Cross-layer Betweenness Centrality in Multiplex Networks with Applications

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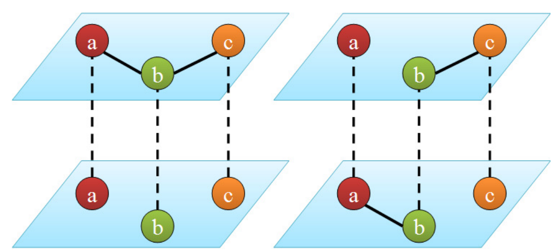
***Abstract*—Several real-life social systems witness the presence of multiple interaction types (or layers) among the entities, thus establishing a collection of co-evolving networks, known as *multiplex networks*. More recently, there has been a significant interest in developing certain centrality measures in multiplex networks to understand the influential power of the entities (to be referred as *vertices* or *nodes* hereafter). In this paper, we consider the problem of studying how frequently the nodes occur on the shortest paths between other nodes in the multiplex networks. As opposed to simplex networks, the shortest paths between nodes can possibly traverse through multiple layers in multiplex networks. Motivated by this phenomenon, we propose a new metric to address the above problem and we call this new metric *cross-layer betweenness centrality* (*CBC*).**

Our definition of CBC measure takes into account the interplay among multiple layers in determining the shortest paths in multiplex networks. We propose an efficient algorithm to compute CBC and show that it runs much faster than the naïve computation of this measure. We show the efficacy of the proposed algorithm using thorough experimentation on two real-world multiplex networks. We further demonstrate the practical utility of CBC by applying it in the following three application contexts: discovering non-overlapping community structure in multiplex networks, identifying interdisciplinary researchers from a multiplex co-authorship network, and the initiator selection for message spreading. In all these application scenarios, the respective solution methods based on the proposed CBC are found to be significantly better performing than that of the corresponding benchmark approaches.

I. Introduction

Several real-life social systems witness the presence of multiple interaction types (or layers) among the entities (such as individuals, organizations, etc.), thus establishing a collection of co-evolving networks, known as *multiplex networks* or *multi-layer* or *heterogeneous* or *cross-layer* networks [1], [2], [3], [4]. An example of a multiplex network is online social networks where the same individuals interact with others through various forms of social media such as Facebook, LinkedIn, and Twitter. Historically, the term *multiplex* has been adopted to indicate the presence of more than one interaction type among the same entities in a complex network [5]. Most of the existing literature in network science [6], [7], [8] assume that the entities (to be referred as *vertices* or *nodes* hereafter) are connected to each other by a single interaction type that captures the underlying dynamics in the network. This oversimplifying assumption, which neglects the presence of multiple interaction types among the entities, might alter the topology of the network leading to overestimation or underestimation of crucial properties of nodes, such as centrality [9], [10], [11]. This motivated the network science research community to focus its attention on the multiplex character of real-world systems including the time-varying and multi-layer nature of the networks [12], [13], [3], [14]. Figure 1 shows two stylized examples of multiplex networks with two layers where vertices a, b, and c exist in both the layers. The interconnectivity between them is different in both layers. Note that each layer in a multiplex network might not contain all the vertices; therefore in that case, one-to-one correspondence might not be possible. However, one can assume the presence of an isolated vertex corresponding to each missing vertex to draw the one-to-one correspondence between vertices in different layers as shown in Figure 1.



(a) (b)

Fig. 1: (Color online) Stylized examples of multiplex networks.

More recently, there has been a significant interest to develop certain centrality measures for multiplex networks in order to understand the influential power of the vertices [15], [1], [13], [3], [16], [17], [18]. These studies handle the layered structure of multiplex networks in three different approaches [19], [20], [3], [17], [21]:

- **Approach 1**: Different layers in the multiplex network are projected into a single layer to form an aggregated simple network and the metrics are computed over this aggregated network [22], [23].
- **Approach 2**: Each layer is treated independently in terms of evaluating the metrics and then these layer-wise results are aggregated later in multiple ways [3], [16].
- **Approach 3**: The computation of metrics in multiplex networks is performed directly by taking into account the effects of the interdependencies among the layers [15], [13].

In certain scenarios, it is evident from the literature that Approach 3 is superior to both Approach 1 and Approach 2

[15]. *In this paper, we consider the problem of studying how frequently the vertices appear on the shortest paths between other vertices in multiplex networks. We propose a new metric for this purpose and we call it* cross-layer betweenness centrality *(*CBC*). This is an important problem as the shortest paths between vertices can possibly traverse through multiple layers in multiplex networks and this phenomenon does not appear in simplex networks (that deals with only single interaction type). As we present in Section I-A, the existing techniques in network analysis for simplex networks are inadequate to accurately measure CBC.*

Our proposed approach to calculate CBC does not perform any type of prior aggregation on the inherent structure of the layers (as is done in **Approach 1**) and it also does not perform layer-wise computation to aggregate the results (as is done in **Approach 2**). Rather, our proposed approach follows **Approach 3** and thus it directly takes into account the interplay among the layers in determining the shortest paths in multiplex networks. In what follows, we provide the motivation for our approach in this paper.

A. Basic Definitions and Motivation

In the literature on network science, centrality measures are aimed to quantify the relative importance of a vertex, an edge, or a subgraph. Betweenness centrality [9] is a well known measure developed for simple networks (i.e., those networks wherein the vertices are connected by a single interaction type or layer). Informally, the betweenness centrality of a vertex v is defined as the fraction of shortest paths between all pairs of vertices which pass through vertex v. More formally, let V be the set of all vertices in a simple network and betweenness centrality of a vertex $v \in V$ is defined as

$$C(v) = \sum_{\substack{x \neq y \neq v \\ \forall x,y \in V}} \frac{\sigma_{x,y}(v)}{\sigma_{x,y}} \tag{1}$$

where $\sigma_{x,y}$ is the number of shortest paths between x and y; and $\sigma_{x,y}(v)$ is the number of shortest paths between x and y that pass through vertex v.

Now, we look at how this well-known definition of betweenness centrality can be utilized for multiplex networks using **Approach 1** and **Approach 2**. Following **Approach 1**, all the layers are projected into a single layer, and two vertices are connected in the aggregated network if they are connected among each other in at least one of the layers [22]. For instance, Figure 2(b) is the aggregated network for the multiplex network shown in Figure 2(a). It is easy to see that the values of betweenness centrality for the vertices a, b and c in the aggregated network are the same and this value is higher than any other vertex in the network. Following **Approach 2**, we first compute the betweenness centrality values of the vertices in each layer independently and then we aggregate these values in some fashion. For instance, consider the multiplex network shown in Figure 2(a). In both the layers, the values of betweenness centrality for the vertices a, b and c are the same and this value is higher than any other vertex in the respective layers. Now, using any aggregation method, it is easy to see that the values of betweenness centrality for the vertices a, b and c are the same and this value is higher than any other vertex. However, from the multiplex network shown in Figure 2(a), *note that vertex* b *possesses a unique power unlike its counterparts, because it acts like a bridge to form (shortest) paths using the two layers in order to connect any vertex in* $\{a, c, g, f, h, i\}$ *to any vertex in* $\{d, e\}$. Thus, it is natural to predict that vertex b should be ranked higher than vertices a and c in this multiplex network. That is, in this stylized example setting, the use of betweenness centrality measure in multiplex networks using **Approach 1** and **Approach 2** leads to erroneous computation of the centrality values. *This motivates us to work with* **Approach 3** *to design a new measure of betweenness centrality for multiplex networks, and to the best of our knowledge, there is no known work in the literature towards this end. In this paper, we precisely address the above research gap.*

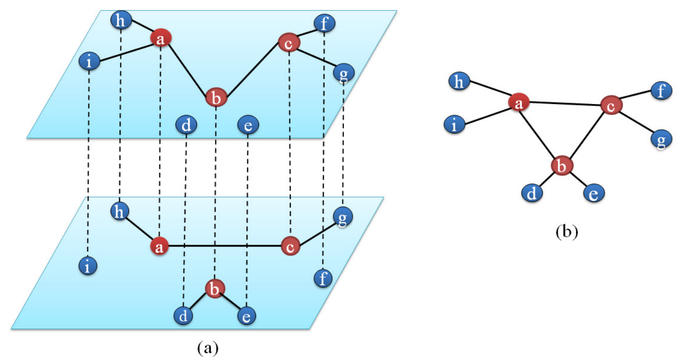


Fig. 2: (Color online) (a) An illustrative example of a multiplex network, where vertex b acts as a bridge between layers; (b) aggregated version of the multiplex network.

