

50 multiple-choice questions

**Question 1** (Level 1) — *What is a sample proportion?*

In a class of 30 students, 12 wear glasses. What is the sample proportion  $\hat{p}$ ?

- (A) 0.4
- (B) 12
- (C) 0.12
- (D)  $\frac{30}{12} = 2.5$

**Question 2** (Level 1) — *Population vs sample*

A country has 25 million people. A pollster surveys 1000 people. Which is the sample?

- (A) The 1000 people surveyed
- (B) The 25 million people
- (C) Both
- (D) Neither

**Question 3** (Level 1) — *Variability between samples*

If you take two different random samples of 50 students, will you always get the same  $\hat{p}$ ?

- (A) No,  $\hat{p}$  varies between samples
- (B) Yes,  $\hat{p}$  is always the same
- (C) Only if the population is large
- (D) Yes, if the samples are random

**Question 4** (Level 1) — *Converting proportion to percentage*

If  $\hat{p} = 0.35$ , what percentage of the sample has the characteristic?

- (A) 35%
- (B) 3.5%
- (C) 0.35%
- (D) 65%

**Question 5** (Level 1) — *Counting from proportion*

In a sample of  $n = 200$ ,  $\hat{p} = 0.15$ . How many in the sample have the characteristic?

- (A) 30
- (B) 15
- (C) 170

(D) 0.15

**Question 6** (Level 1) — *Random sampling*

Why is it important to use a random sample when estimating a population proportion?

- (A) To avoid bias and ensure a fair estimate
- (B) To make the sample larger
- (C) To guarantee  $\hat{p} = p$
- (D) Random sampling is not important

**Question 7** (Level 1) — *Estimate quality*

Which sample size gives a more reliable estimate of the population proportion?

- (A)  $n = 500$
- (B)  $n = 50$
- (C)  $n = 5$
- (D) All are equally reliable

**Question 8** (Level 1) — *Unbiased estimator*

On average, what value does  $\hat{p}$  equal?

- (A) The true population proportion  $p$
- (B) 0.5
- (C) 1
- (D) It depends on the sample

**Question 9** (Level 1) — *Sample proportion range*

What values can  $\hat{p}$  take?

- (A) From 0 to 1
- (B) From  $-1$  to 1
- (C) From 0 to  $\infty$
- (D) Any real number

**Question 10** (Level 1) — *Proportion complement*

If  $\hat{p} = 0.6$ , what proportion does NOT have the characteristic?

- (A) 0.4
- (B) 0.6

(C) 0.36

(D)  $-0.6$

**Question 11** (Level 2) — *Mean of  $p$ -hat*

If  $p = 0.3$  and  $n = 100$ , what is  $E(\hat{p})$ ?

(A) 0.3

(B) 30

(C) 0.03

(D) 0.7

**Question 12** (Level 2) — *Standard error formula*

Write the formula for the standard error of  $\hat{p}$ .

(A)  $\sqrt{\frac{p(1-p)}{n}}$

(B)  $\frac{p(1-p)}{n}$

(C)  $\sqrt{\frac{p}{n}}$

(D)  $\sqrt{np(1-p)}$

**Question 13** (Level 2) — *Calculating SE*

$p = 0.4$ ,  $n = 100$ . Find the standard error of  $\hat{p}$ .

(A) 0.049

(B) 0.24

(C) 0.0024

(D) 4.9

**Question 14** (Level 2) — *Effect of  $n$  on SE*

If  $n$  is quadrupled (from 100 to 400) with  $p = 0.5$ , what happens to SE?

(A) SE is halved

(B) SE is quartered

(C) SE stays the same

(D) SE doubles

**Question 15** (Level 2) — *Sampling distribution shape*

For large  $n$ , the sampling distribution of  $\hat{p}$  is approximately what shape?

- (A) Normal (bell-shaped)
- (B) Uniform
- (C) Binomial
- (D) Skewed right

**Question 16** (Level 2) — *p-hat as  $X/n$* 

If  $X \sim \text{Bi}(n, p)$ , what is the distribution of  $\hat{p} = \frac{X}{n}$ ?

