50 multiple-choice questions

Question 1 (Level 1) — *Derivative equals zero*
If $f'(x) = 2x - 4$, solve $f'(x) = 0$.

(A) $x = 2$

(B) $x = 4$

(C) $x = -2$

(D) $x = 0$

Question 2 (Level 1) — *Stationary point of x*
Find the stationary point of $y = x^2$.

(A) $(0, 0)$

(B) $(1, 1)$

(C) $(0, 1)$

(D) $(2, 4)$

Question 3 (Level 1) — *Minimum or maximum*
The parabola $y = x^2 - 4x + 3$ opens upward. Its stationary point is a:

(A) Minimum

(B) Maximum

(C) Point of inflection

(D) Saddle point

Question 4 (Level 1) — *Stationary point of -x*
Find the stationary point of $y = -x^2 + 6x$.

(A) $(3, 9)$

(B) $(3, -9)$

(C) $(6, 0)$

(D) $(-3, 9)$

Question 5 (Level 1) — *Nature from coefficient*
$y = -3x^2 + 12x - 7$ has a stationary point that is a:

(A) Maximum

(B) Minimum

(C) Point of inflection

(D) Cannot be determined

Question 6 (Level 1) — *Solve f'(x) = 0*
If $f'(x) = 3x^2 - 12$, find the values of x where $f'(x) = 0$.

(A) $x = 2$ or $x = -2$

(B) $x = 4$ or $x = -4$

(C) $x = 2$

(D) $x = \sqrt{12}$

Question 7 (Level 1) — *What is a stationary point?*
A stationary point occurs when:

(A) $f'(x) = 0$

(B) $f(x) = 0$

(C) $f'(x) > 0$

(D) $f''(x) = 0$

Question 8 (Level 1) — *Vertex of parabola*
Find the x-coordinate of the vertex of $y = x^2 - 8x + 15$.

(A) 4

(B) 8

(C) -4

(D) 15

Question 9 (Level 1) — *Minimum value*
Find the minimum value of $y = x^2 - 2x + 5$.

(A) 4

(B) 5

(C) 1

(D) 2

Question 10 (Level 1) — *Gradient sign*
If $f'(x) > 0$, the function is:

(A) Increasing

(B) Decreasing

(C) Stationary

(D) Concave up

Question 11 (Level 2) — *Stationary points of cubic*
Find the x-coordinates of the stationary points of $y = x^3 - 3x$.

(A) $x = 1$ and $x = -1$

(B) $x = 0$ and $x = 3$

(C) $x = \sqrt{3}$ and $x = -\sqrt{3}$

(D) $x = 3$ and $x = -3$

Question 12 (Level 2) — *Nature using second derivative*
If $f'(2) = 0$ and $f''(2) = 5$, the point at $x = 2$ is a:

(A) Local minimum

(B) Local maximum

(C) Point of inflection

(D) Cannot be determined

Question 13 (Level 2) — *Second derivative test*
For $y = x^3 - 3x$, classify the stationary point at $x = 1$.

(A) Local minimum

(B) Local maximum

(C) Point of inflection

(D) Saddle point

Question 14 (Level 2) — *Maximum point*
Find the coordinates of the local maximum of $y = -x^2 + 4x + 1$.

(A) $(2, 5)$

(B) $(4, 1)$

(C) $(2, 1)$

(D) $(-2, -11)$

Question 15 (Level 2) — *Sign diagram*
For $f'(x) = (x - 1)(x - 3)$, $f'(x) < 0$ when:

(A) $1 < x < 3$

(B) $x < 1$ or $x > 3$

(C) $x < 1$

(D) $x > 3$

Question 16 (Level 2) — *Increasing interval*
For what values of x is $y = x^3 - 12x$ increasing?

(A) $x < -2$ or $x > 2$

(B) $-2 < x < 2$

(C) $x > 0$

(D) $x > 2$

Question 17 (Level 2) — *Stationary point coordinates*
Find all stationary points of $y = 2x^3 - 9x^2 + 12x$.

