Quantum Computation 101

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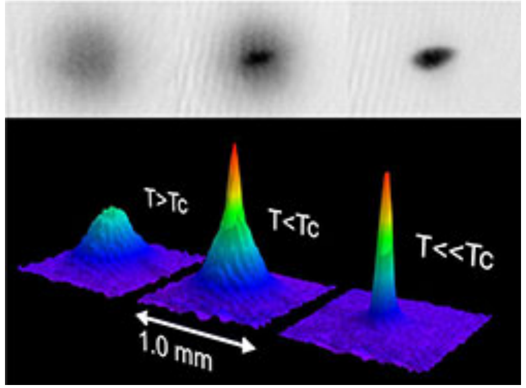
Cheng LI

- Education:

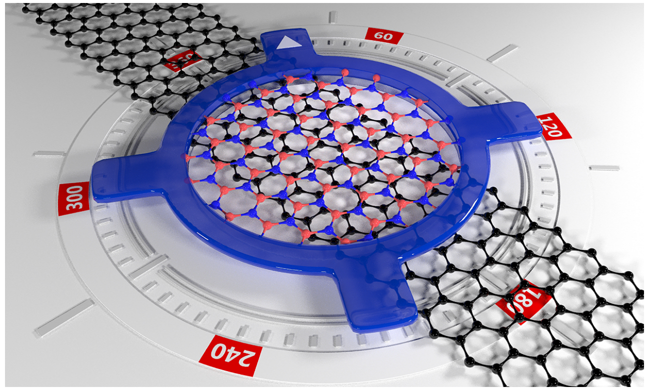
USTC, Hefei, China
2009 - 2013

The OSU, Columbus, OH
2013 – 2020 (expected)

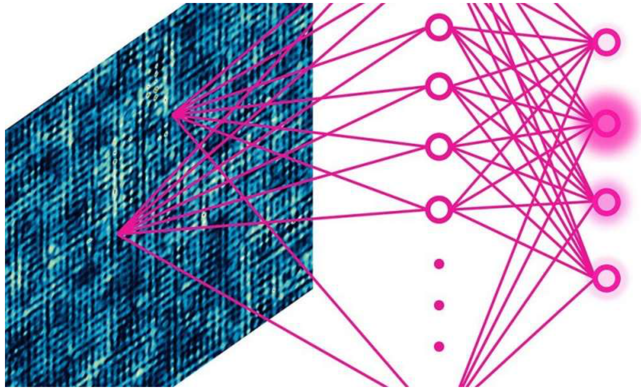
- Research Interest:



Quantum Gases



Twisted 2D Material



Machine Learning in Physics

Outline

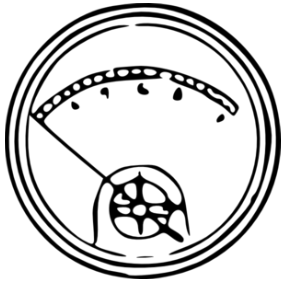
- From Classical State to Quantum State
  - Digital signal
  - Polarization of Light
  - Spin of Electron
- Quantum Gates
  - NOT Gate, CNOT Gate, Toffoli Gate
  - Universal Gate Set
- Topological Quantum Computation

Quantum Physicists V.S. Bayesian Statistician

	Quantum Physics	**Bayesian Statistics**
Philosophy	Uncertainty	Distribution
Foundation	Wavefunction $\psi(x)$	Distribution $\mathrm{p}(x)$
Probability	$\mathrm{P} = \int \lvert\psi(x)\rvert^2 dx$	$\mathrm{P} = \int p(x)dx.$
Observation	$O = \int \psi^*(x)o(x)\psi(x)dx$	$O = \int p(x)o(x)\, dx$

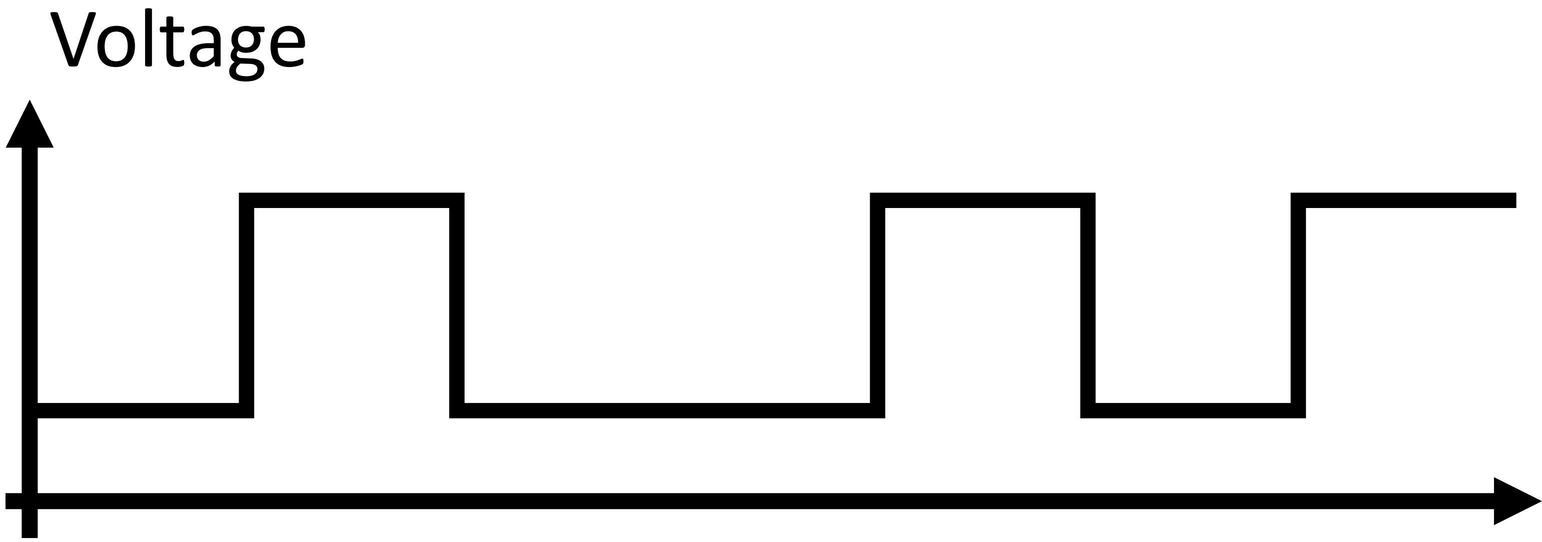
Binary Encoder

Binary

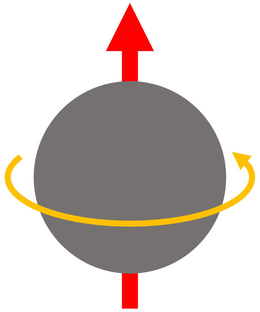
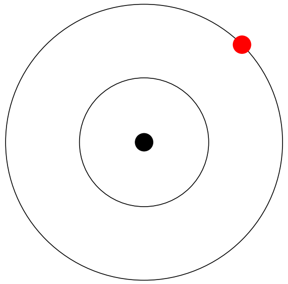
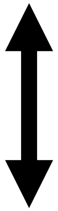
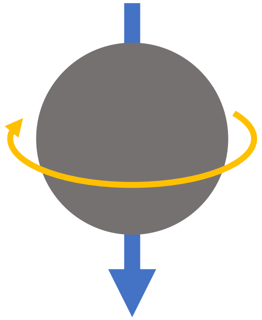
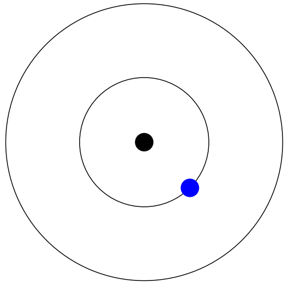


