Influence Maximization for Complementary Goods: Why Parties Fail to Cooperate?

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ABSTRACT

We consider the problem where companies provide different types of products and want to promote their products through viral marketing simultaneously. Most previous works assume products are purely competitive. Different from them, our work considers that each product has a pairwise relationship which can be from strongly competitive to strongly complementary to each other's product. The problem is to maximize the spread size with the presence of different opponents with different relationships on the network. We propose *Interacting Influence Maximization* (IIM) game to model such problems by extending the model of the *Competitive Influence Maximization* (CIM) game studied by previous works, which considers purely competitive relationship. As for the theoretical approach, we prove that the Nash equilibrium of highly complementary products of different companies may still be very inefficient due to the selfishness of companies. We do so by introducing a well-known concept in game theory, called *Price of Stability* (PoS) of the extensive-form game. We prove that in any k selfish players symmetric complementary IIM game, the overall spread of the products can be reduced to as less as $1/k$ of the optimal spread. Since companies may fail to cooperate with one another, we propose different competitive objective functions that companies may consider and deal with separately. We propose a scalable strategy for maximizing influence differences, called TOPBOSS that is guaranteed to beat the first player in a single-round two-player second-move game. In the experiment, we first propose a learning method to learn the ILT model, which we propose for IIM game, from both synthetic and real data to validate the effectiveness of ILT. We then exhibit that the performance of several heuristic strategies in the traditional influence maximization problem can be improved by acquiring the knowledge of the existence of competitive/complementary products in the network. Finally, we compare the TOPBOSS with different heuristic algorithms in real data and demonstrate the merits of TOPBOSS.

Keywords

Influence maximization; game theory; social network

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1. INTRODUCTION

As the popularity of online social and information networks increases, more and more researchers have studied the diffusion phenomenon therein. Ideas, topics, and products are all possible diffusion items to be spread or adopted by nodes, i.e. users, in a social network. Several interesting applications are targeted to facilitate [11] or block [10] the diffusion. For example, in viral marketing, a company would most like to maximize the spread size of its products. Hence, the influence maximization problem [11] has been introduced, which aims to select a set of initial seed users which are assumed to adopt the products (e.g. via free samples) in order to maximize the spread of a product in a social network.

Meanwhile, there may be other companies with competitive products and they want to promote their products via viral marketing. In order to do so, a company may consider blocking the spread of its competitor's product. Consequently, Competitive Influence Maximization (CIM) problem is naturally introduced and studied [3–6, 10, 12, 14, 18]. Their models vary by different settings such as known or unknown opponent's strategy, single-round or multi-round of seed choice, and sequential or non-sequential decision order. In these works, once a node chooses to adopt a product or an idea, it can no longer adopt another competitive product or idea.

However, in the real world, the relationship between two products or ideas can be not so simple and extreme most of the time [17]. Some products or ideas may weaken others' propagation, but not block them completely. For example, if you have bought some cookies already, your willingness to buy another pack of candies may decrease. However, you may still buy it if you find out that most of your friends have bought another pack. Furthermore, some products or ideas may neither be competitive nor independent. They may even help the spread of each other [9]. For example, if you have bought a video game player, your willingness of buying its related video games may increase. The strength of cooperation may vary from strongly associated (e.g., printers and ink cartridges) to almost independent (e.g., printers and cokes) as well.

From the economic perspective [2], these two kinds of products are called *Substitute Goods* (i.e. competitive products) and *Complementary Goods* (i.e. cooperative products). As for the information perspective, two topics diffusing in a network may be either competitive, cooperative or independent to each other. Such a phenomenon has been modeled in [16] in order to reach a better spread prediction on real data. Note, however, that these works have not brought the concept into multi-party influence maximization yet.

Intuitively, one may argue that the influence maximization problem of different companies with complementary goods is just the same as the influence maximization problem of one company with multiple complementary products, which is proposed in [8]. However, in the paper, we prove that this is not true by introducing a well-known concept in game theory called *Price of Stability* (PoS), which is first introduced in [1] to demonstrate how the selfishness of different players reduces the overall benefit of all players. However, such a game-theoretic approach [1] to tackle the influence maximization problem is established for the purely competitive products in [3] but does not consider the existing of cooperative relationships in the real world, which requires more complicated analysis.

To the best of our knowledge, this is the first work to not only model relationships between products from competitive to cooperative for the influence maximization problem with multiple parties, but also provide the rigorous game theoretical analysis on the upper bound of the worst case of the total spread size of all parties. Specifically, we propose an Interactive Influence Maximization (IIM) game to consider additional cooperative pairwise relationships between products by extending the previous CIM game [3] that considers only purely competitive pairwise relationships. We further propose an Interactive Linear Threshold (ILT) model by extending the Competitive Linear Threshold (CLT) model proposed by the previous work [10].

Fig. 1 presents how our IIM game differs from previous CIM game. Suppose that we have a simple network, party 1 and party 2 have activated node 1 and node 2 separately and node 1 can influence node 2. In the traditional CIM model, once node 2 has been activated by party 2, party 1 can never activate node 2 no matter how strong the influence is. In our IIM game, the activation of node 2 toward party 2 will only weaken the influence coming from party 1, or even strengthen it if products of two parties are in a complementary relationship. Node 2 may be activated by both parties at the same time if node 1 has a strong enough influence on it, whereas it's impossible in CIM.

Our Contributions:

- We propose IIM game that models products from multiple parties with different pairwise relationships including not only competitive but also complementary relationships.
- We prove that even if all products from different companies are highly cooperative, the sum of influence spread sizes of all products can still be relatively small, compared to that of all products owned by just one company. We prove that, in a k-player IIM game, ILT model with symmetric complementary relationships between all products, the overall spread size can be reduced to as less as $1/k$ of the optimal one.
- We propose a learning method to learn the parameters of ILT, which is effective based on the experimental results of adoption prediction on both synthetic and real data.
- We demonstrate that the performance of several common heuristic strategies in the traditional influence maximize problem can be improved by knowing extra information, i.e. the existence of other products, for IIM in the experiment.
- We propose a scalable heuristic strategy called TOPBOSS that is guaranteed to beat the first player in one-round two-player second-move IIM game. TOPBOSS runs much more efficient than the greedy algorithm and is very effective than several heuristic methods.

