ECON 712 (Macro Heterogeneity) Project

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Question 2 (Partial Equilibrium)

2.1 Formulation

$$V(a,y) = \max_{c,a'} U(c) + \beta \sum_{y'} \pi(y'|y) V(a',y')$$
$$\text{s.t. } c + a' = y + (1+r)a$$

Stochastic Euler Equation

$$U'(c) = \beta \sum_{y'} \pi(y'|y)(1+r')U'(c')$$
$$\text{where } c = y + (1+r)a - a'$$

2.2 value function iteration (infinite horizon)

My value function iteration is written in 'myVFIUBI.' I start from the guess $V^0(a,y) = 0, \forall a, y$. First, I made a matrix for flow utility. Given today's state (a,y), and today's choice (a'), today's consumption is $c = wy + (1+r)a - a'$, where wage is 1 in partial equilibrium. Second, given the guess of tomorrow's value ($V^{j-1}(a',y')$), we can compute today's value for each choice and maximize over the choice ($a' \in \{a_1, ..., a_N\}$). I update tomorrow's value ($V^j(a,y)$) and iterate until it converges.

For simulation, I used the command 'simulate'. First, I define a Markov transition matrix for assets and income (a,y). Second, I tell Matlab this is the Markov chain matrix by command 'dtmc'. Then Matlab can simulate the path of 61 periods.

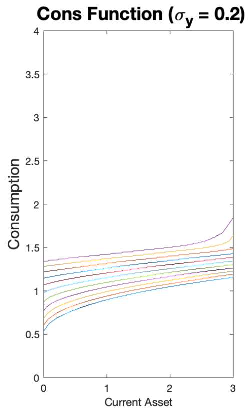
2.3 value function iteration (finite horizon)

Set the terminal condition: $V_T(a,y) = 0, \forall a, y$. Then I do the same procedure (iteration backwards) as in 2.2.

2.4 Comparison between $\sigma_y = 0.2, 0.4$ when $T = \infty$

When the income shock is large, income in a high state is also large. (Income ranges from 0.53 to 1.78 for $\sigma_y = 0.2$, and ranges from 0.27 to 3.04 for $\sigma_y = 0.4$). Because of this, consumption function ranges more if $\sigma_y = 0.4$. The consumption function is concave in asset when the agent is close to the borrowing constraint, which shows the precautionary saving behavior. (Figure 1)

[*]I discussed with Leon, Min.



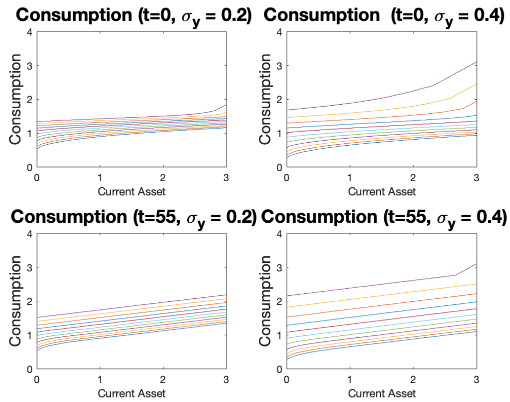
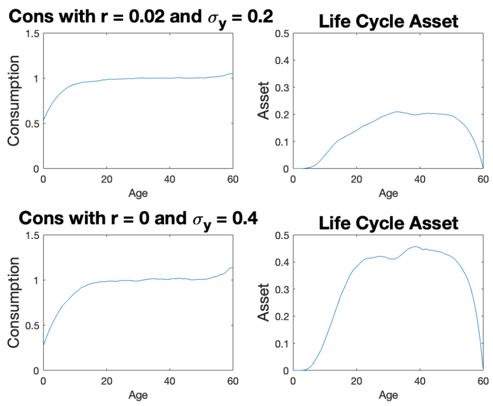


Figure 1: Comparison for σ_y where $T = \infty$ Figure 2: Comparison for σ_y where $T = 60$



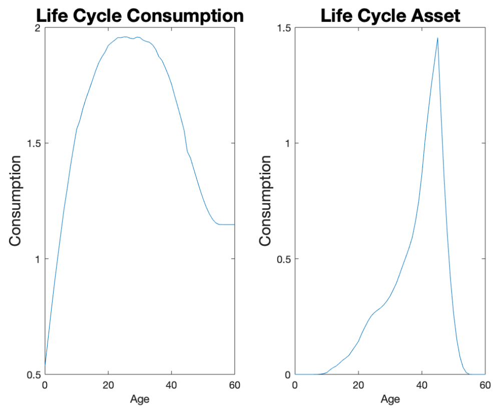


Figure 3: Life Cycle without $\bar{y}_t$

Figure 4: Life Cycle with $\bar{y}_t$

2.5 Compare between $\sigma_y = 0.2, 0.4$ when $T = 60$

When we compare between young and old households, consumption function is less varied in younger age, meaning agents save if they have high income today, and cannot decumulate even if they face low income today because they don't have much savings. (Figure 2)

2.6 Life Cycle Consumption

I tried with different r and σ_y, but I couldn't obtain a hump shape consumption profile (Figure 3). In later ages, consumption increases a little. Intuition would be that agents save assets at younger ages, and they need to consume before they face the end of life. I plotted the life cycle of asset holdings. (For asset holding, I can see the hump shape, reflecting the precautionary saving and the decumulation when approaching the end of life.)

2.7 Life Cycle Consumption with deterministic part ($\bar{y}_t$)

With the deterministic part of wage and mortality risk, I got the hump-shaped consumption profile (Figure 4). This is because wage goes down when agents get old, so they need to reduce the consumption. Also, because of mortality risk (the probability of death increases as they get old), agents start to decumulate assets before facing the end of life (because if they die, they cannot consume the remaining assets).

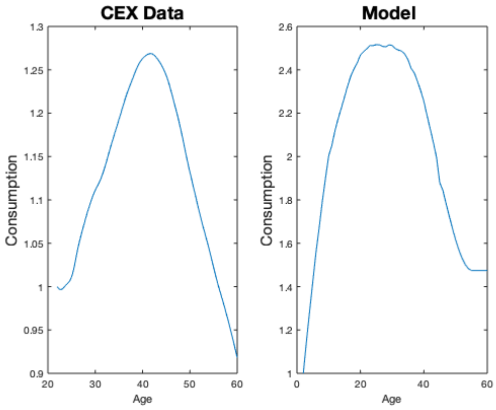


Figure 5: Compare with the data

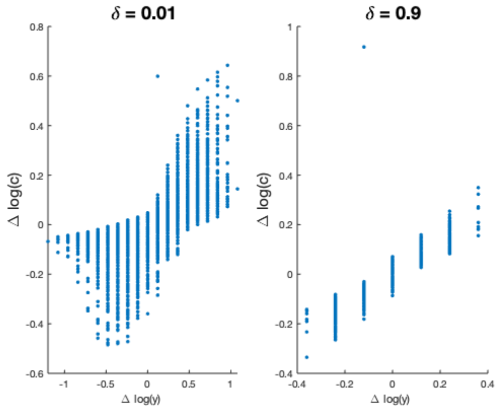


Figure 6: Life Cycle with $\bar{y}_t$

2.8 Life Cycle Consumption

The hump shape is very similar between the data and the model (Figure 5). The small difference is that in the model, consumption increases very rapidly at a young age. This is because agents don't have an asset at all at a young age, and they need to accumulate assets to prepare for future negative income shocks. To cure this problem, we could introduce a positive asset in the first period (age 20), this can be bequest or financial support from parents. Then, people don't have to save much at a young age, and the consumption profile could be even closer to the data.

2.9 Consumption Insurance

Consumption insurance coefficient is 0.8428 for $\delta = 0.01$ and 0.3326 for $\delta = 0.9$, meaning people insure against the income shock better when the persistence of income shock is low. This is because people expect that an income shock is temporary and they can decumulate asset (if it is low income shock) for a moment, and recover the asset level after income goes back to the normal level. I did exercise with the infinite horizon ($T = \infty$). Figure 6 is a plot of $\Delta \log(c)$ and $\Delta \log(y)$. We can see income fluctuations (x-axis) more with $\delta = 0.01$, but consumption does not respond one to one with income shock. Especially for large negative shock, consumption does not drop so much because of the decumulation of assets.

Question 3 (General Equilibrium)

In this section, I set the following parameter values:

$$\rho = 0.04, \ \sigma = 1, \ \delta = 0.8(\text{persistency of income shock}), \ \sigma_y = 0.2.$$

Also, I set the number of grid for capital to be 300 ranging from 0 to 30 ($a \in \{a_1, ..., a_{300}\}$ with $a_1 = 0, a_{300} = 30$). The number of grid for income is 11 ($y \in \{y_1, ..., y_{11}\}$).

3.1 Compute Equilibrium r

I assume Cobb-Douglas production function

$$Y = K^{\alpha} L^{1-\alpha} + (1 - depreciate)K$$

where $\alpha = 0.36, depreciate = 0.08$ following Aiyagari (1994).

