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The Homework 2 of statistical computation

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Exercise one

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Remark:first,we use seqential inversion to generate (U1,U2)~clayton copula($\theta = 2$);second we use inversion of normal to generate (N1,N2)

Some pre-computation:

Suppose (X1,X2)~clayton copular($\theta = 2$) ,obviously, X1~U(0,1) ,$f_1(x_1) = 1$,

Then,when give X1,how about X2 ?

PDF

$$f_{2|1(x_2,x_1)} = \frac{f(x_2,x_1)}{f_1(x_1)} = \frac{3(x_1x_2)^{-3}(x_1^{-2}+x_2^{-2}-1)^{-\frac{5}{2}}}{1}$$

CDF

$$F_{2|1}(x_2|x_1) = \int_0^{x_2} f_{2|1}(t|x_1)\,dt = x_1^{-3}(x_1^{-2}+x_2^{-2}-1)^{-\frac{3}{2}}$$

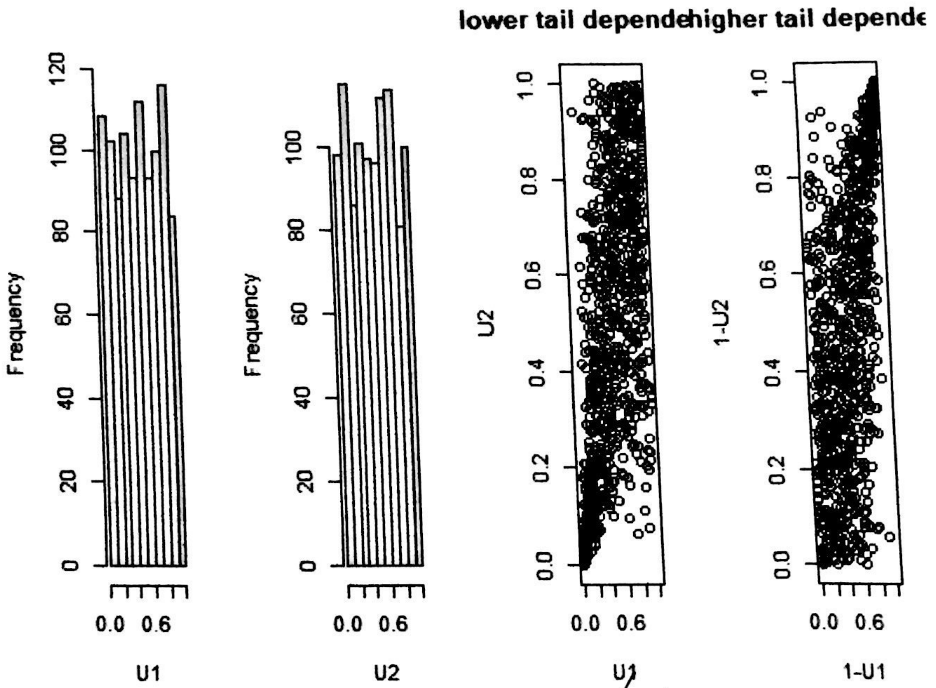
CDF^{-1}

$$X_2 = F_{2|1}^{-1}(U_2|X_1) = [(U_2X_1^3)^{-\frac{2}{3}} + 1 - U_1^{-2}]^{-\frac{1}{2}}$$

