Symplectic Geometry

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Abstract

This is a note on Ana Cannas's *Lectures on Symplectic Geometry*. However, it's not just a copy. It contains my understanding, questions and solutions to homework.

Contents

1 Notations

In order to keep the text short, common used notations are introduced here.

- V be an m-dimensional vector space over $\mathbb{R}$.
- $\Omega : V \times V \to \mathbb{R}$ be a bilinear map.

2 Symplectic Forms

Def 1. The map Ω is **skew-symmetric** if $\Omega(u, v) = -\Omega(v, u), \forall u, v \in V$.

Thm 1 (Standard Form for Skew-symmetric Bilinear Map). *$\exists$ a basis $u_1, \cdots, u_k, e_1, \cdots, e_n, f_1, \cdots, f_n$ of V s.t. $\forall i, j$ and $v \in V$*

$$\Omega(u_i, v) = \Omega(e_i, e_j) = \Omega(f_i, f_j) = 0, \quad \Omega(e_i, f_j) = \delta_{ij}$$

Remark 1. 1. The basis is not unique, though it is traditionally also called a "canonical" basis.

2. In matrix notation with respect to such basis, we have

$$\Omega(u, v) = \begin{bmatrix} - & u & - \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & I_n \\ 0 & -I_n & 0 \end{bmatrix} \begin{bmatrix} | \\ v \\ | \end{bmatrix}$$

and all normed linear independt enginvectors are basis.

Proof. This induction proof is a skew-symmetric version of the Gram-Schmidt process. Let $U := \{u \in V \mid \Omega(u,v) = 0, \forall v \in V\}$. Choose a basis $u_1, \cdots, u_k$, and choose a complemetary space W, $V = U \oplus W$ □

2.1 Skew-Symmetric Bilinear Maps

2.2 Symplectic Vector Space

2.3 Symplectic Manifolds

2.4 Symplectomorphisms

2.5 Homework

3 Symplectic Form on the Cotangent Bundle

3.1 Cotangent Bundle

3.2 Tautological and Canonical Forms in Coordinates

3.3 Coordinate-Free Definitions

3.4 Naturality of the Tautological and Canonical Forms

3.5 Homework