TEDI: Efficient Shortest Path Query Answering on Graphs

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ABSTRACT

Efficient shortest path query answering in large graphs is enjoying a growing number of applications, such as ranked keyword search in databases, social networks, ontology reasoning and bioinformatics. A shortest path query on a graph finds the shortest path for the given source and target vertices in the graph.

Current techniques for efficient evaluation of such queries are based on the pre-computation of compressed Breadth First Search trees of the graph. However, they suffer from drawbacks of scalability. To address these problems, we propose TEDI, an indexing and query processing scheme for the shortest path query answering. TEDI is based on the tree decomposition methodology. The graph is first decomposed into a tree in which the node (a.k.a. bag) contains more than one vertex from the graph. The shortest paths are stored in such bags and these local paths together with the tree are the components of the index of the graph. Based on this index, a bottom-up operation can be executed to find the shortest path for any given source and target vertices.

Our experimental results show that TEDI offers orders-of-magnitude performance improvement over existing approaches on the index construction time, the index size and the query answering.

Categories and Subject Descriptors

H.3.3 [**Information Search and Retrieval**]: Search process; H.3.1 [**Content Analysis and Indexing**]: Indexing methods

General Terms

Algorithms, Design

Keywords

graphs, indexing, shortest path, tree decomposition

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1. INTRODUCTION

Querying and manipulating large scale graph-like data have attracted much attention in the database community, due to the wide application areas of graph data, such as ranked keyword search, XML databases, bioinformatics, social network, and ontologies. The shortest path query answering in a graph is among the fundamental operations on the graph data.

In a ranked keyword search scenario over structured data, people usually give scores by measuring the link distance between two connected elements. If more than one path exists, it is desirable to retrieve the shortest distance between them, because shorter distance normally means higher rank of the connected elements [19, 20, 12]. In a social network application such like Facebook [1], registered users can be considered as vertices and edges represent the friend relationship between them. Normally two users are connected through different paths with various lengths. The problem of retrieving the shortest path among the users efficiently is of great importance.

Let $G = (V, E)$ be an undirected graph where $n = |V|$. Given $u, v \in V$, the shortest path problem finds a shortest path from u to v with respect to the length of paths from u to v. One obvious solution for the shortest path problem is to execute the Breadth First Search (BFS) on the graph. The time complexity of the BFS method is $O(n)$. If the graph is of large size, the efficiency of query answering is expected to be improved.

The query answering can be performed in constant time, if the BFS tree for each vertex of the graph is pre-computed. However, the space overhead is n^2. This is obviously not affordable if n is of large value. Therefore, appropriate indexing and query scheme for answering shortest path queries must find the best trade-off between these two extreme methods.

Xiao et al.[23] proposed the concept of *compact BFS-trees* where the BFS-trees are compressed by exploiting the symmetry property of the graphs. It is shown that the space cost for the compact BFS trees is reduced in comparison to the normal BFS trees. Moreover, shortest path query answering is more efficient than the classic BFS-based algorithm. However, this approach suffers from scalability. Although the index size of the compact BFS-trees can be reduced by 40% or more (depending on the symmetric property of the graph), the space requirement is still prohibitively high. For instance, for a graph with 20 K vertices, the compact BFS-tree has the size of 744 MB, and the index construction takes

[1] `http://www.facebook.com`

more than 30 minutes. Therefore, this approach can not be applied for graphs with larger size which contain hundreds of thousands of vertices.

1.1 Our Contributions

To overcome these difficulties, we propose **TEDI (TreE Decomposition based Indexing)**, an indexing and query processing scheme for the shortest path query answering. Briefly stated, we first *decompose* the graph G into a tree in which each node contains a set of vertices in G. These tree nodes are called *bags*. Different from other partitioning based methods, there are overlapping among the bags, i.e., for any vertex v in G, there can be more than one bag in the tree which contains v. However, it is required that all these related bags constitute a connected subtree (see Definition 1 for the formal definition).

Based on the decomposed tree, we can execute the shortest path search in a bottom-up manner and the query time is decided by the height and the bag cardinality of the tree, instead of the size of the graph. If both of these two parameters are small enough, the query time can be substantially improved. Of course, in order to compute the shortest paths along the tree, we have to pre-compute the *local* shortest paths among the vertices in every bag of the tree. This constitutes the major part of the index structure of the TEDI scheme. Our main contributions are the following:

- **Solid theoretical background.** TEDI is based on the well-known concept of the tree decomposition, which is proved being of great importance in computational complexity theory. The abundant theoretical results provide a solid background for designing and correctness proofs of the TEDI algorithms.
- **Linear time tree decomposition algorithm.** In spite of the theoretical importance of the tree decomposition concept, many results are practically useless due to the fact that finding a tree decomposition with optimal treewidth is an NP-hard problem, w.r.t. the size of the graph. To overcome this difficulty, we propose in TEDI the simple heuristics to achieve a linear time tree decomposition algorithm. To the best of our knowledge, TEDI is the first scheme that applies the tree decomposition heuristics dealing with graphs of large size, and based on that, enables efficient query answering algorithms to be developed.
- **Flexibility of balancing the time and space efficiency.** From the proposed tree decomposition algorithm, we discover an important correlation between the query time and the index size. If less space for the index is available, we can reduce the index size by increasing some parameter k during the tree decomposition process, while the price to pay is longer query time. On the other hand, if the space is less critical, and the query time is more important, we can decrease k to achieve higher efficiency of the query answering. This flexibility offered by TEDI enables the users to choose the best time/space trade-off according to the system requirements.

Finally we conduct experiments on real and synthetic datasets, and compare the results with the compact BFS-tree approach. The experimental results confirm our theoretical analysis by demonstrating that TEDI offers orders-of-magnitude performance improvement over existing algorithms. Moreover, we conduct the experiments over large graphs such as DBLP and road networks. The encouraging results show that TEDI scales well on large datasets.

The rest of the paper is organized as follows: in Section 2 we introduce the formal definitions of the tree decomposition and prove the theoretical results regarding to the shortest path query answering. Section 3 contains the detailed algorithms and the complexity analysis. In Section 4 we present the experimental results. We survey related works in Section 5 and conclude in Section 6.

2. GRAPH INDEXING WITH TREE DECOMPOSITION

2.1 Tree Decomposition

An undirected graph is defined as $G = (V, E)$, where $V = \{0, 1, \ldots, n-1\}$ is the vertex set and $E \subseteq V \times V$ is the edge set. Let $n = |V|$ be the number of vertices and $m = |E|$ be the number of edges. In this paper, we consider only undirected graphs, where $\forall u, v \in V : (u, v) \in E \Leftrightarrow (v, u) \in E$ holds. Moreover, we assume that the edges are not labeled. In the rest of the paper, we will use the term *graph* to denote *undirected graph*.

Definition 1 (Tree Decomposition). *A tree decomposition of $G = (V, E)$, denoted as T_G, is a pair $(\{X_i \mid i \in I\}, T)$, where $\{X_i \mid i \in I\}$ is a collection of subsets of V and $T = (I, F)$ is a tree such that:*

1. $\bigcup_{i \in I} X_i = V$.
2. *for every $(u, v) \in E$, there is $i \in I$, s.t. $u, v \in X_i$.*
3. *for all $v \in V$, the set $\{i \mid v \in X_i\}$ induces a subtree of T.*

A tree decomposition consists of a set of tree nodes, where each node contains a set of vertices in V. We call the sets X_i *bags*. It is required that every vertex in V should occur in at least one bag (condition 1), and for every edge in E, both vertices of the edge should occur together in at least one bag (condition 2). The third condition is usually referred to as the *connectedness condition*, which requires that given a vertex v in the graph, all the bags which contain v should be connected.