The rest of the paper is arranged as follows. Section 2 discusses the related work and Section 3 introduces the problem of IIM game and the ILT model. Section 4 demonstrates why the inefficiency of the IIM game happens and how much influence spread may lose. In Section 5, we discuss the difference of objective functions between the traditional CIM game and our IIM game and further propose the efficient strategy TOPBOSS. In Section 6, we propose the learning

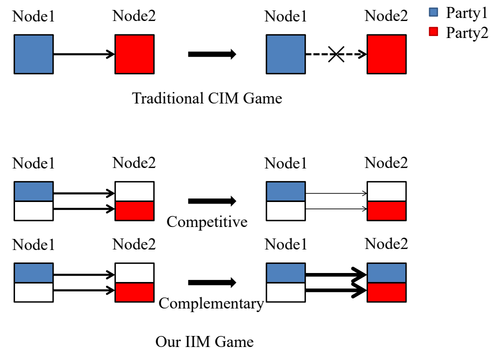


Figure 1: Difference between CIM and IIM games

method of ILT model and show experimental results of adoption prediction and influence maximization. Finally, we conclude in Section 7.

2. RELATED WORKS

In the section, we review related works on the CIM problem and cooperative influence propagation.

2.1 Competitive Influence Maximization

The traditional influence maximization problem contains one player only and thus is to maximize the player's influence spread. The problem, based on independent cascade or linear threshold model, is proved to be NP-hard [11] and has an $(1 - \frac{1}{e})$-approximation solution [11] by a greedy algorithm due to submodularity. For the Competitive Influence Maximization (CIM) problem, there are at least two players with purely competitive products to maximize their own influence spread. We then review the works on CIM.

2.1.1 Theoretical Approach

Bharathi et. al [3] consider a 2-player 1-round game with the extensive form (players making decisions one by one, not simultaneously). In their work, they prove that such a game has a *price of competitive* ratio, i.e. 2, by applying theorem 3.4 in [19]. For the strategy of second-move player, it has given a $(1 - \frac{1}{e})$ approach to the optimal choice by a greedy algorithm proved by [3]. As for the first player's strategy, they assume that the-second player's strategy is unknown and hence the corresponding strategy tries to maximize the expectation value of the first-player's influence only.

In [7], they propose a model DICE, based on CIM, which generalize several other models. They further study both extensive and non-extensive forms of 2-player 1-round CIM game with DICE, based on game theory. In their extensive form, they have proved that when the first-move player's strategy is chosen, there exists submodularity for the second-move player's payoff in DICE. As for non-extensive form, they have determined how many pure/mixed strategies exist for the game under some circumstance.

All above theoretical studies aim for a purely competitive scenario, while our work also considers complementary products.

2.1.2 Strategic Approach

There are several works that model the CIM problems in different settings, including knowing or not knowing the opponent's strategy, two or more players, one round or multiple rounds, and extensive or non-extensive form.

First, most of the works assume that the opponent's strategy is fixed and known. One representative CIM problem is called *Influence Blocking Maximization* (IBM), whose objective function is to minimize the opponent's influence after the opponent has already selected seeds [4, 5, 10]. For these studies, the IBM problem has proved to be NP-hard. Since the opponent's strategy is assumed to be fixed and known, it can treat their influence spread effect as part of the network topology and as a one-player game. Yet this assumption may not be realistic for the reason that we have no way to know the opponent's strategy and the opponent may adjust it after observing our seed choice, making the strategy neither known nor fixed.

As for works which assume the other party's strategies are unknown, most of them model the game with 2-player and 1-round extensive/non-extensive form. This can be very challenging due to the fact that we know very little about the opponent. Kostka et al. [3] prove that the seed selection task in 2 -player extensive form is NP-hard for each player and prove theoretical results mentioned in the previous section. However, they do not propose a practical solution to tackle 2 -player extensive form of CIM game.

Tsai et al. [18] further propose some practical solution for 2-player non-extensive form of CIM game. They propose a framework called Double Oracle by using EXACT, APPRX, LSMI, and PageRank as the oracle strategies and keep updating the strategy choice, based on the opponent's temporal choices. This is a searching strategy for the global equilibrium of a 1-round game. However, it is not applicable to multi-round cases.

Our work studies an extension of CLT model [10] that has never been considered and focuses on extensive from with multiple rounds, where the previous choice of the opponent is known by observation. However, the future strategy of the opponent is still unknown.

2.2 Cooperative Influence Propagation

For related work on cooperative influence propagation, the work [16] models both cooperation and competition in diffusion from the information perspective and shows that such a phenomenon does exist through experiments, which thus supports the consideration of our work. However, this work focuses on applying such a model to reaching a better prediction on information spread, instead of maximizing the spread size by seed choice.

The closest related work is [15]. To the best of our knowledge, the paper is the first and the only work to consider complementary diffusion so far. However, it has several vital differences with our work, including models, theoretical approaches and experiment directions. First, from the aspect of modeling, the work [15] extends the Independent Cascades (IC) model to a model called Comparative Independent Cascade (Com-IC) and adds an extra mechanism to model the reconsideration of adopting products after the adoption of other complementary goods for a node. In our model, we extend the LT model, which considers the collaborative influence from in-neighbors for each possible product adoption, whereas IC just considers the influence from exactly one in-neighbor in each time. Moreover, the accumulative and dynamic nature of our model embeds the reconsideration of products naturally and the special case of our model is exactly the same as the original competitive LT model [10].

Furthermore, our work argues that two complementary products from two different companies may fail to cooperate with each other due to their selfishness, hence reducing the total spread size, based on rigorous game-theoretical analysis.

We analyze the improvement of knowing the existence of opponent under different relationships in our IIM game through experiment. Other than that, due to the fact that companies of complementary products may still be competitive to each other, we further consider different competitive objective functions of different scenarios and conduct thoughtful experiments.

3. PROBLEM STATEMENT

In the section, we first propose the Interacting Linear Threshold (ILT) model to consider diffusion of multiple ideas, e.g. products, from multiple parities, e.g. companies, where the relationship between pairwise products may be *from fully competitive to complementary*, which is different from the traditional CLT model [10] that considers only purely competitive pairwise relationships between ideas. We then formulate the problem of *Interacting Influence Maximization* (IIM) with multiple parties to promote their products with various pairwise relationships.

