50 multiple-choice questions

Question 1 (Level 1) — *Bell curve shape*
What shape does a normal distribution curve have?

(A) Symmetric, bell-shaped

(B) Skewed to the right

(C) Uniform (flat)

(D) U-shaped

Question 2 (Level 1) — *Mean and centre*
A normal distribution has mean $\mu = 50$. Where is the centre of the distribution?

(A) At $x = 50$

(B) At $x = 0$

(C) At $x = 100$

(D) Cannot be determined

Question 3 (Level 1) — *68% rule basic*
Heights are normally distributed with mean 170 cm and standard deviation 5 cm. Approximately what percentage of people have heights between 165 cm and 175 cm?

(A) 68%

(B) 95%

(C) 50%

(D) 99.7%

Question 4 (Level 1) — *95% rule basic*
Scores are normally distributed with $\mu = 100$, $\sigma = 10$. What range contains approximately 95% of scores?

(A) 80 to 120

(B) 90 to 110

(C) 70 to 130

(D) 0 to 200

Question 5 (Level 1) — *Symmetry property*
For a normal distribution, $\Pr(X > \mu) =$?

(A) 0.5

(B) 1

(C) 0.68

(D) 0

Question 6 (Level 1) — *Standard deviation effect*
Two normal distributions have the same mean but $\sigma_1 = 2$ and $\sigma_2 = 5$. Which is taller and narrower?

(A) $\sigma_1 = 2$ (taller and narrower)

(B) $\sigma_2 = 5$ (taller and narrower)

(C) Both have the same shape

(D) Cannot tell without the mean

Question 7 (Level 1) — *Total area under curve*
What is the total area under a normal distribution curve?

(A) 1

(B) 100

(C) ∞

(D) 0.5

Question 8 (Level 1) — *99.7% rule*
Test scores: $\mu = 75$, $\sigma = 8$. Almost all (99.7%) scores are between what values?

(A) 51 to 99

(B) 59 to 91

(C) 67 to 83

(D) 0 to 150

Question 9 (Level 1) — *Above or below mean*
For $X \sim N(60, 5^2)$, is a score of 55 above or below the mean?

(A) Below the mean by 1 standard deviation

(B) Above the mean by 1 standard deviation

(C) At the mean

(D) Below the mean by 5 standard deviations

Question 10 (Level 1) — *Identifying normal data*
Which data set is most likely normally distributed?

(A) Heights of adult women

(B) Number of siblings

(C) Household income

(D) Number of pets owned

Question 11 (Level 2) — *Z-score calculation*
$X \sim N(50, 8^2)$. Find the z-score for $x = 66$.

(A) 2

(B) 16

(C) -2

(D) 0.5

Question 12 (Level 2) — *Negative z-score*
$X \sim N(100, 15^2)$. Find the z-score for $x = 70$.

(A) -2

(B) 2

(C) -30

(D) -0.5

Question 13 (Level 2) — *Using 68-95-99.7 above*
$X \sim N(200, 25^2)$. Find $\Pr(X > 250)$ using the empirical rule.

(A) 0.025

(B) 0.05

(C) 0.16

(D) 0.975

Question 14 (Level 2) — *X from z-score*
$X \sim N(80, 12^2)$. A student has z-score = 1.5. What is their raw score?

(A) 98

(B) 92

(C) 68

(D) 93.5

Question 15 (Level 2) — *Comparing z-scores*
Alice scored 85 on a test with $\mu = 70$, $\sigma = 10$. Bob scored 90 on a test with $\mu = 80$, $\sigma = 5$. Who performed better relative to their class?

(A) Bob ($z = 2$ vs $z = 1.5$)

(B) Alice ($z = 1.5$ vs $z = 2$)

(C) Alice (higher raw score)

(D) Equal performance

Question 16 (Level 2) — *Middle 68%*
$X \sim N(500, 100^2)$. The middle 68% of values lie between which two values?

(A) 400 to 600

(B) 300 to 700

(C) 450 to 550

(D) 200 to 800

Question 17 (Level 2) — *Percentile approximation*
Using the empirical rule for $X \sim N(\mu, \sigma^2)$, approximately what percentile is $\mu + 2\sigma$?

(A) 97.5th percentile

(B) 95th percentile

(C) 84th percentile

(D) 99.85th percentile

Question 18 (Level 2) — *Standard normal*
What are the mean and standard deviation of the standard normal distribution Z?

