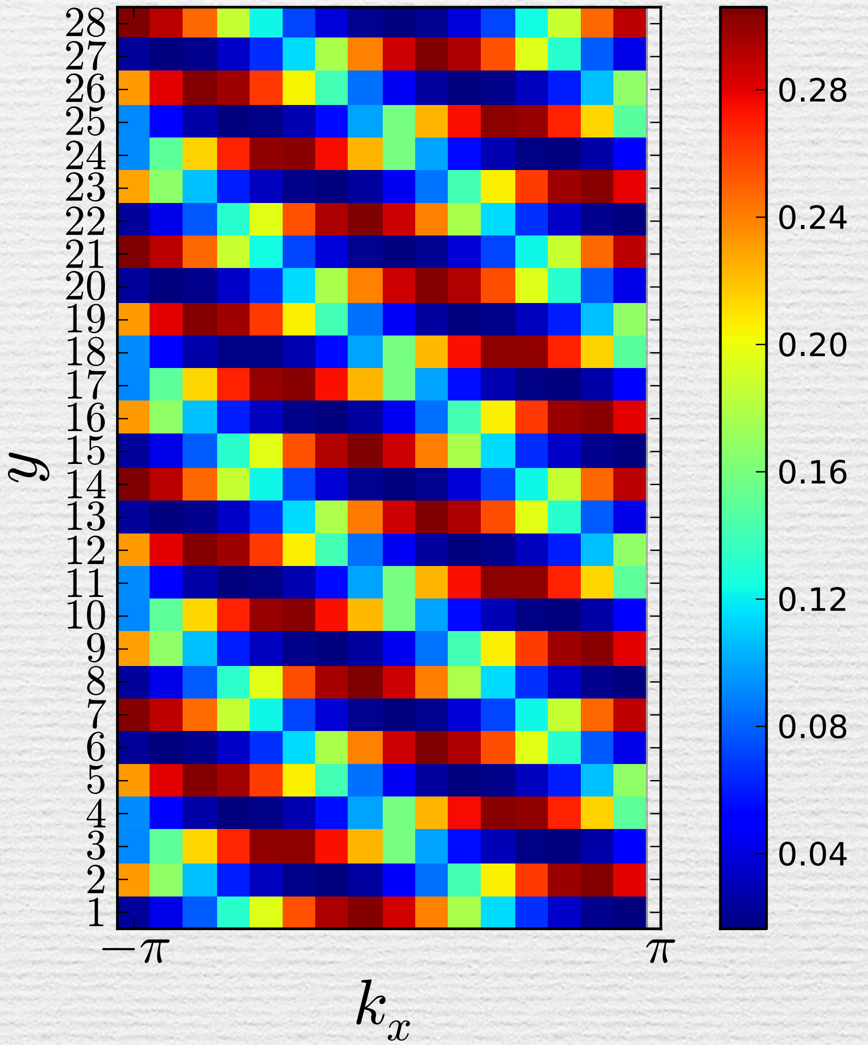
$\nu = 1$



$\nu = 1/2$



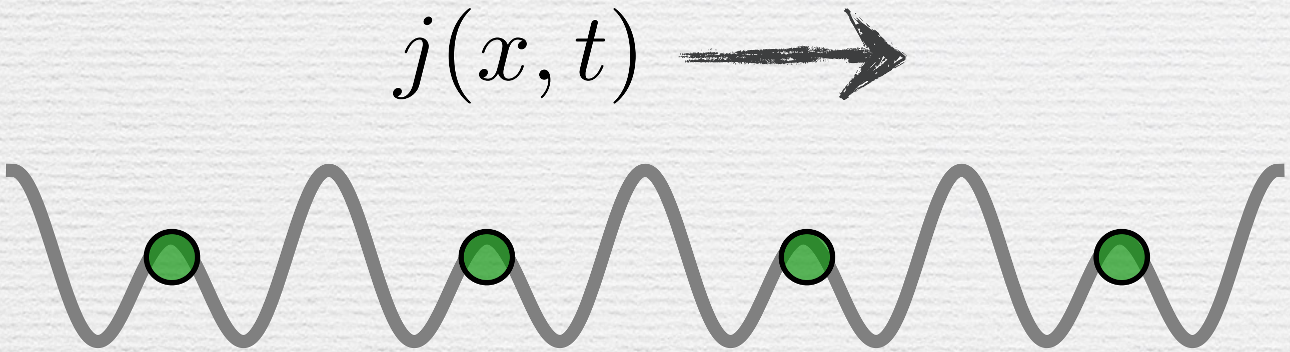
$\nu = 1/3$

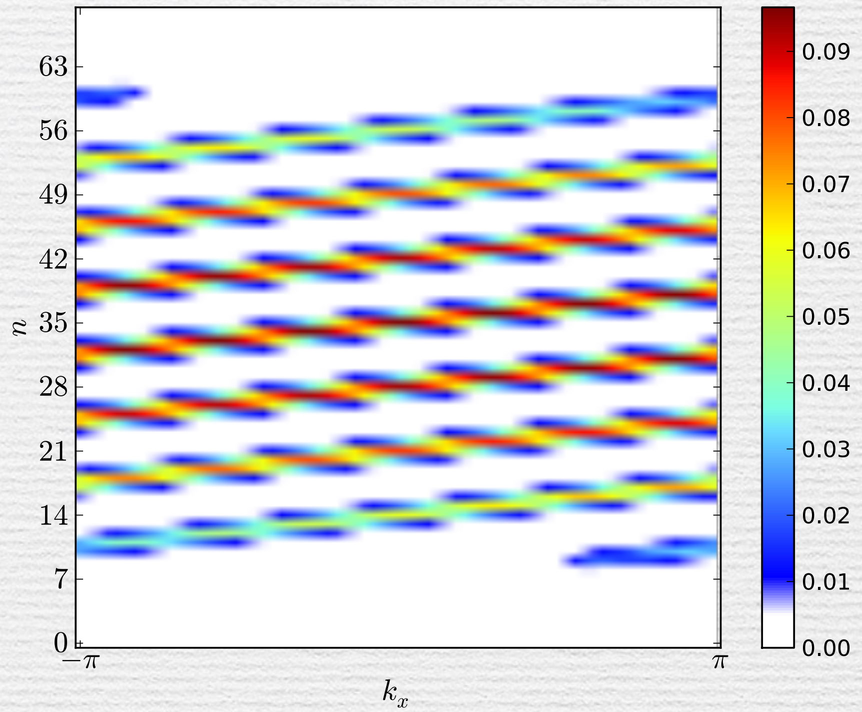


HTOF is also useful to detect FQHE state !

