50 multiple-choice questions

Question 1 (Level 1) — *Total area under pdf*
What must the total area under a probability density function equal?

(A) 1

(B) 0

(C) 100

(D) It depends on the function

Question 2 (Level 1) — *Non-negative pdf*
Which of the following is a requirement for a valid pdf $f(x)$?

(A) $f(x) \geq 0$ for all x

(B) $f(x) > 0$ for all x

(C) $f(x) \leq 1$ for all x

(D) $f(x) = 1$ for all x

Question 3 (Level 1) — *Uniform distribution area*
$f(x) = \frac{1}{4}$ for $0 \leq x \leq 4$, and 0 otherwise. Find $\Pr(1 \leq X \leq 3)$.

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{3}{4}$

(D) 2

Question 4 (Level 1) — *Point probability*
For a continuous random variable X, what is $\Pr(X = 3)$?

(A) 0

(B) $f(3)$

(C) 1

(D) It depends on the distribution

Question 5 (Level 1) — *Triangular pdf area*
A triangular pdf has base from 0 to 4 and height $\frac{1}{2}$. What is the total area?

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) 4

Question 6 (Level 1) — *Reading a pdf graph*
$f(x) = \frac{1}{5}$ for $0 \leq x \leq 5$. Find $\Pr(X > 3)$.

(A) $\frac{2}{5}$

(B) $\frac{3}{5}$

(C) $\frac{1}{5}$

(D) 3

Question 7 (Level 1) — *Pdf vs probability*
A pdf has $f(2) = 0.8$. Does this mean $\Pr(X = 2) = 0.8$?

(A) No, $f(2)$ is the density, not a probability

(B) Yes, $\Pr(X = 2) = 0.8$

(C) Yes, but only if $f(2) \leq 1$

(D) No, $\Pr(X = 2) = 1 - 0.8 = 0.2$

Question 8 (Level 1) — *Uniform expected value*
X is uniformly distributed on $[2, 8]$. What is $E(X)$?

(A) 5

(B) 4

(C) 6

(D) 3

Question 9 (Level 1) — *Can pdf exceed 1?*
Can a pdf value $f(x)$ be greater than 1?

(A) Yes, as long as the total area is 1

(B) No, probabilities cannot exceed 1

(C) Only for the normal distribution

(D) Only if the domain is less than 1 unit wide

Question 10 (Level 1) — *Simple integration area*
$f(x) = \frac{1}{3}$ for $0 \leq x \leq 3$. Find $\Pr(0 \leq X \leq 2)$.

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) 2

(D) $\frac{1}{2}$

Question 11 (Level 2) — *Finding k for valid pdf*
$f(x) = kx$ for $0 \le x \le 4$, 0 otherwise. Find k.

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{1}{16}$

(D) 8

Question 12 (Level 2) — *Probability from linear pdf*
$f(x) = \frac{x}{8}$ for $0 \le x \le 4$. Find $\Pr(X > 2)$.

(A) $\frac{3}{4}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{8}$

Question 13 (Level 2) — *Expected value with integration*
$f(x) = 2x$ for $0 \le x \le 1$. Find $E(X)$.

(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) 1

Question 14 (Level 2) — *CDF from pdf*
$f(x) = \frac{1}{2}$ for $0 \le x \le 2$. Find the CDF $F(x)$ for $0 \le x \le 2$.

(A) $F(x) = \frac{x}{2}$

(B) $F(x) = 2x$

(C) $F(x) = \frac{x^2}{2}$

(D) $F(x) = x$

Question 15 (Level 2) — *Median of uniform*
X is uniform on $[0, 10]$. Find the median.

(A) 5

(B) 10

(C) 2.5

(D) 0

Question 16 (Level 2) — *Finding k quadratic pdf*
$f(x) = kx^2$ for $0 \le x \le 3$, 0 otherwise. Find k.

(A) $\frac{1}{9}$

(B) $\frac{1}{3}$

(C) $\frac{1}{27}$

(D) 9

Question 17 (Level 2) — *Probability from quadratic*
$f(x) = 3x^2$ for $0 \le x \le 1$. Find $\Pr(X < 0.5)$.

(A) 0.125

(B) 0.5

(C) 0.75

(D) 0.25

Question 18 (Level 2) — *Complementary probability*
$f(x) = \frac{3}{8}x^2$ for $0 \le x \le 2$. Find $\Pr(X > 1)$.