B. Our Contributions

We summarize the contributions of this paper as follows:

1) The primary contribution of this paper is to propose the cross-layer betweenness centrality (CBC) for the multiplex networks.
2) The naïve computation of the proposed centrality measure takes $\mathcal{O}(N^3L^3)$ time where N is the number vertices and L is the number of layers in the multiplex network, which takes significantly large amount of running time in practice. We propose a faster algorithm to compute the cross-layer betweenness centrality. The computational complexity of this faster algorithm is $\mathcal{O}(NLE)$ for multiplex networks.
3) We show the efficacy of the proposed algorithm using thorough experimentation using two real-world multiplex network datasets, namely an airlines transportation network comprising three layers and a coauthorship network with two layers.
4) We further demonstrate the practical utility of CBC by using three applications: discovering non-overlapping community structure in multiplex networks, identifying interdisciplinary researchers from a multiplex co-authorship network, and the initiator selection for message spreading. For the scenario of discovering the community structure, we design an efficient community detection algorithm for multiplex

networks by extending the definition of CBC for edges. It turns out that this algorithm outperforms the benchmark community detection algorithms for multiplex networks. We also obtain similar findings in the scenario of identifying interdisciplinary researchers. Finally, we notice that CBC-based strategy is the most efficient among three other baselines for disseminating messages to all the vertices in the network.

We make the data sets and the implementations (computer programs) in our experiments available in the spirit of reproducible research at
https://github.com/centrality-multiplex/MBC.git.

C. Organization of the paper

The rest of this paper is organized as follows. In Section II, we discuss the related work in the field of multiplex networks. Next, we present our proposed metric CBC in Section III. In Section IV, we present a faster algorithm to compute CBC. Then we present the experimental results to demonstrate the efficacy of CBC in Section V. We consider three applications in Section VI to show the utility of our proposed measure. We conclude the paper in Section VII with a few interesting future directions.

II. Related Work

A considerable amount of effort has been devoted to model and characterize the multiplex networks with the aim of creating a consistent mathematical framework to analyze such systems effectively [1], [12], [23], [2], [13], [3], [4], [24], [25]. Here we present an overview of a few key research topics in the context of multiplex networks. However, we refer the interested reader to the above references to explore further on multiplex networks.

Models for Multiplex Networks: Though adjacency matrices are useful to represent the traditional simple networks, such a representation is inadequate for the description and analysis of multiplex networks. Hence, there is a need to develop more general mathematical models to deal with the multiplex networks. Towards this end, several useful models are proposed in the literature [1], [26], [3], [27].

Understanding the Structure of Multiplex Networks: Battiston *et al.* [2] study certain structural properties of multiplex networks such as vertex degree, clustering coefficient and transitivity. Boccaletti *et al.* [1] provide a comprehensive review of the structural and dynamical properties of multiplex networks.

Centrality Measures for Multiplex Networks: Of late, there has been active initiative from the research community to reformulate (or extend) the centrality measures designed for simple network to the setting of multiplex networks. Along these lines, Sola *et al.* [28] propose new eigenvector centrality measure for multiplex networks. Arda *et al.* [15] draw on the idea of biased random walks to define the Multiplex PageRank centrality measure. Ng *et al.* [18] design a co-ranking scheme for objects and relations in multi-relational data. Zhou *et al.* [29] propose a novel method for co-ranking authors and their publications in publication dataset. Li *et al.* [30] design a framework to study the hub and authority scores of objects, and the relevance scores of relations in multi-relational data for query search. A systematic review of such centrality measures proposed for the multiplex networks can be obtained in [31]. Jung *et al.* [32] propose a divide-and-conquer approach to measure centrality based on semantic alignment function, separating the multiplex social networks with respect to concepts describing the relationship. Solé-Ribalta *et al.* [33] recently propose a first version of betweenness centrality measure by adapting the Brandes algorithms for multiplex network. However, their approach is essentially an aggregated approach (similar to Approach 1), i.e., measuring the betweenness centrality in each layer and aggregating it together for each vertex. To the best of our knowledge, our approach is the first attempt to consider the inter-layer dynamics in the formulation of betweenness centrality measure for multiplex networks. Moreover, we design several applications to show the efficacy of our measure in real-world scenario.

Diffusion Processes and Influence in Multiplex Networks: Gomez *et al.* [34] propose a mathematical framework that allows to analyze the emergent diffusion time scales in multiplex networks. Granell *et al.* [35] investigate the interplay between awareness and epidemic spreading in multiplex networks in the presence of two strong assumptions and later the authors [36] analyze the spreading processes by relaxing those assumptions.

Community Detection in Multiplex Networks: Community detection of multiplex networks needs to deal with information from all the simple networks. Kuncheva and Montana [37] propose LART, a community detection algorithm for the detection of communities that are shared by either some or all the layers in the multiplex. Zhu and Li [38] present a unified model to detect community structure by grouping the nodes based on a unified matrix transferred from multiplex network. Hmimida and Kanawati [39] adopt a seed-centric algorithm for community detection in multiplex networks. Mucha *et al.* [40] propose a modularity-based optimization technique to discover community structure in time-dependent, multiscale, and multiplex networks.

Other Important Aspects of Multiplex Networks: De Domenico *et al.* [41] describe an open-source software (muxViz) that contains a set of useful algorithms to analyze the multiplex networks. The resilience analysis of multiplex networks to random failures is studied in De Domenico *et al.* [23]. The evolution of cooperation via evolutionary game dynamics on multiplex networks is studied in [42]. Jiang and Li [43] study information cascade in multiplex networks. Sergey *et al.* [44] present an analytical solution for the catastrophic cascade of failures in interdependent networks.

There also present work in the literature to predict social links for new users [45], discovering correspondence of entities across multiple layers [46], to determine outliers [47] in multiplex networks.

III. Cross-Layer Betweenness Centrality (CBC)

In this section, we formally define our proposed model. Consider a set V ($N = |V|$) of vertices and a multiplex network containing these N vertices with their instances in different layers. Let us assume that $G_m = (V_m, E_m, L_m)$

represents a multiplex network constituting three tuples: V_m is the set of vertices, E_m is the set of unweighted and undirected edges and L_m is the set of layers. Also let $L = |L_m|$. Any element in V_m is represented as x^α where x is any vertex in V and α is any layer in L_m. For the ease of understanding, we assume that each layer always contains equal number of vertices; and without loss of generality, it is the set of vertices in V. Then, $|V_m| = N \times L$. If a vertex is absent in any of the layers, we assume that it is present in that layer as an isolated vertex (for example, as shown in Figure 2). We define a path, $p_{x^\alpha \to y^\beta}$, in G_m as an ordered sequence of vertices which starts from vertex x in layer α and terminates at vertex y in layer β (if $\alpha = \beta$, then vertices x and y are present in the same layer), with the restriction that an edge exists between every pair of consecutive vertices in $p_{x^\alpha \to y^\beta}$. Let $P_{x^\alpha \to y^\beta}$ be the set of all possible paths between vertex x in layer α and vertex y in layer β. For every path $p_{x^\alpha \to y^\beta}$, we define a distance function $d(p_{x^\alpha \to y^\beta})$ as the number of traversed edges in the path. For any $\alpha, \beta \in L_m$ and for any $x, y \in V$, the set of shortest-paths $P^*_{x^\alpha \to y^\beta}$, from vertex x^α to vertex y^β, in the multiplex network is defined as follows:

$$P^*_{x^\alpha \to y^\beta} = \underset{p_{x^\alpha \to y^\beta} \in P_{x^\alpha \to y^\beta}}{\operatorname{argmin}} d(p_{x^\alpha \to y^\beta}) \tag{2}$$

It essentially indicates that a shortest-path between two vertices, in a multiplex network, is a path having minimum length that starts from the source vertex and reaches the destination vertex.

Considering Equation (2), the cross-layer betweenness centrality of vertex v in layer l, call it $C_B(v^l)$, is defined as the sum, for every possible origin-destination pair (x^α, y^β), of the fraction of times that vertex v on layer l occurs on a path in $P^*_{x^\alpha \to y^\beta}$. Specifically, the shortest-path betweenness centrality of a vertex in a layer in the multiplex network is obtained by:

$$C_B(v^l) = \sum_{\substack{x,y \in V \\ x \neq y}} \left(\gamma \sum_{\alpha \in L_m} \frac{\sigma^{v^l}_{x^\alpha y^\alpha}}{\sigma_{x^\alpha y^\alpha}} + (1-\gamma) \sum_{\substack{\alpha,\, \beta \in L_m \\ \alpha \neq \beta}} \frac{\sigma^{v^l}_{x^\alpha y^\beta}}{\sigma_{x^\alpha y^\beta}} \right) \tag{3}$$

where $\sigma_{x^\alpha y^\beta} = |P^*_{x^\alpha \to y^\beta}|$ is the number of shortest-paths from x^α to y^β, and $\sigma^{v^l}_{x^\alpha y^\beta}$ is the number of times vertex v^l is on a shortest-path from x^α to y^β. Note that, γ is a tuning parameter which balances the importance between same-layer and cross-layer shortest paths. The less the value of γ, the more the cross-layered links are preferred. We show the effect of γ in the experimental evaluation in Section V. The value of γ needs to be chosen according to the problem under consideration.