Note that from now on, the node in the graph G is referred to as *vertex*, and the node in the tree decomposition is referred to as *tree node* or simply *node*. For each tree node i, there is a bag X_i consisting of vertices. To simplify the presentation, we will sometimes use the term *node* and its corresponding *bag* interchangeably. Given a tree decomposition T_G, we denote its root as R.

Given any graph G, there may exist many tree decompositions which fulfill all the conditions in Definition 1. However, we are interested in those tree decompositions with smaller bag sizes. We call the cardinality of a bag the *width* of the bag.

Definition 2 (Width, Treewidth). *Let $G = (V, E)$ be a graph.*

- *The* width *of a tree decomposition $(\{X_i \mid i \in I\}, T)$ is defined as $max\{|X_i| \mid i \in I\}$* [2].

[2]The original definition of the width is $max\{|X_i| \mid i \in I\} - 1$, due to technical reasons.

- *The* treewidth *of G is the minimal width of all tree decompositions of G. It is denoted as $tw(G)$ or simply tw.*

EXAMPLE 1. *Consider the graph illustrated in Figure 1(a). One of the tree decompositions is shown in Figure 1(b). The width of this tree decomposition is 3. It is not hard to check that this tree decomposition is optimal, thus the treewidth of the graph is 3.*

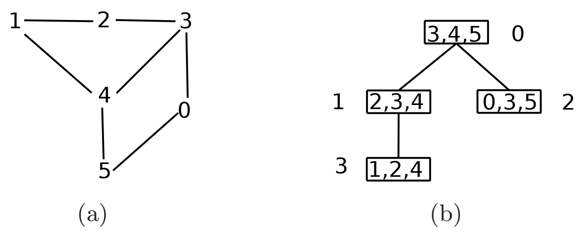


Figure 1: The graph G (a) and one tree decomposition T_G (b) with $tw = 3$

Induced subtree. Let $G = (V, E)$ be a graph and $T_G = (\{X_i \mid i \in I\}, T)$ a tree decomposition of G. Due to the third condition in Definition 1, for any vertex v in V there exists an induced subtree of T_G in which every bag contains v. We call it the *induced subtree* of v and denote it as T_v. Furthermore, we denote the root of T_v as r_v and its corresponding bag as X_{r_v}. For instance, X_0, X_1 and X_2 constitute the bags of the induced subtree of vertex 3 in Figure 1(b), because vertex 3 occurs precisely in these three bags. Since X_0 is the root of the induced subtree, we have $r_3 = 0$.

2.2 Tree Path

Let $G = (V, E)$ be a graph, and $u, v \in V$. v is reachable from vertex u, denoted as $u \to v$, if there is a path starting from u and ending at v with the form $\{u, v_1, \ldots, v_n, v\}$, where (u, v_1), $(v_i, v_{i+1})_{(1 \leq i \leq n-1)}$, $(v_n, v) \in E$. The shortest distance from vertex u to vertex v is denoted as $sdist(u, v)$. Note that in this paper, we consider only *simple paths*. Obviously, for undirected graphs, $u \to v \Leftrightarrow v \to u$.

Let us consider the graph vertices in the tree nodes. Since a vertex in G may occur in more than one bag of T_G, it is identified with $\{v, i\}$, where v is the vertex and i the node in the tree, meaning that vertex v is located in the bag X_i. We denote it as *tree vertex*.

Now we define the so-called *internal edge* in the tree decomposition. Recall that the second condition in Definition 1 requires that for every edge $(u, v) \in E$, both u and v should occur in some bag of T_G. We call these edges *inner edges* in the tree decomposition.

DEFINITION 3 (INNER EDGE). *Let $G = (V, E)$ be a graph and $T_G = (\{X_i \mid i \in I\}, T)$ a tree decomposition of G. The* inner edges *of T_G are the pairs of tree vertices defined as follows:*

$$\{(\{u, i\}, \{v, i\}) \mid (u, v) \in E, u, v \in X_i (i \in I)\}$$

Intuitively, the set of inner edges consists of precisely those edges in E, with the extra information of the bags in which the edges are located. For instance, the inner edges of the tree decomposition of the graph in Example 1 are: $(\{0, 2\}, \{5, 2\})$, $(\{1, 3\}, \{2, 3\})$, $(\{2, 1\}, \{3, 1\})$, $(\{3, 2\}, \{0, 2\})$, $(\{4, 1\}, \{3, 1\})$, $(\{4, 0\}, \{3, 0\})$, $(\{4, 3\}, \{1, 3\})$, $(\{4, 0\}, \{5, 0\})$, ... Note that it is possible that the same pair of vertices occurs in more than one bag. For instance, the edge $(4, 3)$ occurs in both bags X_0 and X_1. Thus there are two inner edges: $(\{4, 1\}, \{3, 1\})$ and $(\{4, 0\}, \{3, 0\})$.

In order to traverse from one bag to another in the tree decomposition, we need to define the second kind of edge, the *inter edge*. Note that two neighboring bags in the tree are in fact connected by the common vertices they share, according to the connectedness condition in Definition 1.

DEFINITION 4 (INTER EDGE). *Let $G = (V, E)$ be a graph and $T_G = (\{X_i \mid i \in I\}, T)$ a tree decomposition of G. Let $v \in X_i$ and $v \in X_j$, where either $(i, j) \in F$ or $(j, i) \in F$ holds. We call the edge from vertex $\{v, i\}$ to $\{v, j\}$ the* inter edge *and denote it as $(\{v, i\}, \{v, j\})$.*

For instance, in Example 1, $(\{5, 0\}, \{5, 2\})$ is an inter edge, as well as $(\{5, 2\}, \{5, 0\})$, because vertex 5 occurs in bags X_0 and X_2, where 0 is the parent node of 2. Now we are ready to define the *tree path* on the tree decomposition with inner and inter edges.

DEFINITION 5 (TREE PATH). *Let $G = (V, E)$ be a graph and $T_G = (\{X_i \mid i \in I\}, T)$ a tree decomposition of G. Let $u, v \in V$. Let further $\{u, i\}$ and $\{v, j\}$ be tree vertices in T_G. A* tree path *from $\{u, i\}$ to $\{v, j\}$ is a sequence of tree vertices connected with either inter or inner edges.*

LEMMA 1. *Let $G = (V, E)$ be a graph and $T_G = (\{X_i \mid i \in I\}, T)$ a tree decomposition of G. Let $u, v \in V$. Let further $\{u, i\}$ and $\{v, j\}$ be tree vertices in T_G. There is a path from u to v in G if and only if there is a tree path from $\{u, i\}$ to $\{v, j\}$.*

PROOF. The "if" direction is trivial: given a tree path from $\{u, i\}$ to $\{v, j\}$, we only need to consider the inner edges. Since for each inner edge $\{u, i\}, \{v, i\}$, there is an edge $(u, v) \in E$, the path from u to v can be easily constructed.

Now we prove the "only if" direction: assume that there is a path from u to v in G. We prove it by induction on the length of the path.

- Basis: if u reaches v with a path of length 1, that is, $(u, v) \in E$. Then there exists a node k in the tree decomposition, s.t. $u \in X_k$ and $v \in X_k$. We start from $\{u, i\}$, traverse along the induced subtree of u, till we reach $\{u, k\}$. Since the induced subtree is connected, the path from $\{u, i\}$ to $\{u, k\}$ can be constructed with inter edges. Then we reach from $\{u, k\}$ to $\{v, k\}$ with an inner edge. Now we traverse from $\{v, k\}$ to $\{v, j\}$ along the induced subtree of v, which can again be constructed with inter edges. The tree path from $\{u, i\}$ to $\{v, j\}$ is thus completed.