(A) $(1, 5)$ and $(2, 4)$

(B) $(1, 5)$ and $(3, 9)$

(C) $(0, 0)$ and $(2, 4)$

(D) $(1, 4)$ and $(2, 5)$

Question 18 (Level 2) — *Local max of cubic*
Classify the stationary point at $x = -1$ for $y = x^3 - 3x$.

(A) Local maximum

(B) Local minimum

(C) Point of inflection

(D) Neither

Question 19 (Level 2) — *Number of stationary points*
How many stationary points does $y = x^4 - 4x^2$ have?

(A) 3

(B) 2

(C) 4

(D) 1

Question 20 (Level 2) — *Decreasing interval*
For what values of x is $y = x^2 - 6x + 8$ decreasing?

(A) $x < 3$

(B) $x > 3$

(C) $x < 0$

(D) $x < 6$

Question 21 (Level 3) — *Inflection point*
Find the point of inflection of $y = x^3 - 6x^2 + 9x + 1$.

(A) $(2, 3)$

(B) $(1, 5)$

(C) $(3, 1)$

(D) $(2, 1)$

Question 22 (Level 3) — *Complete analysis*
Find and classify all stationary points of $y = x^3 - 3x + 2$.

(A) Max $(-1, 4)$, Min $(1, 0)$

(B) Min $(-1, 4)$, Max $(1, 0)$

(C) Max $(1, 0)$, Min $(-1, 4)$

(D) Max $(-1, 2)$, Min $(1, 2)$

Question 23 (Level 3) — *Quartic stationary points*
Find the x-coordinates of the stationary points of $y = x^4 - 8x^2$.

(A) $x = 0, 2, -2$

(B) $x = 0, 4, -4$

(C) $x = 2, -2$

(D) $x = 0, 8, -8$

Question 24 (Level 3) — *Stationary inflection*
Show that $y = x^3$ has a stationary point of inflection at the origin. What is the nature?

(A) Stationary point of inflection

(B) Local minimum

(C) Local maximum

(D) Not a stationary point

Question 25 (Level 3) — *Global maximum on interval*
Find the global maximum of $f(x) = -x^2 + 4x$ on $[0, 5]$.

(A) 4

(B) 5

(C) 0

(D) -5

Question 26 (Level 3) — *Sign diagram for cubic*
Given $f'(x) = 3(x+2)(x-1)$, where is f increasing?

(A) $x < -2$ or $x > 1$

(B) $-2 < x < 1$

(C) $x > 1$

(D) $x > -2$

Question 27 (Level 3) — *Nature of x = 0 for x*
Classify the stationary point of $y = x^4$ at $x = 0$.

(A) Local minimum

(B) Point of inflection

(C) Local maximum

(D) Cannot be determined

Question 28 (Level 3) — *Minimum value of cubic*
Find the local minimum value of $y = x^3 - 6x^2 + 9x + 2$.

(A) 2

(B) 6

(C) 0

(D) 4

Question 29 (Level 3) — *Concavity*
Where is $y = x^3 - 3x^2 + 4$ concave up?

(A) $x > 1$

(B) $x < 1$

(C) $x > 0$

(D) $x > 3$

Question 30 (Level 3) — *Global min on interval*
Find the global minimum of $f(x) = x^3 - 3x$ on $[-2, 2]$.

(A) -2

(B) -3

(C) 0

(D) 2

Question 31 (Level 4) — *Stationary points of e function*
Find the stationary point of $y = xe^{-x}$.

(A) $(1, e^{-1})$

(B) $(0, 0)$

(C) $(e, 1)$

(D) $(-1, -e)$

Question 32 (Level 4) — *Nature of xe*
Classify the stationary point of $y = xe^{-x}$ at $x = 1$.

