I. Equilibrium set-up

The environment is an equilibrium economy, with a representative consumer and a unit continuum of firms. Each of these infinitely-lived firms uses capital and labor in a stochastic, decreasing returns technology to generate value-added according to

$$z^{\nu}\left(K^{\alpha}L^{1-\alpha}\right)^{\theta}, \tag{1}$$

where K is the stock of capital, L is labor, z is a profitability shock, and where we normalize the parameter ν to be $1-(1-\alpha)\theta$. The profitability shock, z, is lognormally distributed and follows a process given by:

$$\ln(z') = \rho\ln(z) + \sigma_z\varepsilon', \qquad \varepsilon' \sim \mathcal{N}(0,1), \tag{2}$$

where a prime indicates the subsequent period, and no prime indicates the current period.

Investment in capital, I, is defined by a standard capital stock accounting identity:

$$K' \;\equiv\; (1-\delta)K + I, \tag{3}$$

in which δ is the rate of capital depreciation. The price of the capital good has been normalized to one. Adjusting the capital stock incurs quadratic costs that take the form:

$$\psi(K,K') \;=\; \frac{\psi(K'-(1-\delta)K)^2}{2K} \tag{4}$$

where ψ is a parameter that governs the magnitude of adjustment costs.

Let P denote the outstanding stock of debt, r denote the risk-free rate of interest, and τ denote the process for the corporate tax rate. We assume that the tax rate can take one of two values, τ^h and τ^l, with $\tau^h > \tau^l$, and that it follows a two-state Markov process

The firm's cash flow, $E^*(K,P,K',P',z,\tau)$, is then its after-tax operating income plus net debt issuance, minus net expenditure on investment, and minus tax-deductible interest payments on debt, as follows:

$$\begin{aligned} E^*(K,P,K',P',z,\tau) \;&=\; (1-\tau)\left(z^{\nu}\left(K^{\alpha}L^{1-\alpha}\right)^{\theta} - wL\right) \\ &-\; (K'-(1-\delta)K) - \psi(K,K') + P' - P(1+r(1-\tau)), \end{aligned} \tag{5}$$

where w is the wage rate. Motivated by the dynamic contracting literature, we assume this debt is secured by capital, that is, we allow a fraction, ξ, of the stock of capital to be used as collateral. The collateral constraint can thus be expressed as:

$$P' \le \xi(1-\delta)K'. \tag{6}$$

Cash flows to shareholders, $E(K,P,K',P',z,\tau)$, are defined in terms of the firm's cash flows, $E^*(K,P,K',P',z,\tau)$. A positive firm cash flow is distributed to its stockholders, while a negative cash flow implies that the firm instead obtains funds from shareholders. In this case, the firm pays a linear cost, λ. Thus, shareholder cash flows are given by:

$$\begin{aligned} E^* \geq 0 \quad &\Rightarrow \quad E = E^* \\ E^* < 0 \quad &\Rightarrow \quad E = E^*(1+\lambda). \end{aligned} \tag{7}$$

Because labor is costlessly adjustable, we can solve for it analytically and plug the solution into the firm's problem, which follows as:

$$\Pi(K, P, z, \tau) = \max_{K', P'} \left\{ E(K, P, K', P', z, \tau) + \frac{1}{1+r} \mathbb{E}\Pi\left(K', P', z', \tau'\right) \right\}, \tag{8}$$

subject to (3) and (6).

The economy also contains an infinitely lived representative consumer, who chooses consumption and labor each period to maximize the expected present value of her utility, discounted at the risk-free rate r. Her one-period utility function is given by $\ln(C) + \chi(1-L)$, in which C is consumption, L is the supply of labor, and χ is a parameter that governs the utility of leisure. Her budget constraint is given by:

$$C + P_d' - P_d(1+r) = wL_s + E(\cdot), \tag{9}$$

Let ζ be the stationary distribution over the firm's states, (z, τ, K_t, K_r, P). We define equilibrium in this economy as follows.

Definition 1 *A competitive equilibrium consists of (i) optimal firm policies for both types of capital and debt, $\{K'(z, \tau, K_t, K_r, P_s), P'(z, \tau, K_t, K_r, P_s)\}$, (ii) allocations to the consumer of consumption, C, and labor, L, and (iii) prices, (w, r), such that:*

1. *All firms solve the problem given by (8).*
2. *The consumer maximizes her utility, subject to (9).*
3. *The labor, bond, and output markets clear.*

II. Possible parameterization

To start, let's try

$\alpha = 0.3$
$\theta = 0.8$
$\rho = 0.7$ I change the rho to 0.9 in the code. A rho lower than that would give a collateral constraint that is always binding
$\sigma_z = 0.2$
$\delta = 0.15$
$\psi = 0.15$
$\xi = 0.5$
$\lambda = 0.05$

Let $r = 0.04$, and calibrate the tax process so that it changes about once every 5 years, with the low and high tax rates being 0.1 and 0.3. Later on we can think about making the tax process correlated with the productivity shock. Finally, pick a number for χ that makes workers work about 1/3 of the time. Some number between 1.5 and 2.5 should work.