- Induction: assume that the lemma holds with paths whose length is less than or equal to $n - 1$, we prove that it holds for paths with length of n. Assume that there is a path from u to v with length n, where u reaches w with length $n - 1$ and $(w, v) \in E$. From induction hypothesis, we know that there is a tree path form $\{u, i\}$ to $\{w, l\}$ in the tree decomposition, where l is a node in the induced subtree of w. Since $(w, v) \in E$, there is a node n such that $w \in X_n$ and $v \in X_n$. Thus $\{w, n\}$ can be reached from $\{w, l\}$ with inter edges.

Then $\{w, n\}$ can reach $\{v, n\}$ with an inner edge. Finally $\{v, n\}$ can reach $\{v, j\}$ with a sequence of inter edges. This completes the proof.

□

EXAMPLE 2. *Consider the graph in Figure 1(a). Vertex 4 reaches vertex 0 with the path* $(4, 1, 2, 3, 0)$. *In the tree decomposition in Figure 1(b), there is a tree path from* $\{4, 1\}$ *to* $\{0, 2\}$ *as follows:* $(\{4, 1\}, \{4, 3\}, \{1, 3\}, \{2, 3\}, \{2, 1\}, \{3, 1\}, \{3, 0\}, \{3, 2\}, \{0, 2\})$.

2.3 Shortest Path Query Answering on Tree Decomposition

With the definition of the tree path, we are able to map every path from u to v in the graph G into a tree path in the corresponding tree decomposition T_G. Since all the paths can be traced in the tree decomposition, after inspecting all the corresponding tree nodes, we are able to compute the shortest path from u to v. Our goal is now to restrict the attention of the tree nodes to those that every tree path would pass through.

There is a well-known property of trees that for any two nodes i and j in a tree, there exists a unique simple path, denoted as $SP_{i,j}$, such that every path from i to j contains all the nodes in $SP_{i,j}$. In the following we show that this property can also be applied to tree paths.

Given a tree path P in the tree decomposition, we say P visits a node i , if there is a tree vertex $\{v, i\}$ in P.

LEMMA 2. *Let* $\{u, i\}$ *and* $\{v, j\}$ *be two tree vertices, and* $SP_{i,j}$ *be the simple path between tree nodes* i *and* j. *Let* P *be a tree path from* $\{u, i\}$ *to* $\{v, j\}$. *Then* P *visits every node in* $SP_{i,j}$.

EXAMPLE 3. *The tree path from* $\{4, 1\}$ *to* $\{0, 2\}$ *shown in Example 2 visits the nodes in a sequence of* $(1, 3, 1, 0, 2)$. *It contains not only all the nodes in the simple path, but also node* 3*, which is not in the simple path.*

THEOREM 1. *Let* $G = (V, E)$ *be a graph and* $T_G = (\{X_i \mid i \in I\}, T)$ *a tree decomposition of* G. *Let* $u, v \in V$, *and* r_u *(resp.* r_v*) be the root node of the induced subtree of* u *(resp.* v*). Then for every node* w *in* SP_{r_u,r_v}, *there is one vertex* $t \in X_w$, *such that* $sdist(u, v) = sdist(u, t) + sdist(t, v)$.

PROOF. Let p be the shortest path from u to v in G that generates $sdist(u, v)$. From Lemma 1, we know that there is a corresponding tree path P of p from $\{u, r_u\}$ to $\{v, r_v\}$ in T_G. According to Lemma 2, P visits all the nodes in SP_{r_u,r_v}. Then for every node n in SP_{r_u,r_v}, there is a vertex $\{t, n\}$ in P. Therefore, t is in p. Then we have $sdist(u, v) = sdist(u, t) + sdist(t, v)$. □

Theorem 1 shows that for every path from u to v in G, although the tree path from $\{u, r_u\}$ to $\{v, r_v\}$ may possibly visit any node in the tree, we only need to concentrate on those vertices which occur in the simple path SP_{r_u,r_v}. More precisely, we can simply take *any* node w from SP_{r_u,r_v}, and for each $t \in X_w$, compute $sdist(u, t)$ and $sdist(t, v)$ respectively. Then, the minimal value of $sdist(u, t) + sdist(t, v)$ is the shortest distance from u to v. However, this is obviously not a solution, since we have to compute the shortest path for $2c$ times, where c is the cardinality of the bag X_w.

The following question thus arises: *Can we compute the shortest path from u to the vertices (respectively from the vertices to v) in the simple path more efficiently*? The answer is yes. Intuitively, we can compute the shortest distance from u to the vertices occur in the simple path bag after bag, in a bottom-up manner. Let us assume that the current bag be X_c and its parent bag be X_p. Assume further that for all the vertices $t \in X_c$, we have obtained $sdist(u, t)$. Now we start processing X_p. For any vertex $s \in X_p \setminus X_c$, the following property holds:

$$sdist(u, s) = \{min(sdist(u, t) + sdist(t, s)) \mid t \in X_c \cap X_p\}$$

This is because *every* path from u to s has to pass through one vertex in $X_c \cap X_p$ (formal proof given in Theorem 2). Therefore, the shortest path must be among them.

Clearly, in order to enable the bottom-up operation, we need to store the shortest distances for each bag in the tree decomposition. That is, in every bag X, for every pair of vertices $x, y \in X$, we pre-compute $sdist(x, y)$ and store them locally. In addition to the local shortest distance, we need to store the vertices in $sdist(x, y)$, denoted as $path(x, y)$, to trace back the vertices along the shortest path later on.

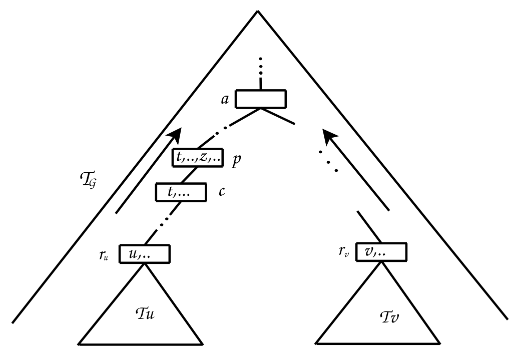


Figure 2: Bottom-up processing on the simple tree path

Given the tree decomposition of G and the local shortest distance, as well as u and v, the shortest path query answering is sketched in Theorem 2. See Figure 2 for a graphical illustration.

THEOREM 2. *Let* $G = (V, E)$ *be a graph and* $u, v \in V$. *Let* tw *be the treewidth of* G *and* $T_G = (\{X_i \mid i \in I\}, T)$ *be the corresponding tree decomposition. The shortest distance from* u *to* v *can be computed in* $O(tw^2 h)$, *where* h *is the height of* T_G.

PROOF. We show that the bottom-up operation can be executed on T_G, so that the shortest distance (as well as the corresponding shortest path) from u to v can be computed.

Let us make the following assumptions of T_G: T_u (resp. T_v) is the induced subtree of u (resp. v) where r_u (resp. r_v) is the root. a is the youngest common ancestor of r_u and r_v, Assume further that the shortest distance ($sdist$) and shortest path ($path$) for the pairs of vertices in every bag in T_G are pre-computed. We explain the process only for the side of u, the bottom-up process from the v side is identical.