(A) $\frac{7}{8}$

(B) $\frac{1}{8}$

(C) $\frac{3}{4}$

(D) $\frac{3}{8}$

Question 19 (Level 2) — *E(X) for uniform*
X is uniform on $[3, 7]$. Find $E(X)$ and $\mathrm{Var}(X)$.

(A) $E(X) = 5$, $\mathrm{Var}(X) = \frac{4}{3}$

(B) $E(X) = 5$, $\mathrm{Var}(X) = 4$

(C) $E(X) = 4$, $\mathrm{Var}(X) = \frac{4}{3}$

(D) $E(X) = 5$, $\mathrm{Var}(X) = \frac{16}{12}$

Question 20 (Level 2) — *Checking valid pdf*
Is $f(x) = 2 - 2x$ for $0 \le x \le 1$ a valid pdf?

(A) Yes, $f(x) \geq 0$ and area $= 1$

(B) No, $f(0) = 2 > 1$

(C) No, $f(1) = 0$

(D) No, the area is 2

Question 21 (Level 3) — *E(X) and Var(X) from pdf*
$f(x) = 2(1-x)$ for $0 \leq x \leq 1$. Find $E(X)$.

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{1}{4}$

Question 22 (Level 3) — *Finding the median*
$f(x) = 2x$ for $0 \leq x \leq 1$. Find the median m.

(A) $\frac{\sqrt{2}}{2} \approx 0.707$

(B) 0.5

(C) $\frac{2}{3}$

(D) $\frac{1}{4}$

Question 23 (Level 3) — *CDF from linear pdf*
$f(x) = \frac{x}{8}$ for $0 \leq x \leq 4$. Find $F(x)$.

(A) $F(x) = \frac{x^2}{16}$

(B) $F(x) = \frac{x}{8}$

(C) $F(x) = \frac{x^2}{8}$

(D) $F(x) = \frac{x}{16}$

Question 24 (Level 3) — *Variance calculation*
$f(x) = 2x$ for $0 \leq x \leq 1$. Given $E(X) = \frac{2}{3}$, find $\mathrm{Var}(X)$.

(A) $\frac{1}{18}$

(B) $\frac{1}{9}$

(C) $\frac{1}{6}$

(D) $\frac{2}{9}$

Question 25 (Level 3) — *Finding k with bounds*
$f(x) = k(4 - x^2)$ for $-2 \le x \le 2$, 0 otherwise. Find k.

(A) $\frac{3}{32}$

(B) $\frac{1}{32}$

(C) $\frac{3}{16}$

(D) $\frac{1}{4}$

Question 26 (Level 3) — *Piecewise pdf*
$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 < x \le 2 \end{cases}$. Find $\Pr(X > 1.5)$.

(A) 0.125

(B) 0.25

(C) 0.5

(D) 0.375

Question 27 (Level 3) — *E(X) for decreasing pdf*
$f(x) = 2(1 - x)$ for $0 \le x \le 1$. Find $E(X^2)$.

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{12}$

Question 28 (Level 3) — *CDF to find probability*
The CDF is $F(x) = x^3$ for $0 \le x \le 1$. Find $\Pr(0.5 < X < 0.8)$.

(A) 0.387

(B) 0.512

(C) 0.125

(D) 0.300

Question 29 (Level 3) — *Pdf from CDF*
$F(x) = 1 - e^{-2x}$ for $x \ge 0$. Find $f(x)$.

(A) $2e^{-2x}$

(B) e^{-2x}

(C) $-2e^{-2x}$

(D) $1 - 2e^{-2x}$

Question 30 (Level 3) — *Mode of pdf*
$f(x) = 6x(1-x)$ for $0 \le x \le 1$. Find the mode.

(A) $\frac{1}{2}$

(B) 0

(C) 1

(D) $\frac{1}{3}$

Question 31 (Level 4) — *E(X) and Var(X) complete*
$f(x) = 6x(1-x)$ for $0 \le x \le 1$. Find $E(X)$ and $\mathrm{Var}(X)$.