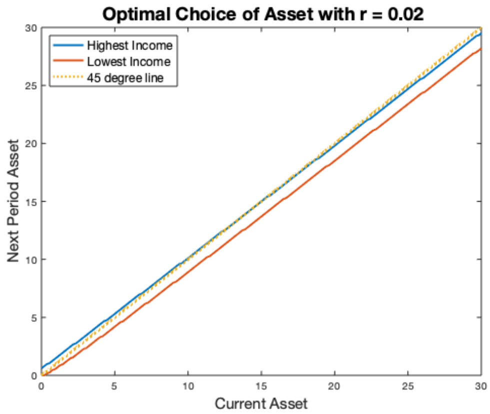


Figure 7: Intersection with 45 degree line

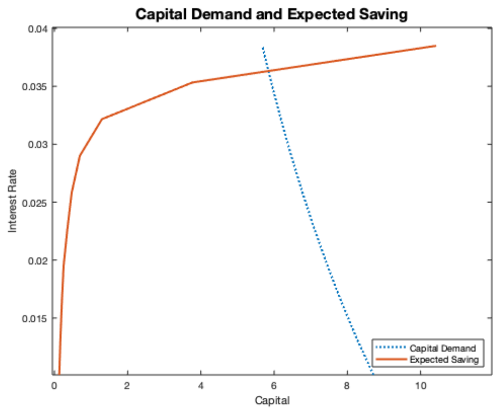


Figure 8: Capital Demand and Savings

3.1 (b) $K(r)$ and $w(r)$

By the FOC of firm's problem (assume perfect competition):

$$K(r) = \left(\frac{\alpha}{r + dep}\right)^{\frac{1}{1-\alpha}}$$

$$w(r) = (1-\alpha)\left(\frac{\alpha}{r + dep}\right)^{\frac{\alpha}{1-\alpha}}$$

3.1 (d) Stationary Distribution

From Figure 7, we can see $a'(a, y)$ intersects with 45 degree line when $r = 0.02$. For finding stationary distribution, I found out that computing eigenvalue and eigenvectors takes a lot of time. So in the later excercises I computed the stationary distribution by multiplying Markov transition matrix 100 times with the vector $(1, 1, ..., 1)/N$, where N is the number of elements in the vector (number of income states $\times$ number of gids for asset).

3.1 (f) Compute $d(r)$

We can see from Figure 8 that the difference is monotonically decreasing with r. So there is a unique r such that capital demand is equal to capital savings from the household. By 'fzero' I found such r (which is 0.0366).

3.2 Compare with Aiyagari (1994)

The result is in Table 1. I obtained qualitatively similar results. As σ (CRRA parameter) increases from 1 to 5, the interest rate goes down, because agents are more risk-averse and save more. As σ_y (standard deviation of income shock) goes up from 0.2 to 0.4, interest rate declines because agents face more risk and save more. These two features are common with Aiyagari (1994). However, my result shows that as δ (persistence of income shock) increases from 0 to 0.9, interest rate goes up. I couldn't find out the reason.

	Risk Aversion(sigma)= 1 or 5	
sigmaY = 0.2, delta = 0	0.03	-0.0111
sigmaY = 0.2, delta = 0.9	0.0362	0.0227
sigmaY = 0.4, delta = 0	0.03	-0.0111
sigmaY = 0.4, delta = 0.9	0.0328	0.0034

Table 1: Comparison with Aiyagari (Intereste rate for different paramters)

3.3 Introduce UBI

(i) Setup

Now agent endogenously chooses whether to work or not. The value function is as follows:

$$\begin{aligned} V(a,y) &= \max\{V^W(a,y), V^U(a,y)\} \\ \text{where } V^W(a,y) &= \max_{a'}\ U((1-\tau)wy + (1+r)a - a') - \kappa + \beta \sum_{y'} \pi(y'|y)V(a',y') \\ V^U(a,y) &= \max_{a'}\ U((1+r)a - a') + \beta \sum_{y'} \pi(y'|y)V(a',y') \end{aligned}$$

I define $V^W(a,y)$ as value when the agent works, and $V^W(a,y)$ as value when the agent doesn't work. Then by value function iteration I get solution $V(a,y),\ a'(a,y),\ l(a,y),\ c(a,y)$. After making a Markov transition matrix, I obtain the starionary distribution $\Phi(a,y)$.

(ii) Estimate κ

I use a bisection method to estimate κ (see the function 'KappaEst3'). I set the initial interval $[\kappa_{min}, \kappa_{max}] = [0.7, 1]$. (After some try and error, I found out that κ is in the range from 0.8 to 0.9.) For each iteration, I set $\kappa^j = 0.5 * \kappa^j_{min} + 0.5 * \kappa^j_{max}$. Given this κ^j, I solve for equlibrium interest rate (r). This time (with labor decision), both K and L are endogenously determined. So we could solve for both K and L. However, because (K/L) is a sufficient statistic for r and w and it's better to reduce the number of endogenous variables, I solve equilibrium K/L by fzero. After obtaining equilibrium r and w, I solve VFI and compute stationary distribution. Given this, I can compute the labor participation rate $\int l(a,y)d\Phi(a,y)$. If this is larger than 0.8, κ is too low, so I set $[\kappa^{j+1}_{min}, \kappa^{j+1}_{max}] = [\kappa^j, \kappa^j_{max}]$. If larbor participation is less than 0.8, I set $[\kappa^{j+1}_{min}, \kappa^{j+1}_{max}] = [\kappa^j_{min}, \kappa^j]$. After some iterations, I got $\kappa = 0.8875$.

(iii) Estimate τ

I estimate τ which balances the government budget. I start from a guess of τ (=0.1) given $\lambda = 0.2$ (See function 'TauEst3'). As in (ii), I solved for equilibrium (K/L), and compute the solution for value function iteration and the stationary distribution. Then I update the guess of τ by

$$\tau = \frac{\lambda}{w \int yl(a,y)d\Phi(a,y)}$$

I iterate until it converges. I got the estimate of $\tau = 0.2364$.

Table 2: Comparioson between Standard and UBI

Model	Y	K	C	Wage	r	Gini (a)	Gini (Income)	Gini (C)	Welfare
Standard ($\lambda = 0$)	1.646	5.899	1.209	1.212	0.036	0.545	0.257	0.071	-14.165
UBI ($\lambda = 0.2$)	1.445	5.994	1.009	1.219	0.034	0.477	0.360	0.069	-15.250

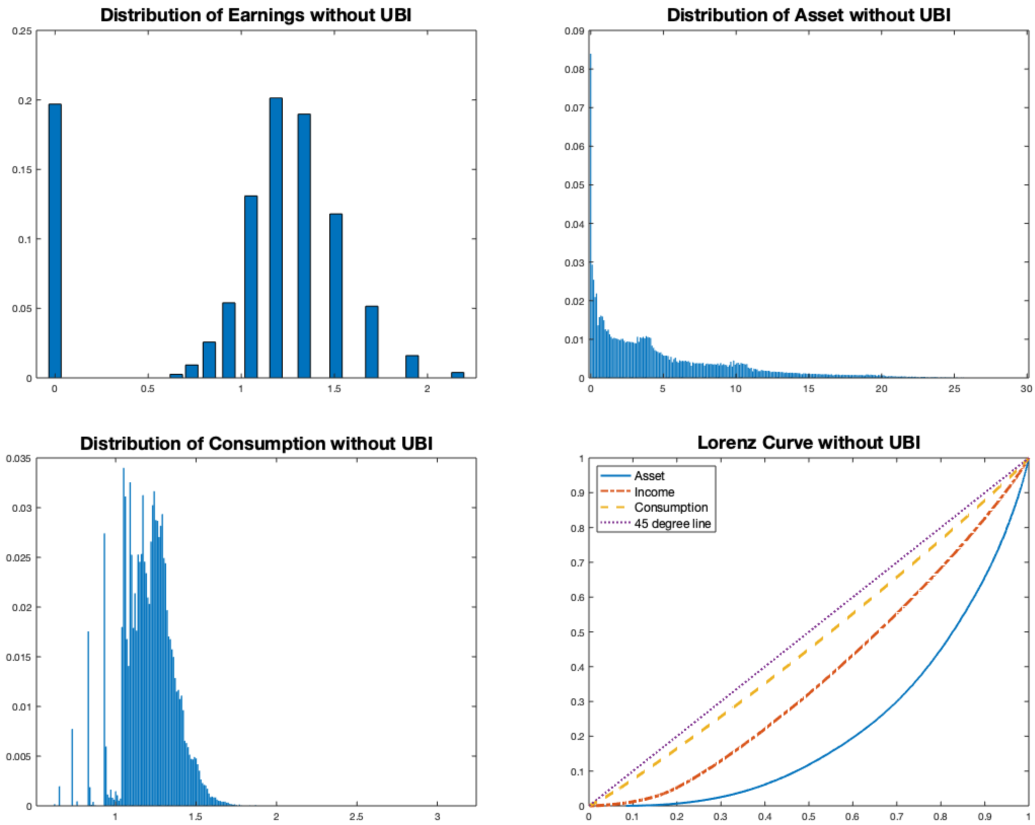


Figure 9: Equilibrium without UBI ($\lambda = 0$)

(iv) Comparison between the two Steady States

We can see from Table 2 that in my parameter choice, welfare is worse in the UBI case (-14.2 without UBI compared to -15.3 with UBI). I computed welfare as the expected value. Capital is roughly the same level, but because of UBI, fewer people work, and aggregate output is less than the standard (without UBI) case. When we look at Gini coefficients, assets and consumption are less unequal with UBI. This is because people have less incentive for precautionary savings (so less inequality for asset holdings), and thanks to UBI agents with a small amount of assets and low income can still consume a reasonable amount of goods. Income is unequal with UBI because around 35% of agents do not work, and they obtain zero wage. (This income does not include UBI benefit.)

code

I attached the main code with the result ("published" from Matlab) in the following pages. For each function, please see m.files in the folder.