Eventually, the shortest-path betweenness of a vertex $v \in V$, in the multiplex network, can be obtained as follows:

$$C_B(v) = \sum_{l \in L_m} C_B(v^l) \tag{4}$$

Note that, if $\gamma < 0.5$ and we use the above definition of cross-layer betweenness centrality to rank the vertices in Figure 2, vertex b is ranked as the first which is followed by vertices a and c.

IV. Faster Algorithm To Compute Cross-layer Betweenness Centrality

In the literature, most of the algorithms for measuring betweenness centrality first compute the shortest paths (in terms of predecessor list), then for each shortest-path the corresponding betweenness centrality is computed for the traversed vertices on that path. This procedure yields to $\mathcal{O}(N^3)$ algorithm. Later on, few faster algorithms have been proposed to compute betweenness centrality [48], [49].

With the above background, it is not difficult to see that the cost of the naïve computation of the cross-layer betweenness centrality is $\mathcal{O}(N^3L^3)$. This takes significantly large time in practice for multiplex networks having several thousands of vertices. To alleviate this computational challenge, we propose a faster algorithm to compute CBC as outlined in Algorithm 1. This algorithm is inspired by Brandes algorithm[1] [48], which is a faster algorithm to compute betweenness centrality in simplex networks. Unlike in the simplex networks, the challenge in the design of an algorithm to compute CBC is that the shortest paths can traverse through multiple layers in the multiplex network.

Before going into the details of the algorithm, let us define the network construction used in this algorithm. Recall that each layer l in L_m contains equal number of vertices (i.e., N), and there are L $(= |L_m|)$ layers in the multiplex network. Multiplex degeneracy of a vertex, i.e., different IDs of a single vertex in different layers, can be determined by the modulus operation on the vertex ID. For instance, if $N = 10$ and $L = 3$, a vertex x which has a vertex ID (say,) 5 in 1^{st} layer is represented by vertex ID 15 in 2^{nd} layer and vertex ID 25 in 3^{rd} layer. Therefore, a modulus of N on the vertex ID can solve the multiplex degeneracy problem, and thus different vertex IDs of a same vertex can be recognized properly.

The algorithm begins with computing the single source all destination shortest paths (lines 19 - 25). Given a source vertex n, it computes the shortest paths to all the other vertices. The shortest-path acyclic graph is stored in variable SP as well as the number of shortest paths that pass through each vertex in variable σ. The initialization of the Breadth First Search differs from the classical one to consider that the source vertex n can be localized in any layer. Thus, the neighbors of source vertex n are the union of the neighbors of n in all layers (line 25). Besides, note that the shortest paths are computed considering the destination vertices corresponding to different entities in the different layers. In the following step, the equivalence of entities in the different layers is performed in the computation of the betweenness. To correctly account for these equivalences, we keep track of the first accessed vertex (independently of the layer) through the variable V_s as well as the shortest path distance in the multiplex network through variable d_M. The two variables d_m and d are used for two different purposes. The variable d stores the distance to every vertex in every layer, whereas d_M keeps track of the first time a vertex is accessed independently of the layer as well as to account for multiplex path degeneracy. However, d is still necessary since the shortest path search procedure

[1] Due to the lack of space, we do not elaborate the Brandes algorithm here. The readers may be interested to go through the Brandes algorithm before understanding our proposed algorithm.

Algorithm 1: Computation of Cross-layer Betweenness Centrality

Data: Graph G_m, γ: tuning parameter
Result: CBC: vector containing betweenness centrality values of all vertices

1 **Initialization:**
2 SP: a $1 \times NL$ matrix storing the shortest-path acyclic
3 graph
4 σ: A $1 \times NL$ matrix storing the number of shortest
5 paths passing through each vertex
6 d: A $1 \times NL$ matrix storing the distance to every node
7 in every layer
8 CBC: A $1 \times N$ matrix storing betweenness centrality
9 values of all vertices
10 **for** $n \in 1...N$ **do**
11     $S \leftarrow \phi$;
12     $SP \leftarrow \phi$;
13     $\sigma \leftarrow 0$; $\sigma[w] \leftarrow 1$, $n \equiv w\%N$;
14     $d \leftarrow -1$; $d[w] \leftarrow 0$, $n \equiv w\%N$;
15     $d_M[1...NL] \leftarrow -1$; $d_M[w] \leftarrow 0$, $n \equiv w\%N$;
16     $V_s[1...N] \leftarrow \phi$;
17     $Q \leftarrow \phi$;
18   enqueue $n \rightarrow Q$;
19   **while** $Q \; != \; \phi$ **do**
20   dequeue $v \leftarrow Q$;
21   push $v \rightarrow S$;
22   **if** $v \neq s$ **then**
23   W =neighbors of v in G_m;
24   **else**
25             $W = \bigcup_{\substack{v' \equiv n\%N \\ v' \in \{1...NL\}}}$ neighbors of v';
26   **for** $w \in W$ **do**
27   **if** $d[w] < 0$ **then**
28   enqueue $w \rightarrow Q$;
29                 $d[w] \leftarrow d[v] + 1$;
30   **if** $d_M[w\%N] < 0 \bigvee d_M[w\%N] == d[w]$ **then**
31                     $d_M[w\%N] = d[w]$;
32   add $w \rightarrow V_s[w\%N]$;
33   **if** $d[w] == d[v] + 1$ **then**
34                 $\sigma[w] \leftarrow \sigma[w] + \sigma[v]$;
35   add $v \rightarrow P[w]$
36   **for** $w \in \{1...N\}$ **do**
37         $\sigma_M[w] \leftarrow 0$;
38   **for** $v \in v_s[w]$ **do**
39             $\sigma_M[w] \leftarrow \sigma_M[w] + \sigma[v]$;
40   **while** S *not empty* **do**
41   pop $w \leftarrow S$;
42   **for** $v \in SP[w]$ **do**
43   **if** $w \in V_s[w\%N]$ **then**
44                 $\delta[v] \leftarrow \delta[v] + (1-\gamma)\frac{\sigma[v]}{\sigma[w]}(\frac{\sigma[w]}{\sigma_M[w]} + \delta[w])$;
45   **else**
46                 $\delta[v] \leftarrow \delta[v] + \gamma\frac{\sigma[v]}{\sigma[w]}\delta[w]$;
47   **if** $w \neq n$ **then**
48                 $CBC[w\%N] \leftarrow CBC[w\%N] + \delta[w]$

must travel through the different layers. Once all the shortest paths are found, to correctly account for the multiplex path degeneracy the number of shortest paths that pass though each vertex independently of the layer is computed (lines 36 to 39).

In the second half of the algorithm (lines 40 - 48), each shortest path in SP contributes to the computation of cross-layer betweenness vector CBC. To account for all the shortest path contributions in an efficient way, the shortest path acyclic graph is traversed starting from the farthest vertex to the source, i.e., the shortest-path acyclic graph is traversed in a backtracking fashion. In a single layer graph, where only classical path degeneracy needs to be accounted, at each traversed vertex w, the paths that go thought w plus the path that starts at w are correctly distributed among the predecessors (i.e., v) considering the number of paths that reach w and the number of paths that reach each predecessor v ($\sigma[w]$ and $\sigma[v]$). Each fraction of the paths is accumulated in each $\delta[v]$. Eventually, when all vertices farther than w to the source are explored, $\delta[w]$ can be safely accumulated in the betweenness of w. However, in a multiplex network this procedure is substantially more complex since we need to account not only for the first vertex it is accessed independently of the layer but also for multiplex path degeneracy. To illustrate these particularities, consider Figure 3, which represents a possible acyclic graph (in black solid arrows) that generates all shortest paths from vertex labeled 1, independently on the layer, to all other vertices. There are two possible shortest-paths (shown in dashed red arrows) from vertex 1 to vertex 4 in the acyclic graph, $\{1^{L1}; 2^{L1}; 4^{L1}\}$ and $\{1^{L1}; 2^{L1}; 4^{L1}; 4^{L2}\}$. However, the shortest-path reaching vertex 4 in layer 2 is not a valid one, since there exists a shorter path that reaches vertex 4 in layer 1. To avoid counting these paths, we only consider a new path that starts at w if w is the first accessed vertex considering its replicas in the different layers (see this check in line 43 of Algorithm 1). Another aspect that we need to account for is the multiplex path degeneracy; for instance, the case of vertex 5 (in layers 1 and 2) in Figure 3. There are two shortest-paths (shown in dotted blue) that reach vertex 5; one reaches vertex 5 in layer 1 and the other reaches vertex 5 in layer 2. Thus, for walks that end at vertex 5, the betweenness contribution to its predecessors, such as vertex 3 in layer 1, corresponds to the number of times we reach 5 considering the layer where it was reached divided by the times we reach 5 independently of the layer where it was reached. The contribution to predecessor v of w of the path that ends at w is given by $\frac{\sigma[w]}{\sigma_M[w]}$ (see line 44 in Algorithm 1). Note that in lines 44 and 46, there is a tuning parameter γ (as defined in Equation 3) which controls the weight between same-layer and cross-layer traversals. Unless otherwise mentioned, we assign γ=0.25 for the rest of the experiments. The theoretical computational complexity of the algorithm developed here is $\mathcal{O}(NLE)$ for multiplex networks.