3.1 Interactive Linear Threshold Model

Given a directed graph $G = (V, E)$ and products from multiple companies, we consider that the products propagating in G are not exclusive, which means if a node $v \in V$ is activated by a certain party, it can be activated by another party again, while previous diffusion models for CIM problems assume that one node can adopt only one company' product. We then define ILT model, based on the CLT model, as follows.

Definition 1. **Interactive Linear Threshold (ILT) model.** A node v is activated by party P_i, at round t when

$$\sum_{u \in O_t^i} w_{u,v}^{i,t} > \theta_v^{i,t}.$$

The notation $w_{u,v}^{i,t} \in \mathbb{R}$ denotes the influence weight on edge (u, v) toward P_i at t^{th} round. O_t^i is the set of nodes activated by P_i before t^{th} round. The $\theta_v^{i,t} \in \mathbb{R}$ is the activation threshold of node v toward party i at t^{th} round. Thus, a node v can be activated by multiple parties.

Note that in LT, edge weights and node thresholds are preferred to be normalized in $[0, 1]$. Nevertheless, we do not follow the convention for the convenience of the design of update mechanisms introduced later.

After propagation in each round, the edge weights and node thresholds will be updated, depending on the activation status of neighborhood in the network, which reflects the dynamics of adoption behaviors.

Here, we introduce a *competitive and cooperative coefficient* $C_{i,j}$ to model the pairwise relationship between two parties P_i and P_j, which represents how intensively P_i and P_j are cooperative or competitive to each other. Note that if the context is clear, we only use the *coefficient* to mean $C_{i,j}$. Depending on $C_{i,j}$, the relationship of two products, from P_j to P_i, can be:

$C_{i,j}$	relationship of two products, from P_j to P_i
R^-	competitive (*substitute goods*)
0	independent (neither competitive nor cooperative)
R^+	cooperative (*complementary goods*)

The coefficient is asymmetric, i.e. $C_{i,j}$ does not require to equal $C_{j,i}$. Hence, it can model situations such as one product boosting another product's spread greatly but not vice versa. We then introduce the update mechanism as follows.

Definition 2. **ILT Update Mechanism.** After propagation in each round, the edge weights and node thresholds for each party

P_i are updated by:

$$\theta_v^{i,t+1} = \theta_v^{i,t} - \sum_{\substack{i \neq j \\ v \in O_t^j, v \notin O_{t-1}^j}} C_{i,j} \theta_v^{j,t} \quad (1)$$

and

$$w_{v,u}^{i,t+1} = w_{v,u}^{i,t} + \sum_{\substack{i \neq j \\ v \in O_t^j \vee u \in O_t^j \\ v \notin O_{t-1}^j \wedge u \notin O_{t-1}^j}} C_{i,j} w_{v,u}^{j,0}. \quad (2)$$

Eq. 1 and 2 are used to model adoption behaviors for different pairwise relationships between products. The first equation is to model that once a user has adopted a certain product, the willingness of adopting another complementary product will increase. The second equation is to model that, for the neighbors of the user that have adopted a certain complementary product, the corresponding outgoing and incoming influence edge weights over the current complementary product will be more likely to increase.

For example, consider that P_j has a product, PS4, and P_i has a product, a video game for PS4, where two products are complementary. If a node v is activated by P_j, i.e. buying PS4, node threshold $\theta_v^{i,t+1}$ will decrease (Eq. 1) and thus v is more likely to buy the video game from P_i. Furthermore, both the outgoing and incoming weights will increase toward P_i (Eq. 2), making it easier for P_i to activate node v as well. Note that with the setting given above, each edge weight can be updated only once toward each $C_{i,j}$. By the design Eq. 2, if the successor or predecessor of an edge has been activated by another party, it cannot be updated by that party again. In contrast, both outgoing and incoming weights will decrease in P_i if $C_{i,j}$ is competitive, making it harder for P_i to activate node v. If the coefficient is zero, then P_j has no effect on P_i's propagation.

Note that the Competitive Linear Threshold (CLT) model [10] is a special case of the ILT model when $C_{i,j} = -\infty$. Two models are equivalent in such a case since a node's thresholds of other parties will become ∞ once a certain party has chosen the node and, hence, no other parties can activate it. If a party chooses it as the seed, then it will have $-\infty$ influence on its neighbor. Therefore, no party should choose the node as a seed once it is activated, assuming that all parties are rational and should maximize their own spread size.

3.2 Interactive Influence Maximization Problem

We define the multi-round interactive influence maximization problem with ILT model as follows.

Definition 3. **Interactive Influence Maximization (IIM).** Given a directed graph $G = (V, E)$ with ILT diffusion model and a decision order, e.g. from P_1 to P_n, each party $P_i (1 \leq i \leq n)$, starting with P_1 one by one according to the decision order at round t, observes previous parties' decisions and chooses s seed nodes, denoted by $S(i, t)$, from V. After that, influence propagation is conducted with ILT model at round t, given their seed choices.

Note that for each round, the influence only propagates to L layers of neighbors (L is fixed and can be set from 1 to ∞). Then, the edge weights and node thresholds are updated based on Eq. 1 and 2 before the next round starts.

After T rounds, all parties stop choosing their seeds. The influence propagation is conducted continuously until no new nodes can be activated. The objective function of each party have several different settings, which will be discussed in the later sections.

Since we are considering a multi-round scenario, a party's previous choice can be observed by other parties before they make decisions, but the future decisions are unknown. This indicates that such a game is an extensive-form game with perfect information in game theory.

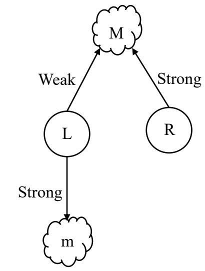


Figure 2: An example of Volunteer's Dilemma on IIM game

Table 1: The payoff matrix of the example in Fig. 2

		Party 2	
		L	R
Party 1	L	(m, m)	$(M + m, M)$
	R	$(M, M + m)$	(M, M)

4. INEFFICIENCY CAUSED BY SELFISHNESS

In this section, we demonstrate why two parties with highly complementary goods may still fail to cooperate with each other due to their selfishness. We first discuss non-extensive-form version of IIM games, which means that all players, i.e. parties, make their decisions simultaneously. We demonstrate the inefficiency of this type of the game by a well-known dilemma in game theory called Volunteer's Dilemma. The dilemma is about a situation in which each player faces the decision of either making a small sacrifice from which all will benefit or free-riding on other players' sacrifices. We also show that this type of the game may not exist a pure Nash equilibrium.

