50 multiple-choice questions

Question 1 (Level 1) — *Identify the degree*
What is the degree of $3x^4 - 2x^2 + 7$?

(A) 4

(B) 3

(C) 2

(D) 7

Question 2 (Level 1) — *Evaluate a polynomial*
If $f(x) = x^2 - 3x + 2$, find $f(1)$.

(A) 0

(B) 6

(C) -2

(D) 4

Question 3 (Level 1) — *Expand a product*
Expand $(x + 2)(x + 3)$.

(A) $x^2 + 5x + 6$

(B) $x^2 + 6x + 5$

(C) $x^2 + 5x + 5$

(D) $x^2 + x + 6$

Question 4 (Level 1) — *Factorise a simple quadratic*
Factorise $x^2 + 7x + 12$.

(A) $(x + 3)(x + 4)$

(B) $(x + 2)(x + 6)$

(C) $(x + 1)(x + 12)$

(D) $(x + 3)(x - 4)$

Question 5 (Level 1) — *Leading coefficient*
What is the leading coefficient of $-5x^3 + 2x^2 - x + 9$?

(A) -5

(B) 5

(C) 3

(D) 9

Question 6 (Level 1) — *Constant term*
What is the constant term of $2x^3 - 4x + 11$?

(A) 11

(B) -4

(C) 2

(D) 0

Question 7 (Level 1) — *Number of terms*
How many terms does the polynomial $4x^3 - x^2 + 6x - 1$ have?

(A) 4

(B) 3

(C) 5

(D) 6

Question 8 (Level 1) — *Zeroes from factored form*
Find the zeroes of $y = (x - 1)(x + 5)$.

(A) $x = 1$ and $x = -5$

(B) $x = -1$ and $x = 5$

(C) $x = 1$ and $x = 5$

(D) $x = -1$ and $x = -5$

Question 9 (Level 1) — *Classify a polynomial*
What type of polynomial is $3x^2 - 7x + 1$?

(A) Quadratic

(B) Linear

(C) Cubic

(D) Quartic

Question 10 (Level 1) — *Y-intercept*
Find the y-intercept of $y = 2x^2 + 3x - 5$.

(A) $(0, -5)$

(B) $(0, 5)$

(C) $(0, 3)$

(D) $(0, 2)$

Question 11 (Level 2) — *Solve by factoring*
Solve $x^2 - x - 6 = 0$.

(A) $x = 3$ or $x = -2$

(B) $x = -3$ or $x = 2$

(C) $x = 6$ or $x = -1$

(D) $x = 3$ or $x = 2$

Question 12 (Level 2) — *Difference of squares*
Factorise $4x^2 - 9$.

(A) $(2x - 3)(2x + 3)$

(B) $(4x - 3)(x + 3)$

(C) $(2x - 9)(2x + 1)$

(D) $(2x - 3)^2$

Question 13 (Level 2) — *Vertex of a parabola*
Find the coordinates of the vertex of $y = x^2 - 6x + 5$.

(A) $(3, -4)$

(B) $(-3, -4)$

(C) $(3, 4)$

(D) $(6, 5)$

Question 14 (Level 2) — *Quadratic formula*
Solve $2x^2 + 3x - 2 = 0$ using the quadratic formula.

(A) $x = \frac{1}{2}$ or $x = -2$

(B) $x = 2$ or $x = -\frac{1}{2}$

(C) $x = \frac{3}{2}$ or $x = -1$

(D) $x = 1$ or $x = -2$

Question 15 (Level 2) — *Perfect square trinomial*
Factorise $x^2 - 10x + 25$.

(A) $(x - 5)^2$

(B) $(x+5)^2$

(C) $(x-5)(x+5)$

(D) $(x-25)(x-1)$

Question 16 (Level 2) — *Axis of symmetry*
Find the axis of symmetry of $y = 2x^2 + 8x - 3$.

(A) $x = -2$

(B) $x = 2$

(C) $x = -4$

(D) $x = 4$

Question 17 (Level 2) — *Sum and product of roots*
If the roots of $x^2 - 7x + 10 = 0$ are α and β, find $\alpha + \beta$.

(A) 7

(B) 10

(C) -7

(D) 5

Question 18 (Level 2) — *Expand with a coefficient*
Expand $3(x-2)(x+4)$.

(A) $3x^2 + 6x - 24$

(B) $3x^2 + 2x - 8$

(C) $3x^2 - 6x - 24$

(D) $3x^2 + 6x + 24$

Question 19 (Level 2) — *Number of x-intercepts*
How many x-intercepts does $y = x^2 + 4x + 4$ have?