1 5V

0 0V



Binary Encoder

Binary	Polarization of Light	Spin of Electron	Two Level System
1			
0			

Polarization of Light

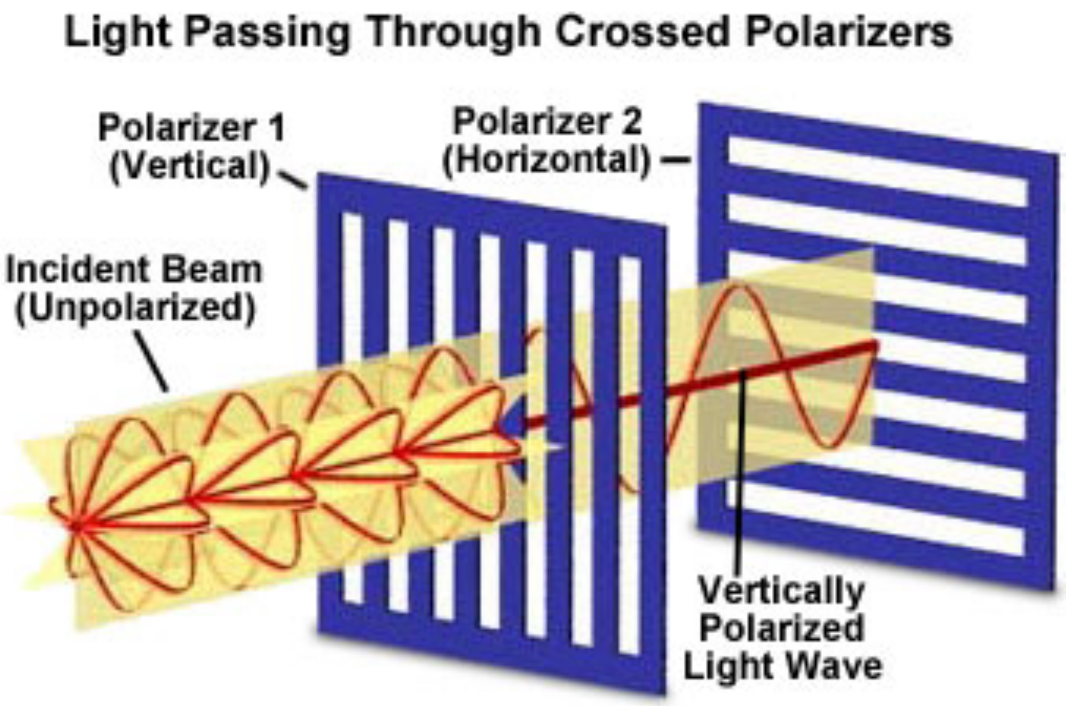


Figure 1

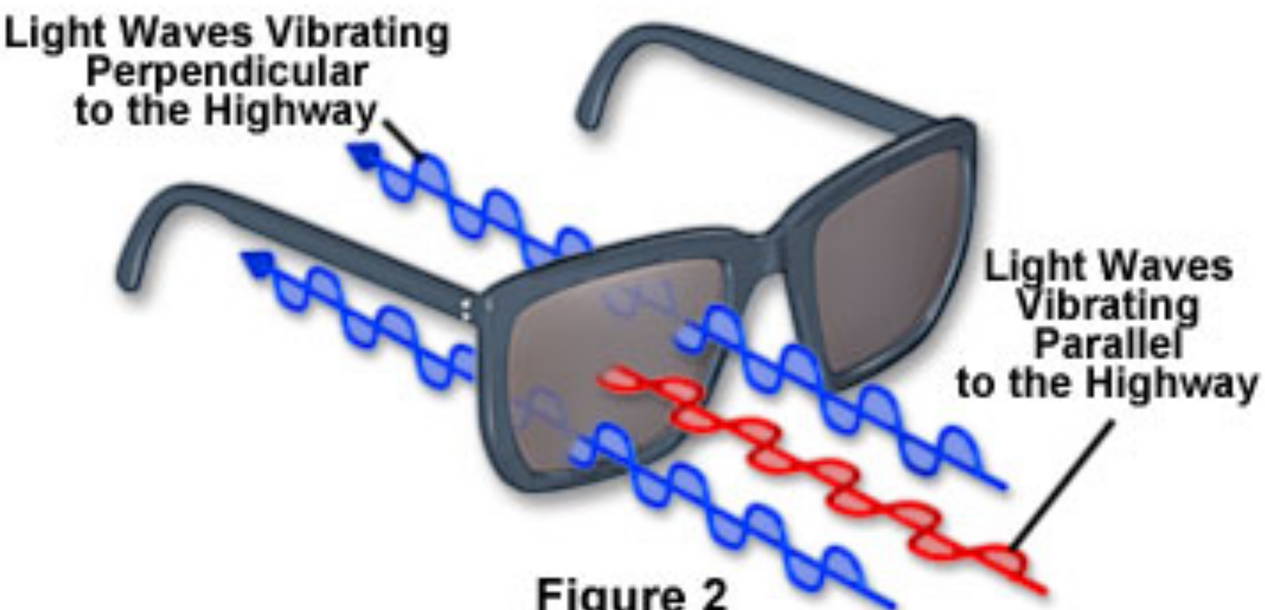
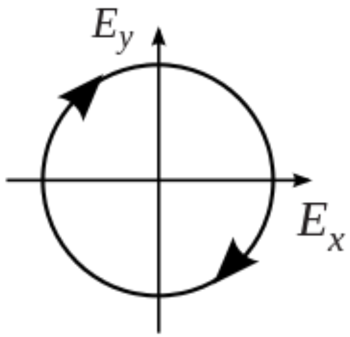


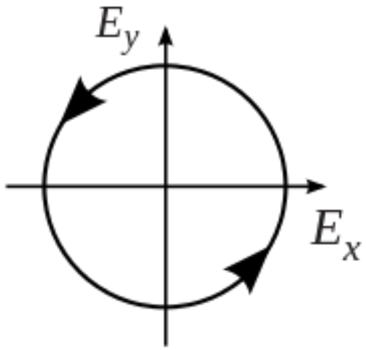
Figure 2

https://www.olympus-lifescience.com/en/microscope-resource/primer/lightandcolor/polarization/

Polarization of Light



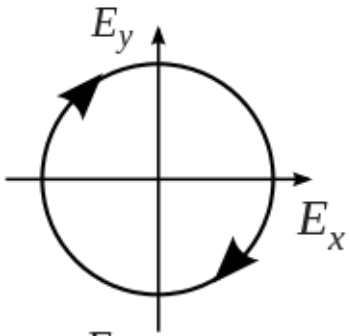
$$= \leftrightarrow + \updownarrow$$



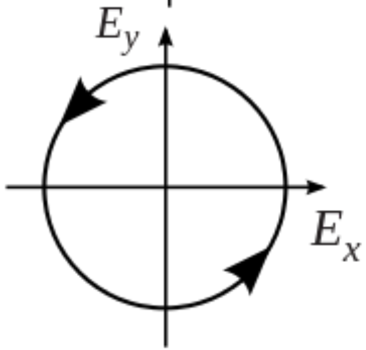
$$= \leftrightarrow - \updownarrow$$

In the basis

$$\leftrightarrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |1\rangle \qquad \updownarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |0\rangle$$



$$= \begin{pmatrix} +1 \\ +1 \end{pmatrix} = |1\rangle + |0\rangle$$



$$= \begin{pmatrix} +1 \\ -1 \end{pmatrix} = |1\rangle - |0\rangle$$

Polarization of Light

TLTR!