Next, we demonstrate the extensive form of IIM games with perfect information, which means that all players have a given decision order and know each other's previous move. We prove that for the symmetric relationships of complementary products, the total spread of all players can be reduced to as less as $1/k$ of the optimal case. This ratio is known as Price of Stability (PoS) in game theory, which is often used for measuring the efficiency of the Nash equilibrium in graph-like games. In other word, it measures how much total value is reduced from the optimal solution to the worst Nash equilibrium.

4.1 Non-Extensive-Form Games

In non-extensive-form games, we use an example, inspired by the Volunteer's Dilemma, to demonstrate why two parties may fail to cooperate due to their selfishness. Suppose that there are two parties, they both want to maximize their own spread size on the network in Fig. 2. Each of them has exactly one seed budget. The clouds labeled by M and m are two clusters of nodes. Each node of M receives a high influence weight from R and a relative low weight from L. Each node of m receives a high weight from R. Cluster M contains M nodes and cluster m contains m nodes, given that $M \gg m$. Two reasonable seed choices will be node L and node R.

If any of them chooses node R, another party can choose node L and get more spread size $(M + m)$, compared to choosing R with getting only (M). However, if they both choose L, both of them can

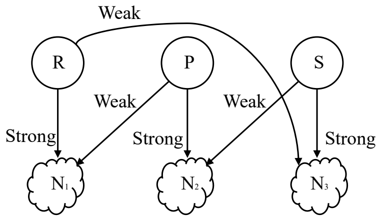


Figure 3: The Rock-Paper-Scissors network on IIM game

Table 2: The payoff matrix of the game in Fig. 3

		Party 2 R	P	S
Party 1	R	(N,N)	$(N,2N)$	$(2N,N)$
	P	$(2N,N)$	(N,N)	$(N,2N)$
	S	$(N,2N)$	$(2N,N)$	(N,N)

only activate m nodes. The payoff matrix of this game is presented as Table 1. When $M \geq m$, one asymmetric pure Nash equilibrium of this game happens if one player chooses L and another selects R. The total spread size then equals $(2M + m)$, the same result if two products belong to the same company.

However, the mixed Nash equilibrium will have a chance (m^2/M^2), which becomes higher as m increases and ends up with both players choosing L. The total influence will become $(2m)$, much less than the total influence of the optimal value $(2M + m)$. Moreover, if the temptation is large enough, e.g. given that $M < m$, even the pure Nash equilibrium will end up with both players choosing R, resulting in less total spread size if $2M > m$.

This game can be extended to a multi-party scenario where all parties may get m as their payoff due to selfishness. In the mixed-strategy Nash equilibrium, the increasing of the number of players will decrease the likelihood that at least one player, as a volunteer, chooses R, which is a result of the bystander effect.

Despite the inefficiency, non-extensive form games may not exist a pure Nash equilibrium as well. Take a game similar to Rock-Paper-Scissors game, shown in Fig. 3, as an example. Table 2 presents the payoff matrix of this game. There is no pure-strategy Nash equilibrium for this game, thus the cost of selfishness is defined as infinite.

Both Volunteer's Dilemma and the existence of equilibrium can be solved by assuming that each party has a given decision order, which transforms the game to an extensive-form game with perfect information in game theory. For the Volunteer's Dilemma, the first-move player will simply choose L, and the second-move player will have to choose R after observing the decision of the first-move player, which is fixed. As for the existence of equilibrium, one of the equilibrium in extensive-form games with perfect information, known as Sub-game Perfect Nash Equilibrium (SPNE), has a mathematical guarantee that there exists one and only one equilibrium when no tie-break occurs.

However, forming the game as the extensive form does not eliminate all inefficiency caused by the selfishness of each party. The famous *Prisoner's Dilemma* situation may still occur in certain networks, which will be demonstrated later.

4.2 Upper Bound of the Reduced Spread Size

In game theory, the Price of Stability (PoS) is used to measure the cost of selfishness of a game. It denotes the ratio of the best objective function value for the best Nash equilibrium of the game to the optimal outcome. Price of anarchy (PoA) denotes that ratio of the value corresponding the worst equilibrium to the optimal outcome.

Since our IIM game is extensive-form with perfect information, there exists one and only one *Sub-game perfect Nash equilibrium* and thus PoS and PoA are the same. We will use PoS_{SPNE} to denote such a ratio as follows.

Definition 4. PoS_{SPNE} of IIM game is defined as

$$PoS_{SPNE} = \frac{F(SPNE)}{F(Optimal)},$$

where *F(SPNE)* and *F(Optimal)* denote the total spread size of the equilibrium and maximum spread size respectively.

Next, we will prove that the upper bound of PoS_{SPNE} in IIM game is $1/k$ by demonstrating a Prisoner's Dilemma like example. To make the example easier to understand, we start with a two-player example first. Prisoner's Dilemma is a situation that each player faces the decision of either to betray or cooperate with another player. The dilemma exhibits how two rational players may fail to cooperate.

Consider the following situation. If player 1 chooses to betray player 2 and player 2 chooses to cooperate, player 1 will gain a higher payoff than the situation where both two cooperate with each other, while player 2 will gain a minimum payoff. However, both players will gain the second minimum payoff if they both choose to betray each other. In game theory, it indicates that in the situation, all rational players will choose to betray each other. We will validate that this dilemma exists in IIM game as well.

LEMMA 1. *The PoS_{SPNE} of two-party multi-round non-negative symmetric interactive influence maximization (IIM) game has the following upper bound:*

$$\exists \text{ IIM Games s.t. } PoS_{SPNE} \leq \frac{1}{2} + \epsilon.$$

We will prove the lemma by demonstrating an example in IIM game, inspired by Prisoner's Dilemma, where the spread size of the equilibrium is reduced as about $\frac{1}{2}$ of total nodes of the optimal spread.

In the game presented in Fig. 4, there is just one round and each party can select only one seed. The numbers in parentheses represent how all the thresholds of nodes in the cluster change before and after the nodes in the cluster are activated by another party. The numbers on edges represent how influence change before and after the nodes that the edges point from/to are activated by another party. In this example, $C_{1,2} = C_{2,1} = 0.5$.

For convenience, M_1 denotes a group containing N nodes, where all of them have exactly the same edge influence weights from L and R respectively, M_2 denotes a group with another N nodes, where all of them have exactly the same weight from R, and m denotes a group with $N + \epsilon$ nodes, where all of them have exactly the same weight from L.

