Finance 937
Corporate, Sovereign and Household Debt

Joao Gomes
The Wharton School

October 1, 2019

International Financing Patterns - Rajan & Zingales 1995

Country	External Financing as a Fraction of Total Financing		Composition of External Financing			
			Net Debt Issuance		Net Equity Issuance	
	Global Vantage	OECD[a] Data	Global Vantage	OECD Data	Global Vantage	OECD Data
United States	0.20	0.23	1.02	1.34	−0.02	−0.34
Japan	0.50	0.56	0.80	0.85	0.20	0.15
Germany		0.33		0.87		0.13
France		0.35		0.39		0.61
Italy		0.33		0.65		0.35
United Kingdom	0.36	0.49	0.55	0.72	0.45	0.28
Canada	0.30	0.42	0.62	0.72	0.38	0.28

International Financing Patterns - Rajan & Zingales 1995

Country	Domestic Bank Credit to the Private Sector as a Fraction of GDP (%)	Stock Market Capitalization ($ billion)	Stock Market Capitalization as a Fraction of GDP (%)	Bond Market Capitalization ($ billion)	Bond Market Capitalization as a Fraction of GDP (%)
United States	70.90	2128.00	49.85	993.20	23.27
Japan	104.22	1794.29	85.31	99.62	4.74
Germany	86.58	257.68	25.79	1.34	0.13
France	80.03	153.42	19.54	44.18	5.63
Italy	33.04	140.24	21.17	4.48	0.68
United Kingdom	53.85	472.90	83.70	14.01	2.48
Canada	44.21	185.20	50.56	27.17	7.42

International Financing Patterns - Rajan & Zingales 1995

	United States	Japan	Germany	France	Italy	United Kingdom	Canada
ASSETS							
Cash and short-term investments	11.2	18.4	8.8	10.3	10.5	11.4	8.2
Account receivable/debtors	17.8	22.5	26.9	28.9	29.0	22.1	13.0
Inventories	16.1	13.9	23.6	17.4	15.6	17.7	11.0
Current assets-other	2.9	3.0	0.1	1.7	1.6	3.7	1.9
Current assets—total	48.0	57.7	59.4	58.3	56.5	54.7	33.2
Fixed assets (tangible)	36.3	28.7	32.7	24.4	32.4	41.3	51.6
Investments and advances—equity	1.4	1.4	1.4	3.4	1.9	1.5	4.8
Investment and advances—other	3.1	9.4	3.4	4.9	4.1	1.2	2.9
Intangible assets	7.6	0.8	2.4	8.5	2.6	0.9	4.7
Assets—other	5.8	2.9	0.7	0.7	3.3	0.5	3.7
Assets—total	100.0	100.0	100.0	100.0	100.0	100.0	100.0
LIABILITIES							
Debt in current liabilities	7.4	16.4	9.9	11.6	16.2	9.6	7.3
Accounts payable/creditors	15.0	15.4	11.5	17.0	14.7	13.7	13.3
Current liabilities—other	11.0	10.4	8.7	17.0	12.2	16.7	2.8
Current liabilities—total	33.4	42.2	30.0	43.4	43.2	40.0	23.1
Deferred taxes	3.2	0.1	0.8	1.3	1.5	0.9	4.4
Long-term debt	23.3	18.9	9.8	15.7	12.1	12.4	28.1
Minority interest	0.6	0.9	1.6	3.9	3.4	1.1	2.0
Reserves—untaxed	0.0	0.0	1.7	0.0	0.0	0.0	0.0
Liabilities—other	5.8	4.8	28.7	6.3	7.8	3.4	2.6
Liabilities—total	66.1	66.8	72.0	68.8	67.4	57.8	60.3
Shareholders equity	34.1	33.2	28.0	31.2	32.6	42.2	39.7
Total liabilities and shareholders equity	100.0	100.0	100.0	100.0	100.0	100.0	100.0

International Leverage and Growth - Arellano et al 2009

	Firm-Level Datasets				
	Mean Asset	Median Asset	Mean Leverage	Mean Growth	No. Firms
Netherlands	263724	24124	0.24	0.03	5077
United Kingdom	71260	849	0.60	0.09	67748
Portugal	17939	787	0.51	0.03	19784
Iceland	2017	85	0.59	0.29	4096
Germany	198267	3205	0.44	0.05	20225
Ireland	128762	3417	0.39	0.06	1807
Spain	5694	365	0.22	0.11	437405
Sweden	12296	323	0.36	0.04	93116
France	7621	220	0.32	0.04	637764
Italy	7740	659	0.14	0.06	414447
Belgium	22789	393	0.46	−0.01	41995
Greece	9484	1535	0.50	0.08	20191
Finland	16201	284	0.40	0.06	26154
Estonia	560	37	0.33	0.47	34187
Croatia	656	115	0.46	0.02	6922
Slovakia	9649	1556	0.39	0.07	4511
Hungary	375	30	0.01	0.45	207207
Czech Republic	3436	221	0.27	0.30	40429
Latvia	3712	588	0.59	0.10	3142
Bosnia	2791	379	0.47	0.05	2660
Poland	23451	3624	0.38	0.02	8044
Serbia	1300	70	0.52	0.11	29385
Bulgaria	1463	91	0.36	0.32	17894
Lithuania	8556	1738	0.49	0.17	2237
Russia	2484	55	0.43	0.48	237639
Ukraine	6618	705	0.28	0.05	15594
Romania	326	14	0.00	0.68	269044

Leverage and the Modigliani Miller Theorem

Models of firms with debt need to deviate from the basic Modigliani-Miller neutrality results. There are several possible reasons for this:

- Differential tax rates for equity and debt cash flows
- The possibility of default on debt
- Related to or motivated by default we can assume collateral constraints or commitment problems
- Costs of issuing or recalling securities

The Early Capital Structure Literature

Earlier models focus on the optimal choice of debt and equity independently of the optimal scale of production.

