50 multiple-choice questions

Question 1 (Level 1) — *Gradient of a line*
What is the gradient of $y = 3x + 2$?

(A) 3

(B) 2

(C) 5

(D) $\frac{2}{3}$

Question 2 (Level 1) — *Rate of change meaning*
A car travels 60 km in 2 hours. What is its average rate of change of distance with respect to time?

(A) 30 km/h

(B) 60 km/h

(C) 120 km/h

(D) 2 km/h

Question 3 (Level 1) — *Gradient from two points*
Find the gradient of the line through $(1, 3)$ and $(3, 7)$.

(A) 2

(B) 4

(C) $\frac{1}{2}$

(D) 5

Question 4 (Level 1) — *Gradient of horizontal line*
What is the gradient of $y = 5$?

(A) 0

(B) 5

(C) 1

(D) Undefined

Question 5 (Level 1) — *Negative gradient*
What is the gradient of $y = -2x + 4$?

(A) -2

(B) 2

(C) 4

(D) -4

Question 6 (Level 1) — *Tangent line concept*
A tangent to a curve at a point is a line that:

(A) Touches the curve at that point and has the same gradient

(B) Crosses the curve at two points

(C) Is always horizontal

(D) Is perpendicular to the curve

Question 7 (Level 1) — *Constant function gradient*
If $f(x) = 7$, what is $f'(x)$?

(A) 0

(B) 7

(C) 1

(D) -7

Question 8 (Level 1) — *Derivative of x*
If $f(x) = x$, what is $f'(x)$?

(A) 1

(B) x

(C) 0

(D) $2x$

Question 9 (Level 1) — *Secant gradient*
On the curve $y = x^2$, find the gradient of the secant through $(1, 1)$ and $(3, 9)$.

(A) 4

(B) 8

(C) 2

(D) 5

Question 10 (Level 1) — *Derivative of linear function*
If $f(x) = 4x - 1$, what is $f'(x)$?

(A) 4

(B) $4x$

(C) -1

(D) $4x - 1$

Question 11 (Level 2) — *Power rule: x*
Find $\dfrac{d}{dx}(x^2)$.

(A) $2x$

(B) x^2

(C) 2

(D) x

Question 12 (Level 2) — *Power rule: x*
Find $\dfrac{d}{dx}(x^3)$.

(A) $3x^2$

(B) $3x^3$

(C) x^2

(D) $2x^3$

Question 13 (Level 2) — *Derivative with coefficient*
Find $f'(x)$ if $f(x) = 5x^2$.

(A) $10x$

(B) $5x$

(C) $10x^2$

(D) 5

Question 14 (Level 2) — *Sum rule*
Find $\dfrac{d}{dx}(x^2 + 3x)$.

(A) $2x + 3$

(B) $2x + 3x$

(C) $x^2 + 3$

(D) $2x$

Question 15 (Level 2) — *Gradient at a point*
If $f(x) = x^2$, find the gradient at $x = 3$.

(A) 6

(B) 9

(C) 3

(D) 2

Question 16 (Level 2) — *Polynomial derivative*
Find $\dfrac{d}{dx}(x^3 - 2x^2 + x)$.

(A) $3x^2 - 4x + 1$

(B) $3x^2 - 2x + 1$

(C) $3x^2 - 4x$

(D) $x^2 - 4x + 1$

Question 17 (Level 2) — *First principles concept*
The derivative $f'(x)$ is defined as $\lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$. For $f(x) = x^2$, what is $f(x+h)$?

(A) $x^2 + 2xh + h^2$

(B) $x^2 + h^2$

(C) $x^2 + 2h$

(D) $x^2 + h$

Question 18 (Level 2) — *Derivative of negative power (intro)*
If $f(x) = x^{-1} = \dfrac{1}{x}$, what is $f'(x)$?