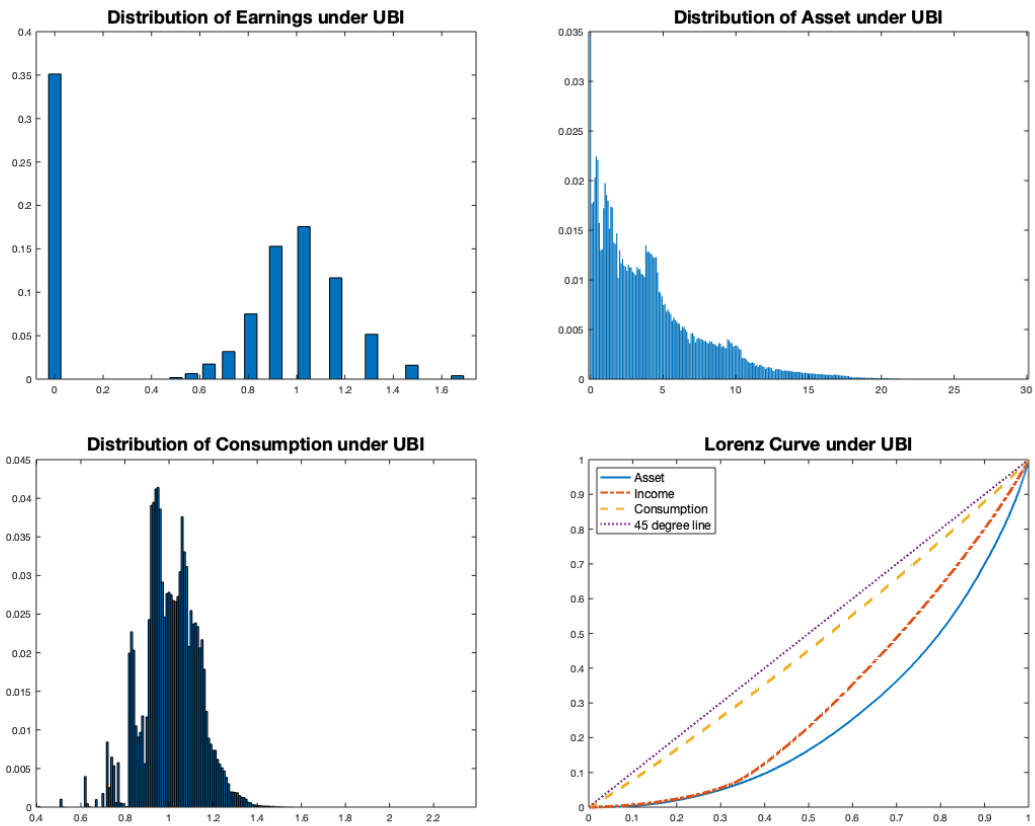


Figure 10: Equilibrium with UBI ($\lambda = 0.2$)

Contents

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Yoshiki Ando

```
clear


% Parameters
Ngrid     = 300; % Number of Grids for Asset
Nincome   = 11;  % Numnber of Grids for Income
Persist=0.8;     % Persistent Parameters for AR(1) Income Process
Mu=0;            % Mean of AR(1) Process
SigmaY=0.2;      % Standard deviation for Shocks in AR(1) Process
rr = 0.02;       % interest rate in Partial Equilibrium Model
rrho = 0.04;     % discount factor
ssigma = 1;      % CRRA parameter (meaning log utility)
Wage = 1 ;       % Wage in Partial Equilibrium Model
MaxA = 3;        % make a grid for asset from 0 to 3
llambda = 0; kkappa = 0; ttau = 0; % without UBI system


% Tauchen
% myTauchenAR1(Persist, Mu, Sigma, NN) => vState, mPi, vStationary
% where y_t+1 = Persist * y_t + (1-Persist^2)^0.5 * Eps
% with Eps follows Normal(0, Sigma^2)
[vState, mPi, vStationary] = myTauchenAR1(Persist, Mu, SigmaY, Nincome);

% Normalize the grid of Income such that E[Income] = 1
vIncome = exp(vState) ./ (exp(vState) * vStationary);
```

2.2 Value Function Iteration (Infinite Case)

```
[mValue, mAssetLocation,mAssetChoice, ~, mConsChoice] =...
    MultigridUBI(Ngrid, Nincome, rr, Wage, ...
    ttau, llambda, kkappa,rrho,  ssigma,Persist, SigmaY, MaxA  );

% Path of Consumption and Asset Holdings
rng(5) % set seed
% make a Markov Chain Matrix for Asset and Consumption
mMarkovJoint = zeros(Ngrid*Nincome) ; % [Grid y_1, Grid y_2, ..., Grid y_N]
mIdentity    = eye(Ngrid);
for jj = 1:Ngrid
 for ii = 1:Nincome
 currentRow =  jj + Ngrid * (ii-1) ;
 vRow = [] ; % make a vector for the current row
 for kk = 1:Nincome % next period income is in y_kk
  vRow = [vRow, mIdentity(mAssetLocation(jj,ii),:) * mPi(ii,kk)] ;
 end
 mMarkovJoint(currentRow,:) = vRow;

 end
end % fill the each row

% simulation (starting from state 1 (a_0 and y_0)
MarkovChain       = dtmc(mMarkovJoint);
mIdenNgridNincome = eye(Ngrid*Nincome);
SimuIniState      = mIdenNgridNincome(1,:);
Path1 = simulate(MarkovChain,60,'X0',SimuIniState);
display('first 10 periods of simulation')
Path1(1:10)'
```

Question 2.3 (Finite Model)

```
Nperiod = 60;
[seqValue, seqAsset, seqCons, seqAssetLocation] =...
 myVFIfinite(Ngrid, Nincome, Nperiod,rr, rrho, ssigma, Persist,SigmaY ) ;
```

Question 2.4 (Compare small and large income shocks)

```
[mValue1, mAssetLocation1,mAssetChoice1, ~, mConsChoice1] =...%SigmaY = 0.2
    MultigridUBI(Ngrid, Nincome, rr, Wage, ...
    ttau, llambda, kkappa,rrho,  ssigma,Persist, SigmaY, MaxA  );
[mValue2, mAssetLocation2,mAssetChoice2, ~, mConsChoice2] =...%SigmaY = 0.4
    MultigridUBI(Ngrid, Nincome, rr, Wage, ...
    ttau, llambda, kkappa,rrho,  ssigma,Persist, 0.4, MaxA  );


vCurrentAsset    = linspace(0,MaxA,Ngrid);

figure;
subplot(1,2,1)
pl=plot(vCurrentAsset,mConsChoice1);
xla=xlabel('Current Asset');
yla=ylabel('Consumption','FontSize',15);
ylim([0 4])
```

```
tit=title('\fontsize{18} Cons Function (\sigma_y = 0.2)');

subplot(1,2,2)
pl=plot(vCurrentAsset,mConsChoice2);
xla=xlabel('Current Asset');
yla=ylabel('Consumption','FontSize',15);
ylim([0 4])
tit=title('\fontsize{18} Cons Function (\sigma_y = 0.4)');
```

Question 2.5 (Consumption Function with T=60)

```
[seqValue1, seqAsset1, seqCons1, seqAssetLocation1] =...
    myVFIfinite(Ngrid, Nincome, Nperiod, rr, rrho, ssigma, Persist,0.2 );
[seqValue2, seqAsset2, seqCons2, seqAssetLocation2] =...
    myVFIfinite(Ngrid, Nincome, Nperiod, rr, rrho, ssigma, Persist,0.4 );


figure;
subplot(2,2,1)
pl=plot(vCurrentAsset,seqCons1(:,:,1));
xla=xlabel('Current Asset');
yla=ylabel('Consumption','FontSize',15);
ylim([0 4])
tit=title('\fontsize{18} Consumption (t=0, \sigma_y = 0.2)');

subplot(2,2,2)
pl=plot(vCurrentAsset,seqCons2(:,:,1) );
xla=xlabel('Current Asset');
yla=ylabel('Consumption','FontSize',15);
ylim([0 4])
tit=title('\fontsize{18} Consumption  (t=0, \sigma_y = 0.4)');

subplot(2,2,3)
pl=plot(vCurrentAsset,seqCons1(:,:,56));
xla=xlabel('Current Asset');
yla=ylabel('Consumption','FontSize',15);
ylim([0 4])
tit=title('\fontsize{18} Consumption (t=55, \sigma_y = 0.2)');

subplot(2,2,4)
pl=plot(vCurrentAsset,seqCons2(:,:,56) );
xla=xlabel('Current Asset');
yla=ylabel('Consumption','FontSize',15);
ylim([0 4])
tit=title('\fontsize{18} Consumption (t=55, \sigma_y = 0.4)');
```

Question 2.6 (Average Consumption Path)

```
Nobs = 1000; % Number of Agents I will simulate

% Try with rr = 0.02 and SigmaY = 0.2
[seqLifeCons1, seqLifeAsset1] =...
 myLifeCycle(Ngrid, Nincome, Nperiod, rr, rrho,...
 ssigma, Persist, SigmaY, Nobs);
```

```
% Take an average over Nobs Agents
vAveCons1 = mean(seqLifeCons1,2 ) ;
vAveAsset1= mean( seqLifeAsset1, 2);
vAge      = 0:Nperiod ;

% Try with rr = 0 and SigmaY = 0.4
[seqLifeCons2, seqLifeAsset2] =...
 myLifeCycle(Ngrid, Nincome, Nperiod, 0, rrho,...
 ssigma, Persist,0.4, Nobs);

vAveCons2 = mean(seqLifeCons2,2 ) ;
vAveAsset2= mean( seqLifeAsset2, 2);


figure ;
subplot(2,2,1)
pl=plot(vAge,vAveCons1 );
xla=xlabel('Age');
yla=ylabel('Consumption','FontSize',15);
ylim([0 1.5])
tit=title('\fontsize{18}  Cons with r = 0.02 and \sigma_y = 0.2 ');

subplot(2,2,2)
pl=plot(vAge,vAveAsset1 );
xla=xlabel('Age');
yla=ylabel('Asset','FontSize',15);
ylim([0 .5])
tit=title('\fontsize{18} Life Cycle Asset ');

subplot(2,2,3)
pl=plot(vAge,vAveCons2 );
xla=xlabel('Age');
yla=ylabel('Consumption','FontSize',15);
ylim([0 1.5])
tit=title('\fontsize{18}  Cons with r = 0 and \sigma_y = 0.4  ');

subplot(2,2,4)
pl=plot(vAge,vAveAsset2 );
xla=xlabel('Age');
yla=ylabel('Asset','FontSize',15);
ylim([0 .5])
tit=title('\fontsize{18} Life Cycle Asset ');
```

