论文分享

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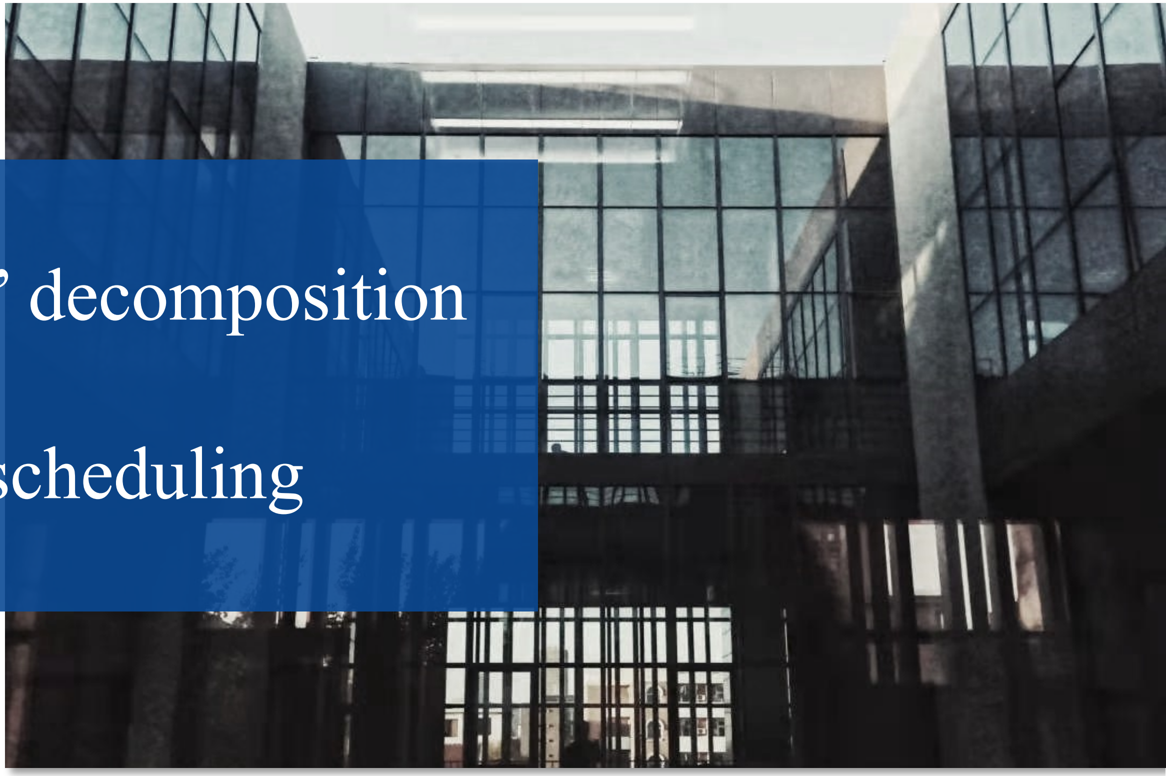
Propagating logic-based Benders' decomposition approaches for distributed operating room scheduling



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Outline

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Discrete Optimization

Propagating logic-based Benders' decomposition approaches for distributed operating room scheduling

Vahid Roshanaei [a], Curtiss Luong [a], Dionne M. Aleman [a b c], David Urbach [d]

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- 多家医院共享医疗资源，统一调度：存在调度中心。
- 目标为最小化手术室运营成本和患者等待成本。

Vahid Roshanaei, Curtiss Luong, Dionne M. Aleman, David Urbach,Propagating logic-based Benders' decomposition approaches for distributed operating room scheduling,European Journal of Operational Research,Volume 257, Issue 2,2017,Pages 439-455,ISSN 0377-2217, https://doi.org/10.1016/j.ejor.2016.08.024.

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介绍——问题描述

- DORS 是一个考虑患者优先级的多医院集中手术安排调度问题，涉及概念如下：
- 一组协作医院 ($h \in H$)、每家医院有多个手术室 ($r \in R_h$)、规划期 ($d \in D$)，网络中 OR 的数量 为$|H| \times |D| \times |R_h|$，每个 OR 的每日可用时间$B_{hd}$。
- 手术室的开放成本（surgical suites and ORs）：套间开放成本、手术室的开放成本（医护人员成本等）
- 患者 ($p \in P$)。 考虑了患者手术的等待时间 (α_p) 和患者手术紧急程度评分 (ρ_p)。
- 患者等待成本：等待时间 * 紧急程度评分
- DORS 安排医院等候名单中的全部或部分现有患者。 任何未安排在当前计划范围内的患者都将添加到要安排在下一个计划范围内的患者池中。
- 安排在当前规划期内手术的患者分等待成本权重和未安排在当前规划期内手术的患者的等待权重不同（患者手术预备成本）。

MODEL

✓Model介绍

✓Model实现

MODEL介绍—重要假设

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（1） 所有 OR 和外科医生在每个外科专业中的功能都相同；

（2）主医师以及护士和麻醉师分配到 ORs 已执行；

（3）每个 OR 在一天内仅分配给一名外科医生（患者到 OR 的分配等同于患者到外科医生的分配）；

（4） 医院拥有相同数量的手术室（参考真实数据集）；

（5）仅在一个外科专科内进行协同调度；

（6）为简化计算，手术持续时间被认为是确定性的；

（7）单次手术持续时间 (Tp) 小于最大 OR 开放时间(Bhd)。

MODEL介绍——参数、目标及约束条件OP

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Table 1
IP notation.