- (A) Approximately  $N\left(p, \frac{p(1-p)}{n}\right)$  for large  $n$
- (B)  $\text{Bi}(n, p)$
- (C)  $N(0, 1)$
- (D) Uniform on  $[0, 1]$

**Question 17** (Level 2) — *Multiple samples*

$p = 0.5$ ,  $n = 64$ . If you take many samples, about 95% of  $\hat{p}$  values will lie within what range?

- (A) (0.375, 0.625)
- (B) (0.4375, 0.5625)
- (C) (0.25, 0.75)
- (D) (0.3125, 0.6875)

**Question 18** (Level 2) — *SE vs SD*

What is the difference between the standard deviation of  $X$  and the standard error of  $\hat{p}$  when  $X \sim \text{Bi}(n, p)$ ?

- (A)  $\text{SE}(\hat{p}) = \frac{\text{SD}(X)}{n}$ ; SE measures proportion variability
- (B) They are the same thing
- (C)  $\text{SE} = \text{SD}^2$
- (D)  $\text{SE} = n \times \text{SD}$

**Question 19** (Level 2) — *Simulating sampling*

A coin has  $p = 0.5$ . You flip it 100 times and get 55 heads. Is  $\hat{p} = 0.55$  unusual?

- (A) No,  $z = 1$  is within normal variation
- (B) Yes,  $0.55 \neq 0.5$
- (C) Yes,  $z = 1$  is significant
- (D) Cannot determine

**Question 20** (Level 2) — *Which  $p$  maximises  $SE$ ?*

For fixed  $n$ , at what value of  $p$  is  $SE(\hat{p})$  maximised?

- (A)  $p = 0.5$
- (B)  $p = 0$
- (C)  $p = 1$
- (D)  $p = 0.25$

**Question 21** (Level 3) — *CLT for proportions*

$p = 0.6$ ,  $n = 225$ . Find  $\Pr(\hat{p} > 0.64)$ . Given  $\Pr(Z < 1.23) \approx 0.891$ .

- (A) 0.109
- (B) 0.891
- (C) 0.040
- (D) 0.500

**Question 22** (Level 3) — *Range for  $p$ -hat*

$p = 0.5$ ,  $n = 400$ . Find the interval that contains the middle 95% of  $\hat{p}$  values.

- (A) (0.451, 0.549)
- (B) (0.475, 0.525)
- (C) (0.400, 0.600)
- (D) (0.450, 0.550)

**Question 23** (Level 3) — *Checking CLT conditions*

$p = 0.05$ ,  $n = 100$ . Can we use the normal approximation for  $\hat{p}$ ?

- (A) No,  $np = 5 < 10$
- (B) Yes,  $n = 100$  is large enough
- (C) Yes,  $n(1 - p) = 95 > 10$
- (D) No,  $p < 0.5$

**Question 24** (Level 3) — *Probability of  $p$ -hat in range*

$p = 0.7$ ,  $n = 200$ . Find  $\Pr(0.66 < \hat{p} < 0.74)$ . Given  $\Pr(Z < 1.23) = 0.891$ .

- (A) 0.782
- (B) 0.891
- (C) 0.218
- (D) 0.680

**Question 25** (Level 3) — *Required  $n$  for SE*

How large must  $n$  be so that  $\text{SE}(\hat{p}) \leq 0.02$  when  $p = 0.5$ ?

- (A) 625
- (B) 250
- (C) 2500
- (D) 100

**Question 26** (Level 3) — *Unusual sample proportion*

$p = 0.25$ ,  $n = 400$ . Is  $\hat{p} = 0.30$  significantly different from  $p$ ? Given  $\Pr(Z < 2.31) = 0.990$ .

- (A) Yes,  $z \approx 2.31$  gives  $p$ -value  $\approx 0.02$
- (B) No, 0.30 is close to 0.25
- (C) Yes, but only at 1% level
- (D) No,  $n$  is too large

**Question 27** (Level 3) — *Comparing two sample sizes*

Sample A:  $n = 100$ ,  $\text{SE} = 0.05$ . Sample B:  $n = 900$ , same  $p$ . Find SE for B.