(A) $-\dfrac{1}{x^2}$

(B) $\dfrac{1}{x^2}$

(C) $-\dfrac{1}{x}$

(D) $\dfrac{-2}{x}$

Question 19 (Level 2) — *Where gradient is zero*
Find where the gradient of $f(x) = x^2 - 4x$ is zero.

(A) $x = 2$

(B) $x = 4$

(C) $x = 0$

(D) $x = -2$

Question 20 (Level 2) — *First principles for 3x*
Using first principles, find the derivative of $f(x) = 3x$.

(A) 3

(B) $3x$

(C) 0

(D) $3h$

Question 21 (Level 3) — *First principles for x*
Use first principles to find $f'(x)$ when $f(x) = x^2$.

(A) $2x$

(B) $2x + h$

(C) x^2

(D) $2xh$

Question 22 (Level 3) — *Fractional power*
Find $\dfrac{d}{dx}(\sqrt{x})$.

(A) $\dfrac{1}{2\sqrt{x}}$

(B) $\dfrac{1}{\sqrt{x}}$

(C) $2\sqrt{x}$

(D) $\dfrac{\sqrt{x}}{2}$

Question 23 (Level 3) — *Equation of tangent*
Find the equation of the tangent to $y = x^2$ at $x = 1$.

(A) $y = 2x - 1$

(B) $y = 2x + 1$

(C) $y = x - 1$

(D) $y = 2x$

Question 24 (Level 3) — *Negative power derivative*
Find $\dfrac{d}{dx}\left(\dfrac{3}{x^2}\right)$.

(A) $-\dfrac{6}{x^3}$

(B) $\dfrac{6}{x^3}$

(C) $-\dfrac{3}{x^3}$

(D) $-\dfrac{6}{x^2}$

Question 25 (Level 3) — *First principles for x*
Using first principles, show that $\dfrac{d}{dx}(x^3) = 3x^2$. What is $\dfrac{f(x+h)-f(x)}{h}$ before taking the limit?

(A) $3x^2 + 3xh + h^2$

(B) $3x^2 + h$

(C) $3x^2$

(D) $3x^2h + 3xh^2 + h^3$

Question 26 (Level 3) — *Gradient at a point on cubic*
Find the gradient of $y = x^3 - 3x$ at $x = 2$.

(A) 9

(B) 2

(C) 12

(D) 6

Question 27 (Level 3) — *Derivative of polynomial*
Find $f'(x)$ if $f(x) = 2x^4 - x^2 + 6$.

(A) $8x^3 - 2x$

(B) $8x^3 - 2x + 6$

(C) $8x^4 - 2x$

(D) $2x^3 - 2x$

Question 28 (Level 3) — *Horizontal tangent*
Find the x-values where $y = x^3 - 12x$ has horizontal tangents.

(A) $x = 2$ and $x = -2$

(B) $x = 4$ and $x = -4$

(C) $x = 2$ only

(D) $x = 0$

Question 29 (Level 3) — *Tangent parallel to given line*
For what value of x is the tangent to $y = x^2 + 1$ parallel to the line $y = 6x$?

(A) $x = 3$

(B) $x = 6$

(C) $x = \dfrac{1}{6}$

(D) $x = 2$

Question 30 (Level 3) — *Derivative of reciprocal square root*
Find $\dfrac{d}{dx}\left(\dfrac{1}{\sqrt{x}}\right)$.

(A) $-\dfrac{1}{2x\sqrt{x}}$

(B) $\dfrac{1}{2\sqrt{x}}$

(C) $-\dfrac{1}{\sqrt{x}}$

(D) $-\dfrac{1}{2\sqrt{x}}$

Question 31 (Level 4) — *Derivative of e*
Find $\dfrac{d}{dx}(e^x)$.

(A) e^x

(B) xe^{x-1}

(C) e^{x-1}

(D) $\dfrac{e^{x+1}}{x+1}$

Question 32 (Level 4) — *Derivative of ln(x)*
Find $\dfrac{d}{dx}(\ln(x))$ for $x > 0$.