(A) Local maximum

(B) Local minimum

(C) Point of inflection

(D) Neither

Question 33 (Level 4) — *Stationary point of ln function*
Find the stationary point of $y = x - \ln(x)$ for $x > 0$.

(A) $(1, 1)$

(B) $(e, e - 1)$

(C) $(1, 0)$

(D) $(0, 0)$

Question 34 (Level 4) — *Trig stationary points*
Find the x-coordinates of the stationary points of $y = \sin(x) + \cos(x)$ for $0 \leq x \leq 2\pi$.

(A) $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$

(B) $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

(C) $x = 0$ and $x = \pi$

(D) $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$

Question 35 (Level 4) — *Parameter for stationary point*
If $y = x^3 - kx$ has a stationary point at $x = 2$, find k.

(A) 12

(B) 6

(C) 8

(D) 4

Question 36 (Level 4) — *Inflection of x - 3x + 2*
Find the inflection point of $y = x^3 - 3x^2 + 2$.

(A) $(1, 0)$

(B) $(1, 2)$

(C) $(0, 2)$

(D) $(3, 2)$

Question 37 (Level 4) — *ex quadratic factor*
Find the stationary points of $y = (x^2 - 1)e^x$.

(A) $x = -1 \pm \sqrt{2}$

(B) $x = \pm 1$

(C) $x = 1 \pm \sqrt{2}$

(D) $x = -2 \pm \sqrt{2}$

Question 38 (Level 4) — *Absolute max on closed interval*
Find the absolute maximum of $f(x) = x^3 - 3x + 1$ on $[-2, 2]$.

(A) 3

(B) 1

(C) 2

(D) -1

Question 39 (Level 4) — *Curve sketching info*
For $y = x^4 - 4x^3$, find the x-coordinate of the point of inflection closest to the origin.

(A) $x = 0$

(B) $x = 2$

(C) $x = 1$

(D) $x = 3$

Question 40 (Level 4) — *Nature with parameters*
If $f(x) = ax^3 + bx^2$ has a stationary point at $(2, 4)$, find a and b.

(A) $a = -1, b = 3$

(B) $a = 1, b = -3$

(C) $a = -2, b = 6$

(D) $a = -1, b = -3$

Question 41 (Level 5) — *Stationary points with parameter*
For $f(x) = x^3 - 3kx + 2$, find the values of k for which f has two distinct stationary points.

(A) $k > 0$

(B) $k \geq 0$

(C) $k > 1$

(D) $k \neq 0$

Question 42 (Level 5) — *Local max value of xe*
Find the maximum value of $f(x) = x^2 e^{-x}$ for $x > 0$.

(A) $4e^{-2}$

(B) $2e^{-2}$

(C) e^{-2}

(D) $2e^{-1}$

Question 43 (Level 5) — *Cubic with given stationary values*
If $f(x) = 2x^3 + ax^2 + bx + 1$ has stationary points at $x = 1$ and $x = -\frac{1}{3}$, find $a + b$.

(A) -4

(B) -2

(C) 0

(D) 4

Question 44 (Level 5) — *No stationary points condition*
For $f(x) = x^3 + px + q$, find the condition on p so that f has no stationary points.

(A) $p > 0$

(B) $p < 0$

(C) $p \geq 0$

(D) $p = 0$

Question 45 (Level 5) — *Exponential stationary analysis*
Find the coordinates of the stationary point of $y = (x - 1)^2 e^x$ and determine its nature.

(A) Local maximum at $(-1, 4e^{-1})$

(B) Local minimum at $(-1, 4e^{-1})$

(C) Local maximum at $(1, 0)$

(D) Local minimum at $(1, 0)$

Question 46 (Level 5) — *Inflection of x*
Find the number of inflection points of $y = x^4 - 6x^2 + 8x + 1$.

(A) 2

(B) 1

(C) 3

(D) 0

Question 47 (Level 5) — *Trig max value*
Find the maximum value of $y = 2\sin(x) + \cos(2x)$ for $0 \leq x \leq 2\pi$.