The process starts with the node r_u. From the information of shortest distance in the tree node X_{r_u}, we can simply obtain $sdist(u, t)$ for each $t \in X_{r_u}$.

Next, we consider r_u as the child node and process its parent node, with the available *sdist* information. This process is executed till a is reached.

We show that at each step of the processing, the shortest distance from u to all the vertices in the current bag can be computed in w^2 time, where w is the width of the current bag. Assume p is the current node, c its child node, and we have obtained $sdist(u,t)$ for each $t \in X_c$.

Now we have to compute the shortest distance from u to every vertex in X_p. Let z be a vertex in X_p. We want to compute $sdist(u,z)$. We consider the following two cases:

1. $z \in X_p$ and $z \in X_c$. Since at the child node $sdist(u,z)$ is already obtained, the value $sdist(u,z)$ remains unchanged.

2. $z \in X_p$ and $z \notin X_c$. This is a more complex case. We show that there is a vertex t, where $t \in X_p$ and $t \in X_c$, such that $sdist(u,z) = sdist(u,t) + sdist(t,z)$. In other words, the shortest path from u to z has to pass through some vertex t which occurs in both X_c and X_p.

   Since z does not occur in X_c, according to the connectedness condition, z does not occur in any bag in the subtree rooted with c. Thus the induced subtrees of u and z do not share any common node in T_G. Since $u \to z$, there is a tree path from $\{u, r_u\}$ to $\{z, r_z\}$, and $c, p \in SP_{r_u,r_z}$. The tree path from $\{u, r_u\}$ to $\{z, r_z\}$ must contain an inter edge of the form $(\{t,c\},\{t,p\})$, where $t \in X_p, X_c$, because this is the only possible edge to traverse from c to p. Therefore, the shortest path from u to z must pass through one vertex $t \in X_p, X_c$.

Given the shortest distance from u to every vertex in X_c, we compute the shortest distance from u to the vertices in X_p as follows: First for each vertex $t \in X_c \cap X_p$, set $sdist(u,t)$ the same value as in X_c. Then for each vertex $t \in X_p \setminus X_c$, set $sdist(u,t) = min(sdist(u,s)+sdist(s,t))$ where $s \in X_c \cap X_p$. Clearly the time consumption is in the worst case $O(w^2)$ where w is the width of X_p.

To sum up, at each step, the time cost of updating the shortest distance is $O(tw^2)$, where tw is the treewidth of G. and there are maximally $2h$ steps where h is the height of the tree decomposition. Hence the overall time cost is $O(tw^2 h)$. $\square$

3. ALGORITHMS AND COMPLEXITY RESULTS

In this section, we present the detailed algorithms for both the index construction and the shortest path query answering. In Section 3.1 we begin with the introduction of algorithmic issues on the tree decomposition from a complexity theory perspective, and then justify our choice of an efficient but suboptimal decomposing algorithm. In Section 3.2 we first analyze the shortest path query answering algorithm proposed in Theorem 2 from the previous section. Then, we point out that the time and space improvement can be made to achieve more efficiency of our algorithm.

3.1 Index Construction via Tree Decomposition

The index construction algorithm consists of three steps (see Algorithm 1). Step 1 and 2 constitute the major part of the index construction task: decomposing the graph G.

Algorithm 1 *index_construction*(G)

Input: $G = (V, E)$
Output: T_G and local shortest paths.
1: *graph_reduction*(k, G); {output the vertex stack S and reduced graph G'}
2: *tree_decomposition*(S, G, G'); {output the tree decomposition T_G}
3: *local_shortest_paths*(G, T_G); {compute local shortest paths in T_G}

Since its introduction by Robertson and Seymour [18], the concepts of tree decomposition has been proved to be of great importance in computational complexity theory. The interested readers may refer to an introductory survey by Bodlaender [4]. The theoretical significance of the tree decomposition based approach lies in the fact that many intractable problems can be solved in polynomial time (or even in linear time) for graphs with treewidth bounded by a constant. Problems which can be dealt with in this way include many well known NP-complete problems, such as the Independent Set, the Hamiltonian Circuits, etc. Recent applications of tree decomposition based approaches can be found in Constraint Satisfaction [15] and database design [10].

However, the practical usefulness of tree decomposition based approaches has been limited due to the following two problems: (1) Computing the treewidth of a graph is hard. In the last section, we have always assumed that a tree decomposition is given. In fact, to obtain a tree decomposition with the minimal width (treewidth) is a optimization problem. Determining whether the treewidth of a given graph is at most a given integer w is NP-complete [2]. Although for a fixed w, linear time algorithms exist to solve the decision problem "treewidth $\leq w$", there is a huge hidden constant factor, which prevents it from being useful in practice. There exist many heuristics and approximation algorithms for determining the treewidth, unfortunately few of them can deal with graphs containing more than 1000 vertices [17]. (2) The second problem lies in the fact that even if the treewidth can be determined, good performance for solving intractable problems by using the tree decomposition based approaches can only be achieved if the underlying structure has *bounded* treewidth (i.e. less than 10). Because the time complexity of most of the algorithms is exponential to the treewidth.

As far as the efficiency is concerned, we can only search for an approximate solution, which yields a suboptimal treewidth. On the other hand, we can tolerate a treewidth which is not bounded. As we have seen from Theorem 2, the time complexity is in the worst case quadratic of the treewidth (width). We will show later in this section that our query answering algorithm does not depend on the treewidth, but with some parameter which can be enforced to be bounded, due to the nice property of our dedicated decomposing algorithm.

Inspired by the so-called pre-processing methods from Bodlaender et al. [5], we apply the reduction rules on the graph by reducing stepwise a graph to another one with fewer vertices, due to the following simple fact.

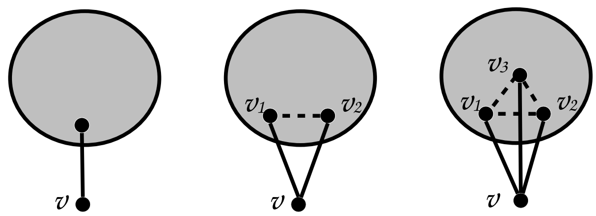
Definition 6 (Simplicial). *A vertex v is simplicial in a graph G if the neighbors of v form a clique in G.*

Proposition 1. *If v is a simplicial vertex in a graph G,* *computing the treewidth of G is equivalent to computing the treewidth of $G - v$.*

Proof. Let T_{G-v} be the tree decomposition of $G - v$. Let further $\{v_1, \dots, v_l\}$ be the neighbors of v. We construct a bag $X_c = \{v, v_1, \dots, v_l\}$. Since $\{v_1, \dots, v_l\}$ form a clique, there exists a bag in T_{G-v} consisting of all the vertices in $\{v_1, \dots, v_l\}$. We denote it as X_p. We can then construct T_G by adding X_c as a child of X_p. It is easy to verify that all the conditions of the tree decomposition are fulfilled. Thus this new tree is also a tree decomposition.

Now consider the treewidth of T_G. X_c has the size of $l+1$. It is easy to show that the size of X_p is equal to or greater than $l + 1$, thus by adding the new bag X_c the treewidth remains to be the same. $\square$

Figure 3 shows some special cases. If a vertex v has degree of one (Figure 3(a)), then we can remove v without increasing the treewidth. Figure 3(b), 3(c) illustrate the cases of degree 2 and 3 respectively.



