Homework II, Computational Economics

Be sure to include all the material that shows your work and to present your results in a way that it is easy to understand. Remember: ignorance and inability to communicate are observationally equivalent.

All the exercises will be based on the following model. There is a representative household with preferences:

$$U_t = \left[\left(\log c_t - \eta \frac{l_t^2}{2} \right)^{0.5} + 0.99 \left(\mathbb{E}_t U_{t+1}^{-9} \right)^{-\frac{0.5}{9}} \right]^{\frac{1}{0.5}}$$

The household consumes, saves, and works, with a budget constraint (notation is self-explanatory):

$$c_t + i_t = w_t l_t + r_t k_t$$

There is a production function:

$$c_t + i_t = e^{z_t} k_t^{\alpha_t} l_t^{1-\alpha_t}$$

with a law of motion for capital:

$$k_{t+1} = 0.9 k_t + i_t$$

where $i_t \geq 0$ and a technology level z_t that follows a Markov Chain that takes values in:

$$z_t \in \{-0.0673, -0.0336, 0, 0.0336, 0.0673\}$$

with transition matrix:

$$\begin{pmatrix} 0.9727 & 0.0273 & 0 & 0 & 0 \\ 0.0041 & 0.9806 & 0.0153 & 0 & 0 \\ 0 & 0.0082 & 0.9836 & 0.0082 & 0 \\ 0 & 0 & 0.0153 & 0.9806 & 0.0041 \\ 0 & 0 & 0 & 0.0273 & 0.9727 \end{pmatrix}$$

and

$$\alpha_t \in \{0.25, 0.3, 0.35\}$$

with transition matrix:

$$\begin{pmatrix} 0.9 & 0.07 & 0.03 \\ 0.05 & 0.9 & 0.05 \\ 0.03 & 0.07 & 0.9 \end{pmatrix}$$

You can think about α_t as the reduced form of some technological change that modifies the elasticity of output with respect to capital.

1. Social Planner (5 points)

Find the associated social planner's problem to this model and write it recursively.

2. Steady State (5 points)

Compute the deterministic steady state of the model when $z_{ss} = 0$ and $\alpha_{ss} = 0.3$.

3. Value Function Iteration with a Fixed Grid (10 points)

Fix a grid of 250 points of capital, centered around k_{ss} with a coverage of $\pm 30\%$ of k_{ss} and equally spaced. Iterate on the Value function implied by the Social Planner's Problem using linear interpolation until the change in the sup norm between to iterations is less than 10^{-6}. Compute the Policy function. Describe the responses of the economy to a technology shock and to a shock to α_t.

4. Value Function Iteration with an Endogenous Grid (10 points)

Repeat previous exercise with an endogenous grid.

Compare the solutions in 2) and 3) in terms of: 1) accuracy, 2) computing time, and 3) complexity of implementation. Present evidence to support your claims.

5. Accelerator (5 points)

Recompute your solution to 2) using an accelerator, i.e., skipping the max operator in the Bellman equation 9 out of each 10 times. Compare accuracy and computing time between the simple grid scheme implemented in 2) and the results from the accelerator scheme. Present evidence to support your claims.

6. Multigrid (5 points)

Implement a multigrid scheme (Chow and Tsitsiklis, 1991) for a Value function iteration, with the grid centered around k_{ss} with a coverage of $\pm 30\%$ of k_{ss} and equally spaced (you can keep the grid of investment fixed).

You will have 100 capital grid points in the first grid, 500 capital grid points in the second, and 5000 capital grid points in the third.

Compare accuracy and computing time between the simple grid scheme implemented in 2) and the results from the multigrid scheme. Present evidence to support your claims.

7. Chebyshev (10 points)

Compute the solution to the previous model when you use 8 Chebyshev polynomials on capital. Compare the solution with the one from previous questions.

8. Projection (10 points)

Compute the solution to the previous model using a third-order approximation and plot the IRFs initilized at the mean of the ergodic distribution. You need to substitute the process for productivity by:

$$z_t = 0.95 z_{t-1} + 0.005 \varepsilon_t^1$$

where ε_t^1 is a normalized gaussian innovation and the process for α by:

$$\alpha_t = 0.03 + 0.9 \alpha_t + 0.01 \varepsilon_t^2$$

where ε_t^2 is a normalized gaussian innovation.