Question 1 [1 mark]

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Question 2 [1 mark]

The graph of $y = \frac{1}{(x+3)^2} + 4$ has a horizontal asymptote with the equation

Question 3 [1 mark]

The gradient of the graph of $y = e^{3x}$ at the point where the graph crosses the vertical axis is equal to

Question 4 [1 mark]

Which one of the following functions is not continuous over the interval $x \in [0, 5]$?

Question 5 [1 mark]

The largest value of a such that the function $f : (-\infty, a] \to R$, $f(x) = x^2 + 3x - 10$, where f is one-to-one, is

Question 6 [1 mark]

Which of the pairs of functions below are **not** inverse functions?

Question 7 [1 mark]

The graph of $y = f(x)$ is shown (a curve with a local minimum to the left of the y-axis and a local maximum to the right, with an inflection point near the origin). The graph of $y = f'(x)$, the first derivative of $f(x)$ with respect to x, could be

Question 8 [1 mark]

If $\int_0^b f(x)\,dx = 10$ and $\int_0^a f(x)\,dx = -4$, where $0 < a < b$, then $\int_a^b f(x)\,dx$ is equal to

Question 9 [1 mark]

Let $f : [0, \infty) \to R$, $f(x) = \sqrt{2x + 1}$.

The shortest distance, d, from the origin to the point (x, y) on the graph of f is given by

Question 10 [1 mark]

An organisation randomly surveyed 1000 Australian adults and found that 55% of those surveyed were happy with their level of physical activity.

An approximate 95% confidence interval for the percentage of Australian adults who were happy with their level of physical activity is closest to

Question 11 [1 mark]

If $\frac{d}{dx}(x \cdot \sin(x)) = \sin(x) + x \cdot \cos(x)$, then $\frac{1}{k} \int x \cos(x)\,dx$ is equal to

Question 12 [1 mark]

A bag contains three red pens and x black pens. Two pens are randomly drawn from the bag without replacement.

The probability of drawing a pen of each colour is equal to

Question 13 [1 mark]

The function $f(x) = \log_e\left(\frac{x+a}{x-a}\right)$, where a is a positive real constant, has the maximal domain

Question 14 [1 mark]

A continuous random variable, X, has a probability density function given by

$$f(x) = \begin{cases} \frac{2}{9}xe^{-\frac{1}{9}x^2} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The expected value of X, correct to three decimal places, is

Question 15 [1 mark]

The maximal domain of the function with rule $f(x) = \sqrt{x^2 - 2x - 3}$ is given by

Question 16 [1 mark]

The function $f(x) = \frac{1}{3}x^3 + mx^2 + nx + p$, for $m, n, p \in R$, has turning points at $x = -3$ and $x = 1$ and passes through the point $(3, 4)$.

The values of m, n and p respectively are

Question 17 [1 mark]

A function g is continuous on the domain $x \in [a, b]$ and has the following properties:

• The average rate of change of g between $x = a$ and $x = b$ is positive. • The instantaneous rate of change of g at $x = \frac{a+b}{2}$ is negative.

Therefore, on the interval $x \in [a, b]$, the function must be

Question 18 [1 mark]

If X is a binomial random variable where $n = 20$, $p = 0.88$ and $\Pr(X \geq 16 \mid X \geq a) = 0.9175$, correct to four decimal places, then a is equal to

Question 19 [1 mark]

A box is formed from a rectangular sheet of cardboard, which has a width of a units and a length of b units, by first cutting out squares of side length x units from each corner and then folding upwards to form a container with an open top.

The maximum volume of the box occurs when x is equal to

Question 20 [1 mark]

A soccer player kicks a ball with an angle of elevation of θř, where θ is a normally distributed random variable with a mean of 42ř and a standard deviation of 8ř.

The horizontal distance that the ball travels before landing is given by the function $d = 50\sin(2\theta)$.

The probability that the ball travels more than 40 m horizontally before landing is closest to

Question 1a [1 mark]

The diagram shows part of the graph of $y = f(x)$, where $f(x) = \frac{x^2}{12}$.
State the equation of the axis of symmetry of the graph of f.

Question 1b [1 mark]

State the derivative of f with respect to x.

Question 1c [2 marks]

The tangent to f at point M has gradient -2.
Find the equation of the tangent to f at point M.

Question 1d.i [1 mark]

Find the equation of the line perpendicular to the tangent passing through point M.

