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Question 2 [3 marks]
Solve $2\log_e(x) - \log_e(x+2) = \log_e(3)$ for x.

Question 3 [2 marks]
Differentiate $x^2 \sin(2x)$.

Question 4 [2 marks]
Find the area bounded by $y = e^x$, the x-axis, $x = 0$ and $x = 2$.

Question 5 [3 marks]
Let $X \sim \text{Bi}(n, p)$. If $E(X) = 12$ and $\text{Var}(X) = 4.8$, find n and p.

Solutions

Question 1

$R \setminus \{3\}$

Marking guide:

- Identify horizontal asymptote at $y = 3$.

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$R \setminus \{3\}$

Marking guide:

- Identify horizontal asymptote at $y = 3$.

Question 2

$x = 6$

Marking guide:

- Combine LHS: $\log_e(\frac{x^2}{x+2})$
- Equate arguments: $x^2 = 3(x + 2)$
- Solve quadratic $x^2 - 3x - 6 = 0$, reject negative solution.

Question 3

$2x\sin(2x) + 2x^2\cos(2x)$

Marking guide:

- $u = x^2, v = \sin(2x)$
- $u' = 2x, v' = 2\cos(2x)$
- Apply $u'v + uv'$

Question 4

$e^2 - 1$

Marking guide:

- Integral $\int_0^2 e^x dx$
- $[e^x]_0^2 = e^2 - e^0$

Question 5

$n = 20, p = 0.6$

Marking guide:

- $np = 12$
- $np(1 - p) = 4.8 \implies 12(1 - p) = 4.8$
- $1 - p = 0.4 \implies p = 0.6$
- $n(0.6) = 12 \implies n = 20$