V. Experimental Evaluation

In this section, we conduct thorough experimental evaluation of CBC. We first describe two real-world multiplex networks that we utilize in the experiments and then we present the experimental results.

A. Dataset Description

We consider the following two multiplex networks:

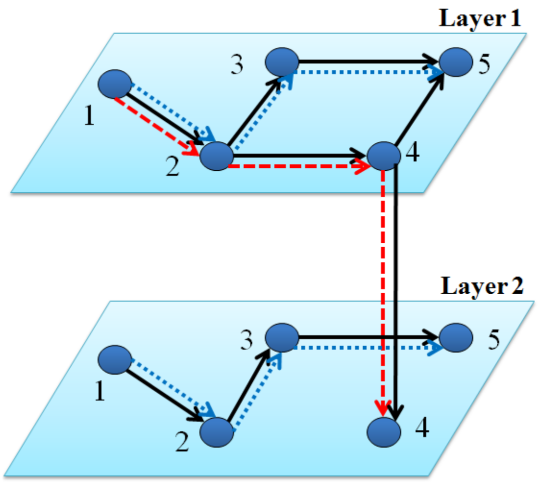


Fig. 3: (Color online) A stylized example to demonstrate that multiple paths might reach the same vertex in different layers.

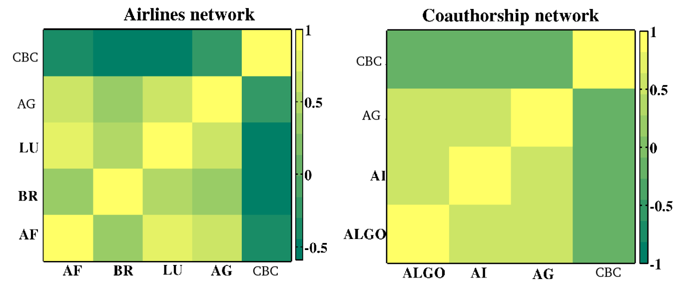


Fig. 4: (Color online) The heat maps representing Kendall τ correlation coefficient between the ranking of vertices using betweenness centrality on the aggregated network and cross-layer betweenness centrality on the multiplex network (ALGO: Algorithm, AI: Artificial Intelligence, AF: Air France, BA: British Airways, LU: Lufthansa, AG: aggregated betweenness centrality and CBC: cross-layer betweenness centrality).

TABLE I: Description of airlines and coauthorship multiplex networks

Airlines network		
Layers	# vertices	# edges
Air France	59	138
British Airways	65	132
Lufthansa	106	488
Coauthorship network		
Layers	# vertices	# edges
Algorithms	20,021	92,681
Artificial Intelligence	30,028	69,229

(i) Airlines Network: The dataset of the airlines network[2] is a multiplex network composed of the airlines operating in Europe. Here, vertices represent airports, while links stand for direct flights between two airports. Note that each commercial airline corresponds to each layer, containing all the connections operated by the same company. For the sake of conciseness, we take only the three major airlines: Air France, Lufthansa and British Airways [50]. Therefore, it forms a multiplex network having three layers and Table I describes the same.

(ii) Coauthorship Network: We use one of the largest bibliographic datasets developed by Chakraborty *et al.* [51], [52]. Each paper is also associated with a variety of bibliographic information – title of the paper, a unique paper index, the named-entity disambiguated author(s) of the paper, year and venue of publication, related field(s) that the paper contributes to, abstract and keyword(s) of the paper, and the list of research papers that it cites. For the sake of conciseness, we take those papers belonging to two fields – Algorithms and Artificial Intelligence. We represent these two fields as two layers to construct a coauthorship multiplex network. Here vertices correspond to authors and edges represent collaborations between pair-wise authors if they collaborate at least once in a paper (see further details in Table I).

[2]http://complex.unizar.es/~atnmultiplex/

B. Experimental Results

In Figure 4, we plot the heat maps representing the Kendall τ correlation coefficient[3] [53] between the ranking of vertices based on aggregated network (following *Approach 1*) and CBC in coauthorship and airlines networks respectively. From this figure, we observe that in each case the overall ranking obtained using CBC measure is negatively correlated with that obtained from the betweenness centrality of the aggregated network.

In co-authorship and airlines networks, 87% and 68% vertices obtain different ranking respectively with respect to the relationship of the rankings obtained using CBC and the aggregated method. The lower differences are observed in the first ranked and last ranked entities. Last ranked vertices have zero centrality in both rankings. The maximum difference on the rankings obtained using CBC and the aggregated methods is an increase of 36% in the ranking for the coauthorship network and a decrease of 28% in the ranking for airlines network. Therefore, we can conclude that the betweenness centrality provided by aggregated method is usually different than the CBC-based method. Thus, to obtain accurate betweenness centrality rankings it is crucial to compute these centralities directly on the multiplex structure.

We further conduct the following experiment to understand two aspects: (i) the fraction of common vertices within the top $n\%$ of vertices from these two ranking schemes, and (ii) the relationship between the top $n\%$ of vertices with highest centralities and the resulting correlation τ. Towards this end, we sort the vertices based on their aggregated and cross-layer betweenness centrality scores. Then we construct a chart showing the value of τ for n varying between 10% to 100% as well as the number of common vertices for a

[3]Kendall rank correlation coefficient τ is a statistic used to measure the association between two measured quantities. Let (x_1, y_1), (x_2, y_2),..., (x_n, y_n) be a set of observations of the joint random variables X and Y respectively. Any pair of observations $(x_i,\, y_i)$ and $(x_j,\, y_j)$ are said to be "concordant" if the ranks for both elements agree: i.e., if both $x_i > x_j$ and $y_i > y_j$ or if both $x_i < x_j$ and $y_i < y_j$; otherwise, They are said to be "discordant". If $x_i = x_j$ or $y_i = y_j$, the pair is neither concordant nor discordant. Then $\tau = \frac{N_c - N_d}{\frac{n(n-1)}{2}}$, where N_c (N_d) is the number of concordant (discordant) pairs.

given fraction of n. The results of this experiment for airlines and coauthorship networks are shown in Figure 5. We notice that for smaller values of n, while most of the elements are common in top ranks for both the rankings in airlines network, the fraction of common vertices is significantly low for coauthorship network (solid lines). Further, we notice two completely different patterns of the rank correlation for two networks (broken lines). While in airlines network, the rank correlation seems to be increasing with the increase of n, it tends to be falling in coauthorship network. It essentially indicates that although two ranking schemes tend to bring mostly different authors at the top, the common authors among them seem to be placed in relatively similar way. The ranking tends to become dissimilar with the increase of n in coauthorship network. We conclude that there is a significant difference between the ranking obtained by CBC and the others. The reason for this difference is that CBC takes into account the inter-layer dynamics while computing the shortest paths.

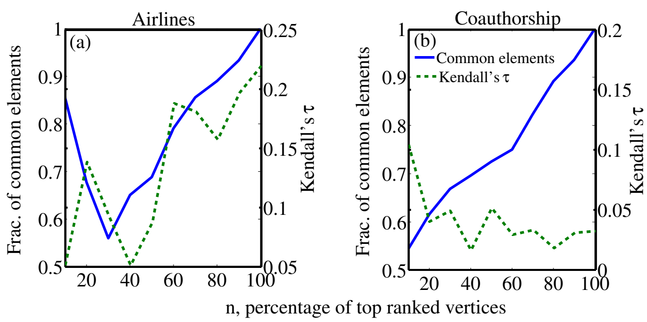


Fig. 5: (Color online) Relation between n (percentage of top vertices) and (i) the fraction of common elements among the top n elements, (ii) Kendall's τ correlation coefficient of common elements obtained from aggregated and cross-layer between centralities for (a) airlines and (b) coauthorship networks.

Effect of γ. It is crucial to understand the effect of γ in the ranking of vertices based on CBC. As mentioned earlier, decreasing value of γ corresponds to more preference to the cross-layer edges than same-layer edges. In Figure 6, we plot the correlation of the rankings (measured by Kendall's τ correlation coefficient) obtained from the aggregated method and from CBC with the change in γ for both the airlines and coauthorship networks. We use dense ranking scheme to rank vertices. We notice that for both the datasets, two rankings are completely independent for the lower values of γ. However, the correlation tends to increase sharply at $\gamma = 0.4$ and keeps rising almost linearly till $\gamma = 0.6$, which is followed by a sub-linear increase. As expected, the correlation becomes 1 at $\gamma = 1$ when the CBC converges to the aggregated method by omitting the effect of cross-layer edges.

