50 multiple-choice questions

Question 1 (Level 1) — *Inverse of adding*
If $f(x) = x + 5$, find $f^{-1}(x)$.

(A) $x - 5$

(B) $x + 5$

(C) $5 - x$

(D) $\dfrac{x}{5}$

Question 2 (Level 1) — *Inverse of multiplying*
If $f(x) = 3x$, find $f^{-1}(x)$.

(A) $\dfrac{x}{3}$

(B) $3x$

(C) $x - 3$

(D) $\dfrac{3}{x}$

Question 3 (Level 1) — *Evaluate inverse at a point*
If $f(x) = x + 7$ and $f(a) = 10$, find a.

(A) 3

(B) 17

(C) -3

(D) 7

Question 4 (Level 1) — *Inverse of subtracting*
If $f(x) = x - 4$, find $f^{-1}(x)$.

(A) $x + 4$

(B) $x - 4$

(C) $4 - x$

(D) $4x$

Question 5 (Level 1) — *Inverse of halving*
If $f(x) = \dfrac{x}{2}$, find $f^{-1}(x)$.

(A) $2x$

(B) $\dfrac{x}{2}$

(C) $x + 2$

(D) $\dfrac{2}{x}$

Question 6 (Level 1) — *Reflection line*
The graph of f^{-1} is a reflection of the graph of f in which line?

(A) $y = x$

(B) $y = -x$

(C) the x-axis

(D) the y-axis

Question 7 (Level 1) — *Point on inverse graph*
If the point $(2, 8)$ lies on $y = f(x)$, what point must lie on $y = f^{-1}(x)$?

(A) $(8, 2)$

(B) $(2, 8)$

(C) $(-2, -8)$

(D) $(-8, -2)$

Question 8 (Level 1) — *Inverse of linear function*
If $f(x) = 2x + 1$, find $f^{-1}(x)$.

(A) $\dfrac{x - 1}{2}$

(B) $\dfrac{x + 1}{2}$

(C) $2x - 1$

(D) $\dfrac{1}{2x + 1}$

Question 9 (Level 1) — *Domain and range swap*
If f has domain $\{1, 2, 3\}$ and range $\{4, 5, 6\}$, what are the domain and range of f^{-1}?

(A) Domain $\{4, 5, 6\}$, range $\{1, 2, 3\}$

(B) Domain $\{1, 2, 3\}$, range $\{4, 5, 6\}$

(C) Domain $\{-4, -5, -6\}$, range $\{-1, -2, -3\}$

(D) Domain $\{1, 2, 3\}$, range $\{1, 2, 3\}$

Question 10 (Level 1) — *Verify an inverse*
If $f(x) = x + 3$ and $g(x) = x - 3$, verify that $f(g(x)) = x$.

(A) $f(g(x)) = x$, so $g = f^{-1}$

(B) $f(g(x)) = x + 6$

(C) $f(g(x)) = x - 6$

(D) $f(g(x)) = 2x$

Question 11 (Level 2) — *Inverse of linear with fraction*
Find the inverse of $f(x) = \dfrac{3x - 2}{4}$.

(A) $\dfrac{4x + 2}{3}$

(B) $\dfrac{4x - 2}{3}$

(C) $\dfrac{3x + 2}{4}$

(D) $\dfrac{4}{3x - 2}$

Question 12 (Level 2) — *Graph reflection check*
The graph of $f(x) = 2x + 3$ passes through $(0, 3)$. Through which point does f^{-1} pass?

(A) $(3, 0)$

(B) $(0, 3)$

(C) $(0, -3)$

(D) $(-3, 0)$

Question 13 (Level 2) — *Inverse of cube root*
Find $f^{-1}(x)$ if $f(x) = \sqrt[3]{x}$.

(A) x^3

(B) $\sqrt[3]{x}$

(C) $3x$

(D) $\dfrac{1}{x^3}$

Question 14 (Level 2) — *Domain of inverse*
If $f : [0, \infty) \to [2, \infty)$ is defined by $f(x) = x + 2$, state the domain of f^{-1}.

(A) $[2, \infty)$

(B) $[0, \infty)$

(C) $\mathbb{R}$

(D) $(-\infty, 2]$

Question 15 (Level 2) — *Inverse of negative linear*
Find $f^{-1}(x)$ if $f(x) = -2x + 6$.

