Autocorrelation sidelobe suppression in MIMO radar OFD-LFM waveforms

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***Abstract*—Aiming at the problem of high grating sidelobes in the autocorrelation function of orthogonal frequency-division linear frequency modulation (OFD-LFM) signals synthesized, this paper proposes a novel multiple-input-multiple-output (MIMO) radar OFD-LFM waveform design method, which aims to suppress the grating sidelobes of autocorrelation function effectively. First, the signal bandwidth of each sub-channel is the total bandwidth; second, each sub-channel transmit waveform of a MIMO radar is split into multiple subcarriers, and each subcarrier is assigned an unequal modulation bandwidth as well as an unequal timewidth. A genetic algorithm is used to determine the optimal solution by constructing a model designed to jointly optimize the subcarrier modulation bandwidth and subcarrier modulation time for each channel. Simulation results show that the emergence of the high grating sidelobes is effectively suppressed by jointly selecting the freedom of choice for optimizing the modulation bandwidth and modulation time of the transmit signal.**

***Index Terms*—multiple input multiple output radar, waveform design, orthogonal frequency division-linear frequency modulation signal, sidelobe suppression**

I. Introduction

Multiple-Input Multiple-Output (MIMO) radar is an advanced radar technology that uses multiple antennas at the transmitter and receiver ends to achieve high-precision detection and localization of targets. MIMO radar has greater degrees of freedom than traditional phased-array radar, and it utilizes spatial diversity and beam-forming techniques to enhance the performance of the radar system, improving resolution and detection capability. Frequency-division orthogonal waveforms are a commonly used technique in MIMO radar systems, which achieves orthogonality [1] by having each channel transmit signals of different carrier frequencies. However, when there is a linear relationship between the carrier frequency and the array element number, it may cause a distance-azimuth coupling effect [2] that affects the performance of the radar system. In order to eliminate this coupling effect and improve the performance of orthogonal waveforms, the literature [3], [4]proposes the use of Discrete Frequency Coded (DFCW) waveforms. DFCW waveforms achieve orthogonality by encoding signals at different frequencies, thus improving the performance of the waveforms.

Further, in order to improve the Doppler tolerance of the waveforms, the literature [5], [6] introduces linear frequency modulation (LFM) signals into the DFCW waveforms and optimizes their autocorrelation performance by a genetic algorithm or a hybrid genetic and simulated annealing algorithm. However, although this approach can improve the autocorrelation performance, the DFCW+LFM waveform still has some limitations in terms of Doppler tolerance. Considering the Doppler stability of linear frequency modulation (LFM) signals, the performance of orthogonal waveforms in terms of Doppler tolerance can be significantly improved by transmitting complete LFM signals with different frequencies in MIMO radar systems. Therefore, several researchers have started to explore the study of orthogonal frequency-division linear frequency modulation (OFD-LFM) signals. Literature [7] analyzes the mutual ambiguity function of OFD-LFM signals, determines the location and amplitude of the peaks of the cross-correlation, and reveals the parametric relationships that need to be satisfied in order to minimize the cross-correlation, but it does not take into account the autocorrelation characteristics of OFD-LFM signals. Literature [8] analyzed the output of OFD-LFM signals after matched filtering, pointed out that the problem of high discrete sidelobes in the autocorrelation function originated from the equal bandwidth and non-overlapping spectrum of the transmitted signals, and proposed a method of designing the waveforms with non-uniform bandwidths and overlapping frequency bands. Nevertheless, the waveforms designed by this method still face serious near-area sidelobe problems in the matched filtered output.