(A) $E(X) = \frac{1}{2}$, $\mathrm{Var}(X) = \frac{1}{20}$

(B) $E(X) = \frac{1}{2}$, $\mathrm{Var}(X) = \frac{1}{12}$

(C) $E(X) = \frac{1}{3}$, $\mathrm{Var}(X) = \frac{1}{20}$

(D) $E(X) = \frac{1}{2}$, $\mathrm{Var}(X) = \frac{1}{4}$

Question 32 (Level 4) — *Finding k with exponential*
$f(x) = ke^{-3x}$ for $x \ge 0$. Find k.

(A) 3

(B) $\frac{1}{3}$

(C) 1

(D) e^3

Question 33 (Level 4) — *Median from CDF*
$F(x) = 1 - (1-x)^3$ for $0 \le x \le 1$. Find the median.

(A) $1 - \frac{1}{\sqrt[3]{2}} \approx 0.206$

(B) 0.5

(C) $\frac{1}{3}$

(D) $1 - \frac{1}{\sqrt{2}} \approx 0.293$

Question 34 (Level 4) — *E(X) for exponential*
$f(x) = 2e^{-2x}$ for $x \ge 0$. Find $E(X)$.

(A) $\frac{1}{2}$

(B) 2

(C) 1

(D) $\frac{1}{4}$

Question 35 (Level 4) — *Quartiles from pdf*
$f(x) = 3x^2$ for $0 \le x \le 1$. Find the first quartile Q_1.

(A) $\frac{1}{\sqrt[3]{4}} \approx 0.630$

(B) 0.25

(C) 0.5

(D) $\frac{1}{\sqrt{4}} = 0.5$

Question 36 (Level 4) — *Conditional probability from pdf*
$f(x) = 2e^{-2x}$ for $x \ge 0$. Find $\Pr(X > 2|X > 1)$.

(A) $e^{-2} \approx 0.135$

(B) $e^{-4} \approx 0.018$

(C) $\frac{1}{2}$

(D) $1 - e^{-2} \approx 0.865$

Question 37 (Level 4) — *Two-part k and probability*
$f(x) = k(x - x^3)$ for $0 \le x \le 1$. Find k and $\Pr(X > 0.5)$.

(A) $k = 4$, $\Pr(X > 0.5) = \frac{9}{16}$

(B) $k = 4$, $\Pr(X > 0.5) = \frac{1}{2}$

(C) $k = 2$, $\Pr(X > 0.5) = \frac{9}{16}$

(D) $k = 4$, $\Pr(X > 0.5) = \frac{7}{16}$

Question 38 (Level 4) — *Var(X) for exponential*
$f(x) = 3e^{-3x}$ for $x \ge 0$. Find $\mathrm{Var}(X)$.

(A) $\frac{1}{9}$

(B) $\frac{1}{3}$

(C) 3

(D) 9

Question 39 (Level 4) — *IQR from pdf*
$f(x) = 2(1 - x)$ for $0 \le x \le 1$. $F(x) = 2x - x^2$. Find the IQR.

(A) ≈ 0.366

(B) 0.5

(C) 0.25

(D) ≈ 0.134

Question 40 (Level 4) — *E(g(X))*
$f(x) = 2x$ for $0 \le x \le 1$. Find $E(3X + 1)$.

(A) 3

(B) $\frac{7}{3}$

(C) 2

(D) $\frac{8}{3}$

Question 41 (Level 5) — *Parameter from E(X)*
$f(x) = (n + 1)x^n$ for $0 \le x \le 1$, $n > 0$. Given $E(X) = \frac{3}{4}$, find n.

(A) 2

(B) 3

(C) 1

(D) 4

Question 42 (Level 5) — *Var(X) full calculation*
$f(x) = 12x^2(1 - x)$ for $0 \le x \le 1$. Find $\mathrm{Var}(X)$.

(A) $\frac{1}{25}$

(B) $\frac{2}{25}$

(C) $\frac{1}{20}$

(D) $\frac{3}{50}$

Question 43 (Level 5) — *Median of Beta-like pdf*
$f(x) = 12x^2(1 - x)$ for $0 \le x \le 1$. The CDF is $F(x) = 4x^3 - 3x^4$. Find the median m by solving $4m^3 - 3m^4 = 0.5$. Which is closest?