(A) $\dfrac{1}{x}$

(B) $\ln(x)$

(C) $\dfrac{1}{\ln(x)}$

(D) e^x

Question 33 (Level 4) — *Derivative of sin(x)*
Find $\dfrac{d}{dx}(\sin(x))$.

(A) $\cos(x)$

(B) $-\cos(x)$

(C) $\sin(x)$

(D) $-\sin(x)$

Question 34 (Level 4) — *Derivative of cos(x)*
Find $\dfrac{d}{dx}(\cos(x))$.

(A) $-\sin(x)$

(B) $\sin(x)$

(C) $\cos(x)$

(D) $-\cos(x)$

Question 35 (Level 4) — *Gradient of exponential at x=0*
Find the gradient of $y = 3e^x$ at $x = 0$.

(A) 3

(B) $3e$

(C) 1

(D) 0

Question 36 (Level 4) — *Tangent to exponential curve*
Find the equation of the tangent to $y = e^x$ at $x = 1$.

(A) $y = ex$

(B) $y = ex - 1$

(C) $y = e(x - 1)$

(D) $y = ex + e$

Question 37 (Level 4) — *Combined polynomial and trig*
Find $\dfrac{d}{dx}(x^2 + \sin(x))$.

(A) $2x + \cos(x)$

(B) $2x - \cos(x)$

(C) $x^2 + \cos(x)$

(D) $2x + \sin(x)$

Question 38 (Level 4) — *Gradient of ln at a point*
Find the gradient of $y = \ln(x)$ at $x = e$.

(A) $\dfrac{1}{e}$

(B) e

(C) 1

(D) $\dfrac{1}{e^2}$

Question 39 (Level 4) — *First principles for e (conceptual)*
Using the definition, $\dfrac{d}{dx}(e^x) = \lim_{h\to 0} \dfrac{e^{x+h} - e^x}{h}$. This simplifies to:

(A) $e^x \cdot \lim_{h\to 0} \dfrac{e^h - 1}{h} = e^x$

(B) $\lim_{h\to 0} \dfrac{e^h}{h} = 1$

(C) $e^x \cdot e^h = e^{2x}$

(D) $e^x \cdot h = xe^x$

Question 40 (Level 4) — *Where tangent is horizontal*
Find all x where the tangent to $y = x^3 - 6x^2 + 9x$ is horizontal.

(A) $x = 1$ and $x = 3$

(B) $x = 0$ and $x = 3$

(C) $x = 1$ only

(D) $x = 2$ and $x = 4$

Question 41 (Level 5) — *First principles for 1/x*
Use first principles to differentiate $f(x) = \dfrac{1}{x}$.

(A) $-\dfrac{1}{x^2}$

(B) $\dfrac{1}{x^2}$

(C) $-\dfrac{2}{x^3}$

(D) $-\dfrac{1}{x}$

Question 42 (Level 5) — *Tangent that passes through origin*
Find the equation of the tangent to $y = e^x$ that passes through the origin.

(A) $y = ex$

(B) $y = x$

(C) $y = e^2 x$

(D) $y = \dfrac{x}{e}$

Question 43 (Level 5) — *Average vs instantaneous rate*
For $f(x) = x^3$, find the average rate of change over $[1, 1+h]$ and verify it approaches $f'(1)$ as $h \to 0$.

(A) $3 + 3h + h^2$, which approaches 3

(B) $3h + 3h^2 + h^3$, which approaches 0

(C) $1 + 3h$, which approaches 1

(D) $3 + h$, which approaches 3

Question 44 (Level 5) — *Tangent meets curve again*
The tangent to $y = x^3$ at $x = 1$ meets the curve again at which point?

(A) $(-2, -8)$

(B) $(-1, -1)$

(C) $(2, 8)$

(D) $(-2, 8)$

Question 45 (Level 5) — *Differentiability at a point*
Is $f(x) = |x|$ differentiable at $x = 0$?