- (A) 0.0167
- (B) 0.005
- (C) 0.15
- (D) 0.025

**Question 28** (Level 3) — *Sampling distribution simulation*

$p = 0.4$ ,  $n = 50$ . Describe the approximate sampling distribution of  $\hat{p}$ .

- (A) Approximately  $N(0.4, 0.0693^2)$
- (B) Approximately  $N(0.5, 0.07^2)$
- (C) Binomial(50, 0.4)
- (D) Approximately  $N(0.4, 0.24^2)$

**Question 29** (Level 3) — *Proportion below threshold*

$p = 0.8$ ,  $n = 150$ . Find  $\Pr(\hat{p} < 0.75)$ . Given  $\Pr(Z < -1.53) \approx 0.063$ .

- (A) 0.063
- (B) 0.937
- (C) 0.126
- (D) 0.050

**Question 30** (Level 3) — *Bias in sampling*

A survey only polls people at a shopping mall. The population proportion of shoppers who prefer Brand A is  $p = 0.3$ . Will this method give an unbiased estimate?

- (A) No, convenience sampling is likely biased
- (B) Yes, as long as  $n$  is large enough
- (C) Yes, any sample gives unbiased estimates
- (D) Only biased if  $n < 30$

**Question 31** (Level 4) — *Full CLT problem*

$p = 0.55$ ,  $n = 500$ . Find  $\Pr(\hat{p} > 0.58)$ . Given  $\Pr(Z < 1.35) = 0.911$ .

- (A) 0.089
- (B) 0.911
- (C) 0.045
- (D) 0.178

**Question 32** (Level 4) — *Inverse problem for  $n$* 

$p = 0.3$ . Find  $n$  so that  $\Pr(|\hat{p} - p| < 0.05) \geq 0.95$ .

- (A) 323
- (B) 322
- (C) 385
- (D) 200

**Question 33** (Level 4) — *Testing a claim*

A company claims 90% satisfaction. In  $n = 200$ ,  $\hat{p} = 0.86$ . Test at 5% significance. Given  $\Pr(Z < -1.89) = 0.029$ .

- (A) Reject;  $z = -1.89$ ,  $p\text{-value} = 0.029 < 0.05$
- (B) Do not reject; 0.86 is close to 0.90
- (C) Reject;  $\hat{p} < p$
- (D) Do not reject;  $p\text{-value} = 0.058 > 0.05$

**Question 34** (Level 4) — *Combining  $p\text{-hat}$  and CI*

$n = 300$ ,  $\hat{p} = 0.65$ . Find the estimated standard error and the 95% CI.

- (A) (0.596, 0.704)
- (B) (0.622, 0.678)

(C) (0.550, 0.750)

(D) (0.605, 0.695)

**Question 35** (Level 4) — *Sampling distribution of difference*

$\hat{p}_1$  from population with  $p_1 = 0.6$  ( $n_1 = 100$ ) and  $\hat{p}_2$  from  $p_2 = 0.5$  ( $n_2 = 100$ ). Find  $E(\hat{p}_1 - \hat{p}_2)$  and  $SE(\hat{p}_1 - \hat{p}_2)$ .

(A)  $E = 0.1$ ,  $SE = 0.07$

(B)  $E = 0.1$ ,  $SE = 0.049$

(C)  $E = 0.55$ ,  $SE = 0.07$

(D)  $E = 0.1$ ,  $SE = 0.007$

**Question 36** (Level 4) — *Increasing precision*

Currently  $SE = 0.04$  with  $n = 150$ . What  $n$  is needed to reduce  $SE$  to 0.02?

(A) 600

(B) 300

(C) 2400

(D) 450

**Question 37** (Level 4) — *Two-sided probability*

$p = 0.45$ ,  $n = 250$ . Find  $\Pr(|\hat{p} - 0.45| > 0.05)$ . Given  $\Pr(Z < 1.59) = 0.944$ .

(A) 0.112

(B) 0.056

(C) 0.944

(D) 0.888

**Question 38** (Level 4) — *Exam proportion question*

A manufacturer claims the defect rate is  $p = 0.02$ . In a sample of 500, 18 are defective. Is this consistent with the claim at 5% significance?

