More PRAM Algorithms

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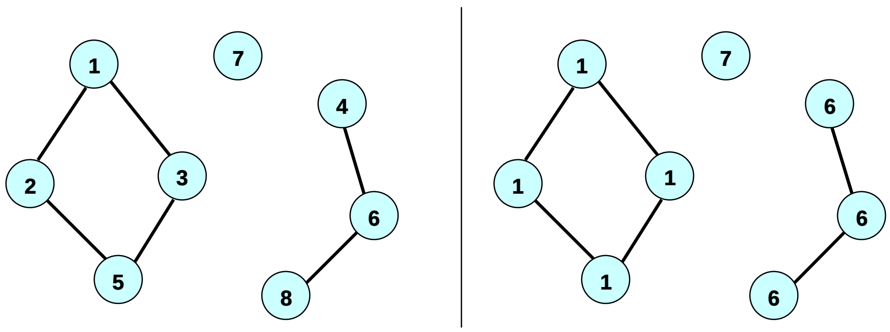
Techniques Covered

- Analysis technique:
  - Brent's scheduling lemma
  - Parallel algorithm is simply characterized by W(n) and S(n)
- Parallel techniques:
  - Scans
  - Pointer doubling
  - Euler tours
  - List ranking and list suffix operations
  - Parallel divide and conquer techniques
- Today:
  - Connected components
  - Sorting algorithms

Connected Components

- Compute the connected components of a graph
- Has many applications: vision, physics simulations, etc.



Sequential Algorithm

- Pretty straightforward:
  - Just perform some kind of traversal of the graph
  - Depth-first search (DFS), breadth-first search (BFS), etc.
  - Label the components

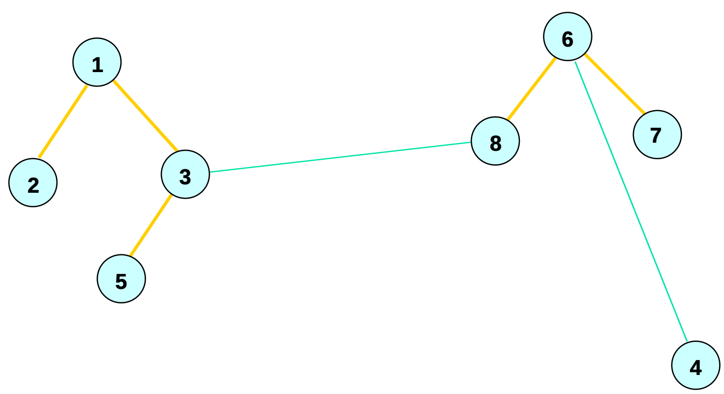
- Performance of sequential algorithm:
  - O(n + e)
  - Cache locality? Sometimes BFS turns out to be a better option than DFS

PRAM Algorithm: high level description

- Proposed by Shiloach and Vishkin
- Start with a forest of singleton vertices
- At each iteration, perform:
  - Hooking: attach a star (or a singleton vertex) with another tree
    - Comes in two forms: conditional and unconditional hooking
  - Pointer doubling: collapse the trees using pointer doubling
- Algorithm terminates when the trees in the forest do not have edges between them

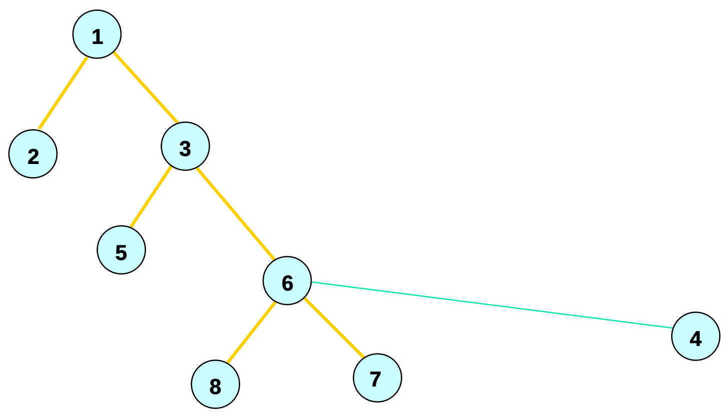
- Parallelism details:
  - There is one processor for each vertex and each edge
  - The edge processors are active for “hooking” and the vertex processors are active for pointer doubling