- In other words they concentrate on the right side of the firm's balance sheet taken the left hand side as given.
- An early exception is Brennan and Schwarz (JF, 1988)

The classic citations about optimal debt are the *ideas* from

- Miller (JF, 1977) about **trade-off**: balancing tax benefits vs default costs
- Myers (JF 1984) about **pecking order**: debt is used only after cheeper inside funds are exhausted

Much empirical work has focused on distinguishing between these two views

- Almost always by looking at the correlation between cash flows and leverage (debt to assets ratio).

Problem of the Firm

Recall the original problem for our baseline firm

$$
\begin{aligned}
& v_0 = \max_{\{k_{t+1}, b_{t+1}\}_{t=0}^{\infty}} \mathrm{E}_0 \left[\sum_{t=0}^{\infty} M_{0,t} d_t \right] \\
\text{s.t.} \quad & d_t + k_{t+1} - k_t = (1 - \tau_c) \left[\pi(z_t, k_t) - (R^b - 1) b_t - \delta k_t \right] \\
& +(b_{t+1} - b_t)
\end{aligned}
$$

Simplifying assumptions (for now):

- The most important is that there is no investment so the stock of capital is a constant, k
- This implies that **exogenous** operating profits (EBIT), $\pi(z_t, k)$

For convenience let also z_t is i.i.d

- There is no link across periods

A Very Simple Debt Model: Miller (1977)

Debt takes the form of a **consol** bond, issued at time $t = 0$

- Pays a **fixed coupon** $c = (R^b - 1)b_t$ every period.
- Thus $b_{t+1} = b_t = c/(R^b - 1)$

The firm defaults on its coupon payment according to an semi-**exogenous** periodic probability $p(c)$, where $p'(c) > 0$

- We will model $p(c)$ below
- After default the firm continues to operate as before.

The firm distributes all its after tax profits to shareholders every period

- By assumption it can't invest or pay its debts.
- There are no costs of issuing equity

We will relax all of these assumptions in a little while.

Institutional Details

Assumptions about default

- Absolute priority rule - senior claimants (debt) are paid first
- Limited liability - the payoff to equity cannot be negative
- Default costs - Debt holders receive $\theta < c$ in a default state

Assumptions about individual taxes

- Shareholders are taxed on distributions at rate τ_d
- Bondholders are taxed on interest income at the rate, τ_i.

Equity and Debt Cash Flows

Each period's expected payoffs for each investor are given by:

- Equity holders:

$$e = \overbrace{(1 - p(c))(1 - \tau_d)(1 - \tau_c)(\mathrm{E}\pi - c)}^{\texttt{not default}} + \overbrace{p(c) \cdot 0}^{\texttt{default}}$$

  where $\mathrm{E}\pi = \int \pi dG(z)$

- Debt holders:

$$b = (1 - \tau_i) \cdot [(1 - p(c))c + p(c)\theta]$$

Optimal Capital Structure - Coupon Choice

Solve the planner's problem:

$$\max_c e$$
$$\text{s.t.} \quad b \geq \bar{b}$$

max the equity holder's value subject to keeping the bond holders happy

Comments:

- If both debt and equity claims are owned by the same agent, they should have an identical weight in social welfare
- The appropriate weight/multiplier on the constraint is 1
- We can simply maximize **total or enterprise** value:

$$\max_c v = e + b$$

What is the value of $\bar{b}$?

- The value of owning a risk free bond

Optimal Capital Structure

Since every period is identical, the firm will pick its coupon, c, to maximize static cash flows:

$$\begin{aligned}\max_c \quad & (1-p(c))\left[(1-\tau_d)(1-\tau_c)(\mathrm{E}\pi - c)\right] + p(c).0 \\ & +(1-\tau_i)\left[(1-p(c))c + p(c)\theta\right]\end{aligned}$$

Combining the tax terms:

$$\begin{aligned}(1-\tau_i) \qquad & \max_c \left\{(1-p(c))\left[\frac{(1-\tau_d)(1-\tau_c)}{(1-\tau_i)}(\mathrm{E}\pi - c) + c\right] + p(c)\theta\right\} \\ & \max_c \left\{(1-p(c))\left[(1-\tau)(\mathrm{E}\pi - c) + c\right] + p(c)\theta\right\}\end{aligned}$$

The Effective Corporate Tax Rate

The "effective" marginal tax rate on corporate distributions combining the various tax rates as:

$$1 - \tau = \frac{(1 - \tau_d)(1 - \tau_c)}{1 - \tau_i}$$

- The highest marginal US the Federal tax rates are approximately $\tau_d = 0.2$ (long term), $\tau_i = 0.35$ and $\tau_c = 0.21$.
- State and local taxes add another 3% to 10% to each rate
- This still ignores many complexities of the tax codes.

