Firms and Equilibrium

1. **Solving the Problem of the Firm in Discrete Time**

Consider the following problem for the firm:

$$\max_{\{k_t,l_t\}_{t=0}^{\infty}} \mathrm{E}_0 \sum_{t=0}^{\infty} \left[\frac{1}{(1+r)^t} d_t \right]$$
$$s.t. \qquad d_t = \pi_t - i_t - \phi_t$$

where d_t is the amount of gross distributions and the other variables obey the usual relationships,

$$i_t = k_{t+1} - (1-\delta)k_t,$$
$$\pi_t = a_t k_t^{\theta_1} l_t^{\theta_2} - W l_t$$

The stochastic process for a_t is given by,

$$\ln(a_t) = (1-\rho)\ln\bar{a} + \rho\ln(a_{t-1}) + \sigma\epsilon_t.$$

where the innovations ϵ_t are standard normal.

Suppose $\rho = 0.8$ and use a grid with 9 points for the shocks. In addition use the following parameter values: $r = 0.02, \delta = 0.1, \theta_1 = 0.3, \theta_2 = 0.6, W = 2, \sigma = 0.1$.

a) Plot the profit function as a function of the capital stock. Do this for the highest and the lowest value of the shock a. This gives you an idea of what the value function should look like.

Suppose that adjustment costs are given by the expression:

$$\phi = b_0 k + b_1 \left(\frac{i}{k} - \delta \right)^2 k$$

For now set $b_0 = 0, b_1 = 0.5$.

b) Use Value Function Iteration on a discrete grid to solve the problem of the firm. Use a grid for capital of 200 points (this is the benchmark for the questions below) and another of 25 points only. For both cases plot the value of the firm against its stock of capital, when $a = \bar{a}$.

c) Plot optimal investment ($i(a, k)$) and financing ($-d(a, k)$ when positive) policies against the current stock of capital. Do this for both the highest, average, and lowest value of the shock a.

d) Show the optimal investment policy (at the mean shock only) when:
(1) $b_0 = 0, b_1 = 0.5$
(2) $b_0 = 0, b_1 = 10$
(3) $b_0 = 0, b_1 = b_1^+ = 0.5$ when $i > \delta k$ and $b_1 = b_1^- = 10$ when $i < \delta k$
(4) $b_1 = 0.5$ and $b_0 = 0.02$ unless $i = \delta k$

2. **Solving the Problem of the Firm in Continuous Time**

Consider the following problem of the firm with Poisson uncertainty:

$$rv(a, k) = \max_{\{i\}} d(a, i, k) + v_k(a, k)(i - \delta k) + \sum_{a'} p(a'|a)(v(a', k) - v(a, k))$$

$$s.t. \qquad d(a, i, k) = ak^\alpha - i - \frac{b}{2}\left(\frac{i}{k} - \delta\right)^2 k$$

Here $p(a'|a)$ is probability to jump from a to a'. Assume that $\alpha = 0.3$, $b = 0.5$, $\delta = 0.05$ and $r = 0.05$.

a) **No uncertainty** Solve the problem of the firm assuming $a = 1$ always. Plot the resulting policies, $i(k)$ and $\dot{k}(k) = i(k) - \delta k$. Verify that $\dot{k} = 0$ at the steady state.

b) **Poisson uncertainty** Now assume that a can take three values: $a \in \{0.9, 1.0, 1.1\}$, and the transition matrix is:

$$P = \begin{pmatrix} 0.5 & 0.5 & 0.0 \\ 0.25 & 0.5 & 0.25 \\ 0.0 & 0.5 & 0.5 \end{pmatrix}$$

Again, plot the resulting policies, $i(a, k)$ and $\dot{k}(a, k)$.

3. **Corporate Investment in Equilibrium - Gomes (2001)**

Consider the following dynamic problem for a single firm,

$$\begin{aligned} v(z,k) &= \max_{k'} \left[d(z,k,k') + M \max\{\mathbb{E}_z v(z',k'), 0\} \right], \\ s.t. \quad & d(z,k,k') = \pi(z,k) - i(k,k') \\ & i(k,k') = k' - (1-\delta)k + 0.5\left(\frac{i}{k} - \delta\right)^2 k \end{aligned}$$

where $d(\cdot)$ is the value of **gross** distributions.

Operating profits obey:

$$\pi = z k^{\alpha_k} l^{\alpha_l} - Wl - f$$

The stochastic process for z is given by:

$$\log(z') = (1-\rho)\log \bar{z} + \rho \log(z) + \sigma\epsilon'$$

where the innovations ϵ follow a truncated standard normal distribution with upper and lower bounds equal to plus and minus 4.

Use the following initial parameter values: $M = 0.95$, $\delta = 0.1$, $\alpha_k = 0.3$, $\alpha_l = 0.65$, $W = 2$, $\rho = 0.95$, $\sigma = 0.02$.

Assume also that the aggregate labor supply curve is given by:

$$L^S = BW^{0.1}$$

For what follows suppose fixed costs f are calibrated so that the the fraction of firms that leave the economy in a stationary equilibrium (the exit rate) is about 2.5%. Assume also that new firms enter this economy with productivities initially unknown and drawn from the long run distribution of z.

a) Compute and plot the stationary distribution when the measure of firms is normalized to 1. What is the value of the constant B that is consistent with this competitive equilibrium? Report the entry rate and the cost of entry implied by your choice of f. Verify that the value of the entry cost is positive.

b) Now suppose instead that the constant $B = 1$. What is the mass of firms that is consistent with a competitive equilibrium with $W = 2$?

4. **Distribution of Firms in Continuous Time**

Consider the investment problem of a firm with capital k. This firm has exactly one investment opportunity that allows it to scale the capital to an exogenous value k'. Per period profits, shocks, and one-time investment cost: ϕ, obey :

$$\pi(z, \hat{k}) = z\hat{k}^{\alpha}, \quad \hat{k} \in \{k, k'\}$$
$$dz = \mu z dt + \sigma z dW$$
$$\phi = b_0 + b_1 (k' - k)$$

In what follows use the following parameters values: $r = 0.05$, $\alpha = 0.9$, $\mu = 0.05$, $\sigma = 0.02$. Assume also that $b_0 = 0.1$ and $b_1 = 1.2$.

a) Derive an analytical expression for the firm's value function after the firm has exercised its investment growth option.

b) Derive an analytical expression for the firm's value function before it exercises its investment option.

c) Compute optimal investment boundary and optimal amount of investment.