Question 2.7 (Life Cycle Income Profile)

```
IncProfileData = readmatrix('incprofile.txt') ;
vIncProfile    = [IncProfileData; ones(16,1)*0.7*IncProfileData(45) ] ;
SurvsProbData  = readmatrix('survs.txt') ;
vSurvsProb     = SurvsProbData(:,1) ;

% Value Function Iteration
[seqValue3, seqAsset3, seqCons3, seqAssetLocation3] =...
  myVFILife(Ngrid, Nincome, Nperiod, rr, rrho, ssigma, Persist,...
  SigmaY,  vIncProfile, vSurvsProb);
```

```
% Simulate 1000 Individuals
[seqLifeCons2, seqLifeAsset2] =...
 myLifeCycle2(Ngrid, Nincome, Nperiod, rr, rrho,...
 ssigma, Persist,SigmaY, Nobs, vIncProfile, vSurvsProb);

% Take an average over Nobs Agents
vAveCons1 = mean(seqLifeCons2,2 ) ;
vAveAsset1= mean(seqLifeAsset2, 2);
vAge      = 0:Nperiod ;

figure ;
subplot(1,2,1)
pl=plot(vAge,vAveCons1 );
xla=xlabel('Age');
yla=ylabel('Consumption','FontSize',15);
%ylim([0 1.5])
tit=title('\fontsize{18} Life Cycle Consumption ');

subplot(1,2,2)
pl=plot(vAge,vAveAsset1 );
xla=xlabel('Age');
yla=ylabel('Consumption','FontSize',15);
%ylim([0 1.5])
tit=title('\fontsize{18} Life Cycle Asset ');
```

Question 2.8 (Compare to CEX Data)

```
CEXData    = readmatrix('consprofile.txt');
CEX2280    = CEXData(1:232 ,3) ;
vAgeCEX    = linspace(22,60,232) ; %Age from 22 to 60
vConsModel = vAveCons1(3:61) / vAveCons1(3); %Age 22-60 ,+ Normalization

figure ;
subplot(1,2,1)
pl=plot(vAgeCEX,CEX2280  );
xla=xlabel('Age');
yla=ylabel('Consumption','FontSize',15);
% ylim([0.5 2.5])
tit=title('\fontsize{18} CEX Data ');

subplot(1,2,2)
pl=plot(vAge(3:61),vConsModel );
xla=xlabel('Age');
yla=ylabel('Consumption','FontSize',15);
% ylim([0.5 2.5])
tit=title('\fontsize{18} Model  ');
```

Question 2.9 (Degree of Consumption Insurance)

```
Nobs = 50000; Ngrid  = 300; Nincome = 11;
rr = 0.02; rrho = 0.04; ssigma = 2;  SigmaY=0.2;

[Phi1, DifLogCons1, DifLogIncome1] = ...
    myPhi(Ngrid, Nincome,  rr, Wage, rrho,ssigma,0.01,SigmaY, Nobs, MaxA) ;
```

```
% Tauchen Method doesn't work well for delta = 0.99
% so I tried with delta = 0.9
[Phi2 DifLogCons2, DifLogIncome2] =...
    myPhi(Ngrid, Nincome,  rr, Wage, rrho,ssigma, 0.9,SigmaY, Nobs, MaxA) ;

display('insurance coefficient for delta=0.01 and 0.9')
display([Phi1 Phi2])

figure;
subplot(1,2,1)
scatter(DifLogIncome1,DifLogCons1,50, '.');
xlabel('\Delta log(y) ');
ylabel('\Delta log(c)','FontSize',15);
title('\fontsize{18} \delta = 0.01 ');

subplot(1,2,2)
scatter(DifLogIncome2,DifLogCons2,50, '.');
xla=xlabel('\Delta log(y) ');
yla=ylabel('\Delta log(c)','FontSize',15);
tit=title('\fontsize{18} \delta = 0.9 ');
```

Part II: General Equilibrium

```
% Parameter in Production Function (Assume Cobb-Douglas)
aalpha = 0.36; depreciate = 0.08;
% Other parameters: the same as before
rrho = 0.04; ssigma = 1; Persist = 0.8; SigmaY=0.2; Ngrid = 300; Nincome = 11;

% start from a guess
rr = 0.02 ;
Kcapital = @(rr) (aalpha / (rr+depreciate) )^(1/(1-aalpha)) ;
Wwage    = @(rr) (1-aalpha) * (aalpha / (rr+depreciate) )^(aalpha/(1-aalpha)) ;

% Solve Value Function Iteration given rr guess
MaxA = 30; Wage = Wwage(rr) ;
tic;
[mValue, mAssetLocation,mAssetChoice, ~, mConsChoice] =...
    MultigridUBI(Ngrid, Nincome, rr, Wage, ...
    ttau, llambda, kkappa,rrho,  ssigma,Persist, SigmaY, MaxA  ) ;
display('time to compute VFI')
toc;

% Check the asset function crosses the 45 degree line
vCurrentAsset    = linspace(0,MaxA,Ngrid);
figure;
pl=plot(vCurrentAsset,mAssetChoice(:,Nincome),...
    vCurrentAsset,mAssetChoice(:,1), vCurrentAsset,vCurrentAsset);
xla=xlabel('Current Asset','FontSize',15);
yla=ylabel('Next Period Asset','FontSize',15);
ylim([0 MaxA])
tit=title('\fontsize{18} Optimal Choice of Asset with r = 0.02');
set(pl,{'LineStyle'},{'-';'-'; ':'});
set(pl,'Linewidth',2);
le=legend({'Highest Income','Lowest Income','45 degree line'},...
    'Location','northwest','FontSize',12);
```

```
% Draw a picture for K(r) and Ea(r)
Ngridrr  = 10 ; % Number of Points for rr to estimate
rrVector = linspace(0.01, 0.0385, Ngridrr)  ;
vKdemand = zeros(1, Ngridrr) ;
vExpectedSaving = zeros(1, Ngridrr) ;
for jj = 1:Ngridrr
    % Demand from Firms
    vKdemand(jj) = Kcapital( rrVector(jj) ) ;

    % Supply from Households
    [vJointStatTemp,vStatAssetTemp, EaTemp, ExcessDeTemp ] =...
    InvDist(Ngrid, Nincome, rrVector(jj),  rrho,...
    ssigma,Persist, SigmaY, MaxA  );

   vExpectedSaving(jj) = EaTemp  ;

end


% Plot Marginal Benefit and Marginal Cost
figure;
pl=plot(vKdemand, rrVector , vExpectedSaving, rrVector );
set(pl,'Linewidth',2);
title('Capital Demand and Expected Saving','FontSize',15)
xlabel('Capital')
ylabel('Interest Rate')
le=legend({'Capital Demand','Expected Saving'},'Location','southeast');
set(pl,{'LineStyle'},{':'; '-'});


% Compute rr which clear the market
tic;
rrint = [0.02, 0.038]    ;
[rr1, fval] = fzero(@(rr) myEqui(Ngrid, Nincome, rr, ttau, llambda,...
    kkappa, rrho,ssigma,Persist, SigmaY, MaxA  ), rrint);
 display([rr1, fval]) %solution by fzero
 display('time to compute equilibrium rr')
toc; % 27.992803 seconds and rr1 = 0.0366 (Ngrid =300; Nincome=11)
```

3.2 Aiyagari Table % Persisit, ssigma (CRRA), SigmaY

```
tic;
ParameterSpace = [0,1,0.2; 0.9,1,0.2; 0, 5, 0.2; 0.9, 5,0.2 ;
    0,1,0.4; 0.9,1,0.4; 0, 5, 0.4; 0.9, 5,0.4 ] ; MaxA = 30;
AiyagariTable  = zeros(size(ParameterSpace,1),2) ;
for jj = 1 : size(ParameterSpace,1)
    ParaTemp = ParameterSpace(jj,:) ; rrint =[-0.04, 0.039];
    [rr1, fval] = fzero(@(rr) myEqui(Ngrid, Nincome, rr, ttau, llambda,...
    kkappa, rrho,ParaTemp(2),ParaTemp(1), ParaTemp(3), MaxA  ), rrint);
 AiyagariTable(jj,: ) = [rr1, fval] ;


end
display('Time to Compute a Table for Aiyagari Comparison')
toc; % 581.884790 seconds
```

```
% 0.0286  0.0357 -0.0161  0.0213  0.0286  0.0324 -0.0161  0.0025

TT = table(ParameterSpace,AiyagariTable) ;
% display(TT)

Table2 = zeros(4,2) ;
Table2(1:2,1) = AiyagariTable(1:2) ;
Table2(1:2,2) = AiyagariTable(3:4) ;
Table2(3:4,1) = AiyagariTable(5:6) ;
Table2(3:4,2) = AiyagariTable(7:8) ;
Sigma =  {'sigma_y = 0.2';'0.2';'0.4';'0.4'};
Rho   =  {'delta = 0';'0.9';'0';'0.9'};
Table2 = round(Table2,4) ;
TT2 = table(Table2, ...
    'VariableNames',{'Risk Aversion(sigma)= 1 or 5'}, ...
    'RowNames',{'sigmaY = 0.2, delta = 0', 'sigmaY = 0.2, delta = 0.9', ...
    'sigmaY = 0.4, delta = 0','sigmaY = 0.4, delta = 0.9'});
display(TT2)
```

