

第四届全国统计物理与复杂系统学术会议暨海峡两岸统计物理会议

2017.7.16-19 陕西师范大学

# Machine Learning for Many-Body Physics

Lei Wang (王磊)

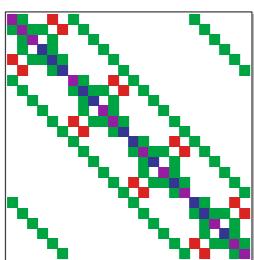
Institute of Physics, CAS

<https://wangleiphy.github.io>

# About me

2006	Bachelor	Nanjing University
2011	PhD	IOP, CAS
2016	Postdoctor	ETH Zurich

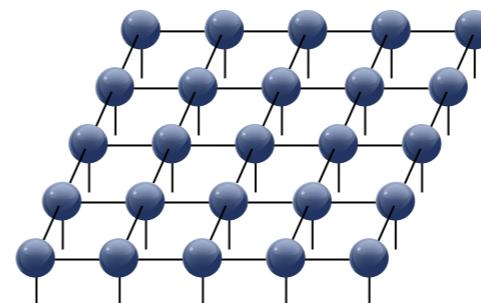
## Computational Quantum Physics



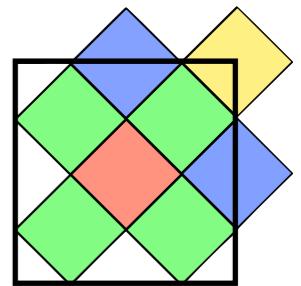
**exact  
diagonalization**



**quantum  
Monte Carlo**



**tensor network  
states**

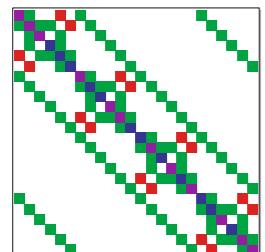


**dynamical mean  
field theories**

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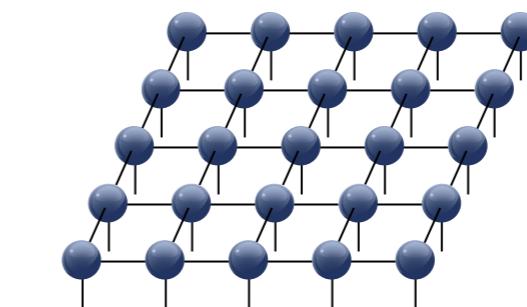
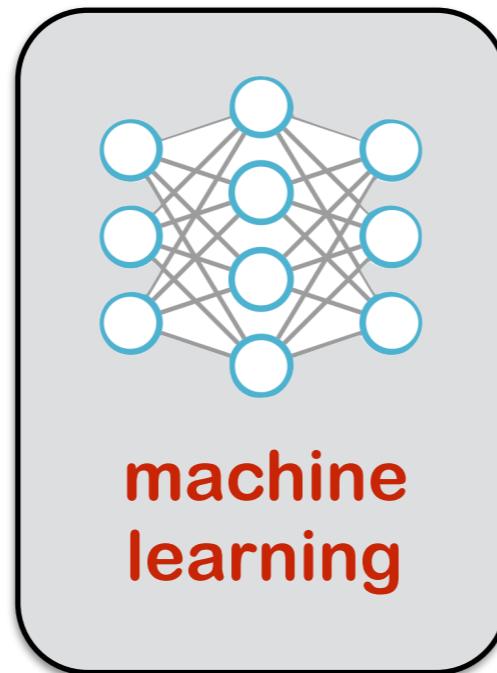
## Computational Quantum Physics