(A) $\dfrac{6 - x}{2}$

(B) $\dfrac{x - 6}{2}$

(C) $-\dfrac{x}{2} + 6$

(D) $\dfrac{x + 6}{2}$

Question 16 (Level 2) — *Self-inverse function*
Show that $f(x) = -x$ is its own inverse.

(A) $f^{-1}(x) = -x$, which equals $f(x)$, so f is self-inverse

(B) $f^{-1}(x) = x$

(C) $f^{-1}(x) = \dfrac{1}{x}$

(D) f has no inverse

Question 17 (Level 2) — *One-to-one test*
Which function has an inverse over its natural domain?

(A) $f(x) = 3x + 1$

(B) $f(x) = x^2$

(C) $f(x) = |x|$

(D) $f(x) = \sin(x)$

Question 18 (Level 2) — *Find x-intercept of inverse*
If $f(x) = 4x - 8$, find the x-intercept of $y = f^{-1}(x)$.

(A) $(-8, 0)$

(B) $(2, 0)$

(C) $(8, 0)$

(D) $(0, -8)$

Question 19 (Level 2) — *Intersection of f and f inverse*
If $f(x) = 2x - 1$, find where $f(x) = f^{-1}(x)$.

(A) $(1, 1)$

(B) $(0, -1)$

(C) $(-1, -1)$

(D) $(2, 3)$

Question 20 (Level 2) — *Inverse from table*
Given $f(1) = 5$, $f(2) = 7$, $f(3) = 9$, find $f^{-1}(7)$.

(A) 2

(B) 7

(C) 5

(D) 3

Question 21 (Level 3) — *Inverse of quadratic (restricted)*
If $f(x) = x^2$ for $x \geq 0$, find $f^{-1}(x)$.

(A) $\sqrt{x}$

(B) $\pm\sqrt{x}$

(C) x^2

(D) $-\sqrt{x}$

Question 22 (Level 3) — *Domain restriction for inverse*
Why does $f(x) = x^2$ (with domain $\mathbb{R}$) not have an inverse?

(A) It is not one-to-one ($f(2) = f(-2) = 4$)

(B) It is not defined for negative x

(C) Its range is all of $\mathbb{R}$

(D) It has no x-intercept

Question 23 (Level 3) — *Inverse of transformed quadratic*
If $f(x) = (x - 1)^2 + 2$ for $x \geq 1$, find $f^{-1}(x)$.

(A) $1 + \sqrt{x - 2}$

(B) $1 - \sqrt{x - 2}$

(C) $\sqrt{x - 2} + 2$

(D) $\sqrt{x - 1} + 2$

Question 24 (Level 3) — *Inverse with cube function*
Find $f^{-1}(x)$ if $f(x) = x^3 + 1$.

(A) $\sqrt[3]{x-1}$

(B) $(x-1)^3$

(C) $\sqrt[3]{x} - 1$

(D) $\sqrt[3]{x+1}$

Question 25 (Level 3) — *Domain and range of inverse*
If $f : [0, 4] \to [1, 9],\ f(x) = 2x + 1$, state the domain and range of f^{-1}.

(A) Domain $[1, 9]$, range $[0, 4]$

(B) Domain $[0, 4]$, range $[1, 9]$

(C) Domain $[0, 9]$, range $[1, 4]$

(D) Domain $[1, 4]$, range $[0, 9]$

Question 26 (Level 3) — *Inverse of reciprocal*
Find $f^{-1}(x)$ if $f(x) = \dfrac{1}{x}$ for $x \neq 0$.

(A) $\dfrac{1}{x}$ (self-inverse)

(B) x

(C) $-\dfrac{1}{x}$

(D) $-x$

Question 27 (Level 3) — *Restricting domain for inverse*
What is the largest domain containing $x = 0$ for which $f(x) = x^2 - 4x$ has an inverse?

(A) $(-\infty, 2]$

(B) $[2, \infty)$

(C) $(-\infty, 0]$

(D) $[0, 4]$

Question 28 (Level 3) — *Inverse of square root function*
If $f(x) = \sqrt{x+3}$ for $x \geq -3$, find $f^{-1}(x)$.

