50 multiple-choice questions

Question 1 (Level 1) — *Minimum of quadratic*
Find the minimum value of $y = x^2 - 4x + 7$.

(A) 3

(B) 7

(C) 2

(D) 4

Question 2 (Level 1) — *Maximum of -x*
Find the maximum value of $y = -x^2 + 10x$.

(A) 25

(B) 10

(C) 50

(D) 5

Question 3 (Level 1) — *Two numbers sum to 10*
Two numbers sum to 10. Their product $P = x(10 - x)$ is maximised when x equals:

(A) 5

(B) 10

(C) 4

(D) 6

Question 4 (Level 1) — *Maximum product*
Two numbers sum to 10. What is their maximum product?

(A) 25

(B) 20

(C) 24

(D) 16

Question 5 (Level 1) — *Minimum of sum of squares*
Find the value of x that minimises $f(x) = x^2 + 4$.

(A) $x = 0$

(B) $x = 4$

(C) $x = -4$

(D) $x = 2$

Question 6 (Level 1) — *Perimeter constraint*
A rectangle has perimeter 20 cm. If one side is x cm, the other side is:

(A) $(10 - x)$ cm

(B) $(20 - x)$ cm

(C) $(20 - 2x)$ cm

(D) $\frac{20}{x}$ cm

Question 7 (Level 1) — *Area of rectangle*
A rectangle has perimeter 20 cm. Express the area A in terms of one side x.

(A) $A = 10x - x^2$

(B) $A = 20x - x^2$

(C) $A = x(20 - x)$

(D) $A = 5x - x^2$

Question 8 (Level 1) — *Max area rectangle*
A rectangle has perimeter 20 cm. Find the maximum area.

(A) 25 cm^2

(B) 20 cm^2

(C) 100 cm^2

(D) 50 cm^2

Question 9 (Level 1) — *Height of ball*
A ball's height is $h = 20t - 5t^2$ metres. Find the maximum height.

(A) 20 m

(B) 40 m

(C) 10 m

(D) 25 m

Question 10 (Level 1) — *Time of max height*
A ball's height is $h = 30t - 5t^2$. At what time does it reach maximum height?

(A) $t = 3$ s

(B) $t = 6$ s

(C) $t = 5$ s

(D) $t = 1.5$ s

Question 11 (Level 2) — *Fencing problem*
A farmer has 100 m of fencing to make a rectangular paddock against a wall (3 sides). If the width is x m, the area is:

(A) $A = 100x - 2x^2$

(B) $A = 100x - x^2$

(C) $A = 50x - x^2$

(D) $A = 50x - 2x^2$

Question 12 (Level 2) — *Max fencing area*
A farmer uses 100 m of fencing for 3 sides of a rectangle (wall on 4th side). Find the maximum area.

(A) 1250 m^2

(B) 2500 m^2

(C) 625 m^2

(D) 1000 m^2

Question 13 (Level 2) — *Minimum sum*
Find two positive numbers whose sum is 20 and whose sum of squares is minimised.

(A) 10 and 10

(B) 5 and 15

(C) 1 and 19

(D) 8 and 12

Question 14 (Level 2) — *Revenue maximisation*
Revenue is $R = 200x - 2x^2$ where x is the number of items sold. Find the value of x that maximises revenue.

(A) 50

(B) 100

(C) 200

(D) 25

Question 15 (Level 2) — *Profit function*
Profit $P = -x^2 + 50x - 400$. Find the maximum profit.

(A) 225

(B) 250

(C) 400

(D) 625

Question 16 (Level 2) — *Minimum distance*
Find the point on $y = x^2$ closest to the point $(0, 1)$. Minimise $D^2 = x^2 + (x^2 - 1)^2$.

(A) $x = \pm\frac{1}{\sqrt{2}}$

(B) $x = 0$

(C) $x = \pm 1$

(D) $x = \pm\frac{1}{2}$

Question 17 (Level 2) — *Box volume*
A 12×12 cm sheet has squares of side x cut from corners and folded up. The volume is:

(A) $V = x(12 - 2x)^2$

(B) $V = x^2(12 - 2x)$

(C) $V = x(12 - x)^2$

(D) $V = (12 - 2x)^3$

Question 18 (Level 2) — *Domain constraint*
For the box from a 12×12 sheet with cut squares of side x, the domain is:

(A) $0 < x < 6$

(B) $0 < x < 12$

(C) $0 < x < 4$

(D) $0 \le x \le 6$

Question 19 (Level 2) — *Max box volume*
Find the value of x that maximises $V = x(12 - 2x)^2$ for $0 < x < 6$.