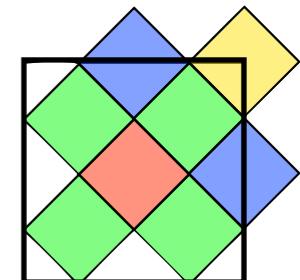
exact  
diagonalization



quantum  
Monte Carlo



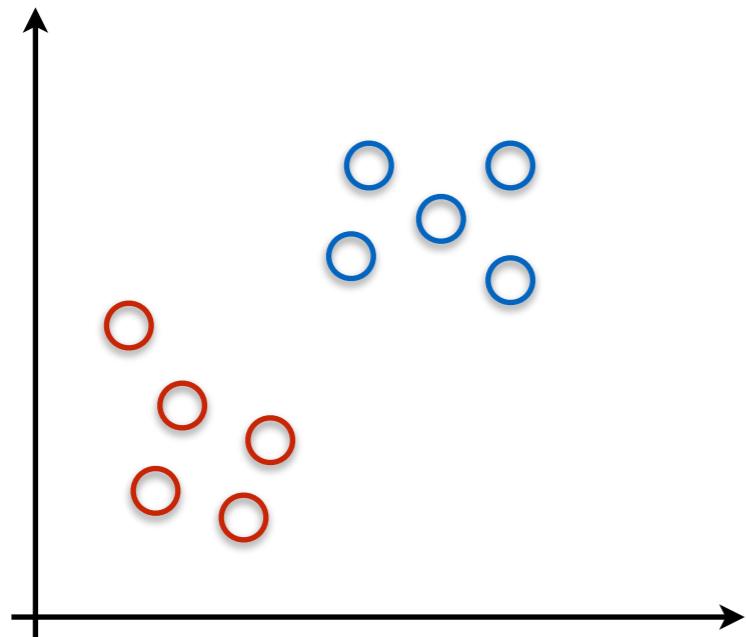
tensor network  
states



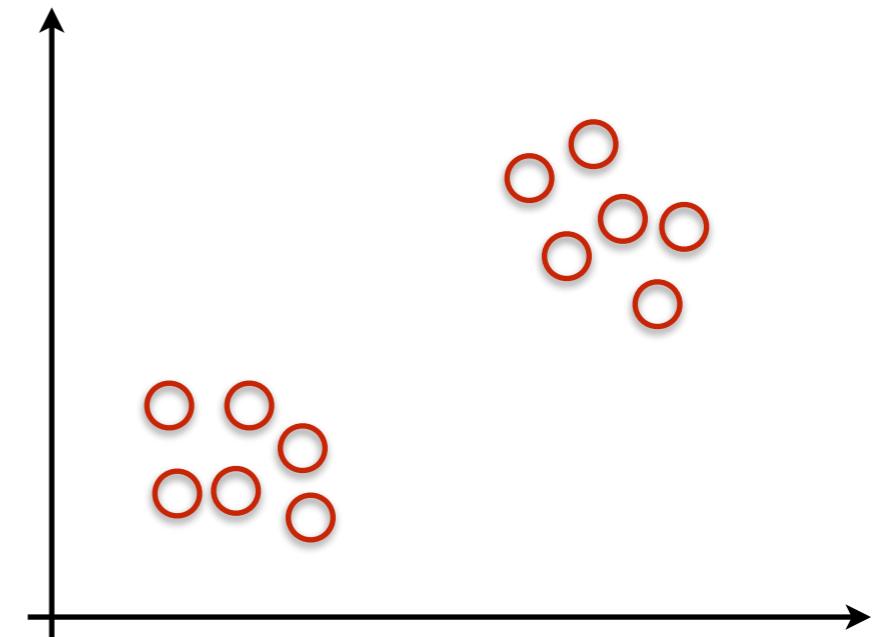
dynamical mean  
field theories

# Machine Learning 101

Supervised learning

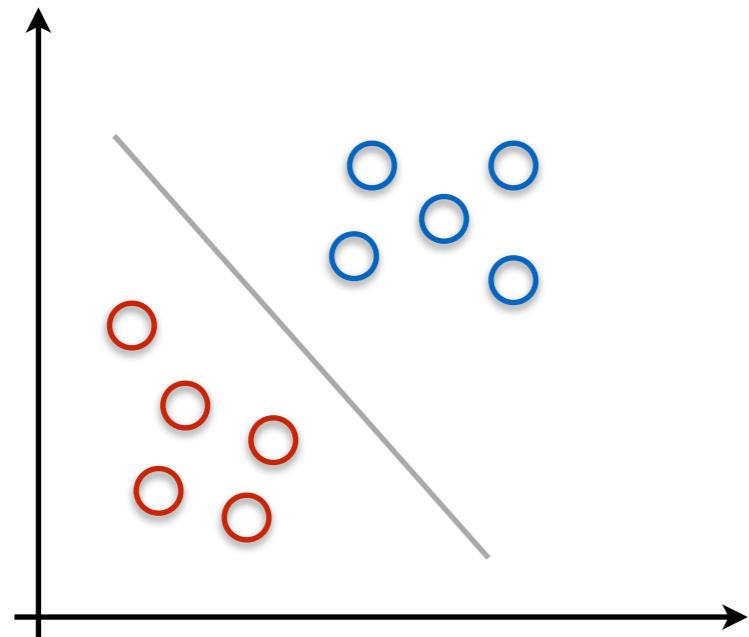


Unsupervised learning

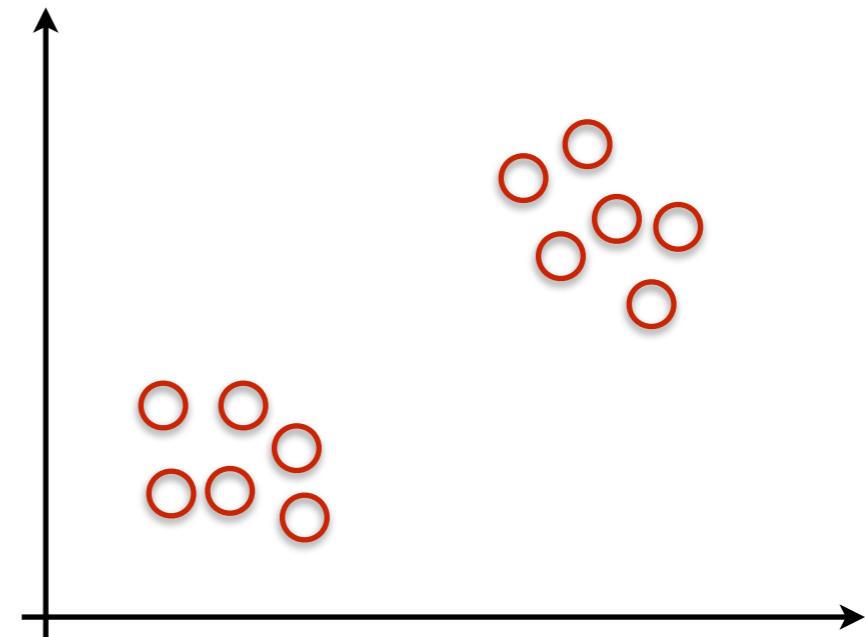


# Machine Learning 101

Supervised learning

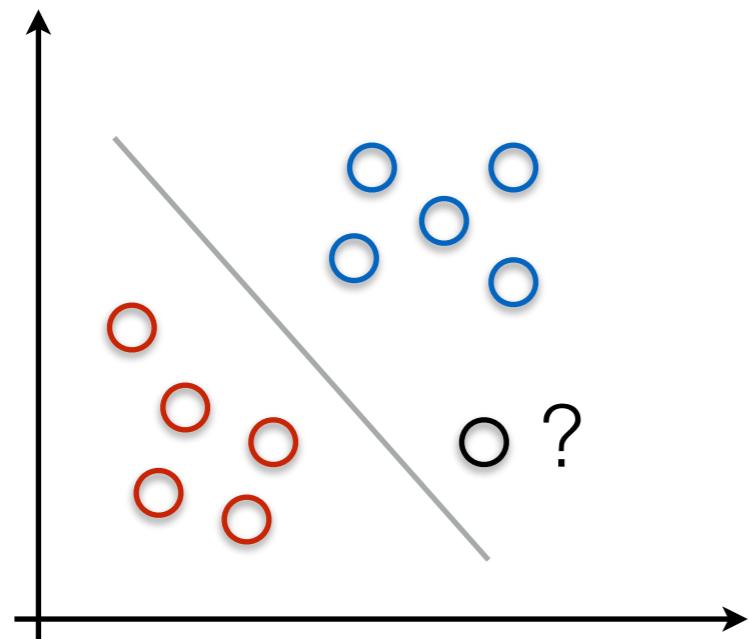


Unsupervised learning

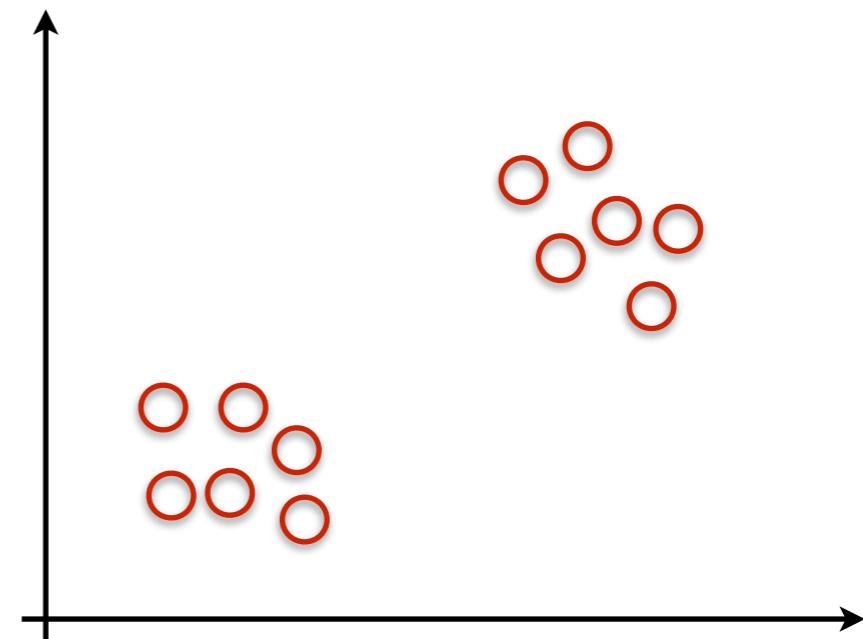


# Machine Learning 101

Supervised learning

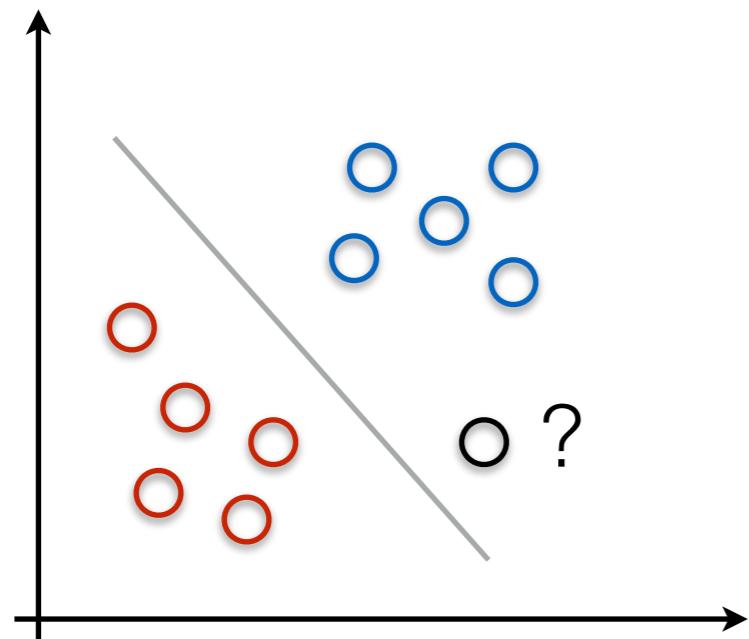


Unsupervised learning

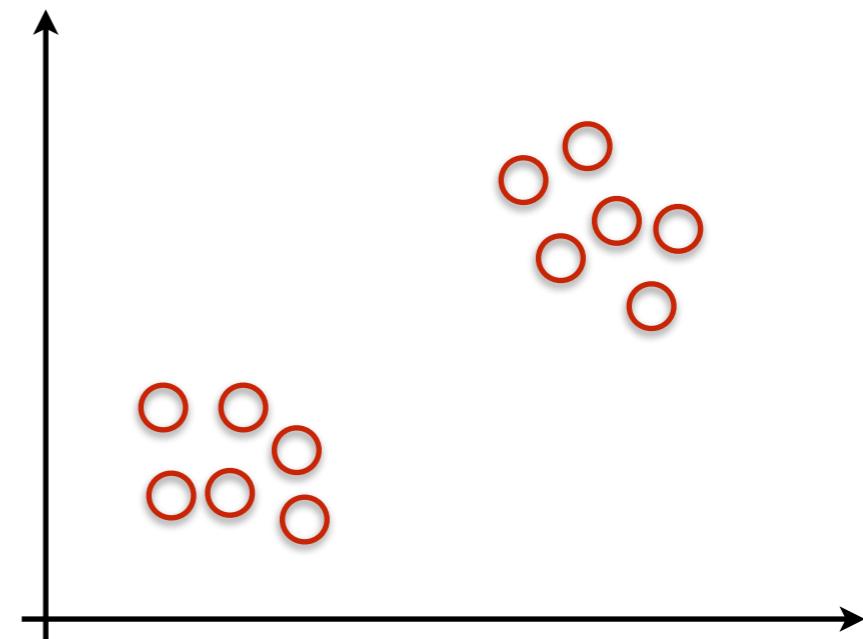


# Machine Learning 101

Supervised learning



Unsupervised learning



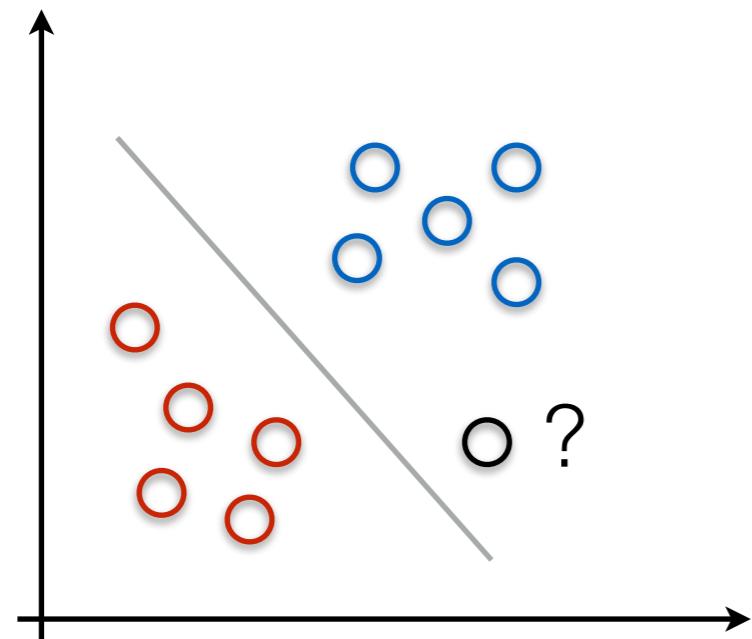
## Classification

Spam detection

Image recognition

# Machine Learning 101

Supervised learning

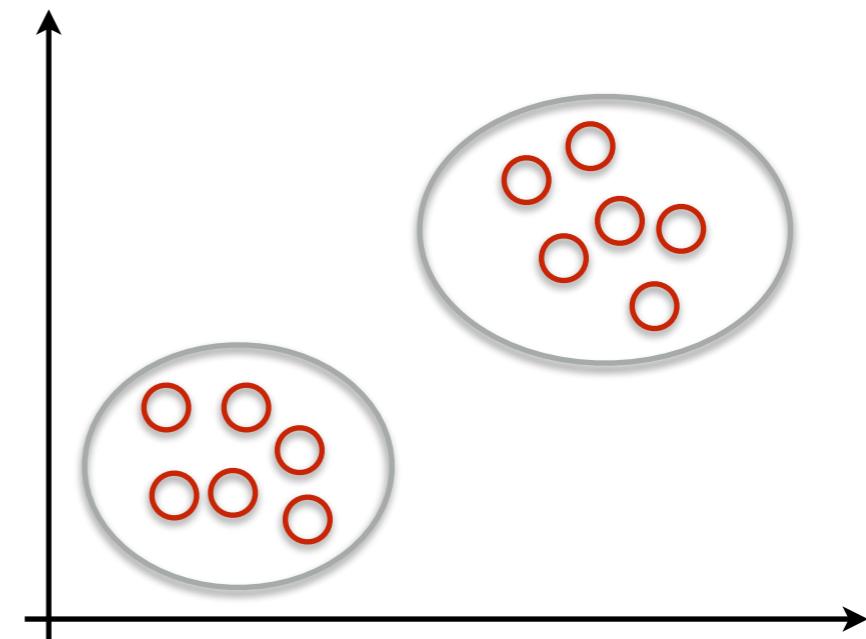


**Classification**

Spam detection

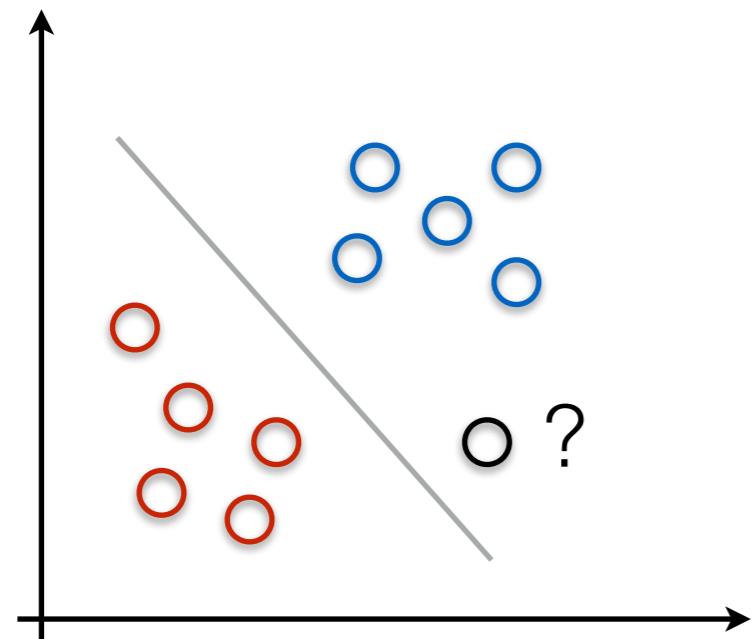
Image recognition

Unsupervised learning



# Machine Learning 101

Supervised learning

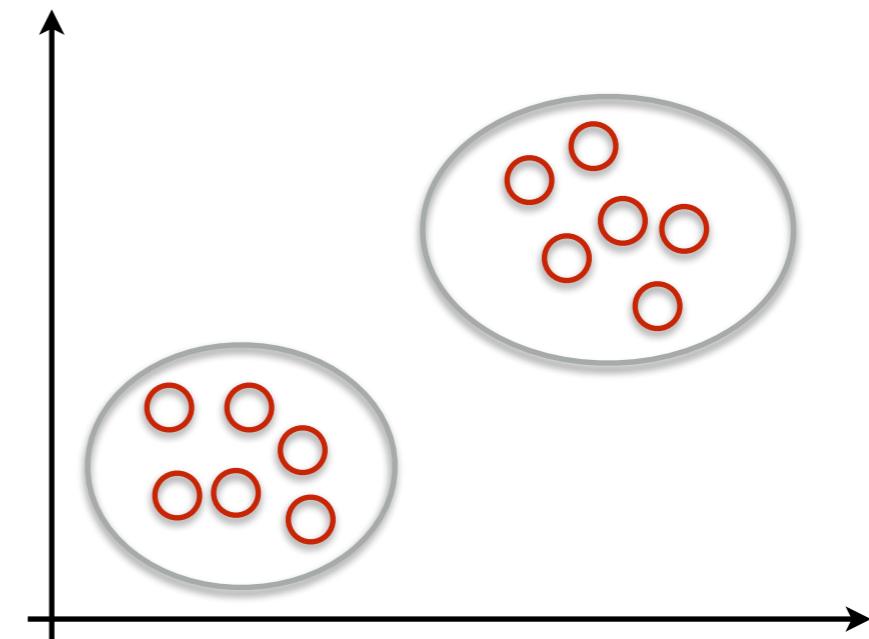


**Classification**

Spam detection

Image recognition

Unsupervised learning

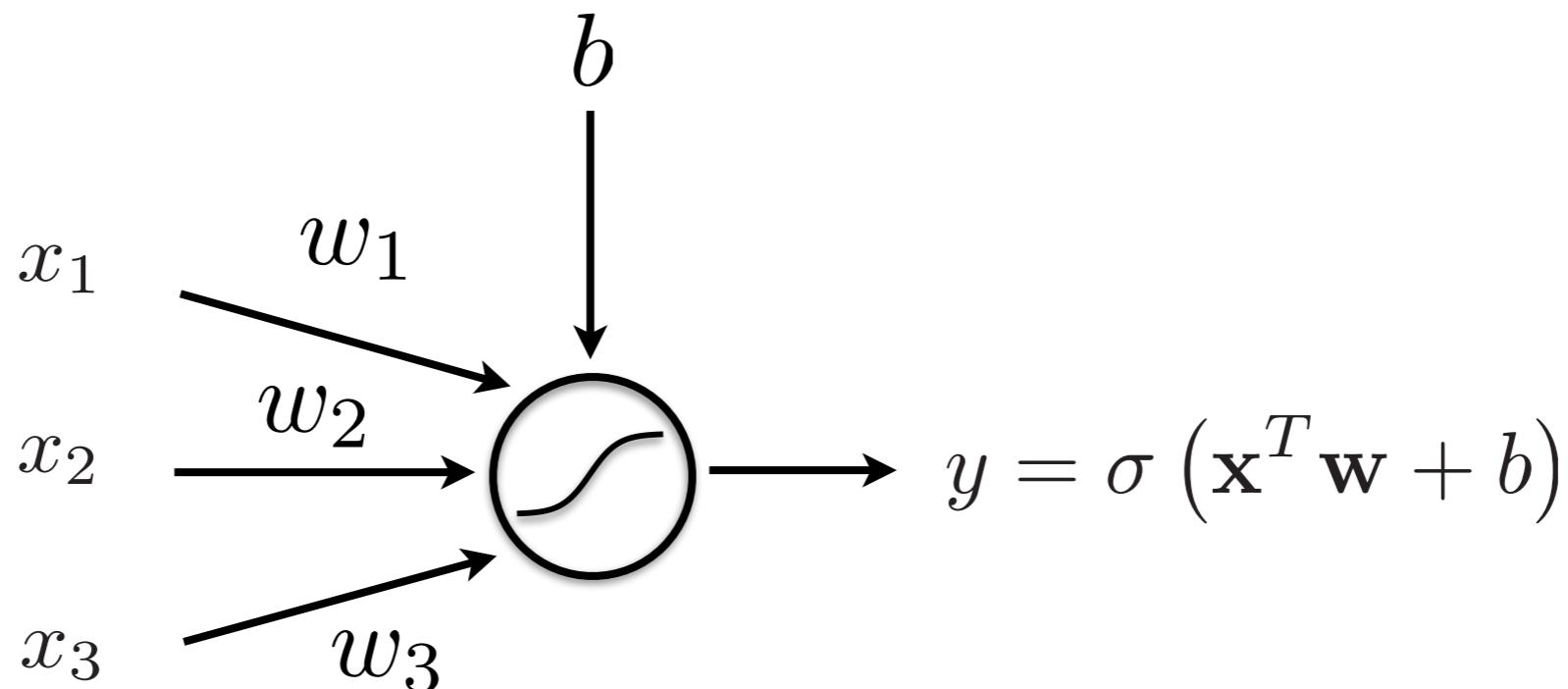