Sets:	
$\mathcal{P}$	Set of patients, $p \in \mathcal{P}$
$\mathcal{P}'$	Set of mandatory patients, $\mathcal{P}' = \{p \mid \rho_p(\lvert\mathcal{D}\rvert - \alpha_p) \le -\Gamma\}$
$\mathcal{H}$	Set of hospitals, $h \in \mathcal{H}$
$\mathcal{D}$	Set of days in the planning horizon, $d \in \mathcal{D}$
$\mathcal{R}_h$	Set of ORs in each hospital's surgical suite, $r \in \mathcal{R}_h$
Parameters:	
G_{hd}	Cost of opening the surgical suite in hospital h on day d
F_{hd}	Cost of opening an OR in hospital h on day d
B_{hd}	Regular operating hours of each OR on day d in hospital h
T_p	Total booked time (preparation time + surgery time + cleaning time) of patient p
ρ_p	Health status score assigned to patient p
α_p	Number of days elapsed from the referral date of patient p
κ_1	Waiting cost for scheduled patients
κ_2	Waiting cost for unscheduled patients
Γ	Health status threshold above which patients have to be operated
Variables:	
x_{hdp}	1 if patient p is assigned to hospital h on day d, 0 otherwise
u_{hd}	1 if the surgical suite in hospital h is opened on day d, 0 otherwise
y_{hdr}	1 if OR r of surgical suite of hospital h is opened on day d, 0 otherwise
x_{hdpr}	1 if patient p is assigned to OR r of hospital h on day d, 0 otherwise
w_p	1 if patient p is not scheduled this horizon, 0 otherwise

$$\text{minimize} \quad \sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}} G_{hd}u_{hd} + \sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}_h} F_{hd}y_{hdr}$$

$$+ \sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}}\sum_{p\in\mathcal{P}}\sum_{r\in\mathcal{R}_h} \kappa_1[\rho_p(d-\alpha_p)x_{hdpr}] \qquad \text{(IP)}$$

$$+ \sum_{p\in\mathcal{P}\setminus\{\mathcal{P}'\}} \kappa_2[\rho_p(\lvert\mathcal{D}\rvert + 1 - \alpha_p)w_p]$$

$$\text{subject to} \quad \sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}_h} x_{hdpr} = 1 \qquad \forall p \in \mathcal{P}' \qquad (1)$$

$$\sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}_h} x_{hdpr} + w_p = 1 \qquad \forall p \in \mathcal{P}\setminus\{\mathcal{P}'\} \qquad (2)$$

$$\sum_{p\in\mathcal{P}} T_p x_{hdpr} \le B_{hd}y_{hdr} \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D};\ r\in\mathcal{R}_h \qquad (3)$$

$$y_{hdr} \le y_{hd,r-1} \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D};\ r\in\mathcal{R}_h\setminus\{1\} \qquad (4)$$

$$y_{hdr} \le u_{hd} \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D};\ r\in\mathcal{R}_h \qquad (5)$$

$$x_{hdpr} \le y_{hdr} \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D};\ p\in\mathcal{P};\ r\in\mathcal{R}_h \qquad (6)$$

$$u_{hd}, y_{hdr}, x_{hdpr} \in \{0,1\} \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D};\ p\in\mathcal{P};\ r\in\mathcal{R}_h$$

$$w_p \in \{0,1\} \qquad \forall p \in \mathcal{P}\setminus\{\mathcal{P}'\}.$$

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MODEL实现——算例生成

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Table 2
Parameter values.

κ_1	50 dollars
κ_2	5 dollars
Γ	500 (6–10% of all patients identified as mandatory)
ρ_p	Uniform distribution [1, 5], where 1 is least urgent 5 is the most urgent
B_{hd}	Uniform distribution [420, 480] minutes in 15-minutes intervals
α_p	Uniform distribution [60, 120] days
F_{hd}	Uniform distribution [4000, 6000]
G_{hd}	Uniform distribution [1500, 2500]

One dataset has three ORs per hospital (yielding easy SPs but a hard MP) and the other has five ORs (yielding hard SPs but an easy MP).

算法

- ✓ 算法介绍
- ✓ 算法实现

算法介绍——引入：BD、LBBD

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分支定价法：分支定界+列生成算法

列生成算法：通过求解子问题（sub problem）来找到可以进基的非基变量，该非基变量在模型中并没有显性的写出来（可以看成是生成了一个变量，每个变量其实等价于一列，所以该方法被称为列生成算法）。如果找不到一个可以进基的非基变量，那么就意味着所有的非基变量的检验数（Reduced Cost，RC）都满足最优解的条件，也就是说，该线性规划的最优解已被找到。

分解算法

Benders Decomposition算法（BD）：将问题划分为Master Problem（MIP） 和 Subproblem，但是Subproblem必须为线性规划模型。算法核心：optimal cut 和 feasible cut。算法常使用于便于分解为两阶段的问题。

Logic-based Benders Decomposition算法（LBBD）：由于BD分解算法对问题的结构要求、Subproblem的要求较高，进而提出了基于问题逻辑的BD分解算法。在LBBD算法中，Master problem 和 Subproblem 的要求降低：MP仍然需要为MIP，但是SUBP可为：LP、MIP、CP、可行性检查等。算法核心：问题拆解、optimal cut 和 feasible cut及其证明。