3.3 Model with UBI

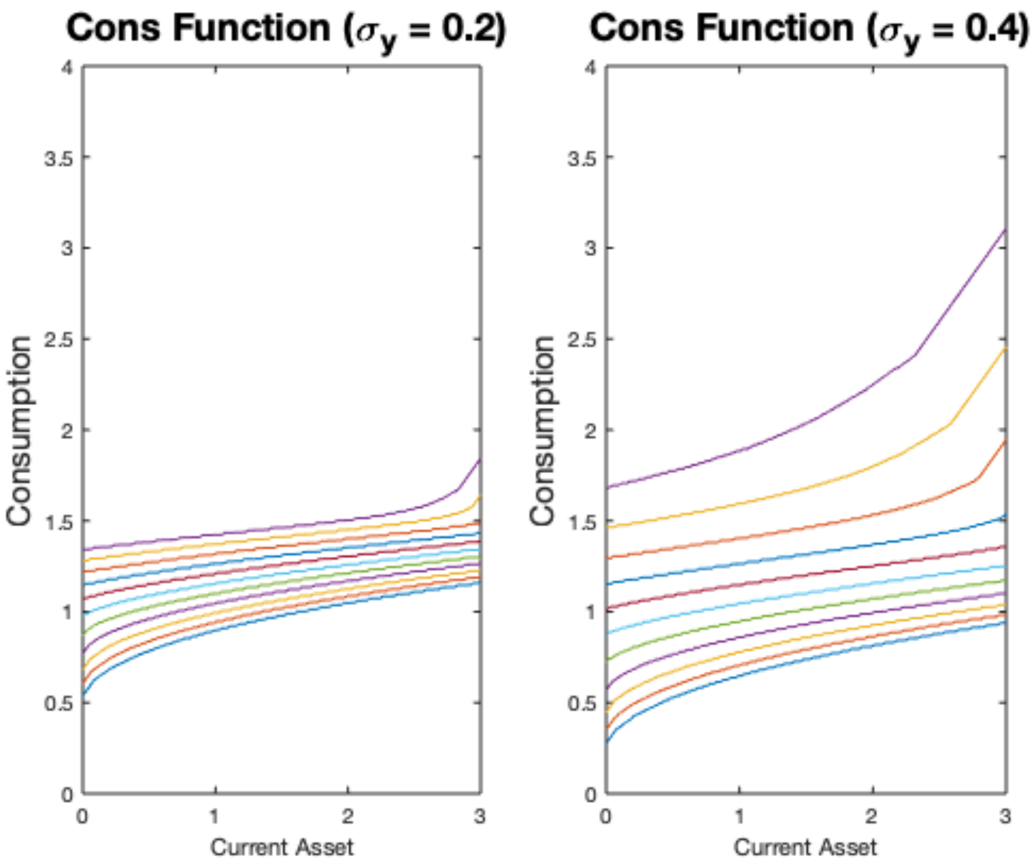
Estimate Kappa such that Labor Force Participation rate is 0.8

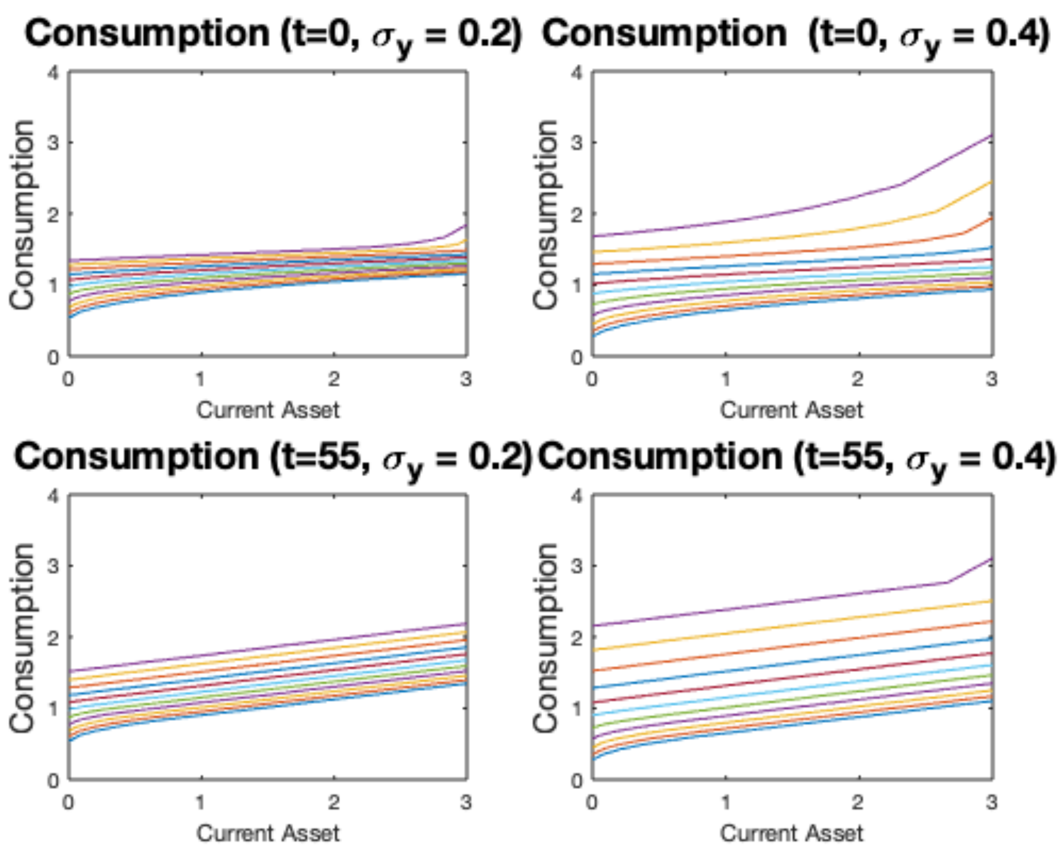
```
LpartTarget = 0.8 ;
tic;
[rrRes1, WageRes1, KappaRes ] =...
    KappaEst3(Ngrid, Nincome,  llambda,  rrho,...
    ssigma,Persist, SigmaY, MaxA, LpartTarget  ) ;
display('rr, Wage, Kappa such that Labor Paticipation =0.8')
[rrRes1, WageRes1, KappaRes ]
display('Time to find Kappa')
toc;


% Estimate Tau with UBI case
tic;
lambdaUBI = 0.2;  KappaSol = KappaRes ;
[rrRes2, WageRes2, TauRes2 ] = TauEst3(Ngrid, Nincome,  lambdaUBI,  rrho,...
    ssigma,Persist, SigmaY, MaxA, KappaSol  ) ;
display('Equilibrium Interest, Wage, tau without UBI')
display([rrRes2, WageRes2, TauRes2 ])
display('time to compute rr and Wage without UBI')
toc;



% Without UBI
x1   = [rrRes1, WageRes1] ; Kkappa = KappaSol;
Eqrr = x1(1); EqWage = x1(2); Eqttau = 0; lambdaNoUBI = 0;

[YY1,KK1,CC1,Wage1,Interest1,EarningDist1, IncomeDist1, ...
    AssetDist1, ConsDist1, GiniAsset1, LorenzAsset1 , ...
    GiniIncome1, LorenzIncome1 ,GiniCons1, LorenzCons1, SocialWelfare1 ] =...
    MacroAggUBI(Ngrid, Nincome, Eqrr, EqWage,Eqttau, lambdaNoUBI, Kkappa,...
  rrho, ssigma,Persist, SigmaY, MaxA  ) ;
```