(A) $x^2 - 3$ for $x \geq 0$

(B) $x^2 - 3$ for all x

(C) $x^2 + 3$ for $x \geq 0$

(D) $(x-3)^2$ for $x \geq 3$

Question 29 (Level 3) — *Graphical verification*
If $f(x) = 3x - 6$, at what point do $y = f(x)$ and $y = f^{-1}(x)$ intersect the line $y = x$?

(A) $(3, 3)$

(B) $(6, 6)$

(C) $(2, 2)$

(D) $(0, -6)$

Question 30 (Level 3) — *Inverse of piecewise*
If $f(x) = 2x + 1$ for $x \geq 0$, what is the domain of f^{-1}?

(A) $[1, \infty)$

(B) $[0, \infty)$

(C) $\mathbb{R}$

(D) $(1, \infty)$

Question 31 (Level 4) — *Inverse of exponential*
Find $f^{-1}(x)$ if $f(x) = e^{2x}$.

(A) $\dfrac{\ln(x)}{2}$

(B) $\ln(2x)$

(C) $2\ln(x)$

(D) $e^{x/2}$

Question 32 (Level 4) — *Inverse of logarithm*
Find $f^{-1}(x)$ if $f(x) = \log_e(x-1) + 3$.

(A) $e^{x-3} + 1$

(B) $e^{x-1} + 3$

(C) $e^{x+3} - 1$

(D) $e^{x-3} - 1$

Question 33 (Level 4) — *Inverse trig: arcsin*
Evaluate $\sin^{-1}\left(\dfrac{1}{2}\right)$.

(A) $\dfrac{\pi}{6}$

(B) $\dfrac{\pi}{3}$

(C) $\dfrac{\pi}{4}$

(D) $\dfrac{5\pi}{6}$

Question 34 (Level 4) — *Domain of inverse trig*
State the domain and range of $y = \cos^{-1}(x)$.

(A) Domain $[-1, 1]$, range $[0, \pi]$

(B) Domain $[-1, 1]$, range $[-\frac{\pi}{2}, \frac{\pi}{2}]$

(C) Domain $[0, \pi]$, range $[-1, 1]$

(D) Domain $\mathbb{R}$, range $[0, \pi]$

Question 35 (Level 4) — *Inverse of transformed exponential*
If $f(x) = 3e^{x+1} - 2$, find $f^{-1}(x)$.

(A) $\ln\left(\dfrac{x+2}{3}\right) - 1$

(B) $\ln\left(\dfrac{x-2}{3}\right) + 1$

(C) $\ln(x+2) - \ln(3) + 1$

(D) $\dfrac{\ln(x+2)}{3} - 1$

Question 36 (Level 4) — *Inverse of restricted quadratic*
If $f : (-\infty, 3] \to [-4, \infty),\ f(x) = (x-3)^2 - 4$, find $f^{-1}(x)$.

(A) $3 - \sqrt{x+4}$

(B) $3 + \sqrt{x+4}$

(C) $3 - \sqrt{x-4}$

(D) $-3 + \sqrt{x+4}$

Question 37 (Level 4) — *Evaluate inverse trig: arctan*
Evaluate $\tan^{-1}(1)$.

(A) $\dfrac{\pi}{4}$

(B) $\dfrac{\pi}{2}$

(C) $\dfrac{3\pi}{4}$

(D) 1

Question 38 (Level 4) — *Graph of inverse exponential*
The graph of $f(x) = e^x$ has a horizontal asymptote at $y = 0$. What is the asymptote of $f^{-1}(x) = \ln(x)$?

(A) Vertical asymptote $x = 0$

(B) Horizontal asymptote $y = 0$

(C) Vertical asymptote $x = 1$

(D) No asymptote

Question 39 (Level 4) — *Composite of function and inverse*
If $f(x) = 2\log_e(x)$ for $x > 0$, find $f^{-1}(f(3))$.

(A) 3

(B) $2\ln(3)$

(C) e^3

(D) $\ln(3)$

Question 40 (Level 4) — *Inverse trig evaluation*
Find the exact value of $\cos\left(\sin^{-1}\left(\dfrac{3}{5}\right)\right)$.