Hence effective corporate tax rate in the US is now about

$$\tau \in [0.03, 0.05]$$

Optimal Capital Structure

what tax does

The objective function can be written as:

$$(1-\tau)\mathrm{E}\pi + \boxed{\tau c - p(c)[(1-\tau)(\mathrm{E}\pi - c) + (c-\theta)]}$$

The optimal capital structure **trades off** the tax benefits of debt against the costs of default:

- The first term is the same (after tax) operating profit we have used so far in models without debt
- The second term is the value of the tax benefits of debt
- The final term is the **expected cost** of bankruptcy in a given period
  - The forgone loss of dividends to shareholders, $\mathrm{E}\pi - c$
  - The losses for bondholders, $c - \theta$

The Miller Tax-Tradeoff Formula

The first order condition for optimal debt is:

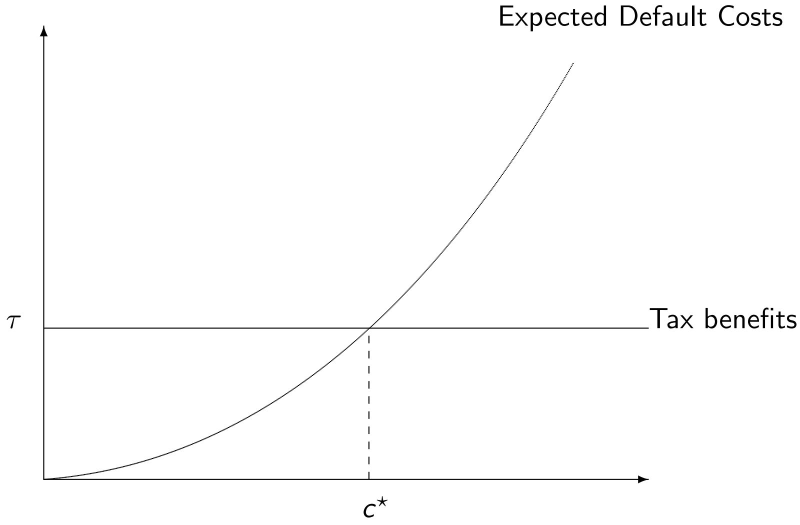
$$\tau(1 - p(c)) - p'(c)\left[(1-\tau)(\mathrm{E}\pi - c) + (c - \theta)\right] = 0$$

This can be rewritten to yield the optimal value for debt, $c^\star$

$$\tau = \frac{p'(c^\star)}{1 - p(c^\star)}\left[(1-\tau)(\mathrm{E}\pi - c^\star) + (c^\star - \theta)\right]$$

- The left hand side is the marginal benefit of extra debt
- The **hazard** ratio, $\frac{p'(c^\star)}{1-p(c^\star)}$, captures the marginal increase in the probability of default
- The term in brackets is the **expected** deadweight loss from default

Optimal Debt: Taxes and Default Trade Off



Endogenous Default

Endogenizing the default decision is essentially about assigning **bargaining** or property rights.

- There are two broad types of models of endogenous default
  1. Equity holders choose when to default **optimally** - equivalent optimal exit in the pure investment models
  2. Debt holders enforces default when **covenants** are violated
- Covenant default is generally not ex-post optimal (formally not **renegotiation-proof**)
- It is usually motivated by enforcement of some ex-ante optimal contract which is rarely modeled explicitly

Optimal Default in the Simple Model

The condition for optimal default is that equity value is driven to 0.

- Assuming $\pi = zk$, the optimal default threshold is given by the cutoff rule:

$$\pi(z^d, k) = z^d k = c \implies z^d = c/k$$

- Higher levels of leverage require higher levels of profits or productivity for the firm to continue to operate

The **endogenous** probability of default is then equal to

$$p(c) = Prob(z < z^d = c/k)$$

As a result $p'(c) > 0$ as we assumed above.

Covenant Default in the Simple Model

Alternatively, we can assume default occurs when the firm violates some arbitrary rule, or covenant:

$$z^c k = v.c, \qquad v \geq 1$$

- For example v can be interpreted as some minimal interest coverage imposed by the debt contract
- It is also the case that z^c is increasing in leverage, c/k

Importantly, the probability of default is:

$$p(c) = Prob(z < z^c = v.c/k) > Prob(z < z^d = c/k)$$

- Usually covenant default is **more frequent** than it is optimal.

Empirical Facts About Leverage

Empirical studies focus on describing corporate leverage

- The ratio of debt to assets (b/k)

Almost all find that corporate leverage is

- increasing in firm size
- decreasing in Tobin's Q
- increasing in the fraction of tangible assets
- declining in cash flows (relative to assets).

In addition more recent work shows that capital structure decisions are

- Lumpy and very persistent (Welch (2004), Leary and Roberts (2005))

Determinants of Firm Leverage - Rajan & Zingales 1995

assets: M/B=(e+b)/k
equity: M/B=e/(k-b)   leverage: b/k
tangibility: how much assets are tangible, easier to borrow against that

	Country Variable	United States	Japan	Germany	France	Italy	United Kingdom	Canada
	Panel A: Book Capital							
	Tangibility	0.50***	1.41***	0.42**	0.53**	0.36	0.41***	0.26***
		(0.04)	(0.18)	(0.19)	(0.26)	(0.23)	(0.07)	(0.10)
q	Market-to-book	−0.17***	−0.04	−0.20***	−0.17**	−0.19	−0.13***	−0.11***
		(0.01)	(0.04)	(0.07)	(0.08)	(0.14)	(0.03)	(0.04)
size	Logsale	0.06***	0.11***	−0.07***	0.02	0.02	0.026***	0.08***
		(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.01)	(0.01)
pi/k	Profitability	−0.41***	−4.26***	0.15	−0.02	−0.16	−0.34	−0.46**
		(0.1)	(0.60)	(0.52)	(0.72)	(0.85)	(0.30)	(0.22)
	Number of observations	2079	316	175	117	96	522	264
	Pseudo R^2	0.21	0.29	0.12	0.12	0.05	0.18	0.19