Circular Polarization

Right Circular Polarizing

$$\boldsymbol{E}_r(t) = E_0(\cos(\omega t), +\sin(\omega t))$$

Left Circular Polarizing

$$\boldsymbol{E}_l(t) = E_0(\cos(\omega t), -\sin(\omega t))$$

Linear polarization

Polarizing along $\hat{e}$

$$\boldsymbol{E}_{\hat{e}}(t, \phi) = E_0 \cos(\omega t + \phi)\hat{e}$$

Superposition

$$\boldsymbol{E}_r(t) = \boldsymbol{E}_x(t, 0) + \boldsymbol{E}_y(t, -\pi/2)$$

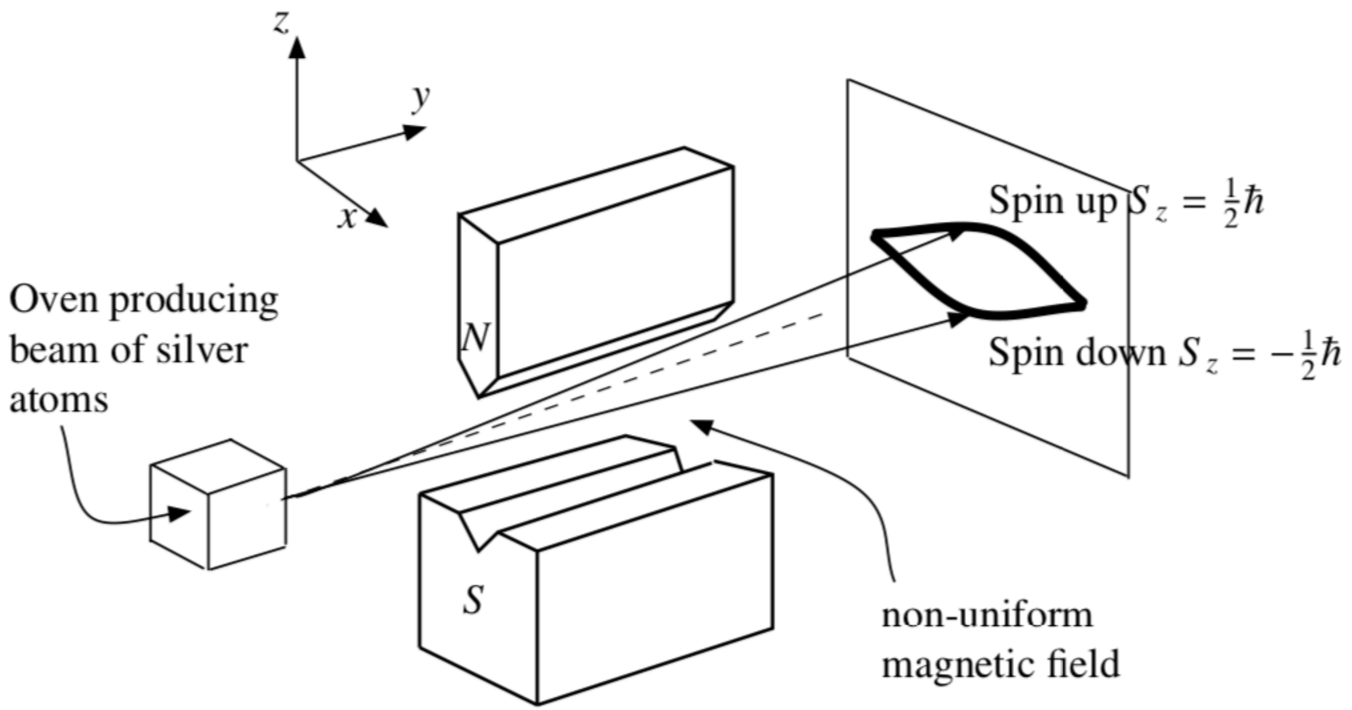
$$\boldsymbol{E}_l(t) = \boldsymbol{E}_x(t, 0) - \boldsymbol{E}_y(t, -\pi/2)$$

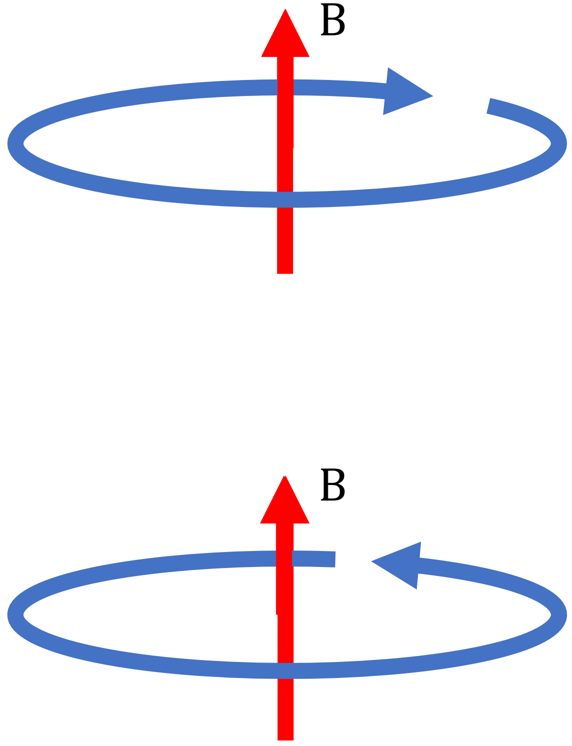
In basis

$$\boldsymbol{E}_x(t, 0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \boldsymbol{E}_y(t, -\pi/2) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

$$\boldsymbol{E}_r(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \boldsymbol{E}_l(t) = \begin{pmatrix} \mathbf{1} \\ -1 \end{pmatrix}$$

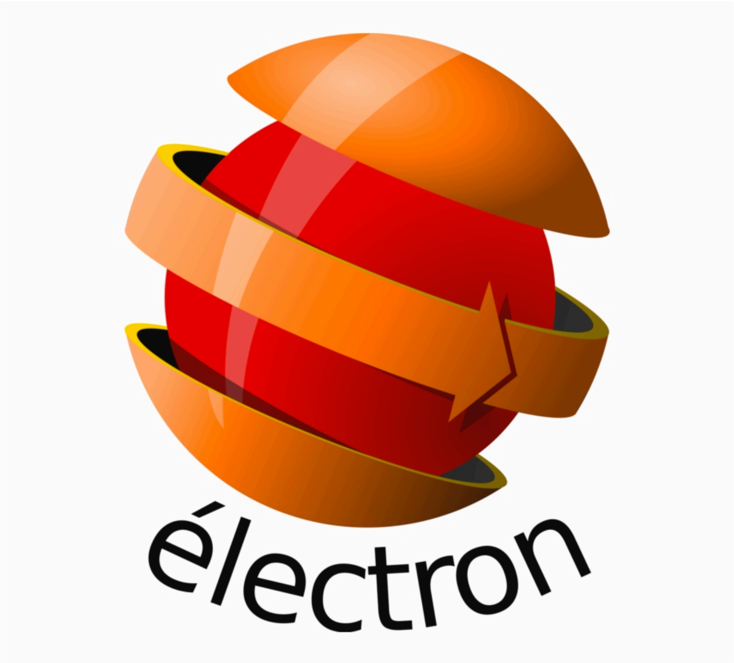
Spin of Electrons





1922, Stern–Gerlach experimen

Spin of Electrons




Ralph Kronig

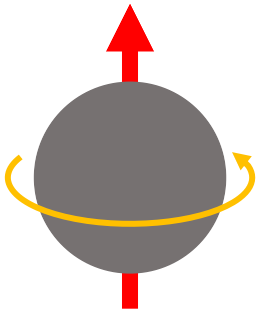

Wolfgang Pauli

$$v = \ 1.45 \times 10^{10}\ m/s$$

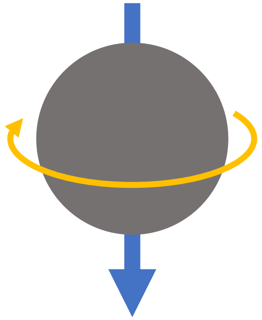

George Uhlenbeck


Samuel Goudsmit

Spin of Electron

$s_z = +1$  $= \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |1\rangle$

$$\langle 1|\sigma_z|1\rangle = +1$$

$s_z = -1$  $= \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |0\rangle$

$$\langle 0|\sigma_z|0\rangle = -1$$

Eigenstates of $\quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

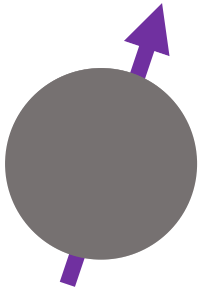
Hermitian matrix

$$H = \alpha \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} + \beta_1 \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} + \beta_2 \begin{pmatrix} & -i \\ i & \end{pmatrix} + \beta_3 \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