(A) Reject;  $z = 2.56$ ,  $p\text{-value} \approx 0.01 < 0.05$

(B) Do not reject; 18 out of 500 is small

(C) Do not reject;  $\hat{p} < 0.05$

(D) Reject;  $\hat{p} > 0.02$

**Question 39** (Level 4) — *Variance of  $p\text{-hat}$*

$p = 0.35$ ,  $n = 200$ . Find  $\text{Var}(\hat{p})$  and  $\text{SD}(\hat{p})$ .



- (A)  $\text{Var} \approx 0.00114$ ,  $\text{SD} \approx 0.0337$
- (B)  $\text{Var} = 0.2275$ ,  $\text{SD} = 0.477$
- (C)  $\text{Var} = 0.0337$ ,  $\text{SD} = 0.00114$
- (D)  $\text{Var} = 0.35$ ,  $\text{SD} = 0.591$

**Question 40** (Level 4) — *Interpreting SE*

$p = 0.5$ ,  $n = 1600$ . The SE of  $\hat{p}$  is 0.0125. In repeated sampling, approximately 99.7% of  $\hat{p}$  values will fall within what range?

- (A) (0.4625, 0.5375)
- (B) (0.475, 0.525)
- (C) (0.4875, 0.5125)
- (D) (0.4250, 0.5750)

**Question 41** (Level 5) — *Exact vs approximate*

$X \sim \text{Bi}(20, 0.5)$ . Find the exact  $\Pr(\hat{p} > 0.7) = \Pr(X > 14)$  and compare with the normal approximation.

- (A) Exact  $\approx 0.021$ ; normal  $\approx 0.037$  (overestimates)
- (B) Both give  $\approx 0.021$
- (C) Exact  $\approx 0.037$ ; normal  $\approx 0.021$
- (D) Both give  $\approx 0.037$

**Question 42** (Level 5) — *Continuity correction*

$X \sim \text{Bi}(100, 0.4)$ . Use the continuity correction to find  $\Pr(X \geq 45)$  via the normal approximation.

- (A)  $\approx 0.179$
- (B)  $\approx 0.154$  (without CC)
- (C)  $\approx 0.500$
- (D)  $\approx 0.046$

**Question 43** (Level 5) — *Deriving the variance of  $p$ -hat*

If  $X \sim \text{Bi}(n, p)$  and  $\hat{p} = \frac{X}{n}$ , derive  $\text{Var}(\hat{p})$ .

- (A)  $\frac{p(1-p)}{n}$
- (B)  $np(1-p)$
- (C)  $\frac{p(1-p)}{n^2}$

(D)  $\frac{p}{n}$

**Question 44** (Level 5) — *Power of a test*

$H_0 : p = 0.5$ ,  $H_1 : p = 0.6$ ,  $n = 100$ . Reject  $H_0$  if  $\hat{p} > 0.58$ . Find the power. Given  $\Pr(Z < -0.41) \approx 0.341$ .

(A) 0.659

(B) 0.341

(C) 0.050

(D) 0.950

**Question 45** (Level 5) — *Finding critical value*

$H_0 : p = 0.4$ , one-sided test at  $\alpha = 0.05$ ,  $n = 250$ . Find the critical value  $\hat{p}_c$  such that  $\Pr(\hat{p} > \hat{p}_c | p = 0.4) = 0.05$ .

(A) 0.451

(B) 0.461

(C) 0.440

(D) 0.431

**Question 46** (Level 5) — *CLT proof sketch*

The Central Limit Theorem states that  $\frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \rightarrow N(0, 1)$  as  $n \rightarrow \infty$ . What is the name of this convergence?

(A) Convergence in distribution

(B) Almost sure convergence

(C) Convergence in probability

(D) Mean square convergence

**Question 47** (Level 5) — *Two-sample z-test*

Sample 1:  $n_1 = 200$ ,  $\hat{p}_1 = 0.45$ . Sample 2:  $n_2 = 300$ ,  $\hat{p}_2 = 0.38$ . Test  $H_0 : p_1 = p_2$  at 5%. Given  $\Pr(Z < 1.63) \approx 0.948$ .