(A) $x = 2$

(B) $x = 3$

(C) $x = 4$

(D) $x = 1$

Question 20 (Level 2) — *Maximum volume value*
The maximum volume of the open box formed from a 12×12 cm sheet (cut $x = 2$) is:

(A) 128 cm^3

(B) 64 cm^3

(C) 192 cm^3

(D) 256 cm^3

Question 21 (Level 3) — *Cylinder surface area*
A closed cylinder has volume 100π cm^3. Express the surface area S in terms of radius r.

(A) $S = 2\pi r^2 + \frac{200\pi}{r}$

(B) $S = 2\pi r^2 + \frac{100\pi}{r}$

(C) $S = \pi r^2 + \frac{200\pi}{r}$

(D) $S = 2\pi r^2 + 200\pi r$

Question 22 (Level 3) — *Min surface area radius*
For $S = 2\pi r^2 + \frac{200\pi}{r}$, find the value of r that minimises S.

(A) $r = \sqrt[3]{50}$

(B) $r = 50$

(C) $r = \sqrt{50}$

(D) $r = 5$

Question 23 (Level 3) — *Maximum area of rectangle under curve*
A rectangle has one side on the x-axis and two corners on $y = 4 - x^2$. Express the area in terms of x (where $x > 0$ is the half-width).

(A) $A = 8x - 2x^3$

(B) $A = 4x - x^3$

(C) $A = 8x - x^2$

(D) $A = 2x(4 + x^2)$

Question 24 (Level 3) — *Max rectangle area value*
Maximise $A = 8x - 2x^3$ for $0 < x < 2$.

(A) $\frac{32\sqrt{3}}{9}$

(B) $\frac{16}{3}$

(C) 8

(D) $\frac{32}{9}$

Question 25 (Level 3) — *Minimum cost*

The cost of running a car at speed v km/h is $C = \frac{v^2}{100} + \frac{100}{v}$ dollars per km. Find the speed that minimises cost.

(A) $\sqrt[3]{5000}$

(B) 50

(C) $\sqrt{5000}$

(D) 100

Question 26 (Level 3) — *Largest rectangle in triangle*

A right triangle has legs 6 and 8. A rectangle is inscribed with one side on the leg of length 8. If the side along the leg is x, the height is $6(1 - \frac{x}{8})$. Find x for max area.

(A) $x = 4$

(B) $x = 3$

(C) $x = 6$

(D) $x = 8$

Question 27 (Level 3) — *Minimum material*

An open-top box with square base has volume 32 cm^3. Express the surface area in terms of base side x.

(A) $S = x^2 + \frac{128}{x}$

(B) $S = x^2 + \frac{32}{x}$

(C) $S = 2x^2 + \frac{128}{x}$

(D) $S = x^2 + 128x$

Question 28 (Level 3) — *Min surface area box*

For $S = x^2 + \frac{128}{x}$, find the x that minimises S.

(A) $x = 4$

(B) $x = 8$

(C) $x = 2$

(D) $x = \sqrt{128}$

Question 29 (Level 3) — *Fence dividing paddock*

A 200 m fence encloses a rectangular area divided into two equal parts by a fence parallel to one side. If width is x, express the area.

(A) $A = 100x - \frac{3x^2}{2}$

(B) $A = 200x - 3x^2$

(C) $A = 100x - x^2$

(D) $A = 50x - \frac{3x^2}{4}$

Question 30 (Level 3) — *Max divided paddock area*
For $A = 100x - \frac{3x^2}{2}$, find the maximum area.

(A) $\frac{5000}{3}$ m^2

(B) 2500 m^2

(C) $\frac{10000}{3}$ m^2

(D) 1250 m^2

Question 31 (Level 4) — *Cone in sphere*
A cone is inscribed in a sphere of radius R. If the cone's height is $h = R + x$ where x is the distance from centre to base, the cone's radius is $r = \sqrt{R^2 - x^2}$. Express the volume in terms of x.

(A) $V = \frac{\pi}{3}(R + x)^2(R - x)$

(B) $V = \frac{\pi}{3}(R^2 - x^2)R$

(C) $V = \pi(R + x)(R - x)^2$

(D) $V = \frac{\pi}{3}(R - x)^2(R + x)$

Question 32 (Level 4) — *Exponential optimisation*
Find the maximum value of $f(x) = xe^{-x}$ for $x \geq 0$.

