50 multiple-choice questions

Question 1 (Level 1) — *What is an estimate?*
A survey of 100 students finds that 40 prefer maths. What is the sample proportion $\hat{p}$?

(A) 0.4

(B) 40

(C) 0.04

(D) $\frac{100}{40} = 2.5$

Question 2 (Level 1) — *Confidence interval meaning*
A 95% confidence interval for a proportion is $(0.35, 0.45)$. What does this mean?

(A) We are 95% confident the true proportion is between 0.35 and 0.45

(B) 95% of the data lies between 0.35 and 0.45

(C) The true proportion is definitely between 0.35 and 0.45

(D) There is a 95% chance the sample proportion is in this range

Question 3 (Level 1) — *Margin of error concept*
A confidence interval is $(0.42, 0.58)$. What is the margin of error?

(A) 0.08

(B) 0.16

(C) 0.50

(D) 0.42

Question 4 (Level 1) — *Centre of CI*
A confidence interval is $(0.30, 0.50)$. What is the point estimate $\hat{p}$?

(A) 0.40

(B) 0.30

(C) 0.50

(D) 0.20

Question 5 (Level 1) — *Sample vs population*
What is the difference between p and $\hat{p}$?

(A) p is the population proportion; $\hat{p}$ is the sample proportion

(B) $\hat{p}$ is the population proportion; p is the sample proportion

(C) They are the same thing

(D) p is always larger than $\hat{p}$

Question 6 (Level 1) — *Larger sample effect*
What happens to the width of a confidence interval when the sample size increases?

(A) It decreases (narrower)

(B) It increases (wider)

(C) It stays the same

(D) It depends on $\hat{p}$

Question 7 (Level 1) — *Confidence level intuition*
Which is wider: a 90% or a 99% confidence interval (same sample)?

(A) 99% CI is wider

(B) 90% CI is wider

(C) They are the same width

(D) Cannot compare without data

Question 8 (Level 1) — *Reading a CI*
A 95% CI for the proportion of left-handers is $(0.08, 0.14)$. Is it plausible that 20% of the population is left-handed?

(A) No, 0.20 is outside the interval

(B) Yes, 0.20 is close enough

(C) Yes, any value is plausible

(D) Cannot tell from a CI

Question 9 (Level 1) — *CI structure*
A confidence interval has the form $\hat{p} \pm E$. What is E called?

(A) Margin of error

(B) Standard error

(C) Confidence level

(D) Significance level

Question 10 (Level 1) — *Interpreting 95%*
If we took 100 different samples and built a 95% CI from each, approximately how many would contain the true p?

(A) 95

(B) 100

(C) 50

(D) 5

Question 11 (Level 2) — *Simple CI calculation*
$\hat{p} = 0.6$, $n = 100$. Find the 95% CI using $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

(A) $(0.504, 0.696)$

(B) $(0.502, 0.698)$

(C) $(0.55, 0.65)$

(D) $(0.404, 0.796)$

Question 12 (Level 2) — *Standard error calculation*
$\hat{p} = 0.3$, $n = 200$. Find the standard error.

(A) 0.0324

(B) 0.021

(C) 0.0458

(D) 0.105

Question 13 (Level 2) — *Margin of error from CI*
A 95% CI is $(0.22, 0.38)$. Find $\hat{p}$ and the margin of error.

(A) $\hat{p} = 0.30$, $\mathrm{ME} = 0.08$

(B) $\hat{p} = 0.30$, $\mathrm{ME} = 0.16$

(C) $\hat{p} = 0.22$, $\mathrm{ME} = 0.08$

(D) $\hat{p} = 0.38$, $\mathrm{ME} = 0.08$

Question 14 (Level 2) — *z-value for 95%*
What z-value is used for a 95% confidence interval?

(A) 1.96

(B) 1.645

(C) 2.576

(D) 2.326

Question 15 (Level 2) — *z-value for 99%*
What z-value is used for a 99% confidence interval?

(A) 2.576

(B) 1.96

(C) 2.326

(D) 3.090

Question 16 (Level 2) — *Effect of confidence level*
A 95% CI is $(0.35, 0.55)$. Would a 99% CI from the same data be narrower or wider?

