50 multiple-choice questions

Question 1 (Level 1) — *Average speed*
A car travels 120 km in 2 hours. What is its average speed?

(A) 60 km/h

(B) 240 km/h

(C) 30 km/h

(D) 120 km/h

Question 2 (Level 1) — *Average rate of change*
If $f(x) = x^2$, find the average rate of change from $x = 1$ to $x = 3$.

(A) 4

(B) 8

(C) 5

(D) 2

Question 3 (Level 1) — *Rate formula*
The average rate of change of f from $x = a$ to $x = b$ is:

(A) $\frac{f(b)-f(a)}{b-a}$

(B) $\frac{f(a)+f(b)}{2}$

(C) $f(b) - f(a)$

(D) $\frac{b-a}{f(b)-f(a)}$

Question 4 (Level 1) — *Average rate of 2x + 1*
Find the average rate of change of $f(x) = 2x + 1$ from $x = 0$ to $x = 5$.

(A) 2

(B) 10

(C) 11

(D) 5

Question 5 (Level 1) — *Temperature change*
Temperature rises from 10řC to 25řC over 3 hours. What is the average rate of change?

(A) 5 řC/hour

(B) 15 řC/hour

(C) $\frac{3}{15}$ řC/hour

(D) 7.5 řC/hour

Question 6 (Level 1) — *Average rate of x*
Find the average rate of change of $f(x) = x^3$ from $x = 0$ to $x = 2$.

(A) 4

(B) 8

(C) 12

(D) 6

Question 7 (Level 1) — *Secant line*
The average rate of change represents the slope of the:

(A) Secant line

(B) Tangent line

(C) Normal line

(D) Asymptote

Question 8 (Level 1) — *Negative average rate*
If $f(x) = -x + 10$, find the average rate of change from $x = 2$ to $x = 8$.

(A) -1

(B) 1

(C) -6

(D) 6

Question 9 (Level 1) — *Zero average rate*
If $f(1) = 5$ and $f(4) = 5$, the average rate of change from $x = 1$ to $x = 4$ is:

(A) 0

(B) 5

(C) $\frac{5}{3}$

(D) 3

Question 10 (Level 1) — *Average rate of constant*
Find the average rate of change of $f(x) = 7$ from $x = 0$ to $x = 100$.

(A) 0

(B) 7

(C) 0.07

(D) 700

Question 11 (Level 2) — *Average vs instantaneous*
How does the instantaneous rate of change differ from the average rate of change?

(A) Instantaneous rate is the derivative at a point; average rate uses two points

(B) They are the same thing

(C) Average rate is always larger

(D) Instantaneous rate uses the integral

Question 12 (Level 2) — *Secant to tangent*
As the two points on a curve get closer together, the secant line approaches the:

(A) Tangent line

(B) Normal line

(C) Horizontal axis

(D) Vertical axis

Question 13 (Level 2) — *Average rate of quadratic*
Find the average rate of change of $f(x) = x^2 - 3x$ from $x = 1$ to $x = 4$.

(A) 2

(B) 6

(C) 3

(D) $\frac{3}{2}$

Question 14 (Level 2) — *Secant line equation*
Find the equation of the secant line to $y = x^2$ through $(1, 1)$ and $(3, 9)$.

(A) $y = 4x - 3$

(B) $y = 4x - 1$

(C) $y = 4x + 1$

(D) $y = 5x - 4$

Question 15 (Level 2) — *Average rate from table*
Given $f(2) = 10$ and $f(5) = 1$, find the average rate of change.

(A) -3

(B) 3

(C) -9

(D) $\frac{11}{3}$

Question 16 (Level 2) — *Comparing rates*
For $f(x) = x^2$, compare the average rate from $x = 0$ to $x = 2$ with the instantaneous rate at $x = 1$.

(A) Both equal 2

(B) Average is 4, instantaneous is 2

(C) Average is 2, instantaneous is 4

(D) Both equal 4

Question 17 (Level 2) — *Population growth*
A population grows from 200 to 800 over 10 years. What is the average rate of growth?

(A) 60 per year

(B) 600 per year

(C) 80 per year

(D) 100 per year

Question 18 (Level 2) — *Rate of height*
A ball's height is $h(t) = 20t - 5t^2$. Find the average rate of change from $t = 0$ to $t = 2$.

(A) 10 m/s

(B) 20 m/s

(C) 5 m/s

(D) 15 m/s

Question 19 (Level 2) — *Average rate of x*
Find the average rate of change of $f(x) = \sqrt{x}$ from $x = 4$ to $x = 9$.