(A) 1

(B) 0

(C) 2

(D) 4

Question 20 (Level 2) — *Solve a monic quadratic*
Solve $x^2 + 2x - 15 = 0$.

(A) $x = -5$ or $x = 3$

(B) $x = 5$ or $x = -3$

(C) $x = -5$ or $x = -3$

(D) $x = 5$ or $x = 3$

Question 21 (Level 3) — *Completing the square*
Write $x^2 + 4x + 1$ in the form $(x + a)^2 + b$.

(A) $(x + 2)^2 - 3$

(B) $(x + 2)^2 + 3$

(C) $(x + 4)^2 - 15$

(D) $(x + 2)^2 - 5$

Question 22 (Level 3) — *Discriminant condition*
For $kx^2 + 4x + 1 = 0$ to have two distinct real solutions, which condition on k is needed (with $k \neq 0$)?

(A) $k < 4$, $k \neq 0$

(B) $k > 4$

(C) $k = 4$

(D) $k < 16$

Question 23 (Level 3) — *Difference of cubes*
Factorise $x^3 - 8$.

(A) $(x - 2)(x^2 + 2x + 4)$

(B) $(x - 2)(x^2 - 2x + 4)$

(C) $(x + 2)(x^2 - 2x + 4)$

(D) $(x - 2)(x^2 + 4)$

Question 24 (Level 3) — *Turning point form*
Express $y = 2x^2 - 12x + 22$ in turning point form $y = a(x - h)^2 + k$.

(A) $2(x - 3)^2 + 4$

(B) $2(x - 3)^2 - 4$

(C) $2(x + 3)^2 + 4$

(D) $2(x - 6)^2 + 22$

Question 25 (Level 3) — *Quadratic with surds*
Solve $x^2 - 6x + 2 = 0$, giving exact answers.

(A) $x = 3 \pm \sqrt{7}$

(B) $x = 3 \pm \sqrt{28}$

(C) $x = 6 \pm \sqrt{7}$

(D) $x = 3 \pm 2\sqrt{7}$

Question 26 (Level 3) — *Product of roots*
For the equation $3x^2 - 5x + 7 = 0$, find the product of the roots.

(A) $\frac{7}{3}$

(B) $\frac{5}{3}$

(C) $\frac{-5}{3}$

(D) $\frac{3}{7}$

Question 27 (Level 3) — *Quadratic inequality*
Solve $x^2 - 5x + 6 \leq 0$.

(A) $2 \leq x \leq 3$

(B) $x \leq 2$ or $x \geq 3$

(C) $-3 \leq x \leq -2$

(D) $2 < x < 3$

Question 28 (Level 3) — *Cubic y-intercept*
Find the y-intercept of $y = 2x^3 - x^2 + 4x - 7$.

(A) $(0, -7)$

(B) $(0, 7)$

(C) $(0, 4)$

(D) $(0, 2)$

Question 29 (Level 3) — *Find a quadratic from roots*
Find a monic quadratic equation with roots $x = -2$ and $x = 5$.

(A) $x^2 - 3x - 10 = 0$

(B) $x^2 + 3x - 10 = 0$

(C) $x^2 - 3x + 10 = 0$

(D) $x^2 - 7x - 10 = 0$

Question 30 (Level 3) — *Long division of polynomials*
When $x^3 + 2x^2 - 5x + 1$ is divided by $(x - 1)$, what is the remainder?

(A) -1

(B) 1

(C) 0

(D) -5

Question 31 (Level 4) — *Factor theorem and full factorisation*
Show that $(x - 2)$ is a factor of $P(x) = x^3 - 3x^2 + 4$, then find all linear factors.

(A) $(x - 2)^2(x + 1)$

(B) $(x - 2)(x + 1)(x + 2)$

(C) $(x - 2)(x^2 - x - 2)$

(D) $(x + 2)^2(x - 1)$

Question 32 (Level 4) — *Remainder theorem*
Find the remainder when $P(x) = 2x^3 + x^2 - 5x + 2$ is divided by $(x + 2)$.

(A) 0

(B) 4

(C) -16

(D) 16

Question 33 (Level 4) — *Cubic graph intercepts*
Find all x-intercepts of $y = x^3 - 4x^2 + x + 6$.

(A) $x = -1, 2, 3$

(B) $x = 1, -2, -3$

(C) $x = -1, -2, 3$

(D) $x = 1, 2, 3$

Question 34 (Level 4) — *Equating coefficients*
Given $x^3 + 6x^2 + 11x + 6 = (x + 1)(x^2 + bx + c)$, find b and c.