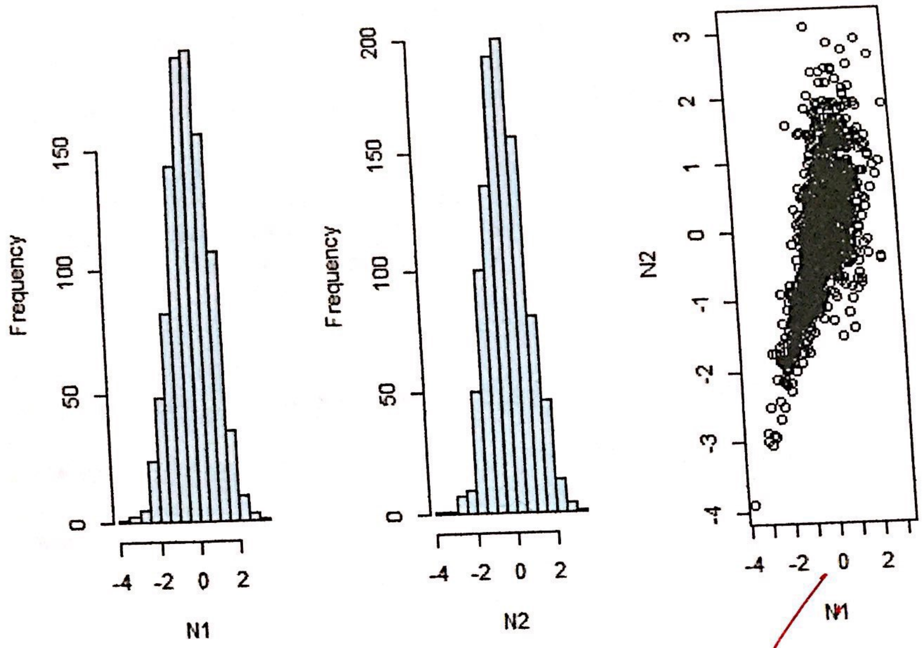
Code Action

```
set.seed(123456)
n=1000    #the number of random variable
U1=runif(n)#the first random variable
U2=runif(n)
temp1=1/(U2*U1^3)^(2/3)#surronding is based on seqential inversion
temp2=1-1/U1^2
temp3=1/(temp1+temp2)^(1/2)#the second random variable
U=cbind(U1,temp3)
###draw some chart
par(mfrow=c(1,4))
hist(U[,1],xlab = 'U1',main = '',col = 'lightblue')#specify the uniform
 distribution
hist(U[,2],xlab = 'U2',main = '',col = 'lightblue')#specify the uniform
 distribution
```

```
plot(U[,1],U[,2],type = 'p',xlab = 'U1',ylab = 'U2',main = 'lower tail
dependence')
plot(1-U[,1],1-U[,2],type = 'p',xlab = '1-U1',ylab = '1-U2',main = 'hig
her tail dependence')
```




```
###draw some graph
N=cbind(qnorm(U[,1]),qnorm(U[,2]))
par(mfrow=c(1,3))
hist(N[,1],xlab = 'N1',main = '',col = 'lightblue')#specify the normal
distribution
hist(N[,2],xlab = 'N2',main = '',col = 'lightblue')#specify the normal
distribution
plot(N[,1],N[,2],type = 'p',xlab = 'N1',ylab = 'N2',main = '')
```



We find that the graph above are awesome,especially the 'lower tail dependence' shows that lower tail dependence clearly.

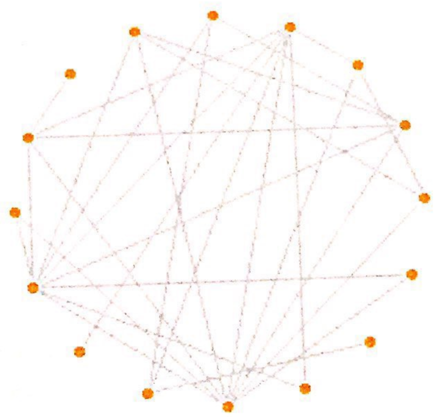
Exercise two

For first question

It's a little complex,but very interesting. Please be patient to read my answer. ok