(a) (b) (c)

Figure 3: A graph containing a vertex v with degree 1 (a), 2 (b) and 3 (c)

The main idea of our decomposition algorithm is to reduce the graph by removing the vertices one by one from the graph, and at the same time push the removed vertices into a stack, so that later on the tree can be constructed with the information from the stack. Each time a vertex v with a specific degree is identified. We first check whether all its neighbors form a clique, if not, we add the missing edges to construct a clique. Then v together with its neighbors are pushed into the stack, which is followed by the deletion of v and the corresponding edges in the graph. See Algorithm 2.

The program begins with removing isolated vertices and vertices with degree 1. Then, the reduction process proceeds by increasing the degree of the vertex. We denote such procedure of removing all the vertices with degree l as *degree-l reduction*.

Example 4. *Consider the graph in Example 1. Figure 4 illustrates the reduction process. The process starts with a degree-2 reduction by removing vertex* 0 *and its edges, after adding the edge between* 3 *and* 5*. Vertex* 0 *and its neighbors are then pushed in the stack. Then vertex* 1 *is removed, following the same principle as of* 0*. After vertex* 2 *is removed, a single triangle is then left.*

Algorithm 2 will terminate if one of the following conditions is fulfilled. (1) The graph is reduced to an empty set. For instance, if the graph contains only simple cycles, it will be reduced to an empty set after degree-2 reductions. This is usually the case for extremely sparse graphs. (2) For graphs which are not sparse, one has to define a upper bound k for the reduction, so that the program stops after the degree-k reduction. Note that as the degree increases, the effectiveness of the reduction will decrease, because in the worst case, we need to add $l(l-1)/2$ edges in order to remove l edges.

Algorithm 2 *graph_reduction*(k, G)

Input: graph G, k is the upper bound for the reduction.
Output: stack S and the reduced graph G'

```
initialize stack S;
for i = 1 to k do
    remove_upto(i);
end for
return S, G;

procedure remove_upto(l)
while TRUE do
    if there exists a vertex v with degree less than l then
        {v_1, ..., v_l} = neighbors of v;
        build a clique for {v_1, ..., v_l} in G;
        push v, v_1, ..., v_l into S;
        delete v and all its edges from G;
    else
        break;
    end if
end while
```

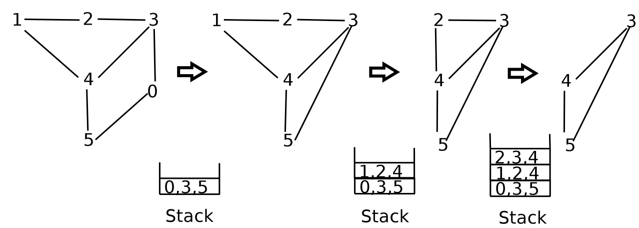


Figure 4: The reduction process on the graph of Example 1

After the reduction process, the tree decomposition can be constructed as follows: (1) At first we collect all the vertices which were not removed by the reduction process and assign this set as the bag of the tree root R. The size of R depends on the structure of the graph (i.e. how many vertices are left after the reduction). For graphs which are not sparse, the bag size of the root is the greatest among all bags in the tree decomposition. (2) The rest of the tree is generated from the information stored in stack S. Let X_c be the set of vertices $\{v, v_1, \dots, v_l\}$ which is popped up from the top of S. Here v is the removed vertex and $\{v_1, \dots, v_l\}$ are the neighbors of v which form a clique. After the parent bag X_p which contains $\{v_1, \dots, v_l\}$ is located in the tree, X_c is added as a child bag of X_p. This process proceeds until S is empty. Algorithm 3 illustrates the process.

The last step of the index construction is to compute and store the shortest distances and paths for every pair of vertices in the bags of the tree decomposition (see Algorithm 4). We apply the standard BFS algorithms. Due to the property of the tree decomposition, there exist redundancies. For instance, both node 1 and 3 in Figure 1(b) contain the vertex pair $(2, 4)$. Therefore, the same shortest distance

Algorithm 3 *tree_decomposition*(S, G, G')

Input: S is the stack storing the removed vertices and their neighbors, G is the graph, G' is the reduced graph of G.
Output: return tree decomposition T_G

```
construct the root of T_G containing all the vertices of G';
while S is not empty do
    pop up a bag X_c = {v, v_1, ..., v_l} from S;
    find the bag X_p containing {v_1, ..., v_l};
    add X_c into T as the child node of X_p;
end while
```

Algorithm 4 *local_shortest_paths*(G, T_G)

Input: G, T_G
Output: local shortest paths in T_G

```
for all bag X in the tree decomposition do
    for every vertex pair x, y ∈ X do
        compute sdist(x, y) and path(x, y)
    end for
end for
```

is stored in two bags. This redundancy can be avoided by storing the shortest distances in a hash table.

3.2 Shortest Path Query Answering

Recall from Theorem 2 that the time complexity of the bottom-up query answering is $O(tw^2h)$. This upper bound is optimal, only if the following two conditions are fulfilled: (1) the treewidth of the underlying graph is bounded (that is, $tw^2 \ll n$), and (2) there is an efficient tree decomposition algorithm for it. The first condition has to be fulfilled, since otherwise the linear time BFS algorithm would be more efficient. Unfortunately, as we have seen in Section 3.1, given an arbitrary graph, neither of them can be guaranteed. Therefore, we have to inspect the tree decomposition heuristics applied in Section 3.1 for improvements.

3.2.1 From Treewidth to $|R|$ and k

According to Algorithm 2, a graph G can be decomposed by the *degree-l reductions* by increasing l from 1 to k. As soon as the *degree-k reduction* is done, all the vertices which are not yet removed are the elements in R of the tree decomposition. Usually if the graph is not extremely sparse, the relationship $k \ll |R|$ holds. In fact, we could even enforce such a relationship by setting k to be small enough in the tree decomposition algorithm. Hence, the resulting tree decomposition has the following properties: (1) the root is of big size ($|R|$), and (2) the rest of the bags have smaller size (the upper bound is k).

If we inspect the bottom-up query processing more carefully, we could observe that the quadratic time computation over the root can be *always* be avoided. To see this, let us consider the vertices u and v and the youngest common ancestor of r_u and r_v is the root R. Assume that X_1 (resp. X_2) is the child node of R which locates in the simple path from r_u (resp. r_v) to R. Consider now that for all $x \in X_1$, $sdist(u, x)$ (resp. all $y \in X_2$, $sdist(y, v)$) have been computed. Clearly, any path from u to v has to pass through a vertex in X_1 and X_2 respectively. Therefore, at the root node R, the *interface* from X_1 to R has to be $X_1 \cap R$, and accordingly, the *interface* from X_2 to R has to be $X_2 \cap R$. Now, we only need to execute a nested loop on $X_1 \cap R$ and $X_2 \cap R$, to decide the final shortest path. Since both $|X_1|$ and $|X_2|$ have the upper bound of k, the overall time consumption is of $O(k^2h)$, thus independent of $|R|$.

The algorithm for the shortest path query answering is presented in Algorithm 5. Comparing with the bottom-up query processing shown in Theorem 2, Algorithm 5 is customized with respect to our dedicated tree decomposition algorithm, in the sense that the query time complexity is adapted to be related to k, instead of the treewidth.