Running Times. We consider the naïve algorithm for computing CBC as the benchmark and refer to this as *standard algorithm*. Table II shows the time taken by Algorithm 1 and the standard algorithm on the airlines and the coauthorship networks respectively. We further show the asymptotic growth

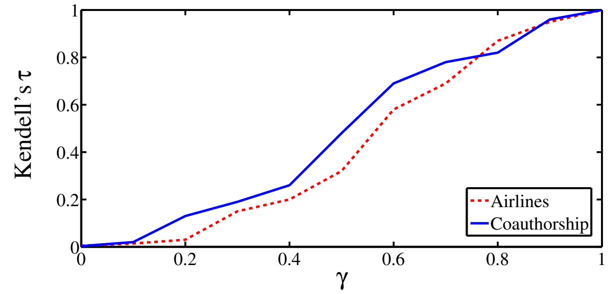


Fig. 6: (Color online) Kendall's τ ranking correlation among the rankings obtained from aggregated method and CBC-based method with the increasing values of γ for both airlines and coauthorship networks.

TABLE II: Execution time (in second) of the standard and our cross-layer betweenness centrality computation algorithms on the real-world graphs.

	Airlines	Coauthorship
Standard Algorithm	0.061	175,383
Our Algorithm 1	0.034	28

of the time taken to compute the centrality value for different sizes of the networks as follows. We generate different instances of random graphs using Erdős-Rényi model[4] [54]. For a fixed number of vertices, we create 10 network instances representing 10 different layers, thus formed a synthetic multiplex network. Building on this, we create such synthetic multiplex networks by increasing the number of vertices while keeping the number of layers constant. Figure 7(a) shows running times for betweenness centrality on random undirected unweighted graphs with 100 to 4000 vertices. Note that, each point on the graph plot is the average value over 500 synthetically generated multiplex networks. Similarly, we increase in the number of layers L from 1 to 20 by keeping the number of vertices N as 2000 and plot the execution times in Figure 7(b). In both the cases, our proposed Algorithm 1 shows significantly better performance than the standard algorithm.

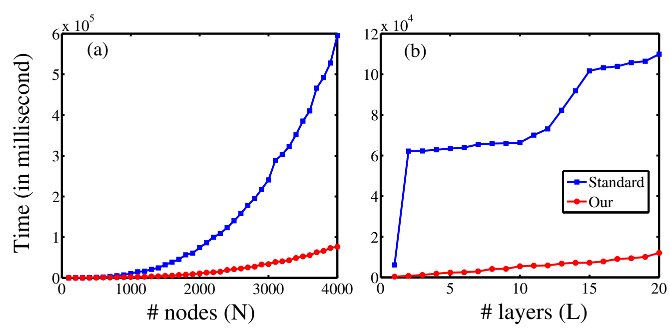


Fig. 7: (Color online) Time taken (in millisecond) by standard algorithm and our proposed algorithm with the increase in (a) the number of vertices and (b) the number of layers in the undirected random graph.

[4]We use matlab implementation: https://www.cs.purdue.edu/homes/dgleich/demos/erdos_renyi/

VI. Applications of Cross-layer Betweenness Centrality

In this section, we present three potential applications of cross-layer betweenness centrality to show its utility in analyzing multiplex networks. The three applications are: (i) Detecting non-overlapping communities from multiplex networks, (ii) Finding authors working in interdisciplinary fields from coauthorship multiplex networks, and (iii) Selecting initiators for message spreading.

A. Community Detection in Multiplex Networks

Community structure is one of the fundamental characteristics of social networks [55]. A network is said to have community structure if the vertices in the network can be easily grouped into sets of vertices such that each set of vertices is densely connected internally. In the particular case of non-overlapping community finding, this implies that the network divides naturally into groups of vertices with dense connections internally and sparse connectivity among groups. Detecting communities has received significant attention in the research of social networks [56], [57]. Most existing methods have been developed to analyze simple networks, where there is only one type of relation between vertices. However these algorithms are inadequate to detect communities in multiplex networks. Though a few studies have investigated community detection in multiplex networks, mostly these are characterized by strong simplifications that reduce community detection in multiplex networks to community detection in simplex networks. Jiang and Jaeger [58] reported on the application of the statistical relational learning modeling framework of Relational Bayesian Networks to this task. Breiger *et al.* [59] described a hierarchical clustering algorithm that is based on an iterative transformation of incidence matrices into a block form, and which can be simultaneously applied to matrices representing multiple relations. Recently, Mucha *et al.* [40] proposed a generalized version of Louvain algorithm [60] to detect community structure in a very general setting encompassing networks that evolve over time, have multiple types of links, and have multiple scales.

Nodes having high betweenness centrality are often located at the fringes of dense subgroups in a network. Therefore, a multiplex network can be partitioned into subgroups using identified bridges if we can utilize bridges as subgroup boundary. Here, we propose a new community detection algorithm utilizing cross-layer betweenness centrality measure. Towards this end, we first specify a method to define the cross-layer betweenness centrality of an edge e as follows [61]:

$$C_B(e) = \frac{d(u)C_B(u) + d(v)C_B(v)}{(d(u)+d(v))(|c(u,v)|+1)},\ e(u,v) \in E \quad (5)$$

where vertices u and v are the two incident vertices to edge e, $d(u)$ is the degree of a node u, $C_B(u)$ is the multiplex betweenness centrality of node u, $c(u,v)$ is the set of common direct neighbor vertices of vertices u and v.

We now present an iterative algorithm as shown in Algorithm 2, we call it *CBC_Cut* (**C**ross-layer **B**etweenness **C**entrality **Cut**), to find communities in a given multiplex network, G_m. The iterative graph clustering algorithm involves three sequential steps:

Algorithm 2: CBC_Cut: Community Detection Algorithm for Multiplex Networks

Data: Graph $G(V, E)$, δ: density threshold
Result: List of communities
G^*: A clone of graph G;
Calculate $C_B(v)$, $\forall v \in V$;
while $G \neq\ empty$ **do**
  Calculate $C_B(e)$, $\forall e \in E$;
  $topE \leftarrow$ The edge with the highest C_B;
  Remove $topE$;
  if *there is a new isolated module* s **then**
    if $Density(s, G^*) > \delta$ **then**
      $Comm.add(s)$;
      $G.remove(s)$;
Return Comm;

Step 1: Compute cross-layer between centrality of all edges G_m as described in Equation 5 and pick an edge e with the highest centrality value.

Step 2: Remove edge e.

Step 3: Identify a subgraph s as a final community and remove from G_m if s is isolated from G_m and the density of s in the actual graph G_m is greater than a threshold, δ.

First, C_B is calculated for all edges in a graph G_m and the highest scored edge e is picked up (Step 1). The highest scoring edge is removed from the graph G (Step 2). Finally, subgraph s that is isolated from G after edge e cut, is recognized as a final cluster if the density of s is more than the threshold (Step 3). These three sequential steps are repeated until G is empty.

1) Complexity Analysis: The most expensive part of the CBC_Cut algorithm is to recompute the cross-layer betweenness centrality for all the nodes in the modified network after deleting highest scored edge. The computational complexity of $C_B(e)$ depends on the computational complexity of calculating C_B of its two end points. We observed earlier that calculating C_B of each vertex takes $\mathcal{O}(NLE)$ for unweighted networks. The average computation time for $C_B(e)$ requires $\mathcal{O}(N(logN)^2)$ because the average degree of nodes is $logN$ in real world networks, e.g., scale-free networks. Thus, the total time complexity for each iteration is bounded by $\mathcal{O}(NLE+N(logN)^2)$. However, designing a faster algorithm for community detection in multiplex networks is not the major goal of this research and thus can be treated as one of the potential future directions.

2) Evaluating the Quality of Community Structure: In the absence of actual ground-truth community structure for a network, the common practice is to use a validation metric to judge the quality of derived communities. Towards this end, Modularity [62] (defined by the fraction of the edges that fall within the given communities minus the expected such fraction if edges were distributed at random) is one of the robust and well-known metrics for evaluating the community structure in simple networks. However, for evaluating community structure in multiplex networks, Mucha *et al.* [40] recently propose a generalized version of modularity as follows:

$$Q_{ms} = \frac{1}{2\mu}\{(A_{ijs} - \gamma_s \frac{k_{is}k_{js}}{2m_s})\delta_{sr} + \delta_{ij}C_{jsr}\}\delta(g_{is}, g_{jr}) \quad (6)$$

where A_{ijs} is the (i, j) entry of the adjacency matrix A in s^{th} layer, γ_s is the resolution parameter (we consider it as 0.5 for all s), k_{is} is the degree of node i in layer s, $2\mu = \sum_{ij} k_{ij}$, m_s is the total number of edges in layer s, C_{jsr} is the inter-slice coupling that connects node j in slice r to itself in slice s, δ is a Kronecker delta, $\delta_{ij} = 1$ if $i = j$ or 0 otherwise and $\delta(g_{is}, g_{jr}) = 1$ if the community assignments g_i and g_j of vertices i and j respectively are the same and 0 otherwise. The more the value of Q_{ms}, the better the performance of the algorithm.