Infrequent Issues - Leary & Roberts 2004

Adjustment Type	Number of Adjustments	Percent of Periods
No Adjustment	92,159	72.39%
Debt Issue	16,021	12.58%
Debt Retirement	10,920	8.58%
Equity Issue	6,867	5.39%
Equity Repurchase	5,723	4.50%

Persistence in Firm Leverage - Welch 2004

TABLE 3

FAMA-MACBETH REGRESSIONS EXPLAINING FUTURE ACTUAL DEBT RATIOS ADR_{t+k} WITH DEBT RATIOS ADR_t AND STOCK RETURN–MODIFIED DEBT RATIOS $IDR_{t,t+k}$

Horizon k (Fama-MacBeth)	Constant	$IDR_{t,t+k}$	ADR_t	R^2 (%)	Cross-Sectional Regressions
		A. Without Intercept			
1-year		102.1 (1.4)	−.5 (1.4)	96.3	37
3-year		94.6 (2.1)	9.5 (2.1)	90.4	35
5-year		86.7 (2.8)	18.7 (2.1)	86.5	33
10-year		68.3 (4.6)	37.7 (1.8)	80.0	28
		B. With Intercept			
1-year	2.7 (.1)	101.4 (1.3)	−5.3 (1.2)	91.3	37
3-year	6.8 (.3)	94.4 (1.5)	−4.2 (1.4)	78.4	35
5-year	9.3 (.4)	86.9 (2.1)	−.5 (1.6)	70.2	33
10-year	13.8 (.6)	70.8 (3.7)	+6.9 (2.7)	56.0	28

NOTE.—The cross-sectional regression specifications are

$$ADR_{t+k} = [\alpha_0 +]\ \alpha_1 \cdot IDR_{t,t+k} + \alpha_2 \cdot ADR_t + \epsilon_{t+k}.$$

Implications of Leverage Facts

Static "trade off" model correctly predicts leverage increases with:

- Size (k), since $\pi = \pi(z, k)$
- Tangible assets - easier to liquidate in default (higher θ)

A negative correlation between profits and leverage is less obvious

- Suggests profits are used to reduce debt instead of providing an incentive to take on more debt
- This is the motivation for the "pecking order" argument of Myers (1984)
- Debt is a costly source of financing - at least at the margin

Infrequent adjustments suggest issuance or adjustment costs are important for capital structure choices

- And rules out a simple static optimization decision
- We will draw on our knowledge about adjustment cost models to generate these facts

Dynamic Capital Structure Literature

The modern dynamic literature essentially starts with Leland (1994).

- A continuous time model that formalizes the impact of taxes and distress on the choice of debt
- The original model uses uses infinite horizon debt, risk neutral investors, and ignores investment and production.
- A better version appears in Leland and Toft (1996) that also allows for debt issues and varying debt maturity.

These models largely ignore investment

- Combining investment and capital structure choices more or less starts with Cooley and Quadrini (AER 2001) and Hennessy and Whited (JF 2005, JFE 2007).
- Abel (2017) offers a beautiful closed form treatment
- But many key results are easier to understand first without investment.

Dynamic Debt Literatures: Institutional Differences

Models of corporate debt usually assume that

- Upon default all debts are liquidated.
- Bondholders become new owners/equity holders and begin to operate the firm which now starts with no debt.
- E.g. Leland (1994), Leland-Toft (1996).

Models of household debt models use existing legal restrictions to calibrate the length of time until borrowing is allowed again

- E.g. Chaterjee et al (2007)

Sovereign default models usually calibrate to the observed time spent outside international capital markets

- E.g. Arellano (2008)

Dynamic Debt Literatures: Default Costs

In corporate models we impose a **debt restructuring** charge equal to a fraction, $1-\theta$, of the value of a zero debt firm

- In corporate finance models we impose a **debt restructuring** charge equal to a fraction, $1-\theta$, of the value of a zero debt firm
- The zero debt firm is the actual recovered value after the bond holder starts as the new owner
- This is a potential deadweight loss of default

For household/sovereign debt models the loss also entails a reduction in the utility or income of the household/sovereign.

Dynamic Models of Corporate Debt

Classic works

- Leland (1994,1998) and Leland and Toft (1996)
- All assume an exogenous stochastic process for the value of a zero debt (unlevered) firm, or "asset value", v^u.

This is equivalent to assuming the model primitive is $\pi = zk$

- Assume productivity/profits follow a GBM

$$dz = \mu z dt + \sigma z dW, \qquad z(0) = z_0$$

- "Asset value" is just a monotonic transformation

$$v^u(z_0) = \mathrm{E}_0 \int_0^\infty \exp(-rt)(1-\tau)z(t)k = \frac{(1-\tau)z_0 k}{r-\mu}$$

- Remember k is fixed - no investment.
- Implicit assumption: default risk is diversifiable, $r^f = r$

Dynamic Models of Corporate Debt

Debt takes the form of a fixed coupon **consol** bond

- Market value depends on when default occurs, $T^\star$

$$b(z(0)) = \mathrm{E}_0 \int_{t=0}^{T^\star} \exp(-rt)c + \exp(-rT^\star)\theta v^u(T^\star)$$

- Upon default, bondholders become owners of a 0 debt firm
- Re-structuring destroys a fraction $1-\theta$ of firm value

Similarly, the market value to the holders of an equity claim on the levered firm is:

$$e(z(0)) = \mathrm{E}_0 \max_{T^\star} \int_0^{T^\star} \exp(-rt)(1-\tau)(zk-c)$$

- After default equity-holders receive 0

This is just an **optimal stopping** time problem (choice of $T^\star$) - easier to solve in continuous time.