Based on the literature [9], [10], [11] analyzed the autocorrelation sidelobe generation causes of OFD-LFM waveforms with spectrally overlapped spectra, and proposed the OFD-LFM waveform design method with joint optimization of frequency coding and initial phase and the OFD-LFM waveform design method with non-uniform frequency interval. The non-uniform frequency interval method avoids the problem of high discrete sidelobes in the matched filter output by directly optimizing the frequency interval, but it cannot guarantee the uniform distribution of the transmitted energy. Literature [13] still designs the frequency interval of different array elements as an integer multiple of the minimum frequency interval $1/T_p$ during frequency coding, thus increasing the degree of freedom of frequency coding, improving the suppression effect of discrete sidelobes, and being able to ensure that the transmit energy distribution is ideally omnidirectional. Literature [12] further divides each channel transmit waveform into multiple subpulses, and each subpulse is assigned an initial phase value, allowing each transmit waveform to build a joint spatio-temporal optimization model within a sub-bandwidth, which effectively reduces radar sidelobe, but radar sidelobe is still maintained at a high level.

Aiming at the above problems in MIMO radar waveform design, a new OFD-LFM waveform design method is proposed in this paper. Firstly, the signal bandwidth of each sub-channel is set to be the total bandwidth; secondly, the transmit waveform of each channel of the MIMO radar is split into multiple subcarriers, and the unequal modulation bandwidth as well as the unequal modulation timewidth are assigned to each sub-signal.

II. OFD-LFM Signal Model

Orthogonal Frequency Division-Linear Frequency Modulation (OFD-LFM) waveforms use a specific set of signals in each transmitting array element of a multiple-input multiple-output (MIMO) radar system. These signals are linear frequency modulated (LFM) signals whose frequency bands may be the same or partially overlap. Assuming that the MIMO radar system has M transmitting array elements, the transmitting signal of the mth array element can be expressed as:

$$S_m(t) = \operatorname{rect}\left(\frac{t}{T_p}\right) \quad \times e^{j2\pi(f_m t + \frac{1}{2}\mu t^2)}, \quad m = 1, 2, \ldots, M \tag{1}$$

Where T_p is the signal pulse width, f_m is the starting frequency of the mth signal, $\mu = \frac{B_s}{T_p}$ is the FM slope, and B_s is the bandwidth of a single signal. For OFD-LFM signals, the frequency interval Δf between adjacent channels is fixed and satisfies $f_m - f_{m-1} = \Delta f$. When the frequency interval Δf is an integer multiple of the reciprocal of the pulse width T_p, orthogonality is achieved between the different signals, which helps to improve the resolution and anti-interference capability of the radar system.

A target in the direction θ receives the signal as:

$$y_\theta(t) = \boldsymbol{a}_t^T(\theta)\boldsymbol{S}(t) = \sum_{m=1}^{M} e^{j2\pi(m-1)d\sin\theta/\lambda} S_m(t) \tag{2}$$

where $\boldsymbol{a}_t(\theta) = [1, e^{j2\pi\frac{d}{\lambda}\sin\theta}, \ldots, e^{j2\pi\frac{(M-1)d}{\lambda}\sin\theta}]^T$ is the emission array's direction vector, d is the spacing between adjacent transmitter array elements, and λ is the wavelength of the signal. $\boldsymbol{S}(t) = [S_1(t), S_2(t), \ldots, S_M(t)]^T$ is the vector of signals emitted by each array element.

In the target detection process, the detection performance is mainly affected by the autocorrelation function of the spatially synthesized signal. When the autocorrelation function has a high autocorrelation, it is easy to cause false alarms. The autocorrelation function of the spatially synthesized signal [10] can be expressed as:

$$\begin{aligned}
f(\theta,\tau) &= \int_{-\infty}^{\infty} y_\theta(t) y_\theta^*(t-\tau)dt \\
&= \int_{-\infty}^{\infty} \sum_{m=1}^{M} \{\exp[j2\pi(m-1)d\sin\theta/\lambda] \cdot S_m(t)\} \cdot \\
&\quad \sum_{i=1}^{M} \{\exp[-j2\pi(i-1)d\sin\theta/\lambda] \cdot S_i^*(t-\tau)\}\, dt \\
&= \int_{-\infty}^{\infty} \left\{ \sum_{m=1}^{M} \left\{ \exp[j2\pi(m-1)d\sin\theta/\lambda] \cdot \exp\left[j2\pi \right.\right.\right. \\
&\quad \left.\left.\left. \left(f_m t + \frac{1}{2}\mu t^2\right)\right] \operatorname{rect}\left(\frac{t}{T_p}\right)\right\} \cdot \left\{ \sum_{i=1}^{M} \{\exp[j2\pi(1-i) \right.\right. \\
&\quad \cdot d\sin\theta/\lambda] \cdot \exp\left[-j2\pi\left(f_i(t-\tau) + \frac{1}{2}\mu(t-\tau)^2\right)\right] \\
&\quad \left.\left. \cdot \operatorname{rect}\left(\frac{t-\tau}{T_p}\right)\right\}\right\} dt \\
&= \sum_{m=1}^{M} \exp\left(j2\pi f_m \tau\right) \xi_0(\tau) + \sum_{v=1}^{M-1} R_v(\tau) + \sum_{n=1}^{M-1} Z_n(\tau)
\end{aligned} \tag{3}$$

Where

$$\begin{aligned}
|R_v(\tau)| &= |\frac{\sin\left[\pi\left(\mu\tau + v\Delta f\right)\left(T_P - |\tau|\right)\right]}{\pi\left(\mu\tau + v\Delta f\right)\left(T_P - |\tau|\right)} \left(T_P - |\tau|\right) \\
&\quad \cdot |\frac{\sin\left[\pi\left(M-v\right)\Delta f\tau\right]}{\sin\left(\pi\Delta f\tau\right)}| \\
&= |\chi_v(\tau)| \cdot |W_v|
\end{aligned} \tag{4}$$

The main sidelobes of the autocorrelation function are determined by (4). By substituting $f_m - f_{m-v} = v\Delta f$ into $R_v(\tau)$, where $|\chi_v(\tau)|$ is a periodic function, maximum values are obtained at $\tau_2 = \pm n/\Delta f$, for $n = 0, 1, \ldots, \frac{T_p}{\Delta f} - 1$. When the peak of $|W_v|$ overlaps with the main lobe of $|\chi_v(\tau)|$, a high grating sidelobe is produced.

III. Waveform design method for joint optimization of unequal bandwidth and unequal time

In the previous section, we discussed the spatio-temporal characteristics of the OFD-LFM signal. Conventional OFD-LFM waveforms have equal or opposite linear FM slopes for the underlying signals and a fixed starting frequency. Since the high sidelobe of the autocorrelation function is due to equal LFM slopes and starting frequencies, various parameters of the sub-linear FM in the waveform are designed to suppress the high autocorrelation sidelobe based on the OFD-LFM.

Since the LFM slopes and initial frequencies are defined by the durations and modulation bandwidths of the sub-channel signals. Conventional methods do not have much freedom for waveform design. We therefore propose to use different sub-linear FM durations and bandwidths to increase the diversity of the linear FM signal, releasing more degrees of freedom in waveform design. We also introduce design techniques to identify optimal parameters for minimizing autocorrelation interference.

The optimized waveform of this paper is shown in Fig. 1.

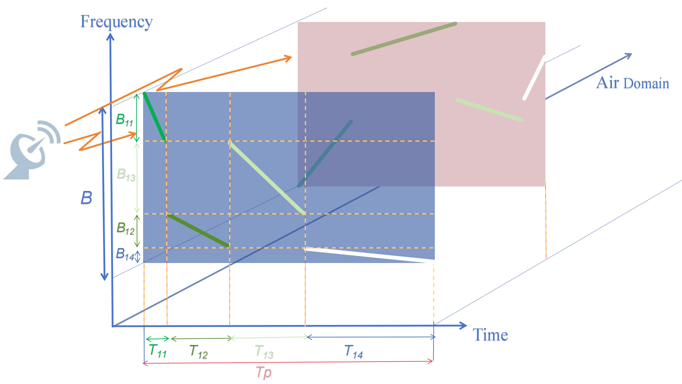


Fig. 1: Optimized waveform design method: Tp represents the total time width and B represents the total bandwidth. Assuming that the MIMO radar has two channels, each channel transmits four subcarriers

