Finance 937
Leverage and Investment

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October 6, 2019

A Model of Corporate Debt and Investment

Simplified version of several papers

- Hennessy and Whited (2005) and Gomes and Schmid (2010) for the benchmark model

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The key novel assumptions are

- Endogenous investment decisions, $k' = k_{t+1}$, in each period.
- Endogenous choice of one period debt, $b' = b_{t+1}$ and distributions, d_t, in each period.
- The unit cost of external equity raised by the firm is $\lambda \geq 0$
- Endogenous profits

Profits, Investment and Debt Issues

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Debt issued at a discount - a face value issue of, b', raises an amount $b'/R^b(b';\cdot)$ today:

- Alternatively, the price of debt is $q^b(b';\cdot) = 1/R^b(b';\cdot)$

Modeling Corporate Income Taxes

Assume (as before) that a single effective tax rate on corporate income, τ, captures all features of the tax code.

- The corporate income **tax base** is:

$$\gamma(z, k, b; R^b_{-1}(b; \cdot)) = \pi(k, z) - \delta k - (R^b_{-1}(b; \cdot) - 1)\frac{b}{R^b_{-1}(b; \cdot)}.$$

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- Corporate tax payments are then defined as:

$$\tau(z,k,b;R^b_{-1}(b;\cdot)) \equiv \tau_c \gamma(z,k,b;R^b_{-1}(b;\cdot))$$

Modeling Corporate Income Taxes

Hennesy and Whited (2005) assume the tax rate is instead a strictly increasing and convex function of the tax base, $\tau_c(\cdot)$, that satisfies:

$$\lim_{\gamma \to \infty} \tau_c(\cdot) = \bar{\tau}_c > 0$$
$$\lim_{\gamma \to -\infty} \tau_c(\cdot) = 0$$

Although complicated, this has several advantages

- Adds realism to the problem since government is no longer subsidizing losses
- Adds extra concavity to the problem making it easier to compute interior solutions

How do we parameterize the function $\tau_c(\cdot)$?

- Could use data from John Graham's web site about firm's marginal tax rates

Net Distributions

Gross distributions to equity (dividends or repurchases) are:

$$\pi(z,k) - \tau(z,k,b;R^b_{-1}(b;\cdot)) - i(k',k) + \frac{b'}{R^b(b';\cdot)} - b$$

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Net distributions to equity are given by:

$$d(z,k,b,k',b';R^b_{-1}(b;\cdot)) \equiv$$

$$[1+\chi\lambda][\pi(z,k) - \tau(z,k,b;R^b_{-1}(b;\cdot)) - i(k',k) + \frac{b'}{R^b(b';\cdot)} - b]$$

where $\chi(\cdot)$ is an indicator function that takes value 1 when the firm issues equity - i.e. $d(\cdot) < 0$.

- This is similar to what we would obtain if the firm faces a strict non-negative dividend constraint, $d(\cdot) \geq 0$.
- In this case λ would be the endogenous multiplier on this constraint and not some exogenous value.

The Problem for Equity Holders

The Bellman equation for the equity holders in this firm is:

$$e(z, k, b; R^b_{-1}(b)) = \max_{k', b'} \left\{ d(\cdot) + \mathrm{E_z} M \max\{e(z', k', b'; R^b(b')), 0\} \right\}.$$

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- In addition, when returns to scale are close to 1 and default probabilities are low, $R^b(b'; \cdot)$ is close to a constant and the problem becomes almost linear in both k' and b' (perfect substitutes).
- Moreover since $R^b_{-1} = R^b_{-1}(z_{-1}, k, b)$ - we actually need to keep track of z_{-1} in the state-space

Simplifications: Collateral Constraints

Many papers impose a collateral constraint to the choice of debt. A popular form for this is (Rampini and Vishwanathan (2012)):

$$b_{t+1} \leq \eta(1-\delta)k_{t+1}$$

- It also matches some evidence that the value of tangible assets is a primary determinant of leverage

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Several computational benefits:

- The constraint imposes an upper bound on debt. If there are tax or some other benefits to debt, leverage will always be exactly at the constraint.
- Ensures risk free debt so that $R_t^b = R_{t-1}^b = R^f$

Collateral Constraints: Alternative Formulations

Fluctuation in collateral values: Kiyotaki and Moore (1997)

$$b_{t+1} \leq \eta p_{I,t}(1-\delta)k_{t+1}$$

- Fluctuations in the current price of capital, $p_{I,t}$, will force the firm to adjust its debt (and/or investment)
- In macro (general equilibrium) models these fluctuations in $p_{I,t}$ are endogenous and often contribute to magnify the effect of any shocks on firm behavior.

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Future cash flows:

$$b_{t+1} \leq \pi(z_{t+1}, k_{t+1}) - \tau(z_{t+1}, k_{t+1}, b_{t+1}) + \eta(1-\delta)k_{t+1}$$

- This is essentially similar to (motivated by) limited commitment models
- E.g. Hennessy and Whited (2005), Gertler and Kiyotaki (2014)

Simplifications: Linearity

Another useful simplification is to assume the profit function and adjustment cost functions are constant returns to scale.