Tractable Models with Finite Maturity

A tractable way to handle finite maturity is to assume that debt matures every period at rate m

- Every period a fraction m of the principal outstanding, P, must be repaid
- Market value of outstanding debt becomes

$$b(z(0)) = \mathrm{E}_0 \int_{t=0}^{T^\star} \exp(-rt)(c + mP) + \exp(-rT^\star)\theta v^u(T^\star)$$

- Similarly, the market value to the holders of an equity claim on the levered firm is:

$$e(z(0)) = \mathrm{E}_0 \max_{T^\star} \int_0^{T^\star} \exp(-rt)[(1-\tau)(zk - c) - mP]$$

Note: This assumes no new debt issues

The Market Value of Debt

HJB equation for debt - before default

$$rb(z) = c + \mu b_z(z)z + \frac{\sigma^2}{2} b_{zz}(z)z^2$$

Solution (replace and check) is:

$$b(z) = \frac{c}{r} + A_1^b z^{\eta_1} + A_2^b z^{\eta_2}$$

- where $\eta_1 > 0 > \eta_2$.

Logically, we have the boundary conditions:

$$\lim_{z\to\infty} b(z_t) \to c/r \implies A_1^b = 0$$
$$b(z_{T^\star}) = b(T^\star) = \theta v^u(T^\star)$$

This is then enough to solve $b(z(t))$ given $z_{T^\star}$.

The Market Value of Equity

Similarly, the market value of equity prior to default obeys the HJB:

$$re(z) = (1 - \tau)(zk - c)\mu e_z(z)z + \frac{\sigma^2}{2} e_{zz}(z)z^2$$

Solution (replace and check) is:

$$e(z) = (1 - \tau)\left[\frac{zk}{r - \mu} - \frac{c}{r}\right] + A_1^e z^{\eta_1} + A_2^e z^{\eta_2}$$

- where again $\eta_1 > 0 > \eta_2$.

Logically, we have the boundary conditions:

$$\lim_{z \to \infty} \left[e(z_t) - (1 - \tau)\left[\frac{zk}{r - \mu} - \frac{c}{r}\right]\right] \to 0 \implies A_1^e = 0$$

$$e(z_{T^\star}) = e(T^\star) = 0$$

This is then enough to solve $e(z(t))$ given $z_{T^\star}$.

Default Decision and Optimal Leverage

Assuming default is chose optimally by the equity holder we get an additional **smooth pasting** condition to pin down $T^{\star}$:

$$e_z(z_{T^{\star}}) = 0$$

Instead we can assume default occurs whenever profits fall below a threshold z_d.

- This is also enough to compute all asset values.

The optimal level of debt, $c^{\star}$, is determined at **time 0** to maximize the total value of the **levered** firm:

$$v(z(t)) = \max_{c \geq 0} \{e(z(t)) + b(z(t))\}$$

Implications for the Pricing of Debt

The **market** value of debt is:

$$b(z) = \frac{c}{r} + \left[\theta v^u(z^\star) - \frac{c}{r}\right] \left(\frac{z}{z^\star}\right)^{\eta_2} < \frac{c}{r} = \bar{b}$$

- Costly default must imply that $\theta v^u(z^\star) < \frac{c}{r}$
- $\bar{b}$ is the market value of risk free consol with the same coupon

Since $\eta_2 < 0$ it follows that the market value of debt is increasing in:

- The **distance of default** $\frac{z}{z^\star}$
- The recovery payoff $\theta v^u(z^\star)$

The Credit Spread

The implied or promised **yield** on (risky) corporate debt is:

$$r^b = \frac{c}{b(c)} > \frac{c}{\bar{b}} = r$$

The default or **credit spread** is:

$$\begin{aligned} r^b - r &= \frac{c/r}{c/r - \left[\theta v^u(z^\star) - \frac{c}{r}\right]\left(\frac{z}{z^\star}\right)^{\eta_2}} \\ &= \frac{1}{1 - \left[\theta v^u(z^\star)/\frac{c}{r} - 1\right]\left(\frac{z}{z^\star}\right)^{\eta_2}} \\ &\approx \left[1 - \frac{\theta v^u(z^\star)}{c/r}\right]\left(\frac{z}{z^\star}\right)^{\eta_2} \end{aligned}$$

Ceteris paribus, this is decreasing in

- The **distance of default** $\frac{z}{z^\star}$
- The **recovery rate** upon default $\theta v^u(z^\star)/(c/r) - 1$

Quantifying the Credit Spread

The implied credit spread in the basic model is:

- Ex-post losses to bondholders upon default are perhaps around 60% for most credit ratings
- Most estimates for broader classes of loans are in the 25%-30% range
- Historical default rates (probabilities) for Baa bonds are about 3% after 5 years but only about 0.3% after one year

Using these values in the expression for the credit spread we get

$$
\begin{aligned}
r^b - r &\approx 0.035 \times 0.6 = 2.1\% \qquad \text{at 5 years} \\
r^b - r &\approx 0.003 \times 0.6 = 0.18\% \qquad \text{at 1 year}
\end{aligned}
$$

