50 multiple-choice questions

Question 1 (Level 1) — *Gradient of $y = x$*
Find the gradient of $y = x^2$ at $x = 3$.

(A) 6

(B) 9

(C) 3

(D) 12

Question 2 (Level 1) — *Gradient of $y = 3x$*
What is the gradient of the line $y = 3x + 1$ at any point?

(A) 3

(B) 1

(C) 4

(D) $3x$

Question 3 (Level 1) — *Derivative of x*
If $f(x) = x^3$, what is $f'(x)$?

(A) $3x^2$

(B) $3x^3$

(C) x^2

(D) $2x^3$

Question 4 (Level 1) — *Gradient at origin*
Find the gradient of $y = x^2$ at $x = 0$.

(A) 0

(B) 1

(C) 2

(D) -1

Question 5 (Level 1) — *Derivative of $5x$*
Differentiate $y = 5x^2$.

(A) $10x$

(B) $5x$

(C) $10x^2$

(D) 5

Question 6 (Level 1) — *Gradient of $y = 4x$*
Find the gradient of $y = 4x^2$ at $x = 1$.

(A) 8

(B) 4

(C) 16

(D) 2

Question 7 (Level 1) — *Tangent line slope*
The tangent to $y = x^2$ at $x = -2$ has gradient:

(A) -4

(B) 4

(C) -2

(D) 8

Question 8 (Level 1) — *Derivative of a constant*
If $y = 7$, what is $\frac{dy}{dx}$?

(A) 0

(B) 7

(C) 1

(D) -7

Question 9 (Level 1) — *Gradient of $y = 2x + 5$*
What is the gradient of the tangent to $y = 2x + 5$ at $x = 10$?

(A) 2

(B) 5

(C) 7

(D) 25

Question 10 (Level 1) — *Derivative of x*
If $f(x) = x$, what is $f'(x)$?

(A) 1

(B) x

(C) 0

(D) 2

Question 11 (Level 2) — *Equation of tangent to $y = x$*
Find the equation of the tangent to $y = x^2$ at $(2, 4)$.

(A) $y = 4x - 4$

(B) $y = 2x - 4$

(C) $y = 4x + 4$

(D) $y = 4x$

Question 12 (Level 2) — *Tangent to cubic at origin*
Find the equation of the tangent to $y = x^3$ at the origin.

(A) $y = 0$

(B) $y = x$

(C) $y = 3x$

(D) $x = 0$

Question 13 (Level 2) — *Gradient of 2x*
Find the gradient of $y = 2x^3$ at $x = -1$.

(A) 6

(B) -6

(C) -2

(D) 12

Question 14 (Level 2) — *Point on tangent line*
The tangent to $y = x^2$ at $x = 1$ passes through which point?

(A) $(0, -1)$

(B) $(0, 1)$

(C) $(2, 2)$

(D) $(1, 0)$

Question 15 (Level 2) — *Tangent to $y = x + 3x$*
Find the equation of the tangent to $y = x^2 + 3x$ at $x = 1$.

(A) $y = 5x - 1$

(B) $y = 5x + 1$

(C) $y = 5x - 4$

(D) $y = 3x + 1$

Question 16 (Level 2) — *Normal gradient*
If the tangent to a curve has gradient 4, what is the gradient of the normal?

(A) $-\frac{1}{4}$

(B) $\frac{1}{4}$

(C) -4

(D) 4

Question 17 (Level 2) — *Horizontal tangent*
At what x-value does $y = x^2 - 6x + 2$ have a horizontal tangent?

(A) $x = 3$

(B) $x = 6$

(C) $x = -3$

(D) $x = 2$

Question 18 (Level 2) — *Tangent y-intercept*
Find the y-intercept of the tangent to $y = x^2$ at $x = 3$.

(A) -9

(B) 9

(C) -3

(D) 6

Question 19 (Level 2) — *Derivative of polynomial*
If $f(x) = 3x^2 - 2x + 1$, find $f'(2)$.

(A) 10

(B) 9

(C) 12

(D) 8

Question 20 (Level 2) — *Tangent at negative x*
Find the equation of the tangent to $y = x^2$ at $x = -1$.

(A) $y = -2x - 1$

(B) $y = -2x + 1$

(C) $y = 2x - 1$

(D) $y = -x - 1$

Question 21 (Level 3) — *Equation of normal*
Find the equation of the normal to $y = x^2$ at $(2, 4)$.

(A) $y = -\frac{x}{4} + \frac{9}{2}$

(B) $y = -4x + 12$

(C) $y = \frac{x}{4} + \frac{9}{2}$

(D) $y = -\frac{x}{4} + 4$

Question 22 (Level 3) — *Tangent to cubic*
Find the equation of the tangent to $y = x^3 - 3x$ at $x = 2$.