Question 1d.ii [2 marks]

The line perpendicular to the tangent at point M also cuts f at point N.
Find the area enclosed by this line and the curve $y = f(x)$.

Question 1e [4 marks]

Another parabola is defined by the rule $g(x) = \frac{x^2}{4a^2}$, where $a > 0$.
A tangent to g and the line perpendicular to the tangent at $x = -b$, where $b > 0$, are shown.
Find the value of b, in terms of a, such that the shaded area is a minimum.

Question 2a.i [1 mark]

On a remote island, there are only two species of animals: foxes and rabbits. The populations increase and decrease in a periodic pattern.

The population of rabbits can be modelled by the rule $r(t) = 1700\sin\left(\frac{\pi t}{80}\right) + 2500$.

One point of minimum fox population, $(20, 700)$, and one point of maximum fox population, $(100, 2500)$, are shown on the graph.

State the initial population of rabbits.

Question 2a.ii [1 mark]

State the minimum and maximum population of rabbits.

Question 2a.iii [1 mark]

State the number of weeks between maximum populations of rabbits.

Question 2b [2 marks]

The population of foxes can be modelled by the rule $f(t) = a\sin\left(\frac{\pi}{60}(t - b)\right) + 1600$.
Show that $a = 900$ and $b = 80$.

Question 2c [1 mark]

Find the maximum combined population of foxes and rabbits. Give your answer correct to the nearest whole number.

Question 2d [1 mark]

What is the number of weeks between the periods when the combined population of foxes and rabbits is a maximum?

Question 2e [4 marks]

The population of foxes is better modelled by the transformation of $y = \sin(t)$ under Q given by

$$Q : \begin{pmatrix} t \\ y \end{pmatrix} \mapsto \begin{pmatrix} \frac{90}{\pi} & 0 \\ 0 & 900 \end{pmatrix} \begin{pmatrix} t \\ y \end{pmatrix} + \begin{pmatrix} 60 \\ 1600 \end{pmatrix}$$

Find the average population during the first 300 weeks for the combined population of foxes and rabbits, where the population of foxes is modelled by the transformation of $y = \sin(t)$ under the transformation Q. Give your answer correct to the nearest whole number.

Question 2f [2 marks]

Over a longer period of time, it is found that the increase and decrease in the population of rabbits gets smaller and smaller.

The population of rabbits over a longer period of time can be modelled by the rule

$$s(t) = 1700e^{-0.003t} \sin\left(\frac{\pi t}{80}\right) + 2500, \quad \text{for all } t \geq 0$$

Find the average rate of change between the first two times when the population of rabbits is at a maximum. Give your answer correct to one decimal place.

Question 2g [2 marks]

Find the time, where $t > 40$, in weeks, when the rate of change of the rabbit population is at its greatest positive value. Give your answer correct to the nearest whole number.

Question 2h [1 mark]

Over time, the rabbit population approaches a particular value.

State this value.

Question 3a.i [1 mark]

Mika is flipping a coin. The unbiased coin has a probability of $\frac{1}{2}$ of landing on heads and $\frac{1}{2}$ of landing on tails.

Let X be the binomial random variable representing the number of times that the coin lands on heads.

Mika flips the coin five times.

Find $\Pr(X = 5)$.

Question 3a.ii [1 mark]

Find $\Pr(X \geq 2)$.

Question 3a.iii [2 marks]

Find $\Pr(X \geq 2 \mid X < 5)$, correct to three decimal places.

Question 3a.iv [2 marks]

Find the expected value and the standard deviation for X.

Question 3b.i [1 mark]

The height reached by each of Mika's coin flips is given by a continuous random variable, H, with the probability density function

$$f(h) = \begin{cases} ah^2 + bh + c & 1.5 \leq h \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

where h is the vertical height reached by the coin flip, in metres, between the coin and the floor, and a, b and c are real constants.

State the value of the definite integral $\int_{1.5}^{3} f(h)\, dh$.

Question 3b.ii [3 marks]

Given that $\Pr(H \leq 2) = 0.35$ and $\Pr(H \geq 2.5) = 0.25$, find the values of a, b and c.

Question 3b.iii [1 mark]

The ceiling of Mika's room is 3 m above the floor. The minimum distance between the coin and the ceiling is a continuous random variable, D, with probability density function g.

The function g is a transformation of the function f given by $g(d) = f(rd + s)$, where d is the minimum distance between the coin and the ceiling, and r and s are real constants.