3) Baseline Algorithms: To evaluate the performance of our proposed Algorithm 2, we use two benchmark multiplex community detection algorithms:

- **ABC_Cut** (**A**ggregated **B**etweenness **C**entrality **Cut**) algorithm where we use aggregated betweenness centrality of vertices as mentioned in Approach 1 to compute the edge betweenness centrality in Equation 5 and use this in Algorithm 2.
- **GenLouvain**[5] [40] is a "generalized Louvain" algorithm for community detection that allows the user to define a quality function in terms of a generalized-modularity null model framework and then follows a two-phase iterative procedure similar to the Louvain method [63], with the important distinction that the algorithm works directly with the modularity matrix, not the adjacency matrix. It is a generalized version to detect communities in multiplex networks.

4) Performance Analysis: We run three algorithms namely, CBC_Cut, ABC_Cut and GenLouvain to determine the community structure in both the airline and the co-authorship networks. We then compare their performances using Q_{ms} as shown in Table III. From the results shown in this table, it is clear that CBC_Cut significantly outperforms the two baseline algorithms. On an average, Q_{ms} of CBC_Cut is 0.569, which is 292.41% higher than that of ABC_Cut and 46.97% higher than that of GenLouvain. We observe that the value of Q_{ms} for our algorithm in coauthorship network is less than that of airline network; the reason being that coauthorship network is very sparse (edge density=0.34 and average clustering coefficient=0.42) and does not possess dense subgroups (see [51]).

TABLE III: Performance of different community detection algorithms. Maximum accuracy is obtained while setting δ=0.60 in Algorithm 2.

Networks	Algorithms	Q_{ms}
Airlines	*ABC_Cut*	0.242
	GenLouvain	0.619
	CBC_Cut	**0.841**
Coauthorship	*ABC_Cut*	0.048
	GenLouvain	0.192
	CBC_Cut	**0.297**

[5]http://netwiki.amath.unc.edu/GenLouvain

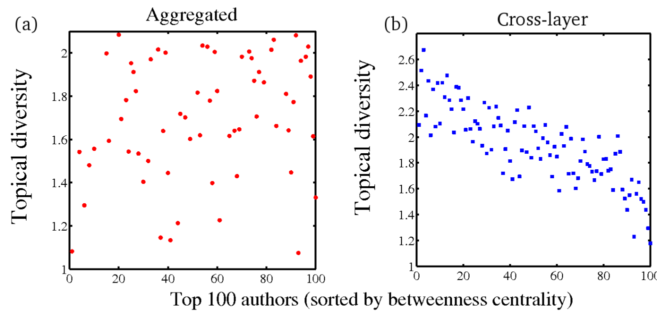


Fig. 8: (Color online) Scatter plots showing the degree of interdisciplinarity of top 100 authors with respect to (a) aggregated and (b) cross-layer betweenness centrality measures. In each panel, the authors are sorted by the decreasing order of corresponding centrality value.

B. Finding Interdisciplinary Authors

In the last two decades, there have been studies claiming that science is becoming ever more interdisciplinary, resulting in the inclination of researchers to work with several areas simultaneously [64]. Leydesdorff [65] mentioned that one of the network features to demarcate the interdisciplinary researchers from the rest would be the betweenness centrality measure. Since the value of betweenness centrality tends to be higher for those vertices which act as bridges, these vertices are also responsible to control the information flow between two communities. Therefore, in the context of multiplex networks, we intend to draw a correlation between the cross-layer betweenness centrality value of each author in the coauthorship multiplex network with his/her degree of interdisciplinary. The rich metadata information in the coauthorship network further allows us to study the extent of interdisciplinary of the authors. Chakraborty *et al.* [66] highlight that the more is the diversity of research topics for an author, the more is the interdisciplinarity of that author. For this, we collect all the abstracts of the papers written by an author and compose a document corresponding to that author. Then we use Latent Dirichlet Allocation (LDA)[6] [67] with its default parameters along with the number of topics being assigned to 100. One of the outputs obtained from LDA contains a document-topic matrix constituting documents as rows and topics as columns. Each document j is represented by a vector of 100 topics $(p_{1j}, p_{2j}, ..., p_{100j})$ where p_{kj} corresponds to the probability that topic i gets reflected from document j. Then the interdisciplinarity $I(j)$ of document j is measured as follows [66]: we take the entropy of the probabilities of the topics associated with document j, i.e., $I(j) = -\sum_i p_{ij} \log(p_{ij})$. Since each document is associated with an author, we thus measure the interdisciplinarity of each author.

Now we compare the scores obtained from the measure of interdisciplinarity with the scores of aggregated betweenness centrality and cross-layer betweenness centrality by plotting them using two scatter diagrams as shown in Figure 8. For the sake of brevity, we consider the top 100 authors based on individual centrality measures, sort them in descending order based on the individual centrality values and plot their degree

[6]http://gibbslda.sourceforge.net/

TABLE IV: Top 10 authors based on aggregated and cross-layer betweenness centrality measures. Common authors in both the lists are italicized.

Aggregated betweenness centrality	Cross-layer betweenness centrality
Witold Pedrycz (University of Alberta)	Didier Dubois (Institut de Recherche en Informatique de Toulouse)
Paolo Dario (Scuola Superiore Sant'Anna)	*Witold Pedrycz* (University of Alberta)
Xin Yao (University of Birmingham)	Wolfram Burgard (University of Fribourg)
Kalyanmoy Deb (Michigan State University)	Erik D. Demaine (Massachusetts Institute of Technology)
Sebastian Thrun (Stanford University)	Micha Sharir (Tel Aviv University)
Nicholas R. Jennings (University of Southampton)	*Leonidas J. Guibas* (Stanford University)
Vijay Kumar (University of Pennsylvania)	Toshio Fukuda (Nagoya University)
Noga Alon (Tel Aviv University)	Fumihito Arai (Nagoya University)
Manuela M. Veloso (Carnegie Mellon University)	Dieter Fox (University of Washington)
Leonidas J. Guibas (Stanford University)	*Xin Yao* (University of Birmingham)

TABLE V: Contingency table showing Kendall τ correlation coefficient among the ranking of vertices obtained from two centrality measures and the interdisciplinarity of all the authors in coauthorship network.

	Aggregated	Cross-layer	Interdisciplinarity
Aggregated	1	0.22	0.31
Cross-layer	0.22	1	**0.72**
Interdisciplinarity	0.31	**0.72**	1

of interdisciplinarity. We observe that with the decrease of cross-layer betweenness centrality, the degree of interdisciplinarity decreases linearly (Figure 8(b)), whereas the same for aggregated betweenness centrality measure does not follow any specific pattern (Figure 8(a)). These results essentially show that our formulation of cross-layer betweenness centrality, quite efficiently, captures the notion of interdisciplinarity in coauthorship networks. Table IV lists the top ten authors based on the aggregated and the cross-layer betweenness centrality measures. We see that only three authors are common in both these lists.

For a broader analysis, we further rank[7] all the authors present in our dataset based on aggregated and cross-layer betweenness centralities and their degree of interdisciplinarity separately. Table V shows a contingency matrix highlighting Kendall τ correlation coefficient among the pair-wise measures. As shown in this table, the correlation is maximum between the ranking obtained from cross-layer betweenness centrality and interdisciplinarity measures (0.72). This once again confirms the merit of our formulation of cross-layer betweenness centrality measure.

C. Initiator Selection for Message Spreading

Message spreading is one of the challenging problems in complex networks [68]. Starting with a set of source vertices/ initiators having a message, the protocol proceeds in a sequence of synchronous rounds with the goal of delivering the message to every node in the network. At every time step, each node in the system having the message communicates

[7]We use dense ranking scheme to rank the authors.

TABLE VI: Time steps required to spread the message using different strategies in two real-world networks. The results are averaged over 500 simulations.

Network	Strategy	Time steps
Airlines	Random	6.66
	Degree	6.15
	Aggregated	5.65
	CBC	5.26
Coauthorship	Random	74.52
	Degree	68.98
	Aggregated	58.62
	CBC	45.26

with one node (not having the message) in its neighborhood and transfers the message. The algorithm terminates when all the vertices in the system have received the message.