(A) $\mu = 0$, $\sigma = 1$

(B) $\mu = 1$, $\sigma = 0$

(C) $\mu = 0$, $\sigma = 0$

(D) $\mu = 1$, $\sigma = 1$

Question 19 (Level 2) — *Tail probability symmetry*
If $\Pr(Z < -1.5) = 0.0668$, what is $\Pr(Z > 1.5)$?

(A) 0.0668

(B) 0.9332

(C) 0.1336

(D) 0.5000

Question 20 (Level 2) — *Between two z-scores*
If $\Pr(Z < 1) = 0.8413$ and $\Pr(Z < -1) = 0.1587$, find $\Pr(-1 < Z < 1)$.

(A) 0.6826

(B) 0.8413

(C) 0.3174

(D) 1.0000

Question 21 (Level 3) — *CDF table lookup*
$X \sim N(40, 6^2)$. Find $\Pr(X < 49)$ given that $\Pr(Z < 1.5) = 0.9332$.

(A) 0.9332

(B) 0.0668

(C) 0.8664

(D) 0.5000

Question 22 (Level 3) — *Probability between two values*
$X \sim N(50, 10^2)$. Find $\Pr(35 < X < 65)$. Use: $\Pr(Z < 1.5) = 0.9332$, $\Pr(Z < -1.5) = 0.0668$.

(A) 0.8664

(B) 0.9332

(C) 0.0668

(D) 0.6826

Question 23 (Level 3) — *Inverse normal basic*
$X \sim N(100, 15^2)$. Find the value x such that $\Pr(X < x) = 0.8413$. Given: $\Pr(Z < 1) = 0.8413$.

(A) 115

(B) 85

(C) 130

(D) 101

Question 24 (Level 3) — *Finding sigma*
$X \sim N(60, \sigma^2)$. Given $\Pr(X > 72) = 0.0228$ and $\Pr(Z > 2) = 0.0228$. Find σ.

(A) 6

(B) 12

(C) 2

(D) 36

Question 25 (Level 3) — *Finding the mean*
$X \sim N(\mu, 4^2)$. Given $\Pr(X < 30) = 0.9772$ and $\Pr(Z < 2) = 0.9772$. Find μ.

(A) 22

(B) 38

(C) 26

(D) 30

Question 26 (Level 3) — *Upper tail probability*
$X \sim N(250, 30^2)$. Find $\Pr(X > 220)$. Given $\Pr(Z < 1) = 0.8413$.

(A) 0.8413

(B) 0.1587

(C) 0.6826

(D) 0.3413

Question 27 (Level 3) — *Percentage in range*
IQ scores follow $N(100, 15^2)$. What percentage of people have IQ between 85 and 130? Use: $\Pr(Z < 1) = 0.8413$, $\Pr(Z < 2) = 0.9772$.

(A) 81.85%

(B) 95%

(C) 68%

(D) 84.13%

Question 28 (Level 3) — *Standardisation practice*
$X \sim N(30, 4^2)$. Express $\Pr(26 < X < 38)$ in terms of Z.

(A) $\Pr(-1 < Z < 2)$

(B) $\Pr(-2 < Z < 1)$

(C) $\Pr(0 < Z < 2)$

(D) $\Pr(-1 < Z < 1)$

Question 29 (Level 3) — *Symmetric interval*
$X \sim N(0, 1)$. Find c such that $\Pr(-c < Z < c) = 0.95$.

(A) 1.96

(B) 2.00

(C) 1.645

(D) 2.576

Question 30 (Level 3) — *Application: quality control*
Bottles are filled to $N(500, 8^2)$ mL. Bottles with less than 484 mL are rejected. What proportion is rejected? Given $\Pr(Z < -2) = 0.0228$.

(A) 2.28%

(B) 5%

(C) 0.15%

(D) 16%

Question 31 (Level 4) — *Inverse normal for percentile*
Exam marks follow $N(65, 12^2)$. The top 10% receive an A. Find the minimum mark for an A. Given $\Pr(Z < 1.2816) = 0.90$.

(A) 80.4

(B) 77.0

(C) 88.4

(D) 71.3

Question 32 (Level 4) — *Two unknowns*
$X \sim N(\mu, \sigma^2)$. Given $\Pr(X < 40) = 0.1587$ and $\Pr(X < 70) = 0.9772$. Find μ and σ. Use: $z_{0.1587} = -1$, $z_{0.9772} = 2$.