(A) $\frac{1}{5}$

(B) $\frac{1}{3}$

(C) 1

(D) 5

Question 20 (Level 2) — *Units of rate*
If distance is in metres and time in seconds, the average rate of change of distance with respect to time has units:

(A) m/s

(B) m/s^2

(C) m^2/s

(D) s/m

Question 21 (Level 3) — *Average rate of e*
Find the average rate of change of $f(x) = e^x$ from $x = 0$ to $x = 1$.

(A) $e - 1$

(B) e

(C) 1

(D) $\frac{e}{2}$

Question 22 (Level 3) — *Average rate of ln(x)*
Find the average rate of change of $f(x) = \ln(x)$ from $x = 1$ to $x = e$.

(A) $\frac{1}{e-1}$

(B) $\frac{1}{e}$

(C) $e - 1$

(D) 1

Question 23 (Level 3) — *Average rate of sin(x)*
Find the average rate of change of $f(x) = \sin(x)$ from $x = 0$ to $x = \frac{\pi}{2}$.

(A) $\frac{2}{\pi}$

(B) $\frac{\pi}{2}$

(C) 1

(D) $\frac{1}{\pi}$

Question 24 (Level 3) — *When avg = instantaneous*
For $f(x) = x^2$ on $[0, 4]$, find the value of c where $f'(c)$ equals the average rate of change.

(A) $c = 2$

(B) $c = 4$

(C) $c = 1$

(D) $c = 3$

Question 25 (Level 3) — *Average rate of 1/x*
Find the average rate of change of $f(x) = \frac{1}{x}$ from $x = 1$ to $x = 4$.

(A) $-\frac{1}{4}$

(B) $\frac{1}{4}$

(C) $-\frac{3}{4}$

(D) $-\frac{1}{3}$

Question 26 (Level 3) — *Secant parallel to tangent*
For $f(x) = x^3$ on $[0, 2]$, find $c \in (0, 2)$ where the tangent is parallel to the secant.

(A) $\frac{2\sqrt{3}}{3}$

(B) $\frac{2}{3}$

(C) $\sqrt{\frac{4}{3}}$

(D) 1

Question 27 (Level 3) — *Average velocity*
A particle's position is $x(t) = t^3 - 6t$. Find the average velocity from $t = 1$ to $t = 3$.

(A) 7

(B) 14

(C) 4

(D) 9

Question 28 (Level 3) — *Difference quotient*
Simplify $\frac{(x+h)^2 - x^2}{h}$.

(A) $2x + h$

(B) $2x$

(C) h

(D) $2xh$

Question 29 (Level 3) — *Average rate of cos*
Find the average rate of change of $f(x) = \cos(x)$ from $x = 0$ to $x = \pi$.

(A) $-\frac{2}{\pi}$

(B) $\frac{2}{\pi}$

(C) $-\frac{\pi}{2}$

(D) 0

Question 30 (Level 3) — *Limit of average rate*
As $h \to 0$, $\frac{f(a+h)-f(a)}{h}$ becomes:

(A) $f'(a)$

(B) $f(a)$

(C) 0

(D) ∞

Question 31 (Level 4) — *Average value of function*
Find the average value of $f(x) = x^2$ on $[0, 3]$.

(A) 3

(B) 9

(C) $\frac{9}{2}$

(D) $\frac{3}{2}$

Question 32 (Level 4) — *Average value of e*
Find the average value of $f(x) = e^x$ on $[0, 2]$.

(A) $\frac{e^2-1}{2}$

(B) $e^2 - 1$

(C) $\frac{e^2}{2}$

(D) $e - 1$

Question 33 (Level 4) — *Average value of sin(x)*
Find the average value of $f(x) = \sin(x)$ on $[0, \pi]$.

(A) $\frac{2}{\pi}$

(B) $\frac{\pi}{2}$

(C) $\frac{1}{\pi}$

(D) 1

Question 34 (Level 4) — *MVT application*
For $f(x) = x^3 - 3x$ on $[0, 2]$, find c guaranteed by the Mean Value Theorem.

(A) $\frac{2\sqrt{3}}{3}$

(B) 1

(C) $\sqrt{\frac{3}{4}}$

(D) $\frac{\sqrt{3}}{2}$

Question 35 (Level 4) — *Average value of 1/x*
Find the average value of $f(x) = \frac{1}{x}$ on $[1, e]$.

(A) $\frac{1}{e-1}$

(B) $\frac{1}{e}$

(C) 1

(D) $\ln(e - 1)$

Question 36 (Level 4) — *Average rate of e on [0,ln2]*
Find the average rate of change of $f(x) = e^x$ from $x = 0$ to $x = \ln 2$.