**Clustering**

Online advertising

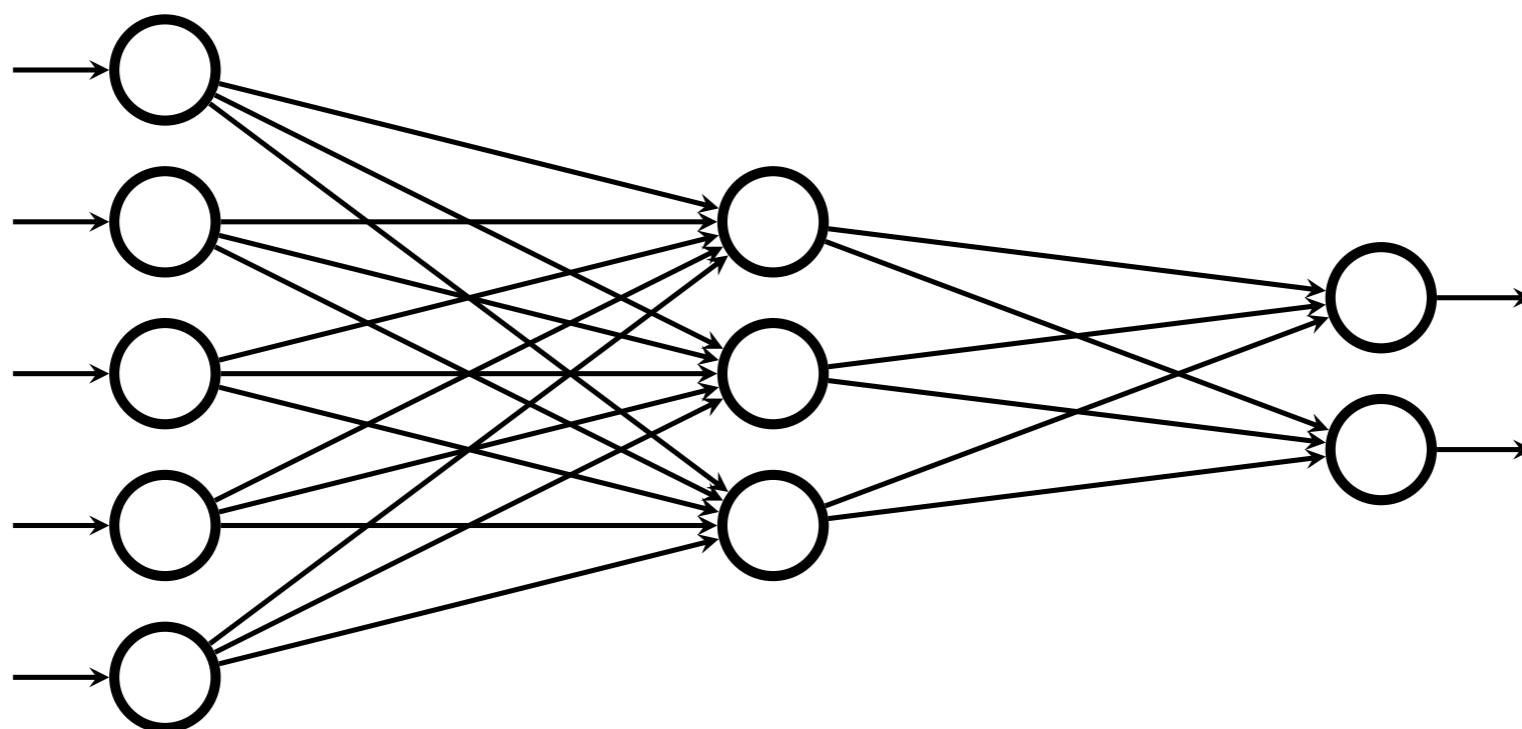
Anomaly detection

# Artificial Neural Networks



Computing unit: artificial neuron

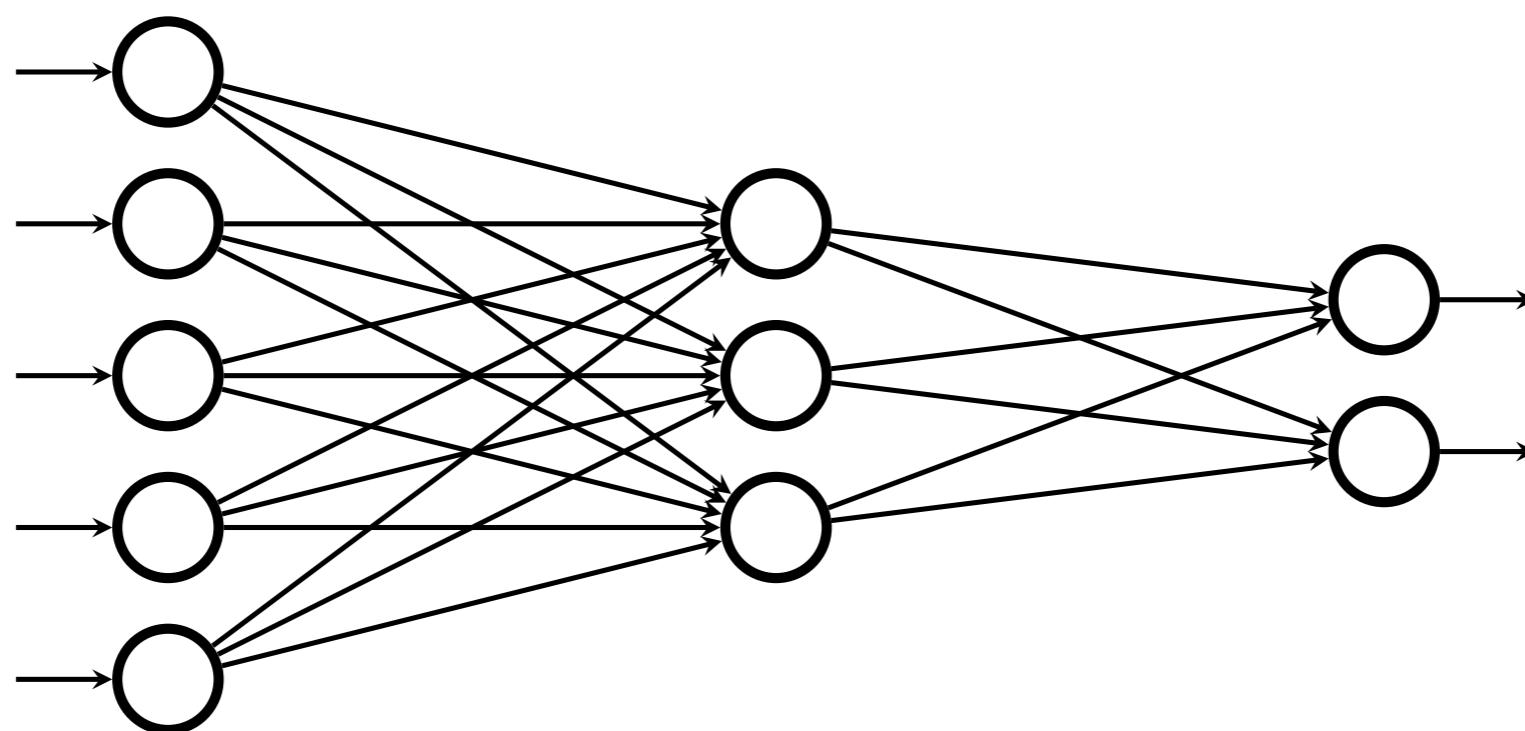
# Artificial Neural Networks



# Artificial Neural Networks



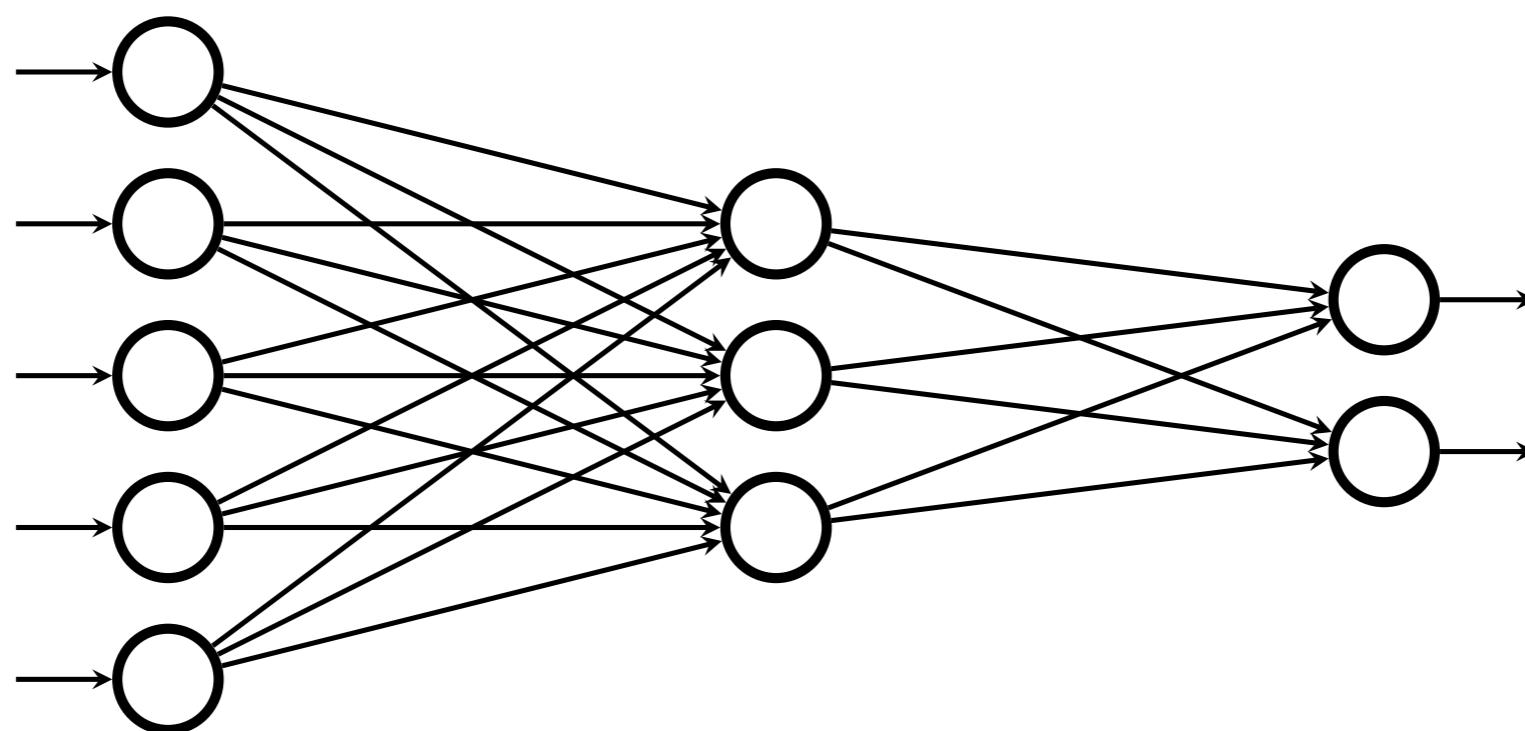
data



# Artificial Neural Networks



data



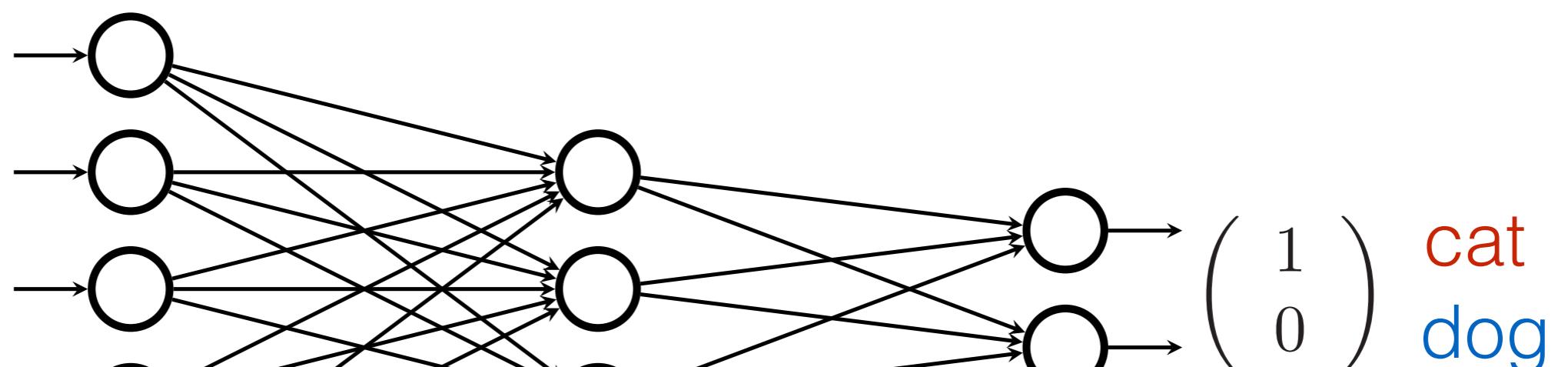
cat  
dog

label

# Artificial Neural Networks



data

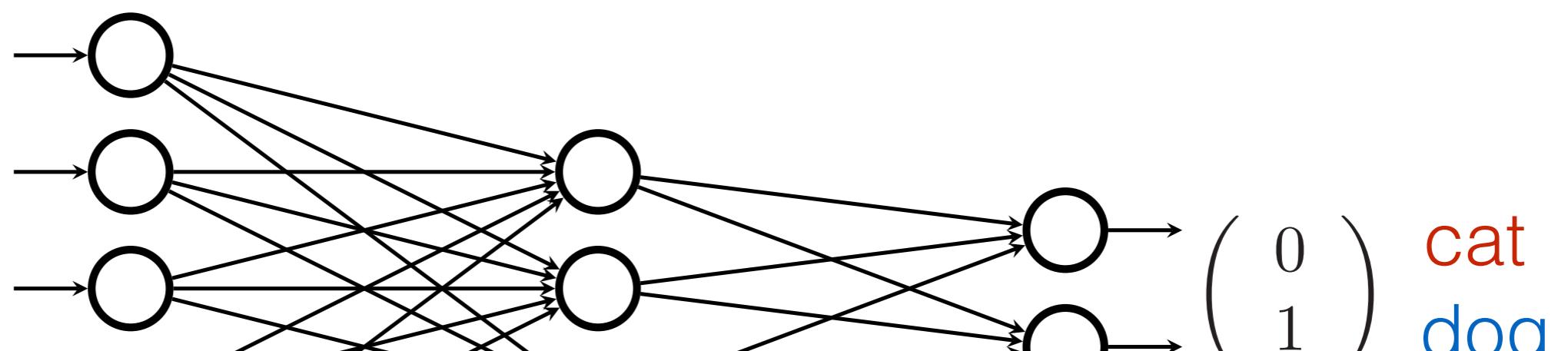


label

# Artificial Neural Networks

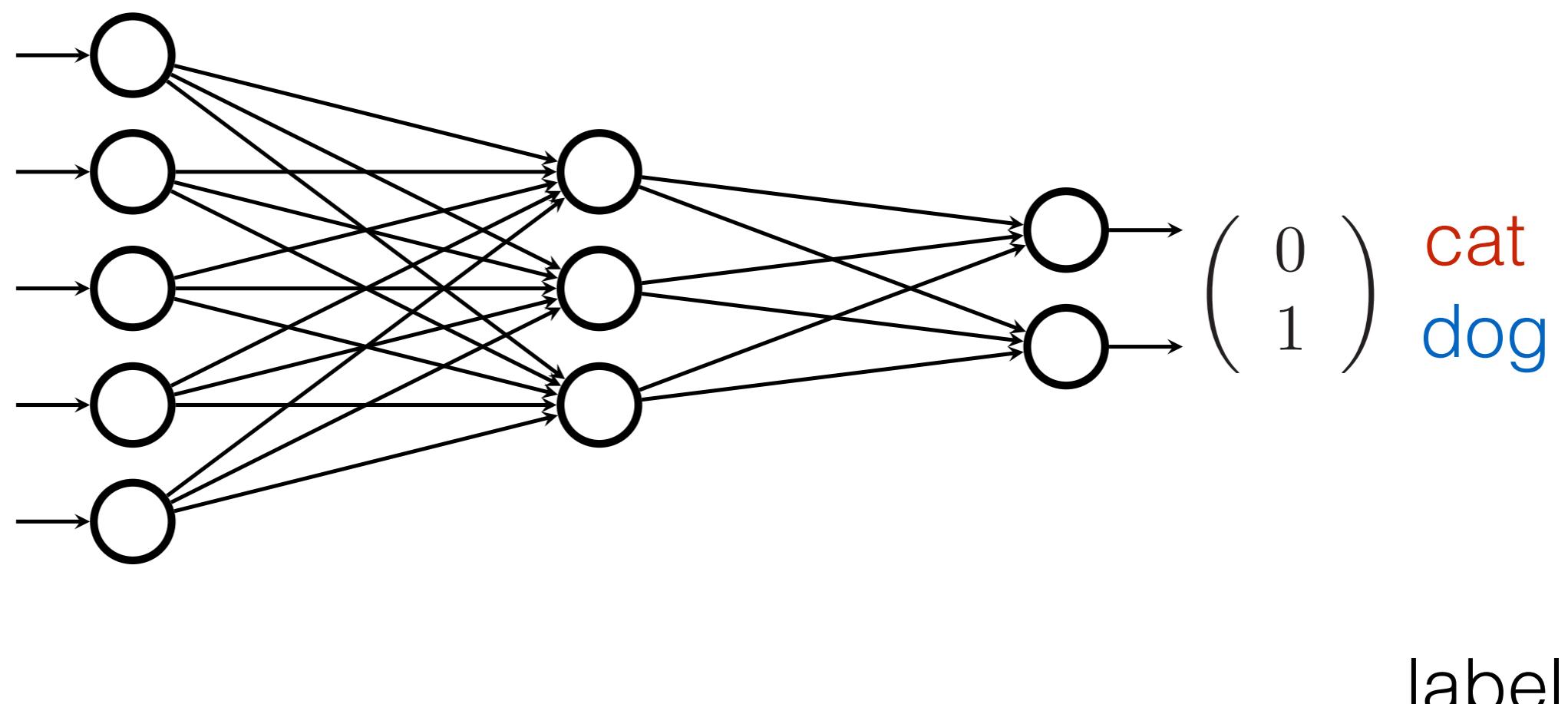


data



label

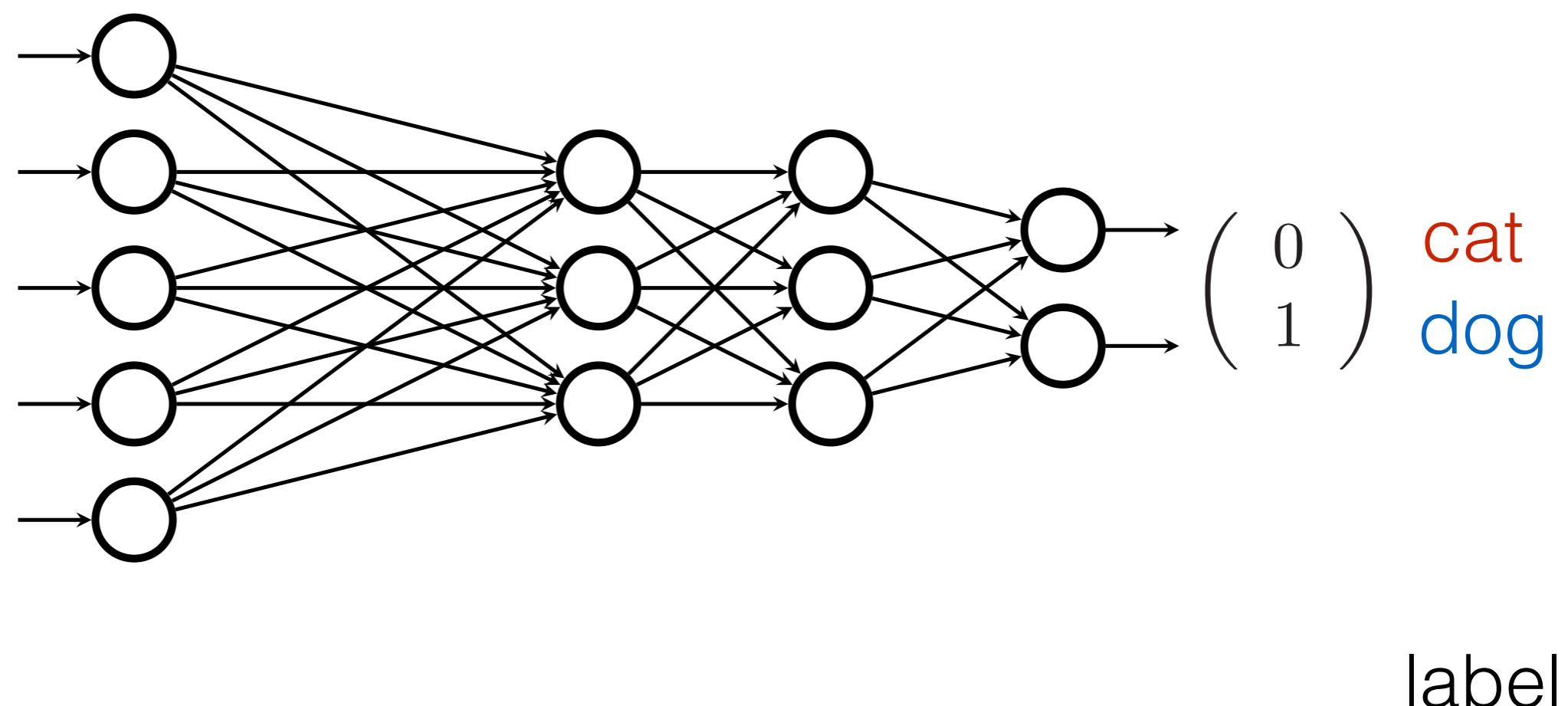
# Artificial Neural Networks



**Universal Function Approximator**

Cybenko 1989  
Hornik, Stinchcombe, White 1989

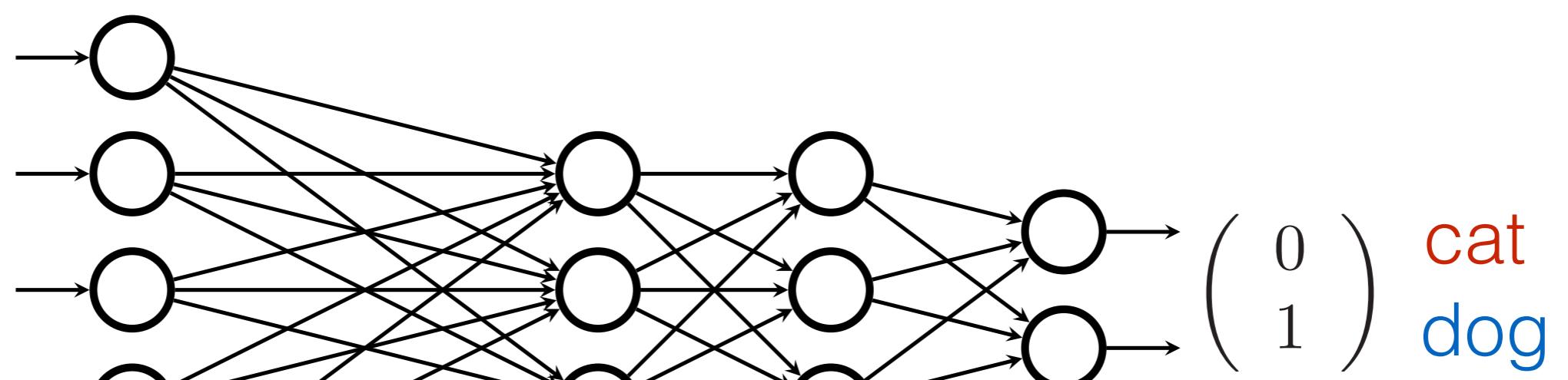
# Artificial Neural Networks



**Universal Function Approximator**

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# Artificial Neural Networks



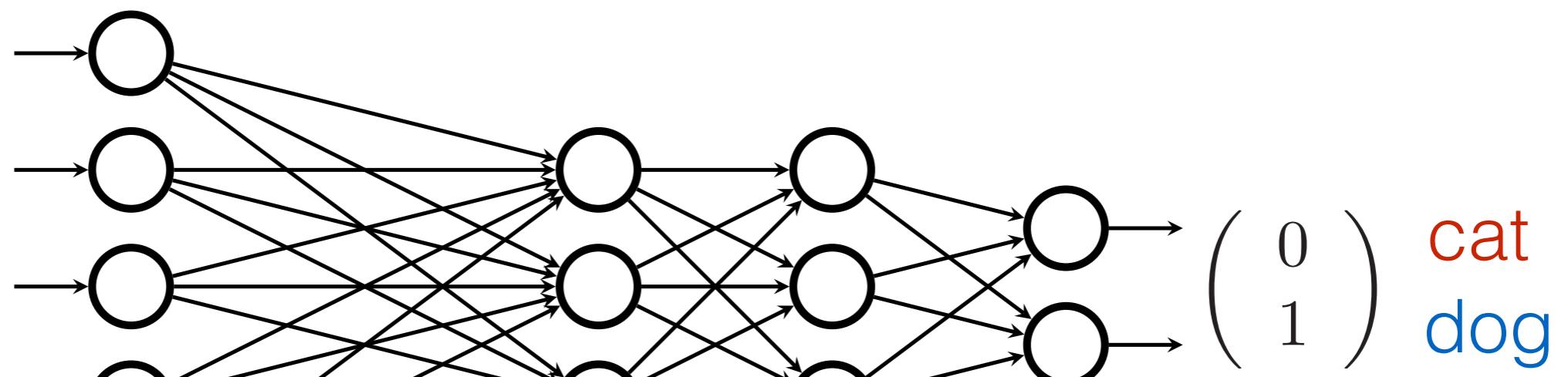
Connections to Renormalization Group ?  
Bény 1301.3124 Mehta and Schwab 1410.3831

label

**Universal Function Approximator**

Cybenko 1989  
Hornik, Stinchcombe, White 1989

# Artificial Neural Networks



data

Connections to Renormalization Group ?

