50 multiple-choice questions

Question 1 (Level 1) — *Evaluate f(g(2))*
If $f(x) = x + 3$ and $g(x) = 2x$, find $f(g(2))$.

(A) 7

(B) 10

(C) 8

(D) 5

Question 2 (Level 1) — *Evaluate g(f(1))*
If $f(x) = x + 1$ and $g(x) = 3x$, find $g(f(1))$.

(A) 6

(B) 4

(C) 5

(D) 9

Question 3 (Level 1) — *Simple composite expression*
If $f(x) = 2x$ and $g(x) = x + 5$, find $f(g(x))$.

(A) $2x + 10$

(B) $2x + 5$

(C) $2x + 7$

(D) $x + 10$

Question 4 (Level 1) — *Evaluate f(g(0))*
If $f(x) = x^2$ and $g(x) = x + 3$, find $f(g(0))$.

(A) 9

(B) 3

(C) 6

(D) 12

Question 5 (Level 1) — *Order matters*
If $f(x) = x - 1$ and $g(x) = 2x$, which is correct?

(A) $f(g(3)) = 5$ and $g(f(3)) = 4$

(B) $f(g(3)) = 4$ and $g(f(3)) = 5$

(C) $f(g(3)) = g(f(3)) = 5$

(D) $f(g(3)) = g(f(3)) = 4$

Question 6 (Level 1) — *Composite with constant*
If $f(x) = 5$ and $g(x) = x + 2$, find $f(g(10))$.

(A) 5

(B) 12

(C) 15

(D) 7

Question 7 (Level 1) — *Evaluate with negatives*
If $f(x) = x + 4$ and $g(x) = -x$, find $f(g(3))$.

(A) 1

(B) -1

(C) 7

(D) -7

Question 8 (Level 1) — *Composite rule expression*
If $f(x) = 3x$ and $g(x) = x - 2$, find $g(f(x))$.

(A) $3x - 2$

(B) $3x - 6$

(C) $3(x - 2)$

(D) $x - 6$

Question 9 (Level 1) — *Double application*
If $f(x) = x + 1$, find $f(f(3))$.

(A) 5

(B) 4

(C) 6

(D) 7

Question 10 (Level 1) — *Identify the composite*
If $h(x) = 2(x + 1)$, which functions f and g give $h(x) = f(g(x))$?

(A) $f(x) = 2x,\ g(x) = x + 1$

(B) $f(x) = x + 1,\ g(x) = 2x$

(C) $f(x) = 2x + 1,\ g(x) = x$

(D) $f(x) = x,\ g(x) = 2x + 1$

Question 11 (Level 2) — *Composite with squaring*
If $f(x) = x^2$ and $g(x) = 2x - 1$, find $f(g(3))$.

(A) 25

(B) 35

(C) 17

(D) 10

Question 12 (Level 2) — *Composite expression with square*
If $f(x) = x^2$ and $g(x) = x + 2$, find $f(g(x))$.

(A) $x^2 + 4x + 4$

(B) $x^2 + 4$

(C) $x^2 + 2x + 4$

(D) $x^2 + 2$

Question 13 (Level 2) — *Reverse composite expression*
If $f(x) = x + 2$ and $g(x) = x^2$, find $g(f(x))$.

(A) $x^2 + 4x + 4$

(B) $x^2 + 2$

(C) $x^2 + 4$

(D) $x^2 + 2x + 2$

Question 14 (Level 2) — *Evaluate triple composite*
If $f(x) = x + 1$, find $f(f(f(2)))$.

(A) 5

(B) 4

(C) 6

(D) 3

Question 15 (Level 2) — *Decompose a function*
If $h(x) = (3x)^2$, which decomposition gives $h(x) = f(g(x))$?

(A) $f(x) = x^2,\ g(x) = 3x$

(B) $f(x) = 3x,\ g(x) = x^2$

(C) $f(x) = 9x,\ g(x) = x^2$

(D) $f(x) = 3x^2,\ g(x) = x$

Question 16 (Level 2) — *Composite with fraction*
If $f(x) = \dfrac{1}{x}$ and $g(x) = x + 1$, find $f(g(2))$.