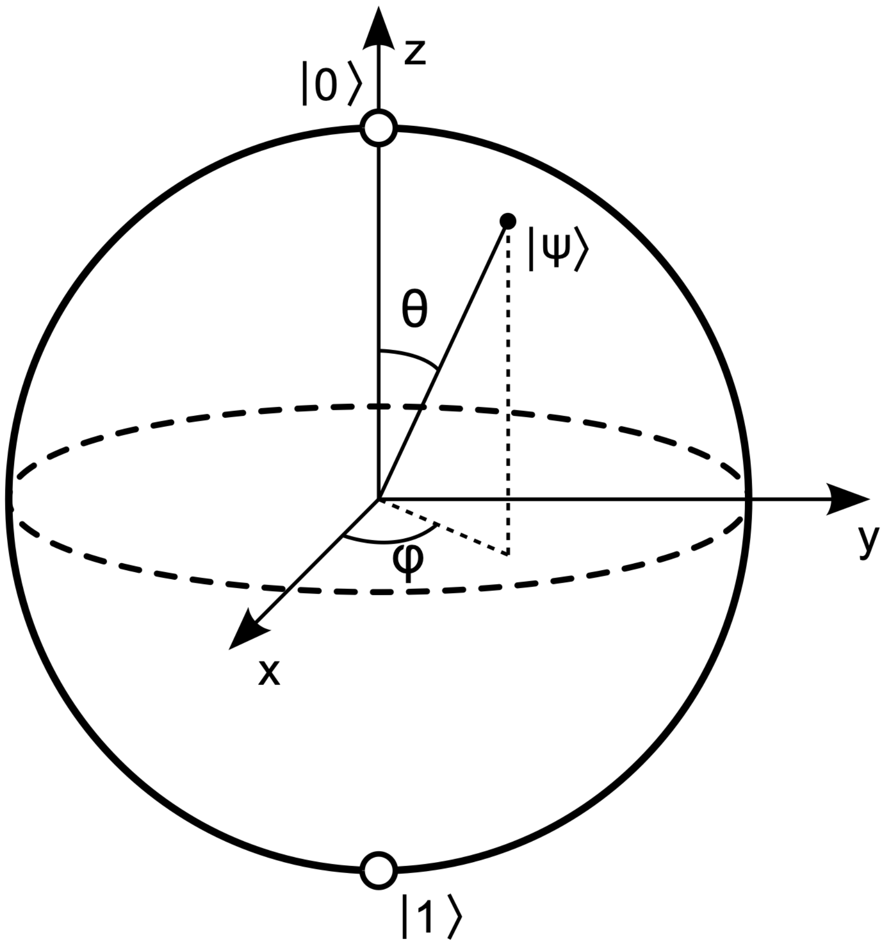
$$\sigma_x \qquad\qquad \sigma_y \qquad\qquad \sigma_z$$

$$|s_x = +1\rangle = \begin{pmatrix} +1 \\ +1 \end{pmatrix} / \sqrt{2} \qquad |s_y = +1\rangle = \begin{pmatrix} +1 \\ +i \end{pmatrix} / \sqrt{2}$$

$$|s_x = -1\rangle = \begin{pmatrix} +1 \\ -1 \end{pmatrix} / \sqrt{2} \qquad |s_y = -1\rangle = \begin{pmatrix} +1 \\ -i \end{pmatrix} / \sqrt{2}$$

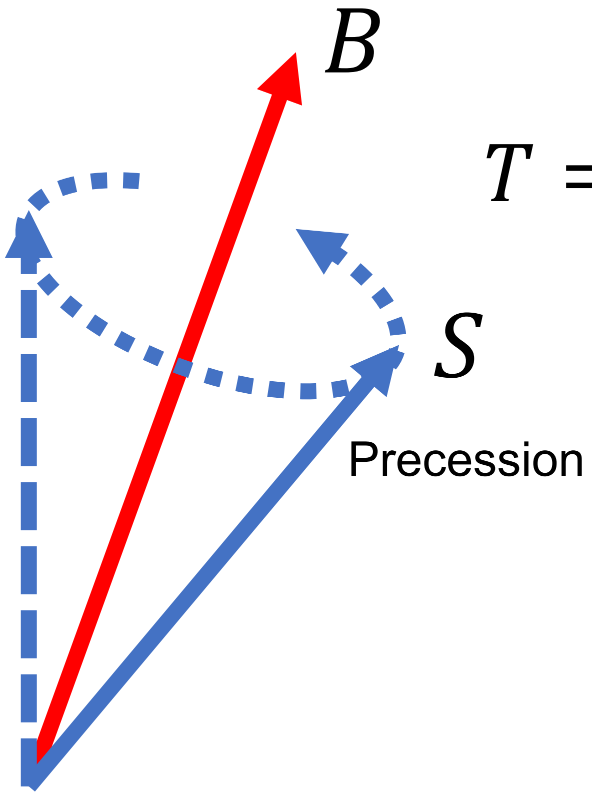


$$= \begin{pmatrix} \cos\dfrac{\theta}{2}\, e^{-i\phi/2} \\ \sin\dfrac{\theta}{2}\, e^{+i\phi/2} \end{pmatrix}$$

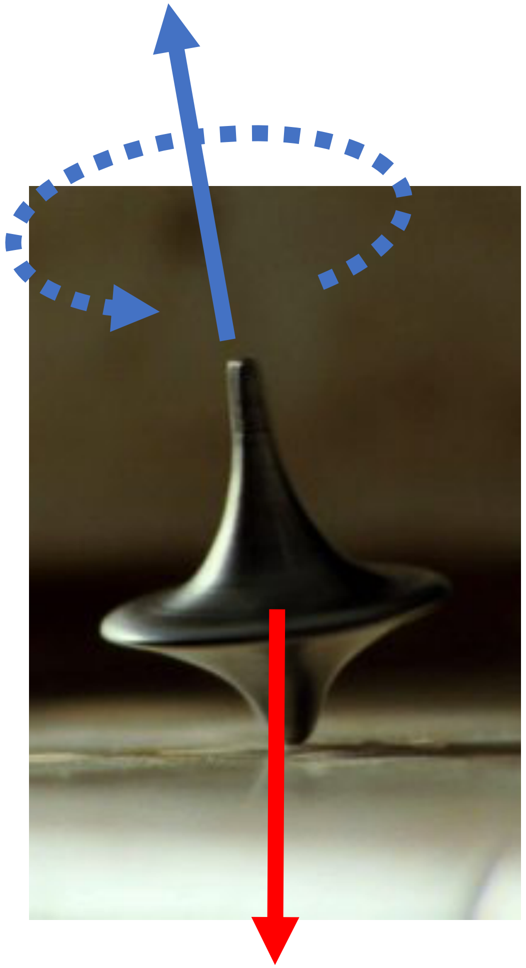


$$s_x = \langle\psi|\sigma_x|\psi\rangle = \sin\theta\cos\phi,$$
$$s_y = \langle\psi|\sigma_y|\psi\rangle = \sin\theta\sin\phi,$$
$$s_z = \langle\psi|\sigma_z|\psi\rangle = \cos\theta.$$