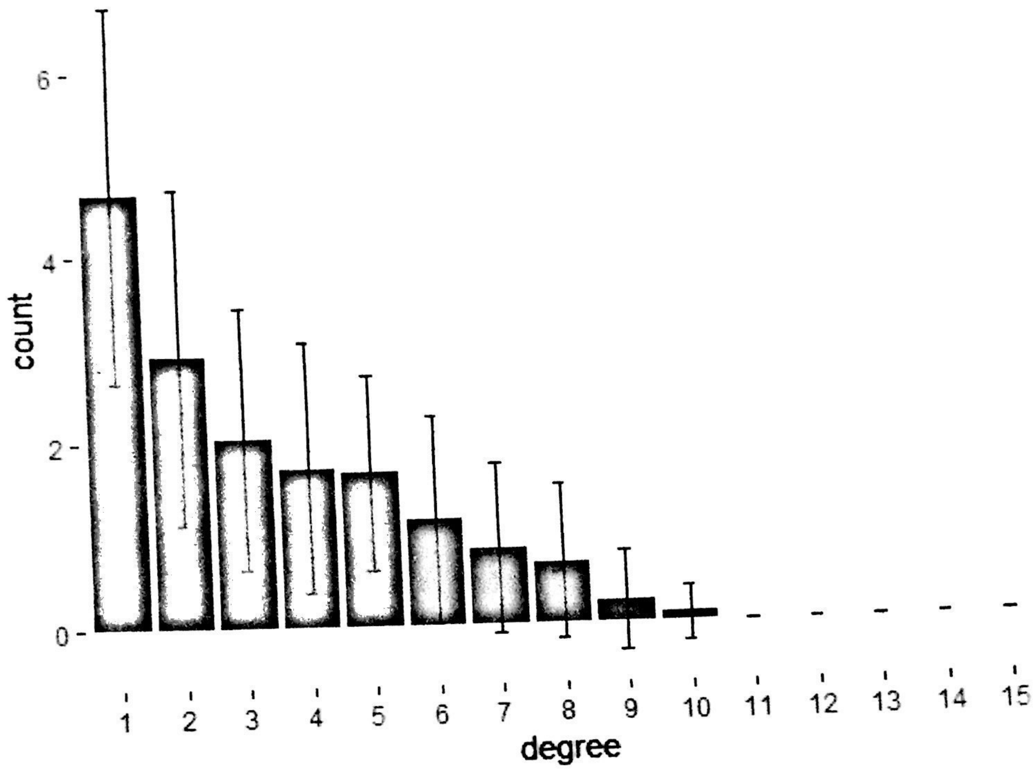
```
library('igraph')
library('ggplot2')

#prepare
a=1 #parameters
n=16 #number of vertex(node)
set.seed(123456) #you know that
G=list() #just be used to contain the graph generated
degree_count_dis=matrix(nrow = 100,ncol = 16) #will be used to record t
he distribution of degree of one graph,notice in some circumstance : 0
is included,but it is very unusual.SO finally we will abandon 0.
index=matrix(nrow = 100,ncol = 3)#will be used to record 'edge_density',
'transitivity','mean_distance' of a graph

#Loop
for (i in 1:100) {
  degs <- sample(1:(n-1),n, replace=TRUE, prob=(1:(n-1))^-a)#generate d
egree sample
```

```r
  if (sum(degs) %% 2 != 0) { degs[1] <- degs[1] + 1 }#for the corridenc
e of a graph
  G[[i]] <- sample_degseq(degs,method = 'simple')# generate the graph
  G[[i]]=simplify(G[[i]]) #get rid of the loop edge and multi-edge

  degree_count_dis[i,]=table(factor(degree(G[[i]]),levels = 0:15))#as w
e have said,it's a little complex,but it's right,will be used to record
 the distribution of degree of this graph
  index[i,1]=edge_density(G[[i]])#as we have said
  index[i,2]=transitivity(G[[i]])#as we have said
  index[i,3]=mean_distance(G[[i]])#as we have said
}

#some other work
degree_count_dis=degree_count_dis[,2:16]#abandon the 0 edge,it is very
unusual
colnames(index)=c('edge_density','transitivity','mean_distance')#give n
ame
index=as.data.frame(index)#change it into data.frame



#just to draw a graph , for example the 50th graph
i=50
g=G[[i]]
V(g)$size = 8
V(g)$frame.color = "white"
V(g)$color = "orange"
V(g)$label = ""
E(g)$arrow.mode = 0
lay = layout_in_circle(g)
plot(g, layout=lay, xlab="For example ,the 50th graph")
```