Summary

arXiv:1301.7435
PRL 110, 166802





Topological charge pumping is a common thread unifies many features of topological states

Guideline for design and detection of topological phases in cold atom systems

Summary

arXiv:1301.7435
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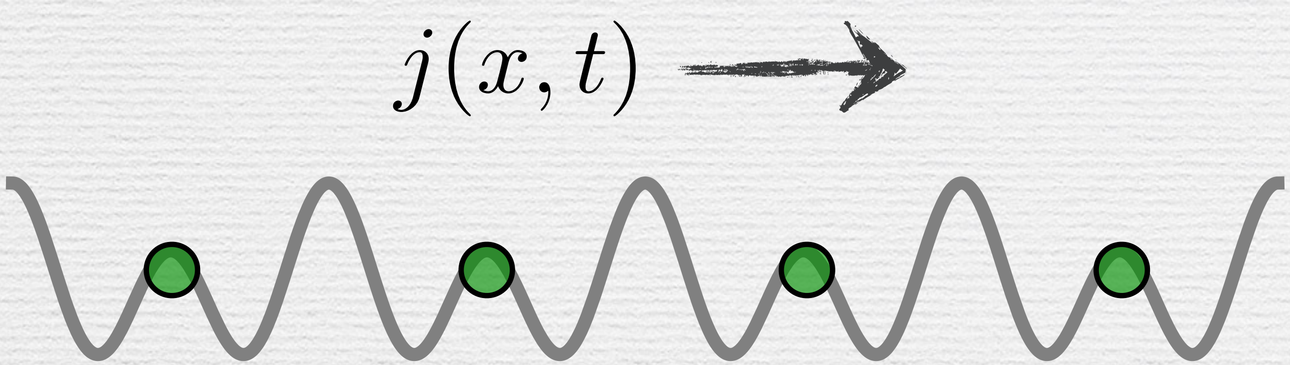
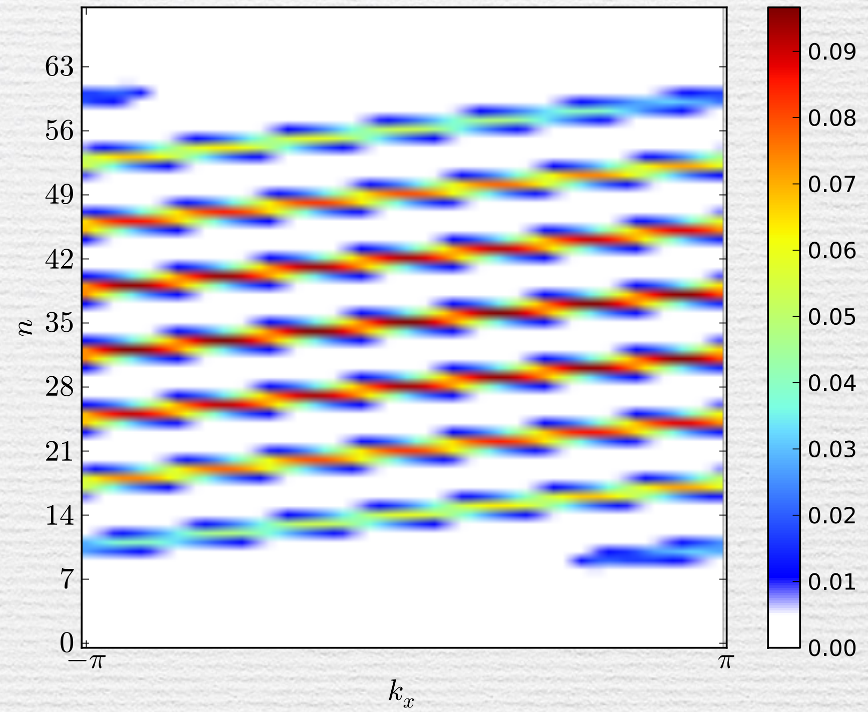


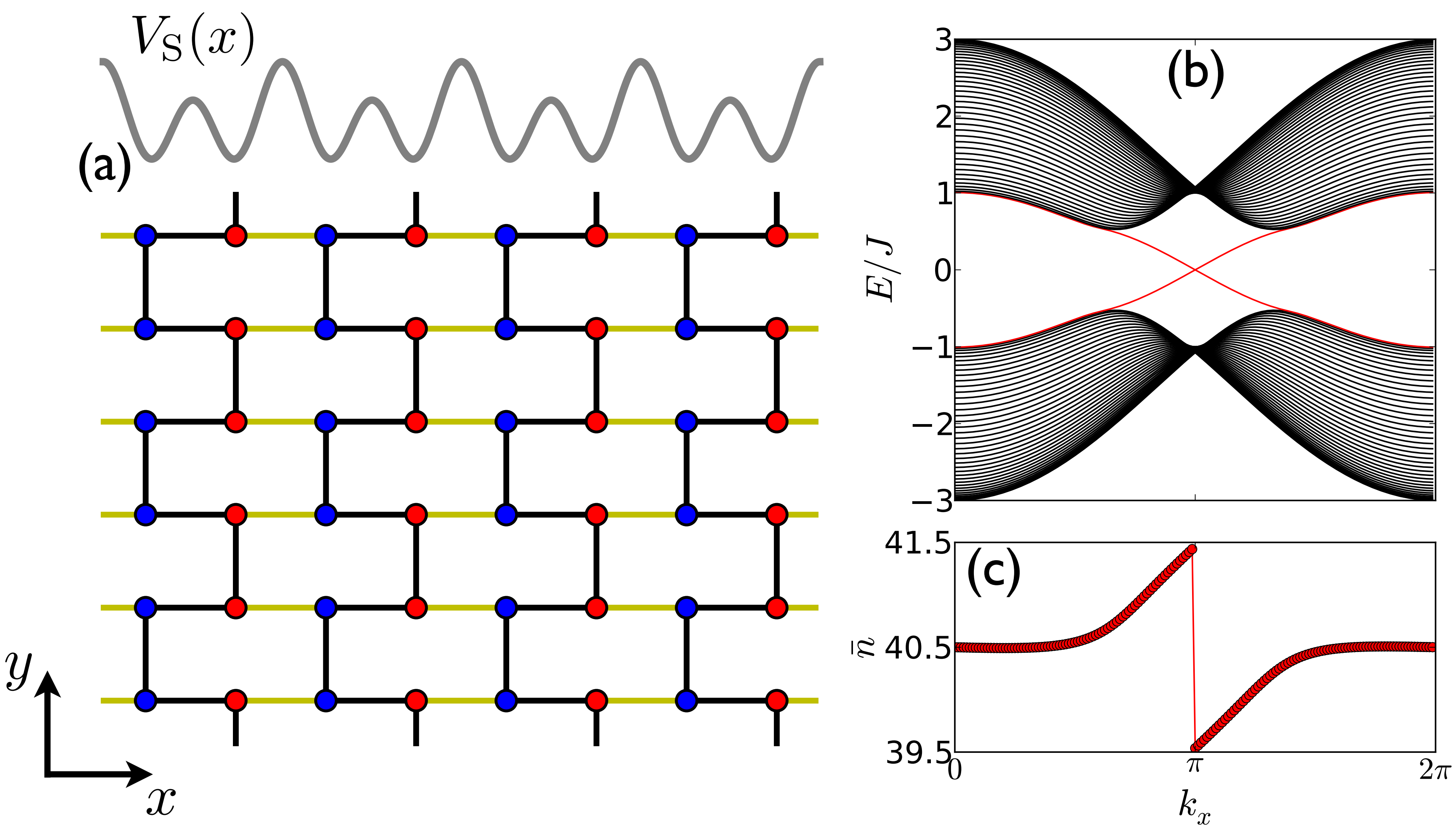


Topological charge pumping is a common thread unifies many features of topological states