Spin in magnetic field

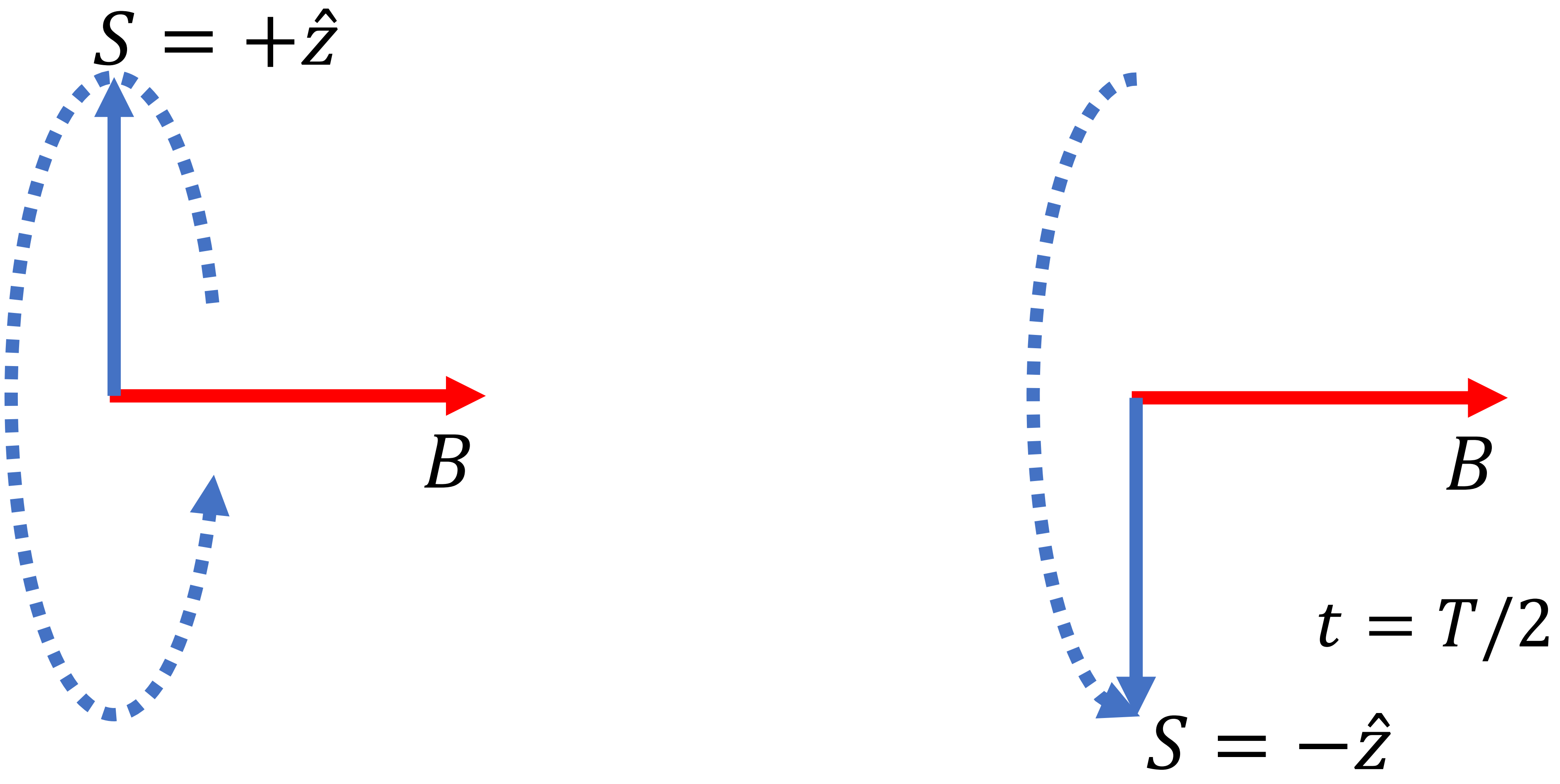


$$T = Const/B$$



Top

Spin $S = \hat{z}$ in Magnetic field $B\hat{x}$



Spin $S = \hat{z}$ in Magnetic field $B\hat{x}$

$$S = +\hat{z} \quad \xleftrightarrow{t = T/2} \quad S = -\hat{z}$$

$$|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |1\rangle \qquad |\psi'\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |0\rangle$$

$$S = (s_x, s_y, s_z) \quad \xleftrightarrow{t = T/2} \quad S = (s_x, -s_y, -s_z)$$

$$|\psi\rangle = \alpha|1\rangle + \beta|0\rangle \qquad |\psi'\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi'\rangle = \beta|1\rangle + \alpha|0\rangle$$

NOT Gate

Classical

	NOT
0	1
1	0

Quantum

$$|\psi\rangle = \alpha|1\rangle + \beta|0\rangle$$

$$\Big\downarrow \text{NOT}$$

$$|\psi'\rangle = \alpha|0\rangle + \beta|1\rangle$$

NOT Gate

$$\mathrm{NOT}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\beta\\ \alpha\end{pmatrix}$$

$$\mathrm{NOT} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$\mathrm{NOT}\cdot\mathrm{NOT}^{\dagger} = 1$$

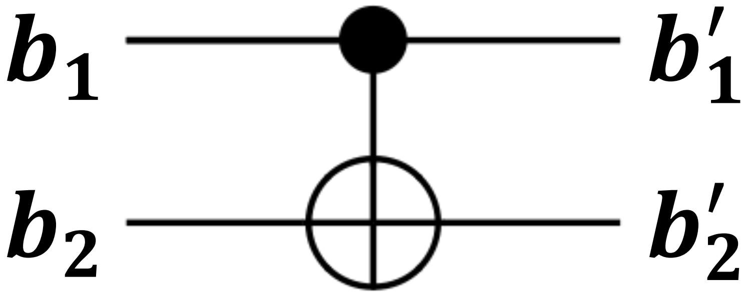
Unitary

Time evolution operators are Unitary

$$|\psi(t)\rangle = U(t,t')|\psi(t')\rangle = e^{-iH(t-t')}|\psi(t')\rangle$$

CNOT Gate

b_1	b_2	b'_1	b'_2
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0



CNOT Gate

Quantum State of Two bits

$|00\rangle, |01\rangle, |10\rangle, |11\rangle$

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

CNOT

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|11\rangle + \delta|10\rangle$$