Example of conditional hooking



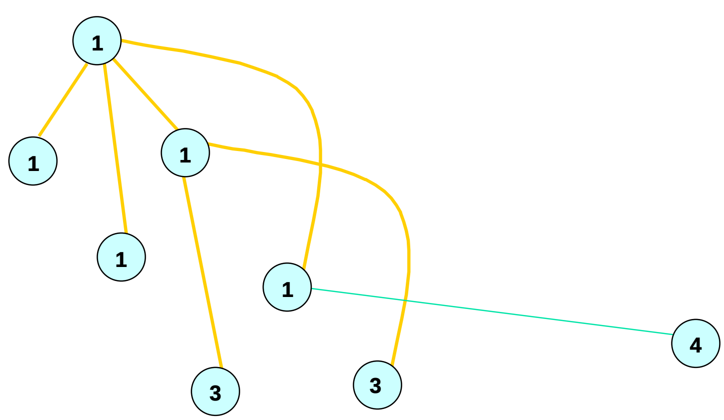
- Conditional hooking:
  - Attaches a star to a tree
  - Only if target tree-vertex has a lower number

Example of conditional hooking



Pointer Doubling

- Decreases the height of trees in the forest
  - Collapses the tree by taking each vertex and making its current grand-parent the new parent
  - Propagate grand-parent's identity to current node



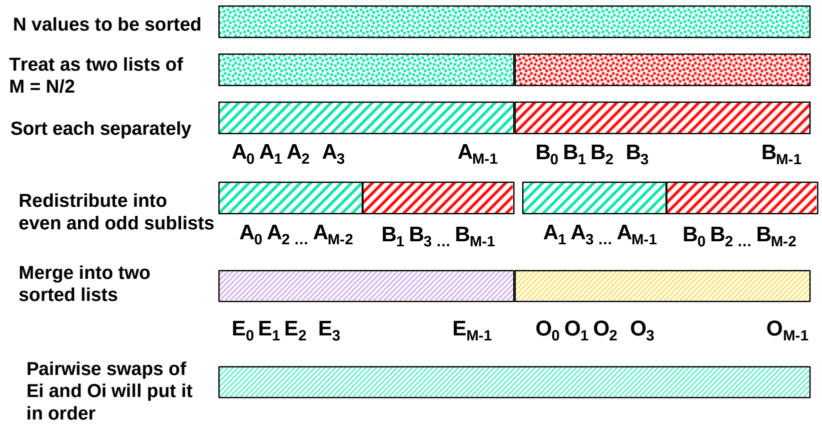
Unconditional hooking

- Just having the conditional hooking and pointer doubling isn't sufficient to have an asymptotically fast (O(log n)) algorithm
- Throw in unconditional hooking
  - Put perform unconditional hooking only on "stagnant" stars
  - Stagnant stars: those stars which had an opportunity to hook up using conditional hooking but failed to do so
  - Eliminate the condition to hook the star during unconditional hooking
- Refined algorithm is loop over:
  - Perform conditional hooking for all stars (using edge processors)
  - For stagnant stars, perform unconditional hooking (with edge-processors)
  - Perform pointer doubling (using vertex-processors)

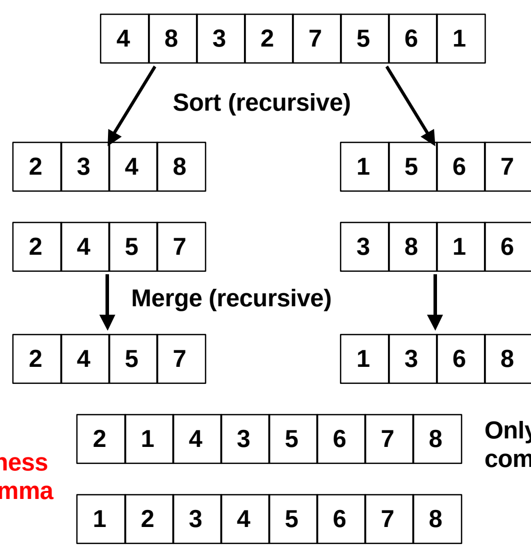
Sorting

- Traditional CS problem
- Sort a sequence of numbers stored in shared memory
- Can we solve it based on the techniques that we have seen so far
  - With n^2 processors and logn time?

