

# Modeling Mass Diffusion in Materials Science Applications

2024 MITE Summer Program  
Hosted by the Wang Materials Group



The University of Texas at Austin  
McKetta Department  
of Chemical Engineering  
Cockrell School of Engineering



# Today's plan

What is Materials Science and Engineering?

Examples

Materials Science Tetrahedron

What is diffusion?

In-person activity

Where is diffusion important?

Examples from everyday

Examples from materials science

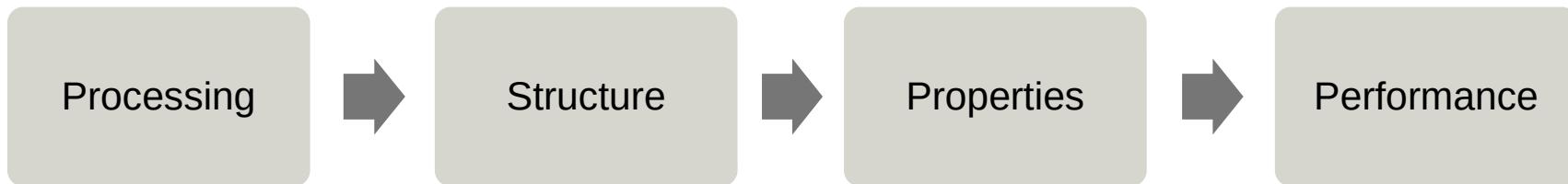
What is computation and simulation?\

Random Walk Diffusion

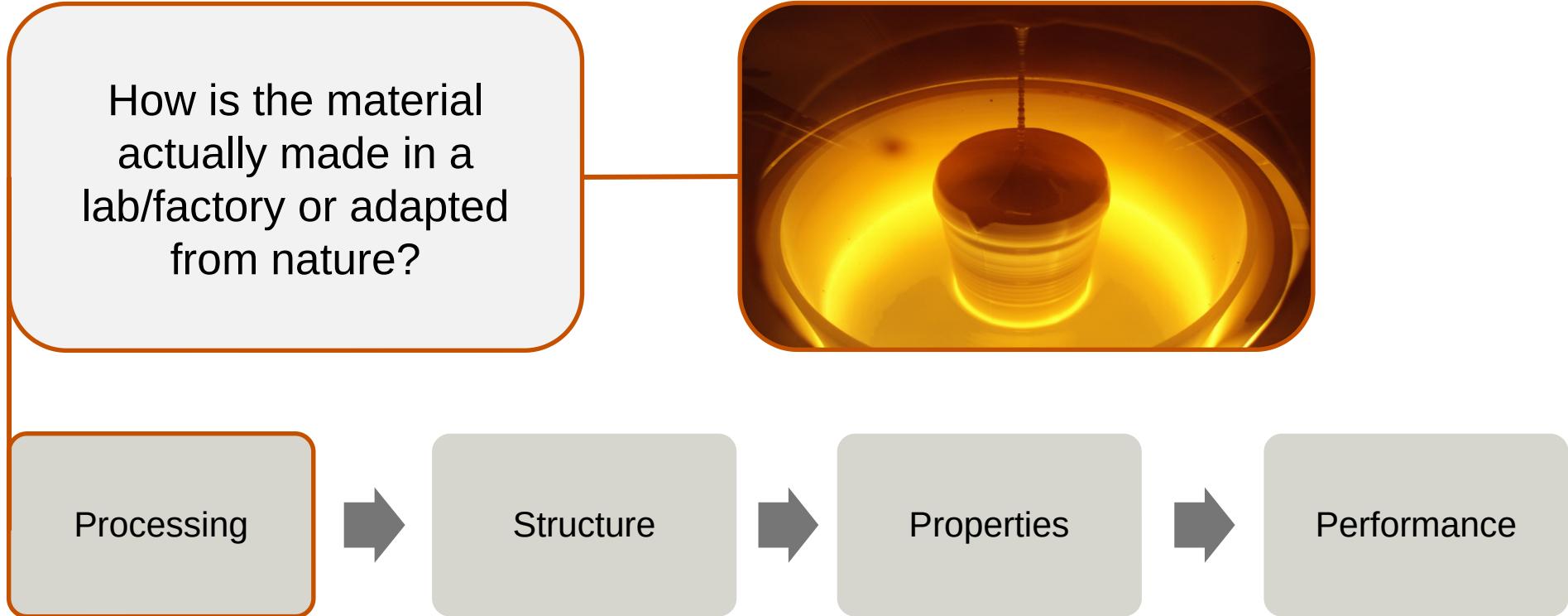
Using simulations to model diffusion

# What is materials science and engineering?

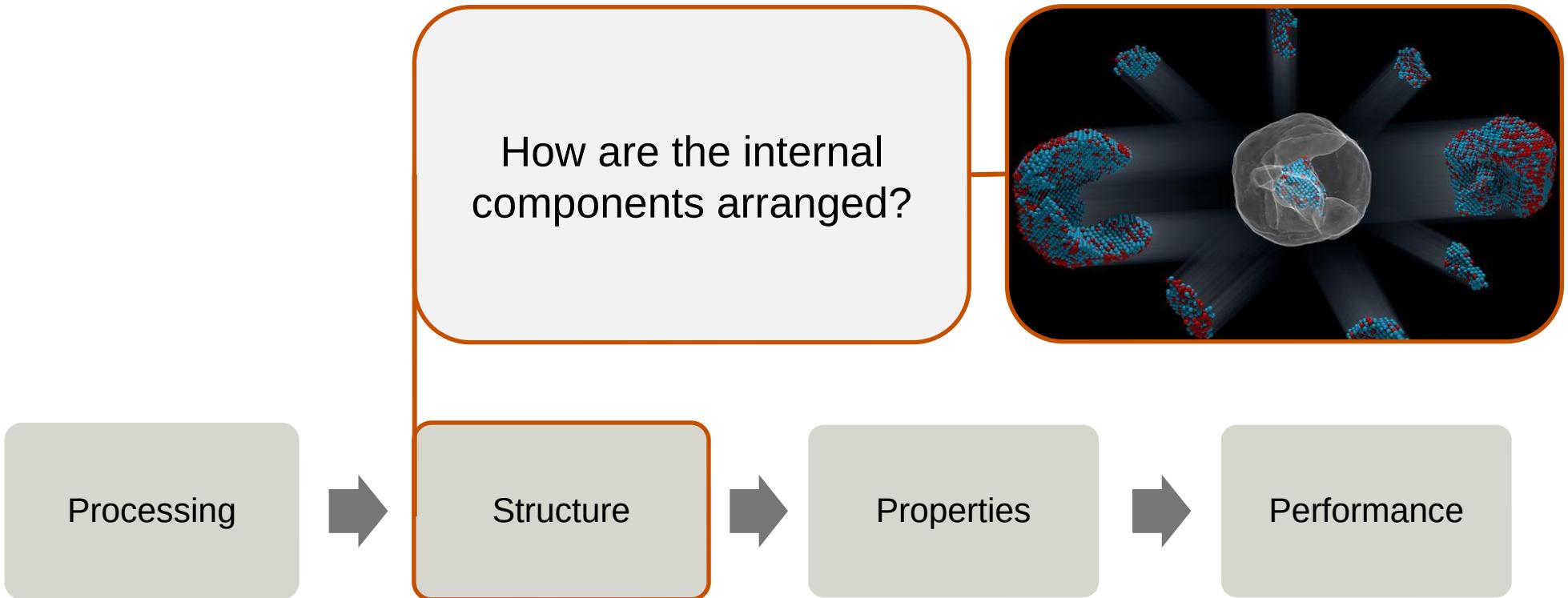
Discovering, understanding, and designing materials (often times solids) through the study and application of processing-structure-property-performance relationships