Recovery Rates on Bonds - Moody's

Average Sr. Unsecured Bond Recovery Rates by Year Prior to Default, 1982-2008[1]					
	Year 1	Year 2	Year 3	Year 4	Year 5
Aaa	n.a.	3.33%[2]	n.a.	97.00%	85.55%
Aa	43.60%	40.15%	43.45%	57.61%	43.40%
A	42.48%	45.45%	44.50%	38.28%	40.95%
Baa	41.85%	44.56%	44.09%	45.44%	42.68%
Ba	48.00%	42.68%	41.58%	41.15%	41.12%
B	36.98%	35.41%	35.88%	36.91%	40.68%
Caa-C	33.96%	33.25%	33.11%	39.59%	41.94%
Investment-Grade	42.05%	44.23%	44.24%	44.57%	43.37%
Speculative-Grade	36.26%	35.71%	36.30%	38.26%	40.90%
All Rated	36.56%	36.65%	37.50%	39.52%	41.51%

Default Rates on Bonds - Moody's

Average Cumulative Issuer-Weighted Global Default Rates, 1920-2008[1]

Rating	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Aaa	0.000	0.009	0.027	0.082	0.164	0.252	0.359	0.514	0.676	0.863
Aa	0.066	0.201	0.310	0.471	0.728	1.027	1.339	1.643	1.935	2.265
A	0.075	0.250	0.529	0.854	1.193	1.555	1.935	2.309	2.715	3.134
Baa	0.280	0.836	1.539	2.306	3.101	3.883	4.643	5.410	6.215	7.021
Ba	1.332	3.183	5.277	7.448	9.533	11.502	13.309	15.084	16.730	18.436
B	3.885	8.595	13.327	17.578	21.290	24.581	27.621	30.136	32.358	34.292
Caa-C	13.153	21.903	28.591	33.568	37.419	40.435	42.759	44.868	46.975	48.972
Inv-Grade	0.143	0.436	0.814	1.243	1.705	2.179	2.654	3.129	3.625	4.133
Spec-Grade	3.561	7.258	10.828	14.043	16.866	19.371	21.616	23.629	25.462	27.225
All Rated	1.398	2.894	4.349	5.673	6.855	7.915	8.875	9.752	10.584	11.396

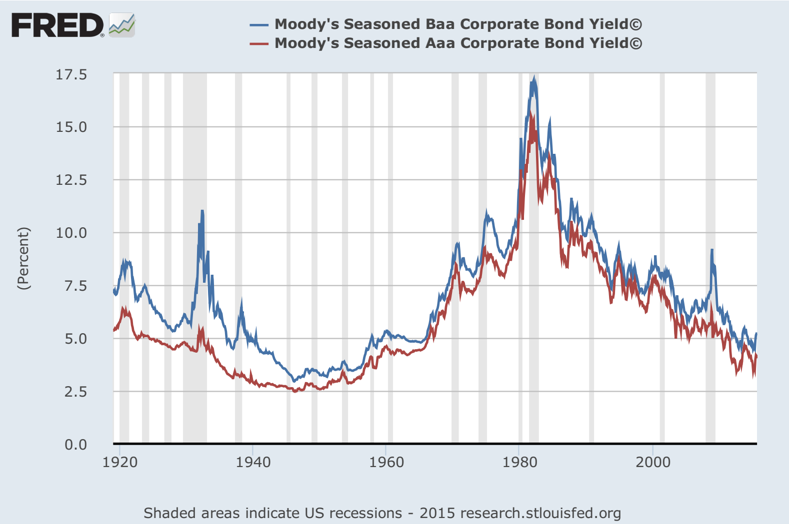
The Credit Spread Puzzle

In the data we observe credit spreads of about 1.0-1.5% at both short and long maturities

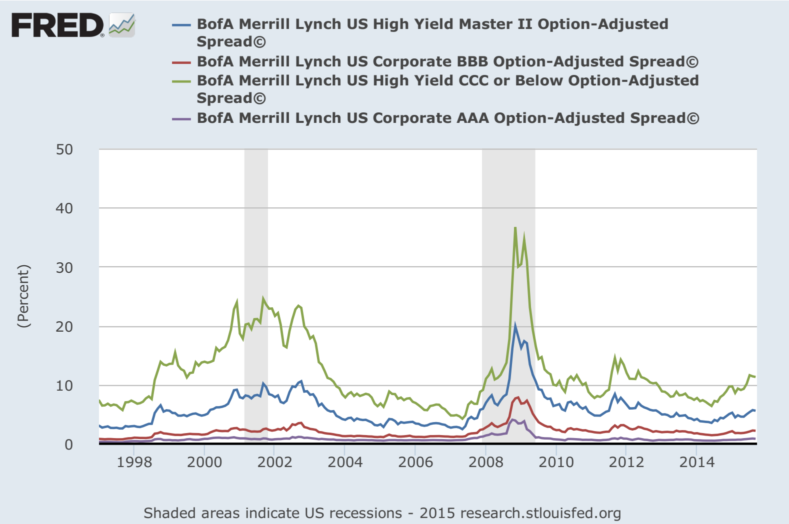
- Measured as the spread between the widely available yields on Baa and Aaa bonds
- The spread between Baa and treasuries is even larger but it is possible that some of that is accounted by liquidity and not default risk
- We have no long term data on even lower rated (junk) bonds which also have much higher yields

The model seems to have trouble generating high credit spreads at fairly short horizons.