(A) $b = 5$, $c = 6$

(B) $b = 6$, $c = 5$

(C) $b = 5$, $c = 11$

(D) $b = 11$, $c = 6$

Question 35 (Level 4) — *Turning points of a cubic*
Find the x-coordinates of the turning points of $y = x^3 - 6x^2 + 9x + 1$.

(A) $x = 1$ and $x = 3$

(B) $x = -1$ and $x = -3$

(C) $x = 2$ and $x = 3$

(D) $x = 0$ and $x = 4$

Question 36 (Level 4) — *Repeated root condition*
For $P(x) = x^3 + ax^2 + 12x - 8$, given that $x = 2$ is a repeated root, find a.

(A) $a = -6$

(B) $a = 6$

(C) $a = -3$

(D) $a = -12$

Question 37 (Level 4) — *End behaviour of a polynomial*
Describe the end behaviour of $y = -2x^4 + 3x^2 - 1$.

(A) $y \to -\infty$ as $x \to \pm\infty$

(B) $y \to +\infty$ as $x \to \pm\infty$

(C) $y \to -\infty$ as $x \to -\infty$ and $y \to +\infty$ as $x \to +\infty$

(D) $y \to +\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to +\infty$

Question 38 (Level 4) — *Sum of cubes*
Factorise $8x^3 + 27$ completely.

(A) $(2x + 3)(4x^2 - 6x + 9)$

(B) $(2x + 3)(4x^2 + 6x + 9)$

(C) $(2x - 3)(4x^2 + 6x + 9)$

(D) $(2x + 3)(2x^2 - 6x + 9)$

Question 39 (Level 4) — *Nature of stationary point*
Determine the nature of the stationary point of $y = x^3 - 3x^2 + 3x - 1$ at $x = 1$.

(A) Stationary point of inflection

(B) Local minimum

(C) Local maximum

(D) Saddle point

Question 40 (Level 4) — *Polynomial from graph features*
A cubic $y = ax^3 + bx^2 + cx + d$ has a y-intercept of 6, and x-intercepts at $x = -1, 1, 6$. Find a.

(A) $a = 1$

(B) $a = -1$

(C) $a = 6$

(D) $a = \frac{1}{6}$

Question 41 (Level 5) — *Quartic solved by substitution*
Solve $x^4 - 5x^2 + 4 = 0$.

(A) $x = \pm 1, \pm 2$

(B) $x = 1, 4$

(C) $x = \pm 1, \pm 4$

(D) $x = \pm\sqrt{5}, \pm 2$

Question 42 (Level 5) — *Unknown coefficients from factor conditions*
The polynomial $P(x) = x^3 + ax^2 + bx - 6$ has factors $(x - 1)$ and $(x + 2)$. Find a and b.

(A) $a = 4$, $b = 1$

(B) $a = 1$, $b = 4$

(C) $a = 5$, $b = 0$

(D) $a = 3$, $b = 2$

Question 43 (Level 5) — *Polynomial with complex zeros*
How many real zeros does $P(x) = x^4 + x^2 + 1$ have?

(A) 0

(B) 2

(C) 4

(D) 1

Question 44 (Level 5) — *Polynomial division and oblique asymptote*
Find the quotient when $x^3 - 2x^2 + 3x - 5$ is divided by $x^2 + 1$.

(A) $x - 2$

(B) $x + 2$

(C) $x^2 - 2$

(D) $x - 5$

Question 45 (Level 5) — *Cubic with parameter*
For $f(x) = x^3 - 3x + k$, find the values of k for which f has three distinct real roots.

(A) $-2 < k < 2$

(B) $k > 2$

(C) $-3 < k < 3$

(D) $k < -2$ or $k > 2$

Question 46 (Level 5) — *Intersection of polynomial and line*
Find the values of m for which $y = mx$ is tangent to $y = x^3 - x$.

(A) $m = -1$

(B) $m = 0$

(C) $m = 1$

(D) $m = -3$

Question 47 (Level 5) — *Sum and product of roots of a cubic*
If α, β, γ are the roots of $2x^3 - 5x^2 + 3x - 1 = 0$, find $\alpha\beta + \alpha\gamma + \beta\gamma$.

(A) $\frac{3}{2}$

(B) $\frac{5}{2}$

(C) 3

(D) $\frac{1}{2}$

Question 48 (Level 5) — *Polynomial identity*
If $P(x) = (x-1)^3 Q(x) + ax + b$ and $P(1) = 3$, $P'(1) = 5$, find a and b.