(A) Do not reject;  $z \approx 1.56$ ,  $p\text{-value} \approx 0.12 > 0.05$

(B) Reject; the proportions differ

(C) Reject;  $z > 1$

(D) Do not reject;  $\hat{p}_1 > \hat{p}_2$

**Question 48** (Level 5) — *Delta method application*

If  $\hat{p} \sim N(p, \frac{p(1-p)}{n})$  approximately, find the approximate distribution of  $g(\hat{p}) = \ln\left(\frac{\hat{p}}{1-\hat{p}}\right)$  (log-odds).

- (A)  $N\left(\ln \frac{p}{1-p}, \frac{1}{np(1-p)}\right)$
- (B)  $N\left(\ln \frac{p}{1-p}, \frac{p(1-p)}{n}\right)$
- (C)  $N\left(p, \frac{1}{np(1-p)}\right)$
- (D)  $N\left(0, \frac{1}{n}\right)$

**Question 49** (Level 5) — *Stratified sampling effect*

A population has two strata: Stratum 1 ( $N_1 = 600$ ,  $p_1 = 0.3$ ) and Stratum 2 ( $N_2 = 400$ ,  $p_2 = 0.7$ ). A proportionally stratified sample of  $n = 100$  is taken. Find  $\text{Var}(\hat{p}_{\text{strat}})$ .

- (A) 0.0021
- (B) 0.0042
- (C) 0.0046
- (D) 0.021

**Question 50** (Level 5) — *Exam multi-step question*

A polling company wants  $\Pr(|\hat{p} - p| < 0.03) \geq 0.99$  for any  $p$ . Find the required sample size.

- (A) 1844
- (B) 1843
- (C) 1068
- (D) 2401

## Solutions

**Q1:** (A)

$$\hat{p} = \frac{12}{30} = 0.4.$$

**Q2:** (A)

The 1000 people surveyed are the sample. The 25 million is the population.

**Q3:** (A)

No. Different random samples will generally give different values of  $\hat{p}$  due to sampling variability.

**Q4:** (A)

$$0.35 \times 100 = 35\%.$$

**Q5:** (A)

$$\text{Count} = 200 \times 0.15 = 30.$$

**Q6:** (A)

Random sampling avoids bias and ensures  $\hat{p}$  is a fair estimate of the population proportion  $p$ .

**Q7:** (A)

A larger sample ( $n = 500$ ) gives a more reliable estimate than a smaller one ( $n = 50$ ).

**Q8:** (A)

On average,  $E(\hat{p}) = p$  (the true population proportion).  $\hat{p}$  is unbiased.

**Q9:** (A)

$\hat{p}$  can take values from 0 to 1 (inclusive), since it is  $\frac{X}{n}$  where  $0 \leq X \leq n$ .

**Q10:** (A)

$$1 - 0.6 = 0.4.$$

**Q11:** (A)

$$E(\hat{p}) = p = 0.3.$$

**Q12:** (A)

$$SE(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}.$$

**Q13:** (A)

$$SE = \sqrt{\frac{0.24}{100}} = \sqrt{0.0024} \approx 0.049.$$

**Q14:** (A)

$$SE \text{ is halved. } SE_{100} = \frac{0.5}{10} = 0.05, SE_{400} = \frac{0.5}{20} = 0.025.$$

**Q15:** (A)

By the Central Limit Theorem,  $\hat{p}$  is approximately normally distributed for large  $n$ .

**Q16:** (A)

$\hat{p}$  has mean  $p$  and variance  $\frac{p(1-p)}{n}$ . For large  $n$ ,  $\hat{p} \approx N\left(p, \frac{p(1-p)}{n}\right)$ .

**Q17:** (A)

$$SE = 0.0625. \text{ 95\% range: } 0.5 \pm 2(0.0625) = (0.375, 0.625).$$

**Q18:** (A)

$SD(X) = \sqrt{np(1-p)}$  measures variability of counts.  $SE(\hat{p}) = \frac{SD(X)}{n} = \sqrt{\frac{p(1-p)}{n}}$  measures variability of the proportion.