- Now $\tau(z, k, b; R^b_{-1})$ is linear in k and b and net distributions can be scaled by capital:

$$d/k \quad = \quad [1 + \chi\lambda][z - \tau(z, b/k) - i(k'/k, 1) + \frac{b'/R^b}{k'}\frac{k'}{k} - b/k]$$

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\end{aligned}
$$

where

- $g = k'/k =$ growth rate of capital
- $\omega = b/k =$ leverage

This requires either $R^b = R^b_{-1}$ is constant or homogenous in (b, k)

- We will see how this can be achieved in some models later on
- For now we assume $R^b = R^f = 1/M$ (e.g. collateral constraint)

The Problem for Equity with Linearity

Rewrite the Bellman equation as

$$e(z,k,b)/k = \tilde{e}(z,\omega) = \max_{g,\omega'} \left\{ d/k + g \mathrm{E}_{\mathrm{z}} M \tilde{e}(z',\omega') \right\}$$

- Note that the only endogenous **state** variable is now ω

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Comments

- Since $d/k(g,\omega,z)$ it follows that the optimal issuance decision can be written as $\chi = \chi(g,\omega,z)$ too.
- Linear homogenous problems are fine if we want to study **smooth** aggregate variables.

Optimal Investment with Linearity

The choice of optimal capital accumulation is now static and, when $\lambda = 0$, obeys the FOC

$$i'(g) = \mathrm{E}_{\mathrm{z}} M \tilde{e}(z', \omega') + \frac{\omega'}{R^b}$$

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- As usual without adjustment costs ($i'(\cdot) = 1$) this equation is indeterminate
- Generally this is not an issue because these problems are usually embedded in a GE setting where consumption saving decisions help to determine investment

Optimal Investment with Linearity and Constraints

With equity issuance costs (or $d \geq 0$ constraint) $\lambda \geq 0$ and the optimal FOC for investment becomes:

$$i'(g) = \frac{\mathrm{E_z} M \tilde{e}(z', \omega')}{1 + \chi\lambda} + \frac{\omega'}{R^b}$$

Everything else the same:

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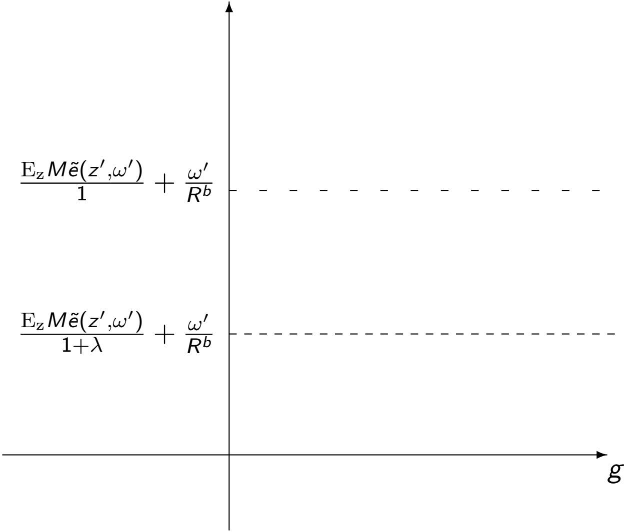
- In the region of positive equity issuance the marginal benefit of investment is below its unconstrained level
- With convex adjustment costs, the optimal investment rate (capital growth) is also lower
- Thus equity issuing firms are *financially constrained*.

Optimal Investment with Costly Equity Issues



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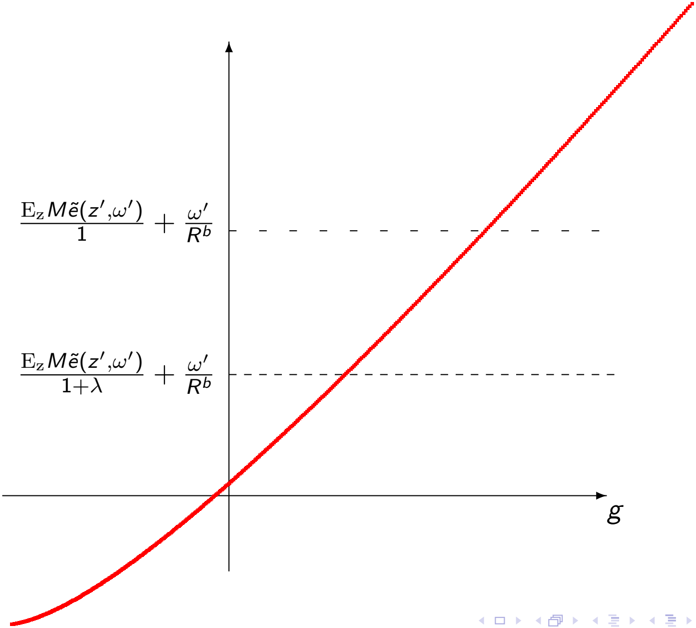