(A) $a = 5$, $b = -2$

(B) $a = 3$, $b = 0$

(C) $a = 5$, $b = 2$

(D) $a = -2$, $b = 5$

Question 49 (Level 5) — *Quartic turning points*
How many turning points does $y = x^4 - 4x^3 + 6x^2 - 4x + 1$ have?

(A) 1

(B) 0

(C) 3

(D) 2

Question 50 (Level 5) — *Rational root theorem*

List all possible rational roots of $3x^3 - 2x^2 + 5x - 6 = 0$, then determine which (if any) is an actual root.

(A) $x = 1$

(B) $x = 2$

(C) $x = \frac{2}{3}$

(D) No rational roots

Solutions

Q1: (A)
The highest power of x is 4 (from $3x^4$), so the degree is $\boxed{4}$.
Q2: (A)
$f(1) = (1)^2 - 3(1) + 2 = 1 - 3 + 2 = \boxed{0}$.
Q3: (A)
$(x+2)(x+3) = x^2 + 3x + 2x + 6 = \boxed{x^2 + 5x + 6}$.
Q4: (A)
$3 \times 4 = 12$ and $3 + 4 = 7$, so $x^2 + 7x + 12 = \boxed{(x+3)(x+4)}$.
Q5: (A)
The highest-degree term is $-5x^3$, so the leading coefficient is $\boxed{-5}$.
Q6: (A)
The constant term is $\boxed{11}$.
Q7: (A)
The terms are $4x^3$, $-x^2$, $6x$, and -1. There are $\boxed{4}$ terms.
Q8: (A)
$x - 1 = 0 \Rightarrow x = 1$ and $x + 5 = 0 \Rightarrow x = -5$. Zeroes are $\boxed{x = 1 \text{ and } x = -5}$.
Q9: (A)
The highest power is 2, so it is a $\boxed{\text{quadratic}}$.
Q10: (A)
When $x = 0$: $y = 0 + 0 - 5 = -5$. The y-intercept is $\boxed{(0, -5)}$.
Q11: (A)
$x^2 - x - 6 = (x-3)(x+2) = 0$, so $x = 3$ or $x = -2$.
Q12: (A)
$4x^2 - 9 = (2x)^2 - 3^2 = \boxed{(2x-3)(2x+3)}$.
Q13: (A)
$x = \frac{6}{2} = 3$. $y = 9 - 18 + 5 = -4$. Vertex is $\boxed{(3, -4)}$.
Q14: (A)
$x = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4}$. So $x = \frac{1}{2}$ or $x = -2$.
Q15: (A)
$x^2 - 10x + 25 = (x-5)^2$ since $2 \times 5 = 10$ and $5^2 = 25$.
Q16: (A)
$x = -\frac{8}{2(2)} = -\frac{8}{4} = \boxed{-2}$.
Q17: (A)
$\alpha + \beta = -\frac{-7}{1} = \boxed{7}$.
Q18: (A)
$(x-2)(x+4) = x^2 + 2x - 8$. So $3(x^2 + 2x - 8) = \boxed{3x^2 + 6x - 24}$.
Q19: (A)
$\Delta = 16 - 16 = 0$. Since $\Delta = 0$, there is $\boxed{1}$ x-intercept (repeated root).
Q20: (A)
$x^2 + 2x - 15 = (x+5)(x-3) = 0$. So $x = -5$ or $x = 3$.
Q21: (A)
$x^2 + 4x + 1 = (x+2)^2 - 4 + 1 = \boxed{(x+2)^2 - 3}$.
Q22: (A)
$\Delta = 16 - 4k > 0 \Rightarrow k < 4$. With $k \neq 0$: $\boxed{k < 4,\ k \neq 0}$.
Q23: (A)
$x^3 - 8 = (x-2)(x^2 + 2x + 4)$.