**Q19:** (A)

$z = 1$ .  $\Pr(|Z| > 1) \approx 0.32$ . Not unusual — well within normal variation.

**Q20:** (A)

$SE = \sqrt{\frac{p(1-p)}{n}}$ . Since  $p(1-p)$  is maximised at  $p = 0.5$ , SE is maximised there.

**Q21:** (A)

$$SE = \sqrt{\frac{0.24}{225}} = \frac{0.4899}{15} \approx 0.03266. \quad z = \frac{0.64 - 0.5}{0.03266} \approx 1.23. \quad \Pr(\hat{p} > 0.64) = 1 - 0.891 = 0.109.$$

**Q22:** (A)

$$SE = \sqrt{\frac{0.25}{400}} = 0.025. \text{ Interval: } 0.5 \pm 1.96(0.025) = 0.5 \pm 0.049 = (0.451, 0.549).$$

**Q23:** (A)

$np = 100 \times 0.05 = 5 < 10$ . The condition fails, so the normal approximation is not appropriate.

**Q24:** (A)

$SE \approx 0.0324$ .  $z_1 = \frac{0.66-0.7}{0.0324} \approx -1.23$ .  $z_2 = \frac{0.74-0.7}{0.0324} \approx 1.23$ .  $Pr = 0.891 - 0.109 = 0.782$ .

**Q25:** (A)

$n \geq \frac{0.25}{0.0004} = 625$ .

**Q26:** (A)

$SE = \sqrt{\frac{0.1875}{400}} = \sqrt{0.000469} \approx 0.02165$ .  $z = \frac{0.30-0.25}{0.02165} \approx 2.31$ .  $p\text{-value} = 2(1 - 0.990) = 0.020 < 0.05$ . Yes, significantly different.

**Q27:** (A)

$SE_B = 0.05 \times \frac{\sqrt{100}}{\sqrt{900}} = 0.05 \times \frac{10}{30} = 0.0167$ .

**Q28:** (A)

$\hat{p} \sim N\left(0.4, \frac{0.24}{50}\right) = N(0.4, 0.0048)$ .  $SD \approx 0.0693$ .

**Q29:** (A)

$SE \approx 0.0327$ .  $z = \frac{0.75-0.8}{0.0327} \approx -1.53$ .  $Pr(\hat{p} < 0.75) \approx 0.063$ .

**Q30:** (A)

No. This is a convenience sample, likely biased. Mall shoppers may differ from the general population.

**Q31:** (A)

$SE = \sqrt{\frac{0.2475}{500}} = \sqrt{0.000495} \approx 0.02225$ .  $z = \frac{0.58-0.55}{0.02225} \approx 1.35$ .  $Pr(\hat{p} > 0.58) = 1 - 0.911 = 0.089$ .

**Q32:** (A)

$1.96 \times \sqrt{\frac{0.21}{n}} \leq 0.05$ .  $\sqrt{\frac{0.21}{n}} \leq 0.02551$ .  $\frac{0.21}{n} \leq 0.000651$ .  $n \geq 322.6$ . So  $n = 323$ .

**Q33:** (A)

$SE = \sqrt{\frac{0.09}{200}} = 0.02121$ .  $z = \frac{0.86-0.90}{0.02121} = -1.89$ . One-tailed  $p\text{-value} = 0.029 < 0.05$ . Reject claim.

**Q34:** (A)

$SE = \sqrt{\frac{0.65 \times 0.35}{300}} = \sqrt{0.000758} \approx 0.02754$ . CI:  $0.65 \pm 1.96(0.02754) = (0.596, 0.704)$ .

**Q35:** (A)

$E = 0.6 - 0.5 = 0.1$ .  $SE = \sqrt{\frac{0.24}{100} + \frac{0.25}{100}} = \sqrt{0.0049} = 0.07$ .

**Q36:** (A)

$\frac{SE_1}{SE_2} = \frac{\sqrt{\frac{n_2}{n_1}}}{\sqrt{\frac{n_1}{n_2}}} \cdot \frac{0.04}{0.02} = \sqrt{\frac{n_2}{150}} \cdot 4 = \frac{n_2}{150}$ .  $n_2 = 600$ .