(A) Wider

(B) Narrower

(C) Same width

(D) Cannot determine

Question 17 (Level 2) — *CI from counts*
In a sample of 250 people, 75 support a policy. Find the 95% CI.

(A) $(0.243, 0.357)$

(B) $(0.271, 0.329)$

(C) $(0.200, 0.400)$

(D) $(0.225, 0.375)$

Question 18 (Level 2) — *Doubling sample size*
If the margin of error is E with sample size n, what is the approximate margin of error with sample size $4n$?

(A) $\frac{E}{2}$

(B) $\frac{E}{4}$

(C) $2E$

(D) $4E$

Question 19 (Level 2) — *Checking if p is in CI*
A 95% CI for a coin's probability of heads is $(0.42, 0.58)$. Is there evidence the coin is unfair?

(A) No, $p = 0.5$ is inside the CI

(B) Yes, the CI is not centred exactly at 0.5

(C) Yes, the CI is too wide

(D) Cannot determine from a CI

Question 20 (Level 2) — *Width of CI formula*

The width of a 95% CI for a proportion is $W = 2 \times 1.96 \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. For $\hat{p} = 0.5$ and $n = 400$, find W.

(A) 0.098

(B) 0.049

(C) 0.196

(D) 0.050

Question 21 (Level 3) — *Sample size for desired ME*

Find the minimum sample size for a 95% CI with margin of error ≤ 0.05, using $\hat{p} = 0.5$.

(A) 385

(B) 384

(C) 400

(D) 196

Question 22 (Level 3) — *99% CI calculation*

$\hat{p} = 0.45$, $n = 500$. Find the 99% CI.

(A) $(0.393, 0.507)$

(B) $(0.406, 0.494)$

(C) $(0.350, 0.550)$

(D) $(0.428, 0.472)$

Question 23 (Level 3) — *Conservative sample size*

Why do we use $\hat{p} = 0.5$ when planning sample size and p is unknown?

(A) Because $\hat{p}(1 - \hat{p})$ is maximised at 0.5, giving the largest n

(B) Because 0.5 is the most common proportion

(C) Because 0.5 minimises the sample size

(D) Because we assume equal probability

Question 24 (Level 3) — *Comparing two CIs*

Survey A: $\hat{p} = 0.52$, $n = 100$. Survey B: $\hat{p} = 0.52$, $n = 1000$. Which has a narrower 95% CI?

(A) Survey B (larger sample)

(B) Survey A (smaller sample)

(C) Both the same (same $\hat{p}$)

(D) Cannot compare

Question 25 (Level 3) — *Interpreting non-overlapping CIs*
Group A: 95% CI $(0.45, 0.55)$. Group B: 95% CI $(0.60, 0.70)$. What can you conclude?

(A) There is a significant difference between the groups

(B) There is no significant difference

(C) Group A is better than Group B

(D) The samples are too small

Question 26 (Level 3) — *Finding n from ME*
A researcher wants ME ≤ 0.03 at 95% confidence with $\hat{p} = 0.2$. Find the minimum n.

(A) 683

(B) 682

(C) 1068

(D) 385

Question 27 (Level 3) — *CI conditions*
What conditions must be met to use the normal approximation CI for a proportion?

(A) Random sample with $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$

(B) $n \geq 30$ only

(C) The population must be normal

(D) $\hat{p} = 0.5$

Question 28 (Level 3) — *Checking conditions*
$n = 50$, $\hat{p} = 0.08$. Can we construct a valid CI using the normal approximation?

(A) No, $n\hat{p} = 4 < 10$

(B) Yes, $n = 50 > 30$

(C) No, n is too small

(D) Yes, all conditions are met

Question 29 (Level 3) — *90% CI*
$\hat{p} = 0.7$, $n = 300$. Find the 90% CI. ($z_{0.95} = 1.645$)

(A) $(0.656, 0.744)$

(B) $(0.648, 0.752)$

(C) $(0.674, 0.726)$

(D) $(0.600, 0.800)$

Question 30 (Level 3) — *Misinterpretation*
Which statement is a CORRECT interpretation of a 95% CI $(0.40, 0.60)$?