Find the values of r and s.

Question 3c.i [1 mark]

Mika's sister Bella also has a coin. On each flip, Bella's coin has a probability of p of landing on heads and $(1 - p)$ of landing on tails, where p is a constant value between 0 and 1.

Bella flips her coin 25 times in order to estimate p.

Let $\hat{P}$ be the random variable representing the proportion of times that Bella's coin lands on heads in her sample.

Is the random variable $\hat{P}$ discrete or continuous? Justify your answer.

Question 3c.ii [1 mark]

If $\hat{p} = 0.4$, find an approximate 95% confidence interval for p, correct to three decimal places.

Question 3c.iii [1 mark]

Bella knows that she can decrease the width of a 95% confidence interval by using a larger sample of coin flips.

If $\hat{p} = 0.4$, how many coin flips would be required to halve the width of the confidence interval found in part c.ii.?

Question 4a [1 mark]

Consider the function f, where $f:\left(-\frac{1}{2},\frac{1}{2}\right)\to R$, $f(x)=\log_e\left(\frac{\frac{1}{2}+x}{\frac{1}{2}-x}\right)$.
State the range of $f(x)$.

Question 4b.i [2 marks]
Find $f'(0)$.

Question 4b.ii [1 mark]
State the maximal domain over which f is strictly increasing.

Question 4c [1 mark]
Show that $f(x)+f(-x)=0$.

Question 4d [3 marks]
Find the domain and the rule of f^{-1}, the inverse of f.

Question 4e.i [1 mark]
Let h be the function $h:\left(-\frac{1}{2},\frac{1}{2}\right)\to R$, $h(x)=k\log_e\left(\frac{\frac{1}{2}+x}{\frac{1}{2}-x}\right)$, where $k\in R$ and $k>0$.
The inverse function of h is defined by $h^{-1}:R\to R$, $h^{-1}(x)=\frac{1}{2}\cdot\frac{e^{kx}-1}{e^{kx}+1}$.
The area of the regions bound by the functions h and h^{-1} can be expressed as a function, $A(k)$.
Determine the range of values of k such that $A(k)>0$.

Question 4e.ii [1 mark]
This question has been redacted following the findings of the Independent Review into the VCAA's Examination-Setting Policies, Processes and Procedures for the VCE.

Question 5a [1 mark]
Consider the composite function $g(x)=f(\sin(2x))$, where the function $f(x)$ is an unknown but differentiable function for all values of x.
Use the following table of values for f and f':
| x | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | |—|—|—|—| | $f(x)$ | -2 | 5 | 3 | | $f'(x)$ | 7 | 0 | $\frac{1}{9}$ |
Find the value of $g\left(\frac{\pi}{6}\right)$.

Question 5b [1 mark]
The derivative of g with respect to x is given by $g'(x)=2\cos(2x)\cdot f'(\sin(2x))$.
Show that $g'\left(\frac{\pi}{6}\right)=\frac{1}{9}$.

Question 5c [2 marks]
Find the equation of the tangent to g at $x=\frac{\pi}{6}$.

Question 5d [2 marks]
Find the average value of the derivative function $g'(x)$ between $x=\frac{\pi}{8}$ and $x=\frac{\pi}{6}$.

Question 5e [3 marks]

Find four solutions to the equation $g'(x) = 0$ for the interval $x \in [0, \pi]$.

Solutions

Question 1

B

Marking guide:

- Period $= \frac{2\pi}{2} = \pi$.

Question 1

B

Marking guide:

- Period $= \frac{2\pi}{2} = \pi$.

Question 2

A

Marking guide:

- As $x \to \pm\infty$, $\frac{1}{(x+3)^2} \to 0$, so $y \to 4$.

Question 3

E

Marking guide:

- $\frac{dy}{dx} = 3e^{3x}$. At $x = 0$: $\frac{dy}{dx} = 3e^0 = 3$.

Question 4

D

Marking guide:

- $\tan\left(\frac{x}{3}\right)$ has a vertical asymptote at $x = \frac{3\pi}{2} \approx 4.71$, which is in $[0, 5]$.

Question 5

C

Marking guide:

- Vertex at $x = -\frac{3}{2} = -1.5$. For one-to-one on $(-\infty, a]$, need $a \leq -1.5$. Largest value is $a = -1.5$.