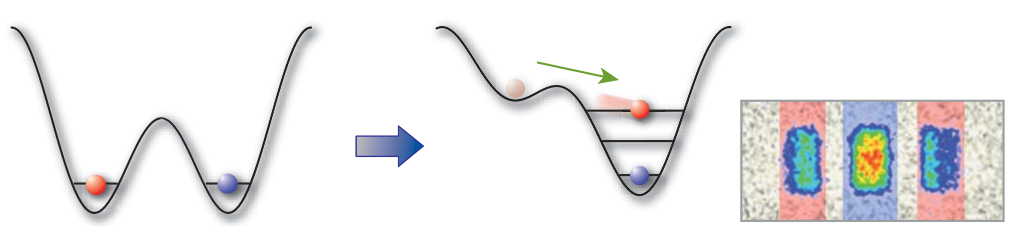
Guideline for design and detection of topological phases in cold atom systems

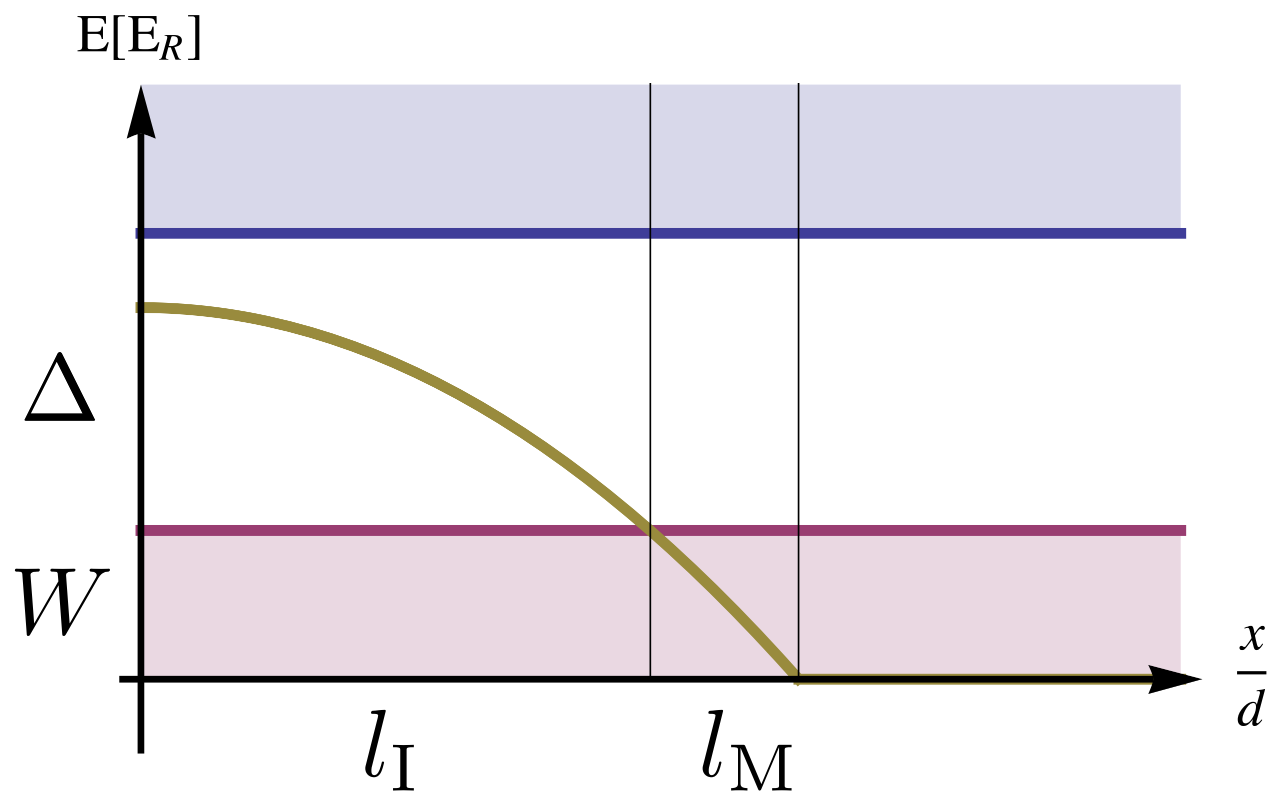
You might try it in your lab !

Thank you!



$V_{\mathrm{S}}(x)$
(a)
y
x
(b)
E/J
3
2
1
0
−1
−2
−3
(c)
$\bar{n}$
41.5
40.5
39.5
0
π
2π
k_x