(A) We are 95% confident the true proportion is between 0.40 and 0.60

(B) There is a 95% probability that p is between 0.40 and 0.60

(C) 95% of all samples have $\hat{p}$ between 0.40 and 0.60

(D) 95% of the population has a value between 0.40 and 0.60

Question 31 (Level 4) — *Full CI problem*
A poll of 600 people finds 372 support a policy. Find the 95% CI for the true proportion.

(A) $(0.581, 0.659)$

(B) $(0.600, 0.640)$

(C) $(0.560, 0.680)$

(D) $(0.571, 0.669)$

Question 32 (Level 4) — *Sample size for 99% CI*
Find n for a 99% CI with ME ≤ 0.04 using $\hat{p} = 0.5$.

(A) 1037

(B) 1036

(C) 601

(D) 2401

Question 33 (Level 4) — *Comparing 95% and 99% CIs*
$\hat{p} = 0.4$, $n = 400$. Find both the 95% and 99% CIs and compare widths.

(A) 95%: $(0.352, 0.448)$; 99%: $(0.337, 0.463)$

(B) 95%: $(0.337, 0.463)$; 99%: $(0.352, 0.448)$

(C) 95%: $(0.376, 0.424)$; 99%: $(0.352, 0.448)$

(D) Both are $(0.352, 0.448)$

Question 34 (Level 4) — *Hypothesis test via CI*
A manufacturer claims 30% defect rate. A sample of 500 finds 120 defective. Does the 95% CI support the claim?

(A) No, 0.30 is outside the CI $(0.202, 0.278)$

(B) Yes, 0.30 is close to the CI

(C) Yes, 0.24 is less than 0.30

(D) Cannot determine

Question 35 (Level 4) — *Effect of p-hat on width*
For fixed n and confidence level, at what value of $\hat{p}$ is the CI widest?

(A) $\hat{p} = 0.5$

(B) $\hat{p} = 0$

(C) $\hat{p} = 1$

(D) $\hat{p} = 0.25$

Question 36 (Level 4) — *Reverse engineering n*
A 95% CI for a proportion is $(0.42, 0.58)$. Given $\hat{p} = 0.50$, find the sample size n.

(A) 150

(B) 100

(C) 385

(D) 200

Question 37 (Level 4) — *Multiple CIs interpretation*
If 20 independent 95% CIs are constructed, how many would you expect NOT to contain the true p?

(A) 1

(B) 0

(C) 5

(D) 19

Question 38 (Level 4) — *CI width comparison*
CI_1 has $n = 100$, $\hat{p} = 0.5$ at 95%. CI_2 has $n = 400$, $\hat{p} = 0.5$ at 95%. What is the ratio of their widths?

(A) $2 : 1$

(B) $4 : 1$

(C) $1 : 2$

(D) $\sqrt{2} : 1$

Question 39 (Level 4) — *CI from raw data*

In 800 trials of a new drug, 640 patients improved. Construct a 95% CI and state whether the drug helps more than 75%.

(A) CI $(0.772, 0.828)$; yes, drug helps more than 75%

(B) CI $(0.772, 0.828)$; no, 0.75 is in the CI

(C) CI $(0.750, 0.850)$; cannot conclude

(D) CI $(0.786, 0.814)$; yes, but barely

Question 40 (Level 4) — *Sample size planning*

A political poll wants ME $\leq 2\%$ at 95% confidence. Using $\hat{p} = 0.5$, find the required sample size.

(A) 2401

(B) 2400

(C) 9604

(D) 1068

Question 41 (Level 5) — *Deriving ME formula*

Derive the formula for sample size n given desired margin of error E, confidence level z, and estimated $\hat{p}$.