Question 6

C

Marking guide:

- C: $f(x) = x^2$ for $x < 0$ and $g(x) = \sqrt{x}$ for $x > 0$. The inverse of $f(x) = x^2, x < 0$ is $g(x) = -\sqrt{x}$, not $\sqrt{x}$.

Question 7

E

Marking guide:

- The original function has a local min (left) and local max (right), so $f'(x) = 0$ at two points, positive between them. Graph E shows a positive hump crossing zero at both turning points, consistent with a cubic-like derivative.

Question 8

E

Marking guide:

- $\int_a^b f(x)\,dx = \int_0^b f(x)\,dx - \int_0^a f(x)\,dx = 10 - (-4) = 14$.

Question 9

D

Marking guide:

- $d = \sqrt{x^2 + y^2} = \sqrt{x^2 + 2x + 1} = \sqrt{(x+1)^2} = x + 1$ (since $x \geq 0$).

Question 10

D

Marking guide:

- $\hat{p} = 0.55$, $n = 1000$. $E = 1.96\sqrt{\frac{0.55 \times 0.45}{1000}} \approx 0.0308$.

- CI: $(0.55 - 0.031, 0.55 + 0.031) \approx (0.519, 0.581)$, as percentage $(51.9, 58.1)$.

Question 11

C

Marking guide:

- From the product rule: $x\cos(x) = \frac{d}{dx}(x\sin(x)) - \sin(x)$.
- $\int x\cos(x)\,dx = x\sin(x) - \int \sin(x)\,dx + c$.
- So $\frac{1}{k}\int x\cos(x)\,dx = \frac{1}{k}\left(x\sin(x) - \int \sin(x)\,dx\right) + c$.

Question 12

A

Marking guide:

- Total pens: $3 + x$. $P = \frac{\binom{3}{1}\binom{x}{1}}{\binom{3+x}{2}} = \frac{3x}{\frac{(3+x)(2+x)}{2}} = \frac{6x}{(2+x)(3+x)}$.

Question 13

C

Marking guide:

- Need $\frac{x+a}{x-a} > 0$. Both factors same sign: $x > a$ or $x < -a$. Domain is $R \setminus [-a, a]$.

Question 14

B

Marking guide:

- $E(X) = \int_0^\infty x \cdot \frac{2}{9}xe^{-\frac{1}{9}x^2}\,dx = \int_0^\infty \frac{2}{9}x^2e^{-\frac{1}{9}x^2}\,dx \approx 2.659$.

Question 15

E

Marking guide:

- Need $x^2 - 2x - 3 \geq 0$, i.e. $(x-3)(x+1) \geq 0$. Solution: $x \leq -1$ or $x \geq 3$. Domain: $(-\infty, -1] \cup [3, \infty)$.

Question 16

B

Marking guide:

- $f'(x) = x^2 + 2mx + n$. Turning points at $x = -3$ and $x = 1$: $f'(x) = (x+3)(x-1) = x^2 + 2x - 3$.
- So $2m = 2 \Rightarrow m = 1$ and $n = -3$.
- $f(3) = 9 + 9 - 9 + p = 4 \Rightarrow p = -5$.

Question 17

A

Marking guide:

- $g(b) > g(a)$ (positive average rate), but $g'\left(\frac{a+b}{2}\right) < 0$ (decreasing at midpoint). The function increases overall but decreases somewhere in between, so it must be many-to-one.

Question 18

B

Marking guide:

- $\Pr(X \geq 16 \mid X \geq a) = \frac{\Pr(X\geq 16)}{\Pr(X\geq a)} = 0.9175$ (since $a < 16$).
- Use CAS to find $\Pr(X \geq 16)$ and solve for a. $a = 12$.

Question 19

D

Marking guide:

- $V = x(a-2x)(b-2x)$. $V' = 12x^2 - 4(a+b)x + ab = 0$.
- $x = \frac{4(a+b)\pm\sqrt{16(a+b)^2-48ab}}{24} = \frac{a+b\pm\sqrt{a^2-ab+b^2}}{6}$.
- Maximum at the smaller root: $x = \frac{a+b-\sqrt{a^2-ab+b^2}}{6}$.

Question 20
A
Marking guide:
- Solve $50\sin(2\theta) > 40$, i.e. $\sin(2\theta) > 0.8$.
- $2\theta > \sin^{-1}(0.8) \approx 53.13\text{ř}$ or $2\theta < 180\text{ř} - 53.13\text{ř} = 126.87\text{ř}$.
- So $26.57\text{ř} < \theta < 63.43\text{ř}$.
- $\Pr(26.57 < \theta < 63.43)$ where $\theta \sim N(42, 8^2)$.
- Using CAS: ≈ 0.969.