算法介绍——框架

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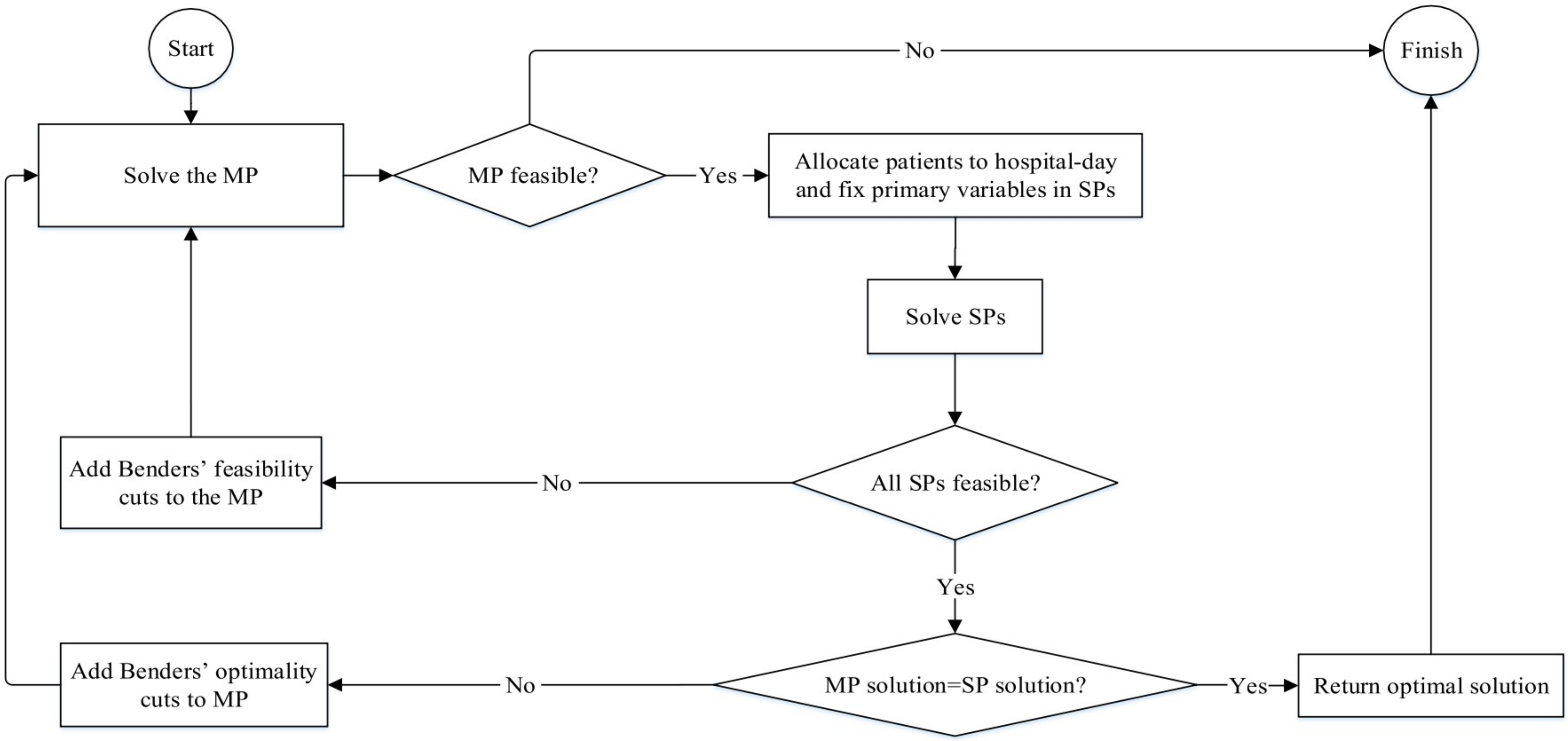


Fig. 1. LBBD flowchart for DORS.

算法介绍—— Location-allocation master problem MP sandiago

Decision: patient → hospital-day， the minimum number of ORs required for each hospital-day.

$$\begin{aligned} \text{minimize} \quad & \sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}} G_{hd}u_{hd} + \sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}} F_{hd}y_{hd} \\ & + \sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}}\sum_{p\in\mathcal{P}} \kappa_1[\rho_p(d-\alpha_p)x_{hdp}] \\ & + \sum_{p\in\mathcal{P}\setminus\{\mathcal{P}'\}} \kappa_2[\rho_p(|\mathcal{D}|+1-\alpha_p)w_p] \end{aligned} \qquad \text{(MP)}$$

$$\text{subject to} \quad \sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}} x_{hdp} = 1 \qquad \forall p\in\mathcal{P}' \qquad (7)$$

$$\sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}} x_{hdp} + w_p = 1 \qquad \forall p\in\mathcal{P}\setminus\{\mathcal{P}'\} \qquad (8)$$

$$x_{hdp} \le u_{hd} \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D};\ p\in\mathcal{P} \qquad (9)$$

$$\boxed{\sum_{p\in\mathcal{P}} T_p x_{hdp} \le |\mathcal{R}_h| B_{hd} u_{hd} \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D}} \qquad (10)$$

$$T_p x_{hdp} \le B_{hd} \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D};\ p\in\mathcal{P} \qquad (11)$$