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \delta|10\rangle + \gamma|11\rangle$$

b_1	b_2	b'_1	b'_2
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

CNOT Gate

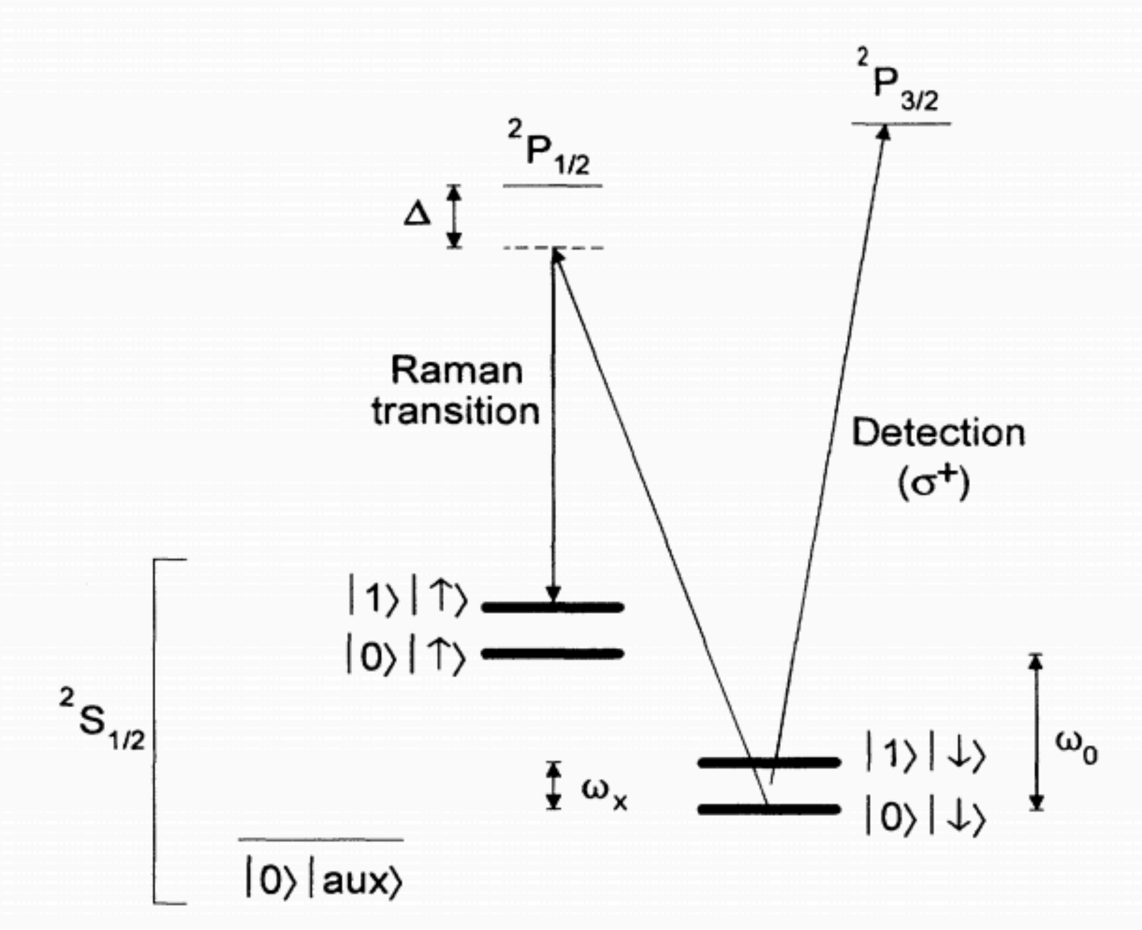
$$\mathrm{CNOT}\begin{pmatrix}\alpha\\ \beta\\ \gamma\\ \delta\end{pmatrix}=\begin{pmatrix}\alpha\\ \beta\\ \delta\\ \gamma\end{pmatrix}$$

$$\mathrm{CNOT}=\begin{pmatrix}1&&&\\&1&&\\&&&1\\&&1&\end{pmatrix}$$

$$\mathrm{CNOT}\cdot\mathrm{CNOT}^{\dagger}=1$$

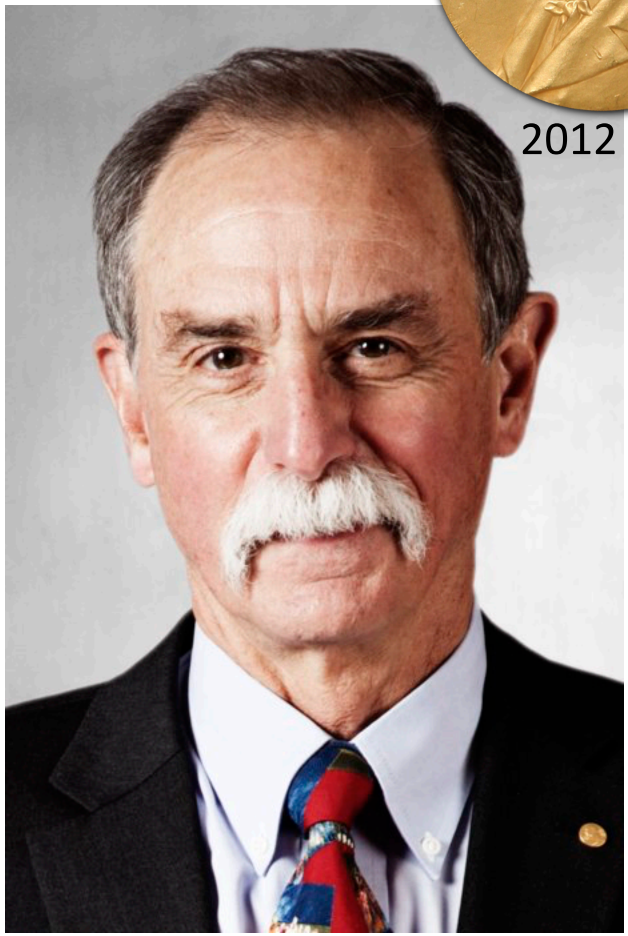
Unitary

CNOT Gate



C. Monroe, D. J. Wineland et al, Phys. Rev. Lett. 75 (1995)

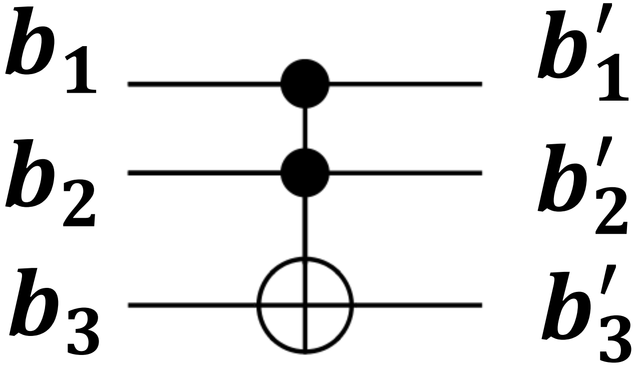
2012 Physics



David J. Wineland

Toffoli gate (controlled-controlled-NOT)

b_1	b_2	b_3	b'_1	b'_2	b'_3
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0



Toffoli gate (controlled-controlled-NOT)

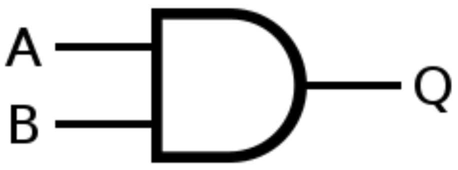
b_1	b_2	b_3	$b_3' = b_1 \text{ AND } b_2$
0	0	0	0
0	1	0	0
1	0	0	0
1	1	0	1

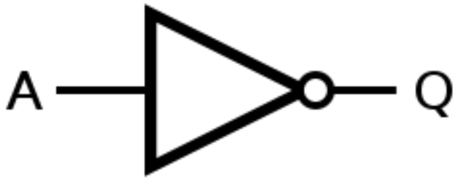
b_1	b_2	b_3	$b_3' = \text{NOT } b_3$
1	1	0	1
1	1	1	0

a $OR\ b = NOT((NOT\ a)\ AND\ (NOT\ b))$

Universal Gate Sets(Classical)