In this paper, the OFD-LFM waveform can be expressed as:

$$S_m(t) = \sum_{n=1}^{N} rect(\frac{t-\sum_{q=0}^{n-1} T_{mq}}{T_{mn}}) \cdot \exp[j2\pi(f_{mn}(t-\sum_{q=0}^{n-1} T_{mq}) + \frac{k_{mn}}{2}(t-\sum_{q=0}^{n-1} T_{mq})^2)] \tag{5}$$

where

$$rect(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

- $S_m(t)$ is the signal transmitted by the m-th channel for $m = 1, 2, \ldots, M$.
- f_{mn} and k_{mn} are the starting frequency and frequency modulation slope of the n-th subcarrier on the m-th channel, respectively.
- The matrix T is of size $M \times N$, where each row represents a different channel and each column represents a different subcarrier. T_{mn} is the duration of the n-th subcarrier on the m-th channel and $T_{m0} = 0$.
- N represents the total number of subcarriers per channel and M represents the total number of channels.

According to (2), a target in the direction θ receives the signal as:

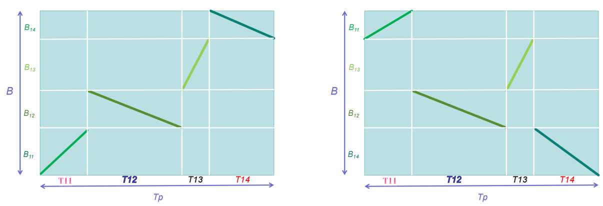
$$\begin{aligned} y_\theta(t) =& \boldsymbol{a}_t^T(\theta)\boldsymbol{S}(t) = \sum_{m=1}^{M} e^{j2\pi(m-1)d\sin\theta/\lambda} S_m(t) \\ &= \sum_{m=1}^{M}\sum_{n=1}^{N} e^{j2\pi(m-1)d\sin\theta/\lambda} rect(\frac{t-\sum_{q=0}^{n-1} T_{mq}}{T_{mn}}) \\ &\cdot \exp[j2\pi(f_{mn}(t-\sum_{q=0}^{n-1} T_{mq}) + \frac{k_{mn}}{2}(t-\sum_{q=0}^{n-1} T_{mq})^2)] \end{aligned} \tag{7}$$

In order to keep the transmit signal of the MIMO radar with low autocorrelation sidelobe level for separation at reception. The autocorrelation function is defined as:

$$\begin{aligned} f(\theta,\tau) &= \int_{-\infty}^{\infty} y_\theta(t) y_\theta^*(t-\tau)dt \\ &= \int_{-\infty}^{\infty} \sum_{m=1}^{M} \{\exp[j2\pi(m-1)d\sin\theta/\lambda] \cdot S_m(t)\} \cdot \\ &\quad \sum_{i=1}^{M} \{\exp[-j2\pi(i-1)d\sin\theta/\lambda] \cdot S_i^*(t-\tau)\} \, dt \\ &= \int_{-\infty}^{\infty} \sum_{m=1}^{M}\sum_{i=1}^{M} \exp[j2\pi(m-1)d\sin\theta/\lambda] \cdot \\ &\quad \exp[-j2\pi(i-1)d\sin\theta/\lambda] \cdot S_m(t) \cdot S_i^*(t-\tau)dt \\ &= \int_{-\infty}^{\infty} \sum_{m-1}^{M}\sum_{i=1}^{M} \exp[j2\pi((m-i)d\sin\theta/\lambda)] \cdot \\ &\quad S_m(t) \cdot S_i^*(t-\tau)dt \\ &= \sum_{m=1}^{M}\sum_{i=1}^{M} \int_{-\infty}^{\infty} \exp[j2\pi((m-i)d\sin\theta/\lambda)] \cdot \\ &\quad (\sum_{n=1}^{N} \text{rect}\left(\frac{t-\sum_{p=0}^{n-1} T_{mp}}{T_{mn}}\right) \exp[j2\pi(f_{mn}(t-\sum_{p=0}^{n-1} T_{mp}) \\ &\quad + \frac{k_{mn}}{2}(t-\sum_{p=0}^{n-1} T_{mp})^2)]) \cdot (\sum_{j=1}^{N} \text{rect}\left(\frac{t-\tau-\sum_{q=0}^{j-1} T_{iq}}{T_{ij}}\right) \cdot \\ &\quad \exp[-j2\pi(f_{ij}(t-\tau-\sum_{q=0}^{j-1} T_{iq}) + \frac{k_{ij}}{2}(t-\tau-\sum_{q=0}^{j-1} T_{iq})^2)])dt \end{aligned} \tag{8}$$