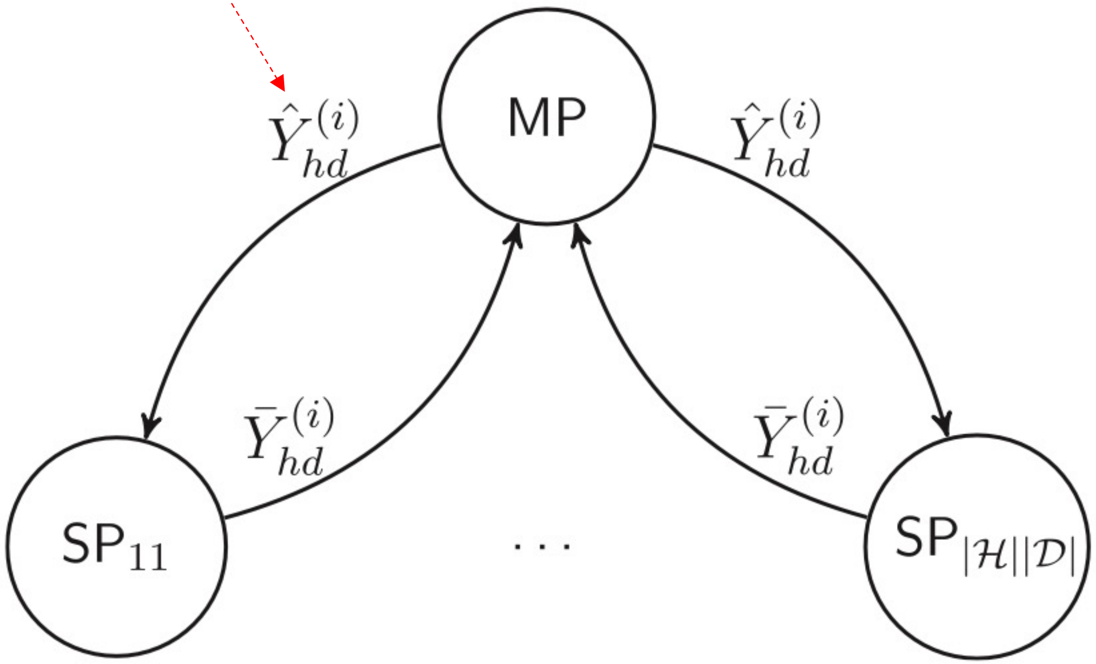
$$\boxed{y_{hd} \ge \frac{\sum_{p\in\mathcal{P}} T_p x_{hdp}}{B_{hd}} \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D}} \qquad (12)$$

$$y_{hd} \le |\mathcal{R}_h| \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D} \qquad (13)$$

$$y_{hd} \in \mathbb{Z}^+ \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D}$$

$$u_{hd}, x_{hdp} \in \{0,1\} \qquad \forall h\in\mathcal{H};\ d\in\mathcal{D};\ p\in\mathcal{P}$$

$$w_p \in \{0,1\} \qquad \forall p\in\mathcal{P}\setminus\{\mathcal{P}'\}.$$



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算法介绍—— OR allocation sub-problems, first-fit sandiago decreasing heuristic algorithm SP

Input： patient → hospital-day $\hat{\mathcal{P}}_{hd}^{(i)}$

Decision : the minimum number of ORs required for each hospital-day, patient → OR (hospital-day)

$$\text{minimize} \quad \bar{Y}_{hd}^{(i)} = \sum_{r \in \mathcal{R}_h} y_r \tag{SP}$$

$$\text{subject to} \quad \sum x_{pr} = 1 \qquad \forall p \in \hat{\mathcal{P}}_{hd}^{(i)} \tag{14}$$

$$\sum_{p \in \hat{\mathcal{P}}_{hd}^{(i)}} T_p x_{pr} \le B_{hd} y_r \qquad \forall r \in \mathcal{R}_h \tag{15}$$

$$x_{pr} \le y_r \qquad \forall p \in \hat{\mathcal{P}}_{hd}^{(i)};\ r \in \mathcal{R}_h \tag{16}$$

$$y_r \le y_{r-1} \qquad \forall r \in \mathcal{R}_h \setminus \{1\} \tag{17}$$

$$x_{pr}, y_r \in \{0, 1\} \qquad \forall p \in \hat{\mathcal{P}}_{hd}^{(i)};\ r \in \mathcal{R}_h. \tag{18}$$

此问题可抽象为背包问题，而此问题已经有较好的启发式算法**FFD**（求解快，效果好），故而可使用**FFD**快速生成可行解并计算目标。 $\bar{F}_{hd}^{(i)}$

$$\hat{Y}_{hd}^{(i)} \le \bar{Y}_{hd}^{(i)} \le \bar{F}_{hd}^{(i)}$$

若 $\bar{F}_{hd}^{(i)}$ $\hat{Y}_{hd}^{(i)}$ 相等，则说明启发式算法求解至最优解（**MP**的输出为**SP**的下界）。不等则使用**IP**求解**SP**。

算法介绍—— Benders cut ： SP 解的情况讨论 sandiago

1. Infeasible SP if $\hat{\mathcal{P}}_{hd}^{(i)}$ is not packable within $|\mathcal{R}_h|$, implying $\bar{Y}_{hd}^{(i)} > |\mathcal{R}_h|$. $\Rightarrow \bar{\mathcal{U}}^{(i)}$
2. Optimal MP and optimal SPs, but $\hat{Y}_{hd}^{(i)} \neq \bar{Y}_{hd}^{(i)}$. $\Rightarrow \bar{\mathcal{J}}^{(i)}$
3. Global optimal solution if $\hat{Y}_{hd}^{(i)} = \bar{Y}_{hd}^{(i)}$.

feasible cut $\bar{\mathcal{U}}^{(i)}$

$$\sum_{p \in \hat{\mathcal{P}}_{hd}^{(i)}} (1 - x_{hdp}) \geq 1 \qquad \forall (h,d) \in \bar{\mathcal{U}}^{(i)}. \tag{19}$$

optimal cut $\bar{\mathcal{J}}^{(i)}$

$$y_{hd} \geq \bar{Y}_{hd}^{(i)} - \sum_{p \in \hat{\mathcal{P}}_{hd}^{(i)}} (1 - x_{hdp}) \qquad \forall (h,d) \in \bar{\mathcal{J}}^{(i)}. \tag{20}$$