(A) $\frac{1}{e}$

(B) e

(C) 1

(D) $\frac{1}{e^2}$

Question 33 (Level 4) — *Minimum of x + 1/x*
Find the minimum value of $f(x) = x + \frac{1}{x}$ for $x > 0$.

(A) 2

(B) 1

(C) e

(D) $\sqrt{2}$

Question 34 (Level 4) — *Trig optimisation*
Find the maximum value of $f(\theta) = \sin\theta\cos\theta$ for $0 \leq \theta \leq \frac{\pi}{2}$.

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{\sqrt{2}}{2}$

(D) $\frac{1}{4}$

Question 35 (Level 4) — *Least material for can*
A cylindrical can holds 500 mL (500 cm^3). Find the radius r that minimises the total surface area $S = 2\pi r^2 + \frac{1000}{r}$.

(A) $r = \left(\frac{250}{\pi}\right)^{1/3}$

(B) $r = \left(\frac{500}{\pi}\right)^{1/3}$

(C) $r = \frac{250}{\pi}$

(D) $r = \sqrt{\frac{250}{\pi}}$

Question 36 (Level 4) — *Maximum area with fixed perimeter*
A window is a rectangle topped by a semicircle. The perimeter is 10 m. If the width is $2r$, express the area.

(A) $A = 10r - 2r^2 - \frac{\pi r^2}{2}$

(B) $A = 10r - 2r^2 + \frac{\pi r^2}{2}$

(C) $A = 10r - r^2(2 + \pi)$

(D) $A = 5r - r^2 - \frac{\pi r^2}{4}$

Question 37 (Level 4) — *Max ln function*
Find the maximum value of $f(x) = \ln(x) - x$ for $x > 0$.

(A) -1

(B) 0

(C) 1

(D) $-e$

Question 38 (Level 4) — *Shortest distance to line*
Find the point on the curve $y = \sqrt{x}$ closest to $(4, 0)$. Minimise $D^2 = (x - 4)^2 + x$.

(A) $x = \frac{7}{2}$

(B) $x = 4$

(C) $x = 3$

(D) $x = \frac{5}{2}$

Question 39 (Level 4) — *Minimum perimeter*
A rectangle has area 36 cm^2. Find the dimensions that minimise the perimeter.

(A) 6×6

(B) 4×9

(C) 3×12

(D) 2×18

Question 40 (Level 4) — *Profit maximisation*
A company sells x items at price $p = 100 - 2x$. Cost is $C = 20x + 100$. Find x for max profit.

(A) 20

(B) 40

(C) 50

(D) 25

Question 41 (Level 5) — *Snell's law type*
A lifeguard at point A (on shore) must reach a swimmer at B (in water). Running speed is 8 m/s, swimming speed is 2 m/s. The shore is 100 m long, A is 40 m from the water line, and B is 30 m from shore. If the lifeguard enters the water at distance x m along the shore from the closest point, the total time is $T = \frac{\sqrt{1600+x^2}}{8} + \frac{\sqrt{900+(100-x)^2}}{2}$. Find $\frac{dT}{dx}$.

(A) $\frac{x}{8\sqrt{1600+x^2}} - \frac{100-x}{2\sqrt{900+(100-x)^2}}$

(B) $\frac{x}{8\sqrt{1600+x^2}} + \frac{100-x}{2\sqrt{900+(100-x)^2}}$

(C) $\frac{1}{8\sqrt{1600+x^2}} - \frac{1}{2\sqrt{900+(100-x)^2}}$

(D) $\frac{x}{4\sqrt{1600+x^2}} - \frac{100-x}{\sqrt{900+(100-x)^2}}$

Question 42 (Level 5) — *Max volume of cylinder in cone*
A cylinder is inscribed in a cone of height H and base radius R. Show that the maximum volume is $\frac{4}{27}\pi R^2 H$. What fraction of the cone's volume is this?

(A) $\frac{4}{9}$

(B) $\frac{4}{27}$

(C) $\frac{1}{3}$

(D) $\frac{2}{9}$

Question 43 (Level 5) — *Minimum angle*
A painting of height 2 m hangs on a wall with its bottom 3 m above eye level. How far from the wall should you stand to maximise the angle subtended? Let $\theta = \arctan\frac{5}{x} - \arctan\frac{3}{x}$.