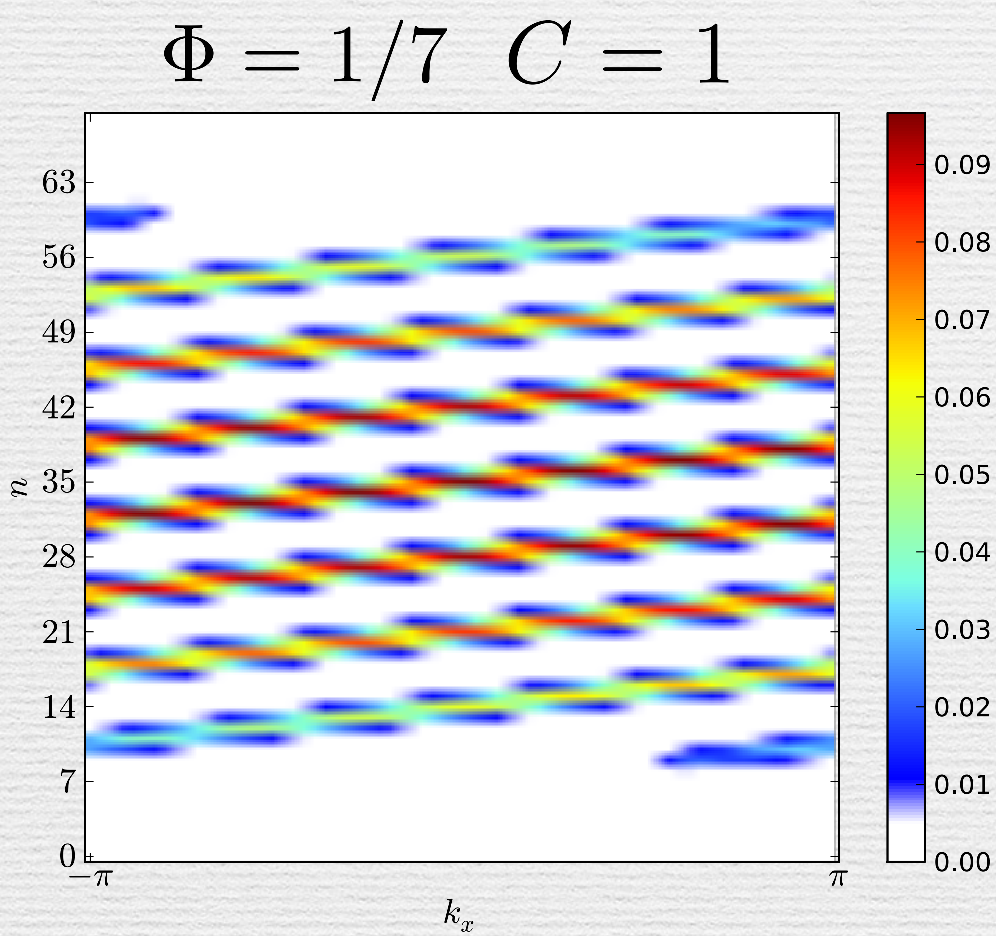


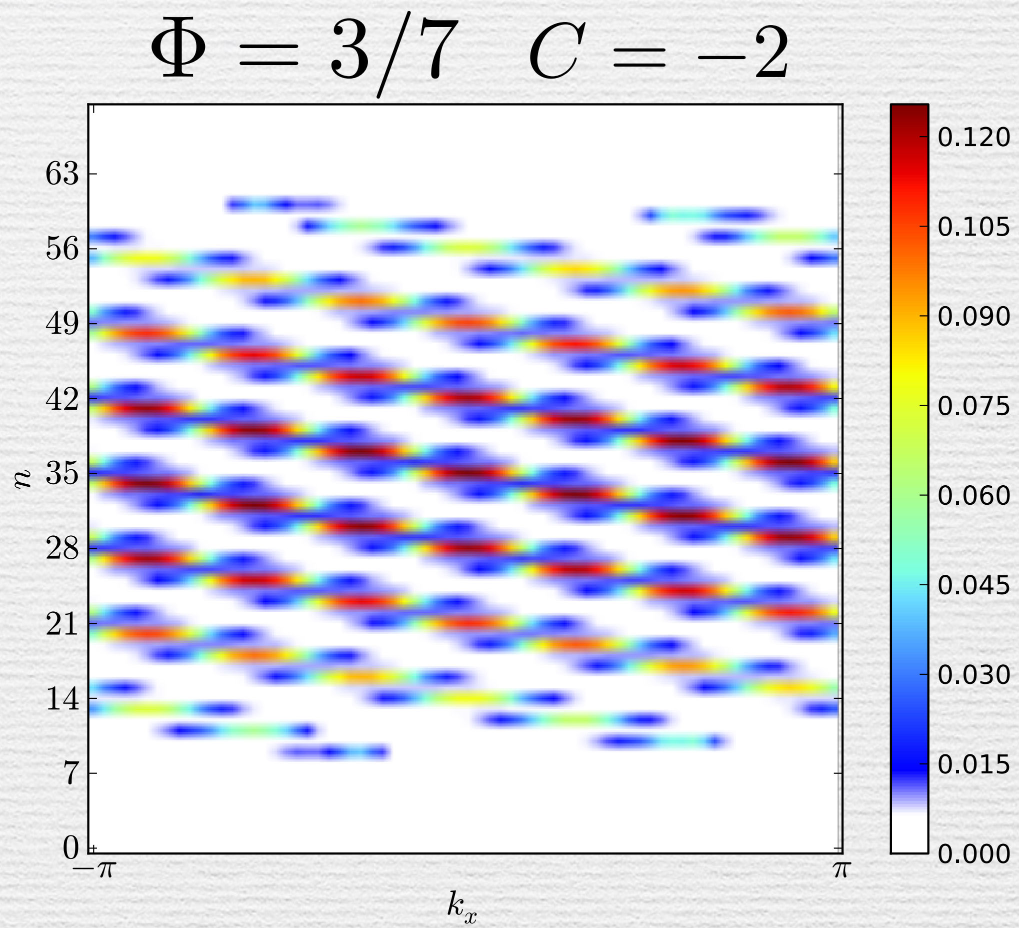


E[E$_R$]
Δ
W
$\frac{x}{d}$
l_{I}
l_{M}

Theoretical Proposal

Wang, Soluyanov and Troyer, PRL 2013

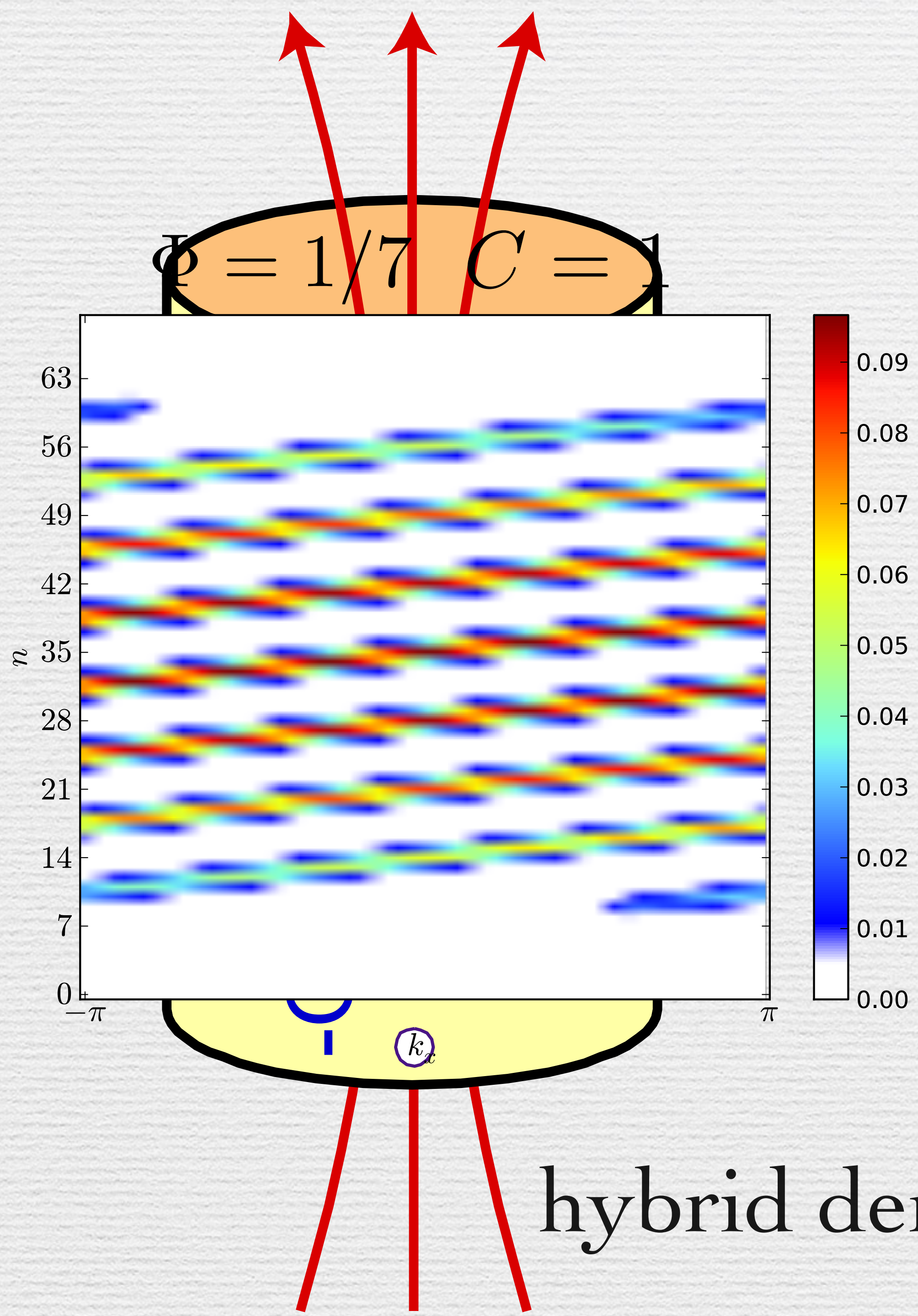