(A) $\mu = 50$, $\sigma = 10$

(B) $\mu = 55$, $\sigma = 15$

(C) $\mu = 40$, $\sigma = 10$

(D) $\mu = 50$, $\sigma = 15$

Question 33 (Level 4) — *Normal approximation to binomial*
$X \sim \text{Bi}(100, 0.4)$. Use a normal approximation to find $\Pr(X \leq 45)$. Given $\Pr(Z < 1.02) \approx 0.846$.

(A) 0.846

(B) 0.500

(C) 0.154

(D) 0.950

Question 34 (Level 4) — *Conditional normal probability*
$X \sim N(50, 10^2)$. Find $\Pr(X > 60 | X > 45)$. Given $\Pr(Z < 1) = 0.8413$, $\Pr(Z < -0.5) = 0.3085$.

(A) 0.230

(B) 0.159

(C) 0.692

(D) 0.533

Question 35 (Level 4) — *Symmetric bounds from probability*
$X \sim N(200, 20^2)$. Find a and b (symmetric about the mean) such that $\Pr(a < X < b) = 0.99$. Given $z_{0.995} = 2.576$.

(A) 148.5 to 251.5

(B) 160 to 240

(C) 140 to 260

(D) 167.1 to 232.9

Question 36 (Level 4) — *Linear transformation*
If $X \sim N(10, 4)$, find the distribution of $Y = 3X + 5$.

(A) $N(35, 36)$

(B) $N(35, 12)$

(C) $N(35, 4)$

(D) $N(30, 36)$

Question 37 (Level 4) — *Probability of range with CAS*
$X \sim N(72, 8^2)$. Find $\Pr(60 < X < 80)$. Given $\Pr(Z < 1) = 0.8413$, $\Pr(Z < -1.5) = 0.0668$.

(A) 0.7745

(B) 0.8413

(C) 0.6826

(D) 0.9082

Question 38 (Level 4) — *Inverse normal two-tailed*
Find c such that $\Pr(|Z| > c) = 0.01$. Given $z_{0.995} = 2.576$.

(A) 2.576

(B) 1.960

(C) 2.326

(D) 3.090

Question 39 (Level 4) — *Application: manufacturing*
Rods have length $X \sim N(20, 0.5^2)$ cm. Rods outside 19 to 21 cm are rejected. Find the rejection rate. Given $\Pr(Z < 2) = 0.9772$.

(A) 4.56%

(B) 2.28%

(C) 5%

(D) 0.26%

Question 40 (Level 4) — *Quartiles of normal*
Find the first quartile (Q_1) of $X \sim N(50, 10^2)$. Given $z_{0.25} = -0.6745$.

(A) 43.3

(B) 40.0

(C) 47.5

(D) 56.7

Question 41 (Level 5) — *Simultaneous equations for parameters*
$X \sim N(\mu, \sigma^2)$. $\Pr(X < 20) = 0.10$ and $\Pr(X > 80) = 0.05$. Find μ and σ. Given $z_{0.10} = -1.282$, $z_{0.95} = 1.645$.

(A) $\mu \approx 46.3$, $\sigma \approx 20.5$

(B) $\mu = 50$, $\sigma = 20$

(C) $\mu \approx 40$, $\sigma \approx 25$

(D) $\mu \approx 46.3$, $\sigma \approx 15$

Question 42 (Level 5) — *Sum of normals*
$X \sim N(30, 16)$ and $Y \sim N(20, 9)$ are independent. Find $\Pr(X + Y > 60)$. Given $\Pr(Z < 2) = 0.9772$.

(A) 0.0228

(B) 0.9772

(C) 0.0456

(D) 0.1587

Question 43 (Level 5) — *Difference of normals*
$X \sim N(100, 25)$ and $Y \sim N(90, 16)$ are independent. Find $\Pr(X - Y > 20)$. Given $\Pr(Z < 1.56) \approx 0.9406$.

(A) 0.059

(B) 0.941

(C) 0.023

(D) 0.159

Question 44 (Level 5) — *Sample mean distribution*
$X \sim N(50, 100)$. A sample of $n = 25$ is taken. Find $\Pr(\bar{X} > 54)$. Given $\Pr(Z < 2) = 0.9772$.

(A) 0.0228

(B) 0.3446

(C) 0.1587

(D) 0.0013

Question 45 (Level 5) — *Normal with calculus*
The pdf of $X \sim N(0, 1)$ is $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. At what value of x does $f(x)$ have inflection points?