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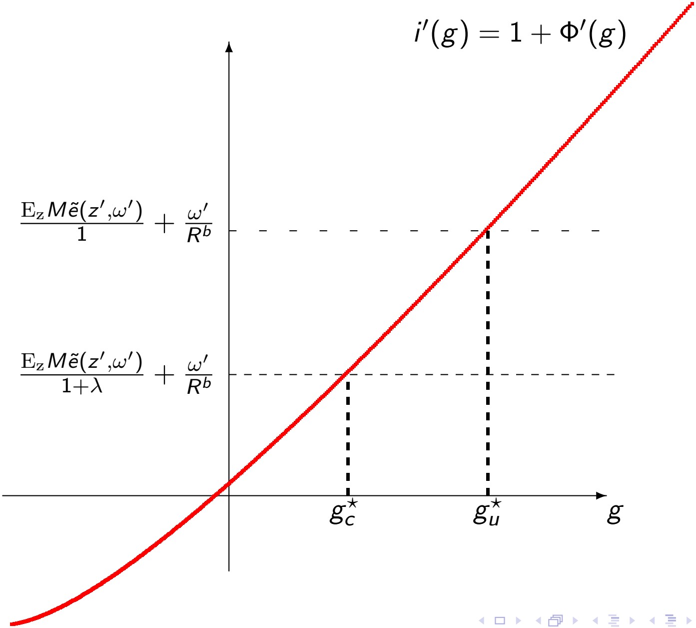

Optimal Leverage with Linearity

With issuance costs/constraints optimal leverage obeys the FOC:

$$(1 + \chi\lambda)/R^b + \mathrm{E_z} M \tilde{e}_\omega(z', \omega') = 0$$

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Together we get the "distorted" Euler equation:

$$1 \;\; = \;\; \mathrm{E}M\left[\frac{1+\chi'\lambda}{1+\chi\lambda}(R^b - \tau_c(R^b - 1))\right] = \frac{1+r^b(1-\tau_c)}{1+r^f}\mathrm{E}\zeta$$

where $\zeta = \frac{1+\chi'\lambda}{1+\chi\lambda}$ and $M = 1/(1+r^f)$

Financial Constraints and Leverage

The wedge ratio

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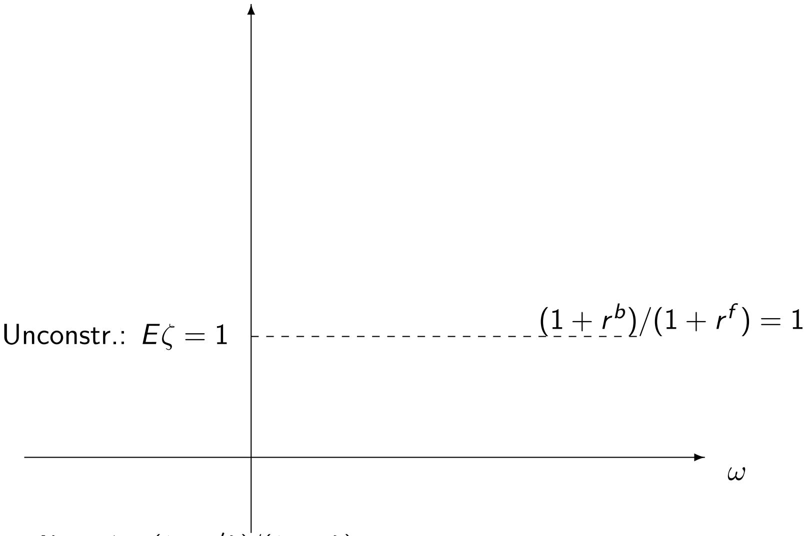
  - In this case we need a collateral constraint to restrict firm leverage
- When a firm issues equity and pays λ today, then $\mathrm{E}\zeta < 1$.
  - This decreases the effective marginal cost of issuing debt today.
- With mean reversion of z it is unlikely that $\mathrm{E}\zeta > 1$

Optimal Leverage With Collateral: No Taxes



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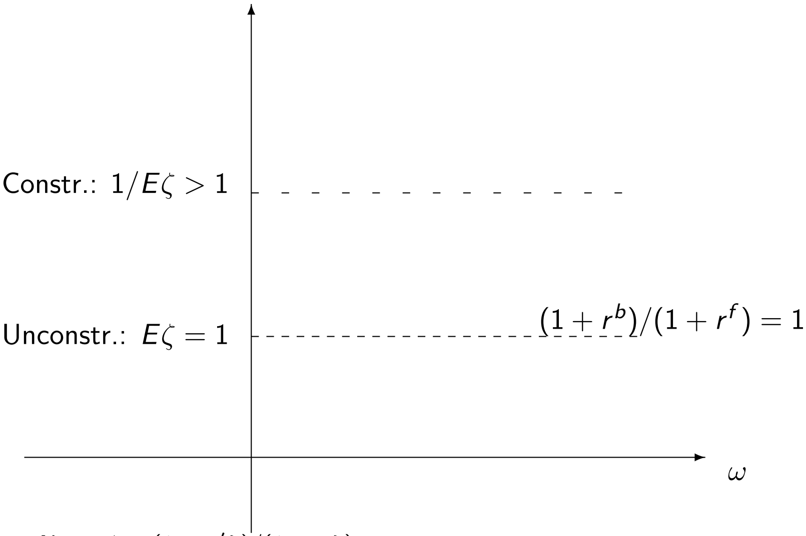