Question 1a
$x = 0$
Marking guide:
- The parabola $f(x) = \frac{x^2}{12}$ is symmetric about the y-axis. Axis of symmetry: $x = 0$.

Question 1b
$f'(x) = \frac{x}{6}$
Marking guide:
- $f'(x) = \frac{2x}{12} = \frac{x}{6}$.

Question 1c
$y = -2x - 12$
Marking guide:
- M1: Find x-coordinate: $f'(x) = -2 \Rightarrow \frac{x}{6} = -2 \Rightarrow x = -12$.
- A1: $f(-12) = \frac{144}{12} = 12$. Tangent: $y - 12 = -2(x + 12) \Rightarrow y = -2x - 12$.

Question 1d.i
$y = \frac{1}{2}x + 18$
Marking guide:
- Perpendicular gradient: $\frac{1}{2}$. Through $M(-12, 12)$: $y - 12 = \frac{1}{2}(x + 12) \Rightarrow y = \frac{1}{2}x + 18$.

Question 1d.ii
$\frac{10976}{9} \approx 1219.6$
Marking guide:
- M1: Find intersection: $\frac{x^2}{12} = \frac{1}{2}x + 18 \Rightarrow x^2 - 6x - 216 = 0 \Rightarrow (x + 12)(x - 18) = 0$. N at $x = 18$.
- A1: Area $= \int_{-12}^{18} \left(\frac{1}{2}x + 18 - \frac{x^2}{12}\right) dx$.

Question 1e
$b = a\sqrt[3]{2}$
Marking guide:
- M1: $g'(x) = \frac{x}{2a^2}$. At $x = -b$: gradient $= -\frac{b}{2a^2}$, perpendicular gradient $= \frac{2a^2}{b}$.
- M1: Find intersection of perpendicular line with $g(x)$, set up area integral.
- M1: Express area as a function of b and differentiate.
- A1: $b = a\sqrt[3]{2}$.

Question 2a.i
2500
Marking guide:
- $r(0) = 1700\sin(0) + 2500 = 2500$.

Question 2a.ii
Min: 800, Max: 4200
Marking guide:
- Min $= 2500 - 1700 = 800$. Max $= 2500 + 1700 = 4200$.

Question 2a.iii
160
Marking guide:
- Period $= \frac{2\pi}{\pi/80} = 160$ weeks.

Question 2b
See marking guide
Marking guide:
- M1: From graph: min fox pop $= 700$, max $= 2500$. Mean $= \frac{700+2500}{2} = 1600$ ✓. Amplitude $a = 2500 - 1600 = 900$.
- A1: Period same as rabbits $= 160$ weeks, so $\frac{2\pi}{\pi/60} = 120$... Actually from graph, period $= 2 \times (100 - 20) = 160$. Min at $t = 20$: $\sin\left(\frac{\pi}{60}(20 - b)\right) = -1 \Rightarrow \frac{\pi(20-b)}{60} = -\frac{\pi}{2} \Rightarrow 20 - b = -30$... Alternatively, max at $t = 100$, min at $t = 20$. $b = 80$.

Question 2c
5339
Marking guide:
- Using CAS, maximise $r(t) + f(t)$. Maximum combined population ≈ 5339.

Question 2d
160
Marking guide:
- Both populations have the same period of 160 weeks, so the combined population also has period 160 weeks.

Question 2e
4100 (to nearest whole number)
Marking guide:
- M1: Under transformation Q: $t_{new} = \frac{90}{\pi}t + 60$, $y_{new} = 900y + 1600$.
- So fox model becomes $f(t) = 900\sin\left(\frac{\pi(t-60)}{90}\right) + 1600$.
- M1: Combined population $= r(t) + f(t) = 1700\sin\left(\frac{\pi t}{80}\right) + 2500 + 900\sin\left(\frac{\pi(t-60)}{90}\right) + 1600$.
- M1: Average $= \frac{1}{300}\int_0^{300}(r(t) + f(t))\,dt$.
- A1: Evaluate using CAS ≈ 4100.