(A) $\frac{1}{\ln 2}$

(B) $\ln 2$

(C) 2

(D) $\frac{2}{\ln 2}$

Question 37 (Level 4) — *Average value of cos(x)*
Find the average value of $\cos(x)$ on $[0, 2\pi]$.

(A) 0

(B) 1

(C) $\frac{1}{\pi}$

(D) $\frac{2}{\pi}$

Question 38 (Level 4) — *MVT for e*
Find the value of $c \in (0, 1)$ where $f'(c)$ equals the average rate of change of $f(x) = e^x$ on $[0, 1]$.

(A) $\ln(e - 1)$

(B) $e - 1$

(C) $\frac{1}{2}$

(D) $1 - \frac{1}{e}$

Question 39 (Level 4) — *Average value of linear*
Find the average value of $f(x) = 3x + 1$ on $[0, 4]$.

(A) 7

(B) 13

(C) $\frac{13}{2}$

(D) 6

Question 40 (Level 4) — *Average value equals midpoint*
For a linear function, the average value on $[a, b]$ equals f at which point?

(A) $x = \frac{a+b}{2}$

(B) $x = a$

(C) $x = b$

(D) $x = \sqrt{ab}$

Question 41 (Level 5) — *Average value of xe*
Set up (but simplify) the average value of $f(x) = x^2$ on $[0, a]$.

(A) $\frac{a^2}{3}$

(B) $\frac{a^2}{2}$

(C) $\frac{a^3}{3}$

(D) a^2

Question 42 (Level 5) — *MVT verification*
For $f(x) = \sin(x)$ on $[0, \pi]$, find c such that $f'(c) = \frac{f(\pi) - f(0)}{\pi}$.

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

Question 43 (Level 5) — *Average rate of composite*
Find the average rate of change of $f(x) = e^{x^2}$ from $x = 0$ to $x = 1$.

(A) $e - 1$

(B) $e^2 - 1$

(C) $2e$

(D) e

Question 44 (Level 5) — *Average value of |sin(x)|*
Find the average value of $|\sin(x)|$ on $[0, 2\pi]$.

(A) $\frac{2}{\pi}$

(B) 0

(C) $\frac{4}{\pi}$

(D) $\frac{1}{\pi}$

Question 45 (Level 5) — *MVT failure*
The Mean Value Theorem requires f to be continuous on $[a, b]$ and differentiable on (a, b). Which function fails MVT on $[-1, 1]$?

(A) $f(x) = |x|$

(B) $f(x) = x^2$

(C) $f(x) = x^3$

(D) $f(x) = \sin(x)$

Question 46 (Level 5) — *Average value equals k*
Find $a > 0$ such that the average value of $f(x) = x^2$ on $[0, a]$ equals 12.

(A) 6

(B) $\sqrt{12}$

(C) 36

(D) 4

Question 47 (Level 5) — *Weighted average rate*
For $f(x) = e^{-x}$, find the average value on $[0, \ln 3]$.

(A) $\frac{2}{3\ln 3}$

(B) $\frac{1}{\ln 3}$

(C) $\frac{2}{3}$

(D) $\frac{1}{3}$

Question 48 (Level 5) — *MVT with ln*
For $f(x) = \ln(x)$ on $[1, e^2]$, find c satisfying the MVT.

(A) $\frac{e^2 - 1}{2}$

(B) e

(C) $\frac{e^2}{2}$

(D) $e^2 - 1$

Question 49 (Level 5) — *Average of squared trig*
Find the average value of $\sin^2(x)$ on $[0, \pi]$.

(A) $\frac{1}{2}$

(B) $\frac{\pi}{2}$

(C) $\frac{2}{\pi}$

(D) 1

Question 50 (Level 5) — *Average rate and area*
Show that for any continuous f on $[a, b]$, the average rate of change of $F(x) = \int_a^x f(t)\,dt$ equals the average value of f. What is this common value for $f(x) = x$ on $[0, 4]$?

(A) 2

(B) 4

(C) 8

(D) $\frac{4}{3}$

Solutions

Q1: (A)
$\frac{120}{2} = 60$ km/h.

Q2: (A)
$\frac{9-1}{2} = 4$.

Q3: (A)
$\frac{f(b)-f(a)}{b-a}$.

Q4: (A)
$\frac{11-1}{5-0} = 2$.

Q5: (A)
$\frac{25-10}{3} = 5$ řC/hour.

Q6: (A)
$\frac{8-0}{2} = 4$.

Q7: (A)
The secant line through the two points.