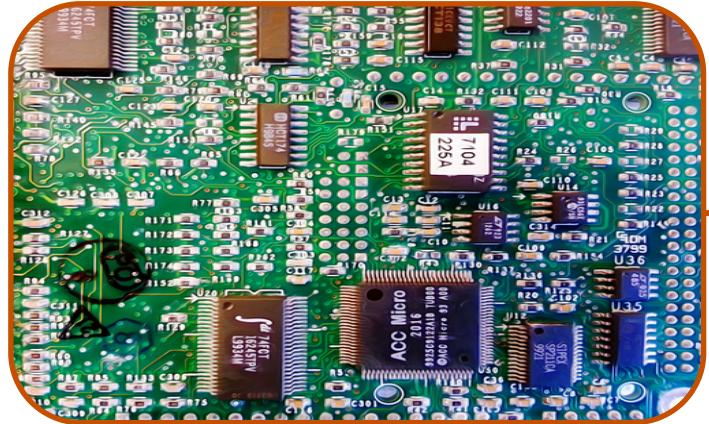
# What is materials science and engineering?



# What is materials science and engineering?



# What is materials science and engineering?



How does it respond to  
an external stimulus?

Mechanical

Electrical

Thermal

Magnetic

Optical

Chemical

Processing



Structure

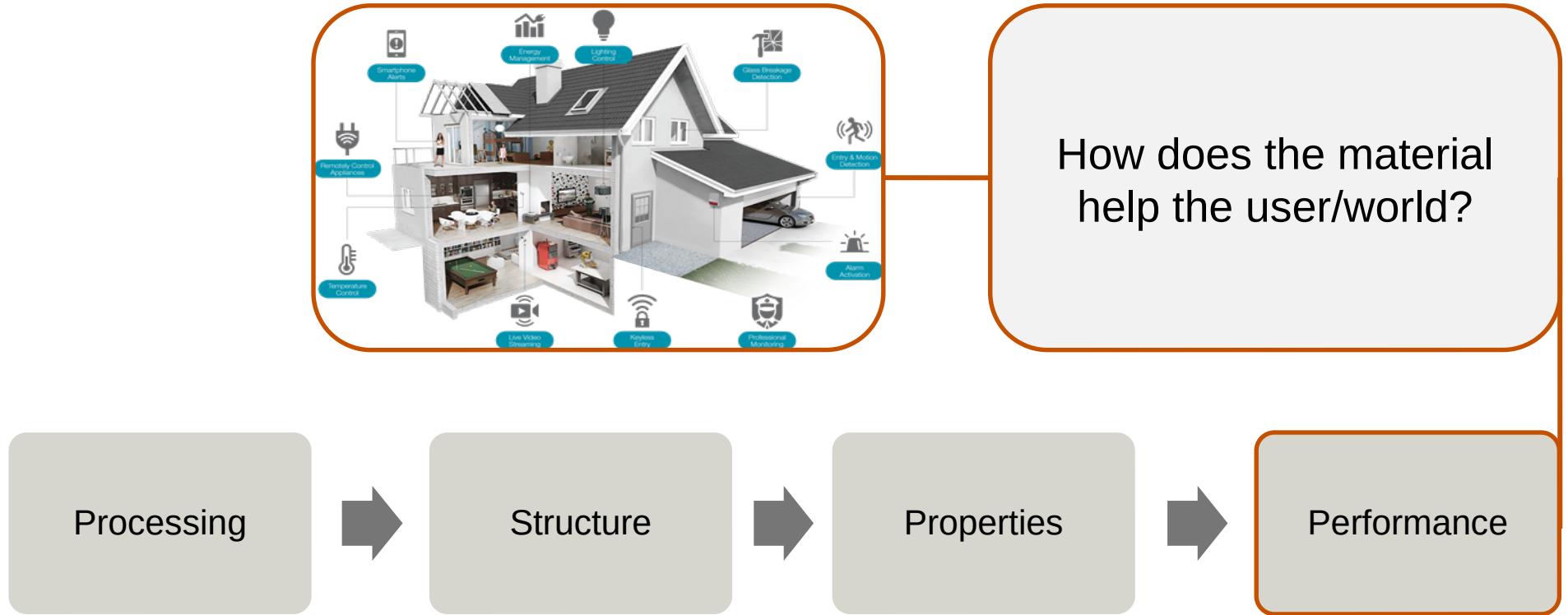


Properties

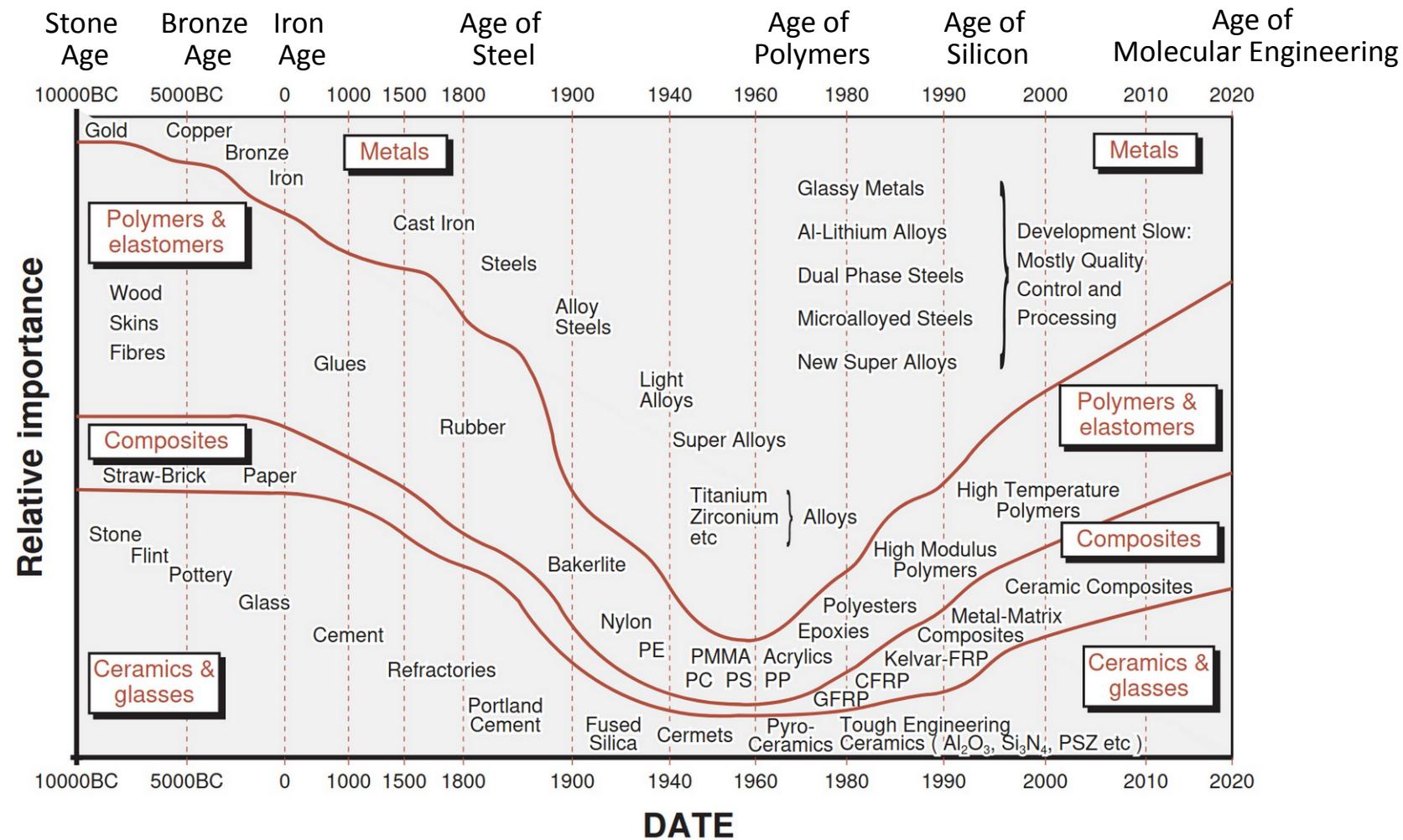


Performance