(A) $\dfrac{4}{5}$

(B) $\dfrac{3}{5}$

(C) $\dfrac{5}{4}$

(D) $\dfrac{\sqrt{7}}{5}$

Question 41 (Level 5) — *Inverse of log with restricted domain*
Let $f : (1, \infty) \to \mathbb{R},\ f(x) = 2\log_e(x - 1) + 1$. Find the rule and domain of f^{-1}.

(A) $f^{-1}(x) = e^{(x-1)/2} + 1$, domain $\mathbb{R}$

(B) $f^{-1}(x) = e^{(x-1)/2} + 1$, domain $(1, \infty)$

(C) $f^{-1}(x) = e^{(x+1)/2} - 1$, domain $\mathbb{R}$

(D) $f^{-1}(x) = e^{x/2} + 1$, domain $(0, \infty)$

Question 42 (Level 5) — *Self-inverse rational function*
Show that $f(x) = \dfrac{3x-2}{x-3}$ (for $x \neq 3$) is self-inverse by finding $f(f(x))$.

(A) $f(f(x)) = x$, so f is self-inverse

(B) $f(f(x)) = \dfrac{1}{x}$

(C) $f(f(x)) = -x$

(D) f is not self-inverse

Question 43 (Level 5) — *Intersection of f and fź*
If $f(x) = e^x - 2$, find the x-coordinate(s) where $y = f(x)$ and $y = f^{-1}(x)$ intersect.

(A) Solutions of $e^x = x + 2$

(B) Solutions of $e^x = 2x$

(C) $x = \ln(2)$ only

(D) No intersection exists

Question 44 (Level 5) — *Derivative of inverse function*
If $f(x) = x^3 + x$ and $f(1) = 2$, find $(f^{-1})'(2)$.

(A) $\dfrac{1}{4}$

(B) 4

(C) $\dfrac{1}{2}$

(D) $\dfrac{1}{3}$

Question 45 (Level 5) — *Restricting domain for inverse existence*
For $f(x) = 2\sin(3x)$, find the largest domain containing $x = 0$ for which f^{-1} exists.

(A) $\left[-\dfrac{\pi}{6}, \dfrac{\pi}{6}\right]$

(B) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

(C) $\left[-\dfrac{\pi}{3}, \dfrac{\pi}{3}\right]$

(D) $[0, \pi]$

Question 46 (Level 5) — *Inverse trig identity*
Simplify $\sin^{-1}(x) + \cos^{-1}(x)$ for $x \in [-1, 1]$.

(A) $\dfrac{\pi}{2}$

(B) π

(C) 0

(D) Depends on x

Question 47 (Level 5) — *Inverse of composite function*
If $f(x) = 2x + 1$ and $g(x) = e^x$, find $(g \circ f)^{-1}(x)$.

(A) $\dfrac{\ln(x) - 1}{2}$

(B) $\dfrac{\ln(x) + 1}{2}$

(C) $\ln\left(\dfrac{x-1}{2}\right)$

(D) $2\ln(x) - 1$

Question 48 (Level 5) — *Area between f and fź*
The function $f : [0, 1] \to [0, 1],\ f(x) = x^2$ and its inverse enclose a region. Find the area.

(A) $\dfrac{1}{3}$

(B) $\dfrac{1}{2}$

(C) $\dfrac{1}{4}$

(D) $\dfrac{2}{3}$

Question 49 (Level 5) — *Inverse trig derivative*
Find $\dfrac{d}{dx}\left[\tan^{-1}(2x)\right]$.

(A) $\dfrac{2}{1 + 4x^2}$

(B) $\dfrac{1}{1 + 4x^2}$

(C) $\dfrac{2}{1 + 2x^2}$

(D) $\dfrac{1}{1 + x^2}$

Question 50 (Level 5) — *Implicit inverse with parameters*
Let $f(x) = ax^3 + b$ where $f(2) = 5$ and $f'(2) = 6$. Find $(f^{-1})'(5)$.