For example ,the 50th graph

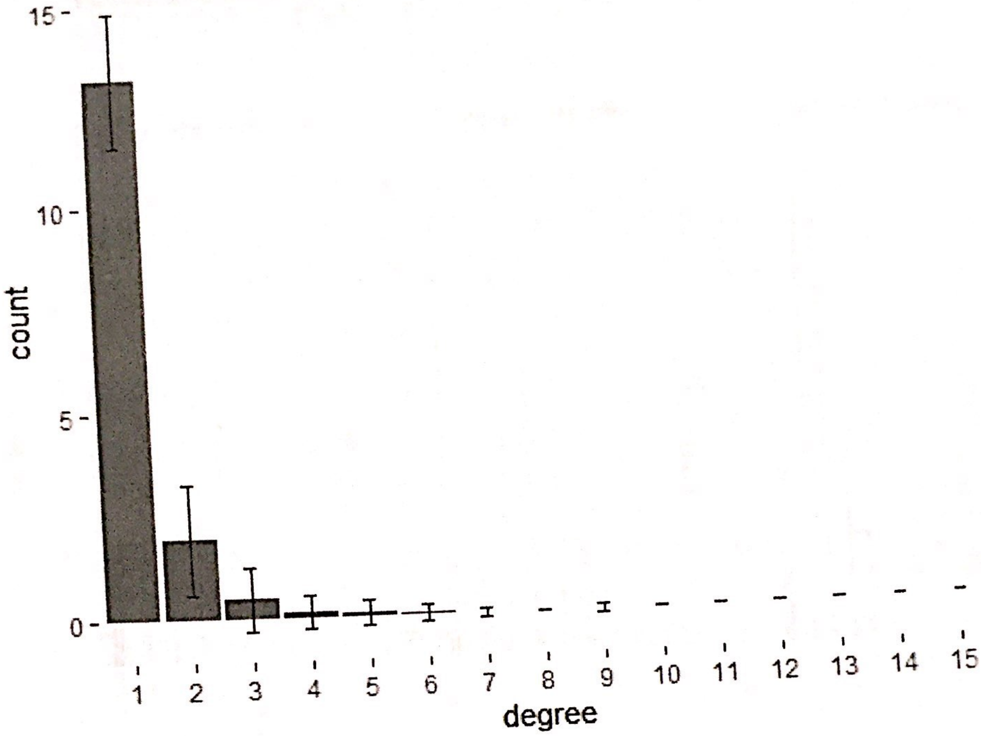
```
###prepare to drawing the most important picture, some pre-work,underst
and it carefully
paras= matrix(nrow = 15,ncol = 2)#will be very useful
for (i in 1:15) {
  paras[i,1]=mean(degree_count_dis[,i])#the mean
  paras[i,2]=sd(degree_count_dis[,i])#the sd
}
paras=as.data.frame(paras)#change into data.frame
paras[,3]=factor(1:15)#the degree
colnames(paras)=c('mean','sd','degree')#give name


#painting!
painting <- ggplot(paras, aes(degree))
painting + geom_bar(aes(weight = mean))+geom_errorbar(aes(ymin=mean-sd,
 ymax=mean+sd), width=.2, position=position_dodge(.9))
```




```
#look at picture ,it's beautiful
```

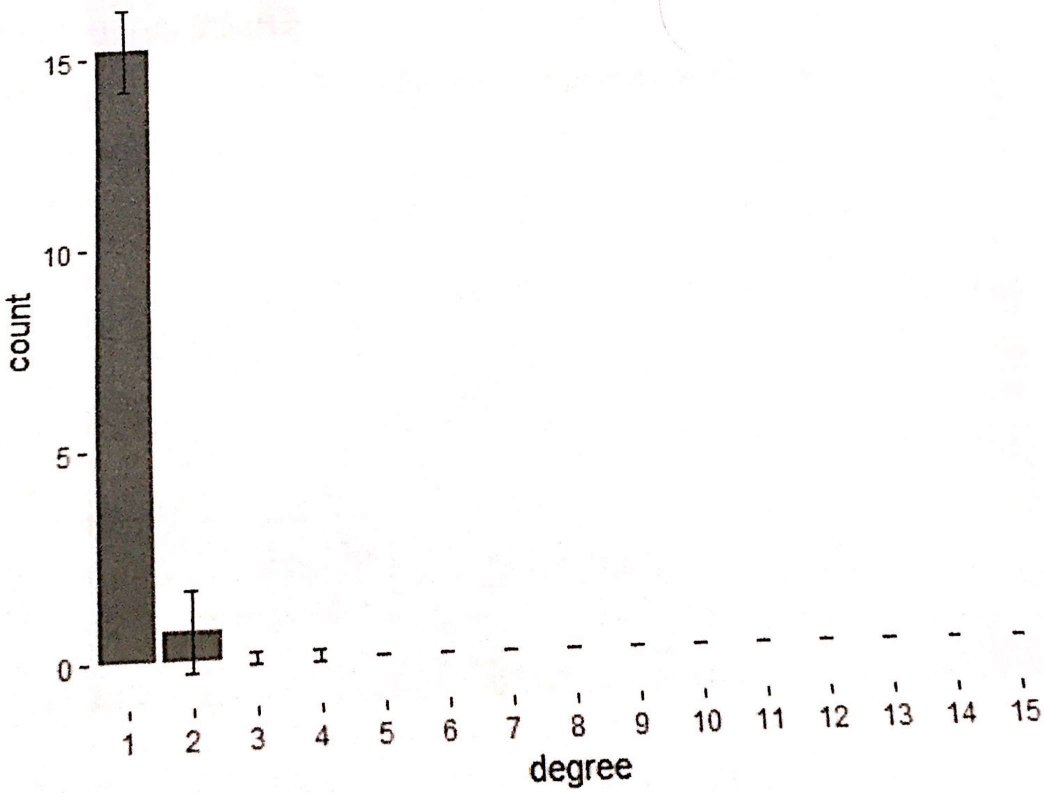
as $\alpha = 3$ or $\alpha = 5$,is in below