(A) $m \approx 0.614$

(B) $m = 0.5$

(C) $m \approx 0.6$

(D) $m \approx 0.75$

Question 44 (Level 5) — *Transformation of variable*
X has pdf $f(x) = 2x$ for $0 \le x \le 1$. Let $Y = X^2$. Find the pdf of Y.

(A) $f_Y(y) = 1$ for $0 \le y \le 1$ (uniform)

(B) $f_Y(y) = 2\sqrt{y}$ for $0 \le y \le 1$

(C) $f_Y(y) = \frac{1}{2\sqrt{y}}$ for $0 \le y \le 1$

(D) $f_Y(y) = 2y$ for $0 \le y \le 1$

Question 45 (Level 5) — *Memoryless property*
$X \sim \text{Exp}(\lambda)$ with $f(x) = \lambda e^{-\lambda x}$. Show that $\Pr(X > s + t | X > s) = \Pr(X > t)$. What is this property called?

(A) Memoryless property

(B) Markov property

(C) Independence property

(D) Stationary property

Question 46 (Level 5) — *E(X) by parts*
$f(x) = xe^{-x}$ for $x \ge 0$. Verify this is a valid pdf and find $E(X)$.

(A) Valid pdf; $E(X) = 2$

(B) Valid pdf; $E(X) = 1$

(C) Not a valid pdf

(D) Valid pdf; $E(X) = \frac{1}{2}$

Question 47 (Level 5) — *Moment generating approach*
$f(x) = 4x^3$ for $0 \le x \le 1$. Find $E(X)$, $E(X^2)$, $E(X^3)$.

(A) $\frac{4}{5}, \frac{2}{3}, \frac{4}{7}$

(B) $\frac{3}{4}, \frac{3}{5}, \frac{1}{2}$

(C) $\frac{4}{5}, \frac{4}{6}, \frac{4}{8}$

(D) $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}$

Question 48 (Level 5) — *Mixed pdf and probability*
$f(x) = c(1 + x)^{-3}$ for $x \ge 0$. Find c and $\Pr(X > 1)$.

(A) $c = 2$, $\Pr(X > 1) = \frac{1}{4}$

(B) $c = 2$, $\Pr(X > 1) = \frac{1}{2}$

(C) $c = 3$, $\Pr(X > 1) = \frac{1}{4}$

(D) $c = 2$, $\Pr(X > 1) = \frac{3}{4}$

Question 49 (Level 5) — *Finding c and E(X) together*
$f(x) = c\sin(x)$ for $0 \le x \le \pi$. Find c and $E(X)$.

(A) $c = \frac{1}{2}$, $E(X) = \frac{\pi}{2}$

(B) $c = \frac{1}{\pi}$, $E(X) = \frac{\pi}{2}$

(C) $c = \frac{1}{2}$, $E(X) = \pi$

(D) $c = 1$, $E(X) = \frac{\pi}{2}$

Question 50 (Level 5) — *Skewness indicator*
$f(x) = 4x^3$ for $0 \le x \le 1$. Given $E(X) = \frac{4}{5}$ and median $m = \left(\frac{1}{2}\right)^{1/4} \approx 0.841$. Compare mean, median, and mode. Is the distribution left or right skewed?

(A) Left skewed (mean < median < mode)

(B) Right skewed (mean > median > mode)

(C) Symmetric

(D) Right skewed (mode < median < mean)

Solutions

Q1: (A)

The total area under any pdf must equal 1, i.e. $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

Q2: (A)

A valid pdf must satisfy $f(x) \geq 0$ for all x.

Q3: (A)

$\Pr(1 \leq X \leq 3) = (3-1) \times \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$.

Q4: (A)

For any continuous random variable, $\Pr(X = c) = 0$ for any specific value c.

Q5: (A)

Area $= \frac{1}{2} \times 4 \times \frac{1}{2} = 1$. This confirms it is a valid pdf.

Q6: (A)

$\Pr(X > 3) = 2 \times \frac{1}{5} = \frac{2}{5}$.

Q7: (A)

No. $f(2) = 0.8$ is the density at $x = 2$. $\Pr(X = 2) = 0$ for continuous variables.

Q8: (A)

$E(X) = \frac{2+8}{2} = 5$.

Q9: (A)

Yes. As long as $f(x) \geq 0$ and the total area equals 1, $f(x)$ can exceed 1.