# What is materials science and engineering?



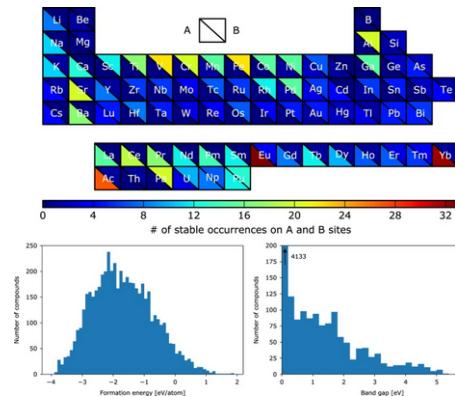
# Why study materials? (Think-pair-share)



# Examples of Materials Science and/or Engineering



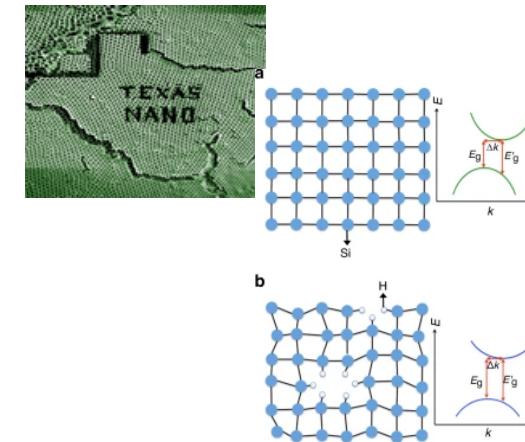
3D printing soft bio-polymers  
Nelson Lab, University of Washington



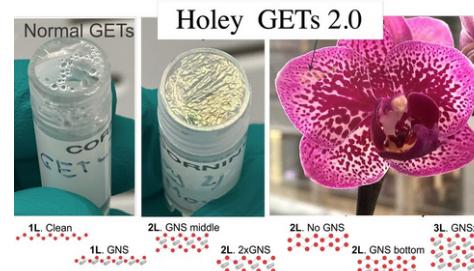
Using supercomputers to discover new and ideal oxides  
Wolverton Research Group, Northwestern