If P_1 chooses node R as a seed (cooperation), P_2 will choose node L as its seed (betrayal) since $2N + \epsilon > 2N$ and P_1 will end up with getting N nodes. However, if P_1 chooses L, P_2 will choose L as well because $N + \epsilon > N$, resulting in both parties getting $N + \epsilon$ nodes. Finally, since $N + \epsilon > N$, P_1 will choose L first and P_2 will also choose L as their final equilibrium.

PROOF. Consider the 2-player IIM game in Fig. 4, where $N \gg \epsilon$, the game tree of this network is presented in Fig. 5, using backward induction, we can see that in $SPNE$, $f(P_1) = f(P_2) =$

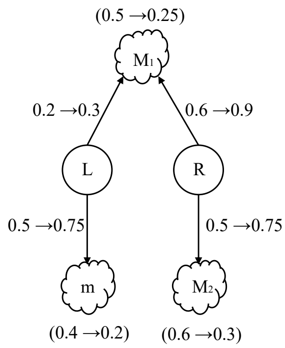


Figure 4: Prisoner's Dilemma on IIM games

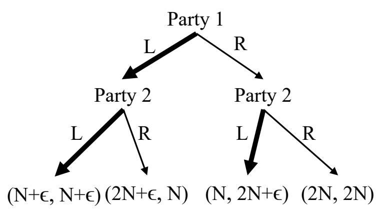


Figure 5: The game tree of the game in Fig. 4

$N+\epsilon$ and in $Optimal$, $f(P_1) = f(P_2) = 2N$, where $f(\cdot)$ denotes the total spread size of one player. Hence, we have

$$\frac{F(SPNE)}{F(Optimal)} = \frac{(N+\epsilon)+(N+\epsilon)}{2N+2N} \approx \frac{1}{2}$$

□

Next, we extend the example to the k-player case.

THEOREM 1. *The PoS_{SPNE} of k-party multi-round non-negative symmetric Interactive Influence Maximization (IIM) Game has the following upper bound:*

$$\exists \text{ IIM Games s.t. } PoS_{SPNE} \leq \frac{1}{k} + \epsilon$$

We prove the theorem by demonstrating an example in IIM game as well, where the ratio of the spread size of the equilibrium to the optimal spread size is about $\frac{1}{k}$.

In the game presented in Fig. 6, again there is just one round and each party can select only one seed. In this example, every party pair (P_i, P_j) have the same coefficient, i.e. $C_{i.j} = C, \forall i \neq j$.

For convenience, M_1, M_2, …, M_k denote k groups, each of which contains N nodes. All nodes in the same group M_i receive exactly the same weights from L and R respectively, denoted by l_i and r_i ($i = 1$ to k). Moreover, m denotes a group with $N+\epsilon$ nodes, where all nodes receive exactly the same weight l_m from L.

The node thresholds and edge weights in each group differ from group to group, but are the same within each group. Edge weight r_i can be any positive number and the threshold for nodes in M_i, denoted by T_i, is given as:

$$T_i = (1+C)^{i-1} r_i - \epsilon. \tag{3}$$

This equation means that group i will be activated by R if and only if at least i parties have chosen R. As for edge weight l_i, we want it to satisfy the inequality:

$$\frac{T_i}{(1+C)^k} > l_i > (1-C)T_i. \tag{4}$$

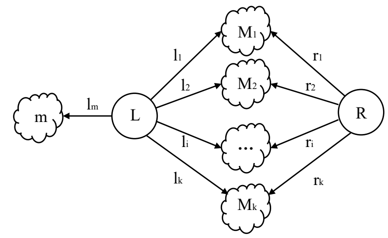


Figure 6: An example of k-player IIM games

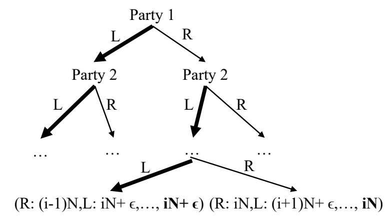


Figure 7: The game tree of the game in Fig. 6

The left part of the inequality means if M_i has not been activated, L can never activate M_i, no matter how many parties have activated L. The right part means once any of the other parties have activated M_i, any party can activate M_i through L. There are many choices of C to satisfy the above inequality. Here, we just show one C exists. Suppose that we have

$$C = 1 - \frac{1}{2^k} + \epsilon. \tag{5}$$

By substituting the C in Eq. 4 with 5 and setting $l_i = \frac{T_i}{2^k}$, we have

$$\frac{T_i}{(2 - \frac{1}{2^k} + \epsilon)^k} > \frac{T_i}{2^k} > (\frac{1}{2^k} - \epsilon)T_i. \tag{6}$$

We can see that the C and l_i we choose satisfy Eq. 4.

Finally, the threshold for nodes in group m is denoted by $T_m = 0$ and edge weight l_m is any positive number, which means m will always be activated by P_i once P_i chooses L. We then start to prove Theorem 1.

PROOF. Consider the k-player IIM game in Fig. 6, where $N \gg \epsilon$. The optimal case will be all parties choosing R with the gain of kN nodes for each party. The game tree is presented in Fig. 7.

For the last-decision party P_k, no matter how many parties have chosen L or R already, choosing L is always the dominant strategy since the payoff is better than choosing R by sacrificing N nodes in common group M_i (for the case that $i-1$ parties choose R) to gain $N+\epsilon$ nodes in group m. We can conclude that the last party will always choose L.

For the second last-decision party P_{k-1}, no matter how many parties have chosen L or R already, choosing L is always the dominant strategy as well since it will not affect P_k's decision and the payoff is always superior to the payoff of choosing R by sacrificing N nodes in common group M_i (for the case that $i-1$ parties choose R) to gain $N+\epsilon$ nodes in group m. The same cases can be inferred from P_{k-2} to P_1. We can conclude that all parties will choose L no matter what the previous parties' choices have been made. Hence, we have:

$$\frac{F(SPNE)}{F(Optimal)} = \frac{k(N+\epsilon)}{k^2 N} \approx \frac{1}{k}.$$

□

As for IIM games with asymmetric relationships between parties, the upper bound of PoS_{SPNE} is infinitely small. For example, if player 1 will greatly boost the spread of player 2's product, but player 2 has almost no effect on player 1's, it is easy to construct games that player 1 helps player 2 gain infinitely large influence spread in the optimal but gain little spread itself. In Nash equilibrium, player 1 will choose another seed that triggers slightly larger spread but does not help player 2 at all. As for asymmetric competitive relationships, the situation for the bound will be the same.