Benders cut 有效性证明

（1）没有 cut 任何原问题OP的可行解； （2） cut 当前MP的最优解

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feasible cut $\bar{\mathcal{U}}^{(i)}$

$$\sum_{p\in\hat{\mathcal{P}}_{hd}^{(i)}}(1-x_{hdp})\geq 1 \qquad \forall(h,d)\in\bar{\mathcal{U}}^{(i)}. \qquad (19)$$

$$\sum_{p\in\hat{\mathcal{P}}_{hd}^{(i)}}(1-x_{hdp}) = \sum_{p\in\hat{\mathcal{P}}_{hd}^{(i)}\cap\tilde{\mathcal{P}}_{hd}}(1-x_{hdp}) + \sum_{p\in\hat{\mathcal{P}}_{hd}^{(i)}\setminus\tilde{\mathcal{P}}_{hd}}(1-x_{hdp})$$

Benders cut 有效性证明

（1）没有 cut 任何原问题OP的可行解；（2） cut 当前MP的最优解

Proof Feasible Cut （至少从当前MP分配至SP_{hd}的患者集合中移除一个患者）：

1） 显而易见，（2）满足

2）proof （1）：OP在（h，d）上的患者集合（S1）和当前MP在（h，d）上的输出集合（S2）存在四种关系：S1 不等于S2，S1包含S2； S1 不等于S2，S2包含S1； S1 不等于S2，S1交S2不为空； S1 不等于S2，S1交S2为空。易知（1）满足

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optimal cut $\bar{\mathcal{J}}^{(i)}$

$$y_{hd} \geq \bar{Y}_{hd}^{(i)} - \sum_{p \in \hat{\mathcal{P}}_{hd}^{(i)}} (1 - x_{hdp}) \qquad \forall (h, d) \in \bar{\mathcal{J}}^{(i)}.$$

$$(20) \quad y_{hd} - \bar{Y}_{hd}^{(i)} + \left(\left| \hat{\mathcal{P}}_{hd}^{(i)} \right| - \sum_{p \in \hat{\mathcal{P}}_{hd}^{(i)}} x_{hdp} \right) \geq 0 \qquad \forall (h, d) \in \bar{\mathcal{J}}^{(i)}.$$

Benders cut 有效性证明

（1）没有 cut 任何原问题OP的可行解；（2） cut 当前MP的最优解

Proof Optiaml Cut （至少从当前MP分配至SP_{hd}的患者集合中移除一个患者或增加OR开放数量）：

1） 显而易见，（2）满足

2）proof （1）：保持开放OR的数量不变，证明过程与feasible cut证明过程相同。保持患者分配不变：设OP开放数量为y1，MP输出为y2，如y1小于等于y2则易知，此解infeasible。必然有y1 大于y2。 式（20）满足。

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算法介绍—— feasible cut增强、Cut propagation for LBBDs

feasible cut $\bar{\mathcal{U}}^{(i)}$

$$\sum_{p \in \mathcal{P}_{hd}^{(i)}} (1 - x_{hdp}) \geq 1 \qquad \forall (h,d) \in \bar{\mathcal{U}}^{(i)}. \tag{19}$$

增强

至少从当前MP分配至SP_{hd}的患者集合中移除一个患者或增加OR开放数量

$$y_{hd} \geq (|\mathcal{R}_h| + 1) - \sum_{p \in \mathcal{P}_{hd}^{(i)}} (1 - x_{hdp}) \qquad \forall (h,d) \in \bar{\mathcal{U}}^{(i)}. \tag{23}$$

Propagation：
If a combination of patients with known surgical times are not feasibly packable within the OR time of a certain hospital-day, it cannot be packed into other hospital-days with less or the same OR time.

算法介绍—— LBBD implementation techniques

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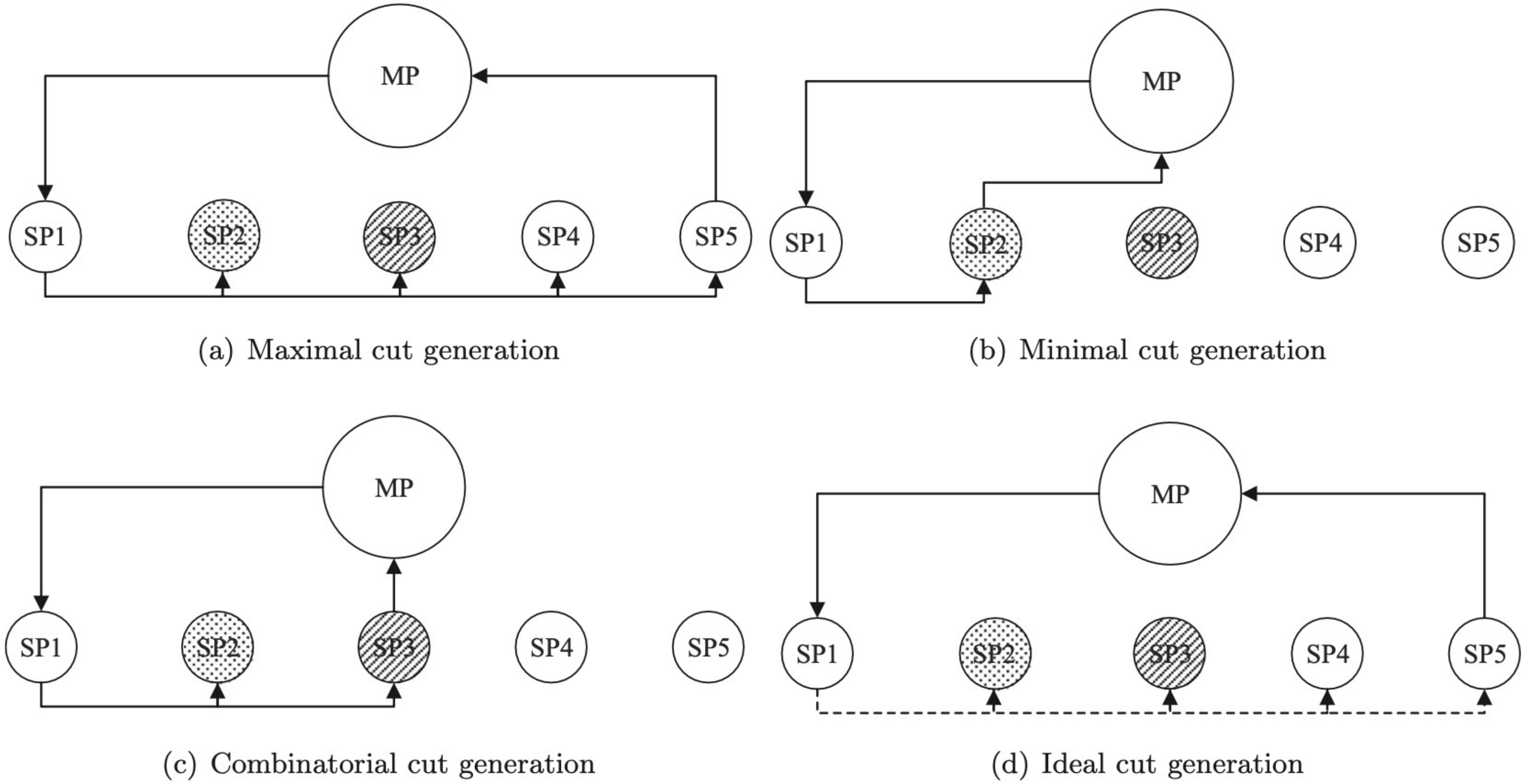