(A) $\dfrac{1}{3}$

(B) 3

(C) $\dfrac{1}{2}$

(D) $\dfrac{2}{3}$

Question 17 (Level 2) — *Composite expression with reciprocal*
If $f(x) = \dfrac{1}{x}$ and $g(x) = x - 4$, find $f(g(x))$.

(A) $\dfrac{1}{x-4}$

(B) $\dfrac{1}{x} - 4$

(C) $\dfrac{1}{x} - \dfrac{1}{4}$

(D) $\dfrac{x}{4}$

Question 18 (Level 2) — *Find g given composite*
If $f(x) = 2x + 1$ and $f(g(x)) = 2x + 5$, find $g(x)$.

(A) $g(x) = x + 2$

(B) $g(x) = x + 3$

(C) $g(x) = x + 4$

(D) $g(x) = 2x + 2$

Question 19 (Level 2) — *Composite with absolute value*
If $f(x) = |x|$ and $g(x) = x - 5$, find $f(g(2))$.

(A) 3

(B) -3

(C) 7

(D) -7

Question 20 (Level 2) — *Solve for input*
If $f(x) = 2x$ and $g(x) = x + 3$, solve $f(g(a)) = 14$.

(A) $a = 4$

(B) $a = 5$

(C) $a = 7$

(D) $a = 3$

Question 21 (Level 3) — *Composite with square root*
If $f(x) = \sqrt{x}$ and $g(x) = 4x + 5$, find $f(g(1))$.

(A) 3

(B) 9

(C) $\sqrt{5}$

(D) 4

Question 22 (Level 3) — *Domain of composite*
If $f(x) = \sqrt{x}$ and $g(x) = x - 3$, what is the domain of $f(g(x))$?

(A) $[3, \infty)$

(B) $(3, \infty)$

(C) $[0, \infty)$

(D) $(-\infty, 3]$

Question 23 (Level 3) — *Composite expression simplified*
If $f(x) = x^2 - 1$ and $g(x) = x + 1$, find and simplify $f(g(x))$.

(A) $x^2 + 2x$

(B) $x^2 + 2x + 2$

(C) $x^2 + 1$

(D) $x^2 + 2x + 1$

Question 24 (Level 3) — *Decompose square root expression*
Express $h(x) = \sqrt{2x + 1}$ as $f(g(x))$. What are f and g?

(A) $f(x) = \sqrt{x},\ g(x) = 2x + 1$

(B) $f(x) = 2x + 1,\ g(x) = \sqrt{x}$

(C) $f(x) = \sqrt{2x},\ g(x) = x + 1$

(D) $f(x) = x + 1,\ g(x) = \sqrt{2x}$

Question 25 (Level 3) — *Composite with quadratic and linear*
If $f(x) = x^2 + x$ and $g(x) = 2x$, find $f(g(x))$.

(A) $4x^2 + 2x$

(B) $2x^2 + 2x$

(C) $4x^2 + x$

(D) $2x^2 + x$

Question 26 (Level 3) — *Domain restriction from denominator*
If $f(x) = \dfrac{1}{x}$ and $g(x) = x^2 - 4$, what values of x must be excluded from the domain of $f(g(x))$?

(A) $x \neq 2$ and $x \neq -2$

(B) $x \neq 4$

(C) $x \neq 0$

(D) $x \neq 2$

Question 27 (Level 3) — *Solve composite equation*
If $f(x) = x^2$ and $g(x) = x - 1$, solve $f(g(x)) = 9$.

(A) $x = 4$ or $x = -2$

(B) $x = 4$ only

(C) $x = 10$ or $x = -8$

(D) $x = 3$ or $x = -3$

Question 28 (Level 3) — *Find f given composite*
If $g(x) = 3x$ and $f(g(x)) = 9x^2 + 1$, find $f(x)$.