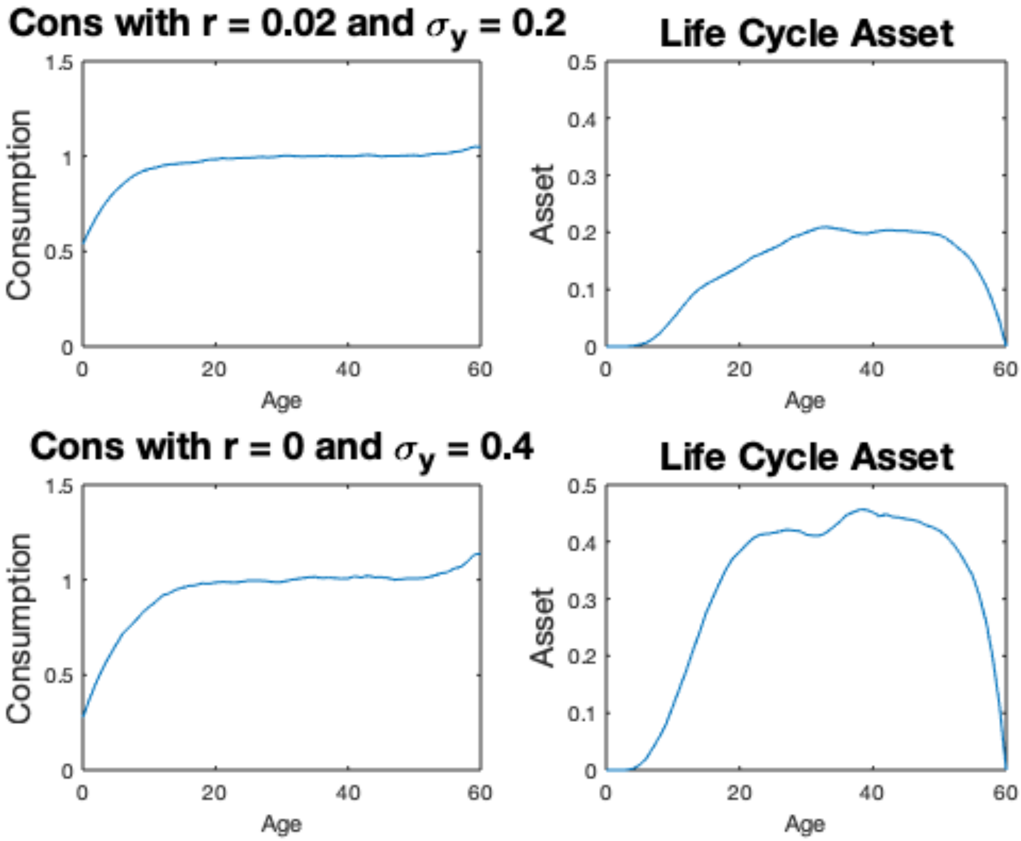
```
display('[YY,KK,CC,Wage,Interest, GiniAsset,GiniIncome, GiniCons, SocialWelfare] without UBI')
display([YY1,KK1,CC1,Wage1,Interest1, GiniAsset1,GiniIncome1, GiniCons1, SocialWelfare1])

% With UBI
x2   = [rrRes2, WageRes2, TauRes2 ] ;
Eqrr = x2(1); EqWage= x2(2); Eqttau = x2(3);

[YY2,KK2,CC2,Wage2,Interest2,EarningDist2, IncomeDist2, ...
    AssetDist2, ConsDist2, GiniAsset2, LorenzAsset2 , ...
    GiniIncome2, LorenzIncome2 ,GiniCons2, LorenzCons2, SocialWelfare2 ] =...
    MacroAggUBI(Ngrid, Nincome, Eqrr,EqWage, Eqttau, lambdaUBI, Kkappa,...
  rrho, ssigma,Persist, SigmaY, MaxA  ) ;
display('[YY,KK,CC,Wage,Interest, GiniAsset,GiniIncome, GiniCons, SocialWelfare] with UBI')
display([YY2,KK2,CC2,Wage2,Interest2, GiniAsset2,GiniIncome2, GiniCons2, SocialWelfare2])

% Welfare Gain from UBI
display('Compare the Welfare with and without UBI')
display([SocialWelfare2, SocialWelfare1])

Table3mat = [YY1,KK1,CC1,Wage1,Interest1, GiniAsset1,GiniIncome1, GiniCons1, SocialWelfare1;
    YY2,KK2,CC2,Wage2,Interest2, GiniAsset2,GiniIncome2, GiniCons2, SocialWelfare2];
VarNameVector2 = {'Y,K,C,Wage,r,Gini(a),Gini(Income),Gini(C),Welfare'} ;
TT3 = table(Table3mat, 'VariableNames',VarNameVector2, 'RowNames',{'Standard','UBI'});
display(TT3)
```

```
first 10 periods of simulation

ans =

  Columns 1 through 6

         1           1         601         601        1201        1201

  Columns 7 through 10

      1201        1501        1501        1201

insurance coefficient for delta=0.01 and 0.9
    0.8428    0.3326

time to compute VFI
Elapsed time is 1.821018 seconds.
    0.0366    0.0011

time to compute equilibrium rr
Elapsed time is 115.446137 seconds.
Time to Compute a Table for Aiyagari Comparison
Elapsed time is 755.100659 seconds.

TT2 =

  4×1 table

                                 Risk Aversion(sigma)= 1 or 5
                                 ____________________________
```

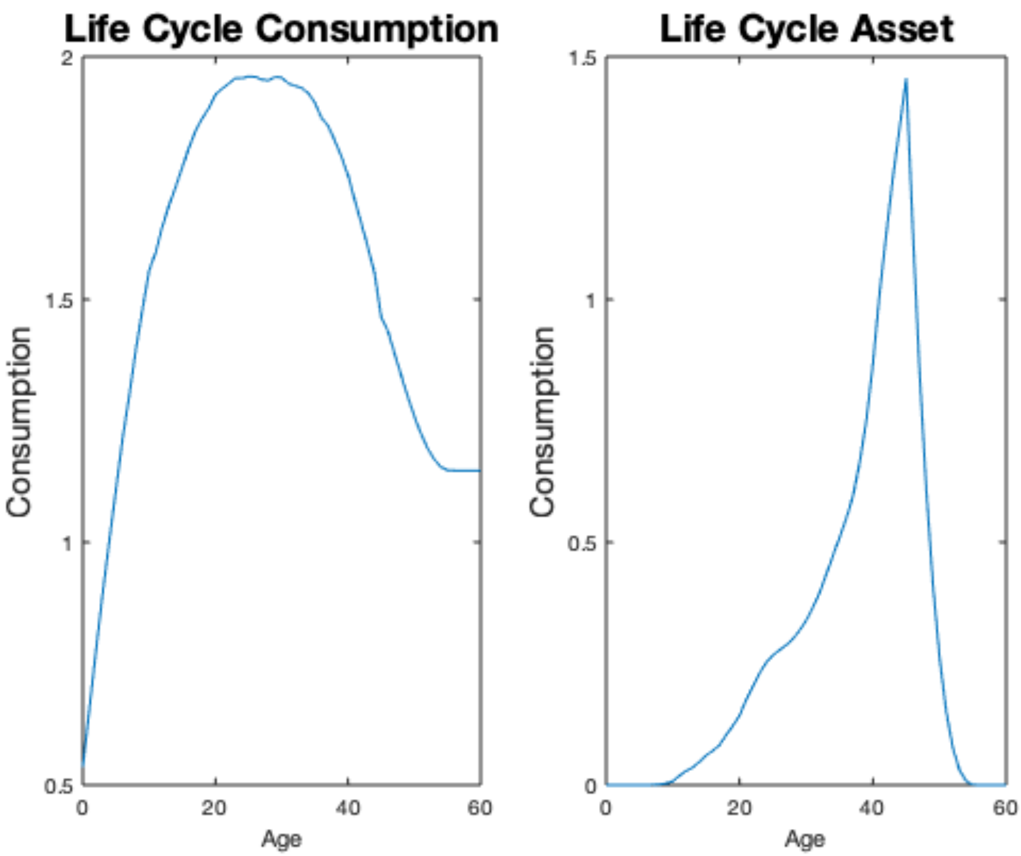
```
    sigmaY = 0.2, delta = 0             0.03    -0.0111
    sigmaY = 0.2, delta = 0.9         0.0362     0.0227
    sigmaY = 0.4, delta = 0             0.03    -0.0111
    sigmaY = 0.4, delta = 0.9         0.0328     0.0034

rr, Wage, Kappa such that Labor Paticipation =0.8

ans =

    0.0356    1.2125    0.8875

Time to find Kappa
Elapsed time is 97.488428 seconds.
Equilibrium Interest, Wage, tau without UBI
    0.0344    1.2194    0.2343

time to compute rr and Wage without UBI
Elapsed time is 65.077830 seconds.
[YY,KK,CC,Wage,Interest, GiniAsset,GiniIncome, GiniCons, SocialWelfare] without UBI
  Columns 1 through 7

    1.6464    5.8993    1.2093    1.2125    0.0356    0.5455    0.2567

  Columns 8 through 9

    0.0710  -14.1647

[YY,KK,CC,Wage,Interest, GiniAsset,GiniIncome, GiniCons, SocialWelfare] with UBI
  Columns 1 through 7

    1.4446    5.9938    1.0090    1.2194    0.0344    0.4769    0.3601

  Columns 8 through 9

    0.0686  -15.2498

Compare the Welfare with and without UBI
  -15.2498  -14.1647


TT3 =

  2×1 table

                Y,K,C,Wage,r,Gini(a),Gini(Income),Gini(C),Welfare
                _________________________________________________

    Standard                     [1×9 double]
    UBI                          [1×9 double]
```