Bény 1301.3124 Mehta and Schwab 1410.3831

label

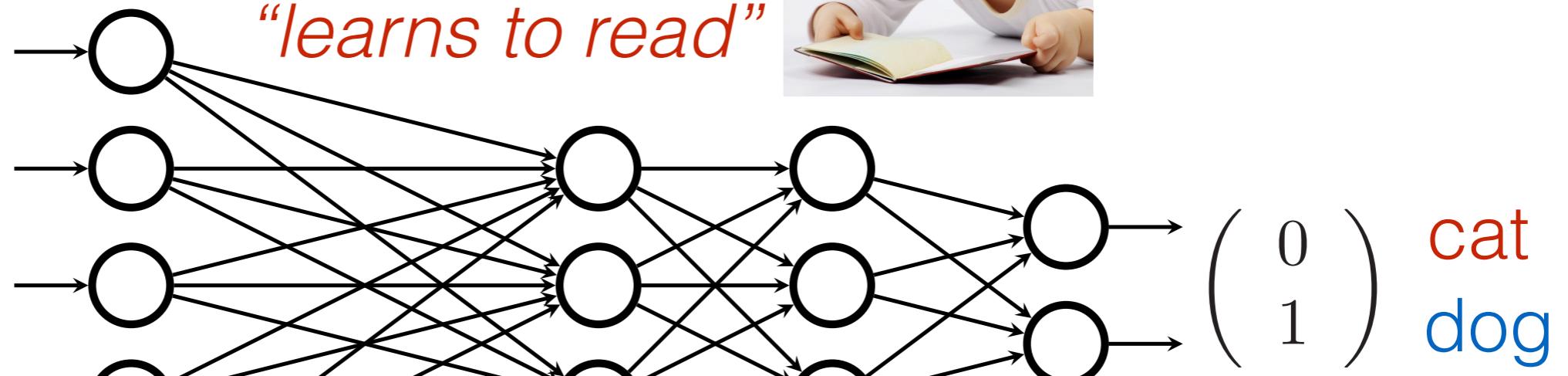
Why deep learning works? Not only a math problem, but also because of the law of physics: symmetry, locality, and compositionality

# Artificial Neural Networks



discriminative  
learning:

*“learns to read”*



data

Connections to Renormalization Group ?

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Why deep learning works? Not only a math problem, but also because of the law of physics: symmetry, locality, and compositionality

# Restricted Boltzmann Machines

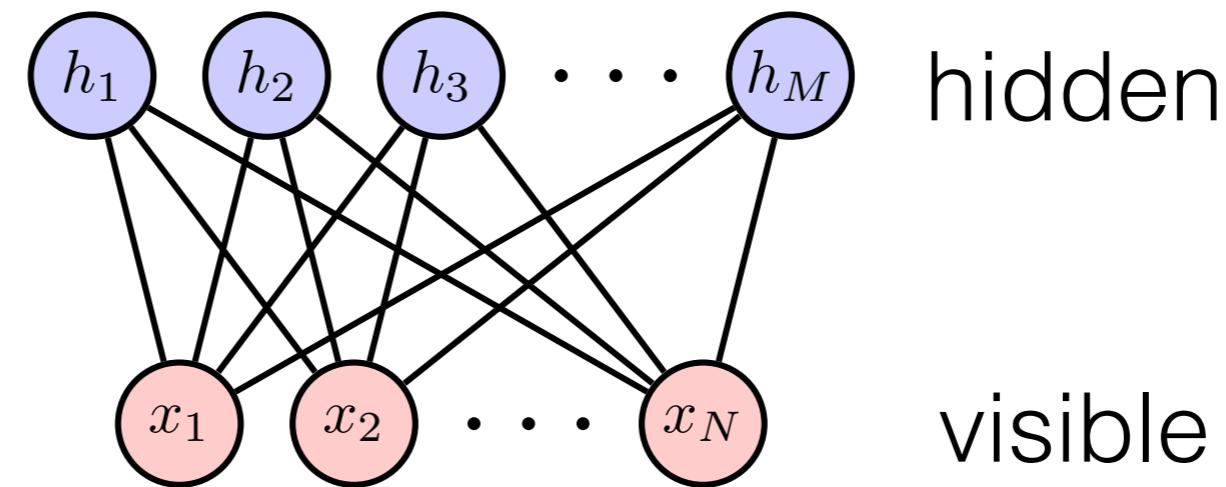
generative  
learning:  
*“learns to write”*



# Restricted Boltzmann Machines

Smolensky 1986 Hinton and Sejnowski 1986

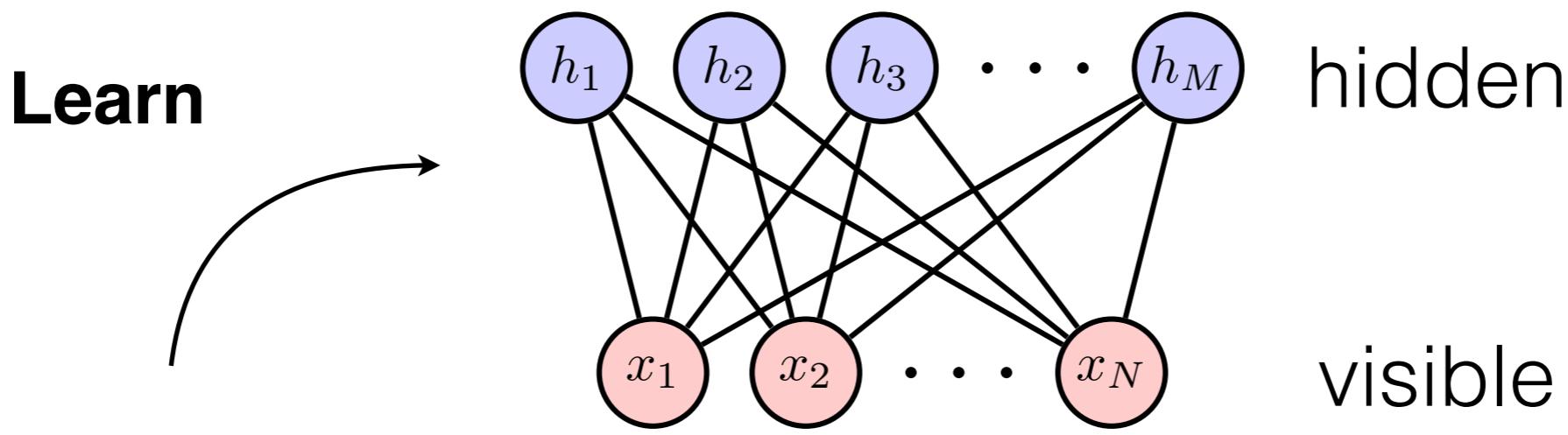
$$E(\mathbf{x}, \mathbf{h}) = - \sum_{i=1}^N a_i x_i - \sum_{j=1}^M b_j h_j - \sum_{i=1}^N \sum_{j=1}^M x_i W_{ij} h_j$$



# Restricted Boltzmann Machines

Smolensky 1986 Hinton and Sejnowski 1986

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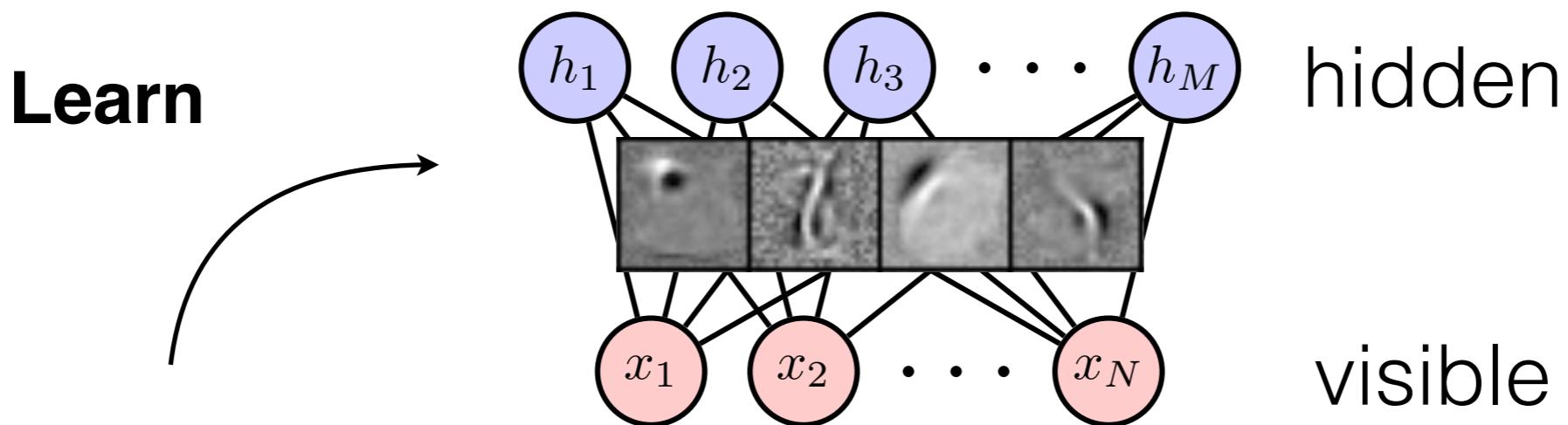


6	2	7	4	2	1	9
1	2	5	3	0	7	5
8	1	8	4	2	6	6
0	7	9	8	6	3	2
7	5	0	5	7	9	5
1	8	7	0	6	5	0
7	5	4	8	4	4	7

# Restricted Boltzmann Machines

Smolensky 1986 Hinton and Sejnowski 1986

$$E(\mathbf{x}, \mathbf{h}) = - \sum_{i=1}^N a_i x_i - \sum_{j=1}^M b_j h_j - \sum_{i=1}^N \sum_{j=1}^M x_i W_{ij} h_j$$



6 2 7 4 2 1 9  
1 2 5 3 0 7 5  
8 1 8 4 2 6 6  
0 7 9 8 6 3 2  
7 5 0 5 7 9 5  
1 8 7 0 6 5 0  
7 5 4 8 4 4 7

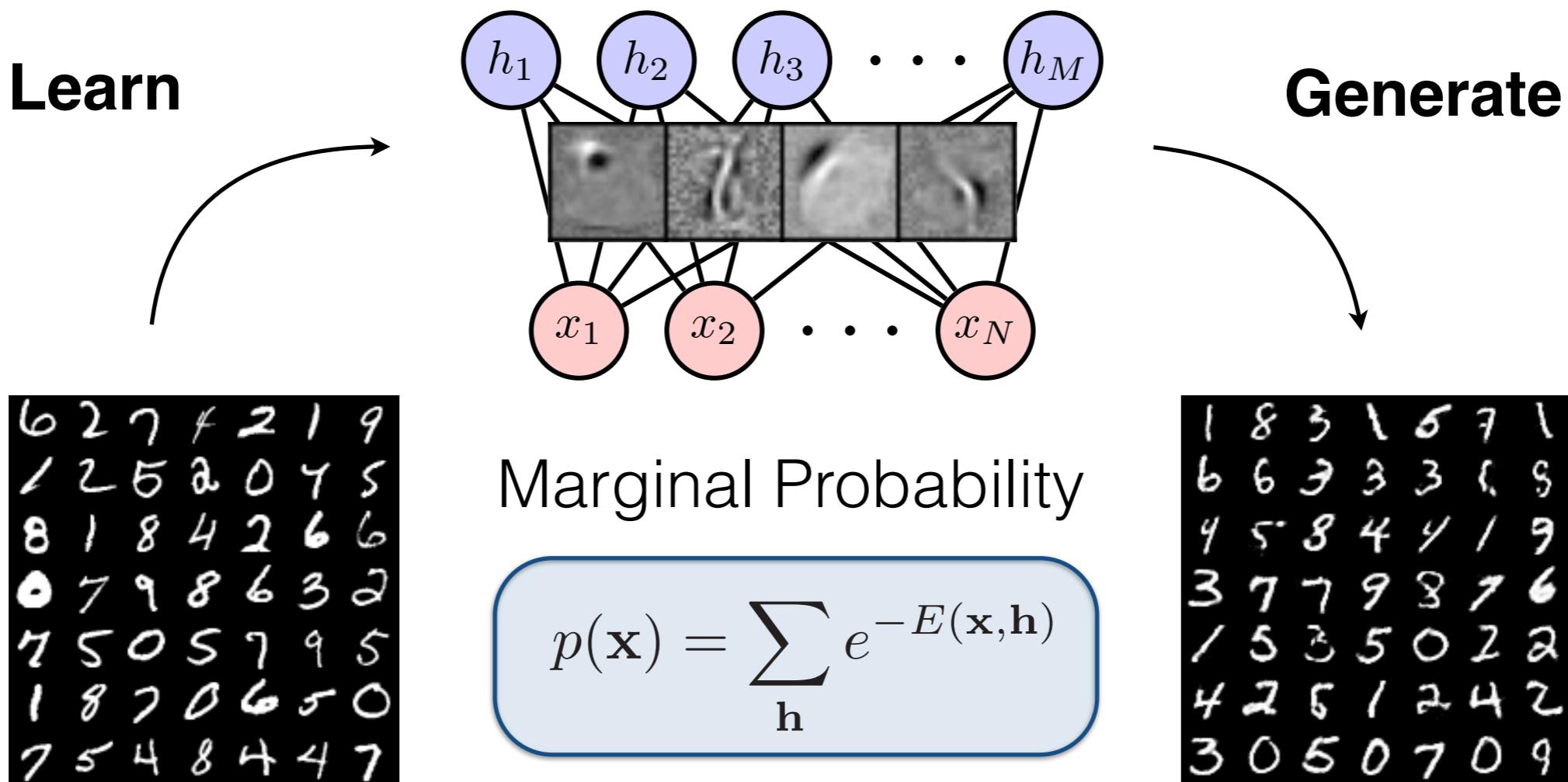
Marginal Probability

$$p(\mathbf{x}) = \sum_{\mathbf{h}} e^{-E(\mathbf{x}, \mathbf{h})}$$

# Restricted Boltzmann Machines

Smolensky 1986 Hinton and Sejnowski 1986

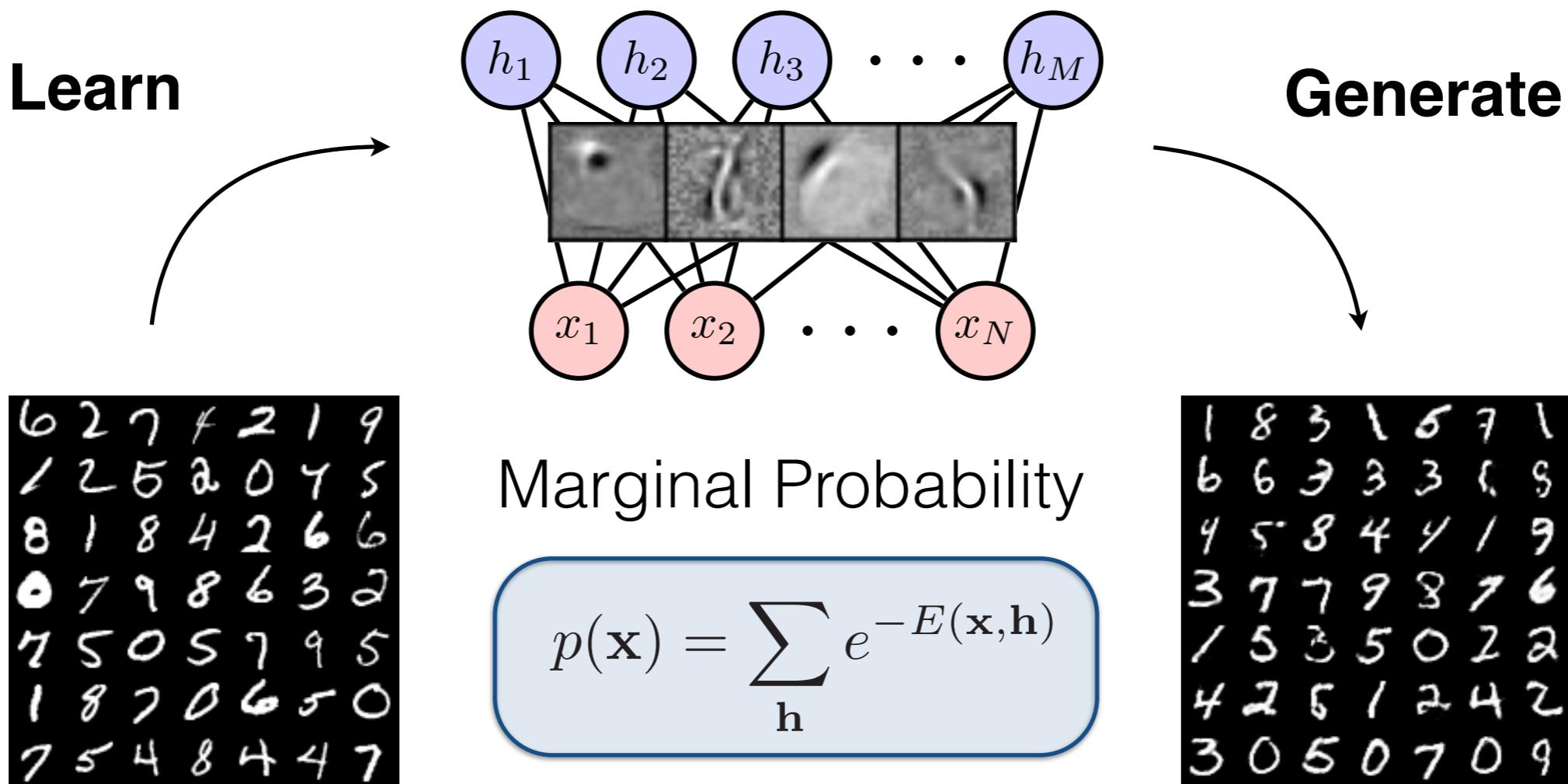
$$E(\mathbf{x}, \mathbf{h}) = - \sum_{i=1}^N a_i x_i - \sum_{j=1}^M b_j h_j - \sum_{i=1}^N \sum_{j=1}^M x_i W_{ij} h_j$$