(A) $x = \pm 1$

(B) $x = 0$

(C) $x = \pm 2$

(D) $x = \pm \frac{1}{\sqrt{2}}$

Question 46 (Level 5) — *Truncated normal expectation*
$Z \sim N(0, 1)$. Find $E(Z|Z > 0)$.

(A) $\sqrt{\frac{2}{\pi}} \approx 0.798$

(B) 0.5

(C) 1

(D) $\frac{1}{\sqrt{2\pi}} \approx 0.399$

Question 47 (Level 5) — *Maximum likelihood*
A random sample from $N(\mu, 9)$ gives values 12, 15, 18, 14, 16. Find the maximum likelihood estimate of μ.

(A) 15

(B) 14

(C) 16

(D) 75

Question 48 (Level 5) — *Chi-squared connection*
If Z_1, Z_2, Z_3 are independent standard normal, what is $E(Z_1^2 + Z_2^2 + Z_3^2)$?

(A) 3

(B) 1

(C) $\sqrt{3}$

(D) 9

Question 49 (Level 5) — *Joint probability problem*
$X \sim N(0,1)$, $Y \sim N(0,1)$ independent. Find $\Pr(X^2 + Y^2 < 1)$.

(A) $1 - e^{-1/2} \approx 0.394$

(B) 0.683

(C) $\frac{\pi}{4} \approx 0.785$

(D) 0.500

Question 50 (Level 5) — *Multi-step exam problem*
Weights follow $N(\mu, 4^2)$. A sample of 16 has $\bar{x} = 52$. Find a 95% confidence interval for μ. Given $z_{0.975} = 1.96$.

(A) $(50.04, 53.96)$

(B) $(44.16, 59.84)$

(C) $(48.08, 55.92)$

(D) $(51.02, 52.98)$

Solutions

Q1: (A)
A normal distribution has a symmetric, bell-shaped curve centred at the mean.
Q2: (A)
The centre is at $x = 50$.
Q3: (A)
$165 = 170 - 5$ and $175 = 170 + 5$, so within 1 SD. About 68%.
Q4: (A)
$100 - 2(10) = 80$ to $100 + 2(10) = 120$.
Q5: (A)
By symmetry, $\Pr(X > \mu) = 0.5$.
Q6: (A)
The distribution with $\sigma_1 = 2$ is taller and narrower.
Q7: (A)
The total area = 1.
Q8: (A)
$75 - 3(8) = 51$ to $75 + 3(8) = 99$.
Q9: (A)
$55 < 60$, so it is below the mean (by 1 standard deviation).
Q10: (A)
Heights of adult women in a large population are approximately normally distributed.
Q11: (A)
$z = \frac{66-50}{8} = 2$.
Q12: (A)
$z = \frac{70-100}{15} = -2$.
Q13: (A)
$z = 2$. $\Pr(X > 250) \approx \frac{1-0.95}{2} = 0.025$.
Q14: (A)
$x = 80 + 1.5 \times 12 = 80 + 18 = 98$.
Q15: (A)
Alice: $z = \frac{85-70}{10} = 1.5$. Bob: $z = \frac{90-80}{5} = 2$. Bob performed better.
Q16: (A)
$500 - 100 = 400$ to $500 + 100 = 600$.
Q17: (A)
$\Pr(X < \mu + 2\sigma) \approx 0.5 + 0.475 = 0.975$, so approximately the 97.5th percentile.
Q18: (A)
$Z \sim N(0, 1)$: mean = 0, standard deviation = 1.
Q19: (A)
By symmetry, $\Pr(Z > 1.5) = \Pr(Z < -1.5) = 0.0668$.
Q20: (A)
$\Pr(-1 < Z < 1) = 0.8413 - 0.1587 = 0.6826$.
Q21: (A)
$z = \frac{49-40}{6} = 1.5$. $\Pr(X < 49) = \Pr(Z < 1.5) = 0.9332$.
Q22: (A)
$\Pr(35 < X < 65) = \Pr(-1.5 < Z < 1.5) = 0.9332 - 0.0668 = 0.8664$.
Q23: (A)
$z = 1$. $x = 100 + 1 \times 15 = 115$.
Q24: (A)
$2 = \frac{72-60}{\sigma} = \frac{12}{\sigma}$. $\sigma = 6$.

