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Problem 1

Given any convex $\mathcal{X} \neq \emptyset$, norm $\| \cdot \|$, and scalar $\epsilon \geq 0$, the following is convex:

$$\mathcal{X}_\epsilon := \{x : \inf_{\overline{x} \in \mathcal{X}} \| x - \overline{x} \| \leq \epsilon\}$$

Proof. Consider $(x_1, x_2) \in \mathcal{X}_\epsilon \times \mathcal{X}_\epsilon$, with the definition of $\mathcal{X}_\epsilon$, we know that

$$\inf_{\overline{x} \in \mathcal{X}} \| x_1 - \overline{x} \| \leq \epsilon \tag{1}$$

$$\inf_{\overline{x} \in \mathcal{X}} \| x_2 - \overline{x} \| \leq \epsilon \tag{2}$$

...
and according to the definition of convex set, $\mathcal{X}_\epsilon$ is convex

$\square$