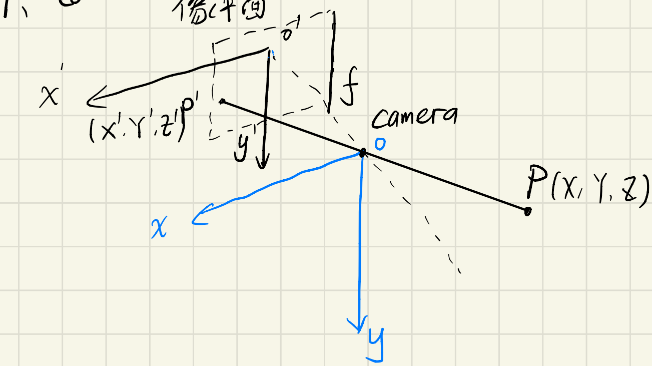
1、①



由相似关系:

$$\frac{Z}{f} = -\frac{X}{X'} = -\frac{Y}{Y'}$$

将成像平面移至相机前方，简化成

$$\frac{Z}{f} = \frac{X}{X'} = \frac{Y}{Y'}$$

$$\Rightarrow \quad X' = f\frac{X}{Z}$$

$$Y' = f\frac{Y}{Z}$$

记像素坐标为 (u,v)，

$$\begin{cases} u = \alpha X' + c_x \\ v = \beta Y' + c_y \end{cases}$$

其中 c_x, c_y 为偏移量；α, β 为缩放比例

$$\Rightarrow \begin{cases} u = \alpha f\frac{X}{Z} + c_x \\ v = \beta f\frac{Y}{Z} + c_y \end{cases} \triangleq \begin{cases} u = f_x\frac{X}{Z} + \cancel{c_y}\, c_x \\ v = f_y\frac{Y}{Z} + c_y \end{cases}$$

②

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \frac{1}{Z}\begin{bmatrix} f_x & 0 & c_x & 0 \\ 0 & f_y & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

2.

$$H = H_S H_A H_P = \begin{bmatrix} sR & t \\ \vec{0}^T & 1 \end{bmatrix}\begin{bmatrix} K & \vec{0} \\ \vec{0}^T & 1 \end{bmatrix}\begin{bmatrix} I & \vec{0} \\ \vec{v}^T & w \end{bmatrix}$$

$$= \begin{bmatrix} sRK + \vec{t}\vec{v}^T & \vec{t}w \\ \vec{v}^T & w \end{bmatrix} = \begin{bmatrix} 8.375 & -0.5 & 6 \\ 16.275 & 15.5 & 12 \\ 4 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow \quad \vec{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \qquad w = 3 \qquad \vec{t} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$sRK = \begin{bmatrix} 0.375 & -4.5 \\ 0.375 & 7.5 \end{bmatrix} \qquad 且 |K| = 1$$

$$\Rightarrow s = \frac{3}{\sqrt{2}} \quad R = \begin{bmatrix} \frac{3}{2} & \frac{-3}{2} \\ \frac{3}{2} & \frac{3}{2} \end{bmatrix} \qquad K = \begin{bmatrix} \frac{1}{4} & 1 \\ 0 & 4 \end{bmatrix}$$

$$\therefore H = \begin{bmatrix} \frac{3}{2} & -\frac{3}{2} & 2 \\ \frac{3}{2} & \frac{3}{2} & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & 1 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 2 & 3 \end{bmatrix}$$

3. ① 仿射变换矩阵 $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

② 设 $A(x_1, y_1)$ $B(x_2, y_2)$ 在 I 上　平行线 $I = (a, b, c)^T$　$I' = (a, b, c')$

$A'(x_3, y_3)$ $B'(x_4, y_4)$ 在 I' 上

有 $x' = a_{11}x + a_{12}y + t_x$　令 $X_1 = x_2 - x_1$　$Y_1 = y_2 - y_1$　; $k = \frac{X_2}{X_1} = \frac{Y_2}{Y_1}$

$y' = a_{21}x + a_{22}y + t_y$　　$X_2 = x_4 - x_3$　$Y_2 = y_4 - y_3$

$$\frac{\cancel{A}[(AB)]^2}{\cancel{A}[(A'B')]^2} = \frac{[a_{11}(x_2 - x_1) + a_{12}(y_2 - y_1)]^2 + [a_{21}(x_2 - x_1) + a_{22}(y_2 - y_1)]^2}{[a_{11}(x_4 - x_3) + a_{12}(y_4 - y_3)]^2 + [a_{21}(x_4 - x_3) + a_{22}(y_2 - y_1)]^2}$$

$$= \frac{[a_{11}^2 + a_{21}^2] X_1^2 + 2(a_{11}a_{12} + a_{21}a_{22}) X_1 Y_1 + (a_{12}^2 + a_{22}^2) Y_1^2}{[a_{11}^2 + a_{21}^2] X_2^2 + 2(a_{11}a_{12} + a_{21}a_{22}) X_2 Y_2 + [a_{12}^2 + a_{22}^2) Y_2^2} = \frac{1}{k^2}$$

$$\Rightarrow \quad \frac{AB}{A'B'} = \frac{1}{k} = \frac{\cancel{d}[AB]}{\cancel{d}[A'B']}$$