Bios 6301: Assignment 3

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Due Thursday, 08 October, 1:00 PM

50 points total.

 $5^{n=day}$ points taken off for each day late.

This assignment includes turning in the first two assignments. All three should include knitr files (named homework1.rmd, homework2.rmd, homework3.rmd) along with valid PDF output files. Inside each file, clearly indicate which parts of your responses go with which problems (you may use the original homework document as a template). Add your name as author to the file's metadata section. Raw R code/output or word processor files are not acceptable.

Failure to properly name files or include author name may result in 5 points taken off.

Question 1

10 points

- 1. Use GitHub to turn in the first three homework assignments. Make sure the teacher (couthcommander) and TA (trippcm) are collaborators. (5 points)
- 2. Commit each assignment individually. This means your repository should have at least three commits. (5 points)

Question 2

15 points

Write a simulation to calculate the power for the following study design. The study has two variables, treatment group and outcome. There are two treatment groups (0, 1) and they should be assigned randomly with equal probability. The outcome should be a random normal variable with a mean of 60 and standard deviation of 20. If a patient is in the treatment group, add 5 to the outcome. 5 is the true treatment effect. Create a linear of model for the outcome by the treatment group, and extract the p-value (hint: see assignment1). Test if the p-value is less than or equal to the alpha level, which should be set to 0.05.

Repeat this procedure 1000 times. The power is calculated by finding the percentage of times the p-value is less than or equal to the alpha level. Use the **set.seed** command so that the professor can reproduce your results.

1. Find the power when the sample size is 100 patients. (10 points)

```
set.seed(100)
n <- 100
pvals <- numeric(n)
# iterate 1000 times, saving each p value
for (i in 1:1000) {
    # Bernoulli distribution
    treat <- rbinom(n,1,0.5)</pre>
```

```
# add 5 to the outcome if the treat is 1
outcome <- rnorm(n, 60, 20) + treat * 5
x <- data.frame(treat, outcome)
pvals[i] <- t.test(outcome ~ treat, dat = x, var.equal = TRUE)$p.value
}
power_100 <- mean(pvals <= 0.05 )*100
power_100</pre>
```

[1] 23.6

1. Find the power when the sample size is 1000 patients. (5 points)

```
set.seed(1000)
n <- 1000
pvals <- numeric(n)
# iterate 1000 times, saving each p value
for (i in 1:1000) {
    # Bernoulli distribution
    treat <- rbinom(n,1,0.5)
    # add 5 to the outcome if the treat is 1
    outcome <- rnorm(n, 60, 20) + treat * 5
    x <- data.frame(treat, outcome)
    pvals[i] <- t.test(outcome ~ treat, dat = x, var.equal = TRUE)$p.value
}
power_1000 <- mean(pvals <= 0.05)*100
power_1000</pre>
```

[1] 96.8

Question 3

15 points

Obtain a copy of the football-values lecture. Save the 2015/proj_rb15.csv file in your working directory. Read in the data set and remove the first two columns.

