概率论第七周作业

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1. $X_n = Y_{2n} = \sum_{j=1}^{2n} Z_j (n \geqslant 1)$

$$P\{X_{n+1} = i_{n+1} | X_1 = i_0, \ldots, X_n = i_n\} = \begin{cases} p^2, i_{n+1} - i_n = 2 \\ 2pq, i_{n+1} = i_n \\ q^2, i_{n+1} - i_n = -2 \end{cases}$$

故其为马尔可夫链，

$p(i,i) = 2pq$,$p(i, i-2) = q^2, p(i, i+2) = p^2$

$P(X_1 = -2) = q^2$

$P(X_1 = 0) = 2pq$

$P(X_1 = 2) = p^2$

3.

$$P\{X_{n+1} = i_{n+1} | X_1 = i_0, \ldots, X_n = i_n\} = \begin{cases} \frac{X_n}{N}(1 - \frac{X_n}{N}), i_{n+1} - i_n = 2 \\ \frac{X_n}{N}(1 - \frac{X_n}{N}), i_{n+1} = i_n \\ \frac{X_n^2}{N} + (1 - \frac{X_n}{N})^2, i_{n+1} - i_n = -2 \end{cases}$$

$p(i, i+1) = \frac{X_i}{N}(1 - \frac{X_i}{N})$

$p(i,i) = \frac{X_i^2}{N^2} + (1 - \frac{X_i}{N}^2)$

$p(i, i-1) = \frac{X_i}{N}(1 - \frac{X_i}{N})$

5.(1)

$$P^2 = \begin{pmatrix} q & 0 & p \\ 0 & p+q & 0 \\ q & 0 & p \end{pmatrix} = \begin{pmatrix} q & 0 & p \\ 0 & 1 & 0 \\ q & 0 & p \end{pmatrix}$$

$$P^4 = P^{2^2} = \begin{pmatrix} q & 0 & p \\ 0 & 1 & 0 \\ q & 0 & p \end{pmatrix}$$

(2)

$$P^n = \begin{cases} P, n\text{为奇数} \\ P^2, n\text{为偶数} \end{cases}$$

6.(1) $\frac{2}{5} \times \frac{1}{3} \times \frac{3}{5} \times \frac{1}{4} \times \frac{3}{5} \times \frac{1}{4} \times \frac{2}{5} = \frac{3}{2500}$

(2) $P\{X_{n+2} = c | X_n = b\} = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$

9.(1) $1 \to 3 \to 2 \to 1$，故1，2，3为闭集、相通

$f_{33} = p_{33} + p_{22}p_{21}p_{13} = 1$故3为常返状态，则1，2，3均为常返状态

$f_{44} = \frac{2}{3}$，则4为非常返

$p_{33} > 0$，则1，2，3非周期，$p_{44} > 0$，故4非周期

(2)$\mu_3 = 1 \cdot 2 \cdot ... f_{11}^n = \frac{1}{2^{n-2}}$

故$\mu_1 = 4$。

10.

$$P^2 = \begin{pmatrix} \frac{5}{12} & \frac{13}{36} & \frac{2}{9} \\ \frac{7}{18} & \frac{7}{18} & \frac{2}{9} \\ \frac{7}{18} & \frac{13}{36} & \frac{1}{4} \end{pmatrix}$$

对于链中所有状态均为正重现和非周期，则该链遍历。

故设$P_{max} = (x, y, z)$,$x + y + z = 1$

$P_{max} \cdot P = P_{max}$

可得$P_{max} = (\frac{14}{35}, \frac{13}{35}, \frac{8}{35})$

12.对于链中所有状态均为正重现和非周期，则该链遍历。

设P_m为极限分布，则$P_m = (\frac{1}{2}, \frac{1}{2})$

故$n \to \infty$,

$$P^n = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

16.

(1)链不可分，非周期，平稳分布$\pi_0 = \frac{1}{5}$,$\pi_k = \frac{3}{4}^{k-1} \times \frac{1}{5}$,链为正常返。

(2)链不可分，周期为2，无平稳分布，所以状态零常返。

(3)链不可分，非周期，无平稳分布，所有状态零常返。

(4)链不可分，非周期，$\pi_k = p^k q$，所有状态正常返。