# MATH 424B - Autumn 2021 - Syllabus

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## Continuity *(Lectures 1-4)*

1. Review of continuity from 327: $\varepsilon - \delta$ definition, sequential definition, composition and sum/product/quotient of continuous functions, Intermediate Value Theorem, Extreme Value Theorem; [*Fitzpatrick 3.1-3.2-3.3-3.5*]

2. Uniform continuity: $\varepsilon - \delta$ definition, sequential definition; [*Fitzpatrick 3.4*]

3. Monotone functions: criterion for continuity for monotone functions, continuity of the inverse; [*Fitzpatrick 3.6*]

4. Limits: $\varepsilon - \delta$ definition, sequential definition, composition and sum/product/quotient properties, function is continuous at a point iff the limit exists and equal the value of the function at that point. [*Fitzpatrick 3.7*]

   [*Chapter 4 in Rudin is also an excellent reference for most of the topics above*]

## Differentiation *(Lectures 7-13)*

5. Derivatives: tangent lines, definition of differentiable function and of its derivative, differentiable functions are continuous, composition and sum/product/quotient of differentiable functions, differentiating of the inverse; [*Fitzpatrick 4.1-4.2*]

6. MVT: Rolle's theorem, Mean Value Theorem, derivative is 0 iff function is constant, criterion for strict monotonicity, maxima and minima, Generalized (Cauchy) MVT, definition of infinite limits and limits to infinity, de l'Hôpital rule (OPTIONAL), Taylor's Theorem; [*Fitzpatrick 4.3 + Thm 4.23 + Rudin Thm 5.15*]

7. Intermediate Value Property: review of IVT, example of a function which is differentiable at a point but the derivative is not continuous, Darboux's Theorem (IVT for derivatives).

   [*Chapter 5 in Rudin up to Thm 5.15 is also an excellent reference for most of the topics above*]

## Integration *(Lectures 15-22)*

8. Partitions; upper and lower Riemann integrals; refinements; equivalent conditions for integrability (qith $\varepsilon$ or sequences); properties of the integral (sum, etc); integrability of step functions; integrability of continuous functions; integrability of monotone functions; composition and integrability of product and absolute value; [*Fitzpatrick 6.1-6.2-6.3*]

9. Fundamental Theorem of Calculus (two statements); integration by parts; change of variables; IVT for integrals; [*Fitzpatrick 6.4-6.5-7.2*]

10. Riemann-Stieltjes integral (OPTIONAL); [*Rudin Chapter 6 up to Thm 6.22*]

11. Improper integrals; principal value integrals (OPTIONAL).

    [*If you set $\alpha(x) = x$, Chapter 6 in Rudin up to Thm 6.22 is also an excellent reference for most of the topics above*]

## Sequences of functions *(Lectures 5-6, 23 + note from Week 5)*

12. Sequences of functions: pointwise convergence, uniform convergence, definition of uniform convergence using supremum, uniformly Cauchy sequences and equivalence to uniform convergence, uniform limit of continuous functions is continuous; [*Fitzpatrick 9.2-9.3 + Thm 9.31*]

13. Sequence of functions and integration; sequences of functions and differentiation; [*Fitzpatrick Thms 9.32 & 9.33, Rudin Thms 7.16 & 7.17*]

14. Equicontinuity; Arzelà-Ascoli Theorem (OPTIONAL). [*HW7*]

    [*Chapter 7 in Rudin up to Thm 7.25 is also an excellent reference for most of the topics above*]

## Series of functions *(Lectures 24-29)*

15. Absolute convergence, uniform convergence; results on uniformly convergent sequences translated for series (sequence of partial sums); Weierstrass $M$-test; [*Rudin Thm 7.10*]

16. Power series: radius of convergence; differentiation of power series; inversion of order of summations; Taylor's theorem; [*Rudin 3.38-3.39-3.40 + Chapter 8 up to Thm 8.4*]

17. Cauchy product: definition, two theorems on convergence. [*Rudin 3.48-3.49-3.50-3.51 (proof on page 175)*]

18. A continuous nowhere differentiable function. [*Fitzpatrick 9.6*]