(A) No, left and right derivatives differ (-1 and 1)

(B) Yes, $f'(0) = 0$

(C) Yes, $f'(0) = 1$

(D) No, because $f(0)$ is undefined

Question 46 (Level 5) — *First principles for x*
Use first principles to find $f'(x)$ where $f(x) = \sqrt{x}$.

(A) $\dfrac{1}{2\sqrt{x}}$

(B) $\dfrac{1}{\sqrt{x}}$

(C) $2\sqrt{x}$

(D) $\dfrac{\sqrt{x}}{2}$

Question 47 (Level 5) — *Tangent perpendicular to a line*
Find the point on $y = \ln(x)$ where the tangent is perpendicular to $y = -2x + 3$.

(A) $(2, \ln 2)$

(B) $(\frac{1}{2}, \ln \frac{1}{2})$

(C) $(e, 1)$

(D) $(1, 0)$

Question 48 (Level 5) — *Derivative of x from first principles*
Using the binomial theorem, the first principles derivative of x^n gives $\dfrac{(x+h)^n - x^n}{h}$. What does this simplify to before taking the limit?

(A) nx^{n-1} + terms with $h \to nx^{n-1}$

(B) $x^{n-1} + h^{n-1}$

(C) nx^n

(D) $(n-1)x^n$

Question 49 (Level 5) — *Number of tangents from external point*
How many tangent lines to $y = x^2$ pass through the point $(0, -1)$?

(A) 2

(B) 1

(C) 0

(D) 3

Question 50 (Level 5) — *Point where gradient equals function value*
Find all points on $y = e^x$ where the gradient equals the y-coordinate.

(A) All points on the curve (since $\dfrac{dy}{dx} = e^x = y$ everywhere)

(B) Only at $x = 0$

(C) Only at $x = 1$

(D) No such point exists

Solutions

Q1: (A)
The gradient is the coefficient of x, which is 3.
Q2: (A)
$\frac{60}{2} = 30$ km/h.
Q3: (A)
$m = \frac{7-3}{3-1} = \frac{4}{2} = 2$.
Q4: (A)
Horizontal lines have gradient 0.
Q5: (A)
The gradient is -2.
Q6: (A)
A tangent touches the curve at that point and has the same gradient as the curve there.
Q7: (A)
The derivative of a constant is 0. So $f'(x) = 0$.
Q8: (A)
$f(x) = x$ is a line with gradient 1, so $f'(x) = 1$.
Q9: (A)
$m = \frac{9-1}{3-1} = \frac{8}{2} = 4$.
Q10: (A)
$f'(x) = 4$.
Q11: (A)
$\frac{d}{dx}(x^2) = 2x$.
Q12: (A)
$\frac{d}{dx}(x^3) = 3x^2$.
Q13: (A)
$f'(x) = 5 \cdot 2x = 10x$.
Q14: (A)
$\frac{d}{dx}(x^2 + 3x) = 2x + 3$.
Q15: (A)
$f'(x) = 2x$. At $x = 3$: $f'(3) = 6$.
Q16: (A)
$3x^2 - 4x + 1$.
Q17: (A)
$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$.
Q18: (A)
$f'(x) = -1 \cdot x^{-2} = -\frac{1}{x^2}$.
Q19: (A)
$f'(x) = 2x - 4 = 0 \Rightarrow x = 2$.
Q20: (A)
$\frac{f(x+h) - f(x)}{h} = \frac{3h}{h} = 3$. So $f'(x) = 3$.
Q21: (A)
$\frac{(x+h)^2 - x^2}{h} = \frac{2xh + h^2}{h} = 2x + h \to 2x$ as $h \to 0$.