Fig. 3. LBBD implementation techniques. Unshaded SPs are optimally solved, dotted SPs are sub-optimally solved, and striped SPs are infeasible. Dashed lines indicate that SP feasibility is checked before an SP is solved.

a） 求解所有SP
b） 求解至出现cut
c） 求解至出现feasible cut
d） 求解至两种cut均出现

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实验分析

- ✓ 论文结果
- ✓ 结果复现
- ✓ 算法前景
- ✓ 个人总结

实验——论文结果

Table 3
Average CPU time (seconds) of LBBD implementations and IP+Gurobi over five trials (bold is the best performance in each test case; superscripts are the average number of unsolved trials per instance; the average CPU time of each implementation is taken over five trials for each of the six LBBDs).

	$\|\mathcal{P}\|$	Maximal	Minimal	Combinatorial	Ideal	LBBD grand average	IP+Gurobi
Five ORs	20	0.39	0.39	0.37	0.39	**0.39**	13.64
	40	7.04	0.93	6.20	7.08	**5.31**	$68.57^{(1.00)}$
	60	7.27	4.03	8.06	7.38	**6.69**	$305.72^{(3.00)}$
	80	21.07	183.55	35.07	15.54	**63.81**	$2233.70^{(3.00)}$
	100	109.43	103.79	140.83	129.63	**120.92**	$******^{(5.00)}$
	120	$307.17^{(0.67)}$	$397.15^{(0.50)}$	$362.76^{(0.50)}$	$364.60^{(0.67)}$	$\mathbf{357.92}^{(0.59)}$	$******^{(5.00)}$
	140	$1367.39^{(1.67)}$	$1227.58^{(1.17)}$	$926.42^{(1.83)}$	$1132.72^{(1.67)}$	$\mathbf{1163.53}^{(1.59)}$	$******^{(5.00)}$
	160	$1263.14^{(1.50)}$	$1953.05^{(1.83)}$	$1744.88^{(1.83)}$	$1496.17^{(2.17)}$	$\mathbf{1614.31}^{(1.83)}$	$******^{(5.00)}$
Three ORs	20	0.49	0.49	0.52	0.45	**0.49**	14.40
	40	8.79	7.08	9.05	9.14	**8.52**	$5500.00^{(3.00)}$
	60	11.17	11.16	11.10	13.98	**11.85**	$2904.60^{(1.00)}$
	80	133.38	299.93	285.60	161.05	**1219.99**	$5718.90^{(4.00)}$
	100	$554.65^{(0.83)}$	$1451.79^{(0.33)}$	$1210.53^{(0.50)}$	$976.55^{(0.83)}$	$\mathbf{1048.38}^{(0.62)}$	$6500.90^{(4.00)}$
	120	$711.76^{(2.00)}$	$1589.68^{(1.17)}$	$1380.80^{(1.83)}$	$1755.94^{(2.00)}$	$\mathbf{1359.55}^{(1.75)}$	$5934.00^{(4.00)}$
	140	$929.89^{(3.17)}$	$1282.93^{(3.67)}$	$731.80^{(3.83)}$	$1970.31^{(3.67)}$	$\mathbf{1228.73}^{(3.59)}$	$4099.70^{(4.00)}$

实验——论文结果……

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Table 4
Five ORs: average CPU time (seconds) of LBBDs over five trials (bold is the best performance in each test case; superscripts are the numbers of unsolved trials; highlight indicates improved performance with cut propagation).