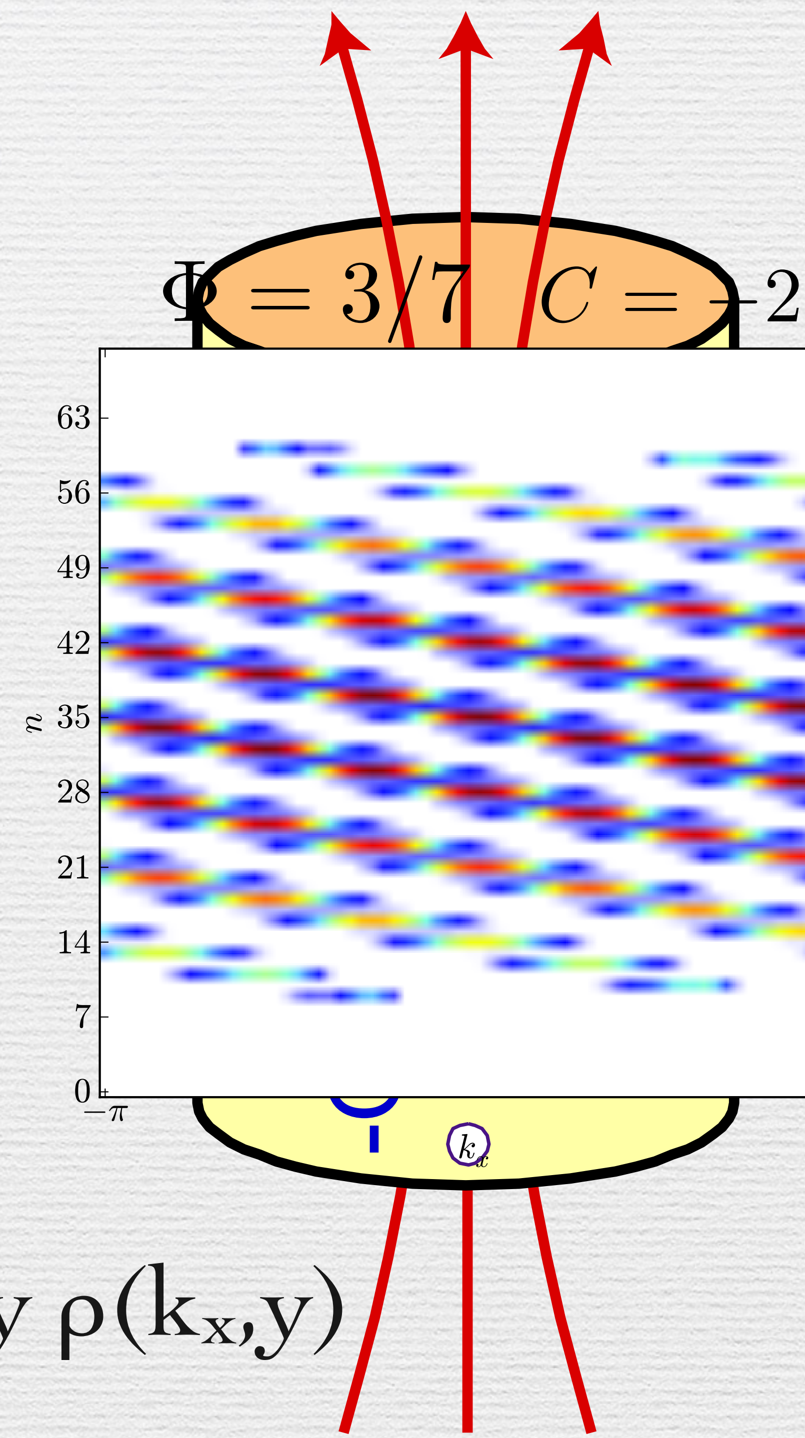


hybrid density $\rho(k_x,y)$

Theoretical Proposal

Wang, Soluyanov and Troyer, PRL 2013



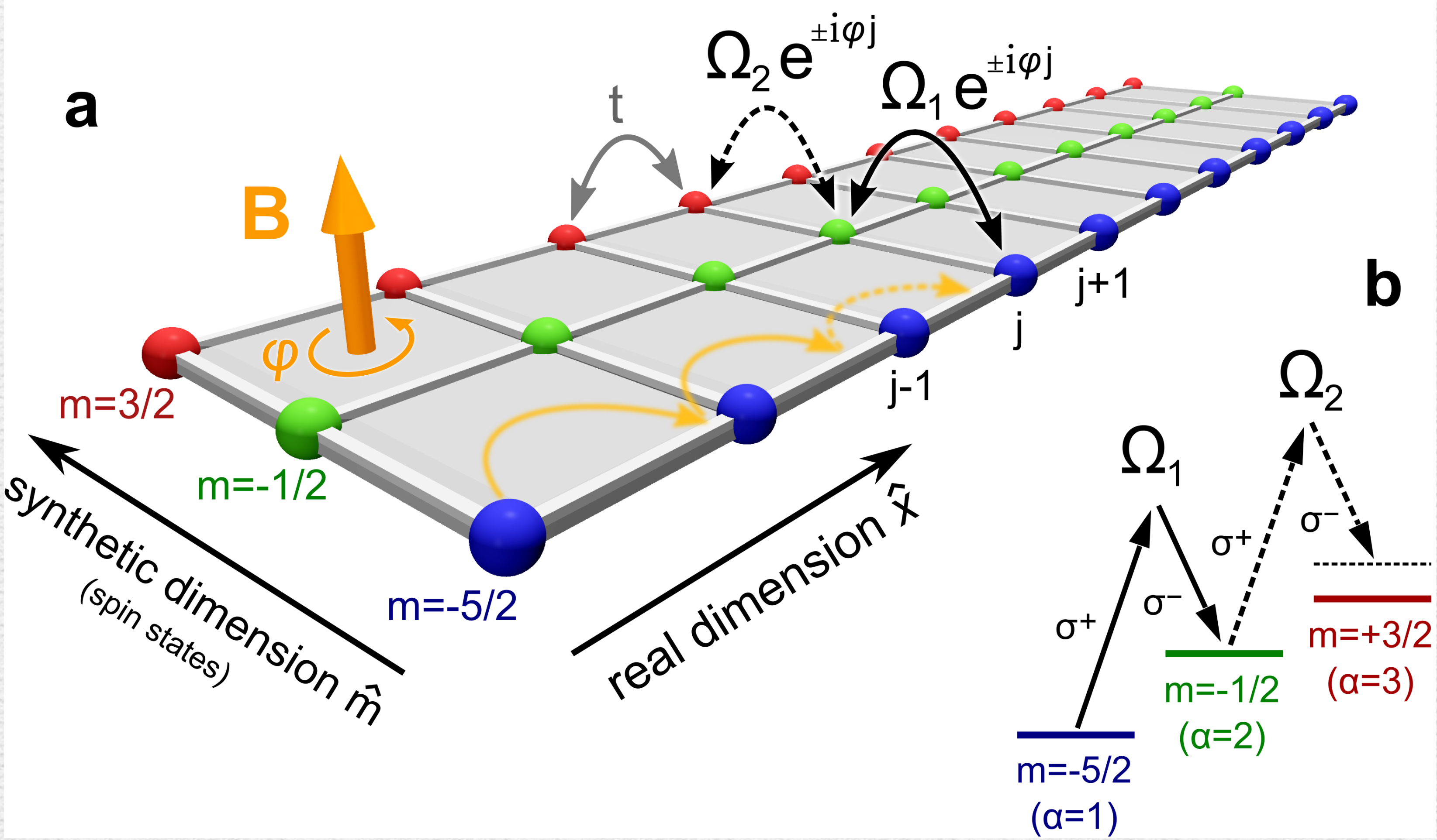


hybrid density $\rho(k_x,y)$

QHE in synthetic dimension

Mancini et al, 1502.02495