AND-OR-NOT





Toffoli gate → AND-OR-NOT

Quantum Universal Gate Sets

CNOT- $R_{\pi/4}$-P

$$R_{\pi/4} = \begin{pmatrix} \cos\pi/8 & -\sin\pi/8 \\ \sin\pi/8 & \cos\pi/8 \end{pmatrix} = e^{-i\sigma_y \pi/8}$$

$$\text{Phase} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

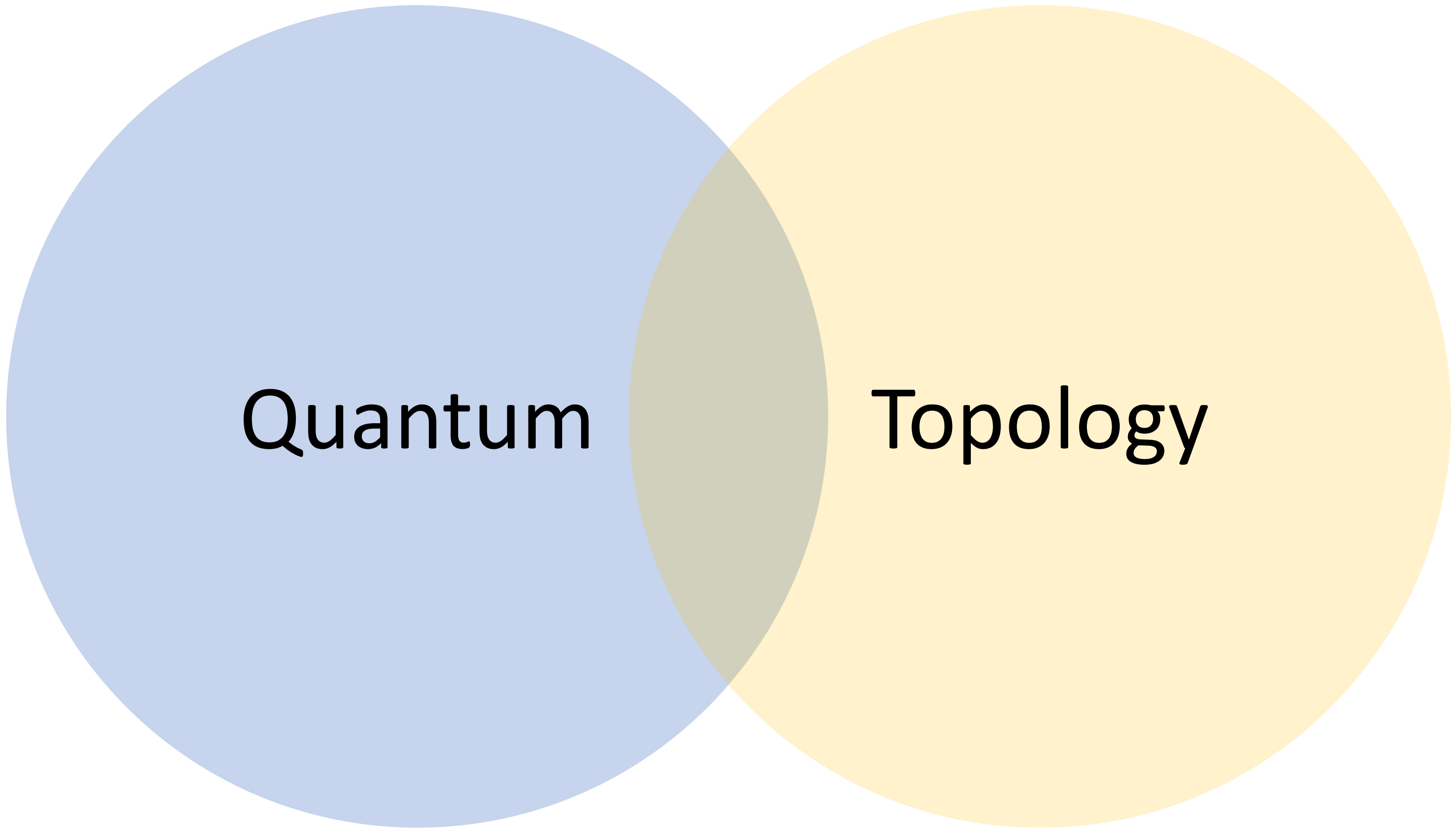
When we say the gate set $\mathcal{G}$ is *universal* we mean that the unitary transformations that can be constructed as quantum circuits using this gate set are *dense* in the unitary group $U(2^n)$, up to an overall phase. That is for any $V \in U(2^n)$ and any $\delta > 0$, there is a unitary $\tilde{V}$ achieved by a finite circuit such that

$$\|\tilde{V} - e^{i\phi}V\|_{\sup} \leq \delta \tag{5.81}$$

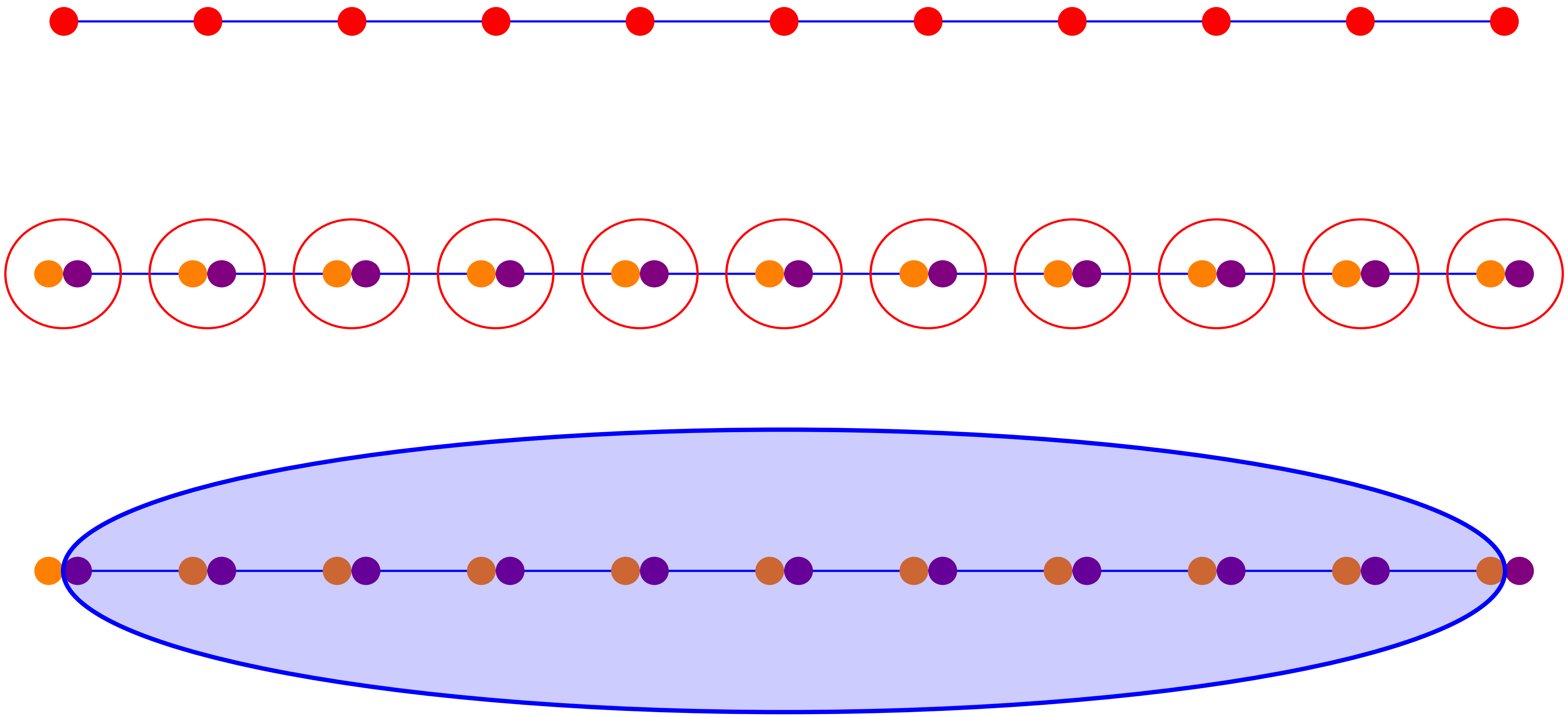
for some phase $e^{i\phi}$.