**Q37:** (A)

$SE = \sqrt{\frac{0.2475}{250}} \approx 0.03146$ .  $z = \frac{0.05}{0.03146} \approx 1.59$ .  $Pr(|\hat{p} - 0.45| > 0.05) = 2(1 - 0.944) = 0.112$ .

**Q38:** (A)

$\hat{p} = 0.036$ .  $SE = \sqrt{\frac{0.0196}{500}} = 0.00626$ .  $z = \frac{0.036-0.02}{0.00626} = 2.56$ .  $p\text{-value} \approx 2 \times 0.005 = 0.01 < 0.05$ . Reject claim.

**Q39:** (A)

$Var(\hat{p}) = \frac{0.35 \times 0.65}{200} = \frac{0.2275}{200} = 0.0011375$ .  $SD(\hat{p}) = \sqrt{0.0011375} \approx 0.0337$ .

**Q40:** (A)

$0.5 \pm 3(0.0125) = 0.5 \pm 0.0375 = (0.4625, 0.5375)$ .

**Q41:** (A)

Exact:  $Pr(X \geq 15) = \frac{\binom{20}{15} + \dots + \binom{20}{20}}{2^{20}} = \frac{15504 + 4845 + 1140 + 190 + 20 + 1}{1048576} = \frac{21700}{1048576} \approx 0.0207$ . Normal:  $z = \frac{0.7-0.5}{\sqrt{0.25/20}} = \frac{0.2}{0.1118} = 1.79$ .  $Pr \approx 0.037$ . The normal overestimates for small  $n$ .

**Q42:** (A)

$\mu = 40$ ,  $\sigma = \sqrt{24} \approx 4.899$ . With CC:  $z = \frac{44.5-40}{4.899} \approx 0.918$ .  $Pr(Z > 0.918) \approx 0.179$ .

**Q43:** (A)

$$\text{Var}(\hat{p}) = \frac{1}{n^2} \text{Var}(X) = \frac{np(1-p)}{n^2} = \frac{p(1-p)}{n}.$$

**Q44:** (A)

Under  $H_1$ :  $\text{SE} = \sqrt{\frac{0.24}{100}} = 0.049$ .  $z = \frac{0.58-0.6}{0.049} = -0.41$ .  $\text{Power} = \Pr(Z > -0.41) = 1 - 0.341 = 0.659$ .

**Q45:** (A)

$$\text{SE} = \sqrt{\frac{0.24}{250}} = 0.03098. \quad \hat{p}_c = 0.4 + 1.645(0.03098) = 0.4 + 0.051 = 0.451.$$

**Q46:** (A)

This is convergence in distribution (also called weak convergence).

**Q47:** (A)

Pooled  $\hat{p} = 0.408$ .  $\text{SE} = \sqrt{0.408 \times 0.592 \left( \frac{1}{200} + \frac{1}{300} \right)} = \sqrt{0.2415 \times 0.00833} = \sqrt{0.002013} \approx 0.04487$ .  $z = \frac{0.45-0.38}{0.04487} = 1.56$ . Two-tailed  $p$ -value  $= 2(1 - 0.941) \approx 0.119 > 0.05$ . Do not reject.

**Q48:** (A)

$$g'(p) = \frac{1}{p(1-p)}. \quad \text{Var}(g(\hat{p})) \approx \frac{1}{[p(1-p)]^2} \cdot \frac{p(1-p)}{n} = \frac{1}{np(1-p)}.$$

**Q49:** (A)

$w_1 = 0.6$ ,  $w_2 = 0.4$ .  $\text{Var} = 0.36 \times \frac{0.21}{60} + 0.16 \times \frac{0.21}{40} = 0.36 \times 0.0035 + 0.16 \times 0.00525 = 0.00126 + 0.00084 = 0.0021$ .

**Q50:** (A)

$2.576 \times \sqrt{\frac{0.25}{n}} \leq 0.03$ .  $\sqrt{\frac{0.25}{n}} \leq 0.01164$ .  $\frac{0.25}{n} \leq 0.0001356$ .  $n \geq \frac{0.25}{0.0001356} = 1843.3$ . So  $n = 1844$ .