(A) $n = \frac{z^2\hat{p}(1-\hat{p})}{E^2}$

(B) $n = \frac{z\hat{p}(1-\hat{p})}{E}$

(C) $n = \frac{z^2}{E^2}$

(D) $n = \frac{E^2}{z^2\hat{p}(1-\hat{p})}$

Question 42 (Level 5) — *Coverage probability*

The true proportion is $p = 0.3$. With $n = 50$, the 95% CI is $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. What is the approximate actual coverage probability?

(A) Approximately 93% (below nominal)

(B) Exactly 95%

(C) Approximately 99%

(D) Approximately 50%

Question 43 (Level 5) — *Wilson score interval*

The Wilson score interval for a proportion uses $\tilde{p} = \frac{X+z^2/2}{n+z^2}$. For $X = 10$, $n = 50$, $z = 1.96$, find $\tilde{p}$. Round to 3 d.p.

(A) 0.221

(B) 0.200

(C) 0.239

(D) 0.186

Question 44 (Level 5) — *CI interpretation in context*
A medical study reports 95% CI for recovery rate as $(0.72, 0.88)$. A rival drug claims 85% recovery. At the 5% level, is there evidence our drug differs from 85%?

(A) No evidence of difference; 0.85 is in the CI

(B) Evidence of difference; $0.85 \neq 0.80$

(C) Evidence of difference; CI is wide

(D) Cannot determine from CI alone

Question 45 (Level 5) — *Two-proportion CI*
Group A: $\hat{p}_1 = 0.6$, $n_1 = 200$. Group B: $\hat{p}_2 = 0.5$, $n_2 = 300$. Find the 95% CI for $p_1 - p_2$.

(A) $(0.012, 0.188)$

(B) $(-0.088, 0.288)$

(C) $(0.05, 0.15)$

(D) $(0.055, 0.145)$

Question 46 (Level 5) — *Required n for narrow CI*
Currently $n = 200$, $\hat{p} = 0.5$, and the 95% CI has ME ≈ 0.069. How large must n be to reduce ME to 0.02?

(A) 2401

(B) 800

(C) 1200

(D) 4802

Question 47 (Level 5) — *CI for small sample*
$n = 20$, $X = 2$ successes. Why is the Wald CI inappropriate here, and what is the issue?

(A) $n\hat{p} = 2 < 10$; normal approximation fails; use Wilson or exact method

(B) It is appropriate; $n = 20 > 10$

(C) It fails because $\hat{p} < 0.5$

(D) It fails because $n < 30$

Question 48 (Level 5) — *Simultaneous CIs*
A researcher constructs 95% CIs for 5 independent proportions. What is the probability that ALL 5 contain the true parameter?

(A) $0.95^5 \approx 0.774$

(B) 0.95

(C) $0.95 \times 5 = 4.75$

(D) $1 - 0.05^5 \approx 1$

Question 49 (Level 5) — *Finite population correction*
A population has $N = 1000$. A sample of $n = 200$ gives $\hat{p} = 0.4$. The FPC is $\sqrt{\frac{N-n}{N-1}}$. Find the adjusted 95% CI.

(A) $(0.339, 0.461)$

(B) $(0.332, 0.468)$

(C) $(0.350, 0.450)$

(D) $(0.340, 0.460)$

Question 50 (Level 5) — *Exam multi-step CI*
In a sample of 1500 voters, 810 prefer candidate A. (a) Find the 99% CI for the true proportion. (b) Is there evidence the candidate has majority support ($p > 0.5$)?