Q10: (A)

$\Pr(0 \leq X \leq 2) = \frac{1}{3} \times 2 = \frac{2}{3}$.

Q11: (A)

$\int_0^4 kx\,dx = k\left[\frac{x^2}{2}\right]_0^4 = 8k = 1$. So $k = \frac{1}{8}$.

Q12: (A)

$\Pr(X > 2) = \int_2^4 \frac{x}{8}\,dx = \frac{1}{8}\left[\frac{x^2}{2}\right]_2^4 = \frac{1}{8}(8-2) = \frac{6}{8} = \frac{3}{4}$.

Q13: (A)

$E(X) = \int_0^1 2x^2\,dx = 2\left[\frac{x^3}{3}\right]_0^1 = \frac{2}{3}$.

Q14: (A)

$F(x) = \int_0^x \frac{1}{2}\,dt = \frac{x}{2}$ for $0 \leq x \leq 2$.

Q15: (A)

For uniform on $[0, 10]$: $F(m) = \frac{m}{10} = 0.5$, so $m = 5$.

Q16: (A)

$k\left[\frac{x^3}{3}\right]_0^3 = k \times 9 = 1$. So $k = \frac{1}{9}$.

Q17: (A)

$\Pr(X < 0.5) = 3\left[\frac{x^3}{3}\right]_0^{0.5} = [x^3]_0^{0.5} = 0.125$.

Q18: (A)

$\Pr(X > 1) = \frac{3}{8}\left[\frac{x^3}{3}\right]_1^2 = \frac{1}{8}[8-1] = \frac{7}{8}$.

Q19: (A)

$E(X) = \frac{3+7}{2} = 5$. $\text{Var}(X) = \frac{(7-3)^2}{12} = \frac{16}{12} = \frac{4}{3}$.

Q20: (A)

$f(0) = 2 \geq 0$, $f(1) = 0 \geq 0$, and $f(x) \geq 0$ on $[0, 1]$. $\int_0^1 (2-2x)\,dx = [2x - x^2]_0^1 = 2 - 1 = 1$. Yes, valid.

Q21: (A)

$E(X) = \int_0^1 (2x - 2x^2)\,dx = \left[x^2 - \frac{2x^3}{3}\right]_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$.

Q22: (A)

$\int_0^m 2x\,dx = [x^2]_0^m = m^2 = 0.5$. So $m = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.707$.