5. INFLUENCE MAXIMIZATION STRATEGIES

As demonstrated in the last section, the IIM game is still a competitive game even if all products are in complementary relationships. Furthermore, the IIM game differs from the previous CIM [3] game in many ways. There exists a dilemma between maximizing self-influence and minimizing opponents' spread size in IIM. Therefore, we need different strategies for solving objective functions of IIM game.

For the traditional CIM game, since a node cannot be activated more than once and parties have to compete with each other in the limited network size, maximizing self-influence and maximizing the opponents' influence can be viewed basically as the same goal. However, in our IIM, with weak competitive or cooperative relationships between parties, one node may be activated by more than one product, which makes the game far from a zero-sum one. Maximizing self-influence and minimizing opponents' influence become two different goals since the self-influence and opponents' influence can be both large or small at the same time, in terms of spread size. The self-influence that a party is willing to sacrifice to reduce its opponents' influence may affect the party's seed choice.

Thus, we propose two objective functions for different reasonable scenarios, i.e. maximizing self-influence and maximizing influence difference, in the following.

5.1 Maximizing Self Influence

If our goal is to maximize our own influence only, we can simply improve most well-known influence maximization strategies by providing extra information about the IIM game. For example, we may replace the max-weight strategy, which chooses inactivate seeds with the maximum out-weight sum in each round, with choosing inactivated seeds with the maximum out-weight sum after edge weights are updated by Eq. 2 in ILT model. Such a replacement can be implemented to most influence maximization strategies such as greedy, PageRank or more complicated strategies by replacing the objective functions of the strategies.

The influence maximization problem is NP-hard. Our problem can be reduced to the traditional influence maximization problem by setting C to 0. Therefore, it must be NP-hard at least. Different heuristic strategies perform differently on different graph topologies [14]. Hence, there is usually no best heuristic strategy, just strategies that win most of the time. The version of each strategy with the replaced objective function will be likely to beat the original version by taking advantage of having more information and producing a strategy that best suits the given network.

However, maximizing self-influence does not guarantee that the opponents will have poor influence spreads. In fact, if we know that opponents with competitive relationships are going to choose some seeds near our seed choices, the best way to maximize our own influence spread is to avoid the conflict with opponents. We may choose some second-best seeds to avoid gaining poor influence spread with competing with the opponents. This sometimes results in that the opponents get more influence spreads than ours since they can choose better seeds with no competitors.

5.2 Maximizing Influence Difference

If our goal is to obtain more influence spread than our opponents' as much as possible, we should consider cooperative opponents and competitive opponents separately.

For cooperative opponents' products, we consider a strategy called *Tracing Opponent Predecessor Strategy* (TOPS). In this strategy, we use the bread-first search to find the predecessor trees of the opponents' seeds that can activate the seeds, including the seeds themselves as candidates. Then, we, as party i, select the best seeds from the candidates in the following order.

1. $\sum_{P_j}(|\textit{party } i\textit{'s spread}| - |\textit{opponent } j\textit{'s spread}|)$
2. with the largest tree height
3. with the maximum out-weight

If all candidates in the tree have been activated by us already, we just choose one inactivated node with the maximum out-weight in the graph unless there's no such one.

The strategy is based on the observation that once a seed is activated by another party with a cooperative relationship, activating that seed will gain all influence spread that the opponent's seed can gain. In the two-player one-round second-move game, if two players have the same budget of seed choice, this strategy is guaranteed to be unbeatable. Since it is second-move, the algorithm will either choose a set of seeds that gain a larger influence spread than the opponent's or choose the same seed as the opponent's and gain the same influence spread. The strategy is also highly scalable for large graphs since the height of an obtained tree is usually very small.

As for competitive opponents, we consider a strategy called *Blocking Opponent Successor Strategy* (BOSS). In this strategy, we use the bread-first search to find the successor trees of the opponents' seeds that the trees will activate, including the seeds themselves as candidates. We, as party i, then select the best seeds from the candidates in the following order.

1. $\sum_{P_j}(|\textit{party } i\textit{'s spread}| - |\textit{opponent } j\textit{'s spread}|)$
2. with the smallest tree height
3. with the maximum out-weight

If all candidates have been activated by us already, we just choose one inactivated node with the maximum out-weight in the graph. This strategy is also guaranteed to be unbeatable in the two-player one- round second-move game and highly scalable for large graphs.

These two strategies have some weaknesses. First, each of them cannot individually handle the diffusion with mixed relationships between parties well. Second, if all opponents choose some bad seeds, although two strategies will still outperform them, they will gain poor performance as well. Finally, they treat opponents with different relationship strengths equally, even when the competitive and cooperative relationship is weak and close to independence.

In order to fix these weaknesses, we combine the two strategies to a strategy called *Tracing Opponent Predecessor-Blocking Opponent Successor Strategy* (TOPBOSS). In this strategy, we use the bread-first search to find the predecessor/successor trees of the cooperative/competitive opponents' seeds and add all of them along with seeds with maximum out-weights into candidates. Then, we, as party i, select the best seeds from the candidates in the following order.

1. $\sum_{P_j}|C_{i,j}|(|\textit{party } i\textit{'s spread}| - |\textit{opponent } j\textit{'s spread}|)$
2. in the predecessor/successor tree
3. with the largest/smallest tree height according to sign of $C_{i,j}(+/-)$
4. with the maximum out-weight

Table 3: Statistics of the datasets

Network	# of nodes	# of edges
Flixster	631,193	7,058,819
Facebook	4,039	88,234
Twitter	81,306	1,768,149
Wiki-Vote	7,115	103,689
Epinions	75,879	508,837

Note that the strategy may choose the opponent's seed directly after the selection and gain the same influence spread as the opponent's seed. For an opponent with a highly competitive coefficient, this may result in that both parties gain poor spreads.

6. EXPERIMENTS

In the section, we first propose a learning method to learn the parameters of ILT and evaluate the effectiveness of adoption prediction on both synthetic and real data. Next, we conduct experiments for the *maximizing self-influence* and *maximizing influence difference* problems to see how much improvement can be achieved by acquiring the knowledge of the existence of other products on different heuristic strategy and the effectiveness of TOPBOSS.