Reference: http://www.theory.caltech.edu/~preskill/ph219/chap5_15.pdf

Topological Quantum Computation

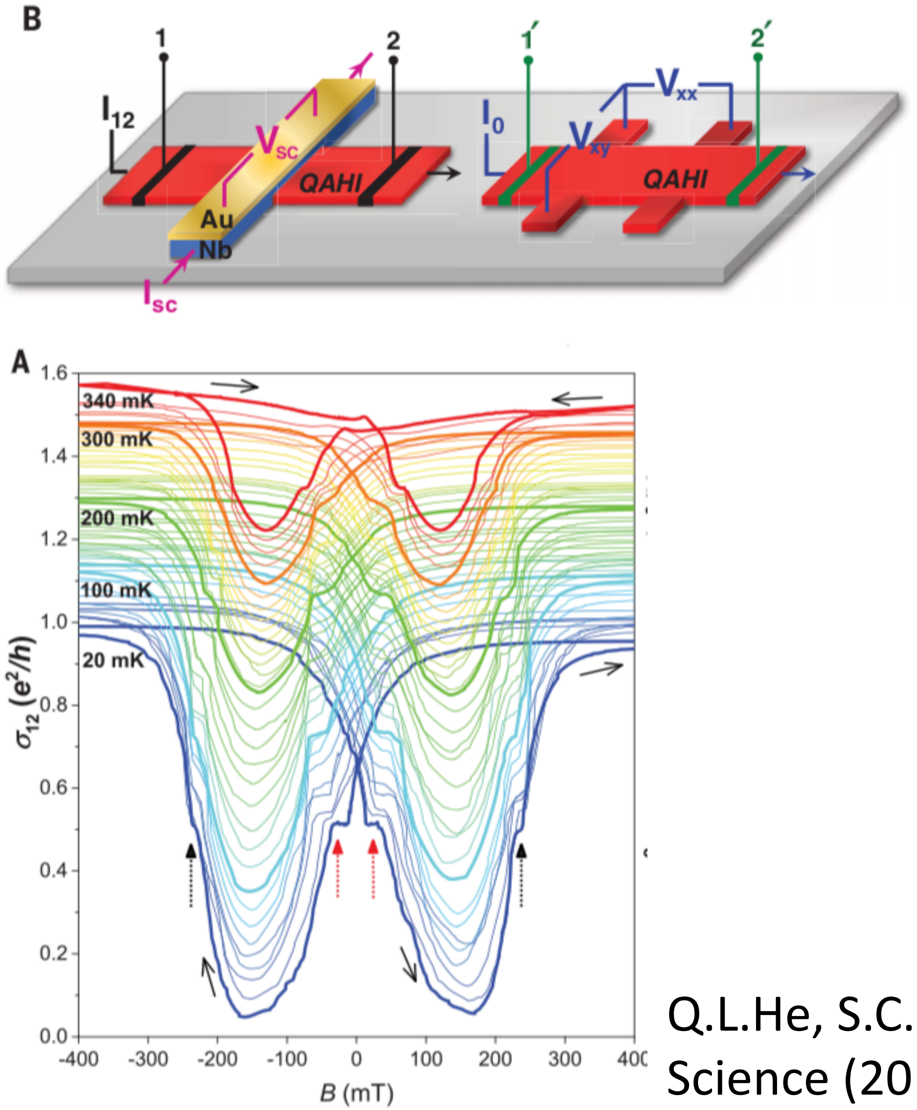


Majorana Fermion: Kitaev 1D Chain

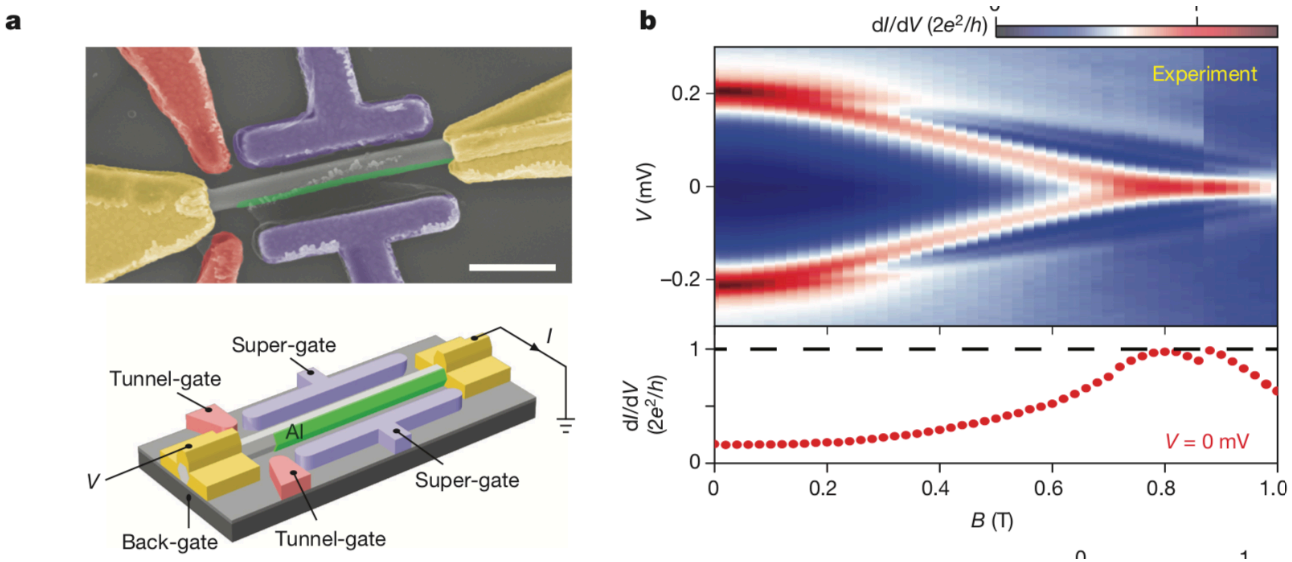


A Yu Kitaev, **Unpaired Majorana fermions in quantum wires**
Physics-Uspekhi, 2001

Current Experiments



Q.L.He, S.C. Zhang, K.L. Wang, et al
Science (2017)



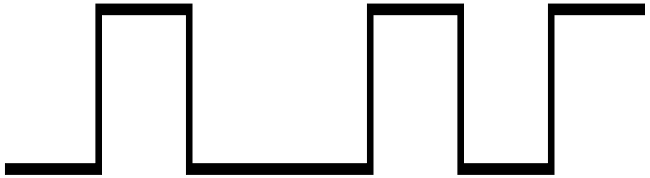
Hao Zhang, Leo p. Kouwenhoven, et al,
Nature (2018)

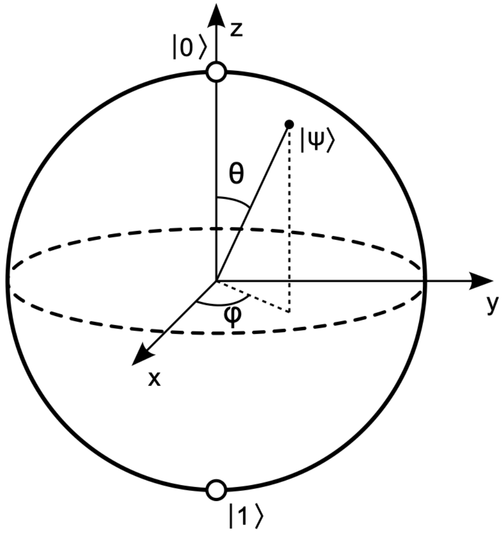
Quantum Computation

Classical Computation

Quantum Computation

Topological Quantum Computation





Topology?