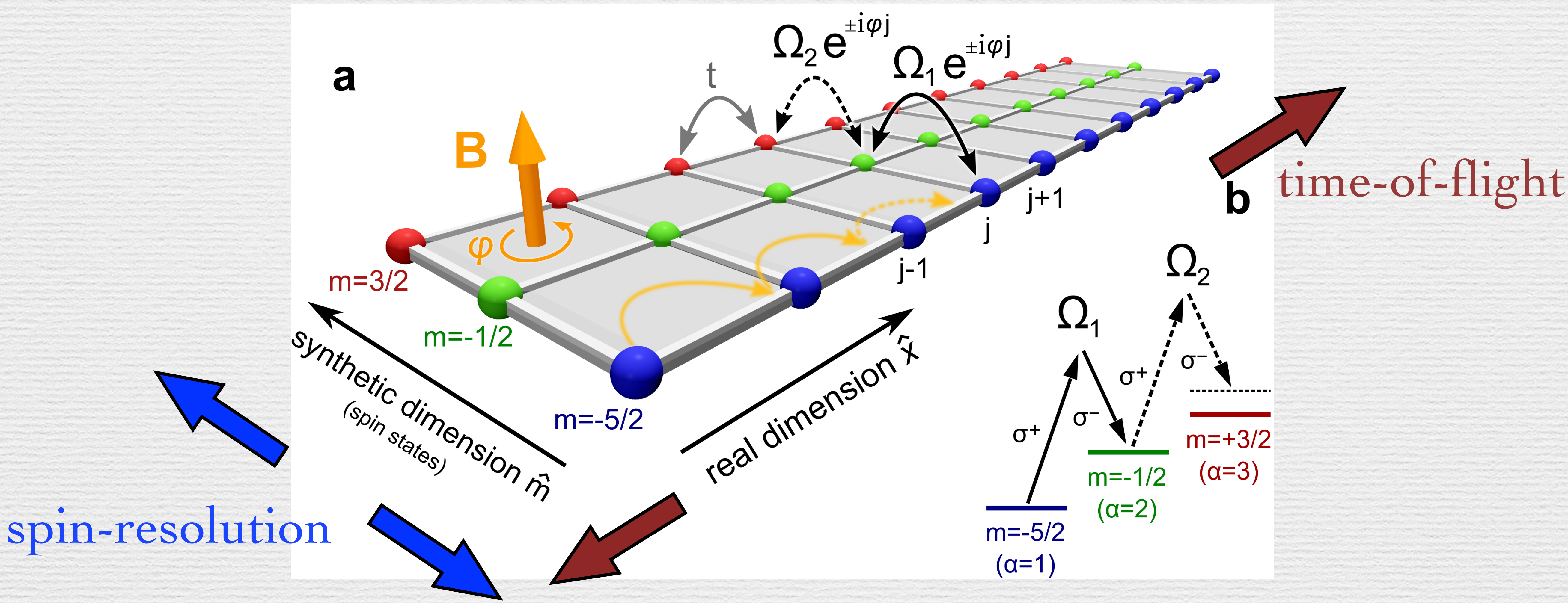
Stuhl et al, 1502.02496



QHE in synthetic dimension

Mancini et al, 1502.02495

Stuhl et al, 1502.02496



QHE in synthetic dimension

Mancini et al, 1502.02495

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