1. Show the correlation matrix of this data set. (3 points)

```
setwd("/Users/ruiwang/Dropbox/Biostatistics/6301/homework")
fb <- data.frame(read.csv("proj_rb15.csv"))
# remove the first two columns
fb <- fb[,c(-1,-2)]
# show the correlation matrix of this data set
cor.fb <- cor(fb)
cor.fb</pre>
```

```
## rush tds 0.9723599 0.9774974 1.0000000 0.7263519 0.6984860 0.5908348
## rec_att 0.7694384 0.7645768 0.7263519 1.0000000 0.9944243 0.8384359
## rec yds 0.7402687 0.7345496 0.6984860 0.9944243 1.0000000 0.8518924
## rec_tds 0.5969159 0.6020994 0.5908348 0.8384359 0.8518924 1.0000000
## fumbles
           0.8589364 0.8583243 0.8526904 0.7459076 0.7224865 0.6055598
            0.9824135 0.9843044 0.9689472 0.8556928 0.8340195 0.7133908
## fpts
                           fpts
##
              fumbles
## rush_att 0.8589364 0.9824135
## rush_yds 0.8583243 0.9843044
## rush_tds 0.8526904 0.9689472
## rec_att 0.7459076 0.8556928
## rec_yds
           0.7224865 0.8340195
## rec_tds 0.6055598 0.7133908
## fumbles 1.0000000 0.8635550
            0.8635550 1.0000000
## fpts
var.fb<-var(fb)</pre>
var.fb
##
                           rush_yds
                                        rush_tds
               rush_att
                                                    rec_att
                                                                rec_yds
## rush_att
             6328.46094
                         27979.7864 192.0590190
                                                  905.61735
                                                             7114.11126
## rush_yds 27979.78642 124314.0880 855.7254243 3988.44050 31286.96872
## rush_tds
              192.05902
                           855.7254
                                       6.1647756
                                                   26.68256
                                                              209.50699
## rec_att
              905.61735
                          3988.4405
                                      26.6825607
                                                  218.89892
                                                             1777.36140
## rec_yds
             7114.11126
                         31286.9687 209.5069913 1777.36140 14593.66553
## rec_tds
               30.99774
                           138.5786
                                       0.9576186
                                                    8.09766
                                                               67.17920
## fumbles
               67.34980
                           298.2894
                                       2.0867780
                                                   10.87760
                                                               86.02767
## fpts
             4712.19384
                         20925.1864 145.0569358
                                                  763.34248
                                                             6074.88603
                rec tds
                            fumbles
                                            fpts
## rush_att 30.9977423
                         67.3497975
                                     4712.19384
## rush_yds 138.5786082 298.2893881 20925.18645
## rush tds
              0.9576186
                          2.0867780
                                       145.05694
## rec att
              8.0976598
                         10.8775990
                                       763.34248
## rec_yds
             67.1791959
                         86.0276733
                                     6074.88603
## rec_tds
              0.4261237
                          0.3896289
                                        28.07858
## fumbles
              0.3896289
                          0.9715215
                                        51.32110
## fpts
             28.0785773
                         51.3211018
                                     3635.46115
mean.fb<-colMeans(fb)
mean.fb
##
      rush_att
                  rush_yds
                              rush_tds
                                            rec_att
                                                        rec_yds
                                                                    rec_tds
##
   63.4651282 271.3948718
                             1.8323077
                                        14.4671795 115.1815385
                                                                   0.5400000
##
       fumbles
                      fpts
##
     0.7912821
               51.2610256
```

1. Generate a data set with 30 rows that has a similar correlation structure. Repeat the procedure 10,000 times and return the mean correlation matrix. (10 points)

```
library(MASS)
loops <- 1e4
keep.1 <- 0</pre>
```

```
set.seed(1)
for (i in seq(loops)) {
    fb.sim <- as.data.frame(mvrnorm(n=30, mu = mean.fb, Sigma=var.fb))
    keep.1 <- keep.1 + cor(fb.sim)/loops
}
# a similar correlation
keep.1
            rush_att rush_yds rush_tds
                                            {\tt rec\_att}
                                                      rec_yds
                                                                rec_tds
## rush att 1.0000000 0.9974500 0.9713472 0.7643291 0.7350341 0.5911347
## rush yds 0.9974500 1.0000000 0.9767243 0.7593050 0.7291431 0.5962737
## rush tds 0.9713472 0.9767243 1.0000000 0.7205943 0.6926162 0.5849183
## rec att 0.7643291 0.7593050 0.7205943 1.0000000 0.9942779 0.8347768
## rec_yds 0.7350341 0.7291431 0.6926162 0.9942779 1.0000000 0.8483749
## rec_tds 0.5911347 0.5962737 0.5849183 0.8347768 0.8483749 1.0000000
## fumbles 0.8545518 0.8538644 0.8478191 0.7410831 0.7175978 0.6001970
           0.9817474 0.9836956 0.9677966 0.8520135 0.8300580 0.7081649
## fpts
             fumbles
                           fpts
## rush_att 0.8545518 0.9817474
## rush_yds 0.8538644 0.9836956
## rush_tds 0.8478191 0.9677966
## rec_att 0.7410831 0.8520135
## rec_yds 0.7175978 0.8300580
## rec_tds 0.6001970 0.7081649
## fumbles 1.0000000 0.8593798
           0.8593798 1.0000000
## fpts
cor(fb)
##
            rush att rush yds rush tds
                                            rec att
                                                      rec yds
## rush_att 1.0000000 0.9975511 0.9723599 0.7694384 0.7402687 0.5969159
## rush yds 0.9975511 1.0000000 0.9774974 0.7645768 0.7345496 0.6020994
## rush_tds 0.9723599 0.9774974 1.0000000 0.7263519 0.6984860 0.5908348
## rec att 0.7694384 0.7645768 0.7263519 1.0000000 0.9944243 0.8384359
## rec_yds 0.7402687 0.7345496 0.6984860 0.9944243 1.0000000 0.8518924
## rec tds 0.5969159 0.6020994 0.5908348 0.8384359 0.8518924 1.0000000
## fumbles 0.8589364 0.8583243 0.8526904 0.7459076 0.7224865 0.6055598
            0.9824135 0.9843044 0.9689472 0.8556928 0.8340195 0.7133908
## fpts
##
             fumbles
## rush_att 0.8589364 0.9824135
## rush_yds 0.8583243 0.9843044
## rush_tds 0.8526904 0.9689472
## rec_att 0.7459076 0.8556928
## rec_yds 0.7224865 0.8340195
## rec_tds 0.6055598 0.7133908
```