Q22: (A)
$\frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$.
Q23: (A)
$y' = 2x$, at $x = 1$: $m = 2$, point $(1, 1)$. $y - 1 = 2(x - 1) \Rightarrow y = 2x - 1$.
Q24: (A)
$\frac{d}{dx}(3x^{-2}) = 3 \cdot (-2)x^{-3} = -\frac{6}{x^3}$.
Q25: (A)
$\frac{(x+h)^3 - x^3}{h} = \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$.
Q26: (A)
$\frac{dy}{dx} = 3x^2 - 3$. At $x = 2$: $3(4) - 3 = 9$.
Q27: (A)
$f'(x) = 8x^3 - 2x$.
Q28: (A)
$\frac{dy}{dx} = 3x^2 - 12 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$.
Q29: (A)
$y' = 2x = 6 \Rightarrow x = 3$.
Q30: (A)
$\frac{d}{dx}(x^{-1/2}) = -\frac{1}{2}x^{-3/2} = -\frac{1}{2x\sqrt{x}}$.
Q31: (A)
$\frac{d}{dx}(e^x) = e^x$.
Q32: (A)
$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$.
Q33: (A)
$\frac{d}{dx}(\sin(x)) = \cos(x)$.
Q34: (A)
$\frac{d}{dx}(\cos(x)) = -\sin(x)$.
Q35: (A)
$y' = 3e^x$. At $x = 0$: $y'(0) = 3e^0 = 3$.
Q36: (A)
Gradient $= e$. Point $(1, e)$. $y - e = e(x - 1) \Rightarrow y = ex$.
Q37: (A)
$\frac{d}{dx}(x^2 + \sin(x)) = 2x + \cos(x)$.
Q38: (A)
$y' = \frac{1}{x}$. At $x = e$: $y' = \frac{1}{e}$.
Q39: (A)
$\frac{e^x(e^h - 1)}{h} = e^x \cdot \lim_{h\to 0} \frac{e^h - 1}{h} = e^x \cdot 1 = e^x$.
Q40: (A)
$y' = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x - 1)(x - 3) = 0$. So $x = 1$ or $x = 3$.
Q41: (A)
$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \frac{-h}{hx(x+h)} = \frac{-1}{x(x+h)} \to -\frac{1}{x^2}$.

Q42: (A)

Tangent at (a, e^a): $y - e^a = e^a(x - a)$. Through origin: $-e^a = -ae^a \Rightarrow a = 1$. Tangent: $y = ex$.

Q43: (A)

$\dfrac{(1+h)^3 - 1}{h} = \dfrac{3h + 3h^2 + h^3}{h} = 3 + 3h + h^2 \to 3$ as $h \to 0$. And $f'(1) = 3(1)^2 = 3$. ✓

Q44: (A)

At $x = 1$: $y = 1$, $y' = 3$. Tangent: $y = 3x - 2$. Solve $x^3 = 3x - 2$: $x^3 - 3x + 2 = 0 \Rightarrow (x-1)^2(x+2) = 0$. Meets again at $x = -2$, $y = -8$.

Q45: (A)

Left derivative: $\lim_{h\to 0^-} \frac{|h|}{h} = -1$. Right derivative: $\lim_{h\to 0^+} \frac{|h|}{h} = 1$. These differ, so f is not differentiable at $x = 0$.

Q46: (A)

$$\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}} \to \frac{1}{2\sqrt{x}}.$$

Q47: (A)

$y' = \frac{1}{x} = \frac{1}{2} \Rightarrow x = 2$. $y = \ln(2)$. Point: $(2, \ln 2)$.

Q48: (A)

$$\frac{nx^{n-1}h + \binom{n}{2}x^{n-2}h^2 + \cdots}{h} = nx^{n-1} + \binom{n}{2}x^{n-2}h + \cdots \to nx^{n-1}.$$

Q49: (A)

At (a, a^2): tangent $y = 2ax - a^2$. Through $(0, -1)$: $-1 = -a^2 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$. Two tangent lines.

Q50: (A)

The gradient of $y = e^x$ is $\dfrac{dy}{dx} = e^x = y$. This holds for all points on the curve. The gradient always equals the y-value.