(A) $\frac{1}{6}$

(B) 6

(C) $\frac{1}{12}$

(D) $\frac{1}{5}$

Solutions

Q1: (A)
Let $y = x + 5$. Swap: $x = y + 5 \Rightarrow y = x - 5$. So $f^{-1}(x) = x - 5$.
Q2: (A)
Let $y = 3x$. Swap: $x = 3y \Rightarrow y = \dfrac{x}{3}$. So $f^{-1}(x) = \dfrac{x}{3}$.
Q3: (A)
$a + 7 = 10 \Rightarrow a = 3$.
Q4: (A)
$y = x - 4 \Rightarrow x = y - 4 \Rightarrow y = x + 4$. So $f^{-1}(x) = x + 4$.
Q5: (A)
$y = \frac{x}{2} \Rightarrow x = \frac{y}{2} \Rightarrow y = 2x$. So $f^{-1}(x) = 2x$.
Q6: (A)
The inverse function is the reflection in the line $y = x$.
Q7: (A)
Swap coordinates: $(8, 2)$ lies on $y = f^{-1}(x)$.
Q8: (A)
$x = 2y + 1 \Rightarrow 2y = x - 1 \Rightarrow y = \dfrac{x - 1}{2}$.
Q9: (A)
Domain of $f^{-1} = \{4, 5, 6\}$ and range of $f^{-1} = \{1, 2, 3\}$.
Q10: (A)
$f(g(x)) = f(x - 3) = (x - 3) + 3 = x$. ✓
Q11: (A)
$x = \frac{3y-2}{4} \Rightarrow 4x = 3y - 2 \Rightarrow y = \frac{4x+2}{3}$.
Q12: (A)
Swap: $(3, 0)$ lies on f^{-1}.
Q13: (A)
$y = \sqrt[3]{x} \Rightarrow x = \sqrt[3]{y} \Rightarrow y = x^3$.
Q14: (A)
Domain of f^{-1} = range of $f = [2, \infty)$.
Q15: (A)
$x = -2y + 6 \Rightarrow 2y = 6 - x \Rightarrow y = \dfrac{6 - x}{2}$.
Q16: (A)
$f(f(x)) = f(-x) = -(-x) = x$. So $f^{-1}(x) = -x = f(x)$.
Q17: (A)
$f(x) = 3x + 1$ is linear with non-zero slope, so it is one-to-one and has an inverse.
Q18: (A)
$f^{-1}(x) = 0$ when $x = f(0) = 4(0) - 8 = -8$. So x-intercept is $(-8, 0)$.
Q19: (A)
$f(x) = x \Rightarrow 2x - 1 = x \Rightarrow x = 1$. Intersection at $(1, 1)$.
Q20: (A)
$f(2) = 7$, so $f^{-1}(7) = 2$.
Q21: (A)
$x = y^2,\ y \geq 0 \Rightarrow y = \sqrt{x}$. So $f^{-1}(x) = \sqrt{x}$.
Q22: (A)
$f(2) = f(-2) = 4$, so f is not one-to-one. A function must be one-to-one to have an inverse.
Q23: (A)
$x = (y - 1)^2 + 2 \Rightarrow (y - 1)^2 = x - 2 \Rightarrow y = 1 + \sqrt{x - 2}$.
Q24: (A)

$x = y^3 + 1 \Rightarrow y^3 = x - 1 \Rightarrow y = \sqrt[3]{x-1}$.

Q25: (A)

Domain of $f^{-1} = [1, 9]$, range of $f^{-1} = [0, 4]$.

Q26: (A)

$x = \frac{1}{y} \Rightarrow y = \frac{1}{x}$. So $f^{-1}(x) = \frac{1}{x}$. It is self-inverse!

Q27: (A)

$f(x) = (x-2)^2 - 4$. Vertex at $x = 2$. Largest domain containing $x = 0$: $(-\infty, 2]$.

Q28: (A)

$x = \sqrt{y+3} \Rightarrow x^2 = y + 3 \Rightarrow y = x^2 - 3$ for $x \geq 0$.

Q29: (A)

$3x - 6 = x \Rightarrow 2x = 6 \Rightarrow x = 3$. Point: $(3, 3)$.

Q30: (A)

Range of f on $[0, \infty)$ is $[1, \infty)$. So domain of $f^{-1} = [1, \infty)$.