Cons Function (σ_y = 0.2)
Cons Function (σ_y = 0.4)
Consumption
Current Asset

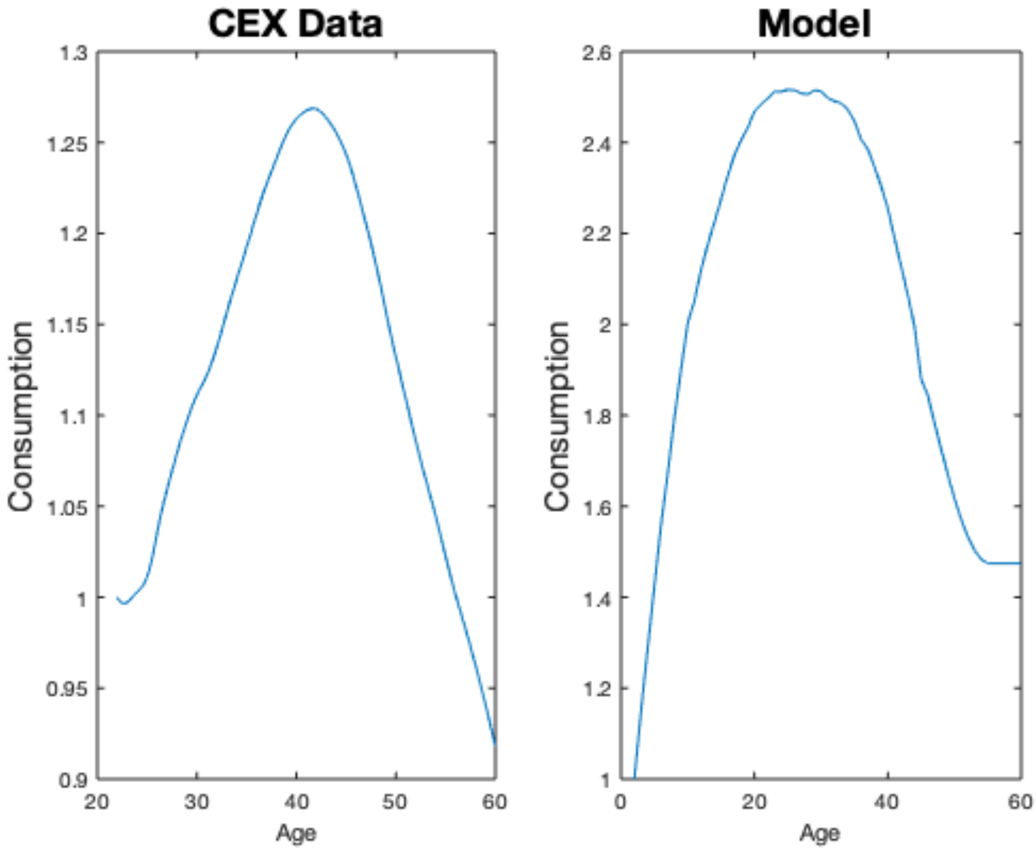

Consumption (t=0, σ_y = 0.2)
Consumption (t=0, σ_y = 0.4)
Consumption (t=55, σ_y = 0.2)
Consumption (t=55, σ_y = 0.4)
Consumption
Current Asset

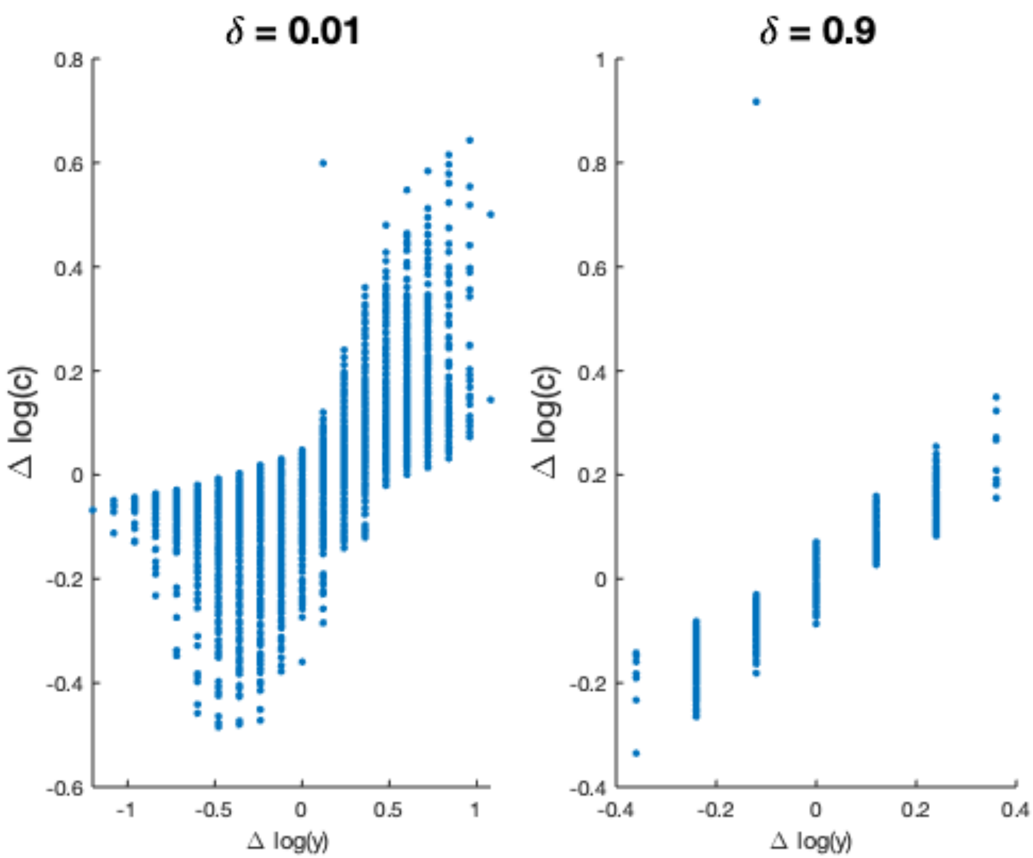


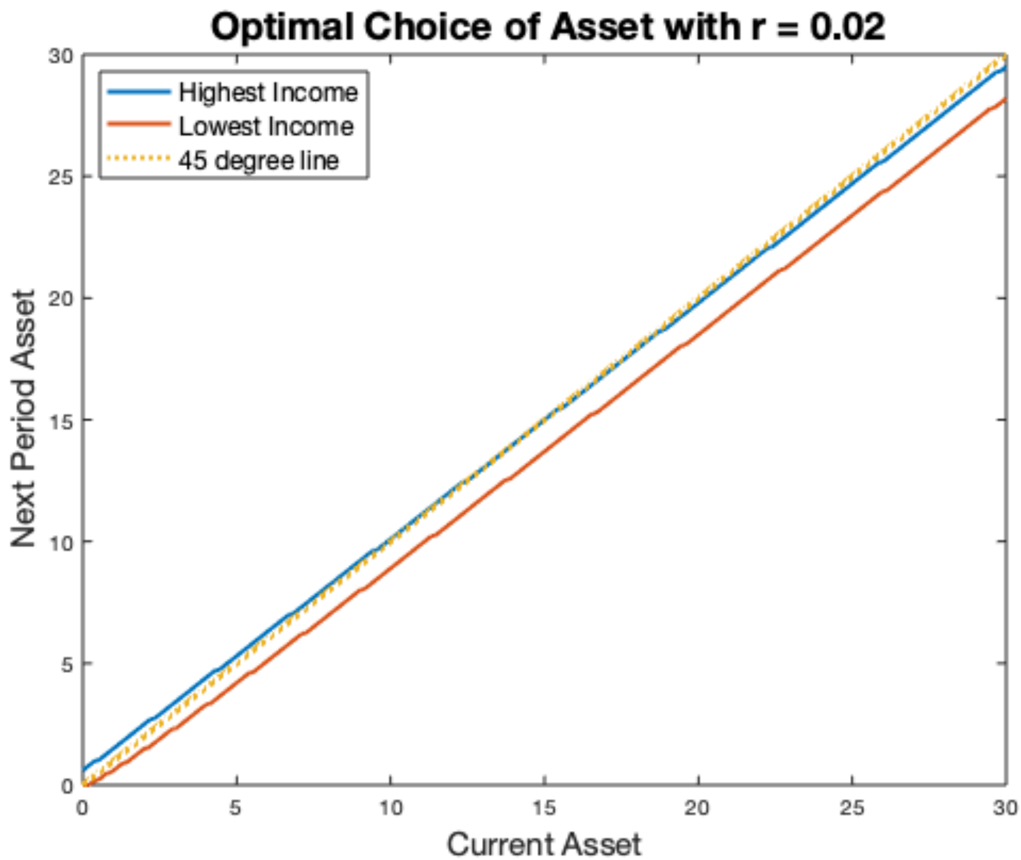
Cons with r = 0.02 and σ_y = 0.2
Consumption
Age
Life Cycle Asset
Asset
Age
Cons with r = 0 and σ_y = 0.4
Consumption
Age
Life Cycle Asset
Asset
Age