	$\|\mathcal{P}\|$	LBBD1	LBBD1$_P$	LBBD2	LBBD2$_P$	LBBD3	LBBD3$_P$
Maximal	20	0.40	**0.38**	**0.38**	**0.38**	0.40	0.39
	40	0.92	0.97	**0.87**	8.49	**0.87**	30.13
	60	5.12	13.10	4.84	**3.30**	4.45	12.85
	80	62.28	18.11	**3.07**	5.59	13.34	24.00
	100	126.74	95.26	**36.73**	44.31	187.17	166.36
	120	309.55	810.01	**113.27**	176.36$^{(1)}$	119.94$^{(1)}$	313.88$^{(2)}$
	140	1770.20$^{(2)}$	**2060.60**	883.87$^{(2)}$	391.04$^{(1)}$	2229.10$^{(3)}$	869.50$^{(2)}$
	160	1164.30$^{(1)}$	1422.80$^{(1)}$	1629.10$^{(1)}$	**372.70**$^{(1)}$	2339.70$^{(3)}$	650.22$^{(2)}$
Average		429.94$^{(3)}$	**552.65**$^{(1)}$	334.02$^{(3)}$	125.27$^{(3)}$	611.87$^{(7)}$	258.42$^{(6)}$
Minimal	20	0.39	**0.38**	**0.38**	**0.38**	0.40	0.40
	40	0.87	**0.61**	0.85	1.05	0.86	1.31
	60	7.08	**1.66**	3.15	2.57	6.84	2.88
	80	4.21	**1.43**	94.08	33.76	726.50	241.34
	100	144.75	65.67	45.00	**39.55**	247.58	80.21
	120	653.11$^{(1)}$	**259.64**	557.61	421.86	126.07$^{(1)}$	364.62$^{(1)}$
	140	2336.90$^{(1)}$	1325.70$^{(1)}$	632.97$^{(1)}$	**400.28**$^{(1)}$	1149.20$^{(2)}$	1520.40$^{(1)}$
	160	**3548.90**	1944.20$^{(1)}$	1373.70$^{(1)}$	275.40$^{(2)}$	3907.30$^{(4)}$	665.82$^{(3)}$
Average		837.03$^{(2)}$	449.91$^{(2)}$	**338.47**$^{(2)}$	146.74$^{(3)}$	770.59$^{(7)}$	359.62$^{(5)}$
Combinatorial	20	0.37	0.37	0.37	**0.36**	0.38	0.37
	40	0.87	0.94	0.86	3.61	**0.85**	30.04
	60	7.41	17.95	3.17	**2.52**	4.43	12.88
	80	**4.04**	41.61	94.13	33.55	13.14	23.96
	100	133.50	272.98	45.05	**39.30**	187.47	166.70
	120	534.61	897.12	200.85	**110.67**	119.67$^{(1)}$	313.63$^{(2)}$
	140	655.00$^{(2)}$	755.93$^{(1)}$	**627.27**$^{(1)}$	430.06$^{(2)}$	2222.10$^{(3)}$	868.18$^{(2)}$
	160	3506.00$^{(1)}$	**2184.10**$^{(1)}$	1468.80$^{(2)}$	274.13$^{(2)}$	2402.60$^{(3)}$	633.66$^{(2)}$
Average		605.23$^{(3)}$	**521.38**$^{(2)}$	269.08$^{(3)}$	111.84$^{(4)}$	618.83$^{(7)}$	256.18$^{(6)}$
Ideal	20	0.40	**0.38**	0.39	0.41	0.40	**0.38**
	40	0.92	0.98	**0.89**	8.64	0.91	30.16
	60	5.16	13.24	4.99	**3.43**	4.51	12.93
	80	**4.30**	37.90	6.92	6.40	13.48	24.26
	100	270.79	**82.22**	31.98	38.24	187.59	166.96
	120	382.33$^{(1)}$	**303.33**	528.36	541.15	119.50$^{(1)}$	312.92$^{(2)}$
	140	**517.50**$^{(1)}$	1443.70$^{(1)}$	605.73$^{(2)}$	1111.10$^{(1)}$	2247.40$^{(3)}$	870.90$^{(2)}$
	160	2788.9$^{(2)}$	2487.90$^{(2)}$	**377.90**$^{(1)}$	253.91$^{(3)}$	2404.50$^{(3)}$	663.91$^{(2)}$
Average		496.29$^{(4)}$	546.33$^{(3)}$	**196.65**$^{(3)}$	245.41$^{(4)}$	622.29$^{(7)}$	260.30$^{(6)}$
Grand average		592.12$^{(3.00)}$	**516.08**$^{(2.00)}$	888.61$^{(2.75)}$	158.54$^{(3.5)}$	655.90$^{(7.00)}$	283.63$^{(5.75)}$

Table 5
Three ORs: Average CPU time (seconds) of LBBDs over five trials (bold is the best performance in each test case; superscripts are the numbers of unsolved trials; asterisks are scenarios with no solved trials; highlight indicates improved performance with cut propagation).