1. Generate a data set with 30 rows that has the exact correlation structure as the original data set. (2 points)

fumbles 1.0000000 0.8635550

0.8635550 1.0000000

fpts

```
# set empirical equals TRUE value, we can have the exact correlation structure
fb.sim <- mvrnorm(n=30, mu = mean.fb, Sigma=var.fb, empirical = TRUE)
cor(fb.sim)</pre>
```

```
##
           rush_att rush_yds rush_tds
                                        rec_att
                                                 rec_yds
                                                          rec_tds
## rush_att 1.0000000 0.9975511 0.9723599 0.7694384 0.7402687 0.5969159
## rush_yds 0.9975511 1.0000000 0.9774974 0.7645768 0.7345496 0.6020994
## rush_tds 0.9723599 0.9774974 1.0000000 0.7263519 0.6984860 0.5908348
## rec_att 0.7694384 0.7645768 0.7263519 1.0000000 0.9944243 0.8384359
## rec_yds 0.7402687 0.7345496 0.6984860 0.9944243 1.0000000 0.8518924
## fumbles 0.8589364 0.8583243 0.8526904 0.7459076 0.7224865 0.6055598
          0.9824135 0.9843044 0.9689472 0.8556928 0.8340195 0.7133908
## fpts
##
            fumbles
                        fpts
## rush_att 0.8589364 0.9824135
## rush_yds 0.8583243 0.9843044
## rush_tds 0.8526904 0.9689472
## rec_att 0.7459076 0.8556928
## rec_yds 0.7224865 0.8340195
## rec_tds 0.6055598 0.7133908
## fumbles 1.0000000 0.8635550
## fpts
          0.8635550 1.0000000
```

cor.fb

```
rush_att rush_yds rush_tds
                                           rec_att
                                                     rec_yds
                                                               rec_tds
## rush_att 1.0000000 0.9975511 0.9723599 0.7694384 0.7402687 0.5969159
## rush_yds 0.9975511 1.0000000 0.9774974 0.7645768 0.7345496 0.6020994
## rush_tds 0.9723599 0.9774974 1.0000000 0.7263519 0.6984860 0.5908348
## rec_att 0.7694384 0.7645768 0.7263519 1.0000000 0.9944243 0.8384359
## rec_yds 0.7402687 0.7345496 0.6984860 0.9944243 1.0000000 0.8518924
## rec_tds 0.5969159 0.6020994 0.5908348 0.8384359 0.8518924 1.0000000
## fumbles 0.8589364 0.8583243 0.8526904 0.7459076 0.7224865 0.6055598
           0.9824135 0.9843044 0.9689472 0.8556928 0.8340195 0.7133908
## fpts
##
             fumbles
                           fpts
## rush_att 0.8589364 0.9824135
## rush_yds 0.8583243 0.9843044
## rush_tds 0.8526904 0.9689472
## rec att 0.7459076 0.8556928
## rec_yds 0.7224865 0.8340195
## rec tds 0.6055598 0.7133908
## fumbles 1.0000000 0.8635550
## fpts
           0.8635550 1.0000000
```

Question 4

10 points

Use LATEX to create the following expressions.

1. Hint: \Rightarrow (4 points)

$$P(B) = \sum_{j} P(B|A_j)P(A_j),$$

$$\Rightarrow P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j} (B|A_j)P(A_j)}$$

$$\begin{split} P(B) &= \sum_{j} P(B|A_{j}) P(A_{j}), \\ \Rightarrow P(A_{i}|B) &= \frac{P(B|A_{i}) P(A_{i})}{\sum_{j} (B|A_{j}) P(A_{j})} \end{split}$$

2. Hint: \zeta (3 points)

$$\hat{f}(\zeta) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ix\zeta}dx$$

$$\hat{f}(\zeta) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ix\zeta} dx$$

3. Hint: \partial (3 points)

$$\mathbf{J} = \frac{d\mathbf{f}}{d\mathbf{x}} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial x_1} & \dots & \frac{\partial \mathbf{f}}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \dots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

$$J = \frac{df}{dx} = \left[\frac{\partial f}{\partial x_1} \cdots \frac{\partial f}{\partial x_n}\right] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$