Odd-Even Merge - classic parallel sort



Example



Only place where comparisons happen!

Proof of correctness by 0-1 sorting lemma

Functional Specification

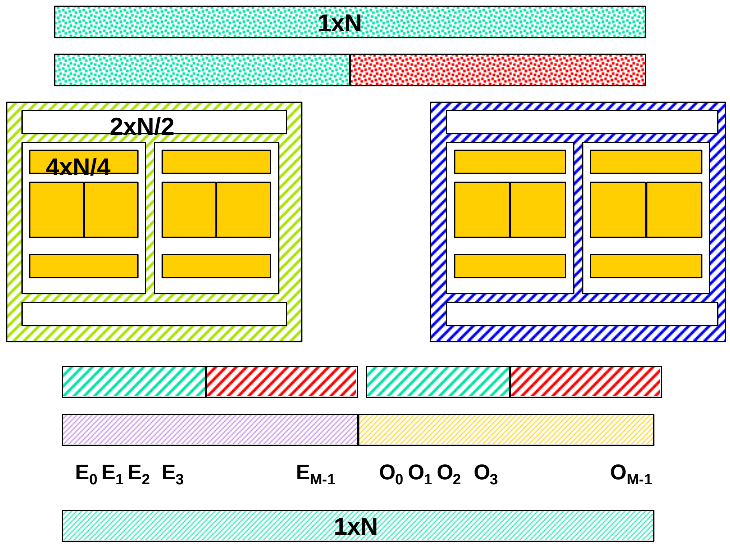
- Sort(S):
  Let S = A || B
  C = Sort(A);   D = Sort(B);
  return Merge(C, D);

- Merge(A, B):
  C = Merge(even(A), odd(B)); D = Merge(odd(A), even(B));
  E = interleave(C, D);
  return pairwise_comp(E);

Proof that Merge works

- Requires the 0-1 sorting lemma:
  - If an algorithm that uses just comparisons works with any sequence of 0's and 1's, then it works with any sequence of numbers
- Merge(A, B) produces the correct output:
  - Let A have x 0's and n/2-x 1's. Let B have y 0's and n/2-y 1's.
  - C then has $\lceil x/2 \rceil + \lfloor y/2 \rfloor$ 0's and D has $\lceil y/2 \rceil + \lfloor x/2 \rfloor$ 0's
  - It follows that the number of 0's in C and D can differ by at most one
  - So pairwise_comp after interleaving should sort the two sequence

Where's the Parallelism?



Complexity Measures

- Analyze merge operation separately:
  - What is work complexity?
  - What is step complexity?
- Sorting is simply a sequence of merge operations:
  - What is work complexity?
  - What is step complexity?

Bitonic Sort

- A bitonic sequence is one that is:
  1. Monotonically increasing and then monotonically decreasing
  2. Or monotonically decreasing and then increasing

- Examples:

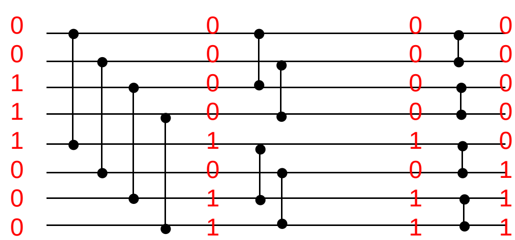
  1 4 7 9 11 8 6 4

  11 9 8 7 4 6 12 13

- Bitonic sequences are "almost" sorted

Half cleaner

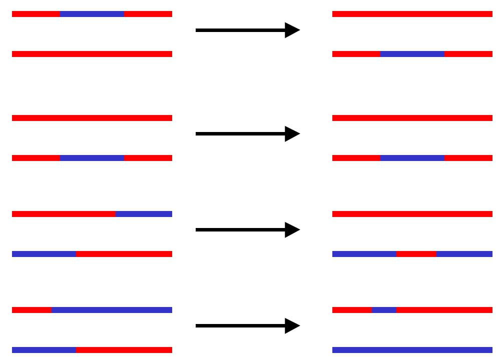
- A half-cleaner takes a bitonic sequence and produces
  1. First half is smaller than smallest element in 2^{nd} half
  2. Both halves are bitonic



Proof

- Consider all possible bitonic sequences of 0's and 1's
- What happens after one level of comparisons:

- Case 1:
- Case 2:
- Case 3:
- Case 4:



Uses of a half-cleaner

- Question: how can we use the half-cleaner to sort a bitonic sequence?
  - In other words accomplish the following: input is a bitonic sequence, output is a sorted sequence

Bitonic Sort

- Problem 1: cleaning a bitonic sequence (solved)
- Problem 2: create a bitonic sequence from two sorted sequences:
  - Reverse the second sequence
  - Concatenate with first
- Sort a sequence: pulling together the pieces

```
Sort (S):
    Let S = A || B
    C = Sort(A);   D = Sort(B);
    E = C || reverse(D)
    return Clean(E)
```

Complexity Issues

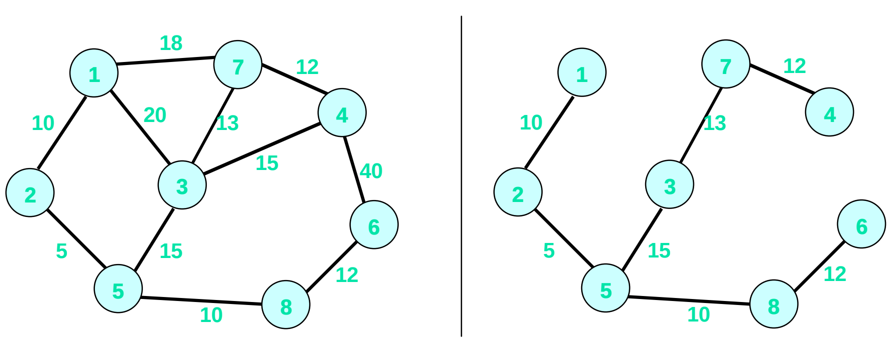
- What is the complexity of the half-cleaner:
  - Number of operations = n
  - Number of steps = 1
- What is the complexity of the cleaner:
  - Number of operations = n logn
  - Number of steps = logn
- What is the complexity of the sorting algorithm:
  - Number of operations = n $\log^2 n$
  - Number of steps = $\log^2 n$

Announcements

- Homework on PRAM algorithms posted on class website
  - Due next Wednesday. Individual work.
- Start doing preparatory work for class project:
  - Topics and pointers to links will be posted on the class website
  - Groups of two students each
  - Start by becoming an “expert” in some topic and then turn it into a semester-long project
- Upcoming lectures:
  - Shared memory architectures and programming models
  - Distributed memory topologies and programming models
  - Distributed algorithms

Minimum Spanning Trees

- Computed the minimum weight spanning tree of a graph
- All the vertices of the graph must be included



Sequential Algorithm

- Start with singleton vertices
- Repeat:
  - Select an arbitrary set
  - Choose an edge with the minimum weight outgoing from this set
  - Combine the two sets
  - Stop when there is just one set left

- Avoid creating cycles
- Kruskal's algorithm: combine along the minimum-weight edge in the graph
- Prim-Djikstra: start with just one distinguished vertex and grow the spanning tree

Parallel Algorithm

- Which of the various techniques that we have studied could be used for designing an efficient parallel algorithm?

- Which sequential algorithm would serve as a good starting point?