From (8), the $S_m(t) \cdot S_i^*(t-\tau)$ can be considered as a form of autocorrelation of the sub-channel signals. When the start frequencies and modulation slopes of the two individual subcarrier are equal, it indicates that the two waveforms contain the same linear FM signal over the FM duration. According to the analysis of (3), the autocorrelation function will show a higher discrete slope edge in this case.

Since the linear FM slope and initial frequency are determined by the modulation time and modulation bandwidth of the sub-linear FM signal, we jointly optimize the modulation time and modulation bandwidth to better suppress the autocorrelation sidelobes.



(a) The starting frequency code for initialization is [1, 2, 3, 4].
(b) The starting frequency code is [4, 2, 3, 1].

Fig. 2: Comparison of signals encoded at different starting frequencies

In the optimization process, we need to select different bandwidth ranges for different time periods within each channel, and we need to indirectly select the starting frequencies. In order to achieve the effect of Fig. 1. First, we calculate the initial starting frequency by the unequal bandwidth, and then select the starting frequency and bandwidth for each subcarrier of the channel by encoding.

The initial starting frequency of each channel is initialized as follows:

$$F_{mn} = \begin{cases} F_{m(n-1)} + B_{m(n-1)} & k_{m(n-1)} > 0, k_{mn} > 0 \\ F_{m(n-1)} + B_{m(n-1)} + B_{mn} & k_{m(n-1)} > 0, k_{mn} < 0 \\ F_{m(n-1)} & k_{m(n-1)} < 0, k_{mn} > 0 \\ F_{m(n-1)} + B_{mn} & k_{m(n-1)} < 0, k_{mn} < 0 \\ F_{m0} & k_{m(n-1)=0} \end{cases} \tag{9}$$

where

- F_{m0} $=f_c$ - $B/2$ where f_c represents the carrier frequency of the channel and B represents the total bandwidth of the channel.
- $B_{mn} = |k_{mn} \cdot T_{mn}|$ is the bandwidth of the n-th subcarrier, which can be different for each subcarrier and B_{m0}=0 and k_{m0} = 0.

We encode the starting frequency of each transmit channel subcarrier and select the starting frequency and corresponding bandwidth by encoding, where the encoding process is as follows:

$$\begin{cases} \quad fcode_{mn} \in 1, 2, ..., N \\ \quad fcode_{mi} \neq fcode_{mj} \\ i \in 1, 2, ..., N, j \in 1, 2, ..., N, i \neq j \end{cases} \tag{10}$$

where

- The matrix $fcode$ is of size $M \times N$, where each row represents a different channel and each column represents sequence of frequency coding.
- The $fcode$ characterizes the position occupied by the waveform.