Using viruses as templates to make more efficient solar cells  
Belcher Lab, MIT



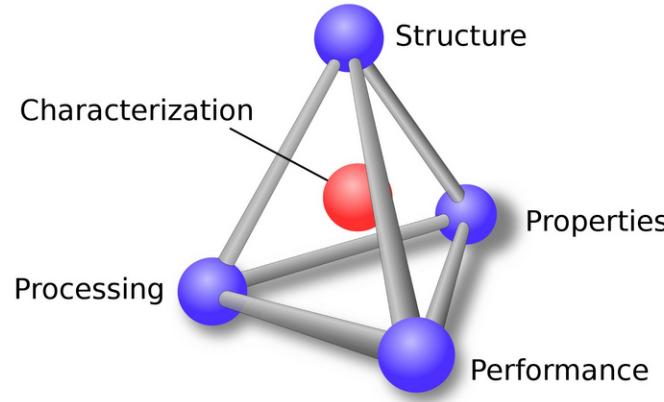
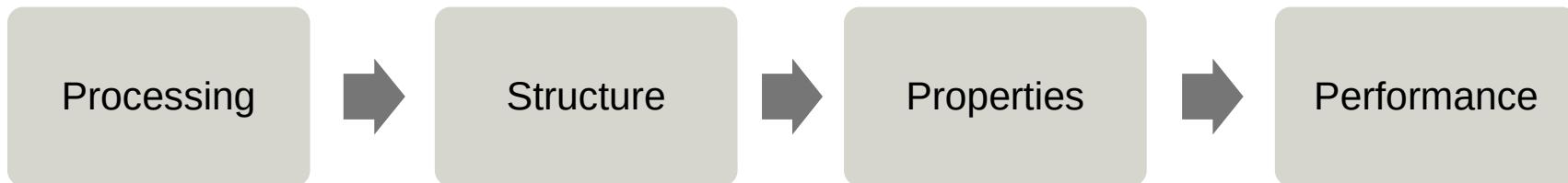
Using defects in nanoparticles for high-efficiency, low-loss nanophotonics  
Korgel & Zheng Research Groups, UT Austin



Breathable graphene tatoos for personalized healthcare  
Akinwande Nano Research Group, UT Austin

# What is materials science and engineering?

Discovering, understanding, and designing materials (often times solids) through the study and application of processing-structure-property-performance relationships



# What is diffusion?



<https://www.youtube.com/watch?v=UlhubVuJvZM>

## In-person activity: Human Diffusion

**Step 0:** Find a space at least twice as large as the space occupied by participants.

What happens to the distribution of the people in the room?

**Step 1:** Pack all participants to one side of the room as close as possible. Each participant faces a different direction.

**Step 2:** Each participant begins shuffling (slowly and carefully!) in the direction they are facing.

**Step 3:** If a participant runs into another object (e.g., wall or other participant), s/he moves in another random direction.

**Step 4:** Repeat for steps 2 and 3 many times.

# What is diffusion? The effect of temperature



What happens to the distribution of the ink in the flask?

\*not strictly mass diffusion

# Everyday examples of diffusion



CO<sub>2</sub> (gas) dissolved  
in flavored liquid

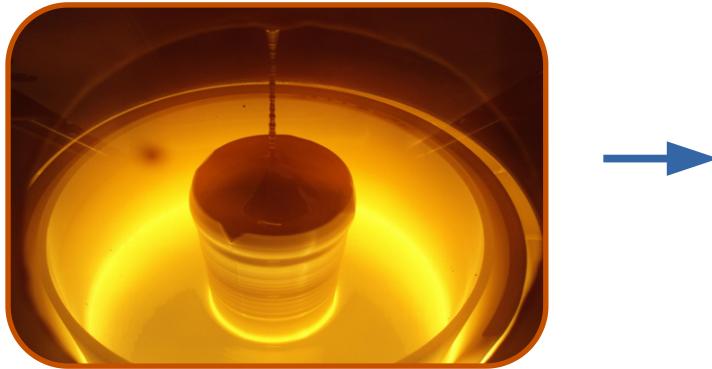


Aerosols, air fresheners, perfumes

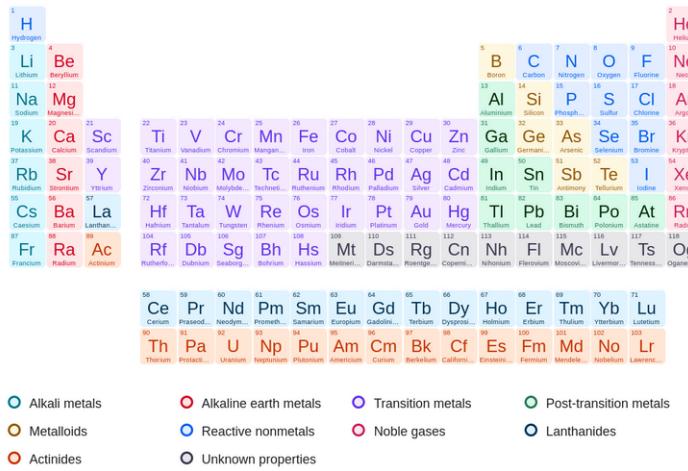
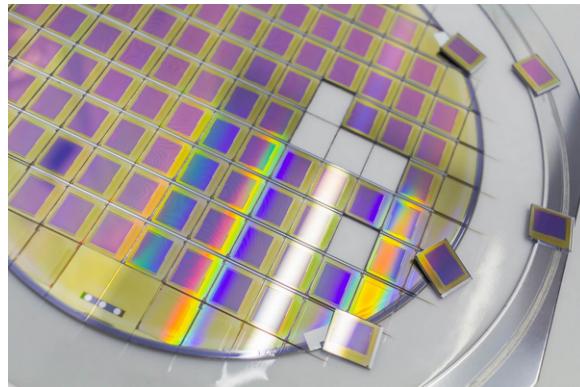