(A) $y = 9x - 16$

(B) $y = 9x - 2$

(C) $y = 9x + 16$

(D) $y = 3x - 4$

Question 23 (Level 3) — *Where gradient equals 8*
Find the x-value where the gradient of $y = x^3 + x$ equals 4.

(A) $x = 1$ or $x = -1$

(B) $x = 1$

(C) $x = \pm 2$

(D) $x = \frac{4}{3}$

Question 24 (Level 3) — *Normal to $y = x$*
Find the gradient of the normal to $y = \sqrt{x}$ at $x = 4$.

(A) -4

(B) 4

(C) $-\frac{1}{4}$

(D) $\frac{1}{4}$

Question 25 (Level 3) — *Tangent parallel to line*
Find the point on $y = x^2$ where the tangent is parallel to $y = 6x + 1$.

(A) $(3, 9)$

(B) $(6, 36)$

(C) $(3, 6)$

(D) $(1, 1)$

Question 26 (Level 3) — *Tangent to 1/x*
Find the equation of the tangent to $y = \frac{1}{x}$ at $x = 1$.

(A) $y = -x + 2$

(B) $y = -x + 1$

(C) $y = x + 2$

(D) $y = -x$

Question 27 (Level 3) — *Tangent intersection*
The tangent to $y = x^2$ at $x = a$ has y-intercept -9. Find a (where $a > 0$).

(A) 3

(B) 9

(C) $\sqrt{3}$

(D) -3

Question 28 (Level 3) — *Tangent to quadratic*
Find the equation of the tangent to $y = 2x^2 - x + 3$ at $x = 1$.

(A) $y = 3x + 1$

(B) $y = 3x - 1$

(C) $y = 4x + 1$

(D) $y = 3x + 4$

Question 29 (Level 3) — *Normal equation*
Find the equation of the normal to $y = x^3$ at $x = 1$.

(A) $y = -\frac{x}{3} + \frac{4}{3}$

(B) $y = -3x + 4$

(C) $y = -\frac{x}{3} + 1$

(D) $y = \frac{x}{3} + \frac{4}{3}$

Question 30 (Level 3) — *Two tangent lines*
How many points on $y = x^3 - 3x$ have a horizontal tangent?

(A) 2

(B) 1

(C) 3

(D) 0

Question 31 (Level 4) — *Tangent to e*
Find the equation of the tangent to $y = e^x$ at $x = 0$.

(A) $y = x + 1$

(B) $y = ex$

(C) $y = x$

(D) $y = e^x$

Question 32 (Level 4) — *Tangent to sin(x)*
Find the gradient of the tangent to $y = \sin(x)$ at $x = \frac{\pi}{3}$.

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) 1

(D) $-\frac{1}{2}$

Question 33 (Level 4) — *Tangent to ln(x)*
Find the equation of the tangent to $y = \ln(x)$ at $x = e$.

(A) $y = \frac{x}{e}$

(B) $y = \frac{x}{e} + 1$

(C) $y = ex - 1$

(D) $y = \frac{1}{e}x - 1$

Question 34 (Level 4) — *Tangent using chain rule*
Find the gradient of the tangent to $y = (2x + 1)^3$ at $x = 0$.

(A) 6

(B) 3

(C) 12

(D) 2

Question 35 (Level 4) — *Tangent to e*
Find the equation of the tangent to $y = e^{2x}$ at $x = 0$.

(A) $y = 2x + 1$

(B) $y = 2x$

(C) $y = x + 1$

(D) $y = 2ex + 1$

Question 36 (Level 4) — *Normal to e*
Find the equation of the normal to $y = e^x$ at $x = 1$.

(A) $y = -\frac{x}{e} + \frac{1+e^2}{e}$

(B) $y = -ex + e^2 + 1$

(C) $y = -\frac{x}{e} + e$

(D) $y = \frac{x}{e} + e$

Question 37 (Level 4) — *Tangent to product*
Find the gradient of the tangent to $y = xe^x$ at $x = 1$.

(A) $2e$

(B) e

(C) e^2

(D) $3e$

Question 38 (Level 4) — *Tangent to cos(x)*
Find the equation of the tangent to $y = \cos(x)$ at $x = \frac{\pi}{2}$.

(A) $y = -x + \frac{\pi}{2}$

(B) $y = x - \frac{\pi}{2}$

(C) $y = -x$

(D) $y = -x + \pi$

Question 39 (Level 4) — *Tangent where gradient = e*
Find the x-value where the tangent to $y = e^x$ has gradient e^2.

(A) $x = 2$

(B) $x = e^2$

(C) $x = e$

(D) $x = \ln 2$

Question 40 (Level 4) — *Tangent to x ln(x)*
Find the gradient of the tangent to $y = x\ln(x)$ at $x = 1$.