(A) $\sqrt{15}$ m

(B) $\sqrt{10}$ m

(C) 3 m

(D) 5 m

Question 44 (Level 5) — *Optimise with constraint*
Find the minimum value of $x + y$ subject to $xy = 16$ and $x, y > 0$.

(A) 8

(B) 16

(C) 4

(D) 10

Question 45 (Level 5) — *Max area of triangle*
A triangle has two sides of length a and b. Find the angle θ between them that maximises the area.

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) π

Question 46 (Level 5) — *Max of xa e-x*
For $f(x) = x^a e^{-x}$ ($a > 0$, $x > 0$), find the x-value where f is maximised.

(A) $x = a$

(B) $x = \frac{a}{e}$

(C) $x = ae$

(D) $x = \frac{1}{a}$

Question 47 (Level 5) — *Optimal price*
Demand is $q = 100e^{-0.02p}$. Revenue $R = pq$. Find the price p that maximises revenue.

(A) 50

(B) 100

(C) 25

(D) $\frac{1}{0.02}$

Question 48 (Level 5) — *Minimum surface area sphere*

Among all closed rectangular boxes with volume V, the one with minimum surface area is a cube. If $V = 27$, what is the minimum surface area?

(A) 54

(B) 36

(C) 27

(D) 81

Question 49 (Level 5) — *Rate of change optimisation*

Water flows into a conical tank (height 10 m, radius 5 m at top) at 2 m^3/min. How fast is the water level rising when the depth is 4 m?

(A) $\frac{1}{2\pi}$ m/min

(B) $\frac{1}{\pi}$ m/min

(C) $\frac{2}{\pi}$ m/min

(D) $\frac{1}{4\pi}$ m/min

Question 50 (Level 5) — *Quickest path*

A person can walk at 5 km/h on a road and 3 km/h across a field. The road goes along the edge of a field. They are 1 km into the field and need to reach a point 4 km along the road. If they walk across the field to a point x km along the road, then along the road, find the optimal x.

(A) $\frac{3}{4}$ km

(B) 1 km

(C) $\frac{4}{3}$ km

(D) $\frac{5}{3}$ km

Solutions

Q1: (A)
$y' = 2x - 4 = 0 \Rightarrow x = 2$. $y(2) = 4 - 8 + 7 = 3$.
Q2: (A)
$x = 5$. $y(5) = -25 + 50 = 25$.
Q3: (A)
$P' = 10 - 2x = 0 \Rightarrow x = 5$.
Q4: (A)
$P(5) = 5(10 - 5) = 25$.
Q5: (A)
$f'(x) = 2x = 0 \Rightarrow x = 0$. Minimum value $= 4$.
Q6: (A)
$2x + 2y = 20 \Rightarrow y = 10 - x$.
Q7: (A)
$A = x(10 - x) = 10x - x^2$.
Q8: (A)
$A' = 10 - 2x = 0 \Rightarrow x = 5$. $A = 25$ cm^2.
Q9: (A)
$t = 2$. $h(2) = 40 - 20 = 20$ m.
Q10: (A)
$t = 3$ seconds.
Q11: (A)
$A = x(100 - 2x) = 100x - 2x^2$.
Q12: (A)
$x = 25$. $A = 100(25) - 2(625) = 1250$ m^2.
Q13: (A)
$S = 2x^2 - 40x + 400$. $S' = 4x - 40 = 0 \Rightarrow x = 10$. Both numbers are 10.
Q14: (A)
$x = 50$.
Q15: (A)
$x = 25$. $P(25) = -625 + 1250 - 400 = 225$.
Q16: (A)
$\frac{d(D^2)}{du} = 2u - 1 = 0 \Rightarrow u = \frac{1}{2}$, so $x^2 = \frac{1}{2}$, $x = \pm\frac{1}{\sqrt{2}}$.
Q17: (A)
$V = x(12 - 2x)^2$.
Q18: (A)
$0 < x < 6$.
Q19: (A)
$V' = 0$: $x = 6$ (boundary) or $x = 2$. So $x = 2$.
Q20: (A)
$V(2) = 2 \times 64 = 128$ cm^3.
Q21: (A)
$h = \frac{100}{r^2}$. $S = 2\pi r^2 + \frac{200\pi}{r}$.
Q22: (A)
$4\pi r = \frac{200\pi}{r^2} \Rightarrow r^3 = 50 \Rightarrow r = \sqrt[3]{50}$.
Q23: (A)
$A = 2x(4 - x^2) = 8x - 2x^3$.
Q24: (A)
$x^2 = \frac{4}{3}$, $x = \frac{2}{\sqrt{3}}$. $A = \frac{16}{\sqrt{3}} - \frac{16}{3\sqrt{3}} = \frac{32}{3\sqrt{3}} = \frac{32\sqrt{3}}{9}$.