(A) $\frac{3}{2}$

(B) 3

(C) 2

(D) 1

Question 48 (Level 5) — *Rational function stationary points*
Find the x-values of the stationary points of $y = \frac{x^2}{x+1}$ (for $x \neq -1$).

(A) $x = 0$ and $x = -2$

(B) $x = 0$ and $x = -1$

(C) $x = -1$ and $x = -2$

(D) $x = 0$ only

Question 49 (Level 5) — *Cubic tangent condition*
For $f(x) = x^3 - 3x^2 + 4$, the local maximum value minus the local minimum value equals:

(A) 4

(B) 2

(C) 8

(D) 3

Question 50 (Level 5) — *Stationary point of x ln(x)*
Find the minimum value of $y = x\ln(x) - x$ for $x > 0$.

(A) -1

(B) 0

(C) $-e$

(D) 1

Solutions

Q1: (A)

$2x - 4 = 0 \Rightarrow x = 2$.

Q2: (A)

$2x = 0 \Rightarrow x = 0$. Stationary point is $(0, 0)$.

Q3: (A)

Since the coefficient of x^2 is positive, the parabola opens upward and the stationary point is a minimum.

Q4: (A)

$-2x + 6 = 0 \Rightarrow x = 3$. $y(3) = 9$. Stationary point is $(3, 9)$.

Q5: (A)

Since the coefficient of x^2 is $-3 < 0$, the parabola has a maximum.

Q6: (A)

$3x^2 = 12 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$.

Q7: (A)

A stationary point occurs when $f'(x) = 0$.

Q8: (A)

$\frac{dy}{dx} = 2x - 8 = 0 \Rightarrow x = 4$.

Q9: (A)

$y' = 2x - 2 = 0 \Rightarrow x = 1$. $y(1) = 1 - 2 + 5 = 4$.

Q10: (A)

When $f'(x) > 0$, the function is increasing.

Q11: (A)

$3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$.

Q12: (A)

$f''(2) = 5 > 0$, so the point is a local minimum.

Q13: (A)

$y''(1) = 6 > 0$, so $x = 1$ is a local minimum.

Q14: (A)

$x = 2$, $y(2) = -4 + 8 + 1 = 5$. Maximum at $(2, 5)$.

Q15: (A)

$f'(x) < 0$ when $1 < x < 3$ (product of one positive and one negative factor).

Q16: (A)

$3x^2 - 12 > 0 \Rightarrow x^2 > 4 \Rightarrow x < -2$ or $x > 2$.

Q17: (A)

$y' = 6(x - 1)(x - 2) = 0$. $x = 1$: $y = 5$; $x = 2$: $y = 4$. Points: $(1, 5)$ and $(2, 4)$.

Q18: (A)

$y'' = 6x$, $y''(-1) = -6 < 0$. So $x = -1$ is a local maximum.

Q19: (A)

$4x(x^2 - 2) = 0$: $x = 0, \pm\sqrt{2}$. There are 3 stationary points.

Q20: (A)

$2x - 6 < 0 \Rightarrow x < 3$.

Q21: (A)

$y'' = 6x - 12 = 0 \Rightarrow x = 2$. $y(2) = 8 - 24 + 18 + 1 = 3$. Inflection at $(2, 3)$.

Q22: (A)

$x = 1$: $y = 0$, $y'' = 6 > 0$ (min). $x = -1$: $y = 4$, $y'' = -6 < 0$ (max). Local max $(-1, 4)$, local min $(1, 0)$.

Q23: (A)

$4x(x - 2)(x + 2) = 0 \Rightarrow x = 0, 2, -2$.