Bond Yields - Baa and Aaa Rated Bonds (Moody's)



Bond Yields - Low and High Yield Bonds (BOFA Indexes)



The Under-Leverage Puzzle

Another way to think about the credit spread puzzle is as a leverage puzzle

- The model predicts a very low cost of debt, r^b, or alternatively a very high quantity of debt, $c^\star$
- This is sometimes called *debt conservatism*
- Leverage ratios are about 30% or so for the average firm in Compustat but often predicted to be about 80% in dynamic debt models
- Without investment or other frictions
- Graham (2000) estimates marginal tax rates for individual firms and argues they are high enough to justify higher leverage ratios

This seems to be a public corporation phenomenon

- Leverage for private firms seems to be much higher

Generating Higher Credit Spreads - Estimating Default Costs Glover (2016)

Estimated recovery rates come from a very small sample of firms/countries/households

- What if there exists ex ante heterogeneity in firms' expected default cost, $1 - \theta$?
- Firms internalize their expected default costs.
- Firms with higher costs optimally choose lower leverage and will have a lower default probability.

Implication

- Estimated default costs are downwardly biased because these defaults should be concentrated on ex ante low-default-cost firms.

Generating Higher Credit Spreads - Estimating Default Costs Glover (2016)

The model generates a cross section of firms with productivity process:

$$dz = \mu z dt + \beta_i \sigma z dW + \sigma_i dW_i,$$

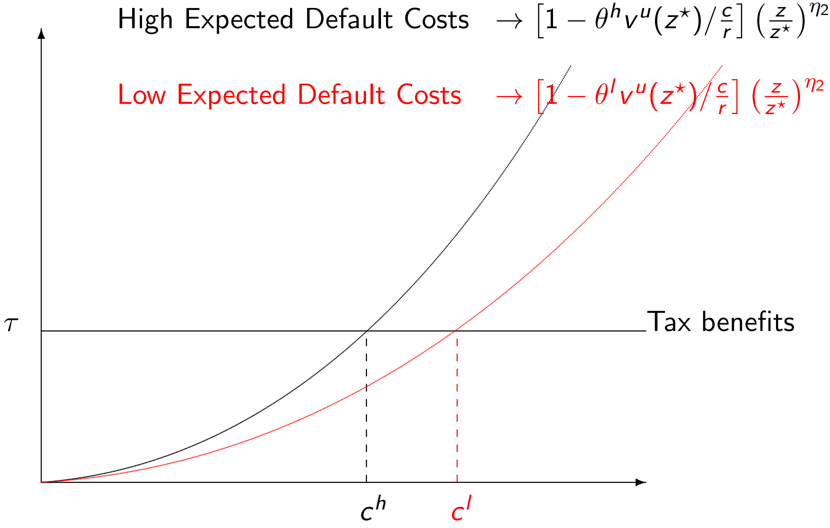
where

- β_i is the sensitivity to the aggregate shock, dW
- dW_i is a firm specific Wiener process

In addition the recoveries in default, θ_i, are now firm specific.

- Estimating these recoveries (and the other idiosyncratic parameters) is the core of the paper.

Variation in Recovery Rates



Estimated Default Costs - Glover (2016)

	Unconditional Estimated $1-\bar{\theta}$	Ex-post Estimated $1-\bar{\theta}$
Mean	0.445	0.246
Std. Dev.	0.270	0.243
Bias		-0.199

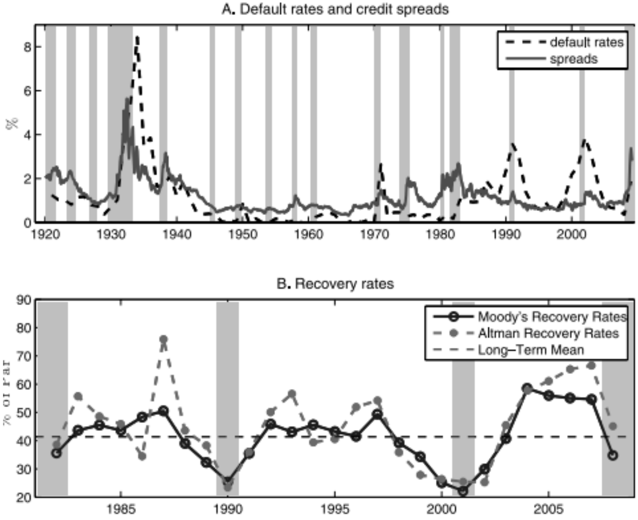
Generating Higher Credit Spreads - Time Varying Risk Premia

Hackbarth et al (2006), Bhamra et al (2010) and Chen (2010) explore the idea that the default risk in corporate bonds

- Cannot be diversified
- Concentrated in periods of high marginal utility - recessions

In addition recovery rates are also lower in recessions.

Time Variation in Risk and Risk Premia



Generating Higher Credit Spreads - Time Varying Risk Premia

A partial equilibrium model

$$dC/C = gdt + \sigma_C dW_C$$

- with the instantaneous correlation between $dW_C dW = \rho dt$.