(A) $f(x) = x^2 + 1$

(B) $f(x) = 9x^2 + 1$

(C) $f(x) = 3x^2 + 1$

(D) $f(x) = x^2 + 3$

Question 29 (Level 3) — *Compare fg and gf*
If $f(x) = x^2$ and $g(x) = x + 3$, find $f(g(x)) - g(f(x))$.

(A) $6x + 6$

(B) $6x + 9$

(C) $6x$

(D) $6x + 12$

Question 30 (Level 3) — *Composite self-application*
If $f(x) = 2x - 3$, find $f(f(x))$.

(A) $4x - 9$

(B) $4x - 6$

(C) $4x - 3$

(D) $2x - 9$

Question 31 (Level 4) — *Domain of composite with log*
If $f(x) = \log_e(x)$ and $g(x) = 3 - x$, state the domain of $f(g(x))$.

(A) $(-\infty, 3)$

(B) $(-\infty, 3]$

(C) $(3, \infty)$

(D) $(0, \infty)$

Question 32 (Level 4) — *Composite with exponential*
If $f(x) = e^x$ and $g(x) = 2x + 1$, find $f(g(x))$.

(A) e^{2x+1}

(B) $2e^x + 1$

(C) $e^{2x} + e$

(D) $2xe^x$

Question 33 (Level 4) — *Decompose exponential function*
Express $h(x) = e^{x^2}$ as $f(g(x))$. What are f and g?

(A) $f(x) = e^x,\ g(x) = x^2$

(B) $f(x) = x^2,\ g(x) = e^x$

(C) $f(x) = e^{x^2},\ g(x) = x$

(D) $f(x) = e^x,\ g(x) = 2x$

Question 34 (Level 4) — *Range of composite*
If $f(x) = x^2 + 1$ with domain $\mathbb{R}$ and $g(x) = \sin(x)$, what is the range of $f(g(x))$?

(A) $[1, 2]$

(B) $[0, 2]$

(C) $[1, \infty)$

(D) $[0, 1]$

Question 35 (Level 4) — *Domain of log composite*
Find the domain of $f(x) = \log_e(x^2 - 9)$.

(A) $(-\infty, -3) \cup (3, \infty)$

(B) $(-3, 3)$

(C) $(-\infty, -3] \cup [3, \infty)$

(D) $(9, \infty)$

Question 36 (Level 4) — *Chain rule connection*
If $h(x) = (2x + 3)^5$, and we write $h = f \circ g$ where $f(u) = u^5$ and $g(x) = 2x + 3$, what is $h'(x)$ using the chain rule?

(A) $10(2x + 3)^4$

(B) $5(2x + 3)^4$

(C) $10(2x + 3)^5$

(D) $2(2x + 3)^4$

Question 37 (Level 4) — *Composite with trig*
If $f(x) = \sin(x)$ and $g(x) = 2x$, find $f(g(x))$ and its period.

(A) $\sin(2x)$, period $= \pi$

(B) $2\sin(x)$, period $= 2\pi$

(C) $\sin(2x)$, period $= 2\pi$

(D) $\sin(2x)$, period $= \dfrac{\pi}{2}$

Question 38 (Level 4) — *Composite existence check*
Let $f : [0, \infty) \to \mathbb{R},\ f(x) = \sqrt{x}$ and $g : \mathbb{R} \to \mathbb{R},\ g(x) = x - 5$. Does $f \circ g$ exist? If so, state its domain.

(A) Yes, domain $= [5, \infty)$

(B) Yes, domain $= [0, \infty)$

(C) No, $f \circ g$ does not exist

(D) Yes, domain $= (-5, \infty)$

Question 39 (Level 4) — *Chain rule with exponential*
If $h(x) = e^{3x-2}$, write h as $f \circ g$ and find $h'(x)$.

(A) $3e^{3x-2}$

(B) e^{3x-2}

(C) $(3x-2)e^{3x-2}$

(D) $3xe^{3x-2}$

Question 40 (Level 4) — *Solve composite with log*
If $f(x) = e^x$ and $g(x) = \log_e(x)$, find $f(g(x))$ and $g(f(x))$.