Given the graph G, its tree decomposition T_G and the pre-computed local shortest paths in the bags on T_G, as well as the vertices u, v, we first locate the root of the induced subtree of u (v), denoted as r_u (r_v). The algorithm considers special cases, according to the relationship between r_u and r_v. (1) if r_u and r_v are located in one bag, then the shortest path can be immediately returned (line 4). (2) Otherwise, we have to locate the youngest common ancestor of r_u and r_v, denoted as *root*. Here a special case is either r_u or r_v is *root*. If this occurs, the bottom-up operation should be only executed from one side. Note that we stop the processing at the child node of the *root* node. At each step of the bottom-up processing, we update the shortest distance value, as well as the *parent* value, for the later backtrace for the shortest path(line 11 and 19). As soon as both the child nodes X_1 and X_2 are reached, we obtain the common values with the *root* node by executing the set intersection operation with it (line 25). At last, a nested loop over the bag X_1 and X_2 is executed, to *stitch the threads* from both sides. Finally, the shortest path can be traced back using the p set we have stored during the bottom-up processing.

3.3 Complexity

Index construction time. For the index construction, we have to (1) generate the tree decomposition, and (2) at each tree node, generate the required shortest paths. For (1), both of the reduction step and the tree construction procedure take time $O(n)$. For (2), we deploy the classic BFS algorithm, which costs in worst case $O(n)$. In fact, we need to run for each vertex in G exactly one BFS procedure. Therefore, the overall index construction time is $O(n^2)$.
Index size. In each bag X, for each pair of vertices u, v in X, we need to store the vertices along the shortest path between u and v. Thus the index size is $l|X|^2$, where l is the average length of all the local shortest paths. Since the relationship $k \ll |R|$ holds, the root size ($|R|$) is dominant among all the bags. Therefore, the index size is $l|R|^2$. The index size consists of the tree structure, constructed by using the tree decomposition algorithm. However, this space overhead is linear to n, thus can be ignored.
Query. The bottom-up query processing for shortest path computation takes time $O(k^2h)$, where k is the number of the reductions and h is the height of the tree decomposition.

3.3.1 The Trade-off between Time and Space

The number of the reductions for the tree decomposition algorithm, k, and the root size $|R|$, represent the trade-off between the time and space consumptions of the TEDI approach. As analyzed above, the time complexity for the query answering depends on k, whereas the index size is decided by $|R|$. Clearly, regarding to the tree decomposition algorithm, $|R|$ is a monotonically decreasing function of k. Usually, $|R|$ decreases drastically as k is small, e.g. from 1 to

Algorithm 5 $reach\_dist(T_G, u, v)$

Input: T_G is the tree decomposition of G and u, v vertices in G. In every bag X of T_G, and any pair of $x, y \in X$, $sdist(x, y)$ stores the shortest distance, and $path(x, y)$ stores the vertices along the path.

Output: the shortest path from u to v.

r_u = root of induced subtree of u;
r_v = root of induced subtree of v;
if $(r_u == r_v)$ **then**
    return $path(u, v)$;
end if
$root$ = youngest common ancestor of r_u and r_v;
if $(r_u == root)$ **then**
    switch u and v; switch r_u and r_v;
end if
for all $x \in X_{r_u}$ **do**
    label x with $d_u(x) = sdist(u, x)$; $p(x) = u$;
end for
$X_1 = X_{r_u}$;
while $X_1.parent \neq root$ **do**
    $X_p = X_1.parent$
    for all $t \in X_p \setminus X_1$ **do**
        for all $s \in X_p \cap X_1$ **do**
            if $d_u(t) > d_u(s) + sdist(s, t)$ **then**
                $d_u(t) = d_u(s) + sdist(s, t)$; $p(t) = s$;
            end if
        end for
    end for
    $X_1 = X_p$;
end while
$X_1 = X_1 \cap root$;
if r_v is the ancestor of r_u **then**
    $X_2 = \{v\}$; $d_v(v) = 0$;
else
    for all $x \in X_{r_v}$ **do**
        label x with $d_v(x) = sdist(v, x)$; $p(x) = v$;
    end for
    $X_2 = X_{r_v}$;
    while $X_2.parent \neq root$ **do**
        $X_p = X_2.parent$
        for all $t \in X_p \setminus X_2$ **do**
            for all $s \in X_p \cap X_2$ **do**
                if $d_v(t) > d_v(s) + sdist(s, t)$ **then**
                    $d_v(t) = d_v(s) + sdist(s, t)$; $p(t) = s$;
                end if
            end for
        end for
        $X_2 = X_p$;
    end while
    $X_2 = X_2 \cap root$;
end if
$mdist = \{min(d_u(x) + d_v(y) + sdist(x, y)) \mid (x \in X_1, y \in X_2\}$;
let $x \in X_1, y \in X_2$ be the vertices along the shortest path;
while $x! = u$ **do**
    output $path(x, p(x))$; $x = p(x)$;
end while
output $path(x, y)$;
while $y! = v$ **do**
    output $path(y, p(y))$; $y = p(y)$;
end while

5. Then, the decreasing of $|R|$ follows diverse trends, which is decided by the characteristics of the underlying graphs. One obvious instance is that for extremely sparse graphs, $|R|$ can be decreased rapidly into zero after 3 or 4 steps of reductions. For instance, the graph "Eva" in Figure 5 belongs to this category. However, this correlation can not be solely reflected by the density of the underlying graph. An exact investigation of such a relationship is left as the future work. Figure 5 depicts the k-$|R|$ relationship for the real datasets we have tested. In this illustration, the Y-axis depicts the value of $|R|/n$ in percentage, namely the proportion of the root size with respect to the graph size. Note that due to the wide range of the value on Y-axis, we plot the vertical axis logarithmically. More details of the dataset are given in Section 4.

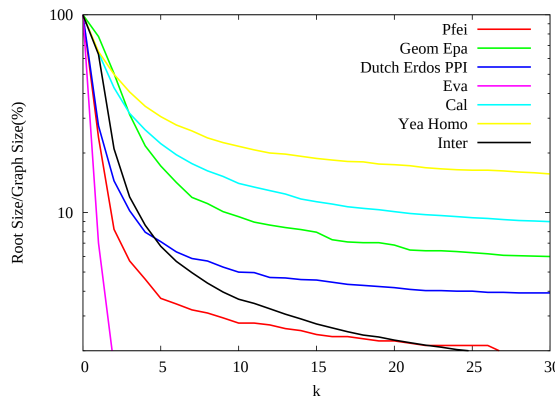


Figure 5: k and $|R|$ relationships for real data

4. EXPERIMENTAL RESULTS

In this section we evaluate the TEDI approach on real, synthetic, and large datasets respectively, for the shortest path query answering. All tests are run on an Intel(R) Core 2 Duo 2.4 GHz CPU, and 2 GB of main memory. All algorithms are implemented in C++ with the Standard Template Library (STL).

We are interested in the following parameters:

- Index size,
- Index construction time, and
- Query time.

The index size consists of two parts: (1) the size of the tree decomposition. This includes the tree structure and the vertices stored in the bags of the tree decomposition. (2) the local shortest paths stored in the hash table, which is dominant comparing to part (1).

The index construction time consists of two parts as well: (1) time cost for the tree decomposition. (2) the time for the local shortest path generation. Here part (2) is dominant.

Besides the standard measurements, we are also interested in the structure of the tree decomposition, which may influence the performance of the algorithm. These are:

- the number of tree nodes (#TreeN),
- the number of all the vertices stored in the bags (#SumV),

- the height of the tree (h),
- the number of vertex reductions (k), and
- the root size of the tree. ($|R|$).

In our experiments, we compare our approach with the compact BFS trees proposed by Xiao et al. [23], which we denote as SYMM. To the best of our knowledge, SYMM is the state-of-the-art implementation for efficient shortest path query answering with pre-computed index structures.

4.1 Real Datasets

We test a variety of real graph data including biological networks (PPI, Yea and Homo), social networks (Pfei, Geom, Erdos, Dutch, and Eva), information networks (Cal and Epa) and technological networks (Inter). All the graph data are provided by the authors of [23].