Air pollutants

# Silicon Wafer Processing: Example of Diffusion in Materials Science



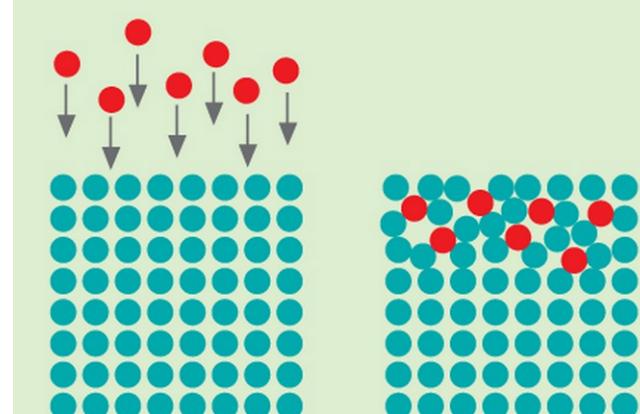
99.99999999% pure Silicon



# Silicon Wafer Processing: Example of Diffusion in Materials Science

**Intentional incorporation of impurities (e.g., boron, phosphorous)**

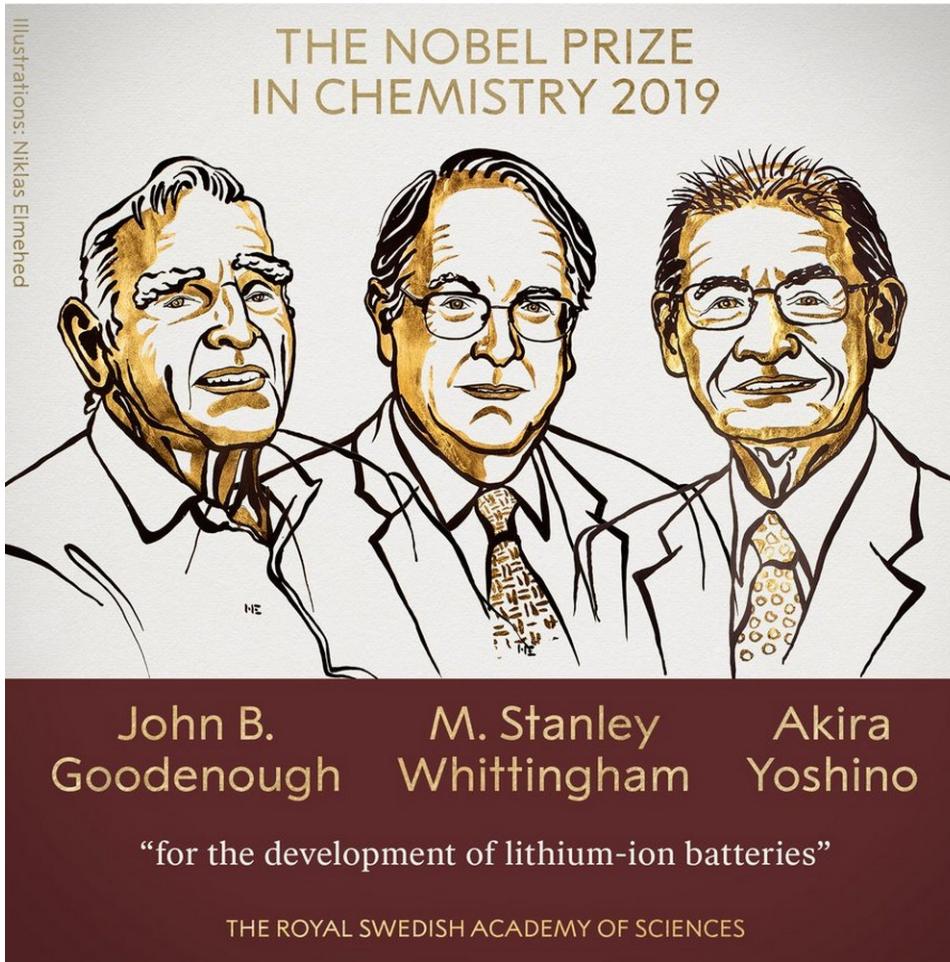
**Step 1:** Steady-state gas diffusion  
or ion implantation



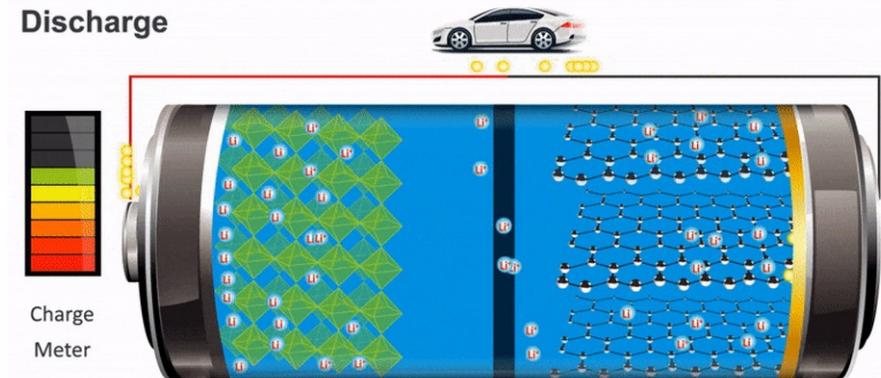
**Step 2:** Drive-in process (higher temperature)  
→ uniform distribution of impurities