# Restricted Boltzmann Machines

Smolensky 1986 Hinton and Sejnowski 1986

$$E(\mathbf{x}, \mathbf{h}) = - \sum_{i=1}^N a_i x_i - \sum_{j=1}^M b_j h_j - \sum_{i=1}^N \sum_{j=1}^M x_i W_{ij} h_j$$



**Universal approximator of probability distributions**

Freund and Haussler, 1989 Le Roux and Bengio, 2008

# Why machine learning for many-body physics ?

- **Conceptual connections**: a new and natural way to think about (quantum) many-body systems
- **Data driven approach**: making scientific discovery based on big datasets
- **Techniques**: neural networks, kernel methods, pattern recognition, feature extraction, dimensional reduction, clustering analysis, probabilistic modeling, recommender systems, hardware acceleration, software frameworks...

# Ideas

# Ideas

A general way to do fittings

Solving inverse problems

Variational wave functions

Quantum state tomography/classifier/decoding

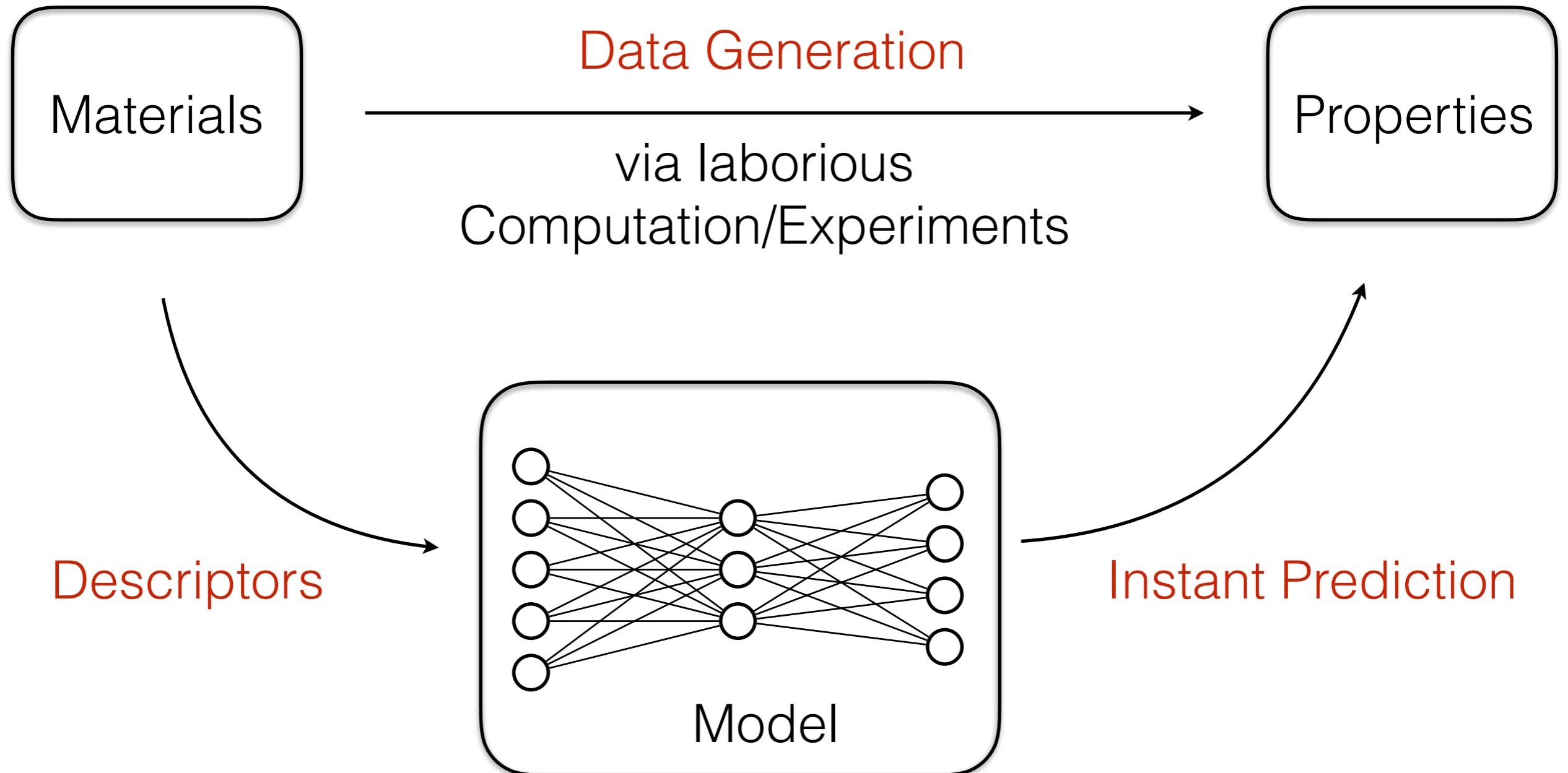
Classification/discovery phases of matter

Connection to tensor networks & RG

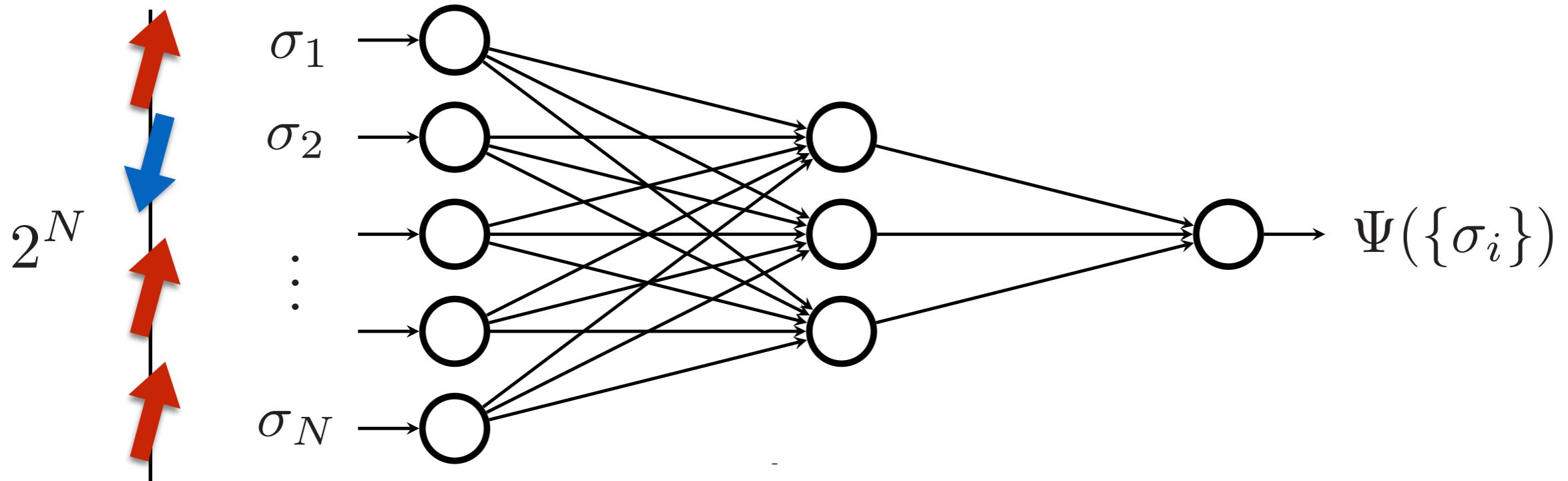
Recommender engines for QMC

# *Function Approximation*

# Material Discovery



# Variational wave functions

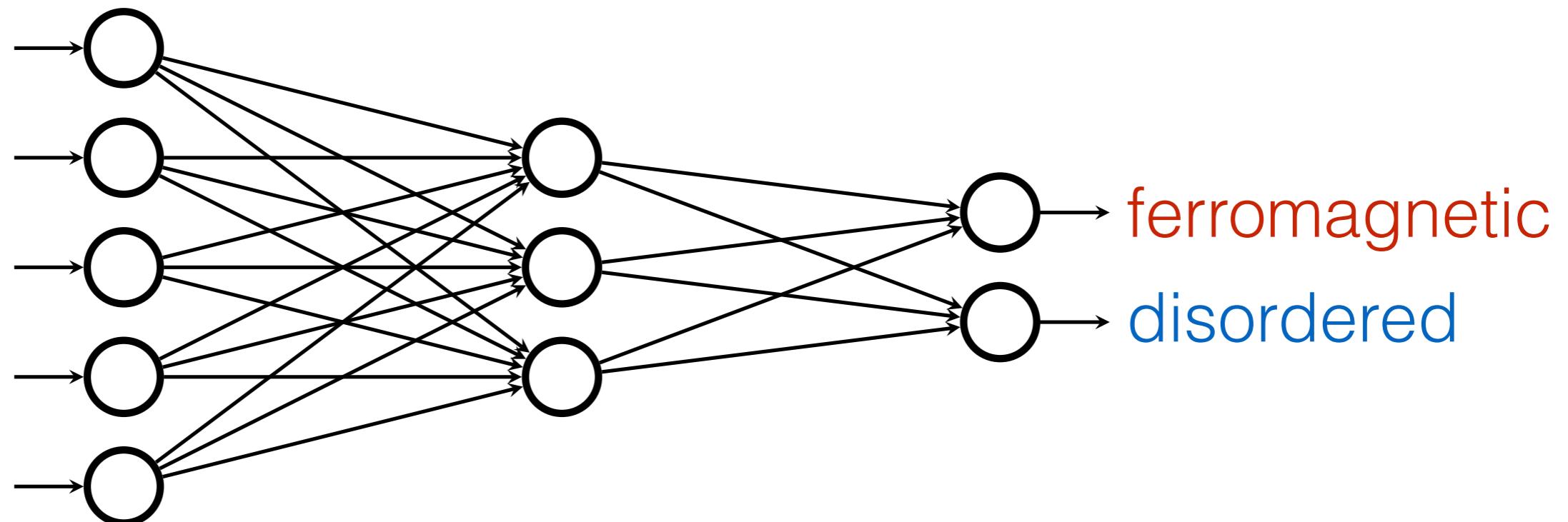
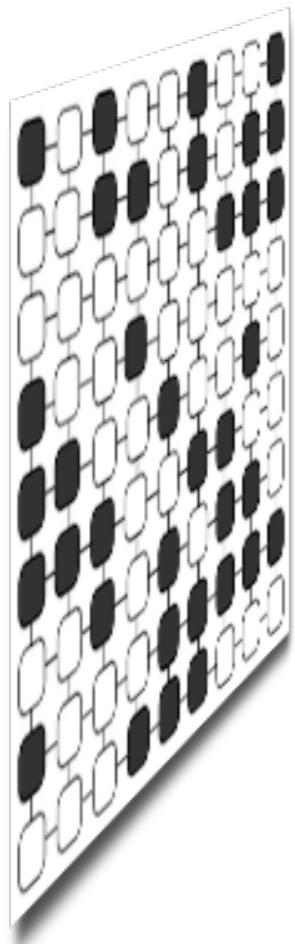


- Neural net as an efficient many-body wave function
- Universal function approximator
- Feature discovery and abstraction power from deep hierarchical structure

*“Phase” Recognition*

# Supervised Approach

Ising configurations



data

“Machine Learning Phase of Matter”

Carrasquilla and Melko, 1605.01735

label

Broecker, Carrasquilla, Melko, Trebst, 1608.07848

Tanaka, Tomiya 1609.09087

Ch'ng, Carrasquilla, Melko, Khatami, 1609.02552

Ohtsuki, Ohtsuki, 1610.00462

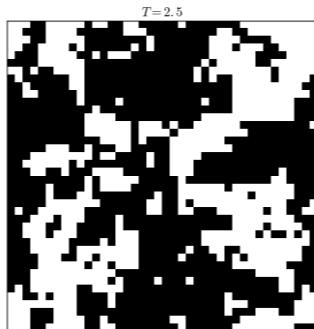
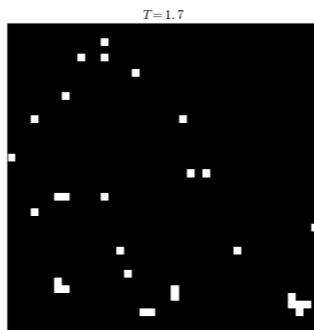
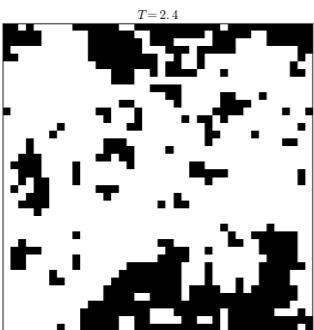
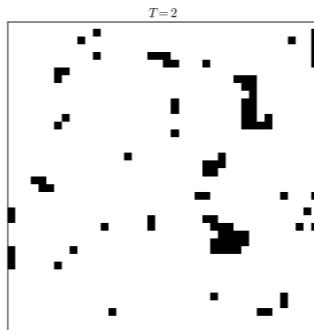
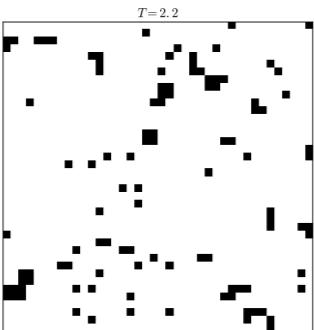
Schindler, Regnault, Neupert, 1704.01578

1612.04909

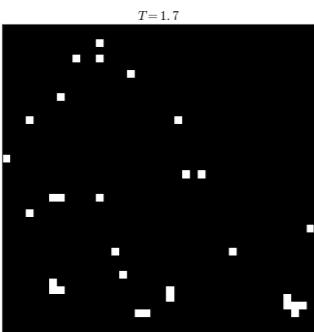
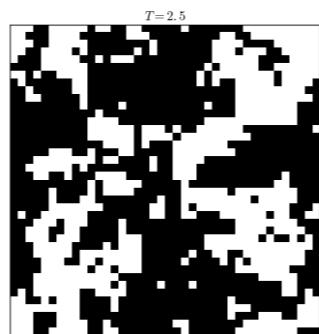
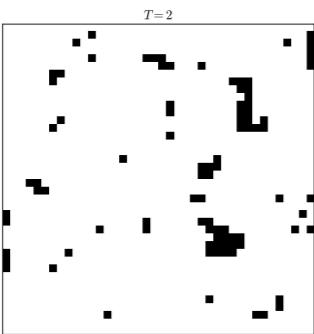
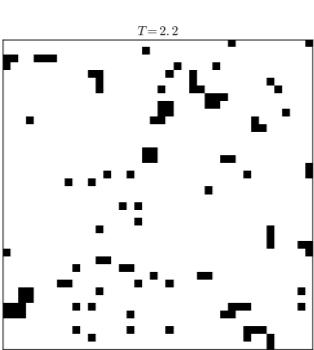
Ponte, Melko, 1704.05848

