BEPP 931 - Solution to Problem Set 1

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Question 1

We compare three ways of solving the system of equations. In the first approach, we use matrix operations:

$$\begin{pmatrix} 64919121 & -159018721 \\ 41869520.5 & -102558961 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 64919121 & -159018721 \\ 41869520.5 & -102558961 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

This approach yields $x = 1.4587\text{e+}08$ and $y = 9550\text{e+}07$ in 0.0014 seconds whereas using Cramer's rule yields the same result but is much faster requiring only 5.8521e-04 seconds. The approach that yields the desired solution of $x = 205117922$ and $y = 83739041$ is to resolve catastrophic cancellation by breaking up big numbers into a sum and changing the order (in 0.0003 seconds):

$$x = \frac{-102558961}{((64919121 * (-102500000) - 41869520.5 * (-159000000)) - 64919121 * 58961 + 41869520.5 * 18721)}$$

Question 2

The direct way of solving the polynomial explicitly yields $-1.0889\text{e+}40$ in 0.0508 seconds. Factoring out such that we solve $y^4(83521y^4 + 578x^2) - x^4(2 - 2x^2 + x^4)$ yields 0 in only 0.000226 seconds (since there are now fewer terms to compute) which is however far off from the correct answer. Changing the order to $[83521y^8 - x^4(2+x^4)] + [x^2(578y^4 + 2x^4)]$ yields $-6.4186\text{e+}39$ in 0.000419 seconds.

Question 3

Operation	speed relative to addition
Multiplication	0.467
Division	0.7311
Exponent	2.7878
Sine	0.6077

Question 4

Using the general stopping rule yields the following number of terms and errors:

ϵ	No. Terms	Error
10^{-2}	7	0.2391
10^{-3}	9	0.0221
10^{-4}	11	0.0014

Using the adaptive rule yields the following number of terms and errors:

ϵ	No. Terms	Error
10^{-2}	11	0.0014
10^{-3}	12	3.2436e-4
10^{-4}	14	1.3465e-5