# What is the materials science behind Li-ion battery winning the 2019 Nobel prize in Chemistry?

Illustrations: Niklas Elmehed

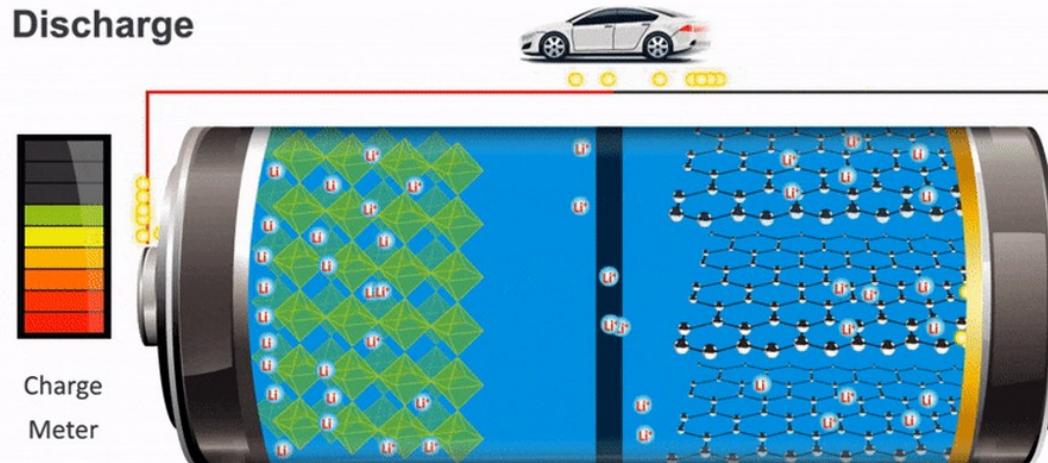


## How Lithium-ion Batteries Work



# Diffusion of Lithium ions in a battery

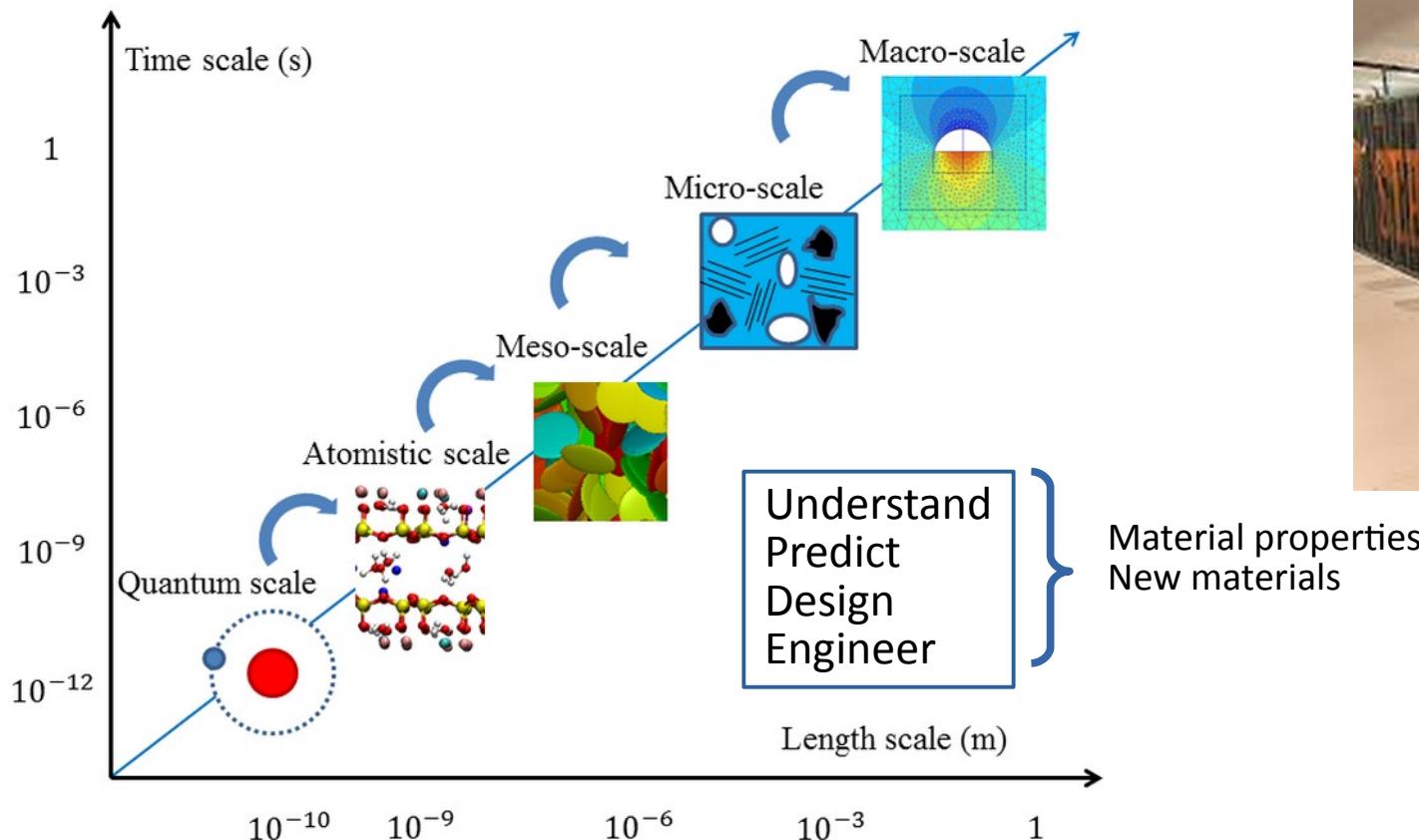
## How Lithium-ion Batteries Work



U.S. DEPARTMENT OF  
**ENERGY** | Office of ENERGY EFFICIENCY  
& RENEWABLE ENERGY

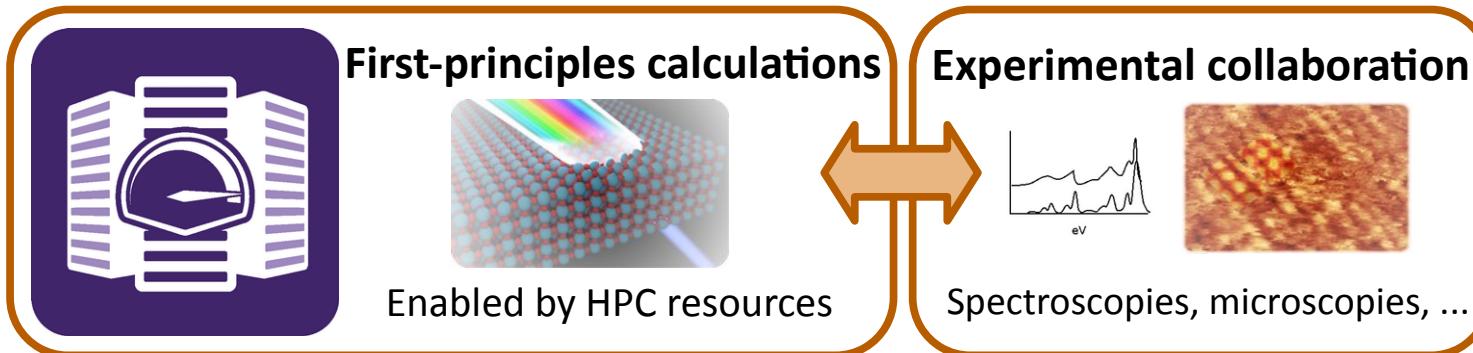
<https://www.energy.gov/science/doe-explainsbatteries>