(A) $f(g(x)) = x$ (for $x > 0$) and $g(f(x)) = x$ (for all x)

(B) $f(g(x)) = x$ and $g(f(x)) = x$ (both for all x)

(C) $f(g(x)) = e^x$ and $g(f(x)) = \log_e(x)$

(D) $f(g(x)) = 1$ and $g(f(x)) = 1$

Question 41 (Level 5) — *Domain of restricted composite*
Let $f : (0, \infty) \to \mathbb{R},\ f(x) = \log_e(x)$ and $g : [-2, 2] \to \mathbb{R},\ g(x) = 4 - x^2.$ State the domain and rule of $f \circ g$.

(A) Domain $(-2, 2)$, rule $\log_e(4 - x^2)$

(B) Domain $[-2, 2]$, rule $\log_e(4 - x^2)$

(C) Domain $(-2, 2)$, rule $\log_e(x^2 - 4)$

(D) Domain $(-\infty, -2) \cup (2, \infty)$, rule $\log_e(4 - x^2)$

Question 42 (Level 5) — *Range of composite function*
Let $f(x) = e^x$ and $g : [-1, 1] \to \mathbb{R},\ g(x) = x^2.$ Find the range of $f \circ g$.

(A) $[1, e]$

(B) $[e^{-1}, e]$

(C) $[0, e]$

(D) $(0, e]$

Question 43 (Level 5) — *Chain rule with trig composite*
If $h(x) = \sin^2(3x)$, find $h'(x)$.

(A) $3\sin(6x)$

(B) $6\sin(3x)\cos(3x)$

(C) $2\sin(3x)\cos(3x)$

(D) $6\sin(3x)$

Question 44 (Level 5) — *Finding composite from graph*
The function $h(x) = 2\log_e(x+1) - 3$ can be expressed as a sequence of transformations of $\log_e(x)$. If $h = T \circ \log_e$ where T is a single linear transformation on the output, what is $T(y)$?

(A) $T(y) = 2y - 3$

(B) $T(y) = 2(y - 3)$

(C) $T(y) = 2y + 3$

(D) $T(y) = y - 3$

Question 45 (Level 5) — *Composite function equation*
If $f(x) = 2x - 1$ and $g(x) = ax + b$ such that $f(g(x)) = g(f(x))$ for all x, find a and b.

(A) $b = 1 - a$ for any $a \in \mathbb{R}$

(B) $a = 2,\ b = -1$ only

(C) $a = 1,\ b = 0$ only

(D) No solution exists

Question 46 (Level 5) — *Chain rule with nested functions*
Find $\dfrac{d}{dx}\left[e^{\sin(x^2)}\right]$.

(A) $2x\cos(x^2)e^{\sin(x^2)}$

(B) $\cos(x^2)e^{\sin(x^2)}$

(C) $2xe^{\sin(x^2)}$

(D) $2x\sin(x^2)e^{\cos(x^2)}$

Question 47 (Level 5) — *Maximal domain of nested composite*
Find the maximal domain of $h(x) = \log_e(\sqrt{4-x} - 1)$.

(A) $(-\infty, 3)$

(B) $(-\infty, 3]$

(C) $(-\infty, 4)$

(D) $(3, 4]$

Question 48 (Level 5) — *Composite function period*
If $f(x) = x^2$ and $g(x) = \cos(\pi x)$, find the period of $h(x) = f(g(x))$.

(A) 1

(B) 2

(C) π

(D) $\frac{1}{2}$

Question 49 (Level 5) — *Functional equation via composite*
If $f(x) = \dfrac{x}{x+1}$ for $x \neq -1$, find a simplified expression for $f(f(x))$.

(A) $\dfrac{x}{2x+1}$

(B) $\dfrac{x^2}{(x+1)^2}$

(C) $\dfrac{x}{x+2}$

(D) $\dfrac{2x}{x+1}$

Question 50 (Level 5) — *Derivative of implicit composite*
Let $y = \log_e(\log_e(x))$ for $x > 1$. Find $\dfrac{dy}{dx}$.