Q31: (A)

$x = e^{2y} \Rightarrow \ln(x) = 2y \Rightarrow y = \dfrac{\ln(x)}{2}$.

Q32: (A)

$x = \ln(y-1) + 3 \Rightarrow x - 3 = \ln(y-1) \Rightarrow y - 1 = e^{x-3} \Rightarrow y = e^{x-3} + 1$.

Q33: (A)

$\sin^{-1}\left(\frac{1}{2}\right) = \dfrac{\pi}{6}$ since $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.

Q34: (A)

Domain: $[-1, 1]$. Range: $[0, \pi]$.

Q35: (A)

$x = 3e^{y+1} - 2 \Rightarrow \frac{x+2}{3} = e^{y+1} \Rightarrow y + 1 = \ln\left(\frac{x+2}{3}\right) \Rightarrow y = \ln\left(\frac{x+2}{3}\right) - 1$.

Q36: (A)

$x = (y-3)^2 - 4 \Rightarrow (y-3)^2 = x + 4 \Rightarrow y - 3 = -\sqrt{x+4}$ (since $y \leq 3$). So $f^{-1}(x) = 3 - \sqrt{x+4}$.

Q37: (A)

$\tan^{-1}(1) = \dfrac{\pi}{4}$ since $\tan\left(\frac{\pi}{4}\right) = 1$.

Q38: (A)

The horizontal asymptote $y = 0$ of e^x reflects to the vertical asymptote $x = 0$ of $\ln(x)$.

Q39: (A)

$f^{-1}(f(3)) = 3$ since applying a function then its inverse returns the input.

Q40: (A)

Let $\theta = \sin^{-1}\left(\frac{3}{5}\right)$. Then $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ (positive since $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$).

Q41: (A)

$x = 2\ln(y-1) + 1 \Rightarrow \frac{x-1}{2} = \ln(y-1) \Rightarrow y = e^{(x-1)/2} + 1$. As $x \to 1^+$, $f(x) \to -\infty$; as $x \to \infty$, $f(x) \to \infty$. Range of $f = \mathbb{R}$. So $f^{-1} : \mathbb{R} \to \mathbb{R}$, $f^{-1}(x) = e^{(x-1)/2} + 1$.

Q42: (A)

$$f(f(x)) = f\left(\tfrac{3x-2}{x-3}\right) = \frac{3 \cdot \frac{3x-2}{x-3} - 2}{\frac{3x-2}{x-3} - 3} = \frac{\frac{9x-6-2(x-3)}{x-3}}{\frac{3x-2-3(x-3)}{x-3}} = \frac{7x}{7} = x.$$

Q43: (A)

Intersections with $y = x$: $e^x - 2 = x$, i.e. $e^x = x + 2$. By inspection or CAS, $x \approx -1.841$ and $x \approx 1.146$. The exact solution requires the Lambert W function. The key point is solving $e^x = x + 2$.

Q44: (A)

$f'(x) = 3x^2 + 1$, so $f'(1) = 4$. $(f^{-1})'(2) = \dfrac{1}{f'(1)} = \dfrac{1}{4}$.

Q45: (A)

Need $3x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so $x \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$.

Q46: (A)
$\sin^{-1}(x) + \cos^{-1}(x) = \dfrac{\pi}{2}$ for all $x \in [-1, 1]$.

Q47: (A)
$g(f(x)) = e^{2x+1}$. $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. $g^{-1}(x) = \ln(x)$, $f^{-1}(x) = \frac{x-1}{2}$. So $(g \circ f)^{-1}(x) = \frac{\ln(x)-1}{2}$.

Q48: (A)
$A = \int_0^1 (\sqrt{x} - x^2)\, dx = \left[\frac{2x^{3/2}}{3} - \frac{x^3}{3}\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$.

Q49: (A)
$\dfrac{d}{dx}[\tan^{-1}(2x)] = \dfrac{1}{1+(2x)^2} \cdot 2 = \dfrac{2}{1+4x^2}$.

Q50: (A)
$f'(x) = 3ax^2$. $f'(2) = 12a = 6 \Rightarrow a = \frac{1}{2}$. $(f^{-1})'(5) = \dfrac{1}{f'(2)} = \dfrac{1}{6}$.