Zhang, Kim, 1611.01518

# Unsupervised Approach



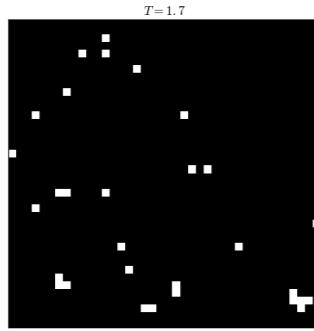
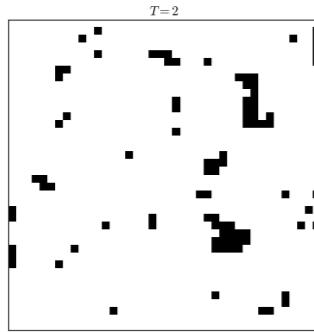
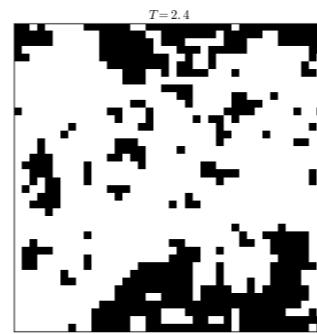
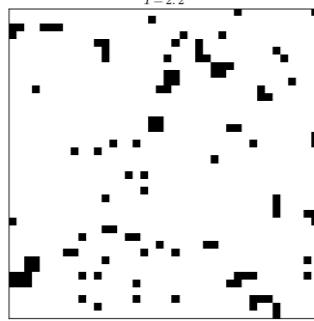
# Unsupervised Approach



ferromagnetic

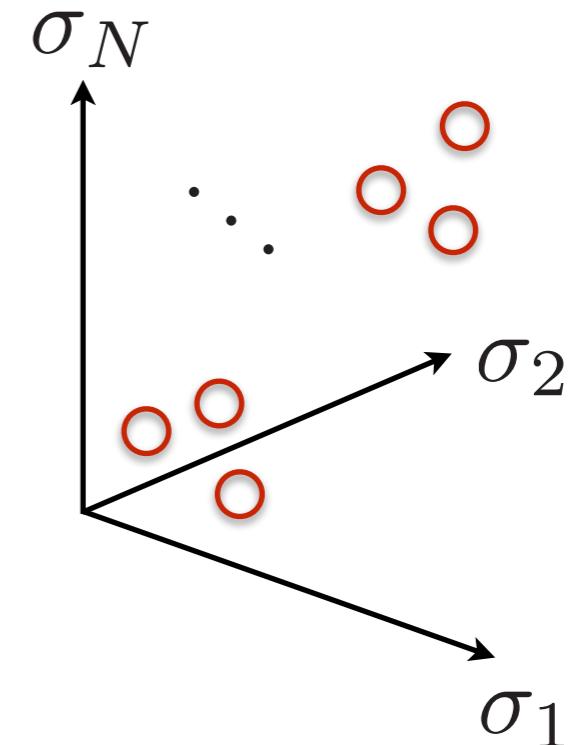
disordered

# Unsupervised Approach



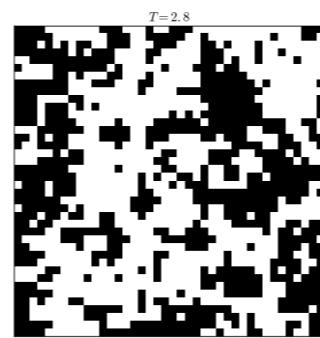
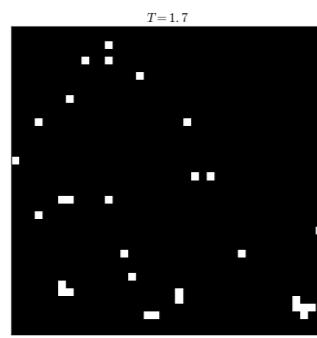
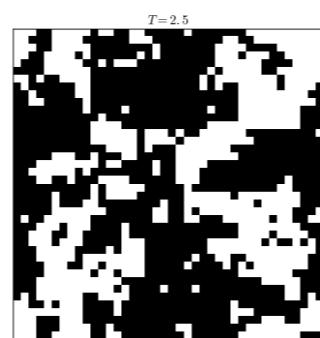
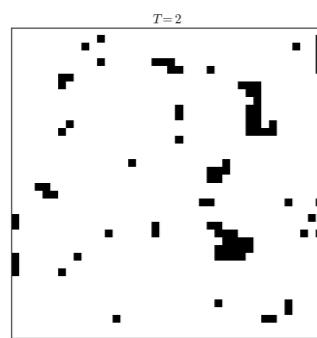
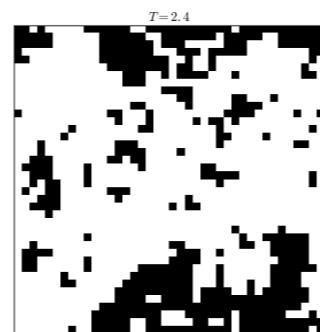
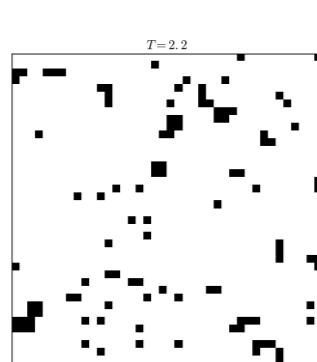
ferromagnetic

disordered



only data, no label

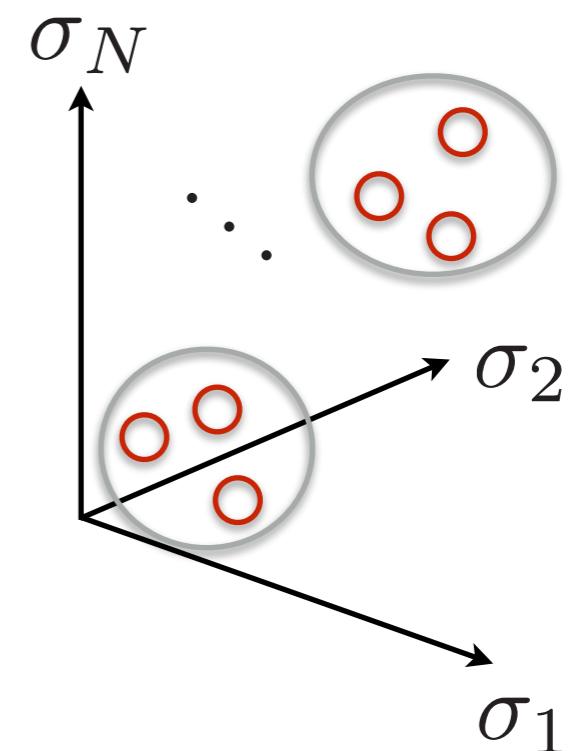
# Unsupervised Approach



ferromagnetic

disordered

LW, 1606.00318  
Discovering phase transition  
with dimensional reduction  
and clustering analysis



only data, no label

Nieuwenburg, Liu, Huber, 1610.02048  
Liu, Nieuwenburg, 1706.08111  
Broecker, Assaad, Trebst, 1707.00663

Wetzel, 1703.02435  
Hu, Singh, Scalettar, 1704.00080  
Wetzel, Scherzer, 1705.05582  
Wang and Zhai, 1706.07977

# *Algorithmic Innovations*

Liu, Qi, Meng, Fu, 1610.03137

Liu, Shen, Qi, Meng, Fu, 1611.09364

Xu, Qi, Liu, Fu, Meng, 1612.03804

Nagai, Shen, Qi, Liu, Fu, 1705.06724

Li Huang and LW, 1610.02746

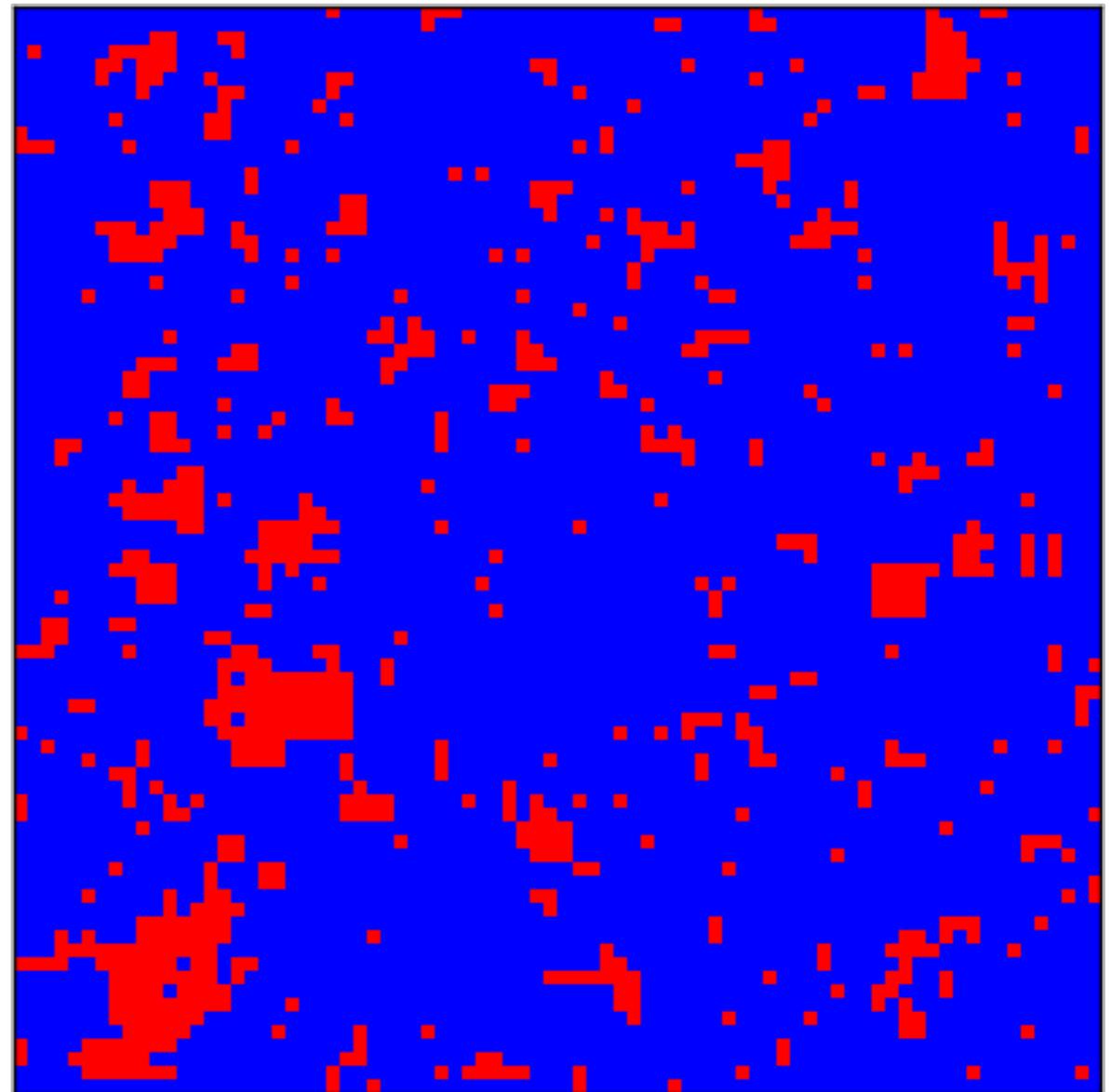
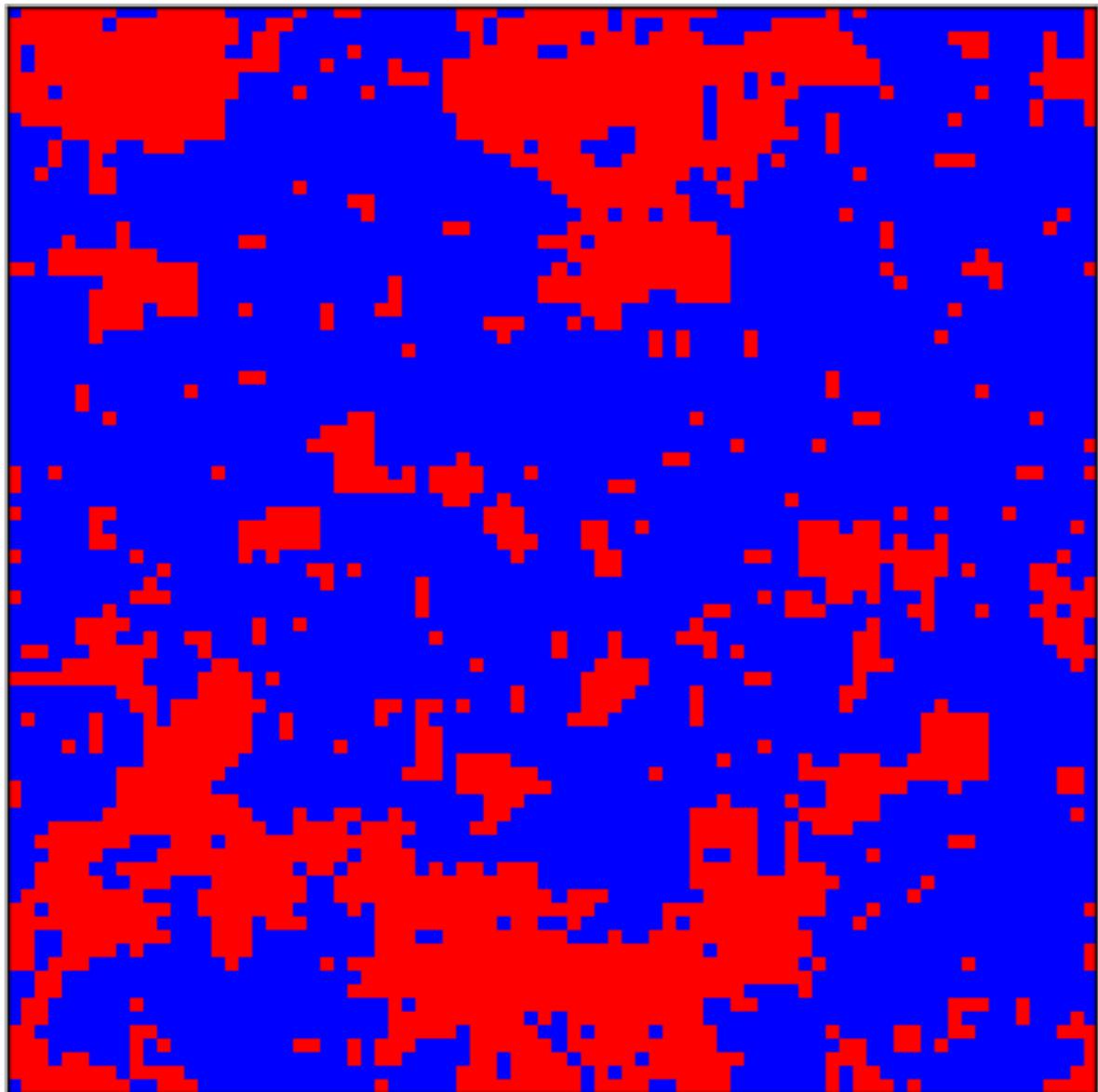
Li Huang, Yi-feng Yang and LW, 1612.01871