Question 2f
-3.0
Marking guide:
- M1: Find the first two maximum points of $s(t)$ using CAS (e.g., $t_1 \approx 40$, $t_2 \approx 200$, or solve $s'(t) = 0$).
- A1: Average rate of change $= \frac{s(t_2)-s(t_1)}{t_2-t_1} \approx -3.0$.

Question 2g
$t \approx 156$ weeks
Marking guide:
- M1: Find $s'(t)$ and maximise it for $t > 40$ using CAS.
- A1: $t \approx 156$ weeks.

Question 2h
2500
Marking guide:
- As $t \to \infty$, $e^{-0.003t} \to 0$, so $s(t) \to 2500$.

Question 3a.i
$\frac{1}{32}$
Marking guide:
- $\Pr(X = 5) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$.

Question 3a.ii
$\frac{13}{16}$
Marking guide:
- $\Pr(X \geq 2) = 1 - \Pr(X = 0) - \Pr(X = 1) = 1 - \frac{1}{32} - \frac{5}{32} = \frac{26}{32} = \frac{13}{16}$.

Question 3a.iii
0.806
Marking guide:
- M1: $\Pr(X \geq 2 \mid X < 5) = \frac{\Pr(2 \leq X < 5)}{\Pr(X<5)} = \frac{\Pr(X \geq 2) - \Pr(X=5)}{1-\Pr(X=5)}$.
- A1: $= \frac{\frac{13}{16} - \frac{1}{32}}{1-\frac{1}{32}} = \frac{\frac{25}{32}}{\frac{31}{32}} = \frac{25}{31} \approx 0.806$.

Question 3a.iv
$E(X) = \frac{5}{2}$, $\text{SD}(X) = \frac{\sqrt{5}}{2}$
Marking guide:
- M1: $E(X) = np = 5 \times \frac{1}{2} = \frac{5}{2}$.
- A1: $\text{Var}(X) = np(1-p) = \frac{5}{4}$. $\text{SD}(X) = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$.

Question 3b.i
1
Marking guide:
- Total area under a PDF is 1.

Question 3b.ii
$a = \frac{16}{15}$, $b = -\frac{16}{3}$, $c = \frac{32}{5}$
Marking guide:
- M1: Set up three equations: $\int_{1.5}^{3}(ah^2+bh+c)\,dh = 1$, $\int_{1.5}^{2}(ah^2+bh+c)\,dh = 0.35$, $\int_{2.5}^{3}(ah^2+bh+c)\,dh = 0.25$.
- M1: Solve the system of three equations in three unknowns.
- A1: $a = \frac{16}{15}$, $b = -\frac{16}{3}$, $c = \frac{32}{5}$.

Question 3b.iii
$r = -1$, $s = 3$
Marking guide:
- If ceiling is 3 m, then $d = 3 - h$, so $h = 3 - d = -d + 3$. Hence $g(d) = f(-d+3) = f(-1 \cdot d + 3)$, so $r = -1$, $s = 3$.

Question 3c.i
Discrete
Marking guide:
- Discrete, because $\hat{P} = \frac{X}{25}$ where X can only take integer values 0, 1, 2, ..., 25. So $\hat{P}$ can only take a countable number of values.

Question 3c.ii
$(0.208, 0.592)$
Marking guide:
- $CI = 0.4 \pm 1.96\sqrt{\frac{0.4 \times 0.6}{25}} = 0.4 \pm 1.96 \times 0.09798 = 0.4 \pm 0.192 = (0.208, 0.592)$.

Question 3c.iii
100
Marking guide:
- To halve the width, need to multiply n by 4 (since width $\propto \frac{1}{\sqrt{n}}$). $4 \times 25 = 100$.

Question 4a
R
Marking guide:
- As $x \to \frac{1}{2}^-$, $f(x) \to +\infty$. As $x \to -\frac{1}{2}^+$, $f(x) \to -\infty$. Range is R.

Question 4b.i
$f'(0) = 4$
Marking guide:
- M1: $f(x) = \log_e\left(\frac{1}{2} + x\right) - \log_e\left(\frac{1}{2} - x\right)$.
- A1: $f'(x) = \frac{1}{\frac{1}{2}+x} + \frac{1}{\frac{1}{2}-x}$. At $x = 0$: $f'(0) = 2 + 2 = 4$.

Question 4b.ii
$\left(-\frac{1}{2}, \frac{1}{2}\right)$
Marking guide:
- $f'(x) > 0$ for all $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ (the entire domain). So f is strictly increasing on its whole domain.

