Homework 5

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Testing a human face recognition algorithm

(Example 2.5 *Computational Statistics* by Givens & Hoeting)

We will fit a logistic regression model to `facerecognition.RData` related to testing a human face recognition algorithm. The experiment used the recognition algorithm to match the first image of each person (called a probe) to one of the remaining 2143 images. Ideally, a match is made to the other image of the same person (called the target). A successful match yielded a response of $y_i = 1$ and a match to any other person yielded a response of $y_i = 0$. The predictor variable `eyediff` measures the absolute difference in eye region pixel intensity between the probe image and its corresponding target.

1. (30') Code your own Newton-Raphson algorithm to fit the logistic regression. Take initial guess $\boldsymbol{\beta}^{(0)} = (\beta_0^{(0)}, \beta_1^{(0)})^\top = (0.96, 0)^\top$ and set the convergence tolerance $\varepsilon = 10^{-5}$. Draw a figure showing the value of $\left\| \frac{\partial \log L(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}} \right\|_2$ at each iteration.
2. (10') Make a plot showing how $\boldsymbol{\beta}^{(t)}$ moves until convergence. Check your result with the estimate from `glm()` function in R (setting `family="binomial"`).
3. (10' bonus) Add underlying contour plot of $\log L(\boldsymbol{\beta})$ to the plot made in 2.