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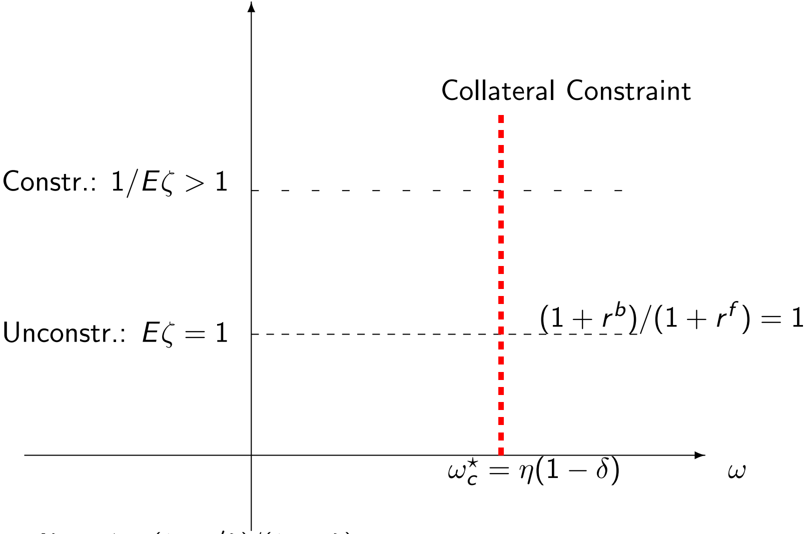

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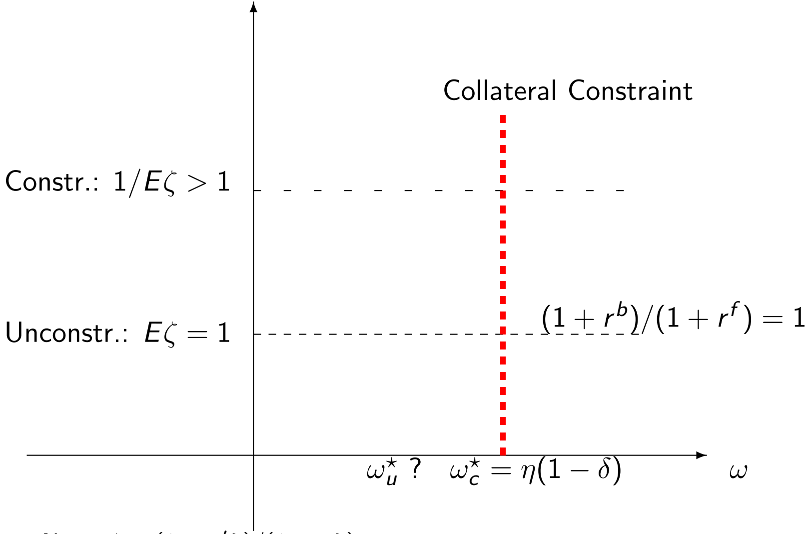

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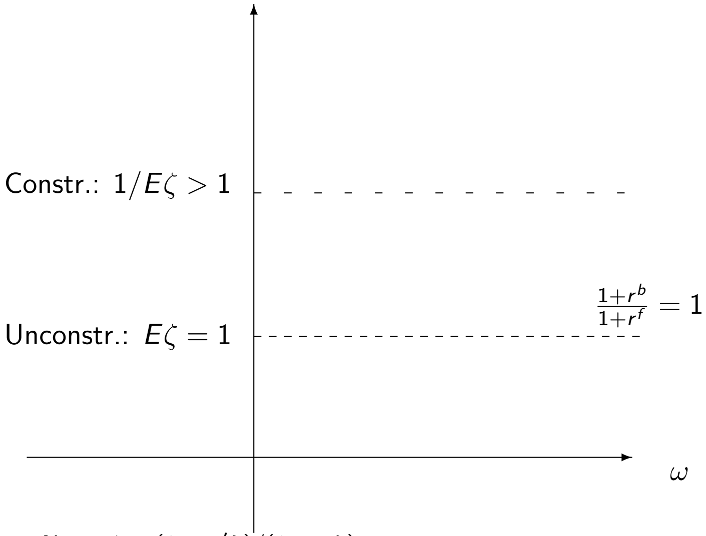