The model optimized in this paper is:

$$\min_{\boldsymbol{T_{mn}}, \boldsymbol{B_{mn}}, \boldsymbol{fcode_{mn}}} \left\{ \frac{\max_{\tau \neq 0} |f(\theta, \tau)|}{|f(\theta, 0)|} \right\}$$
$$\text{s.t.} \begin{cases} \sum_{n=1}^{N} B_{mn} = B \\ \sum_{n=1}^{N} T_{mn} = T_p \end{cases} \tag{11}$$

The optimization process is performed using Genetic Algorithm (GA), which is a global optimization method that mimics the natural selection and genetic mechanisms and is highly robust and insensitive to the initial values.

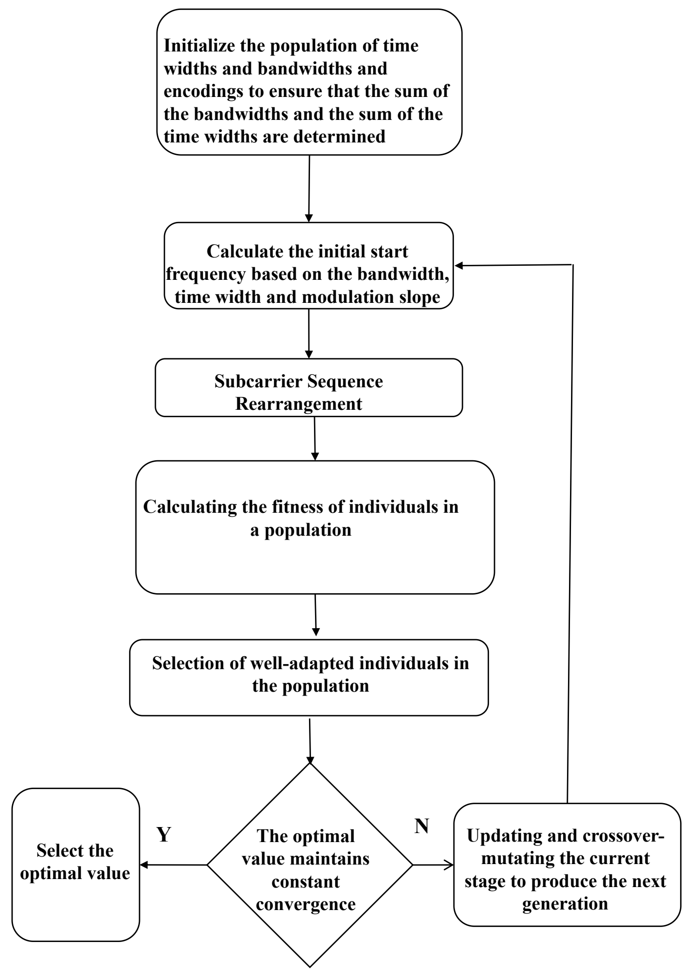


Fig. 3: algorithm flow chart

IV. Simulation and Analysis

To verify the effectiveness of the algorithm proposed in the previous section, this section will assess the algorithm through simulation experiments and compare it with existing methods. The number of array elements in the MIMO radar is set to $M = 8$, assuming that both the transmitting and receiving antennas are uniform linear arrays with an element spacing of $d = \lambda/2$. Each channel emits an OFD-LFM signal with a center frequency of $f_0 = 300$MHz, total bandwidth of $B = 3$MHz, pulse width of $T_p = 100\mu$s, and the number of subcarriers is $N = 8$. The signal bandwidth for each channel, as divided in the reference literature, is $B_s = 2.16$MHz. The angle range for MIMO radar target detection is set from $-90°$ to $90°$, and this paper's algorithm is compared with traditional orthogonal low-frequency modulated signal waveform design methods ([9], [12], [13], [15], and [16]). This method maximally utilizes the signal bandwidth, increasing the selection range of subcarrier frequency bands.