LW, 1702.08586

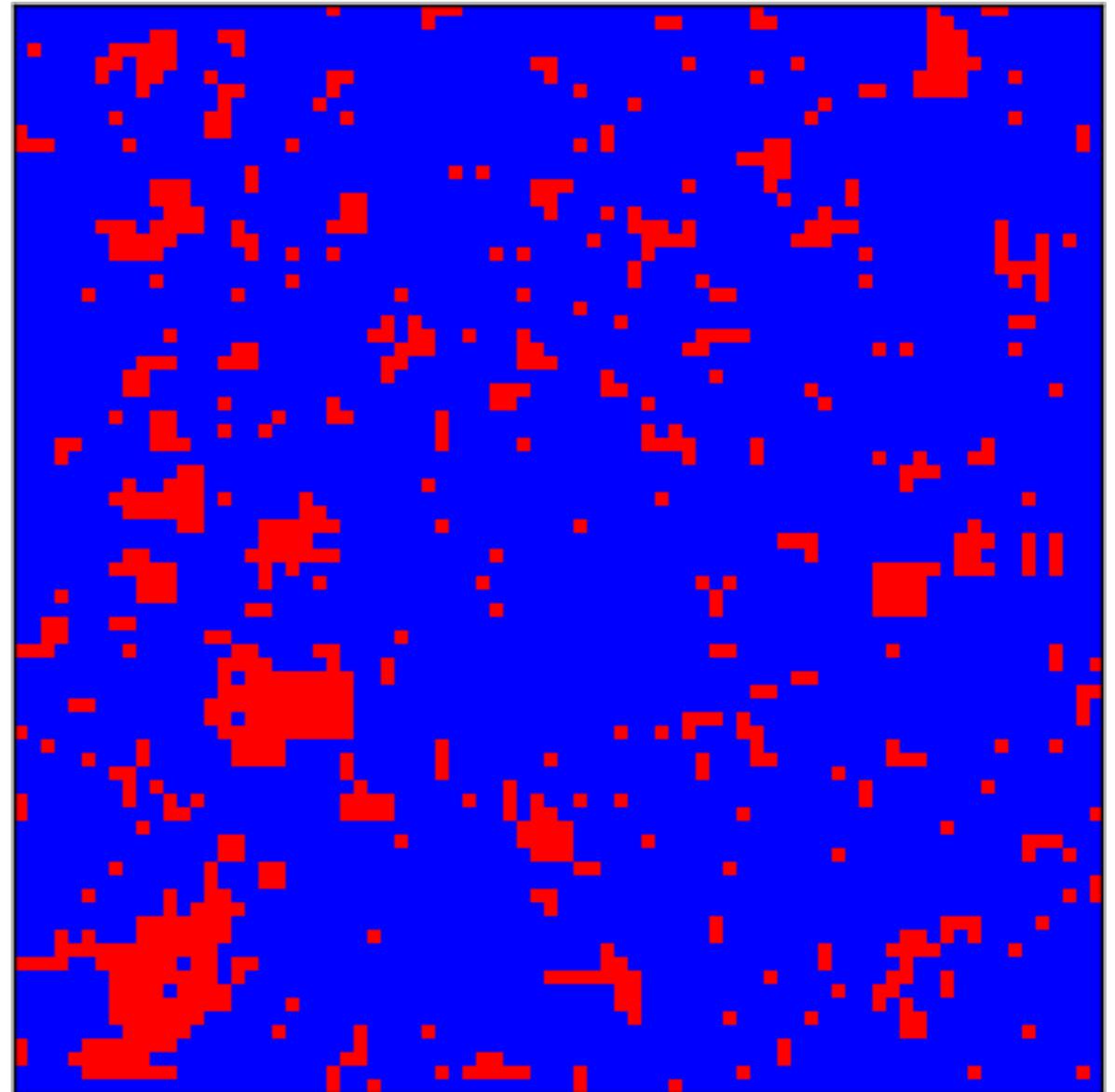
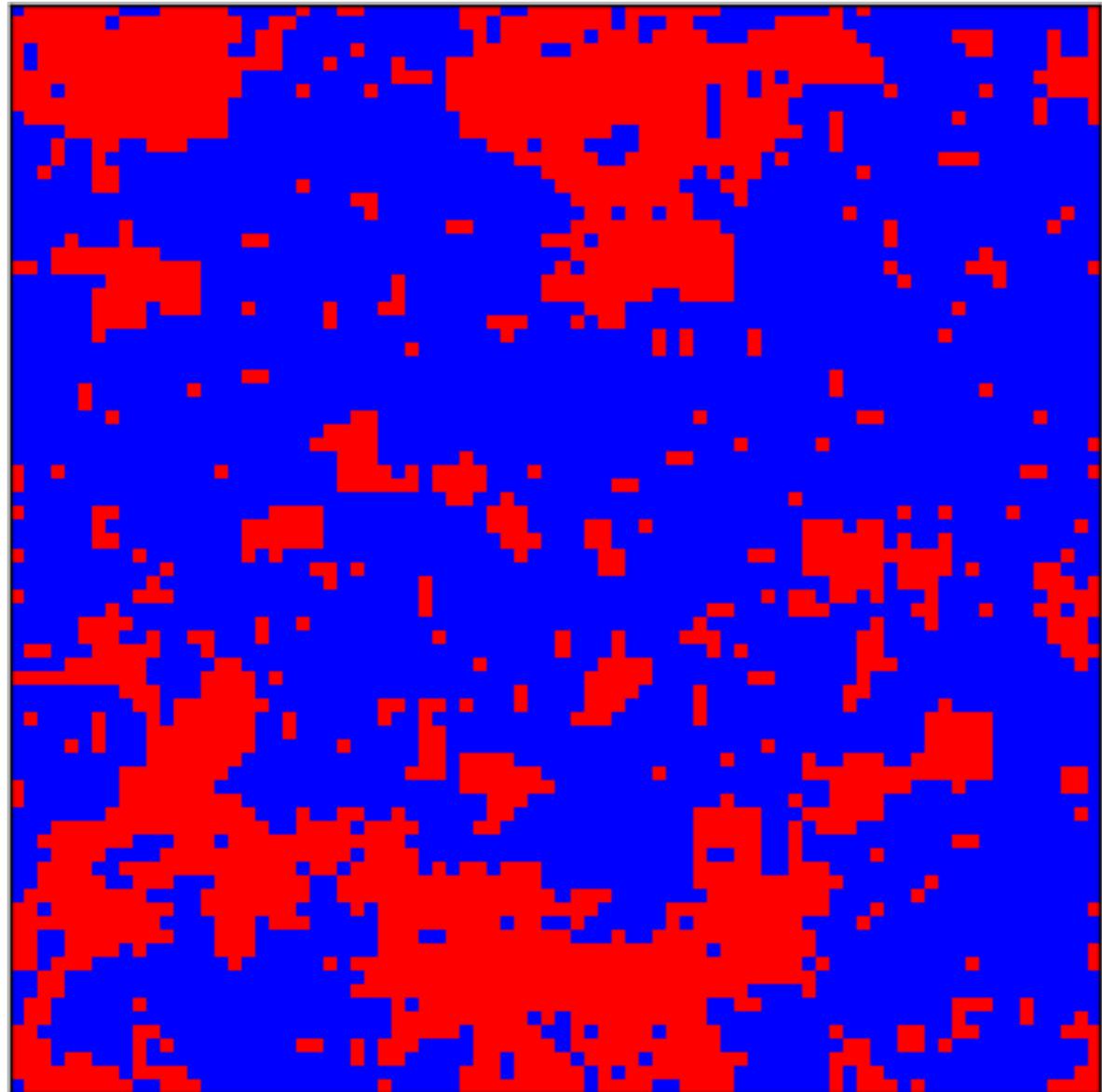
# A Video from Google DeepMind

[http://www.nature.com/nature/journal/v518/n7540/fig\\_tab/nature14236\\_SV2.html](http://www.nature.com/nature/journal/v518/n7540/fig_tab/nature14236_SV2.html)

# Local vs Cluster algorithms



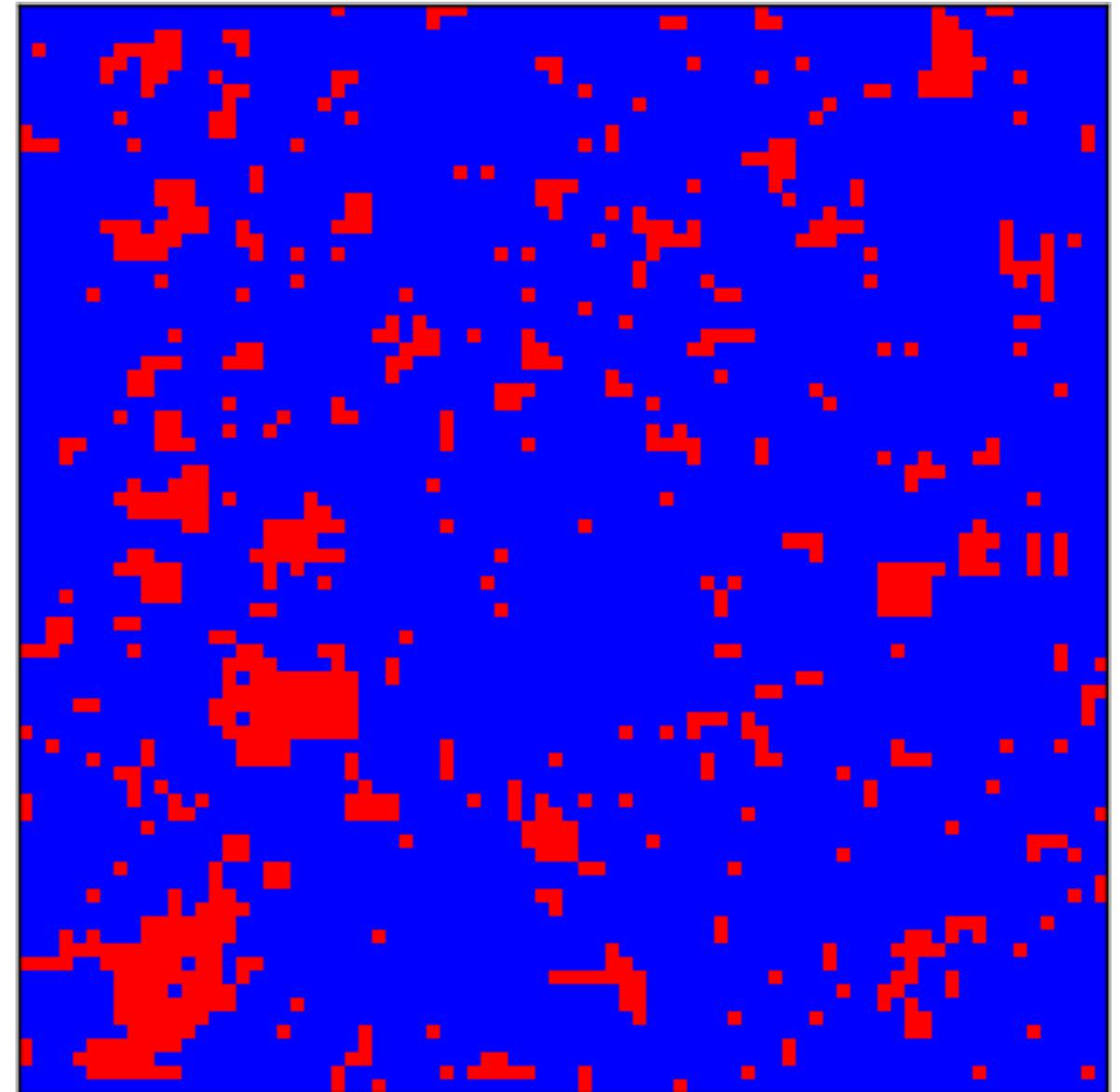
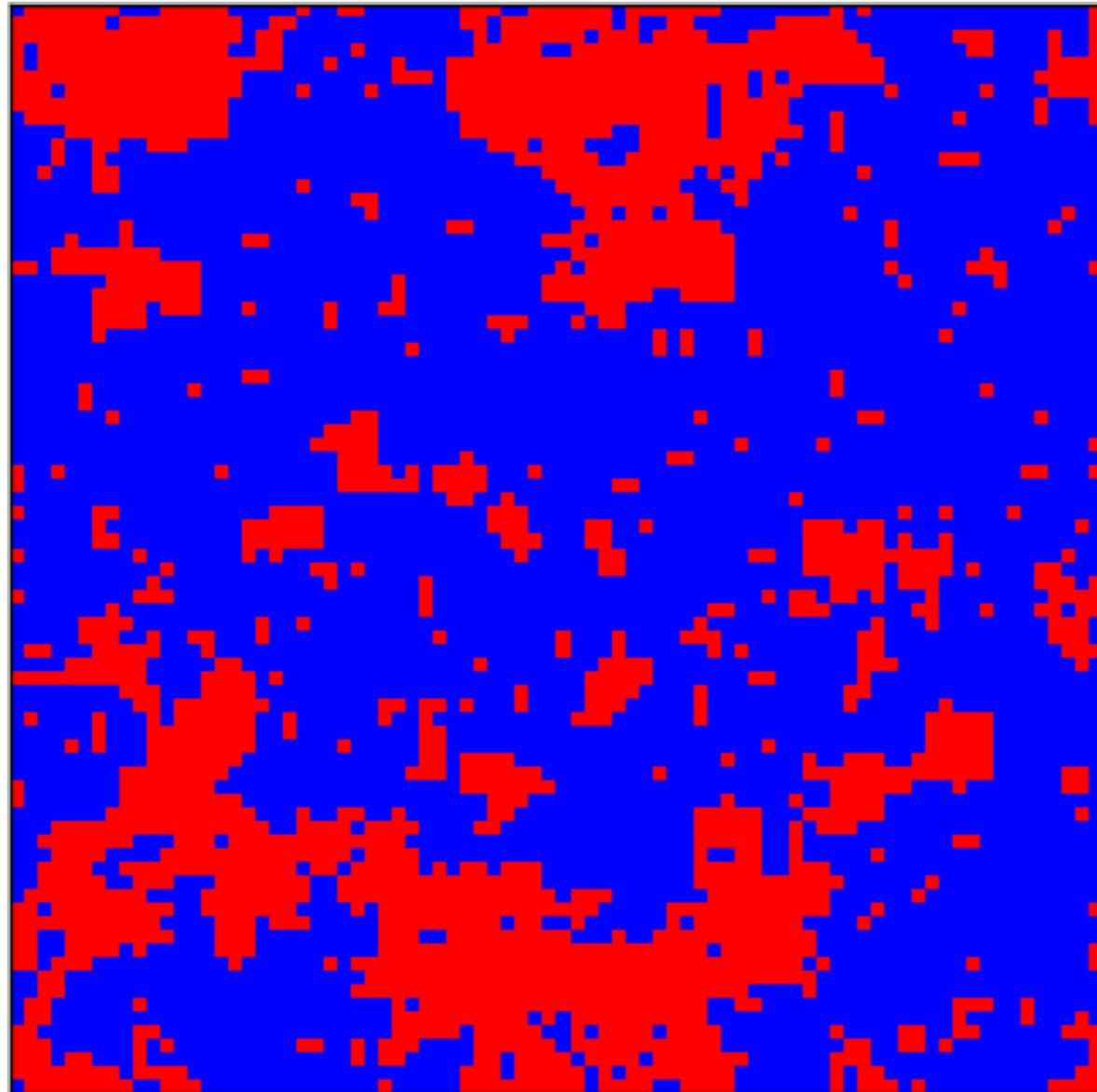
# Local vs Cluster algorithms



is slower than



# Local vs Cluster algorithms



Algorithmic innovation outperforms Moore's law!

# Discovering cluster updates with BM



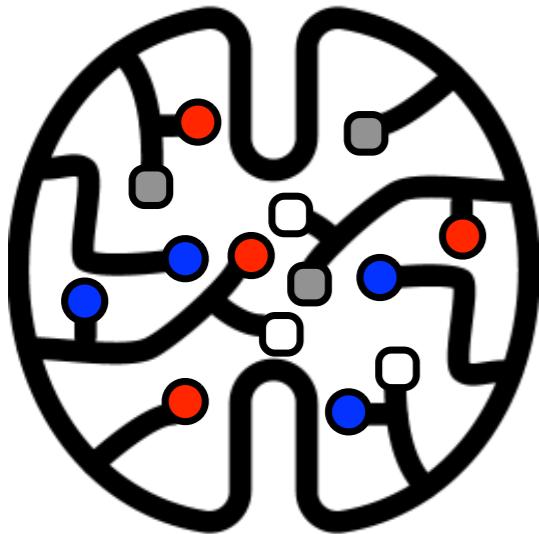
**Learn preferences**



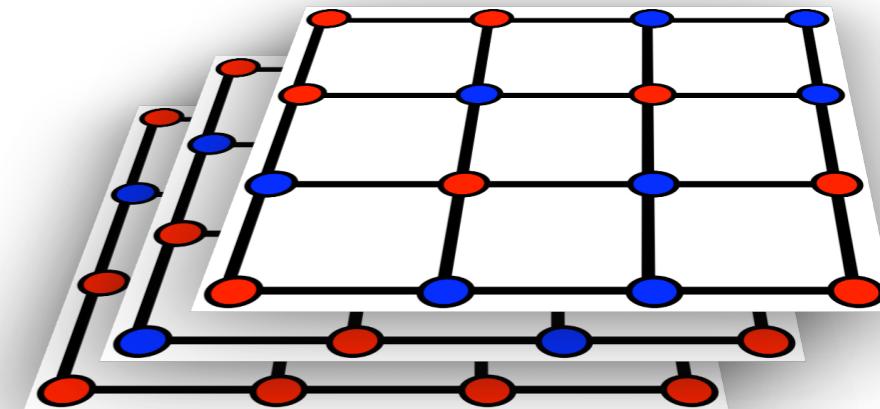
**Recommendations**



# Discovering cluster updates with BM



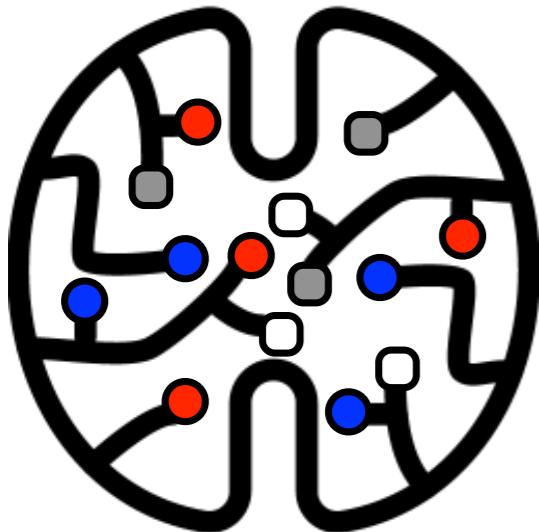
**Learn preferences**  
← →  
**Recommendations**



- Use Boltzmann Machines as **recommender systems** for Monte Carlo simulation

Li Huang and LW, 1610.02746

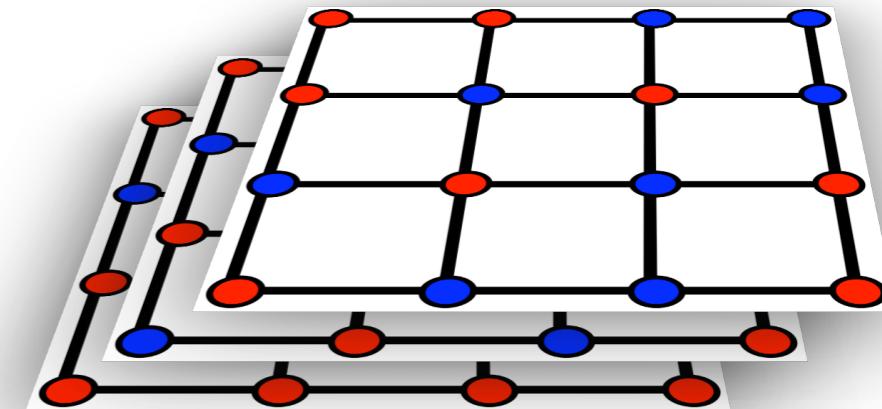
# Discovering cluster updates with BM



**Learn preferences**



**Recommendations**



- Use Boltzmann Machines as **recommender systems** for Monte Carlo simulation

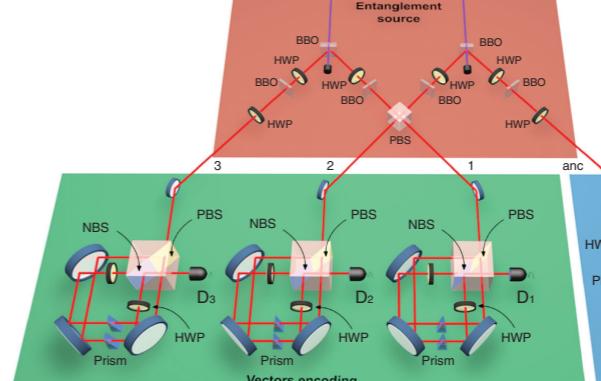
Li Huang and LW, 1610.02746

- Moreover, BM parametrizes Monte Carlo policies and explores **novel algorithms!**