Question 4c
See marking guide
Marking guide:
- A1: $f(-x) = \log_e\left(\frac{\frac{1}{2}-x}{\frac{1}{2}+x}\right) = -\log_e\left(\frac{\frac{1}{2}+x}{\frac{1}{2}-x}\right) = -f(x)$. Therefore $f(x) + f(-x) = 0$.

Question 4d
$f^{-1} : R \to R$, $f^{-1}(x) = \frac{e^x-1}{2(e^x+1)}$
Marking guide:
- M1: Let $y = \log_e\left(\frac{\frac{1}{2}+x}{\frac{1}{2}-x}\right)$. Swap x and y: $x = \log_e\left(\frac{\frac{1}{2}+y}{\frac{1}{2}-y}\right)$.
- M1: $e^x = \frac{\frac{1}{2}+y}{\frac{1}{2}-y}$. Solve for y: $e^x(\frac{1}{2} - y) = \frac{1}{2} + y$, $\frac{e^x}{2} - ye^x = \frac{1}{2} + y$, $y(1 + e^x) = \frac{e^x-1}{2}$.
- A1: $f^{-1}(x) = \frac{e^x-1}{2(e^x+1)}$. Domain of f^{-1} is R.

Question 4e.i
$k \neq 1$ (i.e. $k \in (0,1) \cup (1,\infty)$)
Marking guide:
- When $k = 1$, $h = f$ and $h^{-1} = f^{-1}$. Since f is odd and passes through the origin, h and h^{-1} are reflections in $y = x$ and the bounded regions cancel to zero. For $k \neq 1$, the functions are distinct from each other (not symmetric about $y = x$), so $A(k) > 0$. Range: $k \in (0,1) \cup (1,\infty)$.

Question 4e.ii
Redacted
Marking guide:
- This question was redacted by VCAA. All students were awarded this mark.

Question 5a
3
Marking guide:
- $g\left(\frac{\pi}{6}\right) = f\left(\sin\left(\frac{\pi}{3}\right)\right) = f\left(\frac{\sqrt{3}}{2}\right) = 3$.

Question 5b
See marking guide
Marking guide:
- $g'\left(\frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{3}\right) \cdot f'\left(\sin\left(\frac{\pi}{3}\right)\right) = 2 \times \frac{1}{2} \times f'\left(\frac{\sqrt{3}}{2}\right) = 1 \times \frac{1}{9} = \frac{1}{9}$.

Question 5c
$y = \frac{1}{9}x - \frac{\pi}{54} + 3$ or $y = \frac{1}{9}\left(x - \frac{\pi}{6}\right) + 3$
Marking guide:
- M1: Point: $\left(\frac{\pi}{6}, 3\right)$, gradient: $\frac{1}{9}$.
- A1: $y - 3 = \frac{1}{9}\left(x - \frac{\pi}{6}\right)$.

Question 5d
$\frac{-2}{\frac{\pi}{6}-\frac{\pi}{8}} = \frac{-2}{\frac{\pi}{24}} = \frac{-48}{\pi}$
Marking guide:
- M1: Average value of $g'(x) = \frac{1}{\frac{\pi}{6}-\frac{\pi}{8}} \int_{\pi/8}^{\pi/6} g'(x)\,dx = \frac{g(\pi/6)-g(\pi/8)}{\frac{\pi}{6}-\frac{\pi}{8}}$.
- A1: $g(\pi/8) = f(\sin(\pi/4)) = f(\frac{\sqrt{2}}{2}) = 5$. Average $= \frac{3-5}{\pi/24} = \frac{-2\times 24}{\pi} = -\frac{48}{\pi}$.

Question 5e
$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{8}, \frac{3\pi}{8}$
Marking guide:
- M1: $g'(x) = 2\cos(2x) \cdot f'(\sin(2x)) = 0$. Either $\cos(2x) = 0$ or $f'(\sin(2x)) = 0$.

- M1: $\cos(2x) = 0 \Rightarrow 2x = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$. (Also $2x = \frac{5\pi}{2}$ gives $x = \frac{5\pi}{4} > \pi$.)
- A1: $f'(\sin(2x)) = 0 \Rightarrow \sin(2x) = \frac{\sqrt{2}}{2}$ (from table). $2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \ldots$. So $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}$ (last one $> \pi$). Also $2x = 2\pi + \frac{\pi}{4}$ etc. Four solutions in $[0, \pi]$: $\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{3\pi}{4}$.