**1.5.15** *Binomial trees.* Show that the number of nodes at each level in the worst-case

trees for weighted quick-union are *binomial coefficients*. Compute the average depth of

a node in a worst-case tree with *N* = 2*n* nodes.

Answer:

Let L (n, m) be the number of nodes of the tree with height = n, level = m.

We will prove L (n, m) = C (n, m) by induction.

Base case, when n = 1

L (1, 1) = 1 = C (1, 1)

Suppose it holds true for k – 1, then

For K, i.e. L (k, m)

Because it’s worst case tree, the tree with height = k must be united by 2 equal size h = k – 1 trees. For nodes number in level m, one tree donates its nodes in level m while the other one denotes its node in level m - 1

L (k, m) = L (k – 1, m) + L (k, m – 1)

* L (k, m) = C(k-1,m) + C(k, m-1)
* L (k, m) = C(k, m)

So for any the worst case tree with height = n, the nodes at level = m has nodes C (n, m), that is, *binomial coefficients*

Let *Sn* = be the total depth of all nodes of the binomial tree.

Then

*Sn* = = = = = n(2n – 1) + n - Sn

Sn =