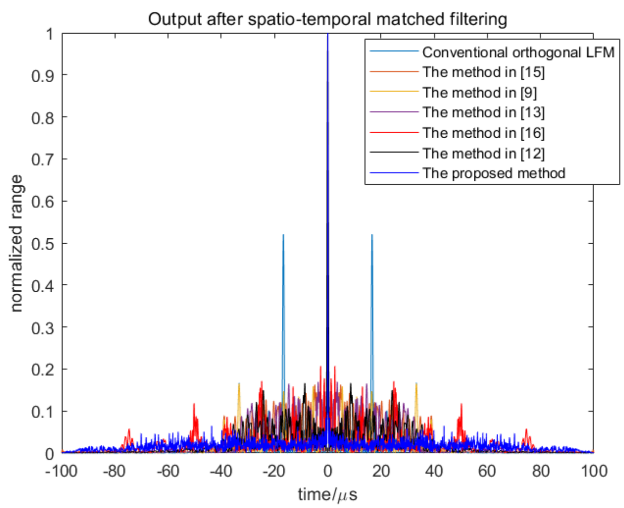


Fig. 4: The autocorrelation function optimized by different methods when $B = 3$MHz

Figure 4 shows the autocorrelation functions for different waveform designs under $B = 3$MHz are shown. The reference literature defines $B_s = 2.16$MHz, while the maximum sub-bandwidth assigned in this paper is $B_s = 3$MHz. The peak value of the sidelobes of the autocorrelation function determines the pulse compression performance, and high sidelobes will mask the echoes of weak targets and increase the probability of false alarms. As can be seen in Figure 4, the waveform autocorrelation function obtained by the traditional orthogonal low-frequency transmit waveform design method has high sidelobes. These high sidelobes reduce the pulse compression performance and cause a certain probability of false alarms, which is not conducive to the detection and processing of radar signals.

To further compare the performance of waveforms designed by different algorithms, Table 1 gives the average sidelobe of the autocorrelation function obtained by the method proposed in this paper, the conventional orthogonal low-frequency modulation waveform design method, and the [9], [12], [13], [15] and [16] methods, under the condition $B = 3$MHz. Peak Ratio (ASP). It can be seen that the ASP of this paper's method is lower than other methods. This is mainly because the method in this paper increases the selection range of the subcarrier bands of each channel, which utilizes the signal bandwidth more fully. Moreover, this paper divides the unequal time width and unequal bandwidth for each signal, so that the modulation slope of the signal has a larger optimization space, and according to the modulation bandwidth to calculate the starting frequency of each signal, optimized the starting frequency of each sub-signal, so that the starting frequency of the signal is staggered so that the signal is conducive to reducing the autocorrelation sidelobe of the signal.

TABLE I: The Average ASP of Different Methods Under the Condition of $B = 3$MHz

LFM Design Method	The average ASP
Conventional orthogonal LFM waveform	0.512
Frequency-phase joint optimization of LFM waveforms [9]	0.227
Composite Signal Orthogonal Waveform Set [16]	0.178
Non-uniform frequency spacing LFM waveform [15]	0.166
Expanded Frequency Coding Space LFM Waveform [13]	0.161
Non-uniform segmented quadrature LFM waveforms [12]	0.138
This Paper's Method	0.108

V. Conclusion

To improve the sidelobe performance of OFD-LFM waveforms in multiple-input multiple-output radars, this paper proposes a waveform optimization strategy. First, the bandwidth of each channel is the total bandwidth; second, the transmit waveform of each channel of a MIMO radar is split into multiple sub-signals, and each sub-signal is assigned a unequal modulation bandwidth as well as unequal modulation time width. By constructing a joint optimization model aiming at selecting the modulation bandwidth with the lowest autocorrelation sidelobe of the transmit waveform as well as the modulation time width, this paper employs a genetic algorithm to determine the optimal solution. Simulation results show that the optimized waveform significantly improves the sidelobe performance and enhances the detection capability of weak targets.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (U2133216), Shenzhen Science and Technology Program (KQTD20190929172704911), Guangdong Provincial Science and Technology Program (2019ZT08X751), and the Science and Technology Planning Project of Key Laboratory of Advanced IntelliSense Technology, Guangdong Science and Technology Department (2023B1212060024).

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