Q23: (A)
$F(x) = \frac{1}{8} \cdot \frac{x^2}{2} = \frac{x^2}{16}$ for $0 \le x \le 4$.
Q24: (A)
$E(X^2) = \int_0^1 2x^3\,dx = \frac{1}{2}$. $\text{Var}(X) = \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$.
Q25: (A)
$\int_{-2}^{2}(4 - x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = (8 - \frac{8}{3}) - (-8 + \frac{8}{3}) = \frac{32}{3}$. $k = \frac{3}{32}$.
Q26: (A)
$\Pr(X > 1.5) = \int_{1.5}^{2}(2 - x)\,dx = \left[2x - \frac{x^2}{2}\right]_{1.5}^{2} = (4 - 2) - (3 - 1.125) = 2 - 1.875 = 0.125$.
Q27: (A)
$E(X^2) = \int_0^1 (2x^2 - 2x^3)\,dx = \left[\frac{2x^3}{3} - \frac{x^4}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$.
Q28: (A)
$\Pr(0.5 < X < 0.8) = F(0.8) - F(0.5) = 0.512 - 0.125 = 0.387$.
Q29: (A)
$f(x) = F'(x) = 2e^{-2x}$ for $x \ge 0$.
Q30: (A)
$f'(x) = 6 - 12x = 0$. $x = \frac{1}{2}$. Since $f''(x) = -12 < 0$, this is a maximum.
Q31: (A)
$E(X) = \int_0^1 (6x^2 - 6x^3)\,dx = [2x^3 - \frac{3x^4}{2}]_0^1 = 2 - 1.5 = 0.5$. $E(X^2) = \int_0^1 (6x^3 - 6x^4)\,dx = \frac{6}{4} - \frac{6}{5} = 1.5 - 1.2 = 0.3$. $\text{Var}(X) = 0.3 - 0.25 = 0.05 = \frac{1}{20}$.
Q32: (A)
$k\left[-\frac{1}{3}e^{-3x}\right]_0^{\infty} = k \cdot \frac{1}{3} = 1$. So $k = 3$.
Q33: (A)
$1 - (1 - m)^3 = 0.5$. $(1 - m)^3 = 0.5$. $1 - m = 0.5^{1/3} \approx 0.794$. $m \approx 0.206$.
Q34: (A)
This is Exp(2), so $E(X) = \frac{1}{2}$.
Q35: (A)
$Q_1^3 = 0.25$. $Q_1 = 0.25^{1/3} = \frac{1}{\sqrt[3]{4}} \approx 0.630$.
Q36: (A)
$\Pr(X > a) = e^{-2a}$. $\Pr(X > 2|X > 1) = \frac{e^{-4}}{e^{-2}} = e^{-2} \approx 0.135$.
Q37: (A)
$k = 4$. $\Pr(X > 0.5) = 4\int_{0.5}^{1}(x - x^3)\,dx = 4\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0.5}^{1} = 4\left[(\frac{1}{2} - \frac{1}{4}) - (\frac{1}{8} - \frac{1}{64})\right] = 4\left[\frac{1}{4} - \frac{7}{64}\right] = 4 \times \frac{9}{64} = \frac{9}{16}$.
Q38: (A)
$\text{Var}(X) = \frac{1}{3^2} = \frac{1}{9}$.
Q39: (A)
$2x - x^2 = 0.25$: $x^2 - 2x + 0.25 = 0$, $x = 1 - \frac{\sqrt{3}}{2} \approx 0.134$. $2x - x^2 = 0.75$: $x = 1 - \frac{1}{2} = 0.5$. IQR $= 0.5 - 0.134 = 0.366$.
Q40: (A)
$E(X) = \frac{2}{3}$. $E(3X + 1) = 3 \times \frac{2}{3} + 1 = 2 + 1 = 3$.
Q41: (A)
$\frac{n+1}{n+2} = \frac{3}{4}$. $4(n + 1) = 3(n + 2)$. $4n + 4 = 3n + 6$. $n = 2$.
Q42: (A)
$E(X) = \int_0^1 12x^3(1 - x)\,dx = 12\left(\frac{1}{4} - \frac{1}{5}\right) = 12 \times \frac{1}{20} = \frac{3}{5}$. $E(X^2) = \int_0^1 12x^4(1 - x)\,dx = 12\left(\frac{1}{5} - \frac{1}{6}\right) = 12 \times \frac{1}{30} = \frac{2}{5}$. $\text{Var}(X) = \frac{2}{5} - \frac{9}{25} = \frac{1}{25}$.
Q43: (A)
Solving numerically: $4m^3 - 3m^4 = 0.5$ gives $m \approx 0.614$.
Q44: (A)

$F_X(x) = x^2$. $F_Y(y) = y$ for $0 \le y \le 1$. $f_Y(y) = 1$ for $0 \le y \le 1$. So $Y \sim \text{Uniform}(0, 1)$.

Q45: (A)

$\Pr(X > s + t | X > s) = \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}} = e^{-\lambda t} = \Pr(X > t)$. This is the memoryless property.

Q46: (A)

$\int_0^\infty xe^{-x}\,dx = \Gamma(2) = 1! = 1$ ✓. $E(X) = \int_0^\infty x^2 e^{-x}\,dx = \Gamma(3) = 2! = 2$.

Q47: (A)

$E(X) = \frac{4}{5}$, $E(X^2) = \frac{4}{6} = \frac{2}{3}$, $E(X^3) = \frac{4}{7}$.

Q48: (A)

$\frac{c}{2} = 1$, so $c = 2$. $\Pr(X > 1) = 2\int_1^\infty (1 + x)^{-3}\,dx = 2 \cdot \frac{1}{2}(1 + 1)^{-2} = (2)^{-2} = \frac{1}{4}$.

Q49: (A)

$c \times 2 = 1$, so $c = \frac{1}{2}$. $E(X) = \frac{1}{2}\int_0^\pi x \sin x\,dx = \frac{1}{2}[\sin x - x\cos x]_0^\pi = \frac{1}{2}(0 + \pi) = \frac{\pi}{2}$.

Q50: (A)

Mean = 0.8, median ≈ 0.841, mode = 1. Since mean < median < mode, the distribution is left (negatively) skewed.