With CRRA preferences this implies a stochastic discount factor:

$$dM/M = \ln \beta dt + dC/C$$

- Although most papers use Epstein-Zin-Weil preferences to greatly amplify risk premia.
- Bad news about consumption growth correlate with increases in default risk

Generating Higher Credit Spreads - Jumps

Continuous time models with pure diffusion processes imply that default rates and credit spreads will converge to 0 as the time interval dt also goes to 0

- Lando (2004) makes this point in his textbook

Alternative is to allow for jumps in the stochastic process for profits/shocks (Chen (2011)):

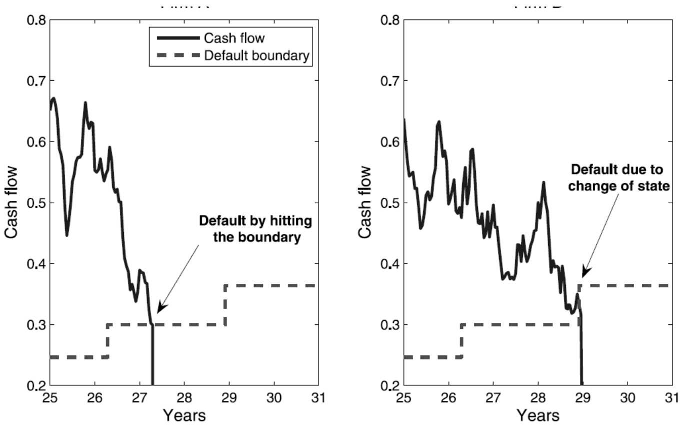
$$dz = \mu(s_t)zdt + \beta_i\sigma(s_t)zdW + \sigma_i dW_i,$$

where s_t is a Poisson growth/volatility state.

- Now there is a default boundaries for each state s_t

Default probabilities are always sizable, even over small intervals.

Generating Higher Credit Spreads - Jumps



Dynamic Models of Household/Sovereign Debt

Some classic references

- Early work includes Eaton and Gersowitz (1983) and Bulow and Rogoff (1989)
- Modern dynamic models are Arellano (2008) and Chaterjee and Eyigungor (2012) for sovereign debt and Chaterjee et al (2007) for models of unsecured household debt

The main agent is a central planner/benevolent government that seeks to maximize the utility of the representative household:

$$V = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

The country is a **small open economy** with access to international capital markets

- Issues defaultable **one period** debt B, at discount price q
- With risk free debt $q = \frac{1}{1+r}$

National income is an endowment, Y, with c.d.f., $F(Y'|Y)$.

Dynamic Models of Sovereign Debt - Arellano (2008)

Budget constraint

$$C = Y + q(Y, B')B' - B$$

- Just like firms earlier, a small country takes international bond prices $q(Y, B')$ as given.

In default the country

- Does not pay (a fraction of) B
- Suffers an output loss of αY
- Is excluded from international capital markets next period with probability θ
- Unlike firms, we need to posit the continuation value for a sovereign nation after default

Dynamic Models of Sovereign Debt - Arellano (2008)

Value functions (discrete time)

- Outside default

$$V^c(Y,B) = \max_{B'} \left\{ u(Y + q(Y,B')B' - B) + \beta \mathrm{E}[V(B',Y')|Y] \right\}$$

- In default

$$V^d(Y) = u((1-\alpha)Y) + \beta \mathrm{E}[\theta V^c(0,Y') + (1-\theta)V^d(Y')|Y]$$

- Aggregate

$$V(Y,B) = \max\{V^c(Y,B), V^d(Y)\}$$

Optimal default requires:

$$V^c(B,Y) \leq V^d(Y) \implies Y \leq Y^d(B)$$

- Note that $Y^d(B)$ is increasing in the level of debt, B

The probability of default is then

$$\int_{Y^d(B)} F(dY'|Y) = F(Y^d(B))$$

Dynamic Models of Sovereign Debt - Arellano (2008)

Market value of debt

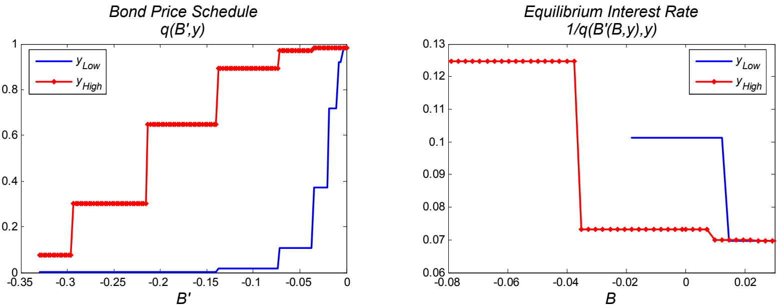
$$q(Y, B) = \frac{1}{1+r}\left[1 - F(Y^d(B))\right]$$

- With i.i.d output, $q(Y, B) = q(B)$ - the market price of debt does not depend on current Y
- It is assume the international lender has access to world risk free rate, i.e., sovereign default is diversifiable

Country credit spread

$$r^b - r = 1/q(Y, B) - 1$$

Bond Prices and Credit Spread



Comments

- Recessions are associated with higher interest rates - countercyclical interest rate schedule
- Defaults are more likely when current debt, B, is higher

Dynamic Models of Household Debt

Unsecured household debt (Chaterjee et al (2007)):

- Budget constraint

$$c = y(s) - \eta(s) + Q(s, b')b' - b$$

  $\eta(s)$ are medical expense (large/jump) shocks

- Default entails reduction in future income, $y(s)$ and being shut down form markets for a period of time

Dynamic Models of Household Debt

Mortgage default (e.g. Campbell and Cocco (2014))

- Preferences for housing services, $u(c, h)$
- Budget constraint for households carrying a mortgage
  $$a' + c = y(s) + (r^b(s,m) + \lambda(s,m)).b + (1 + r^a)a - v.P.h$$
  where
  - v is the user cost of a house
  - $r(s,m)$ and $\lambda(s,m)$ are the interest rate and principal repayment on a mortgage of size b - depends on mortgage type
  - a and r^a are the level of, and interest rate on, household savings.
- Collateral constraint on mortgage origination debt (LTV constraint)
  $$b \leq \theta P.h$$
- Two trigger default: covenant and optimality