Computational Economics HW #1

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Question 1 (Social Planner)

$$
\begin{aligned}
V(k,z,A) = \max_{c_1,c_2,l_1,l_2,l,k'} & \; c_1^{0.5}c_2^{0.5} - \frac{l^2}{2} + 0.96\sum_{z'}\sum_{A'} P(z'|z)P(A'|A)V(k',z',A') \\
\text{s.t. } c_1 + k' - 0.9k &= e^z k^{0.33} l_1^{0.67} \\
c_2 &= Al_2 \\
l &= l_1 + l_2
\end{aligned}
$$

This is equivalent to

$$
V(k,z,A) = \max_{l_1,l_2,k'} [e^z k^{0.33} l_1^{0.67} - k' + 0.9k]^{0.5}(Al_2)^{0.5} - \frac{(l_1+l_2)^2}{2} + 0.96\sum_{z'}\sum_{A'} P(z'|z)P(A'|A)V(k',z',A')
$$

Question 2 (Steady State)

After deriving the Euler equation and FOCs for l_1, l_2 and imposing $(z=0, A=1)$, I have this equation:

$$
\begin{aligned}
1 &= 0.96[0.33k^{-0.67}l_1^{0.67} + 0.9] && \text{(EE in SS)} \\
l_1 + l_2 &= 0.5\left[k^{0.33}l_1^{0.67} - k + 0.9k\right]^{-0.5} 0.67k^{0.33}l_1^{-0.33}l_2^{0.5} && \text{(FOC for } l_1 \text{ in SS)} \\
l_1 + l_2 &= 0.5\left[k^{0.33}l_1^{0.67} - k + 0.9k\right]^{0.5} l_2^{-0.5} && \text{(FOC for } l_2 \text{ in SS)}
\end{aligned}
$$

I solved this system of equation by fsolve. The result is:
$k^* = 0.8302,\ l_1^* = 0.2350,\ l_2^* = 0.2690.$

Question 3 (Value Function Iteration)

We have two FOCs for labor supply

$$
\begin{aligned}
l_1 + l_2 &= 0.5\left[e^z k^{0.33}l_1^{0.67} - k' + 0.9k\right]^{-0.5} 0.67e^z k^{0.33}l_1^{-0.33}(Al_2)^{0.5} && \text{(FOC for } l_1\text{)} \\
l_1 + l_2 &= 0.5\left[e^z k^{0.33}l_1^{0.67} - k' + 0.9k\right]^{0.5} (Al_2)^{-0.5}A && \text{(FOC for } l_2\text{)}
\end{aligned}
$$

Given the state (A, z) and the capital choice (k'), l_1 and l_2 will be determined from the two equations. The maximization problem is as follows:

$$
\begin{aligned}
V^{j+1}(k,z,A) = \max_{k'} & \left[e^z k^{0.33} l_1^*(k')^{0.67} - k' + 0.9k\right]^{0.5}\left[Al_2^*(k')\right]^{0.5} \\
& -\frac{1}{2}\left[l_1^*(k') + l_2^*(k')\right]^2 + 0.96\sum_{z'}\sum_{A'} P(z',A'|z,A)V^j(k',z',A')
\end{aligned}
$$

[*] I discussed with Leon, Min.

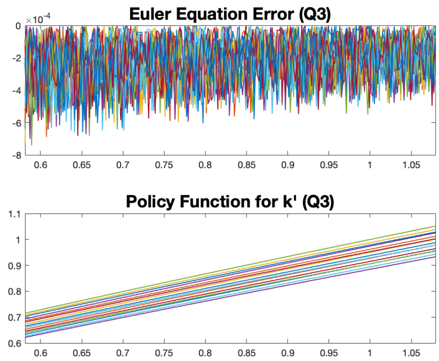

Figure 1: Euler Eq. Error and Policy Function

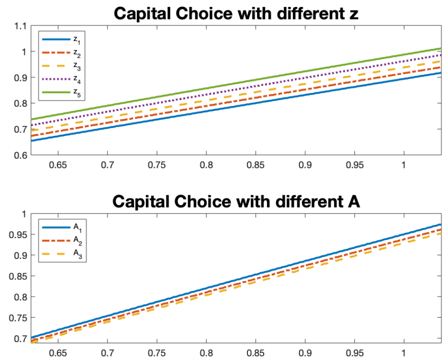

Figure 2: Policy for different shocks

I solved this maximization by fminbnd in Matlab. I set the tolerance level 10^{-4} in this entire problem set because the VFI with tolerance 10^{-6} took too much time to compute. After the convergence with 10^{-4} (it takes 4073.7 seconds for Question 3), I had Euler equation error with -0.00074047. From Figure 2, we can see that capital choice is higher with larger z and lower with higher A. This is because with high$z$ economy has higher output and accumulate capital more. With high A, people enjoy c_2 more and put less resource to produce c_1, inducing less capital accumulation.

Question 4 (Endogenous Grid)

(i) Set initial values of $\tilde{V}(k', z, A) = e^z k'^{0.33}$, exogenous grid $Y_{exo}(k, z, A) = e^z k^{0.33} + 0.9k$
(ii) Compute $c^*(k', z, A) = \left[\frac{\tilde{V}_{k'}(k', z, A)}{0.5(Al_2)^{0.5}}\right]^{-\frac{1}{0.5}}$
(iii) Set endogenous grid of $Y^*(k', z, A) = c^*(k', z, A) + k'$
(iv) Compute today's value $V(Y^*(k', z, A), z, A) = c^*(k', z, A)^{0.5}(Al_2)^{0.5} - \frac{1}{2}(l_1 + l_2)^{0.5} + \tilde{V}(k', z, A)$
(v) Linearly interpolate from Y_{end} to Y_{exo}. Obtain $V(Y(k, z, A), z, A)$.
(vi) Update $\tilde{V}(k', z, A) = \beta \sum_{z', A'} P(z', A'|z, A)V(Y', z', A')$

After the iteration (I solved this iteration for steady state value l_1), I obtained $V(k, z, A)$. I used this as an initial value for usual VFI with endogenous l_1.

Question 5 (Comparison)

- Accuracy
  Euler Equation error in usual VFI: -0.00074047 (largest error for all grid points)
  EEE in endogenous grid: -0.0007411
  The accuracy is almost the same (given that I use the same VFI for endogenous l_1)

- Computing Time
  Usual VFI: 1879 seconds for exogenous l_1. 2194 seconds for endogenous l_1.
  Endogenous Grid: 0.2 seconds for exogenous l_1. 4657 seconds for endogenous l_1.
  The endogenous grid method is very fast, but considering that the second step (endogenous l_1) takes a lot of time, the accuracy in computing the value of each state is not so good.

- Implementation
  It took time to understand the endogenous grid method, but once I understand the mechanism, coding is relatively straightforward. Because I don't have to solve the maximization problem in the endogenous grid method, computation is very fast.

Question 6 (Accelerator)

EEE in usual VFI is -0.00074047, and EEE with accelerator is -0.00075483, so both are very close. Computing time in usual VFI is 4073 seconds, and 1583 seconds with the accelerator. So with the same accuracy, the accelerator is three times faster than the usual VFI.

Question 7 (Multigrid)

EEE in usual VFI is -0.00074047, and EEE with multigrid is -0.0010189, so I couldn't obtain the better accuracy with multigrid. Computing time in usual VFI is 4073 seconds, and 3058 seconds with the multigrid. So multigrid was faster is computing time but couldn't compute in better accuracy.