Some statistics of the graphs w.r.t. the tree decomposition algorithm are shown in Table 1. Note that we have chosen the optimal k, in order to achieve the best query time performance.

Graph	n	#TreeN	#SumV	h	k	$\|R\|$
Pfei	1738	1680	3916	16	6	60
Gemo	3621	3000	9985	10	5	623
Epa	4253	3637	11137	7	7	618
Dutsch	3621	3442	8700	9	5	258
Eva	4475	4457	9303	9	2	75
Cal	5925	5095	18591	14	10	832
Erdos	6927	6690	18979	9	7	405
PPI	1458	1359	3638	11	7	101
Yeast	2284	1770	6708	6	9	516
Homo	7020	5778	24359	10	15	1244
Inter	22442	21757	67519	10	13	687

Table 1: Statistics of real graphs and the properties of the index

We measure the time and space cost of the index construction on the real datasets, and compare our results with SYMM [23]. See Table 2 for details. The index size generated with TEDI has been dramatically reduced comparing with SYMM. In fact, most of the index sizes of TEDI are two orders of magnitude smaller than those generated with SYMM. For the index construction time, improvement can be shown similarly. The index construction time of TEDI on every graph is at least one order of magnitude faster than that with SYMM.

The measurement on the index construction time and the index size confirms the complexity analysis in Section 3.3.1. The index construction time is only dependent on the graph size, whereas the index size is decided by the root size $|R|$. For instance, the graph size of Homo is 3 times small than Inter, while the root size is two times greater than Inter. This is exactly reflected by the index size and index construction time in Table 2 respectively.

We execute the query answering algorithm given in Algorithm 5. For each dataset we randomly generate 10000 pairs of vertices and query the shortest paths between each pair of vertices. To make a fair comparison, we have implemented the naive BFS algorithm. This way, we could compare the speedup of our method w.r.t BFS algorithm to the speedup of the SYMM algorithm w.r.t. BFS. The results are shown in Table 3. Surprisingly, the speedup for all the datasets, except for one graph, is higher than that of SYMM.

In summary, TEDI algorithm is superior to SYMM in *all* aspects.

	TEDI			SYMM [23]
Graph	TEDI (ms)	BFS	Speedup	Speedup
Pfei	0.003420	0.052	15.2	13.04
Gemo	0.002933	0.123	42.4	41.10
Epa	0.002096	0.105	50.0	39.62
Dutsch	0.002655	0.097	37.3	28.21
Eva	0.002299	0.089	38.7	20.20
Cal	0.003325	0.187	56.7	59.31
Erdos	0.002037	0.146	71.9	57.72
PPI	0.002629	0.050	19.2	13.30
Yeast	0.002463	0.071	28.4	25.63
Homo	0.007666	0.226	29.7	N.a.
Inter	0.004178	0.693	169.0	N.a.

Table 3: Comparison between TEDI and SYMM on query time over real dataset.

4.2 Synthetic Data

We have generated synthetic graphs according to the BA model [3], a widely used model to simulate real graphs. To make a fair comparison, we set $k = 1.1$, so that the average degree of the graph generated is $2k$, which is identical to the synthetic datasets in [23]. We vary the graph size from 1000 to 10000 vertices by the step of 1000.

Graph	n	#TreeN	#SumV	h	k	$\|R\|$
1k	1000	808	2131	9	3	194
2k	2000	1730	4786	11	5	272
3k	3000	2641	7362	10	6	361
4k	4000	3559	10131	10	7	443
5k	5000	4460	12758	10	8	542
6k	6000	5355	15371	10	9	612
7k	7000	6292	18626	12	9	710
8k	8000	7201	20790	11	9	801
9k	9000	8089	23497	12	9	913
10k	10000	8983	26224	11	9	1019

Table 4: Statistics of the synthetic graphs and the properties of the index

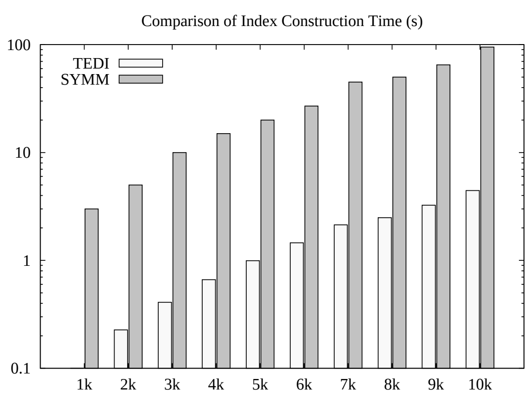
Some statistics of those graphs w.r.t. the tree decomposition algorithm are shown in Table 4. Again, we have chosen the optimal k, in order to achieve the best query time performance. The measurement of the index construction time and index size is similar to those of real dataset.

The index size and the index construction time are shown in Table 5. Figure 6(a) and 6(b) illustrate the comparison of index construction time and the index size on the synthetic datasets with SYMM. Note that due to the wide range of time and space cost, we plot the vertical axis logarithmically. The results in Figure 6(a) and 6(b) show clearly that on both the index size and the index construction time, TEDI outperforms the approach SYMM with the improvement of more than an order of magnitude.

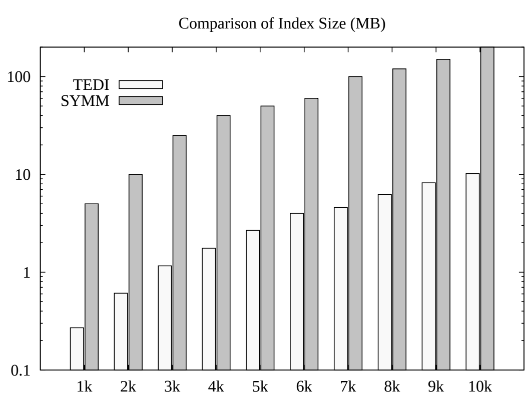
Since SYMM does not present the query time for the synthetic data, we can not make a comparison on that. Instead,

	Index Size (MB)				Index Time (s)			
Graph	paths	tree	TEDI	SYMM [23]	t_{tree}	t_{paths}	TEDI	SYMM [23]
Pfei	0.025	0.008	0.033	7.9243	0.003	0.099	0.102	2.688
Gemo	1.81	0.020	1.830	44.9907	0.068	0.878	0.946	14.859
Epa	1.63	0.022	1.652	28.1992	0.056	0.97	1.026	37.14
Dutsch	0.404	0.016	0.420	20.8559	0.011	0.311	0.322	13.687
Eva	0.026	0.018	0.044	5.5447	0.006	0.239	0.245	289.532
Cal	3.04	0.038	3.078	92.026	0.145	2.535	2.680	34.094
Erdos	0.516	0.018	0.534	32.2695	0.038	0.849	0.887	90.453
PPI	0.052	0.008	0.060	5.954	0.004	0.130	0.134	1.547
Yeast	1.08	0.014	1.094	19.4457	0.019	0.566	0.585	7.578
Homo	6.88	0.048	6.928	21.574	0.198	7.745	7.943	53.985
Inter	1.66	0.136	1.796	744.07478	0.796	15.858	16.654	1709.64

Table 2: Comparison between TEDI and SYMM on index construction of real dataset.