Datasets. For synthetic datasets, we use real graphs provided by Stanford Large Network Dataset Collection [13], including Facebook, Twitter, Wiki-vote and Epinions. Facebook and Twitter are two social networks. Wiki-vote is a voting network and Epinions is a trust network. As for real data, we use the Flixster graph with users' movie rating action logs as our dataset. Flixster[1] is a movie social network where users can rate movies. There are $66,563$ movies in the total of $8,196,077$ action logs. Table 3 shows the statistics of these datasets.

6.1 Learning Parameters of ILT

6.1.1 Learning Method

We then demonstrate how to learn parameters of ILT model, given the complete graph and action logs. For each v, we randomly draw edge weights and node thresholds from 0 to 1. Then, we divide each weight, e.g. $w_{u,v}$, by the in-degree of the direct successor, e.g. v, and normalize all weights of in-edges of a node as most previous works do. Next, we transform the diffusion into a classification problem and thus we want to find a set of parameters of ILT that minimizes classification errors on the given action logs. Let v_i represent activation status of node v on party P_i (if v was activated by P_i, $v_i = 1$; otherwise, $v_i = 0$,). We define the activation function $A(v, P_i)$ as:

$$A(v,P_i) = \sum_{u \in O^i} (w_{u,v}u_i + \sum_{j \neq i} C_{i,j} w_{u,v}(u_j + v_j - u_j v_j)) - (\theta_v - \sum_{j \neq i} C_{i,j} v_j \theta_v), \tag{7}$$

where O^i denotes the set of the activated nodes by P_i. According to the diffusion mechanism of ILT model, v_i will be activated if and only if $A(v, P_i)$ is greater than 0. We further introduce the differentiable sigmoid function as follows.

$$v_i = \sigma(A(v, P_i)), \tag{8}$$

where $\sigma(\cdot)$ denotes the sigmoid function defined as:

$$\sigma(x) = \frac{1}{1 + e^{-x}}. \tag{9}$$

[1]http://www.cs.ubc.ca/ jamalim/datasets/

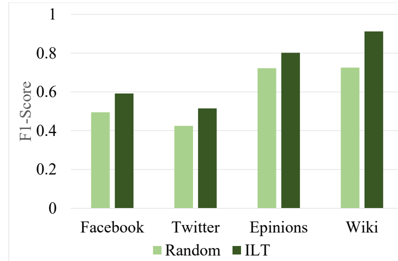


Figure 8: Adoption prediction on synthetic data

Then, we define the error function to minimize as:

$$Err = \frac{1}{2} \sum_{v \in G} \sum_{P_i \in P} (v_i - v_i^{'})^2, \tag{10}$$

where $v_i^{'}$ denotes the actual activation status of node v on the training data. By taking differentiation of the error function with respect to every parameter, we get:

$$\Delta C_{i,j} = \sum_{v \in G} \{\sigma^{'}(A(v,P_i))(v_i - v_i^{'})(v_j\theta_v + \sum_{\sum_{u \in O^i}} w_{u,v}(u_j + v_j - u_j v_j))\} \tag{11}$$

$$\Delta w_{u,v} = \sum_{P_i \in P} \{(\sigma^{'}(A(v,P_i))(v_i - v_i^{'}) \sum_{j \neq i} (u_i + C_{i,j}(u_j + v_j - u_j v_j))\} \tag{12}$$

$$\Delta \theta_v = \sum_{P_i \in P} \{\sigma^{'}(A(v,P_i))(v_i - v_i^{'}) \sum_{j \neq i} (-1 + C_{i,j} v_j)\}. \tag{13}$$

Thus, we can use the gradient decent method to find a set of good parameters to minimize the error function in Eq. 10.

6.1.2 Experiments with Synthetic Action Logs

Since action logs are usually hard to be obtained in the real world, we generate synthetic actions on real graphs, e.g. *Facebook*, *Twitter*, *Wiki-vote* and *Epinions*. We generate edge weights and node thresholds in the same way mentioned above. We set coefficients of two propagation products $C_{1,2}$ and $C_{2,1}$ as 0.75, and then generate 10000 training action logs and 1000 testing action logs for each network. The parameters of ILT are learned on the training data and we use the trained ILT to predict the propagation in testing data. Fig. 8 presents the result of adoption prediction.

The *Random* method makes predictions following the same method to generate the training data. The *ILT* represents our model with the proposed learning method. It can be observed that our model achieves an $11 \sim 26\%$ improvement of the random method for each network.

6.1.3 Experiment with Real Data

We use the *Flixster* dataset, which contains real action logs. First, we divide the network into training (468,954 nodes) and testing (126,239 nodes) parts. We then extract two movie's action logs for testing and use the rest of logs to train ILT. We further divide the extracted action logs of two movies into two parts, where one part (Part A) is corresponding to the training network (this part is used to train the coefficients between two movies), and the other part (Part B) is related to the testing network (this part is used for the testing). We use the first $1/5$ action logs, sorted in chronological

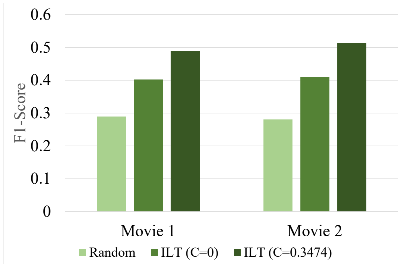


Figure 9: Adoption prediction on real data

order, as the seeds and the rest as diffusion groudtruth for computing the classification error to minimize. We then train both edge weights and node thresholds in training and testing networks by the action logs except for the two extracted movies. Next, we use two movie's action logs in the training network (Part A) to train the coefficients between the two movies. For simplicity, we assume that two coefficients is symmetric and the value is denoted by C. Finally, we use the trained testing network and the learned C to test the two movie's action logs in the testing network (Part B) to evaluate the results of adoption prediction.

Fig. 9 presents our results. The C of two movie learned from the training data is 0.3474. This indicates that two movies we selected are in weak complementary relationship. The method *Random* is the same as above with assuming that two movies are independent ($C = 0$). The setting of trained *ILT* with $C = 0$ assumes that two movies are independent as well. Finally, the setting of trained *ILT* with $C = 0.3474$ uses the learned coefficient.

The improvements of trained *ILT* without learned C, compared to *Random*, are 39% and 46% for the two testing movies respectively. The improvements of trained *ILT* with learned C are 69% and 83% respectively. Thus, by learning the coefficient C, the prediction results are improved significantly.