	$\|\mathcal{P}\|$	LBBD1	LBBD1$_P$	LBBD2	LBBD2$_P$	LBBD3	LBBD3$_P$
Maximal	20	0.64	**0.45**	**0.45**	0.47	0.48	0.46
	40	**2.56**	6.84	10.59	10.73	10.82	11.17
	60	19.18	11.39	9.45	**8.94**	9.10	8.98
	80	183.22	297.55	**53.82**	103.97	55.05	106.65
	100	625.15$^{(2)}$	1581.00$^{(1)}$	**385.59**	135.47$^{(1)}$	402.16	138.54$^{(1)}$
	120	662.36$^{(2)}$	673.15$^{(2)}$	**291.47**$^{(2)}$	1175.5$^{(2)}$	295.80$^{(2)}$	1172.30$^{(2)}$
	140	2546.40$^{(3)}$	2276.30$^{(3)}$	305.51$^{(3)}$	51.95$^{(3)}$	341.57$^{(3)}$	57.60$^{(4)}$
Average		577.07$^{(7)}$	692.38$^{(6)}$	**150.98**$^{(5)}$	212.43$^{(6)}$	159.28$^{(5)}$	213.67$^{(7)}$
Minimal	20	0.65	0.46	**0.44**	0.45	0.45	0.46
	40	7.89	13.99	6.72	**3.49**	6.82	3.55
	60	12.69	16.87	9.41	**9.22**	9.45	9.32
	80	340.15	985.98	**52.44**	183.01	53.16	184.86
	100	2160.60$^{(1)}$	2440.60$^{(1)}$	**756.49**	1300.10	766.33	1286.6
	120	3668.30$^{(3)}$	3033.50$^{(2)}$	1033.10$^{(1)}$	**377.26**	1048.80$^{(1)}$	376.91
	140	******$^{(5)}$	******$^{(5)}$	**1086.40**$^{(3)}$	1476.20$^{(3)}$	1090.90$^{(3)}$	1479.10$^{(3)}$
Average		1031.71$^{(9)}$	1081.90$^{(8)}$	**420.71**$^{(4)}$	478.53$^{(3)}$	425.13$^{(4)}$	477.26$^{(3)}$
Combinatorial	20	0.60	0.52	0.51	0.50	0.50	**0.48**
	40	7.90	14.41	7.17	**3.71**	10.24	10.84
	60	12.96	16.97	9.48	9.23	9.06	**8.92**
	80	340.22	977.06	53.79	185.93	**52.47**	104.12
	100	2200.30$^{(1)}$	2480.00$^{(1)}$	768.24	1289.60	**388.91**	136.11$^{(1)}$
	120	3916.60$^{(3)}$	1280.10$^{(3)}$	950.97$^{(1)}$	**637.10**	300.12$^{(2)}$	1199.90$^{(2)}$
	140	******$^{(5)}$	******$^{(5)}$	1091.40$^{(3)}$	1488.80$^{(3)}$	297.87$^{(3)}$	49.12$^{(4)}$
Average		1079.76$^{(9)}$	794.84$^{(9)}$	411.65$^{(4)}$	**516.41**$^{(3)}$	151.314$^{(5)}$	215.64$^{(7)}$
Ideal	20	0.56	0.45	**0.42**	0.43	0.43	**0.42**
	40	**2.97**	14.56	10.99	6.52	9.98	9.84
	60	31.13	16.78	9.41	8.83	8.96	8.72
	80	243.60	469.33	53.97	**50.55**	51.93	96.91
	100	3147.40$^{(2)}$	1847.10$^{(1)}$	106.90$^{(1)}$	239.69	**386.11**	132.12$^{(1)}$
	120	4191.20$^{(2)}$	2853.50$^{(3)}$	960.99$^{(2)}$	**837.62**$^{(1)}$	309.02$^{(2)}$	1383.30$^{(2)}$
	140	7158.40$^{(4)}$	******$^{(5)}$	1277.30$^{(3)}$	1046.10$^{(3)}$	320.84$^{(3)}$	48.90$^{(4)}$
Average		2110.77$^{(8)}$	866.95$^{(9)}$	345.71$^{(6)}$	**312.82**$^{(4)}$	155.32$^{(5)}$	240.03$^{(7)}$
Grand average		1199.83$^{(8.25)}$	859.02$^{(8.00)}$	332.26$^{(4.75)}$	**380.05**$^{(4.00)}$	222.76$^{(4.75)}$	286.65$^{(6.00)}$

实验——结果复现

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LBBD × origin_problem ×

```
Optimal solution found (tolerance 1.00e-08)
Best objective -4.394050000000e+05, best bound -4.394050000000
Day-Hospital-Room: 0-0-0, Patient: 0, 3, 8,
Day-Hospital-Room: 0-0-1, Patient: 1, 19, 22,
Day-Hospital-Room: 0-0-2, Patient: 17, 27, 33,
Day-Hospital-Room: 0-0-3, Patient: 2, 13, 37,
Day-Hospital-Room: 0-0-4, Patient: 6, 30, 36,
Day-Hospital-Room: 1-2-0, Patient: 12, 14, 34,
Day-Hospital-Room: 1-2-1, Patient: 4, 7, 23,
Day-Hospital-Room: 1-2-2, Patient: 28, 39,
Day-Hospital-Room: 1-2-3, Patient: 9, 11, 21,
Day-Hospital-Room: 1-2-4, Patient: 25, 31, 35,
Day-Hospital-Room: 2-0-0, Patient: 5, 10, 15,
Day-Hospital-Room: 2-0-1, Patient: 16, 32, 38,
Day-Hospital-Room: 2-0-2, Patient: 18, 26,
Day-Hospital-Room: 2-0-3, Patient: 20, 24, 29,
Not Scheduled Patients: cost_suite:  5691.0
cost_room:  61504.0
cost_3:  -506600.0
cost_4:  0.0
objective:  -439405.0
time:  212.51316499710083
```

LBBD × origin_problem ×

```
H    0    0                    5.0000000    5.00000  0.00%    -    0s
     0    0    5.00000    0    5    5.00000    5.00000  0.00%    -    0s

Cutting planes:
  Gomory: 4
  Cover: 5
  MIR: 3
  StrongCG: 3
  Mod-K: 1
  RLT: 4

Explored 1 nodes (201 simplex iterations) in 0.01 seconds (0.00 work units)
Thread count was 8 (of 8 available processors)

Solution count 1: 5

Optimal solution found (tolerance 1.00e-08)
Best objective 5.000000000000e+00, best bound 5.000000000000e+00, gap 0.0000%
----------------------- iter: 14, add_cut_num: 0 ------------------------
------------------------------- optimal ----------------------------------
-------------------------- add_cut_num_all: 18 --------------------------
LBBD: optimal objective: -439405.0
LBBD time: 13.427375078201294
```

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V Roshanaei, C Luong, DM Aleman… - European Journal of …, 2017 - Elsevier

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感谢！
THANKS

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