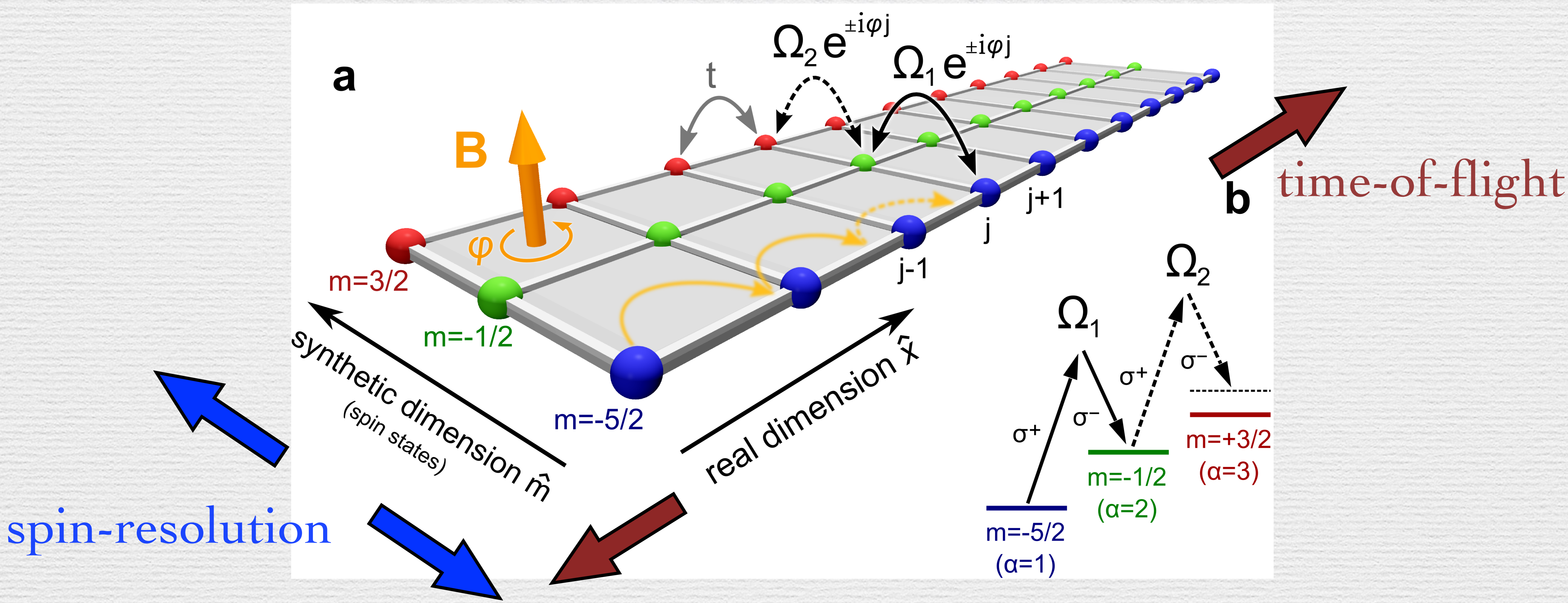
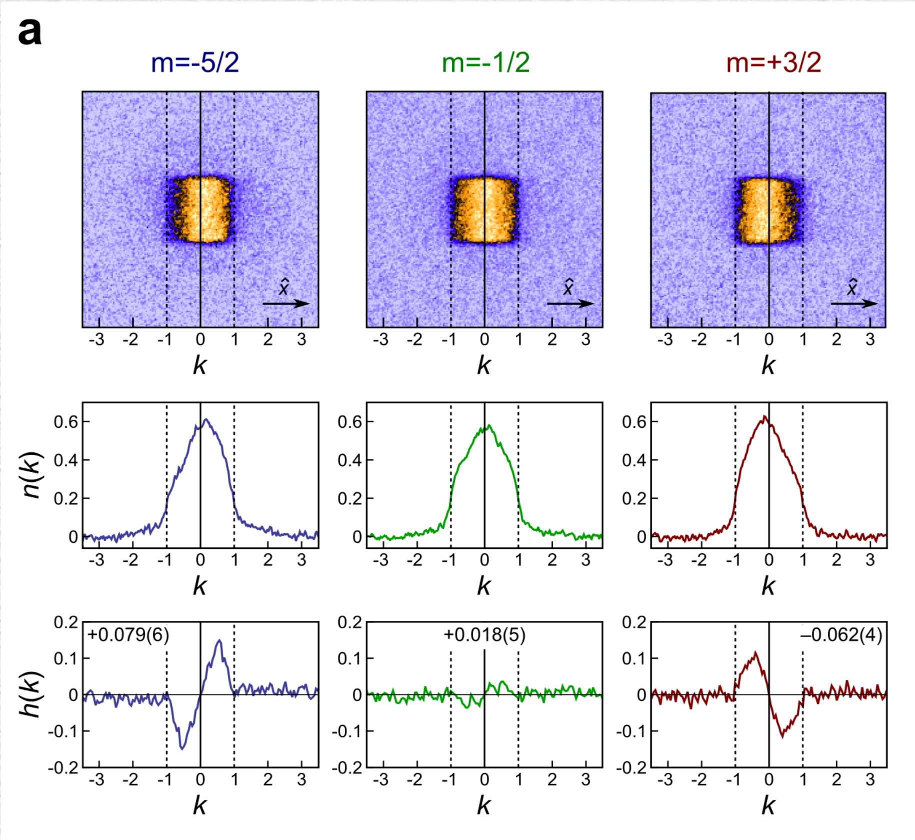


Time-of-flight measures the hybrid density $\rho(k_x, y)$

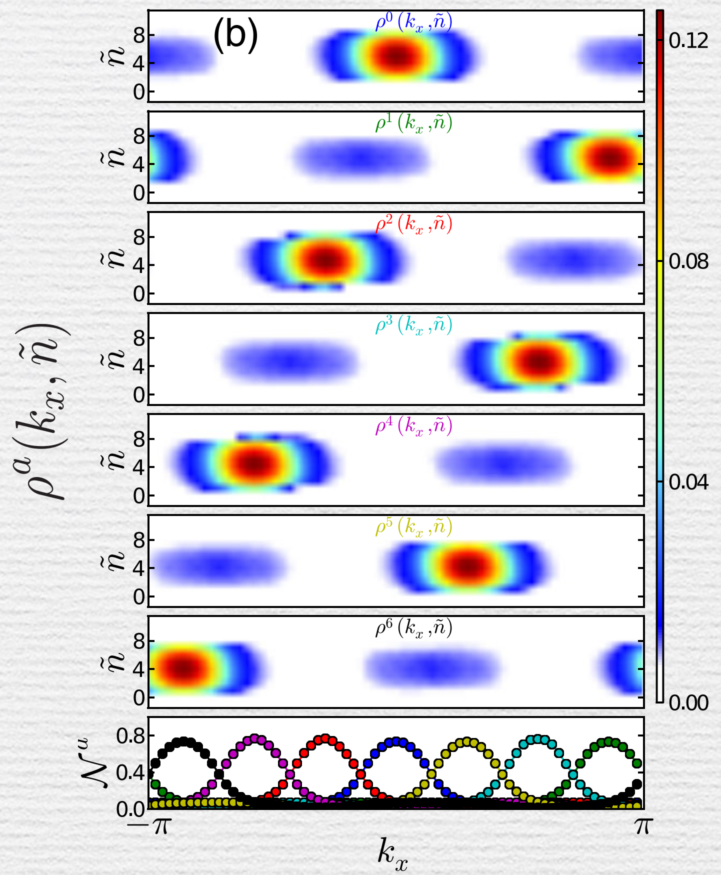
by interpreting the spin states as the spatial dimension