$\alpha = 3$.



count
0
5
10
15
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
degree

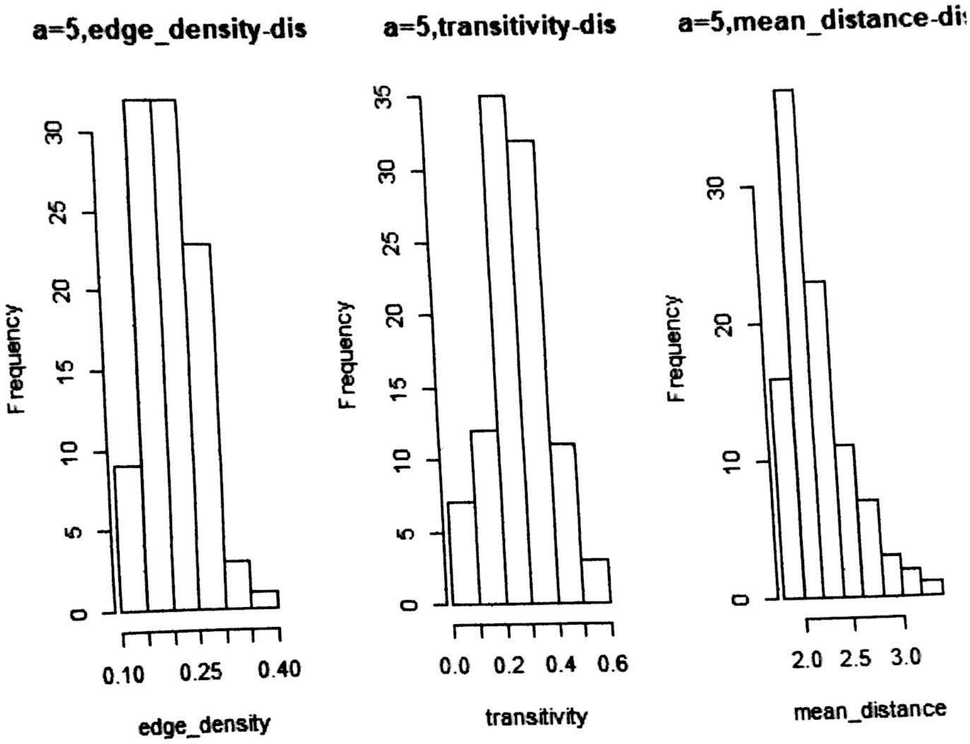
$\alpha = 5$



count
0
5
10
15
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
degree

For second question

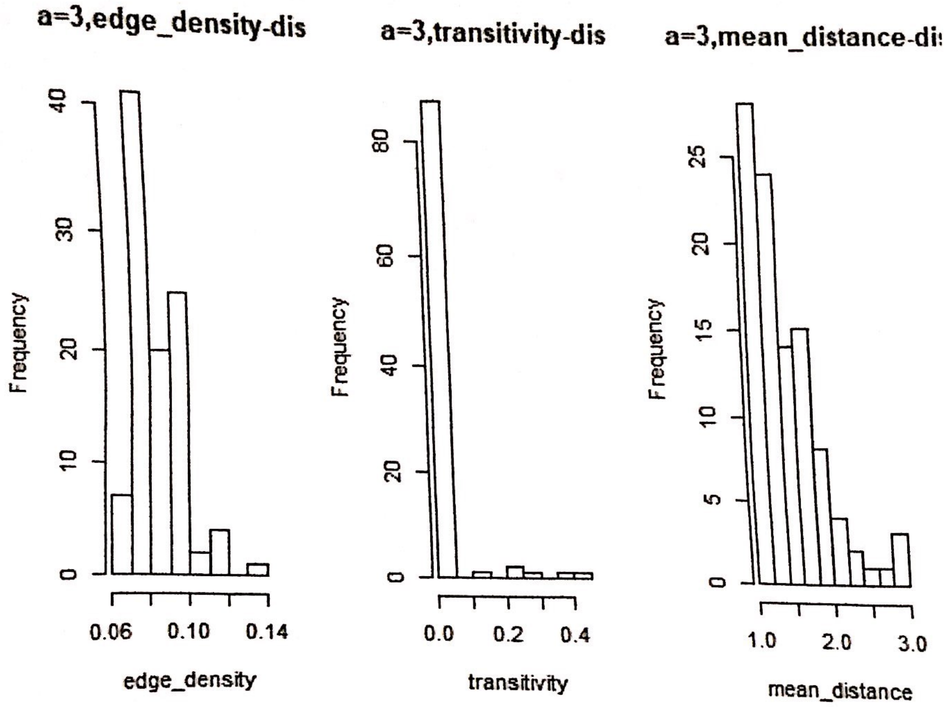
$\alpha = 1$



That's the distribution.