LW, 1702.08586

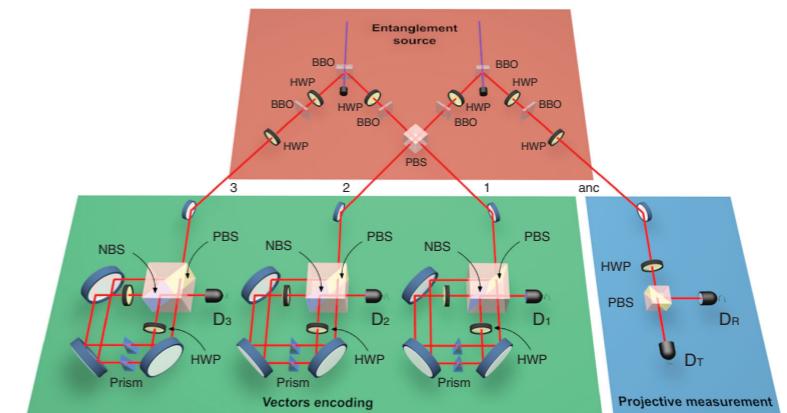
*Quantum Many-Body Physics  
for Machine Learning*

# Quantum Machine Learning

- Use a quantum computer to speed up classical ML subroutines
    - Optimization
    - Linear algebra
    - Sampling
    - Clustering
    - Support vector machine
    - Principal component analysis

Cai et al, PRL **114**, 110504 (2015)

	$^{13}C$	$F_1$	$F_2$	$F_3$
$^{13}C$	15479.9Hz			
$F_1$	-297.7Hz	-33130.1Hz		
$F_2$	-275.7Hz	64.6Hz	-42681.4Hz	
$F_3$	39.1Hz	51.5Hz	-129.0Hz	-56443.5Hz
$T_2^*$	1.22s	0.66s	0.63s	0.61s
$T_1$	7.9	1.1	0.9	1.2



Cai et al, PRL 114, 110504 (2015)

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$T_2$	7.9s	4.4s	6.8s	4.8s

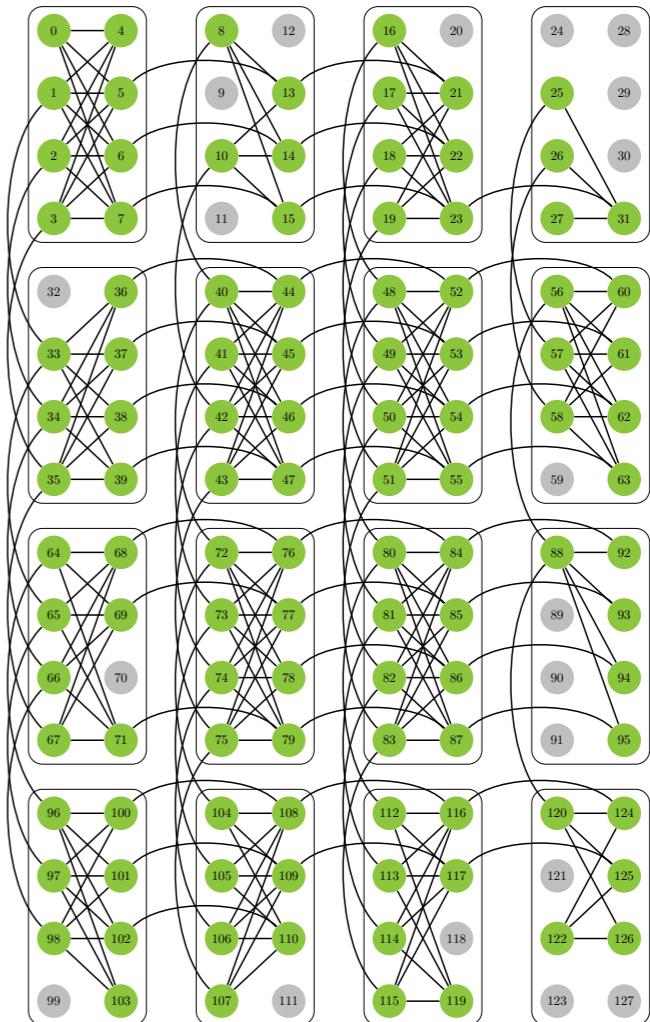
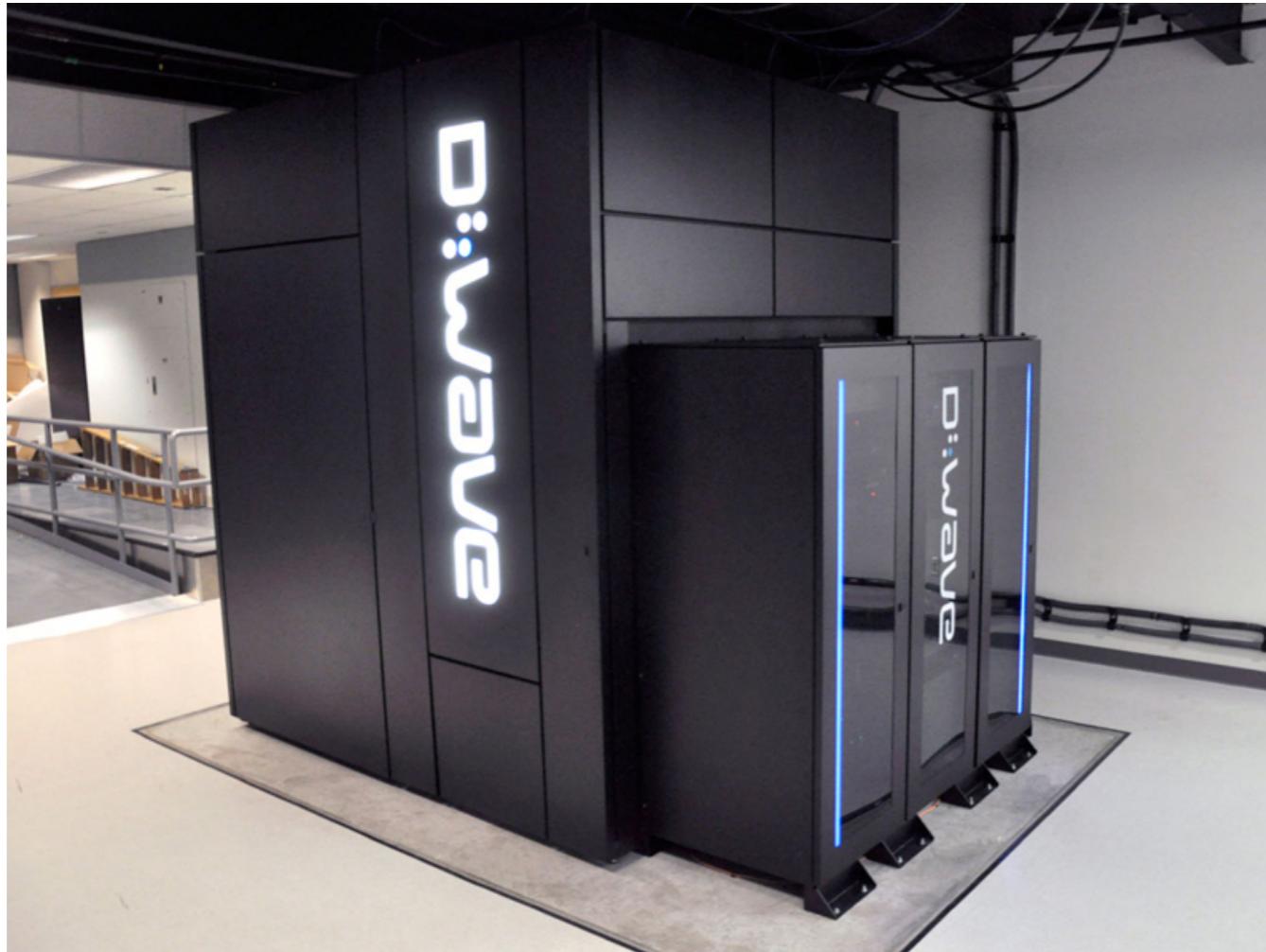
Li et al, PRL 114, 140504 (2015)

- Quantum data and quantum architecture

“Advances in quantum machine learning”, Adcock et al, 1512.02900  
“Quantum machine learning”, Biamonte et al, 1611.09347

# Quantum Boltzmann Machine

\$15 million “quantum Ising simulator”



Is there any advantage of quantum architecture ?

# Quantum entanglement perspective on deep learning

Xun Gao, L.-M. Duan, 1701.05039

Yichen Huang and J. E. Moore, 1701.06246

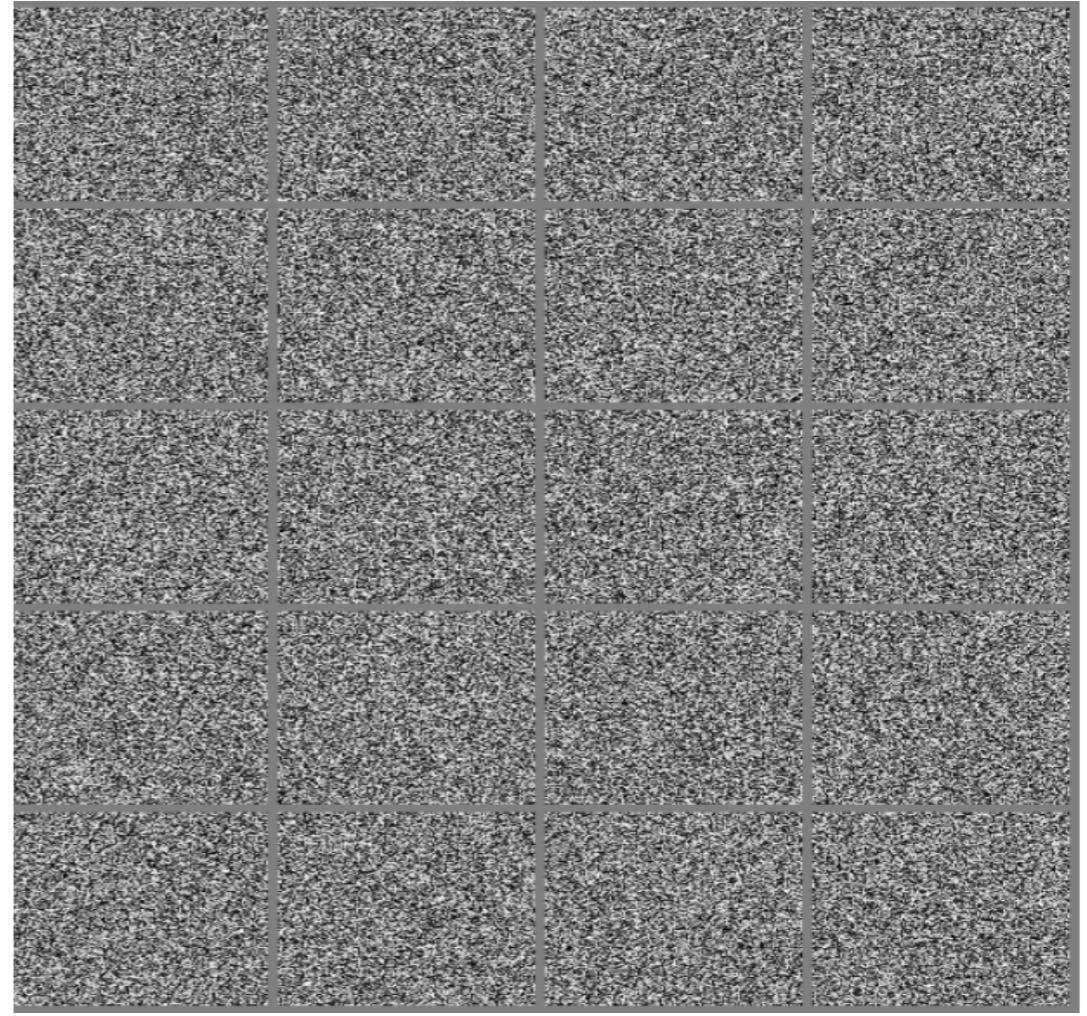
Dong-Ling Deng, Xiaopeng Li and S. Das Sarma, 1701.04844

Jing Chen, Song Cheng, Haidong Xie, LW, and Tao Xiang, 1701.04831

8	9	0	1	2	3	4	7	8	9	0	1	2	3	4	5	6	7	8	6
4	2	6	4	7	5	5	4	7	8	9	2	9	3	9	3	8	2	0	5
0	1	0	4	2	6	5	3	5	3	8	0	0	3	4	1	5	3	0	8
3	0	6	2	7	1	1	8	1	7	1	3	8	9	7	6	7	4	1	6
7	5	1	7	1	9	8	0	6	9	4	9	9	3	7	1	9	2	2	5
3	7	8	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	0
1	2	3	4	5	6	7	8	9	8	1	0	5	5	1	9	0	4	1	9
3	8	4	7	7	8	5	0	6	5	5	3	3	3	9	8	1	4	0	6
1	0	0	6	2	1	1	3	2	8	8	7	8	4	6	0	2	0	3	6
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6	5	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7
8	9	0	1	2	3	4	5	6	7	8	9	6	4	2	6	4	7	5	5
4	7	8	9	2	9	3	9	3	8	2	0	9	8	0	5	6	0	1	0
4	2	6	5	5	5	4	3	4	1	5	3	0	8	3	0	6	2	7	1
1	8	1	7	1	3	8	5	4	2	0	9	7	6	7	4	1	6	8	4
7	5	1	2	6	7	1	9	8	0	6	9	4	9	9	6	2	3	7	1
9	2	2	5	3	7	8	0	1	2	3	4	5	6	7	8	0	1	2	3
4	5	6	7	8	0	1	2	3	4	5	6	7	8	9	2	1	2	1	3
9	9	8	5	3	7	0	7	7	5	7	9	9	4	7	0	3	4	1	4
4	7	5	8	1	4	8	4	1	8	6	6	4	6	3	5	7	2	5	9

MNIST database

from the “Deep Learning” book by Goodfellow, Bengio, Courville <https://www.deeplearningbook.org/>



random images

# Deep Learning and Quantum Entanglement: Fundamental Connections with Implications to Network Design

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## Abstract

Deep convolutional networks have witnessed unprecedented success in various machine learning applications. Formal understanding on what makes these networks so successful is gradually unfolding, but for the most part there are still significant mysteries to unravel. The inductive bias, which reflects prior knowledge embedded in the network architecture, is one of them. In this work, we establish a fundamental connection between the fields of quantum physics and deep learning. We use this connection for asserting novel theoretical observations regarding the role that the number of channels in each layer of the convolutional network fulfills in the overall inductive bias. Specifically, we show an equivalence between the function realized by a deep convolutional arithmetic circuit (ConvAC) and a quantum many-body wave function, which relies on their common underlying tensorial structure. This facilitates the use of quantum entanglement measures as well-defined quantifiers of a deep network's expressive ability to model intricate correlation structures of its inputs. Most importantly, the construction of a deep convolutional arithmetic circuit in terms of a Tensor Network is made available. This description enables us to carry a graph-theoretic analysis of a convolutional network, tying its expressiveness to a min-cut in the graph which characterizes it. Thus, we demonstrate a direct control over the inductive bias of the designed deep convolutional network via its channel numbers, which we show to be related to the min-cut in the underlying graph. This result is relevant to any practitioner designing a convolutional network for a specific task. We theoretically analyze convolutional arithmetic circuits, and empirically validate our findings on more common convolutional networks which involve ReLU activations and max pooling. Beyond the results described above, the description of a deep convolutional network in well-defined graph-theoretic tools and the formal structural connection to quantum entanglement, are two interdisciplinary bridges that are brought forth by this work.



**Thank you!**

# 量子纠缠：从量子物质态到深度学习

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(1 中国科学院物理研究所 北京 100190)

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2017-06-05 收到

† email: wanglei@iphy.ac.cn

DOI: 10.7693/wl20170702

## Quantum entanglement: from quantum states of matter to deep learning

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(2 University of Chinese Academy of Sciences, Beijing 100049, China)

**摘要** 量子纠缠在量子物质态的研究中扮演着日趋重要的角色，它可以标记传统范式难以区分的新奇量子态和量子相变，并指导设计高效的数值算法来精确地研究量子多体问题。最近，随着一些深度学习技术在量子物理问题中的应用，人们惊奇地发现：从量子纠缠的视角审视深度学习，或许有助于反过来理解和解决一些深度学习中的问题。量子纠缠定量化地刻画了现实数据集的复杂度，并指导相应的人工神经网络结构设计。沿着这个思路，物理学家们对于量子多体问题所形成的种种洞察和理论可以以一种意想不到的方式应用在现实世界中。

**关键词** 量子纠缠，张量网络，人工神经网络，深度学习



**《物理》杂志**  
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