Q25: (A)
$\frac{v}{50} = \frac{100}{v^2} \Rightarrow v^3 = 5000 \Rightarrow v = \sqrt[3]{5000} \approx 17.1$.
Q26: (A)
$A' = 6 - \frac{3x}{2} = 0 \Rightarrow x = 4$.
Q27: (A)
$S = x^2 + 4x \cdot \frac{32}{x^2} = x^2 + \frac{128}{x}$.
Q28: (A)
$2x^3 = 128 \Rightarrow x^3 = 64 \Rightarrow x = 4$.
Q29: (A)
$A = x \cdot \frac{200-3x}{2} = 100x - \frac{3x^2}{2}$.
Q30: (A)
$x = \frac{100}{3}$. $A = 100 \cdot \frac{100}{3} - \frac{3}{2} \cdot \frac{10000}{9} = \frac{10000}{3} - \frac{5000}{3} = \frac{5000}{3}$ m^2.
Q31: (A)
$V = \frac{\pi}{3}(R^2 - x^2)(R + x) = \frac{\pi}{3}(R + x)^2(R - x)$.
Q32: (A)
$x = 1$. $f(1) = e^{-1} = \frac{1}{e}$.
Q33: (A)
$x^2 = 1 \Rightarrow x = 1$. $f(1) = 2$.
Q34: (A)
Maximum of $\sin(2\theta)$ is 1, so max of $f = \frac{1}{2}$ at $\theta = \frac{\pi}{4}$.
Q35: (A)
$r^3 = \frac{250}{\pi} \Rightarrow r = \left(\frac{250}{\pi}\right)^{1/3}$.
Q36: (A)
$A = 2r \cdot \frac{10-(2+\pi)r}{2} + \frac{\pi r^2}{2} = 10r - (2+\pi)r^2 + \frac{\pi r^2}{2} = 10r - 2r^2 - \frac{\pi r^2}{2}$.
Q37: (A)
$x = 1$. $f(1) = 0 - 1 = -1$.
Q38: (A)
$2x - 7 = 0 \Rightarrow x = \frac{7}{2}$. Point: $\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$.
Q39: (A)
$x^2 = 36 \Rightarrow x = 6$. Dimensions: 6×6 (square).
Q40: (A)
$P = 80x - 2x^2 - 100$. $P' = 80 - 4x = 0 \Rightarrow x = 20$.
Q41: (A)
$\frac{dT}{dx} = \frac{x}{8\sqrt{1600+x^2}} - \frac{100-x}{2\sqrt{900+(100-x)^2}}$.
Q42: (A)
Maximum cylinder volume $= \frac{4}{27}\pi R^2 H$. Cone volume $= \frac{1}{3}\pi R^2 H$. Fraction $= \frac{4/27}{1/3} = \frac{4}{9}$.
Q43: (A)
$\frac{3}{x^2+9} = \frac{5}{x^2+25}$. Cross-multiply: $3(x^2 + 25) = 5(x^2 + 9)$, $2x^2 = 30$, $x = \sqrt{15}$.
Q44: (A)
$x^2 = 16$, $x = 4$, $y = 4$. Min $= 8$.
Q45: (A)
$\sin\theta$ is maximised when $\theta = \frac{\pi}{2}$.
Q46: (A)
$a - x = 0 \Rightarrow x = a$.
Q47: (A)
$1 - 0.02p = 0 \Rightarrow p = 50$.
Q48: (A)
$x = 3$. $S = 6(9) = 54$.
Q49: (A)

$2 = \frac{\pi(16)}{4} \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{2}{4\pi} = \frac{1}{2\pi}$ m/min.
Q50: (A)
$\frac{5x}{3} = \sqrt{1+x^2}$. $25x^2 = 9(1+x^2)$. $16x^2 = 9$. $x = \frac{3}{4}$.