(A) 1

(B) 0

(C) 2

(D) e

Question 41 (Level 5) — *Tangent from external point*
A tangent to $y = x^2$ passes through the point $(0, -4)$. Find the positive x-coordinate of the point of tangency.

(A) 2

(B) 4

(C) $\sqrt{2}$

(D) 1

Question 42 (Level 5) — *Tangent to e through origin*
The tangent to $y = e^x$ at $x = a$ passes through the origin. Find a.

(A) 1

(B) 0

(C) e

(D) $\frac{1}{e}$

Question 43 (Level 5) — *Common tangent*
Find the value of k such that $y = x^2$ and $y = k - x^2$ share a common tangent at the same point.

(A) 0

(B) 1

(C) 2

(D) -1

Question 44 (Level 5) — *Tangent to implicit curve*
Find the gradient of the tangent to $x^2 + y^2 = 25$ at the point $(3, 4)$.

(A) $-\frac{3}{4}$

(B) $\frac{3}{4}$

(C) $-\frac{4}{3}$

(D) $\frac{4}{3}$

Question 45 (Level 5) — *Tangent to parametric curve*
If $f(x) = x^3 - 6x^2 + 9x$, find the values of x where the tangent is parallel to the x-axis.

(A) $x = 1$ and $x = 3$

(B) $x = 2$ and $x = 3$

(C) $x = 0$ and $x = 3$

(D) $x = 1$ and $x = 4$

Question 46 (Level 5) — *Tangent area triangle*
The tangent to $y = \frac{1}{x}$ at $x = a$ $(a > 0)$ forms a triangle with the axes. Find the area in terms of a.

(A) 2

(B) $2a$

(C) $\frac{1}{a}$

(D) 4

Question 47 (Level 5) — *Tangent to sin at general point*
The tangent to $y = \sin(x)$ at $x = \frac{\pi}{6}$ meets the x-axis at what value of x?

(A) $\frac{\pi}{6} - \frac{1}{\sqrt{3}}$

(B) $\frac{\pi}{6} + \frac{1}{\sqrt{3}}$

(C) $\frac{\pi}{6}$

(D) $-\frac{\pi}{6}$

Question 48 (Level 5) — *Tangent meets curve again*
The tangent to $y = x^3$ at $x = 1$ meets the curve again at what x-value?

(A) -2

(B) 2

(C) -1

(D) -3

Question 49 (Level 5) — *Number of tangents from point*
How many tangent lines to $y = x^3 - x$ pass through the point $(0, 0)$?

(A) 1

(B) 2

(C) 3

(D) 0

Question 50 (Level 5) — *Tangent to ln(x) through origin*
Find the value of x at which the tangent to $y = \ln(x)$ passes through the origin.

(A) e

(B) 1

(C) e^2

(D) $\frac{1}{e}$

Solutions

Q1: (A)
$\frac{dy}{dx} = 2x$. At $x = 3$: gradient $= 2(3) = 6$.
Q2: (A)
The gradient of $y = 3x + 1$ is $m = 3$.
Q3: (A)
$f'(x) = 3x^2$.
Q4: (A)
$\frac{dy}{dx} = 2x$. At $x = 0$: gradient $= 0$.
Q5: (A)
$\frac{dy}{dx} = 10x$.
Q6: (A)
$\frac{dy}{dx} = 8x$. At $x = 1$: gradient $= 8$.
Q7: (A)
$\frac{dy}{dx} = 2(-2) = -4$.
Q8: (A)
$\frac{dy}{dx} = 0$.
Q9: (A)
Gradient is 2 everywhere since it is a linear function.
Q10: (A)
$f'(x) = 1$.
Q11: (A)
$f'(x) = 2x$, so $m = 4$. Tangent: $y - 4 = 4(x - 2)$, giving $y = 4x - 4$.
Q12: (A)
$f'(0) = 0$. The tangent at $(0, 0)$ with gradient 0 is $y = 0$.
Q13: (A)
$\frac{dy}{dx} = 6x^2$. At $x = -1$: $6(-1)^2 = 6$.
Q14: (A)
$f'(1) = 2$, $f(1) = 1$. Tangent: $y - 1 = 2(x - 1) \Rightarrow y = 2x - 1$. At $x = 0$, $y = -1$.
Q15: (A)
$f'(1) = 5$, $f(1) = 4$. Tangent: $y - 4 = 5(x - 1)$, i.e. $y = 5x - 1$.
Q16: (A)
$m_{\text{normal}} = -\frac{1}{4}$.
Q17: (A)
$\frac{dy}{dx} = 2x - 6 = 0 \Rightarrow x = 3$.
Q18: (A)
$y = 6x - 9$. The y-intercept is -9.
Q19: (A)
$f'(x) = 6x - 2$. $f'(2) = 12 - 2 = 10$.
Q20: (A)
$y - 1 = -2(x + 1)$, giving $y = -2x - 1$.
Q21: (A)
$m_{\text{tan}} = 4$, $m_{\text{norm}} = -\frac{1}{4}$. Normal: $y - 4 = -\frac{1}{4}(x - 2)$, i.e. $y = -\frac{x}{4} + \frac{9}{2}$.
Q22: (A)
$f'(2) = 9$. Tangent: $y - 2 = 9(x - 2)$, i.e. $y = 9x - 16$.
Q23: (A)
$3x^2 + 1 = 4 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$.
Q24: (A)
$f'(4) = \frac{1}{4}$. Normal gradient $= -4$.