# What is Modeling and Simulation in Materials Science?

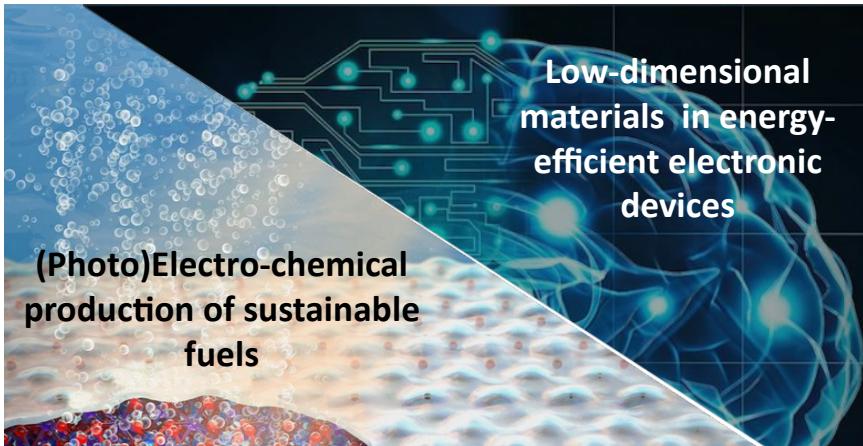


**TACC**  
TEXAS ADVANCED COMPUTING CENTER

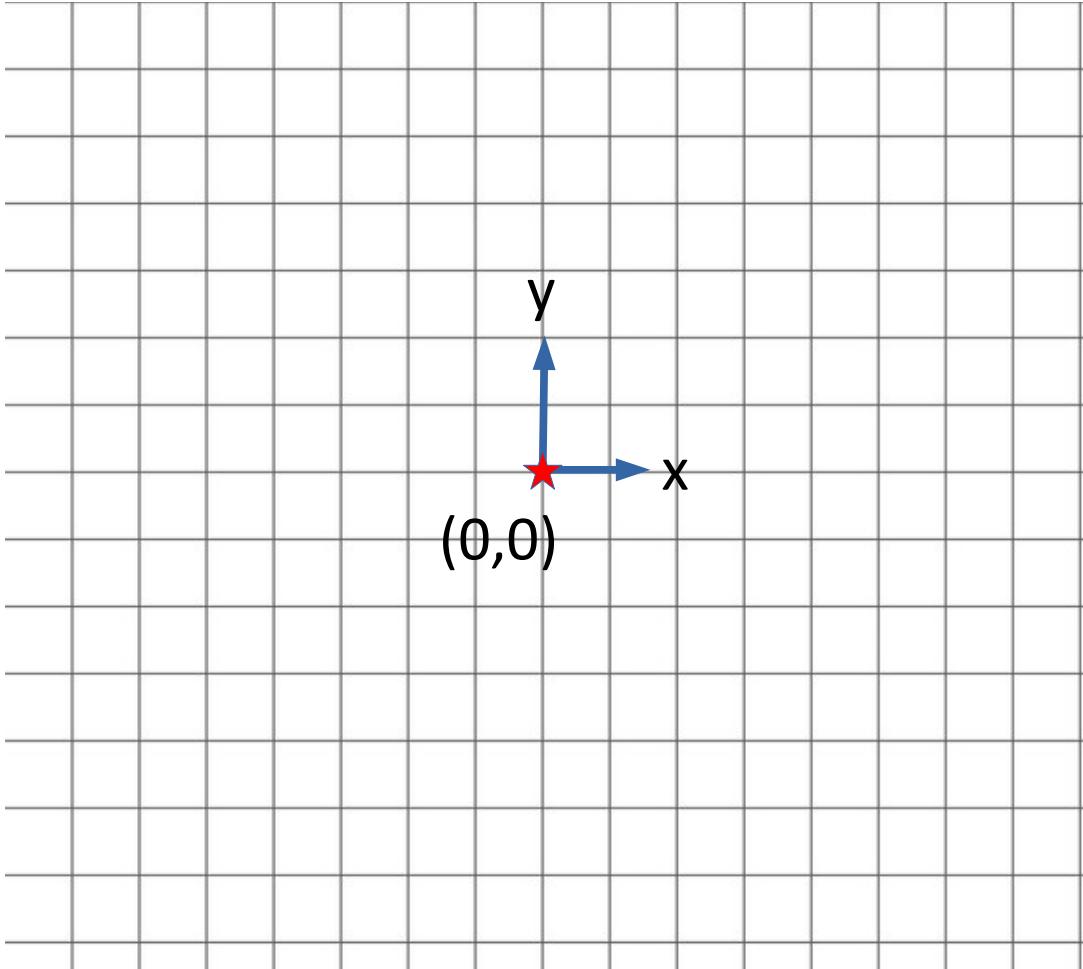
# Research Overview and Philosophy



## Harnessing Materials Imperfections for Energy Sustainability



# Random walk diffusion: an atomic model for diffusion



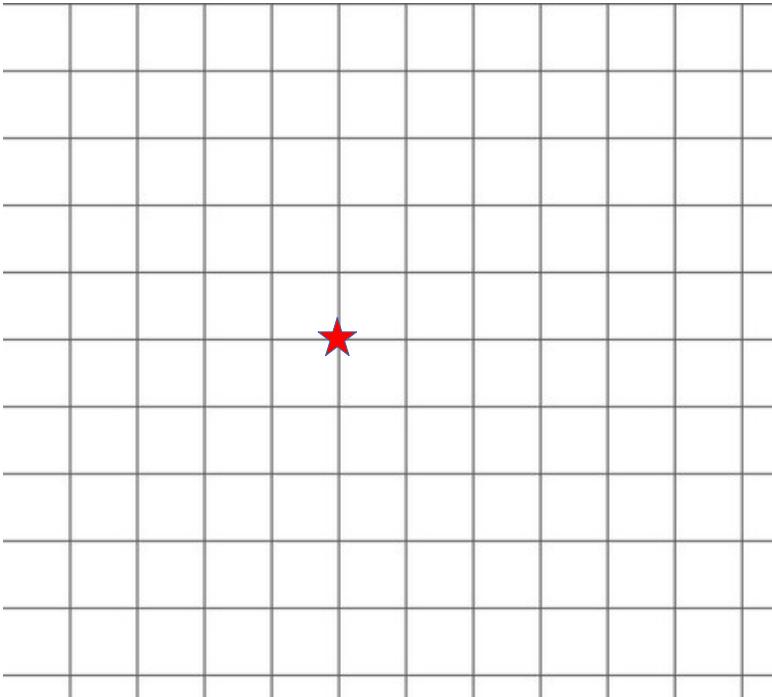
**Rules for the random walker:**

- divide time into  $nt$  discrete steps spaced by  $\Delta t$  time, where  $nt$  is an integer and  $\Delta t$  is a number
- can only move 1 space at each time step
- equal and random probability of moving up, down, left, right

If the random walker starts at the red star, where will it be after many ( $n \gg 1$ ) steps?