Q8: (A)
$\frac{2-8}{8-2} = \frac{-6}{6} = -1$.

Q9: (A)
$\frac{5-5}{4-1} = 0$.

Q10: (A)
$\frac{7-7}{100} = 0$.

Q11: (A)
The instantaneous rate is the limit of the average rate as the interval approaches zero — it is $f'(a)$.

Q12: (A)
The tangent line at that point.

Q13: (A)
$\frac{4-(-2)}{4-1} = \frac{6}{3} = 2$.

Q14: (A)
Slope $= 4$. $y - 1 = 4(x - 1)$, i.e. $y = 4x - 3$.

Q15: (A)
$\frac{1-10}{3} = -3$.

Q16: (A)
Average rate $= 2$, instantaneous rate at $x = 1 = 2$. They are equal.

Q17: (A)
$\frac{800-200}{10} = 60$ per year.

Q18: (A)
$\frac{20-0}{2} = 10$ m/s.

Q19: (A)
$\frac{3-2}{9-4} = \frac{1}{5}$.

Q20: (A)
metres per second (m/s).

Q21: (A)
$\frac{e-1}{1} = e - 1$.

Q22: (A)
$\frac{1-0}{e-1} = \frac{1}{e-1}$.

Q23: (A)
$\frac{1-0}{\frac{\pi}{2}-0} = \frac{2}{\pi}$.

Q24: (A)

$2c = 4 \Rightarrow c = 2$.

Q25: (A)

$\frac{\frac{1}{4}-1}{4-1} = \frac{-\frac{3}{4}}{3} = -\frac{1}{4}$.

Q26: (A)

$3c^2 = 4 \Rightarrow c = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$.

Q27: (A)

$\frac{9-(-5)}{3-1} = \frac{14}{2} = 7$.

Q28: (A)

$\frac{2xh+h^2}{h} = 2x + h$.

Q29: (A)

$\frac{-1-1}{\pi-0} = -\frac{2}{\pi}$.

Q30: (A)

$f'(a)$, the derivative of f at a.

Q31: (A)

$\frac{1}{3}\int_0^3 x^2 \, dx = \frac{1}{3} \cdot 9 = 3$.

Q32: (A)

$\frac{1}{2}[e^x]_0^2 = \frac{e^2-1}{2}$.

Q33: (A)

$\frac{1}{\pi}[-\cos(x)]_0^\pi = \frac{1}{\pi}(1+1) = \frac{2}{\pi}$.

Q34: (A)

$3c^2 = 4 \Rightarrow c = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \approx 1.15$.

Q35: (A)

$\frac{1}{e-1} \cdot 1 = \frac{1}{e-1}$.

Q36: (A)

$\frac{2-1}{\ln 2} = \frac{1}{\ln 2}$.

Q37: (A)

$\frac{1}{2\pi}[\sin(x)]_0^{2\pi} = \frac{0}{2\pi} = 0$.

Q38: (A)

$e^c = e - 1 \Rightarrow c = \ln(e-1)$.

Q39: (A)

$\frac{1}{4}\left[\frac{3x^2}{2} + x\right]_0^4 = \frac{1}{4}(24+4) = 7$.

Q40: (A)

The midpoint $x = \frac{a+b}{2}$.

Q41: (A)

$\frac{1}{a} \cdot \frac{a^3}{3} = \frac{a^2}{3}$.

Q42: (A)

$\frac{0-0}{\pi} = 0$. So $\cos(c) = 0 \Rightarrow c = \frac{\pi}{2}$.

Q43: (A)

$\frac{e-1}{1} = e - 1$.

Q44: (A)

Total area $= 4$. Average $= \frac{4}{2\pi} = \frac{2}{\pi}$.

Q45: (A)

$f(x) = |x|$ is not differentiable at $x = 0$, so MVT does not apply.

Q46: (A)

$a^2 = 36 \Rightarrow a = 6$.

Q47: (A)

$\frac{1}{\ln 3}[-e^{-x}]_0^{\ln 3} = \frac{1}{\ln 3}\left(1 - \frac{1}{3}\right) = \frac{2}{3\ln 3}$.

Q48: (A)

$\frac{\ln(e^2)-\ln(1)}{e^2-1} = \frac{2}{e^2-1}$. So $\frac{1}{c} = \frac{2}{e^2-1}$, giving $c = \frac{e^2-1}{2}$.

Q49: (A)
$\frac{1}{\pi} \cdot \frac{\pi}{2} = \frac{1}{2}$.
Q50: (A)
$\frac{1}{4} \int_0^4 x\, dx = \frac{1}{4} \cdot 8 = 2$.