Q25: (A)
$f'(x) = 2x = 6 \Rightarrow x = 3$. Point is $(3, 9)$.
Q26: (A)
$f'(1) = -1$, $f(1) = 1$. Tangent: $y = -x + 2$.
Q27: (A)
Tangent at (a, a^2): $y = 2ax - a^2$. y-intercept $= -a^2 = -9$, so $a = 3$.
Q28: (A)
$f'(1) = 3$. Tangent: $y - 4 = 3(x - 1)$, i.e. $y = 3x + 1$.
Q29: (A)
Normal: $y - 1 = -\frac{1}{3}(x - 1)$, i.e. $y = -\frac{x}{3} + \frac{4}{3}$.
Q30: (A)
$3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$. There are 2 points.
Q31: (A)
$f'(0) = 1$, $f(0) = 1$. Tangent: $y = x + 1$.
Q32: (A)
$f'\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.
Q33: (A)
$f'(e) = \frac{1}{e}$, $f(e) = 1$. Tangent: $y = \frac{x}{e}$.
Q34: (A)
$\frac{dy}{dx} = 6(2x + 1)^2$. At $x = 0$: $6(1)^2 = 6$.
Q35: (A)
$f'(0) = 2$, $f(0) = 1$. Tangent: $y = 2x + 1$.
Q36: (A)
Normal: $y - e = -\frac{1}{e}(x - 1)$, i.e. $y = -\frac{x}{e} + \frac{1}{e} + e$.
Q37: (A)
$y' = e^x(1 + x)$. At $x = 1$: $2e$.
Q38: (A)
$f'(\frac{\pi}{2}) = -1$, $f(\frac{\pi}{2}) = 0$. Tangent: $y = -(x - \frac{\pi}{2}) = -x + \frac{\pi}{2}$.
Q39: (A)
$e^x = e^2 \Rightarrow x = 2$.
Q40: (A)
$y' = \ln(x) + 1$. At $x = 1$: $\ln(1) + 1 = 1$.
Q41: (A)
Tangent at (a, a^2): $y = 2ax - a^2$. Through $(0, -4)$: $-a^2 = -4$, so $a = 2$.
Q42: (A)
$0 - e^a = e^a(0 - a) \Rightarrow -e^a = -ae^a \Rightarrow a = 1$.
Q43: (A)
From $2x = -2x$: $x = 0$. Then $0 = k - 0$, so $k = 0$. But both curves must meet, so actually at $x = 0$: $0 = k$. Thus $k = 0$.
Q44: (A)
$\frac{dy}{dx} = -\frac{x}{y} = -\frac{3}{4}$.
Q45: (A)
$3(x^2 - 4x + 3) = 0 \Rightarrow (x - 1)(x - 3) = 0$, so $x = 1$ or $x = 3$.
Q46: (A)
Tangent at $(a, \frac{1}{a})$: $y = -\frac{x}{a^2} + \frac{2}{a}$. x-int: $2a$, y-int: $\frac{2}{a}$. Area $= \frac{1}{2}(2a)(\frac{2}{a}) = 2$.
Q47: (A)
Set $y = 0$: $-\frac{1}{2} = \frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$, so $x = \frac{\pi}{6} - \frac{1}{\sqrt{3}}$.
Q48: (A)
$x^3 - 3x + 2 = 0 \Rightarrow (x - 1)^2(x + 2) = 0$. The other intersection is $x = -2$.
Q49: (A)

Through $(0,0)$: $-(a^3-a)=(3a^2-1)(-a)$, so $-a^3+a=-3a^3+a$, giving $2a^3=0$, $a=0$. But also check: the origin is ON the curve, and the tangent at $(0,0)$ has $m=-1$. There are $\boxed{1}$ tangent line(s).

Q50: (A)

$-\ln a=-1$, so $\ln a=1$, giving $a=e$.