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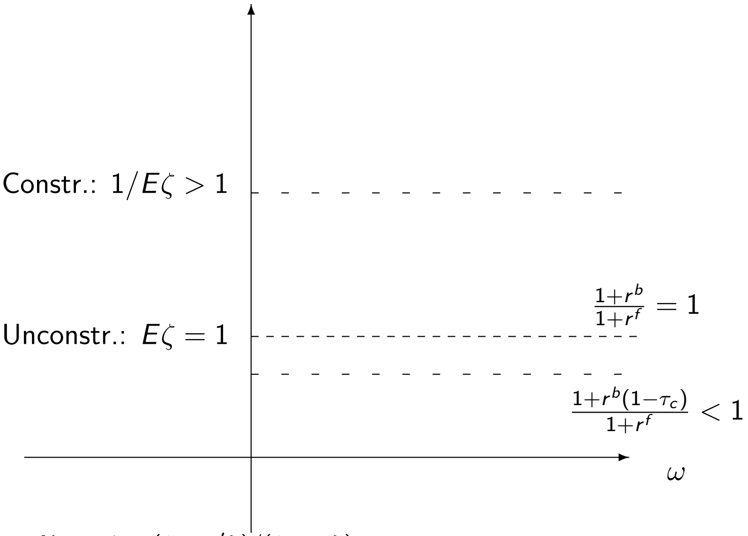

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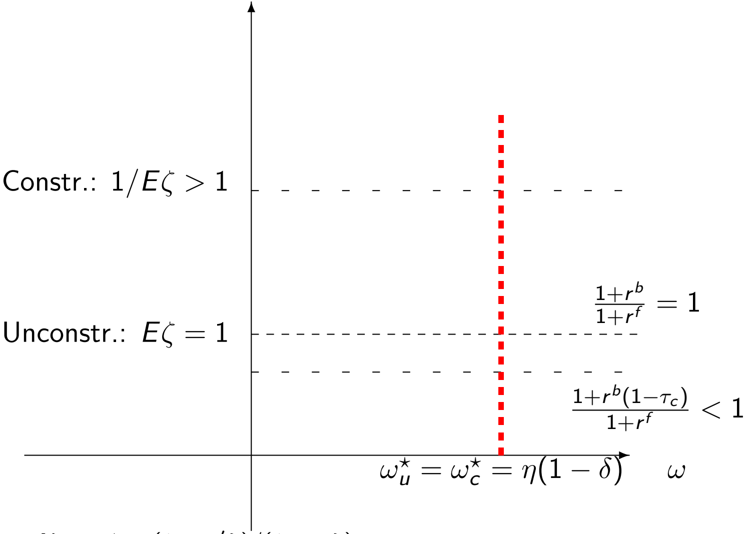

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3. Otherwise solve optimal investment from

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   and compute the implied d.
4. What if $d > 0$ in the last case? Then we must have $d = 0$ and the FOC for investment does not hold.

Solving Models with Collateral Constraints

When there are no taxes, so $\tau_c = 0$:

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4. Otherwise solve optimal investment from

$$i'(g) = \mathrm{E_z} M\tilde{e}(z', \omega)/(1+\lambda) + \omega'/R^b$$

5. Compute the implied d assuming $\omega' = \eta(1-\delta)$.
6. Again, if $d > 0$ we must have $d = 0$ ($\omega' = \eta(1-\delta)$) and the FOC for investment does not hold.

Models with Default

These models are much harder to solve but they offer other interesting insights.

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The easiest way to model this decision is to assume that default takes place because of exogenous **liquidity** reasons.

- A plausible implementation of this idea is is to assume default occurs whenever the firm does not have enough **current** funds to meet its obligations:

$$z - \tau(z, \omega) + (1 - \delta) - \omega < 0 \implies z < \bar{z}(\omega)$$

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Three key implicit assumptions in this formulation:

- The firm cannot use new debt to service old debt
- The firm cannot use new equity issues to pay debt

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The easiest way to model this decision is to assume that default takes place because of exogenous **liquidity** reasons.

- A plausible implementation of this idea is is to assume default occurs whenever the firm does not have enough **current** funds to meet its obligations:

$$z - \tau(z, \omega) + (1 - \delta) - \omega < 0 \implies z < \bar{z}(\omega)$$

Three key implicit assumptions in this formulation:

- The firm cannot use new debt to service old debt
- The firm cannot use new equity issues to pay debt
- Government claims having priority over debt holders

Default and The Cost of Debt

Assume upon default the bondholder recovers a part of the firm's assets (per unit of capital), $\theta(z)$

- The required yield on new debt offered to bond holders, R^b, must obey the arbitrage (zero profit) condition:

$$\mathrm{E} M R^f \omega' = \mathrm{E}\left[M \int_{z' > \bar{z}(\omega')} R^b \omega' + \int_{z' < \bar{z}(\omega')} \theta(z') \right]$$

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- The equilibrium bond yield $R^b(\omega', z; R^f) \geq R^f$ obeys:

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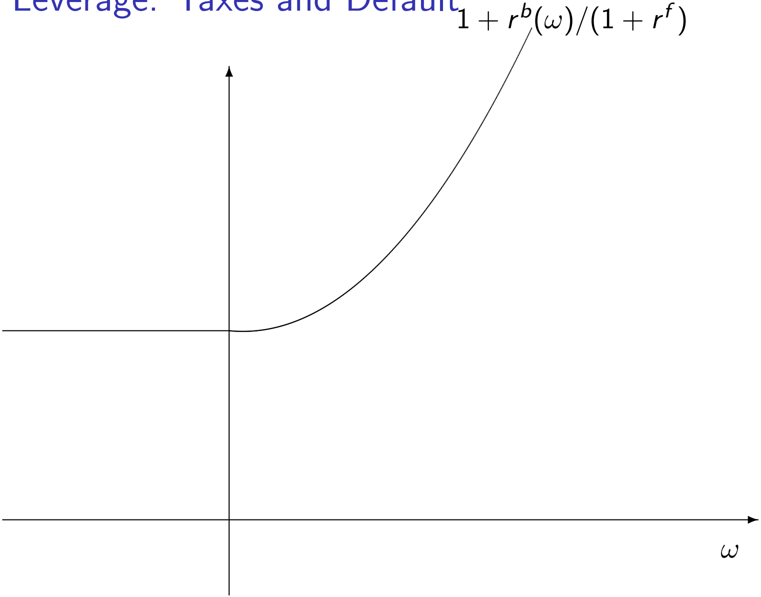
This is

1. an increasing function of the leverage ratio ω'
2. a decreasing function the current productivity shock z

Optimal Leverage: Taxes and Default



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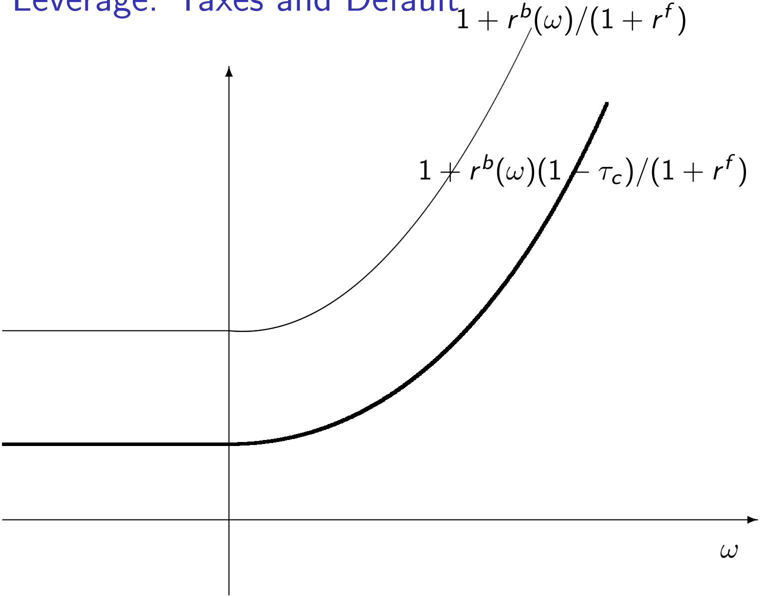

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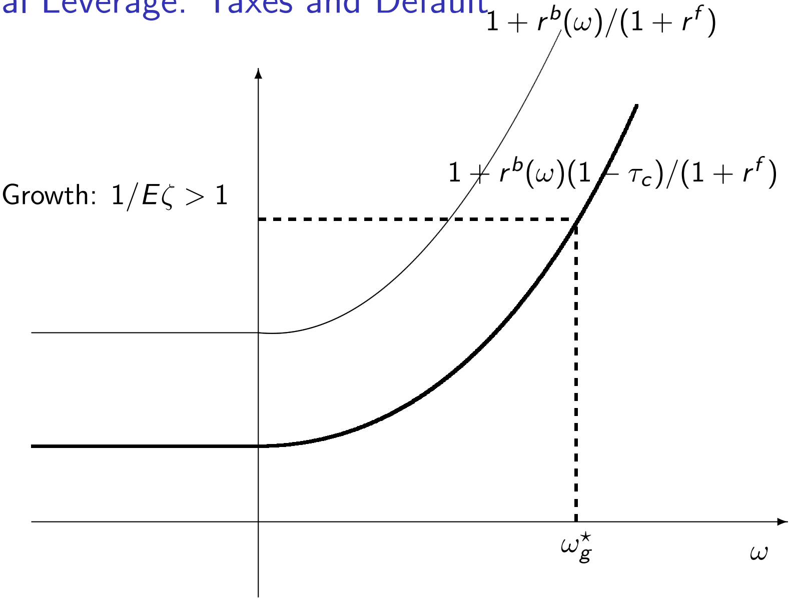