(A) $\dfrac{1}{x\log_e(x)}$

(B) $\dfrac{1}{\log_e(x)}$

(C) $\dfrac{1}{x(\log_e(x))^2}$

(D) $\dfrac{1}{x}$

Solutions

Q1: (A)
$g(2) = 2 \times 2 = 4$. Then $f(4) = 4 + 3 = 7$.
Q2: (A)
$f(1) = 1 + 1 = 2$. Then $g(2) = 3 \times 2 = 6$.
Q3: (A)
$f(g(x)) = f(x + 5) = 2(x + 5) = 2x + 10$.
Q4: (A)
$g(0) = 0 + 3 = 3$. Then $f(3) = 3^2 = 9$.
Q5: (A)
$f(g(3)) = f(6) = 5$ and $g(f(3)) = g(2) = 4$. So $f(g(3)) \neq g(f(3))$.
Q6: (A)
$g(10) = 12$. Then $f(12) = 5$ since f is constant.
Q7: (A)
$g(3) = -3$. Then $f(-3) = -3 + 4 = 1$.
Q8: (A)
$g(f(x)) = g(3x) = 3x - 2$.
Q9: (A)
$f(3) = 4$. Then $f(4) = 5$.
Q10: (A)
Let $g(x) = x + 1$ and $f(x) = 2x$. Then $f(g(x)) = 2(x + 1)$.
Q11: (A)
$g(3) = 2(3) - 1 = 5$. Then $f(5) = 5^2 = 25$.
Q12: (A)
$f(g(x)) = (x + 2)^2 = x^2 + 4x + 4$.
Q13: (A)
$g(f(x)) = (x + 2)^2 = x^2 + 4x + 4$.
Q14: (A)
$f(2) = 3$, $f(3) = 4$, $f(4) = 5$.
Q15: (A)
$g(x) = 3x$ and $f(x) = x^2$. Then $f(g(x)) = (3x)^2$.
Q16: (A)
$g(2) = 3$. Then $f(3) = \frac{1}{3}$.
Q17: (A)
$f(g(x)) = \frac{1}{x - 4}$.
Q18: (A)
$2g(x) + 1 = 2x + 5 \Rightarrow 2g(x) = 2x + 4 \Rightarrow g(x) = x + 2$.
Q19: (A)
$g(2) = 2 - 5 = -3$. Then $f(-3) = |-3| = 3$.
Q20: (A)
$f(g(a)) = 2(a + 3) = 2a + 6 = 14 \Rightarrow 2a = 8 \Rightarrow a = 4$.
Q21: (A)
$g(1) = 4(1) + 5 = 9$. Then $f(9) = \sqrt{9} = 3$.
Q22: (A)
$f(g(x)) = \sqrt{x - 3}$ requires $x - 3 \geq 0$, so $x \geq 3$. Domain: $[3, \infty)$.
Q23: (A)
$f(g(x)) = (x + 1)^2 - 1 = x^2 + 2x + 1 - 1 = x^2 + 2x$.
Q24: (A)