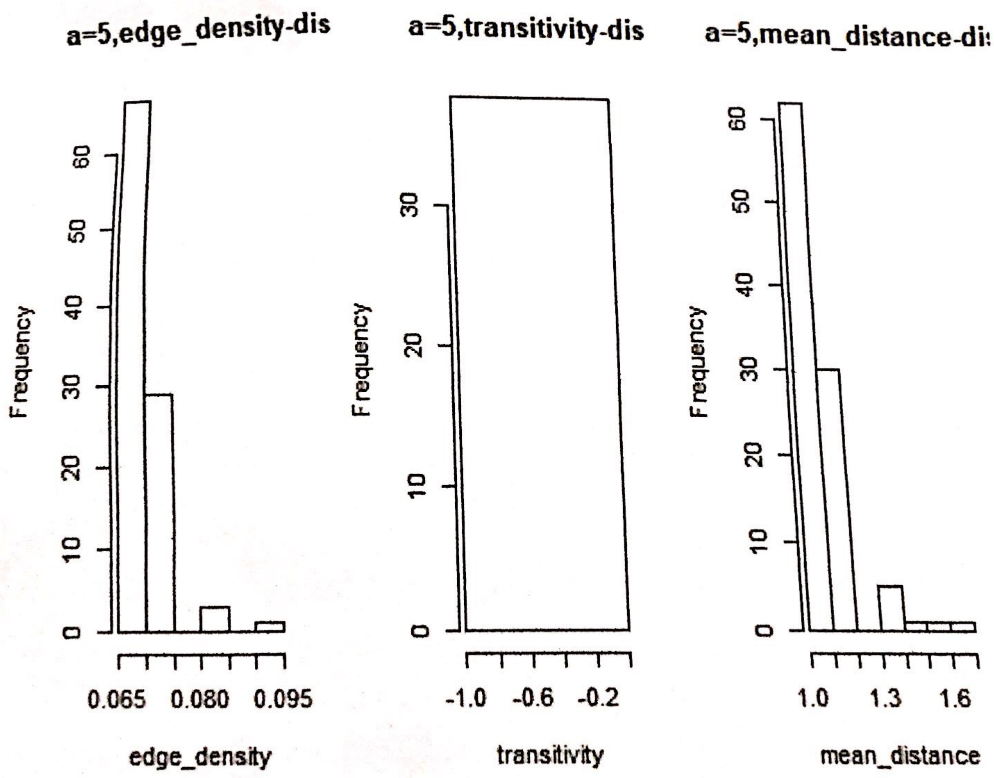
When a=$\alpha = 3$ or $\alpha = 5$,the ditribution picture are below .

$\alpha = 3$:



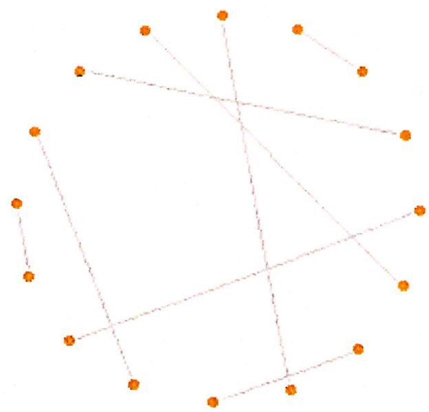
a=3,edge_density-dis
a=3,transitivity-dis
a=3,mean_distance-di:
Frequency
edge_density
transitivity
mean_distance

$\alpha = 5$:



a=5,edge_density-dis
a=5,transitivity-dis
a=5,mean_distance-di:
Frequency
edge_density
transitivity
mean_distance

It's obvious that the trend of edge_density and transitivity and mean_distance is becoming smaller and smaller. Especially when $\alpha = 5$,the transitivity almost is 0. Look at that picture below($\alpha = 5$).



The 50th graph for instance