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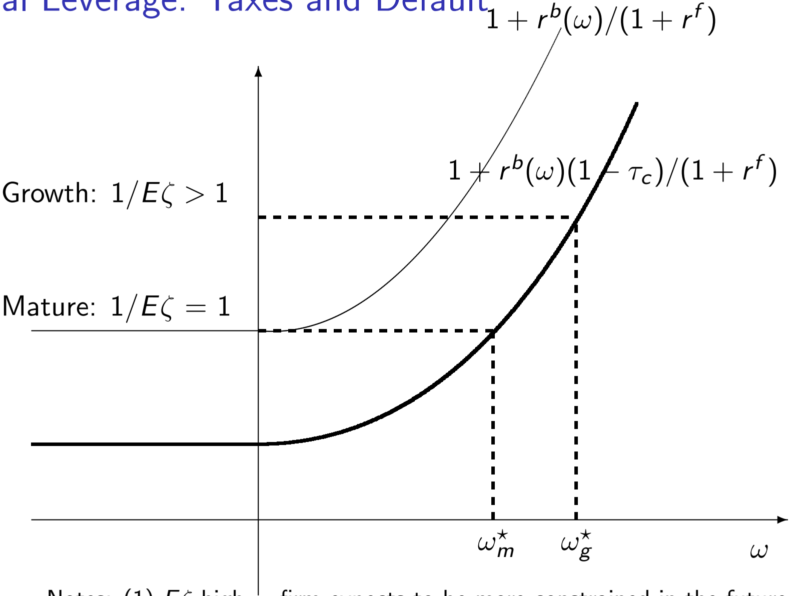
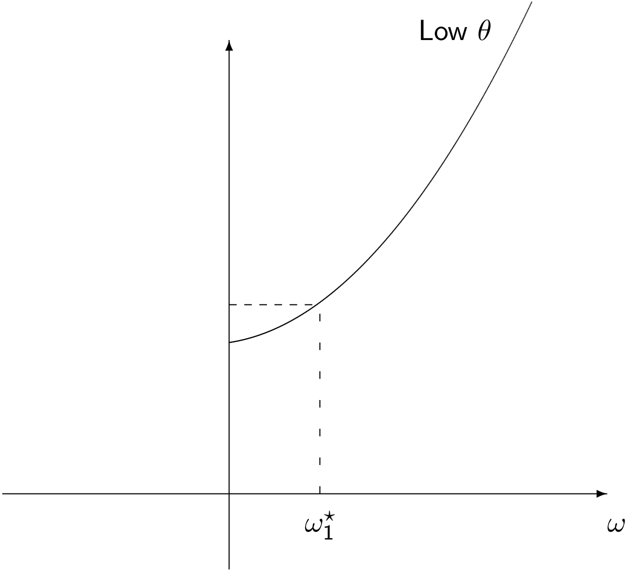


Notes: (1) $E\zeta$ high = firm expects to be more constrained in the future
(2) Now ω' has an interior solution even if $\lambda = 0$

Optimal Leverage: Effect of Higher Recovery θ



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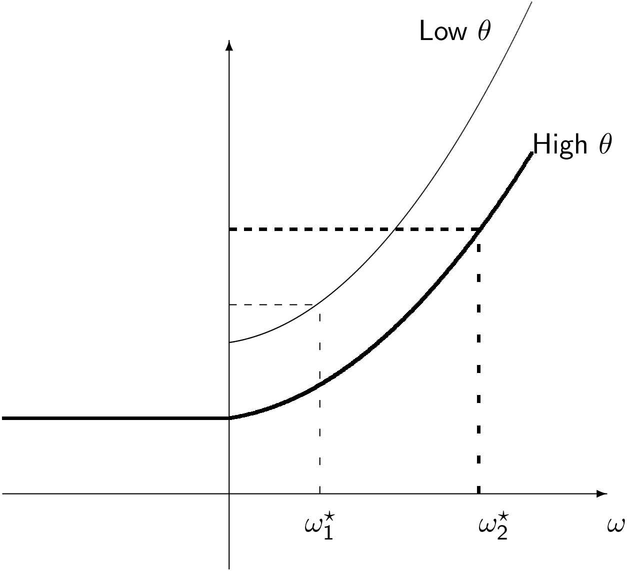

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This is only exactly true when taxes are 0, otherwise the firm almost always prefer to use debt before internal funds.

Empirical Issues: Determinants of Leverage

Collectively then these models should fit the data quite well. Specifically they imply that:

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- Empirically these should be the *high* Q firms

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- Empirically these should be the *high* Q firms
- Leverage decreases with the cost of default: the extent to which assets are tangible or can be collateralized $\theta(z)$
- Empirically there should be a relation between leverage and tangible asses like Plant, Property and Equipment (PP&E)

Determinants of Firm Leverage - Rajan & Zingales 1995

Country Variable	United States	Japan	Germany	France	Italy	United Kingdom	Canada
Panel A: Book Capital							
Tangibility	0.50***	1.41***	0.42**	0.53**	0.36	0.41***	0.26***
	(0.04)	(0.18)	(0.19)	(0.26)	(0.23)	(0.07)	(0.10)
Market-to-book	−0.17***	−0.04	−0.20***	−0.17**	−0.19	−0.13***	−0.11***
	(0.01)	(0.04)	(0.07)	(0.08)	(0.14)	(0.03)	(0.04)
Logsale	0.06***	0.11***	−0.07***	0.02	0.02	0.026***	0.08***
	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.01)	(0.01)
Profitability	−0.41***	−4.26***	0.15	−0.02	−0.16	−0.34	−0.46**
	(0.1)	(0.60)	(0.52)	(0.72)	(0.85)	(0.30)	(0.22)
Number of observations	2079	316	175	117	96	522	264
Pseudo R^2	0.21	0.29	0.12	0.12	0.05	0.18	0.19

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The one big point of debate is the relation between leverage and profits.

