1. Social Planner's problem.

$$\max_{c_1, c_2, l_1, l_2, k'} \left\{ \left(c_1^{0.5} c_2^{0.3} - \frac{l^2}{2} \right)^2 + 0.96 \times V(z', A', k') \right\} = V(z, A, k)$$

s.t. $c_1 + k' = e^z k^{0.33} l_1^{0.67} + 0.9k$

$c_2 = A l_2$

$l = l_1 + l_2$

2. Steady state. - Solved by FOC of the Bellman Equation

In the deterministic state where $z=0$ and $A=1$,

$$v(k) = \max_{l_1, l_2, k'} \left\{ (k^{0.33} l_1^{0.67} + 0.9k - k')^{0.5} l_2^{0.5} - \frac{(l_1+l_2)^2}{2} + 0.96 v(k') \right\}$$

F.O.C $[l_1]$ $l_1 + l_2 = 0.5 \times 0.67 \times l_2^{0.3} \times k^{0.33} \times l_1^{-0.33} \times (k^{0.33} l_1^{0.67} + 0.9k - k')^{-0.5}$

$[l_2]$ $l_1 + l_2 = 0.5 \times l_2^{-0.5} \times (k^{0.33} l_1^{0.67} + 0.9k - k')^{0.5}$

$[k']$ $1 = 0.96 \times (0.9 + l_1^{0.67} \cdot 0.33 k^{-0.67})$

ET

$k = k' = k''$

By Fsolve of Matlab, we get

$k_{ss} = 0.83$

$l_1 ss = 0.23$

$l_2 ss = 0.27$

$C_1 ss = 0.27$

$C_2 ss = 0.27$

$V ss = 0.14$