# An implementation of the 2D Random Walk Diffusion

- Example:  $nt = 10$  time steps,  $\Delta t = 1$  s  
Total time is  $t = nt * \Delta t = 10$  s



Start at origin  $(x,y) = (0,0)$  at time  $t = 0$

Use a random number generator and encode the following:

- 1 = move up
- 2 = move down
- 3 = move left
- 4 = move right

# Random walk diffusion: a small simulation

<https://rwd2d-mercury.runmercury.com/>

MERCURY

## Diffusion on Lattice

Choose a 2D lattice:

square

Enter the number of steps in the simulation:

100

Choose a display type:

static (fast)

Download Share

Random walk in 2D

Random walk trajectory on square lattice with 100 steps



# An implementation of the 2D Random Walk Diffusion

## Objective

User chooses  $nt$  = number of time steps

## Concept

Variable, integer

## Python Representation

`nt`

# An implementation of the 2D Random Walk Diffusion

## Objective

User chooses  $nt$  = number of time steps

---

Keep track of position of random walker  
at each time step.  
Let's assume it starts at the origin.

---

## Concept

Variable, integer

---

Array (list of items)  
e.g., [3, 4.5, 8, -1]

2D →  $x$  and  $y$  coordinate  
for each position

---

## Python Representation

`nt`

---

Use the library numpy,  
shorthand is np:

```
x = np.zeros(nt+1)
y = np.zeros(nt+1)
```

---

# An implementation of the 2D Random Walk Diffusion

## Objective

User chooses  $nt$  = number of time steps

Keep track of position of random walker  
at each time step.  
Let's assume it starts at the origin.

Specify how the position changes at  
each time step.

## Concept

Variable, integer

Array (list of items)  
e.g., [3, 4.5, 8, -1]

2D →  $x$  and  $y$  coordinate  
for each position

## Python Representation

`nt`

Use the library numpy,  
shorthand is `np`:

```
x = np.zeros(nt+1)
y = np.zeros(nt+1)
```

```
delx = np.array([?, ?, ?, ?])
dely = np.array([?, ?, ?, ?])
```

# An implementation of the 2D Random Walk Diffusion

## Objective

User chooses  $nt$  = number of time steps

Keep track of position of random walker at each time step.  
Let's assume it starts at the origin.

Specify how the position changes at each time step.

Save each new position of the diffusion path

## Concept

Variable, integer

Array (list of items)  
e.g., [3, 4.5, 8, -1]

2D →  $x$  and  $y$  coordinate for each position

Index the array  
i.e., access a specific element  
“zero index”

## Python Representation

`nt`

Use the library numpy,  
shorthand is `np`:

```
x = np.zeros(nt+1)
y = np.zeros(nt+1)
```

```
delx = np.array([?, ?, ?, ?])
dely = np.array([?, ?, ?, ?])
```

```
x = [1, 2, 3]
x[0] = 1
x[1] = 2
```

# An implementation of the 2D Random Walk Diffusion

## Objective

Repeat for  $nt$  times

Encode the random number to a change in position of the random walker

## Concept

for loop  
range function

Generate a (pseudo)-random number

## Python Representation

Input:

```
for i in range(3):  
    print(i)
```

Output:

```
0  
1  
2
```

---

```
np.floor(4* np.random.rand(nt))
```

Generate random number b/t 0 and 1

---

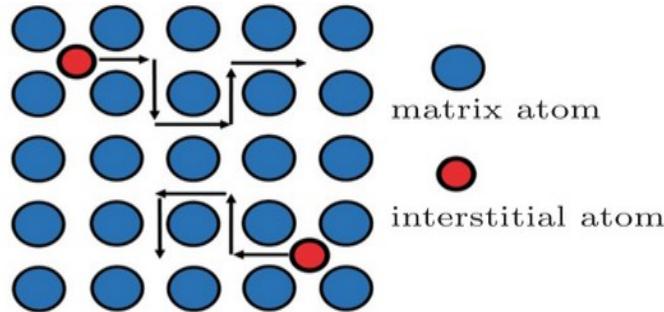
Random number b/t 0 and 4

---

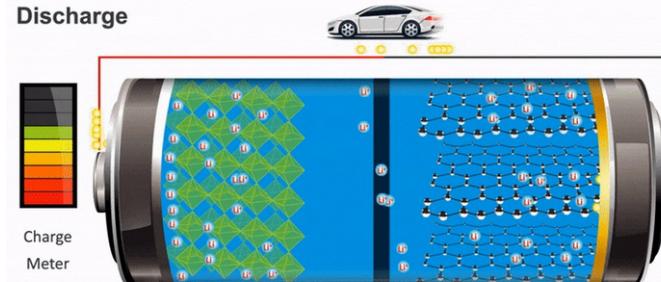
Random integer: 0, 1, 2, 3

# Let's try a little coding

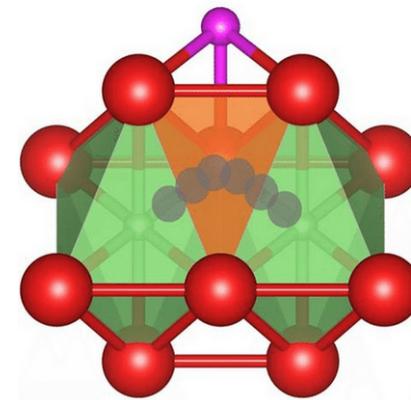
# Does the Random Walk Diffusion model have any correspondence with a real material?



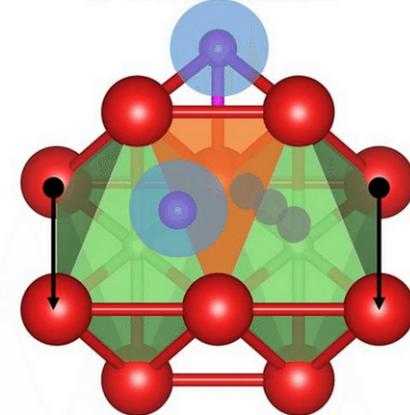
How Lithium-ion Batteries Work



1-TM Diffusion



2-TM Diffusion



Florian Schipper et al 2017 J. Electrochem. Soc. 164 A6220

# Things learned today

Materials Science and Engineering and the Materials Science Tetrahedron

Diffusion for mass transport

Examples of diffusion

Soda, air fresheners

Impurities in semiconductors, Li-ion battery

Computation and simulation of diffusion

Random Walk Diffusion

**More questions?** Reach out at <https://wangmaterialsgroup.com>