$g(x) = 2x + 1$ and $f(x) = \sqrt{x}$. Then $f(g(x)) = \sqrt{2x+1}$.
Q25: (A)
$f(g(x)) = (2x)^2 + 2x = 4x^2 + 2x$.
Q26: (A)
$g(x) \neq 0 \Rightarrow x^2 - 4 \neq 0 \Rightarrow x \neq \pm 2$.
Q27: (A)
$(x-1)^2 = 9 \Rightarrow x - 1 = \pm 3 \Rightarrow x = 4$ or $x = -2$.
Q28: (A)
Let $u = 3x$, so $x = \frac{u}{3}$. Then $f(u) = 9 \cdot \frac{u^2}{9} + 1 = u^2 + 1$. So $f(x) = x^2 + 1$.
Q29: (A)
$f(g(x)) = (x+3)^2 = x^2 + 6x + 9$. $g(f(x)) = x^2 + 3$. Difference $= 6x + 6$.
Q30: (A)
$f(f(x)) = 2(2x-3) - 3 = 4x - 6 - 3 = 4x - 9$.
Q31: (A)
$f(g(x)) = \log_e(3-x)$ requires $3 - x > 0$, so $x < 3$. Domain: $(-\infty, 3)$.
Q32: (A)
$f(g(x)) = e^{2x+1}$.
Q33: (A)
$g(x) = x^2$ and $f(x) = e^x$. Then $f(g(x)) = e^{x^2}$.
Q34: (A)
$g(x) \in [-1, 1]$, so $f(g(x)) = \sin^2(x) + 1 \in [0+1, 1+1] = [1, 2]$.
Q35: (A)
$x^2 - 9 > 0 \Rightarrow (x-3)(x+3) > 0 \Rightarrow x < -3$ or $x > 3$. Domain: $(-\infty, -3) \cup (3, \infty)$.
Q36: (A)
$h'(x) = 5(2x+3)^4 \cdot 2 = 10(2x+3)^4$.
Q37: (A)
$f(g(x)) = \sin(2x)$. Period $= \dfrac{2\pi}{2} = \pi$.
Q38: (A)
Need $g(x) \geq 0$, i.e. $x - 5 \geq 0$, so $x \geq 5$. $f \circ g$ exists with domain $[5, \infty)$.
Q39: (A)
$f(x) = e^x$, $g(x) = 3x - 2$. $h'(x) = e^{3x-2} \cdot 3 = 3e^{3x-2}$.
Q40: (A)
$f(g(x)) = e^{\log_e(x)} = x$ for $x > 0$, and $g(f(x)) = \log_e(e^x) = x$ for all $x \in \mathbb{R}$.
Q41: (A)
Need $4 - x^2 > 0 \Rightarrow x^2 < 4 \Rightarrow -2 < x < 2$. Combined with $\text{dom}(g) = [-2, 2]$: domain of $f \circ g = (-2, 2)$. Rule: $f(g(x)) = \log_e(4 - x^2)$.
Q42: (A)
$g([-1, 1]) = [0, 1]$. Then $f([0, 1]) = [e^0, e^1] = [1, e]$. Range of $f \circ g = [1, e]$.
Q43: (A)
$h'(x) = 2\sin(3x) \cdot \cos(3x) \cdot 3 = 6\sin(3x)\cos(3x) = 3\sin(6x)$.
Q44: (A)
Writing $h(x) = 2\log_e(x+1) - 3$: the output transformation is $T(y) = 2y - 3$.
Q45: (A)
$f(g(x)) = 2(ax+b) - 1 = 2ax + 2b - 1$. $g(f(x)) = a(2x-1) + b = 2ax - a + b$. Equating: $2b - 1 = -a + b \Rightarrow b = 1 - a$. Since coefficients of x match ($2a = 2a$), any a works with $b = 1 - a$. For the special case $a = 1$: $b = 0$, giving $g(x) = x$. But the most general family is $b = 1 - a$. If we require $g \neq f$, and $a = 2$: $b = -1 =$ same as f. So the non-trivial solution is $a = 2, b = -1$.
Q46: (A)

$\dfrac{d}{dx}\left[e^{\sin(x^2)}\right] = e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x = 2x\cos(x^2)e^{\sin(x^2)}$.

Q47: (A)

Need $4 - x \geq 0 \Rightarrow x \leq 4$. Need $\sqrt{4-x} > 1 \Rightarrow 4 - x > 1 \Rightarrow x < 3$. Domain: $(-\infty, 3)$.

Q48: (A)

$h(x) = \cos^2(\pi x) = \frac{1+\cos(2\pi x)}{2}$. The period of $\cos(2\pi x)$ is 1. So the period of h is 1.

Q49: (A)

$f(f(x)) = f\left(\frac{x}{x+1}\right) = \dfrac{\frac{x}{x+1}}{\frac{x}{x+1}+1} = \dfrac{\frac{x}{x+1}}{\frac{2x+1}{x+1}} = \dfrac{x}{2x+1}$.

Q50: (A)

$\dfrac{dy}{dx} = \dfrac{1}{\log_e(x)} \cdot \dfrac{1}{x} = \dfrac{1}{x\log_e(x)}$.