Q24: (A)
$y = 2(x^2 - 6x + 9) + 22 - 18 = 2(x-3)^2 + 4$.
Q25: (A)
$x = \frac{6 \pm \sqrt{36-8}}{2} = \frac{6 \pm \sqrt{28}}{2} = 3 \pm \sqrt{7}$.
Q26: (A)
Product $= \frac{c}{a} = \frac{7}{3}$.
Q27: (A)
$(x-2)(x-3) \leq 0$. Solution: $2 \leq x \leq 3$.
Q28: (A)
$y = 2(0) - 0 + 0 - 7 = -7$. The y-intercept is $(0, -7)$.
Q29: (A)
$(x+2)(x-5) = x^2 - 3x - 10 = 0$.
Q30: (A)
$P(1) = 1 + 2 - 5 + 1 = -1$. The remainder is $\boxed{-1}$.
Q31: (A)
$P(2) = 8 - 12 + 4 = 0$ ✓. Dividing: $P(x) = (x-2)(x^2 - x - 2) = (x-2)(x-2)(x+1) = (x-2)^2(x+1)$.
Q32: (A)
$P(-2) = 2(-8) + 4 + 10 + 2 = -16 + 16 = 0$. The remainder is $\boxed{0}$.
Q33: (A)
$P(-1) = 0$, so $(x+1)$ is a factor. $P(x) = (x+1)(x^2 - 5x + 6) = (x+1)(x-2)(x-3)$. x-intercepts: $x = -1, 2, 3$.
Q34: (A)
$(x+1)(x^2 + bx + c) = x^3 + (b+1)x^2 + (c+b)x + c$. Comparing: $b + 1 = 6 \Rightarrow b = 5$; $c = 6$. So $b = 5, c = 6$.
Q35: (A)
$y' = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3) = 0$. Turning points at $x = 1$ and $x = 3$.
Q36: (A)
$P(2) = 8 + 4a + 24 - 8 = 0 \Rightarrow 4a = -24 \Rightarrow a = -6$. Check: $P'(x) = 3x^2 + 2ax + 12$, $P'(2) = 12 - 24 + 12 = 0$ ✓.
Q37: (A)
Leading term $-2x^4$: as $x \to \pm\infty$, $y \to -\infty$. Both ends point downward.
Q38: (A)
$8x^3 + 27 = (2x+3)(4x^2 - 6x + 9)$.
Q39: (A)
$y' = 3(x-1)^2$, so $y'(1) = 0$. $y'' = 6x - 6$, $y''(1) = 0$. Check sign of y': $y' \geq 0$ for all x, so it is a stationary point of inflection.
Q40: (A)
$y = a(0+1)(0-1)(0-6) = a(1)(-1)(-6) = 6a$. So $6a = 6 \Rightarrow a = 1$.
Q41: (A)
Let $u = x^2$: $(u-1)(u-4) = 0$, so $u = 1$ or $u = 4$. $x^2 = 1 \Rightarrow x = \pm 1$; $x^2 = 4 \Rightarrow x = \pm 2$. Solutions: $x \in \{-2, -1, 1, 2\}$.
Q42: (A)
$P(1) = 1 + a + b - 6 = 0 \Rightarrow a + b = 5$ (1). $P(-2) = -8 + 4a - 2b - 6 = 0 \Rightarrow 4a - 2b = 14 \Rightarrow 2a - b = 7$ (2). Adding: $3a = 12$, $a = 4$, $b = 1$.
Q43: (A)
Let $u = x^2$: $u^2 + u + 1 = 0$. $\Delta = 1 - 4 = -3 < 0$. No real values of u, hence no real zeros. Answer: $\boxed{0}$.
Q44: (A)

$\frac{x^3-2x^2+3x-5}{x^2+1} = (x-2) + \frac{2x-3}{x^2+1}$. The quotient is $x-2$.

Q45: (A)

$f'(x) = 3x^2 - 3 = 0 \Rightarrow x = \pm 1$. Local max at $x=-1$: $f(-1) = -1+3+k = 2+k > 0 \Rightarrow k > -2$. Local min at $x=1$: $f(1) = 1-3+k = k-2 < 0 \Rightarrow k < 2$. So $-2 < k < 2$.

Q46: (A)

$x^3 - (1+m)x = x(x^2-(1+m)) = 0$. For tangency at a non-zero point, we need $x^2 = 1+m$ and the line to touch the curve. Substituting $y = x^3 - x$: $y' = 3x^2 - 1 = m$. So $3x^2 - 1 = m$ and $x^2 = 1+m$. From these: $3(1+m)-1 = m \Rightarrow 2+2m = 0 \Rightarrow m = -1$. Check: also tangent at origin when $1+m = 0$, i.e. $m = -1$. So $m = -1$.

Q47: (A)

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = \frac{3}{2}$.

Q48: (A)

At $x=1$: $P(1) = 0+a+b = a+b = 3$. $P'(1) = 0+0+a = a = 5$. So $a=5$, $b=-2$.

Q49: (A)

$y = (x-1)^4$. $y' = 4(x-1)^3 = 0$ only at $x=1$. Since $y' < 0$ for $x < 1$ and $y' > 0$ for $x > 1$, there is exactly $\boxed{1}$ turning point (a minimum).

Q50: (A)

Possible: $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{3}, \pm\frac{2}{3}$. Testing $x=1$: $3-2+5-6 = 0$ ✓. So $x=1$ is a rational root.