- Empirically the relation between debt and profits is robustly negative (Titman and Wessels (1988), Myers (1993), Rajan and Zingales (1995), and Fama and French (2002))
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- We saw that models without investment and costs of issuing equity/constraints may find it hard to match this fact
- Adding large costs to issue equity (or more debt) **and** investment is endogenous encourages firms to use excess cash flows to reduce debt and lower the probability of paying large issuance costs in the future
- This is a classical precautionary savings argument.

The Model with Optimal Default

These models are (by far) the hardest to solve.

- When default is optimally determined by equity holders it will generally be determined by the value of an outside option $e^0(z)k$ (we normalize it to 0 in these notes):

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The big problem is that the bond pricing equation:

$$EMR^f\omega' = EM\left[\int_{z'>\bar{z}^o(\omega')} R^b\omega' + \int_{z'<\bar{z}^o(\omega')} \theta(z')\right]$$

now requires knowledge of the equity value function $\tilde{e}(z',\omega')$.

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3. Use $\tilde{e}(z,\omega)$ to update $\bar{z}^o$ and R^b
4. Stop upon convergence

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More serious theoretical issues:

- Does this dual recursion converge? It is not obviously a contraction when R^b is endogenous
- Is there a unique solution/equilibrium? We can sometimes construct multiple equilibria. (Chatterjee and Eyigungor (2012))

The Model with Decreasing Returns

In this case we have two endogenous state variables

- To again reduce the state space let us define *net worth*:

$$n = \pi(z,k) - \tau(z,k,b;R^b_{-1}(b)) + (1-\delta)k - b$$

This allows us to write net distributions to equity as:

$$d(z,n,k',b') = [1+\chi\lambda]\left[n - k' + \frac{b'}{R^b(b')}\right]$$

The Bellman equation for the equity holders in this firm is:

$$e(z,n) = \max_{k',b'}\left\{d(z,n,k',b') + \mathrm{E_z} M \max\{e(z',n'),0\}\right\}$$

Now we have one endogenous state, n, and two controls (k',b').

- Adjustment costs to k or b make this trick harder to deploy
- Should think carefully about how to model the costs

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- Only requires at least as many moments as underlying structural parameters.

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   - Weighting matrix is the variance-covariance of estimated moments.

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- This way we can focus the challenging estimation procedure on fewer parameters.

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Another serious problem is its reliance on asymptotic theory for most estimation and testing procedures.

- The existence of a long time series for $\{y_t\}$ is unrealistic in many panels where most of the variation is cross-sectional.

Hennessy and Whited (2007) - Matching Moment Conditions

Panel A: Moments		
	Actual Moments	Simulated Moments
Average Equity Issuance/Assets	0.0892	0.0963
Variance of Equity Issuance/Assets	0.0911	0.0847
Variance of Investment/Assets	0.0068	0.0117
Frequency of Equity Issuance	0.1751	0.2305
Payout Ratio	0.2226	0.2026
Frequency of Negative Debt	0.3189	0.3258
Variance of Distributions	0.0013	0.0037
Average Debt-Assets Ratio (Net of Cash)	0.1204	0.1104
Covariance of Investment and Equity Issuance	0.0004	0.0005
Covariance of Investment and Leverage	-0.0018	-0.0025
Serial Correlation of Income/Assets	0.5121	0.5661
Standard Deviation of the Shock to Income/Assets	0.1185	0.1057

Hennessy and Whited (2007) - Parameter Estimates

Panel B: Parameter Estimates

α	λ_0	λ_1	λ_2	ξ	ϕ	σ_ε	ρ	χ^2
0.627	0.598	0.091	0.0004	0.104	0.732	0.118	0.684	8.018
(0.219)	(0.233)	(0.026)	(0.0008)	(0.059)	(0.844)	(0.042)	(0.349)	(0.091)

parameters, with standard errors in parentheses. λ_0, λ_1, and λ_2 are the fixed, linear, and quadratic costs of equity issuance. ϕ governs the shape of the distributions tax schedule, with a lower value for ϕ corresponding to a lower marginal tax rate. ξ is the bankruptcy cost parameter, with total bankruptcy costs equal to ξ times the capital stock. σ_ε is the standard deviation of the innovation to $\ln(z)$, in which z is the shock to the revenue function. ρ is the serial correlation of $\ln(z)$. χ^2 is a chi-squared statistic for the test of the overidentifying restrictions. In parentheses is its p-value.