(A) CI $(0.507, 0.573)$; yes, evidence of majority

(B) CI $(0.507, 0.573)$; no, too close to 0.5

(C) CI $(0.514, 0.566)$; yes

(D) CI $(0.490, 0.590)$; no evidence

Solutions

Q1: (A)
$\hat{p} = \frac{40}{100} = 0.4$.
Q2: (A)
We are 95% confident that the true population proportion lies between 0.35 and 0.45.
Q3: (A)
Margin of error $= \frac{0.58-0.42}{2} = \frac{0.16}{2} = 0.08$.
Q4: (A)
$\hat{p} = \frac{0.30+0.50}{2} = 0.40$.
Q5: (A)
p is the true (unknown) population proportion. $\hat{p}$ is the sample proportion used to estimate p.
Q6: (A)
The width decreases (the interval becomes narrower) because the estimate is more precise.
Q7: (A)
A 99% confidence interval is wider because we need more certainty.
Q8: (A)
No. 0.20 is outside $(0.08, 0.14)$, so it is not plausible at the 95% level.
Q9: (A)
E is the margin of error.
Q10: (A)
Approximately 95 out of 100 intervals would contain the true p.
Q11: (A)
$\text{SE} = \sqrt{\frac{0.24}{100}} = \sqrt{0.0024} = 0.049$. CI: $0.6 \pm 1.96(0.049) = 0.6 \pm 0.096 = (0.504, 0.696)$.
Q12: (A)
$\text{SE} = \sqrt{\frac{0.3\times0.7}{200}} = \sqrt{\frac{0.21}{200}} = \sqrt{0.00105} \approx 0.0324$.
Q13: (A)
$\hat{p} = \frac{0.22+0.38}{2} = 0.30$. $\text{ME} = \frac{0.38-0.22}{2} = 0.08$.
Q14: (A)
The z-value for 95% CI is 1.96.
Q15: (A)
The z-value for 99% CI is 2.576.
Q16: (A)
A 99% CI would be wider. The z-value increases from 1.96 to 2.576.
Q17: (A)
$\hat{p} = 0.3$. $\text{SE} = \sqrt{\frac{0.3\times0.7}{250}} = \sqrt{0.00084} \approx 0.029$. CI: $0.3 \pm 1.96(0.029) = 0.3 \pm 0.057 = (0.243, 0.357)$.
Q18: (A)
$\text{ME} \propto \frac{1}{\sqrt{n}}$. Quadrupling n halves the ME. New $\text{ME} = \frac{E}{2}$.
Q19: (A)
$p = 0.5$ is inside $(0.42, 0.58)$, so there is no evidence the coin is unfair at the 5% significance level.
Q20: (A)
$W = 2 \times 1.96 \times 0.025 = 0.098$.
Q21: (A)
$n \geq \left(\frac{1.96}{0.05}\right)^2 \times 0.25 = 38.416^2 \times 0.25 = 1536.64 \times 0.25 = 384.16$. So $n = 385$.
Q22: (A)

SE $= \sqrt{\frac{0.45\times0.55}{500}} = \sqrt{0.000495} \approx 0.0222$. CI: $0.45 \pm 2.576(0.0222) = 0.45 \pm 0.057 = (0.393, 0.507)$.

Q23: (A)

$\hat{p}(1-\hat{p})$ is maximised at 0.5, giving the largest (most conservative) sample size needed.

Q24: (A)

Survey B has $n = 1000$, giving SE $= \sqrt{\frac{0.2496}{1000}} \approx 0.0158$ vs SE ≈ 0.0500 for Survey A. Survey B is narrower.

Q25: (A)

The intervals do not overlap, suggesting a significant difference between the two population proportions at the 95% level.

Q26: (A)

$n \geq \left(\frac{1.96}{0.03}\right)^2 \times 0.16 = 4268.44 \times 0.16 = 682.95$. $n = 683$.

Q27: (A)

The sample must be random, and both $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$ (success/failure condition).

Q28: (A)

$n\hat{p} = 50 \times 0.08 = 4 < 10$. The success condition is not met, so the normal approximation is not appropriate.

Q29: (A)

SE $= \sqrt{\frac{0.21}{300}} = \sqrt{0.0007} \approx 0.0265$. CI: $0.7 \pm 1.645(0.0265) = 0.7 \pm 0.044 = (0.656, 0.744)$.

Q30: (A)

We are 95% confident that the true population proportion lies in $(0.40, 0.60)$.

Q31: (A)

$\hat{p} = 0.62$. SE $= \sqrt{\frac{0.62\times0.38}{600}} = \sqrt{0.000393} \approx 0.0198$. CI: $0.62 \pm 1.96(0.0198) = 0.62 \pm 0.039 = (0.581, 0.659)$.