Hybrid density reveals topology



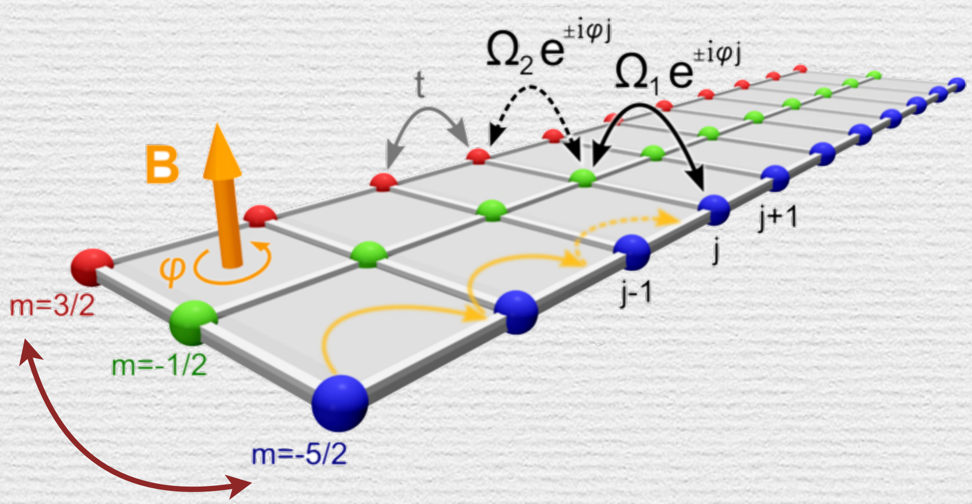
Mancini et al, 1502.02495

spin-resolved densities

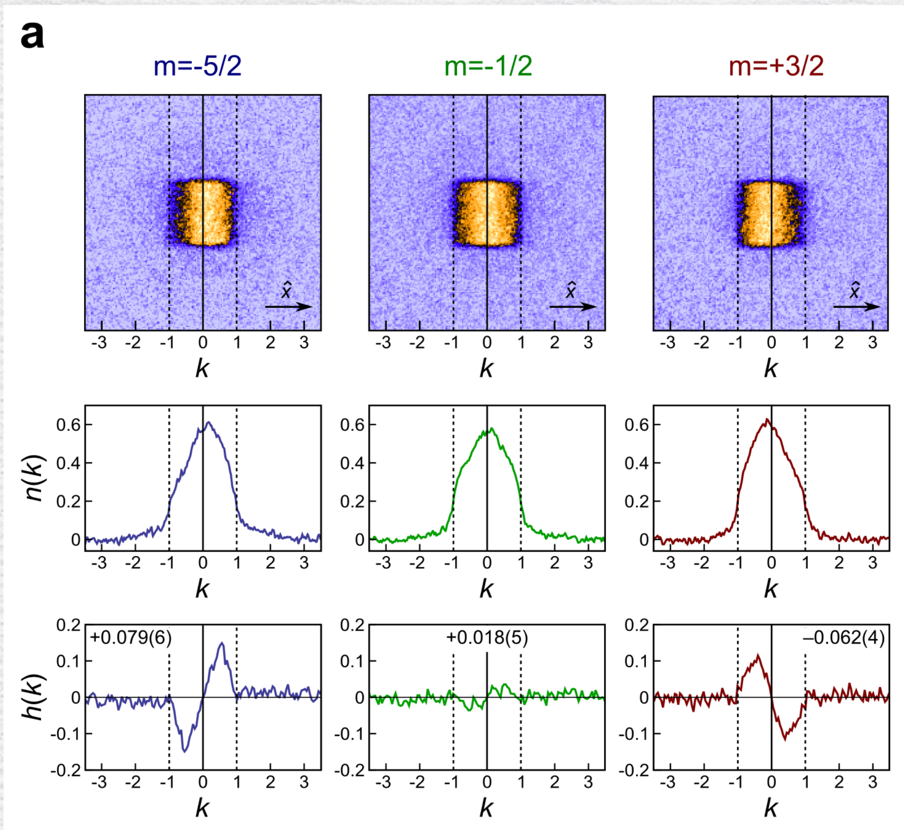


Wang, Soluyanov, Troyer, PRL 2013

sub-lattice densities

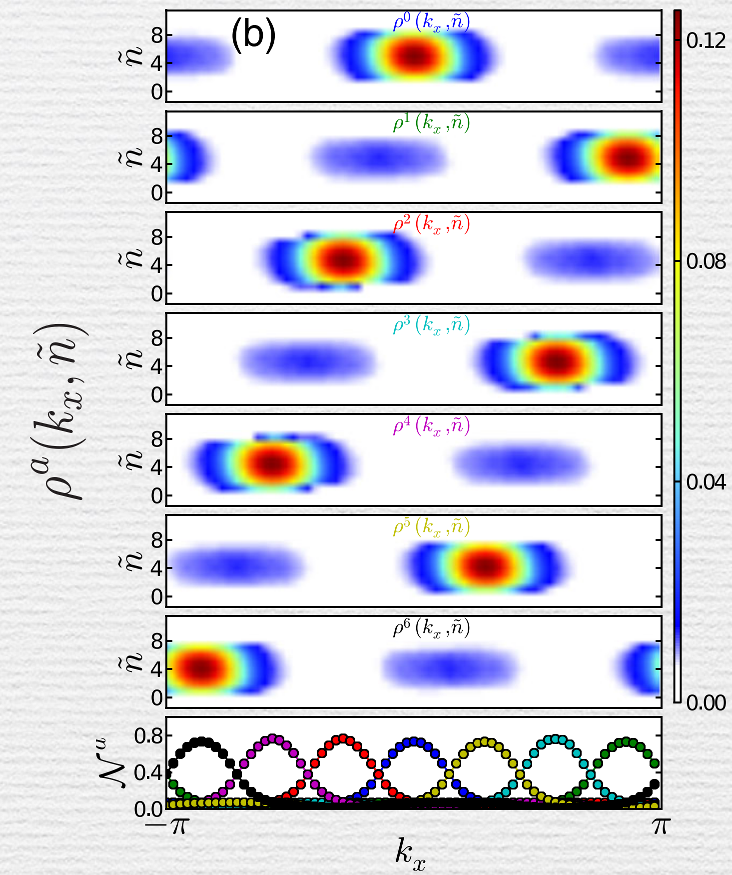


Hybrid density reveals topology



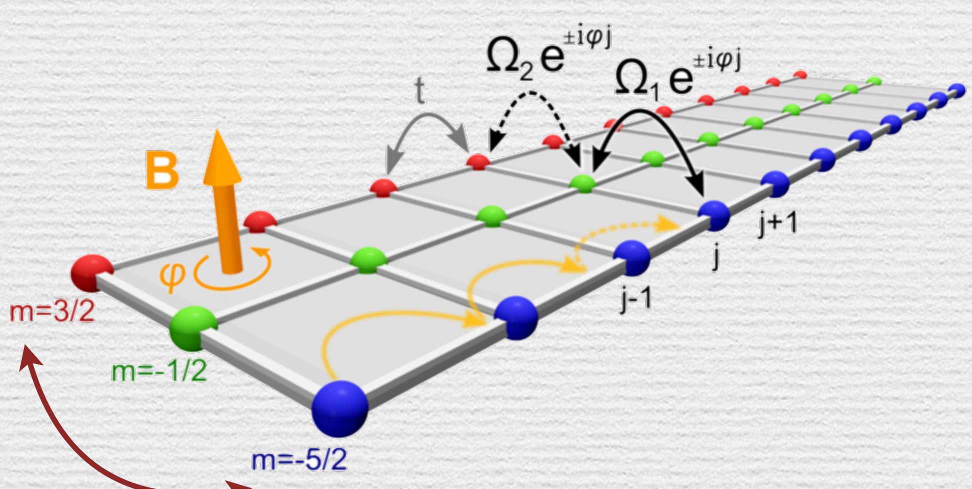
Mancini et al, 1502.02495

spin-resolved densities



Wang, Soluyanov, Troyer, PRL 2013

sub-lattice densities



the signal persists
even if it was a tube