(a)



(b)

Figure 6: Comparison of index construction time and size on synthetic data

	Index Size (MB)			Index Time (s)		
Graph	paths	tree	TEDI	t_{tree}	t_{paths}	TEDI
1k	0.27	0.004	0.274	0.003	0.088	0.091
2k	0.61	0.010	0.620	0.010	0.217	0.227
3k	1.16	0.014	1.174	0.024	0.385	0.409
4k	1.76	0.020	1.780	0.038	0.625	0.663
5k	2.68	0.025	2.705	0.040	0.953	0.993
6k	4.0	0.030	4.030	0.081	1.340	1.456
7k	4.6	0.036	4.636	0.092	1.942	2.134
8k	6.2	0.042	6.242	0.091	2.400	2.491
9k	8.2	0.047	8.247	0.124	3.128	3.252
10k	10.2	0.052	10.052	0.167	4.274	4.441

Table 5: Index Construction of synthetic dataset

we report the query time on synthetic graphs with TEDI and compare the results with the BFS algorithms. The results is given in Table 6. Interestingly, the speedup to the naive BFS method increases, as the size of the graph grows. This observation will be confirmed in the next section, namely on very large datasets, the speedup of the query time to BFS can be increased substantially.

	Query Time		
Graph	TEDI (ms)	BFS	Speedup
1k	0.001545	0.027	17.5
2k	0.002073	0.050	25.0
3k	0.002645	0.078	29.5
4k	0.003286	0.104	31.6
5k	0.003674	0.145	39.5
6k	0.004155	0.168	40.4
7k	0.004450	0.193	43.4
8k	0.004754	0.226	47.5
9k	0.005135	0.257	50.0
10k	0.005722	0.299	52.3

Table 6: TEDI query time on synthetic datasets

4.3 Scalability over Large Datasets

To test the scalability of our approach, we conduct the experiments on much larger datasets. We have chosen two datasets. The first one is DBLP dataset. We first generate an undirected graph from the DBLP N-triple dump [3]. The dataset contains all the `inproceeding` records and all `proceedings` records, with all their persons, publishers, series and relations between persons and inproceedings. To highlight the purpose of shortest path query answering, we removed elements in each paper that are not interesting to keyword search, such as url, ee, etc. Finally, we get a graph containing $593K$ vertices.

The second large dataset is the road network of the San Francisco Bay area [4] denoted as BAY. The graph contains $321K$ vertices.

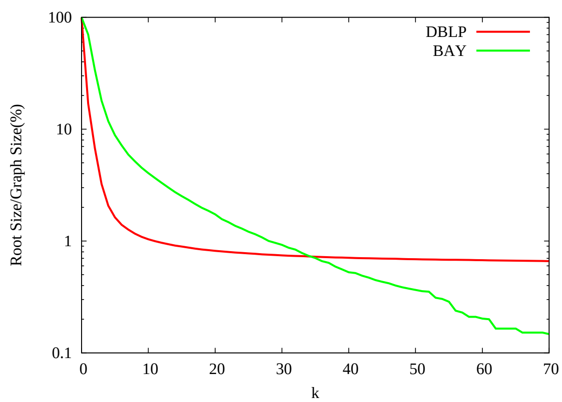


Figure 7: k and $|R|$ relationships for large data

Figure 7 illustrates the relationship of k and $|R|$ w.r.t. the reduction step of the tree decomposition process. Table 7 shows some characteristics of the graphs. The curves in Figure 7 exposes distinct features of the $k - |R|$ relationship on DBLP and BAY. For DBLP, the root size $|R|$ can hardly be reduced after 4000 (approximately 0.67% of the graph size). On the other hand, the root size of BAY remains decreasing continuously. The results of the tree decomposition reflect these differences. The decomposed tree of BAY has a much smaller root size than that of DBLP (245 vs. 3821). However, the height of BAY is correspondingly greater than DBLP (30 vs. 351). This implies that the BAY dataset requires less space for index structure, but the query time is longer. For DBLP, the query time is much shorter because of the smaller height of the tree, but the price to pay is greater index size. All of these observations are reflected in the Table 8 and 9.

Graph	n	#TreeN	#SumV	h	k	$\|R\|$
DBLP	592 983	589 164	1 309 710	30	100	3821
BAY	321 272	321 028	1 298 993	351	80	245

Table 7: Statistics of large graphs and the properties of the index

The experimental result on large graphs demonstrate clearly that the TEDI approach scales well on large dataset. Moreover, the query time speedup is more substantial, in comparison to the relatively smaller graphs.

As far as the index time is concerned, the experimental results are against the complexity analysis of $O(n^2)$. The reason for this is that the average shortest path length of BAY graph is much longer than that of DBLP. Therefore, the time cost of the BFS algorithm for BAY is greater than DBLP.

[3] `http://www4.wiwiss.fu-berlin.de/bizer/d2rq/benchmarks`
[4] `http://www.dis.uniroma1.it/~challenge9/download.shtml`

	Index Size (MB)			Index Time (s)		
Graph	paths	tree	TEDI	t_{tree}	t_{paths}	TEDI
DBLP	117.2	2.6	119.8	102.4	2124.0	2226.4
BAY	24.7	2.6	27.3	182.2	2859.7	3041.9

Table 8: Index construction of large dataset.

	Query Time		
Graph	TEDI (ms)	BFS (ms)	Speedup
DBLP	0.055	32.47	590.0
BAY	0.258	20.54	80.0

Table 9: Comparison of TEDI query time on large datasets to BFS

5. RELATED WORK

The classic shortest path algorithm deploys Breadth First Search, which can be implemented with a queue [16]. In recent years, many efficient algorithms with preprocessing have been proposed for finding the shortest paths. The graphs under consideration have some special constraints such as edge weights. See [8, 11]. A survey paper on various versions of algorithms are presented in [9]. All of these methods are based on heuristics designed specifically on the underlying datasets (like GIS data). It is unknown, whether the algorithms can be extended to dealing the other graph datasets. Moreover, one assumption common to all those algorithms is that the whole graph can be stored in main memory.

Another related graph query problem – which is more intensively studied in the database community – is the reachability query answering. Many approaches have been proposed to first pre-compute the transitive closure, so that the reachability queries can be more efficiently answered comparing to BFS or DFS. The *2-HOP approach* [7] proposes selecting a small amount of vertices that can be stored as landmark to facilitate the query answering. However, the time cost for generating such an optimized vertex set is too high to be practical ($O(n^3)$). Some approximation algorithms to 2-HOP are proposed [19]. Another category of reachability query answering algorithms is so-called *interval labeling approach* [1, 22, 6, 21, 14]. These methods first extract some tree from the graph, then store the transitive closure of the rest of the vertices. Good performance has been obtained for sparse graphs. Recently, Jin et al. have proposed the 3-HOP algorithm for reachability query answering on dense graphs [13]. None of these methods can be extended to cope with the shortest path query answering. Given u, v in a graph, reachability queries require only a boolean answer (yes or no). Therefore, the transitive closure stored in the index can be drastically compressed, as long as the boolean query can be correctly answered. On the other hand, shortest path queries require that the paths to be returned. Therefore, the compression methods with the information loss can not be adapted for answering shortest path queries.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we introduced an indexing and query answering scheme based on the tree decomposition concept for the shortest path query answering. The careful theoretical analysis has shown that our approach is intuitive and efficient. Through extensive experiments over various datasets, we demonstrate that TEDI achieves the improvement of performance by more than an order of magnitude in *all* aspects including query time, index construction time and index size. Moreover, the algorithm scales well over large datasets.

In the future we plan to investigate the following problems: (1) Development more heuristics for the tree decomposition algorithms. (2) The integration of A* heuristics for a more efficient query answering. (3) Maintenance of the index structure. Furthermore, we will consider on-disk algorithms for both index construction and query answering.

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