Attempts to speed up Value Function Iteration

There is a representative household with preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} 0.96^t \left(c_{1,t}^{0.5} c_{2,t}^{0.5} - \frac{l_t^2}{2} \right)$$

The household consumes, saves, and works with a budget constraint (notation is self-explanatory):

$$c_{1,t} + p_{2,t} c_{2,t} + i_t = w_t l_t + r_t k_t$$

There is a production function:

$$c_{1,t} + i_t = e^{z_t} k_t^{0.33} l_{1,t}^{0.67}$$

with a law of motion for capital:

$$k_{t+1} = 0.9 k_t + i_t$$

and a technology level z_t that follows a Markov Chain that takes values in:

$$z_t \in \{-0.0673, -0.0336, 0, 0.0336, 0.0673\}$$

with transition matrix:

$$\begin{pmatrix} 0.9727 & 0.0273 & 0 & 0 & 0 \\ 0.0041 & 0.9806 & 0.0153 & 0 & 0 \\ 0 & 0.0082 & 0.9836 & 0.0082 & 0 \\ 0 & 0 & 0.0153 & 0.9806 & 0.0041 \\ 0 & 0 & 0 & 0.0273 & 0.9727 \end{pmatrix}$$

Also, there is a production function:

$$c_{2,t} = A_t l_{2,t}$$

where where A_t follows a a Markov Chain that takes values in:

$$A_t \in \{0.9, 1, 1.1\}$$

with transition matrix:

$$\begin{pmatrix} 0.9 & 0.1 & 0 \\ 0.05 & 0.9 & 0.05 \\ 0 & 0.1 & 0.9 \end{pmatrix}$$

Note the aggregate labor market constraint

$$l_t = l_{1,t} + l_{2,t}$$

STEP **1. Social Planner**

Find the associated social planner's problem to this model and write it recursively.

STEP **2. Steady State**

Compute the deterministic steady state of the model when $z_{ss} = 0$ and $A_{ss} = 1$.

STEP **3. Value Function Iteration with a Fixed Grid**

Fix a grid of 250 points of capital, centered around k_{ss} with a coverage of $\pm 30\%$ of k_{ss} and equally spaced. Iterate on the Value function implied by the Social Planner's Problem using linear interpolation until the change in the sup norm between to iterations is less than 10^{-6}. Compute the Policy function. Describe the responses of the economy to a technology shock.

STEP **4. Value Function Iteration with an Endogenous Grid**

Repeat previous exercise with an endogenous grid.

STEP **5. Comparison of Grids**

Compare the solutions in 2) and 3) in terms of: 1) accuracy, 2) computing time, and 3) complexity of implementation. Present evidence to support your claims.

STEP **6. Accelerator**

Recompute your solution to 2) using an accelerator, i.e., skipping the max operator in the Bellman equation 9 out of each 10 times. Compare accuracy and computing time between the simple grid scheme implemented in 2) and the results from the accelerator scheme. Present evidence to support your claims.

STEP **7. Multigrid**

Implement a multigrid scheme (Chow and Tsitsiklis, 1991) for a Value function iteration, with the grid centered around k_{ss} with a coverage of $\pm 30\%$ of k_{ss} and equally spaced.

You will have 100 capital grid points in the first grid, 500 capital grid points in the second, and 5000 capital grid points in the third.

Compare accuracy and computing time between the simple grid scheme implemented in 2) and the results from the multigrid scheme. Present evidence to support your claims.

STEP **8. Stochastic Grid**

Implement a stochastic grid scheme (Rust, 1997) for a Value function iteration, with 500 vertex points with a coverage of $\pm 25\%$ of k_{ss} (you can keep the grid of investment fixed). Compare accuracy and computing time between the simple grid scheme implemented in 2) and the results from the multigrid scheme. Present evidence to support your claims.