Q24: (A)
$y'(0) = 0$, $y''(0) = 0$. $y'' < 0$ for $x < 0$ and $y'' > 0$ for $x > 0$: concavity changes. Stationary point of inflection.
Q25: (A)
$f'(x) = -2x + 4 = 0 \Rightarrow x = 2$. $f(0) = 0$, $f(2) = 4$, $f(5) = -5$. Max is 4.
Q26: (A)
$f'(x) > 0$ when $x < -2$ or $x > 1$.
Q27: (A)
$y' = 4x^3$: negative for $x < 0$, positive for $x > 0$. Sign changes $- \to +$: local minimum.
Q28: (A)
Stationary at $x = 1, 3$. $y''(3) = 6 > 0$ (min). $y(3) = 27 - 54 + 27 + 2 = 2$.
Q29: (A)
$6x - 6 > 0 \Rightarrow x > 1$.
Q30: (A)
$f(-2) = -2$, $f(-1) = 2$, $f(1) = -2$, $f(2) = 2$. Global min $= -2$.
Q31: (A)
$e^{-x}(1 - x) = 0 \Rightarrow x = 1$ (since $e^{-x} \neq 0$). $y(1) = e^{-1}$. Point: $(1, e^{-1})$.
Q32: (A)
$y''(1) = e^{-1}(1 - 2) = -e^{-1} < 0$. Local maximum.
Q33: (A)
$1 - \frac{1}{x} = 0 \Rightarrow x = 1$. $y(1) = 1$. Point: $(1, 1)$.
Q34: (A)
$\cos(x) = \sin(x) \Rightarrow \tan(x) = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$.
Q35: (A)
$3(4) - k = 0 \Rightarrow k = 12$.
Q36: (A)
$x = 1$, $y(1) = 1 - 3 + 2 = 0$. Inflection at $(1, 0)$.
Q37: (A)
$x^2 + 2x - 1 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}$.
Q38: (A)
$f(-2) = -1$, $f(-1) = 3$, $f(1) = -1$, $f(2) = 3$. Absolute max $= 3$.
Q39: (A)
$x = 0$ or $x = 2$. Closest to origin: $x = 0$.
Q40: (A)
From $12a + 4b = 0$: $b = -3a$. Sub: $8a - 12a = 4 \Rightarrow a = -1, b = 3$.
Q41: (A)
$3x^2 - 3k = 0 \Rightarrow x^2 = k$. Two distinct solutions when $k > 0$.
Q42: (A)
$f'(x) = 0$ at $x = 0$ or $x = 2$. For $x > 0$, $x = 2$: $f(2) = 4e^{-2}$.
Q43: (A)
Sum $= 1 - \frac{1}{3} = \frac{2}{3} = -\frac{a}{3}$, so $a = -2$. Product $= -\frac{1}{3} = \frac{b}{6}$, so $b = -2$. $a + b = -4$.
Q44: (A)
$3x^2 + p = 0 \Rightarrow x^2 = -\frac{p}{3}$. No real solutions when $p > 0$.
Q45: (A)
$y' = e^x(x - 1)(x + 1) = 0$ at $x = \pm 1$. $y(-1) = 4e^{-1}$: test $y''(-1)$... $y''(x) = e^x(x^2 + 2x - 1)$, $y''(-1) = e^{-1}(-2) < 0$: max at $(-1, 4e^{-1})$.
Q46: (A)
$y'' = 12(x - 1)(x + 1) = 0$ at $x = \pm 1$. Sign changes at both. Two inflection points.
Q47: (A)
$y' = 0$ when $\cos(x) = 0$ or $\sin(x) = \frac{1}{2}$. At $x = \frac{\pi}{6}$: $y = 1 + \frac{1}{2} = \frac{3}{2}$. Maximum $= \frac{3}{2}$.

Q48: (A)
$y' = \frac{x(x+2)}{(x+1)^2} = 0 \Rightarrow x = 0$ or $x = -2$.
Q49: (A)
$f(0) = 4$ (max), $f(2) = 0$ (min). Difference $= 4 - 0 = 4$.
Q50: (A)
$y' = \ln(x) = 0 \Rightarrow x = 1$. $y(1) = 0 - 1 = -1$. Min value $= -1$.