A fundamental issue in message spreading is the selection of initiators [69]. The traditional practice is to select initiators based on the degree of vertices, which was proved to be more useful than the random node selection in terms of average time steps required to broadcast the message [70]. Since CBC produces a ranked list of vertices within a network, we posit that initiator selection based on CBC would help in disseminating the message more quickly. For this, we again consider Erdős-Rényi networks discussed in Section V-B and vary the number of vertices, keeping the other parameters constant (number of layers is fixed as 10). We select 10 initiators based on the following criteria separately: (i) random, (ii) highest degree, (iii) highest betweenness centrality based on aggregated method, and (iv) highest betweenness centrality based on CBC-based method. For each network configuration, we run 500 simulations and, in Figure 9, we report the average number of time steps required for the message to reach all the vertices in the network.

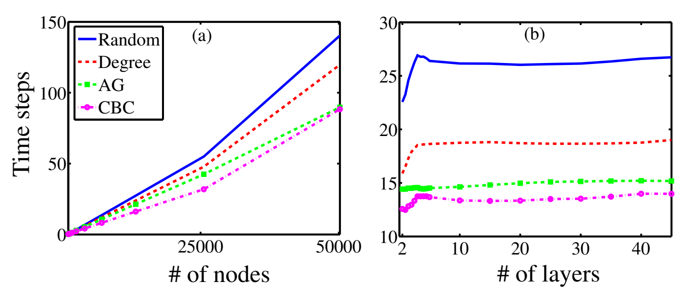


Fig. 9: (Color online) Number of time steps required to spread a message in Erdős-Rényi networks by varying (a) the number of vertices (number of layers is 10), and (b) number of layers (number of vertices is 10,000) (AG: aggregated centrality method, CBC: cross-layered betweenness centrality based method).

We observe, in Figure 9(a), that CBC-based initiator selection requires the least number of time steps to spread the message compared to the aggregated method and degree-based selection for different sizes of the network. For all the cases, the number of time steps is almost linear with the number of vertices. Further, we vary the number of layers from 2 to 50, keeping the number of vertices as 10,000, and report the results in Figure 9(b). Here again the same patterns are observed where CBC-based selection strategy outperforms others. The results in Table VI for two real-world networks also corroborate our earlier observations. As expected, for both the cases random selected performs the least among all the four strategies. These results again highlight the importance of the proposed CBC measure.

VII. Conclusions

In this paper, we proposed cross-layer betweenness centrality measure for the multiplex networks. We also presented a faster algorithm to compute the cross-layer between centrality of vertices in multiplex networks. Simulations of betweenness centrality measure considering the multiplex version of a network exhibited behavior very different from the same computed after considering the aggregated network. Three applications presented here indicated that taking into consideration the multiplex nature of social interaction helps uncover the emergence of rankings of vertices and structural properties that would otherwise remain undetected if only simplex networks were investigated. The proposed algorithm for faster computation of betweenness centrality works for simple unweighted multiplex networks. However, we believe that the adaptation to the weighted multiplex network is straightforward; it is only required to replace the shortest-paths acyclic graph generation for the Dijkstra algorithm instead of the current BFS procedure.

There are several possible extensions to this work. The proposed formulation of CBC is the simplest extension of traditional betweenness centrality designed for simple networks. Instead of considering a linear combination, one can think of further incorporating separate weight for jumping across different layers. For example, one possible extension could be to increase the weight if more layers need to be traversed to reach a destination node from a source node. It is interesting to further extend this definition to sub-graphs in multiplex networks as the sub-graphs can exhibit the synergy that may occur when groups of vertices are considered together. Future work can use more general random graphs such as directed graphs and more general multiplex structures. Moreover, if nodes in a graph are associated with weights indicating some preferences/bias, one can think of reformulating the proposed CBC by taking into consideration node-weight.

References

[1] S. Boccalettia, G. Bianconic, R. Criadod, C. del Geniof, J. Gmez-Gardenesi, M. Romanced, I. Sendina-Nadalj, Z. Wangk, and M. Zaninm, "The structure and dynamics of multilayer networks," *Physics Reports*, vol. 544(1), pp. 1–122, 2014.

[2] V. L. F. Battiston, V. Nicosia, "Structural measures for multiplex networks," *Physical Review E*, vol. 89, 032804, 2014.

[3] M. Kivela, A. Arenas, M. Barthelemy, J. Gleeson, Y. Moreno, and M. Porter, "Multilayer networks," *Journal of Complex Networks*, vol. 2, no. 3, pp. 203–271, 2014.

[4] V. Nicosia, G. Bianconi, V. Latora, and M. Barthelemy, "Non-linear growth and condensation in multiplex networks," in *http://arxiv.org/abs/1312.3683*, 2014.

[5] J. F. Padgett and C. K. Ansell, "Robust Action and the Rise of the Medici, 1400-1434," *The American Journal of Sociology*, vol. 98, no. 6, pp. 1259–1319, 1993.

[6] M. Neumann, *Networks: An Introduction*. Oxford University Press, 2010.

[7] D. Easley and J. Kleinberg, *Networks, Crowds, and Markets: Reasoning About a Highly Connected World*. Cambridge University Press, 2010.

[8] M. Jackson, *Social and Economic Networks*. Princeton University Press, 2008.

[9] L. Freeman, "Centrality in Social Networks: Conceptual Clarification," *Social Networks*, vol. 1, pp. 215–239, 1979.

[10] P. Holme and J. Saramäki, "Temporal networks," *Physics Reports*, vol. 519, no. 3, pp. 97–125, 2012.

[11] H. Jeong, S. Mason, A. Barabasi, and Z. Oltvai, "Lethality and centrality in protein networks," *Nature*, vol. 411, pp. 41–42, May 2001.

[12] E. Cozzo, M. Kivel, M. D. Domenico, A. Sol, A. Arenas, S. Gmez, M. A. Porter, and Y. Moreno, "Clustering coefficients in multiplex networks." *CoRR*.

[13] J. Gomez-Gardenes, I. Reinares, A. Arenas, and L. Mario Floria, "Evolution of cooperation in multiplex networks," *Nature Scientific Reports*, vol. 2, Article N. 620, 2012.

[14] V. Nicosia and V. Latora, "Measuring and modelling correlations in multiplex networks," in *http://arxiv.org/abs/1403.1546*, 2014.

[15] Arda, M. R. J., P. Pietro, and B. G. Halu, "Multiplex pagerank," *PLoS ONE*, vol. 8, no. 10, p. e78293, 10 2013.

[16] G. Kolda, B. W. Bader, and J. P. Kenny, "Higher-order web link analysis using multilinear algebra," in *Proceedings of the Fifth IEEE International Conference on Data Mining*, ser. ICDM '05. Washington, DC, USA: IEEE Computer Society, 2005, pp. 242–249.

[17] K.-M. Lee, J. Y. Kim, W. kuk Cho, K.-I. Goh, and I.-M. Kim, "Correlated multiplexity induces unusual connectivity in multiplex random networks," *CoRR*, vol. abs/1111.0107, 2011.

[18] M. K.-P. Ng, X. Li, and Y. Ye, "Multirank: Co-ranking for objects and relations in multi-relational data," in *ACM SIGKDD*. New York, NY, USA: ACM, 2011, pp. 1217–1225.

[19] F. Battiston, V. Nicosia, and V. Latora, "Metrics for the analysis of multiplex networks." *CoRR*.

[20] A. Halu, S. Mukherjee, and G. Bianconi, "Emergence of overlap in ensembles of spatial multiplexes and statistical mechanics of spatial interacting networks ensembles," *CoRR*, vol. abs/1309.4411, 2013.

[21] V. Nicosia, G. Bianconi, V. Latora, and M. Barthelemy, "Growing multiplex networks," *Phys. Rev. Lett*, vol. 11, no. 058701, 2013.

[22] F. Battiston, V. Nicosia, and V. Latora, "Structural measures for multiplex networks," *Phys. Rev. E*, vol. 89, p. 032804, Mar 2014.

[23] M. D. Domenico, A. Sole-Ribalta, S. Gomez, and A. Arenas, "Navigability of interconnected networks under random failures," *PNAS*, vol. 11, 8351, 2014.

[24] P. Y. Xiangnan Kong, J. Zhang, "Inferring anchor links across multiple heterogeneous social networks," in *CIKM*, Burlingame, CA, USA, 2013, pp. 179–188.

[25] J. Zhang, X. Kong, J. Zhang, and V. Yu, "Predicting social links for new users across aligned heterogeneous social networks," in *ICDM*, Dallas, Texas, 2013, pp. 1289–1294.

[26] M. De Domenico, A. Solé-Ribalta, E. Cozzo, M. Kivelä, Y. Moreno, M. A. Porter, S. Gómez, and A. Arenas, "Mathematical Formulation of Multilayer Networks," *Physical Review X*, no. 4, p. 041022, Dec.

[27] M. Kurant and P. Thiran, "Layered complex networks," *Phys. Rev. Lett.*, vol. 96, p. 138701, Apr 2006.