Q32: (A)

$n \geq (64.4)^2 \times 0.25 = 4147.36 \times 0.25 = 1036.84$. $n = 1037$.

Q33: (A)

SE $= \sqrt{\frac{0.24}{400}} = 0.0245$. 95% CI: $0.4 \pm 0.048 = (0.352, 0.448)$. 99% CI: $0.4 \pm 0.063 = (0.337, 0.463)$. The 99% CI is wider by about 0.030.

Q34: (A)

$\hat{p} = 0.24$. SE $= \sqrt{\frac{0.24\times0.76}{500}} \approx 0.0191$. CI: $(0.202, 0.278)$. Since $0.30 \notin (0.202, 0.278)$, the claim is not supported.

Q35: (A)

$\hat{p}(1-\hat{p})$ is maximised at $\hat{p} = 0.5$, making the CI widest.

Q36: (A)

$0.08 = 1.96 \times \frac{0.5}{\sqrt{n}}$. $\sqrt{n} = \frac{0.98}{0.08} = 12.25$. $n = 150.06$. So $n \approx 150$.

Q37: (A)

Expected number missing $= 20 \times 0.05 = 1$.

Q38: (A)

$\frac{W_1}{W_2} = \frac{\sqrt{400}}{\sqrt{100}} = \frac{20}{10} = 2$. CI_1 is twice as wide.

Q39: (A)

$\hat{p} = 0.80$. SE $= \sqrt{\frac{0.16}{800}} = \sqrt{0.0002} \approx 0.0141$. CI: $0.80 \pm 0.028 = (0.772, 0.828)$. Since $0.75 < 0.772$ (below CI), evidence suggests drug helps more than 75%.

Q40: (A)

$n \geq (98)^2 \times 0.25 = 9604 \times 0.25 = 2401$.

Q41: (A)

$E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. $E^2 = z^2 \cdot \frac{\hat{p}(1-\hat{p})}{n}$. $n = \frac{z^2\hat{p}(1-\hat{p})}{E^2}$.

Q42: (A)

For $n = 50$ and $p = 0.3$, the Wald interval has actual coverage approximately 92–94%, slightly below the nominal 95% due to the discrete nature of the binomial.

Q43: (A)

$\tilde{p} = \frac{10+1.9208}{53.8416} = \frac{11.9208}{53.8416} \approx 0.221$.

Q44: (A)

$0.85 \in (0.72, 0.88)$, so there is no evidence at the 5% level that our drug's recovery rate differs from 85%.

Q45: (A)

$\text{SE} = \sqrt{\frac{0.24}{200} + \frac{0.25}{300}} = \sqrt{0.0012 + 0.000833} = \sqrt{0.002033} \approx 0.0451$. CI: $(0.1 - 0.088, 0.1 + 0.088) = (0.012, 0.188)$.

Q46: (A)

$\frac{\sqrt{n}}{\sqrt{200}} = \frac{0.069}{0.02} = 3.45$. $\sqrt{n} = 3.45\sqrt{200} = 48.79$. $n = 2381$. Or directly: $n = \left(\frac{1.96}{0.02}\right)^2 \times 0.25 = 2401$.

Q47: (A)

$\hat{p} = 0.1$, $n\hat{p} = 2 < 10$. The normal approximation fails. The Wald CI may include negative values. An exact (Clopper–Pearson) or Wilson interval should be used instead.

Q48: (A)

$\Pr = 0.95^5 \approx 0.7738$. Only about 77% chance all five are correct.

Q49: (A)

$\text{SE} = 0.0346$. $\text{FPC} \approx 0.895$. Adjusted $\text{SE} = 0.0346 \times 0.895 = 0.0310$. CI: $0.4 \pm 1.96(0.0310) = (0.339, 0.461)$.

Q50: (A)

$\hat{p} = 0.54$. $\text{SE} = \sqrt{\frac{0.54\times0.46}{1500}} = \sqrt{0.0001656} \approx 0.01287$. CI: $0.54 \pm 2.576(0.01287) = 0.54 \pm 0.033 = (0.507, 0.573)$. Since the entire CI is above 0.5, yes, there is evidence of majority support.