Q25: (A)
$2 = \frac{30-\mu}{4}$. $8 = 30 - \mu$. $\mu = 22$.
Q26: (A)
$z = -1$. $\Pr(X > 220) = \Pr(Z > -1) = \Pr(Z < 1) = 0.8413$.
Q27: (A)
$\Pr(85 < X < 130) = \Pr(-1 < Z < 2) = 0.9772 - 0.1587 = 0.8185$. About 81.85%.
Q28: (A)
$z_1 = -1$, $z_2 = 2$. $\Pr(26 < X < 38) = \Pr(-1 < Z < 2)$.
Q29: (A)
$\Pr(-c < Z < c) = 0.95$ means $\Pr(Z < c) = 0.975$. So $c = 1.96$.
Q30: (A)
$z = -2$. $\Pr(X < 484) = \Pr(Z < -2) = 0.0228$. So 2.28% are rejected.
Q31: (A)
$z = 1.2816$. $x = 65 + 1.2816 \times 12 = 65 + 15.38 = 80.38$. Minimum mark ≈ 80.4.
Q32: (A)
$40 - \mu = -\sigma$ and $70 - \mu = 2\sigma$. Subtract: $30 = 3\sigma$, $\sigma = 10$. $\mu = 50$.
Q33: (A)
$\mu = 40$, $\sigma = \sqrt{24} \approx 4.899$. $z = \frac{45-40}{4.899} \approx 1.02$. $\Pr(X \leq 45) \approx 0.846$.
Q34: (A)
$\Pr(X > 60) = 1 - 0.8413 = 0.1587$. $\Pr(X > 45) = 1 - 0.3085 = 0.6915$. $\Pr(X > 60 | X > 45) = \frac{0.1587}{0.6915} \approx 0.2295$.
Q35: (A)
$a = 200 - 2.576 \times 20 = 148.48$. $b = 200 + 2.576 \times 20 = 251.52$.
Q36: (A)
$E(Y) = 3(10) + 5 = 35$. $\text{Var}(Y) = 9(4) = 36$. $Y \sim N(35, 36)$.
Q37: (A)
$\Pr(60 < X < 80) = 0.8413 - 0.0668 = 0.7745$.
Q38: (A)
$\Pr(Z < c) = 0.995$, so $c = 2.576$.
Q39: (A)
$\Pr(19 < X < 21) = \Pr(-2 < Z < 2) = 0.9772 - 0.0228 = 0.9544$. Rejection rate $= 1 - 0.9544 = 0.0456 = 4.56\%$.
Q40: (A)
$Q_1 = 50 + (-0.6745)(10) = 50 - 6.745 = 43.255 \approx 43.3$.
Q41: (A)
$20 - \mu = -1.282\sigma$ and $80 - \mu = 1.645\sigma$. Subtract: $60 = 2.927\sigma$, $\sigma = 20.50$. $\mu = 20 + 1.282(20.50) = 46.28$.
Q42: (A)
$X + Y \sim N(50, 25)$. $z = \frac{60-50}{5} = 2$. $\Pr(X + Y > 60) = 1 - 0.9772 = 0.0228$.
Q43: (A)
$X - Y \sim N(10, 41)$. $z = \frac{20-10}{\sqrt{41}} \approx \frac{10}{6.403} \approx 1.56$. $\Pr(X - Y > 20) \approx 1 - 0.9406 = 0.0594$.
Q44: (A)
$\bar{X} \sim N(50, 4)$. $z = \frac{54-50}{2} = 2$. $\Pr(\bar{X} > 54) = 0.0228$.
Q45: (A)
$f'(x) = -xf(x)$. $f''(x) = (x^2 - 1)f(x) = 0$. Since $f(x) > 0$, $x^2 = 1$, so $x = \pm 1$.
Q46: (A)
$\int_0^\infty z f(z)\, dz = \frac{1}{\sqrt{2\pi}}$. $E(Z|Z > 0) = \frac{1/\sqrt{2\pi}}{1/2} = \frac{2}{\sqrt{2\pi}} = \sqrt{\frac{2}{\pi}} \approx 0.798$.
Q47: (A)
$\hat{\mu} = \bar{x} = \frac{12+15+18+14+16}{5} = \frac{75}{5} = 15$.
Q48: (A)

$E(Z_i^2) = 1$ for each i. $E(Z_1^2 + Z_2^2 + Z_3^2) = 3$.
Q49: (A)
$\Pr(X^2 + Y^2 < 1) = 1 - e^{-1/2} \approx 1 - 0.6065 = 0.3935$.
Q50: (A)
SE $= \frac{4}{\sqrt{16}} = 1$. CI: $52 \pm 1.96(1) = (50.04, 53.96)$.