[28] L. Sola, M. Romance, R. Criado, J. Flores, A. G. del Amo, and S. Boccaletti, "Eigenvector centrality of nodes in multiplex networks," *Chaos*, vol. 3, no. 033131, 2013.

[29] D. Zhou, S. A. Orshanskiy, H. Zha, and C. L. Giles, "Co-ranking authors and documents in a heterogeneous network," in *ICDM*, 2007, pp. 739–744.

[30] X. Li, M. K. Ng, and Y. Ye, "Har: Hub, authority and relevance scores in multi-relational data for query search." in *SDM*. SIAM / Omnipress, pp. 141–152.

[31] M. D. Domenico, A. Sol-Ribalta, E. Omodei, S. Gmez, and A. Arenas, "Centrality in interconnected multilayer networks." *CoRR*, vol. abs/1311.2906, 2013.

[32] J. J. Jung, K. Juszczyszyn, and N. T. Nguyen, "Centrality measurement on semantically multiplex social networks: divide-and-conquer approach," *Int. J. Intell. Inf. Database Syst.*, vol. 1, no. 3/4, pp. 277–292, Jan. 2007.

[33] A. Solé-Ribalta, M. De Domenico, S. Gómez, and A. Arenas, "Centrality rankings in multiplex networks," in *Proceedings of the 2014 ACM Conference on Web Science*, ser. WebSci '14, 2014, pp. 149–155.

[34] S. Gomez, A. Diaz-Guilera, J. Gomez-Gardenes, C. J. Perez-Vicente, Y. Moreno, and A. Arenas, "Diffusion dynamics on multiplex networks," *Physical Review Letters*, vol. 110, 028701, 2013.

[35] S. G. C. Granell and A. Arenas, "Dynamical interplay between awareness and epidemic spreading in multiplex networks," *Physical Review Letters*, vol. 111, 128701, 2013.

[36] ——, "Competing spreading processes on multiplex networks: awareness and epidemics," *Physical Review E*, vol. 90, 012808, 2014.

[37] Z. Kuncheva and G. Montana, "Community detection in multiplex networks using locally adaptive random walks," in *http://arxiv.org/abs/1507.01890*, 2015.

[38] "A unified model for community detection of multiplex networks," in *Web Information Systems Engineering WISE 2014*, ser. Lecture Notes in Computer Science, B. Benatallah, A. Bestavros, Y. Manolopoulos, A. Vakali, and Y. Zhang, Eds., 2014, vol. 8786.

[39] "Community detection in multiplex networks: A seed-centric approach," *Networks and Heterogeneous Media*, vol. 10, no. 1, pp. 71–85, 2015.

[40] P. J. Mucha, T. Richardson, K. Macon, M. A. Porter, and J.-P. Onnela, "Community structure in time-dependent, multiscale, and multiplex networks," *Science*, vol. 329, no. 5989, p. 277, 2010.

[41] A. A. M. De Domenico, M.A. Porter, "Muxviz: a tool for multilayer analysis and visualization of networks," *Journal of Complex Networks*, pp. 1–18, 2014.

[42] A. A. J. Gomez-Gardenes, I. Reinares and L. M. Floria, "Evolution of cooperation in multiplex networks," *Nature Scientific Reports, 10.1038 srep00620*, 2012.

[43] Z. Li and Y. Jiang, "Cross-layers cascade in multiplex networks," in *Proceedings of the 2014 International Conference on Autonomous Agents and Multi-agent Systems*, ser. AAMAS '14, 2014, pp. 269–276.

[44] S. V. Buldyrev, R. Parshani, G. Paul, H. E. Stanley, and S. Havlin, "Catastrophic cascade of failures in interdependent networks," *Nature*, no. 7291, pp. 1025–1028, Apr.

[45] J. Zhang, X. Kong, and P. S. Yu, "Predicting social links for new users across aligned heterogeneous social networks," in *Proceedings of ICDM*, 2013, pp. 1289–1294.

[46] X. Kong, J. Zhang, and P. S. Yu, "Inferring anchor links across multiple heterogeneous social networks," in *Proceedings of CIKM*, 2013, pp. 179–188.

[47] J. Kuck, H. Zhuang, X. Yan, H. Cam, and J. Han, "Query-based outlier detection in heterogeneous information networks," in *Proceedings of EDBT*, 2015, pp. 325–336.

[48] U. Brandes, "A faster algorithm for betweenness centrality," *Journal of Mathematical Sociology*, vol. 25, no. 163, 2001.

[49] M. E. J. Newman, "Scientific collaboration networks. ii. shortest paths, weighted networks, and centrality," *Physical Review E*, vol. 64, no. 1, p. 016132, Jun. 2001.

[50] A. Cardillo, J. Gómez-Gardeñes, M. Zanin, M. Romance, D. Papo, F. Pozo, and S. Boccaletti, "Emergence of network features from multiplexity," *Scientific Reports*, Feb.

[51] T. Chakraborty, S. Sikdar, V. Tammana, N. Ganguly, and A. Mukherjee, "Computer science fields as ground-truth communities: Their impact, rise and fall," in *ASONAM*, Canada, 2013, pp. 426–433.

[52] T. Chakraborty, S. Sikdar, N. Ganguly, and A. Mukherjee, "Citation interactions among computer science fields: a quantitative route to the rise and fall of scientific research," *Social Netw. Analys. Mining*, vol. 4, no. 1, p. 187, 2014.

[53] M. G. Kendall, "A new measure of rank correlation," *Biometrika*, no. 1, pp. 81–93.

[54] P. Erdos and A. Renyi, "On the evolution of random graphs," *Publ. Math. Inst. Hungary. Acad. Sci.*, vol. 5, pp. 17–61, 1960.

[55] M. E. J. Newman, "Communities, modules and large-scale structure in networks," *Nature Physics*, vol. 8, no. 1, pp. 25–31, 2012.

[56] M. Girvan and M. E. J. Newman, "Community structure in social and biological networks," *Proceedings of the National Academy of Sciences*, vol. 99, no. 12, pp. 7821–7826, 2002.

[57] M. E. J. Newman, "Detecting community structure in networks," *The European Physical Journal B - Condensed Matter and Complex Systems*, vol. 38, no. 2, pp. 321–330, Mar. 2004.

[58] J. Jiang and M. Jaeger, "Community detection for multiplex social networks based on relational bayesian networks," *Foundations of Intelligent Systems*, vol. 8502, pp. 30–39, 2014.

[59] R. L. Breiger, S. A. Boorman, and P. Arabie, "An algorithm for clustering relational data with applications to social network analysis and comparison with multidimensional scaling," *Journal of Mathematical Psychology*, no. 3.

[60] V. D. Blondel, J.-L. Guillaume, R. Lambiotte, and E. Lefebvre, "Fast unfolding of communities in large networks," *J. Stat. Mech.*, vol. 2008, no. 10, p. P10008, Oct. 2008.

[61] W. Hwang, T. Kim, M. Ramanathan, and A. Zhang, "Bridging centrality: Graph mining from element level to group level," in *ACM SIGKDD*. New York, NY, USA: ACM, 2008, pp. 336–344.

[62] M. E. Newman, "Modularity and community structure in networks," *Proc Natl Acad Sci U S A*, no. 23, pp. 8577–8582, Jun.

[63] V. Blondel, J. Guillaume, R. Lambiotte, and E. Mech, "Fast unfolding of communities in large networks," *J. Stat. Mech*, p. P10008, 2008.

[64] "An approach to interdisciplinarity through bibliometric indicators," *Scientometrics*, vol. 51, no. 1.

[65] L. Leydesdorff, "Betweenness centrality as an indicator of the interdisciplinarity of scientific journals," *Journal of the American Society for Information Science and Technology*, no. 9, pp. 1303–1319.

[66] T. Chakraborty, S. Kumar, M. D. Reddy, S. Kumar, N. Ganguly, and A. Mukherjee, "Automatic classification and analysis of interdisciplinary fields in computer sciences." in *SocialCom*. Washington, USA: IEEE, pp. 180–187.

[67] D. M. Blei, A. Y. Ng, and M. I. Jordan, "Latent dirichlet allocation," *J. Mach. Learn. Res.*, vol. 3, pp. 993–1022, Mar. 2003.

[68] F. Chierichetti, S. Lattanzi, and A. Panconesi, "Rumour spreading and graph conductance," in *SODA*. SIAM, pp. 1657–1663.

[69] C. Granell, S. Gómez, and A. Arenas, "Competing spreading processes on multiplex networks: Awareness and epidemics," *Phys. Rev. E*, vol. 90, p. 012808, Jul 2014.

[70] A. Demers, D. Greene, C. Hauser, W. Irish, J. Larson, S. Shenker, H. Sturgis, D. Swinehart, and D. Terry, "Epidemic algorithms for replicated database maintenance," in *PODC*, New York, USA, 1987, pp. 1–12.