6.2 Interactive Influence Maximization

Although our model supports multiple players, the experiments focus on two-player games with various settings of competitive and cooperative coefficients for promoting two players' products. For each round, the propagation step L is set as 1 and the number of seeds for each round is 1 for each player. The number of total rounds T is 5. We conduct the experiments on the Facebook, Twitter, Wiki-vote and Epinions networks with the same parameter setting we generated in the previous section. We set the coefficient from -1 to 1 to analyze the diffusion results of two players.

6.2.1 Maximizing Self-Influence

We aim to demonstrate whether acquiring knowledge of the existence of cooperative/competitive products on the network helps to maximize the spread of one player. We do not aim to propose a better heuristic method to solve the NP-hard influence maximization problem.

We next show that how influence maximization methods are improved by knowing the fact that there are other competitive/cooperative goods diffusing in the network. The methods include *Max-Weight*, which chooses the seed with maximum out-weight sum, *Greedy*, proposed by [11], and *PageRank* algorithms as the second-move player against each other.

Fig. 10 presents the results of average influence improvements for each strategy against different strategies (including itself) on four networks. Since the improvement is always positive, we can conclude that influence maximization strategies acquiring knowledge of the existence of cooperative or competitive products perform better than the original version of strategies, which thus indicates the usefulness of knowing the existence of competitive or complementary products.

6.2.2 Maximizing Influence Difference

Here, we demonstrate the effectiveness of TOPBOSS for competitive and complementary opponents' products. We compare TOPBOSS with Max-Weight, Greedy, and PageRank on various competitive and cooperative coefficients respectively.

Fig. 11 presents the results of average influence differences between two players as the compared strategy plays the second-move player against four influence maximization strategies (including itself). The TOPBOSS outperforms the other strategies most of the time and takes much less computation time than the greedy approach.

6.3 Discussion

For adoption prediction on synthetic data, it is clear that the proposed learning method is effective on all networks. The prediction on Wiki-vote network even reaches 0.91 in F1-score. The adoption prediction on real data validates that the trained ILT with learned coefficients is significantly better than the trained ILT with $C = 0$, i.e. assuming two movies are independent. This indicates that learning the relationships between different products does help the prediction on the real diffusion.

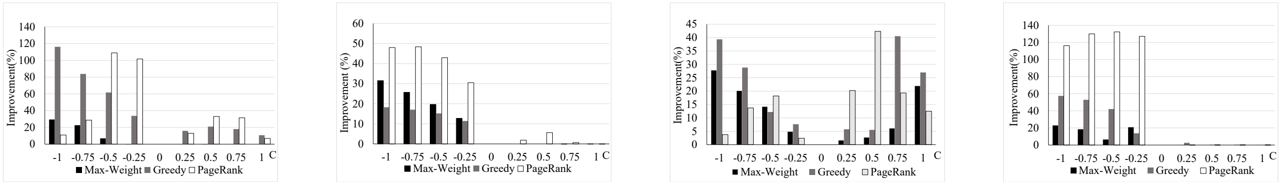
As for influence maximization experiments, some interesting phenomena are observed. First, the improvement is more significant on competitive than cooperative opponents except for the Wiki-vote network. One possible explanation is that in a denser network, e.g. Facebook, Twitter or Epinions, it is more important to avoid competitors than seeking cooperators, because the diffusion effect will be more likely to cover the cooperators and take advantage of them even if one party doesn't know their existence or locations. However, for sparser networks, e.g. Wiki-vote, the diffusion may not spread far enough to the cooperators. Thus, making the information of the existence and locations of cooperators will be more important on sparse networks to facilitate the total spread.

Second, the larger the absolute value of the coefficient is, the larger the improvement usually is, except for PageRank, since two products with stronger independence are the less important to help each other. As for PageRank, a possible explanation is that PageRank is a relatively weak heuristic algorithm. The performance of it is highly dominated by the opponent when the coefficient is large. Therefore, the improvement is more significant when the opponent is not completely dominating the network.

For the second influence maximization experiments, we can see that there is no single influence maximization method whose result dominates the others in every network. However, the results of TOPBOSS are more stable. In other words, it is always the top one or competitive to the top one.

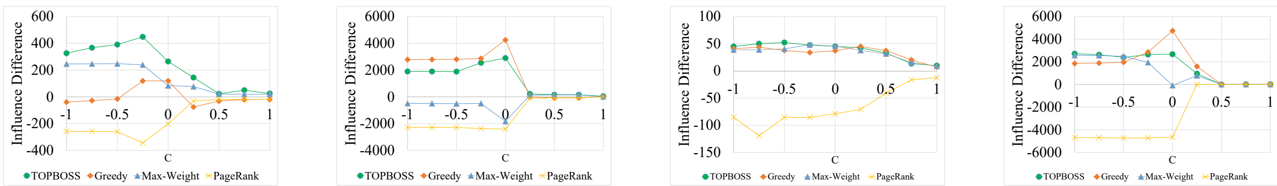
7. CONCLUSIONS

We propose the ILT model to model pairwise relationship between parities, where a relationship can be from competitive to complementary, with considering collaborative influence from friends in a social network for a user to adopt a product from a party. We use game-theoretical concepts to prove that why even two products from two different companies are highly complementary, their total spread may not be as large as the spread of two products from only one company due to their selfishness. We further prove that, given



(a) Facebook (b) Twitter (c) Wiki (d) Epinions

Figure 10: Improvement for heuristic methods on four datasets



(a) Facebook (b) Twitter (c) Wiki (d) Epinions

Figure 11: TOPBOSS and three influence maximization algorithms on influence difference

k symmetric highly complementary products owned by k different companies, the total spread can be reduced to as less as $1/k$ of the total spread size of k products owned by just one company.

We further propose a learning method for ILT model and in the experiment, we show the method is effective on both synthetic and real data. We study Interactive Influence Maximization problem with two objective functions. For maximizing self-influence, we demonstrate that most of the influence maximization methods can be improved with the extra information of the existence of other products. As for maximizing influence difference, we propose TOPBOSS to tackle the problem, which reaches effective results with very efficient computation time.

For future work, we would like to tackle the dynamic change of relationships between pairwise parties. For instance, a product may block other products to spread at the beginning but the effect of the product usually decays as time passes. This requires further studies and more complicated models.

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