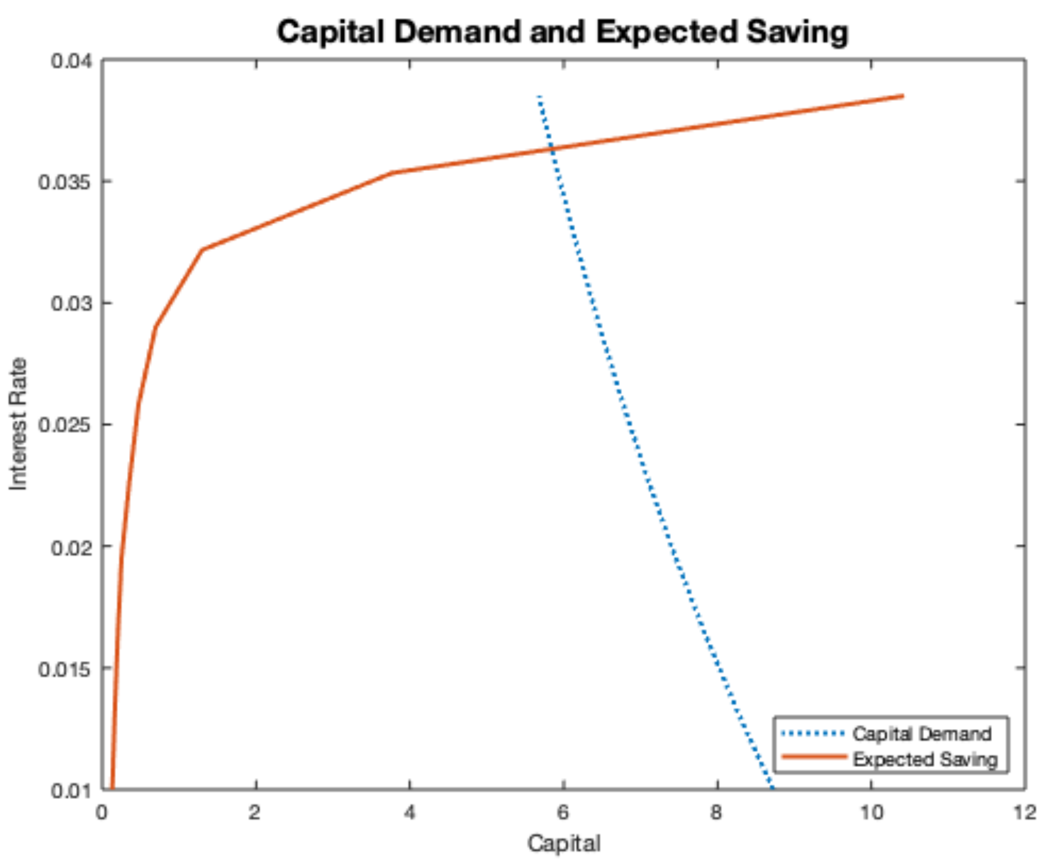


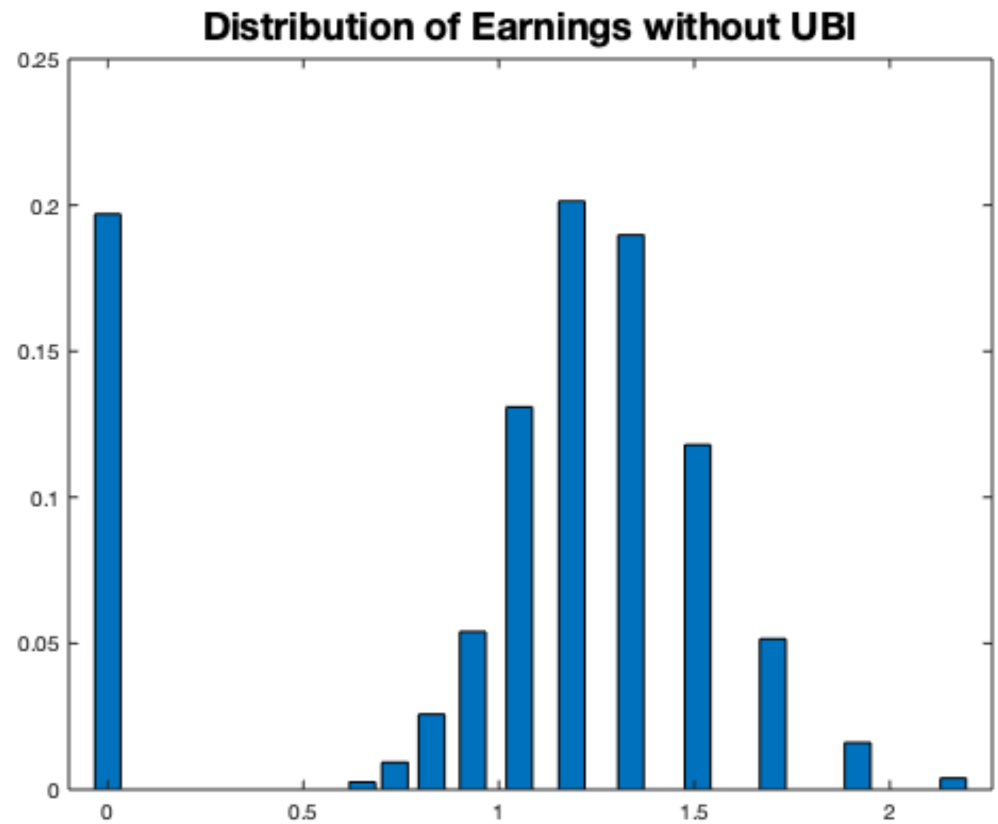
Life Cycle Consumption
Consumption
Age
Life Cycle Asset
Consumption
Age

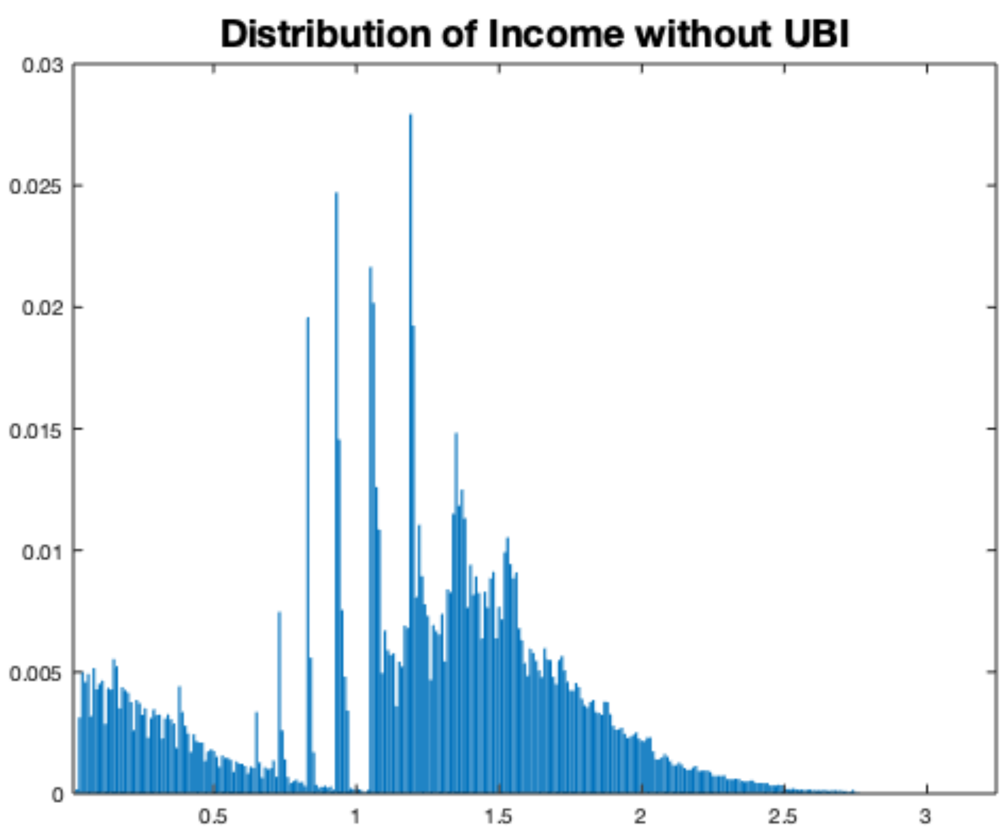

CEX Data
Consumption
Age
Model
Consumption
Age

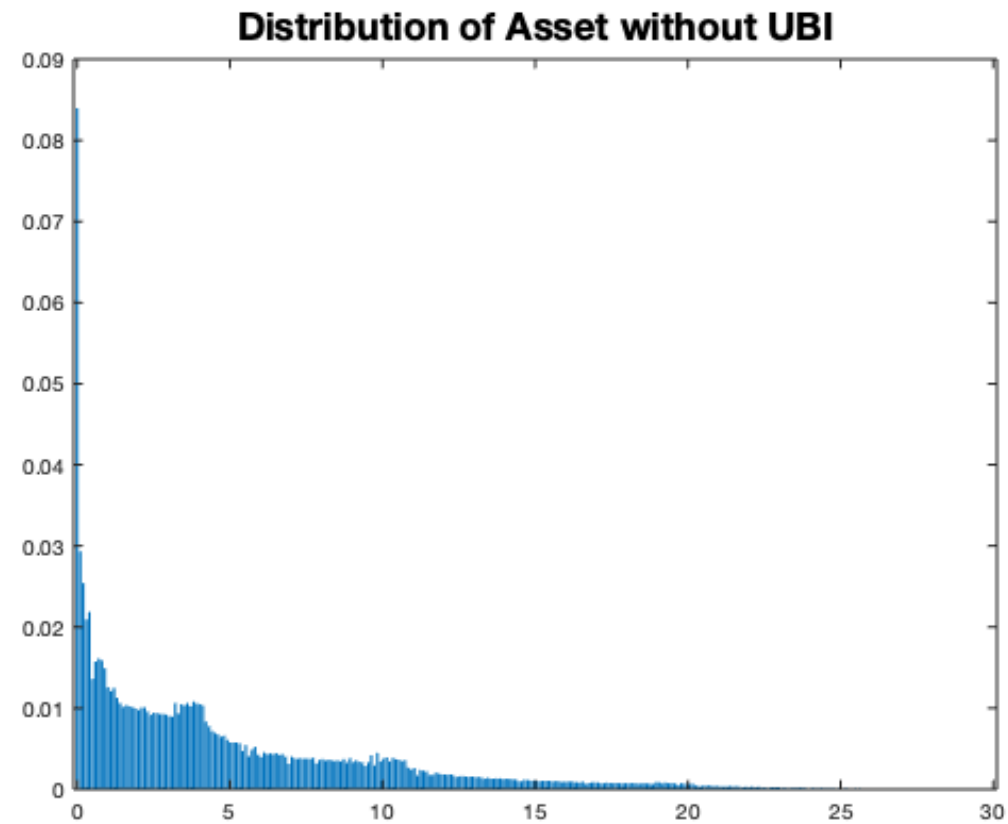

δ = 0.01
Δ log(c)
Δ log(y)
δ = 0.9
Δ log(c)
Δ log(y)

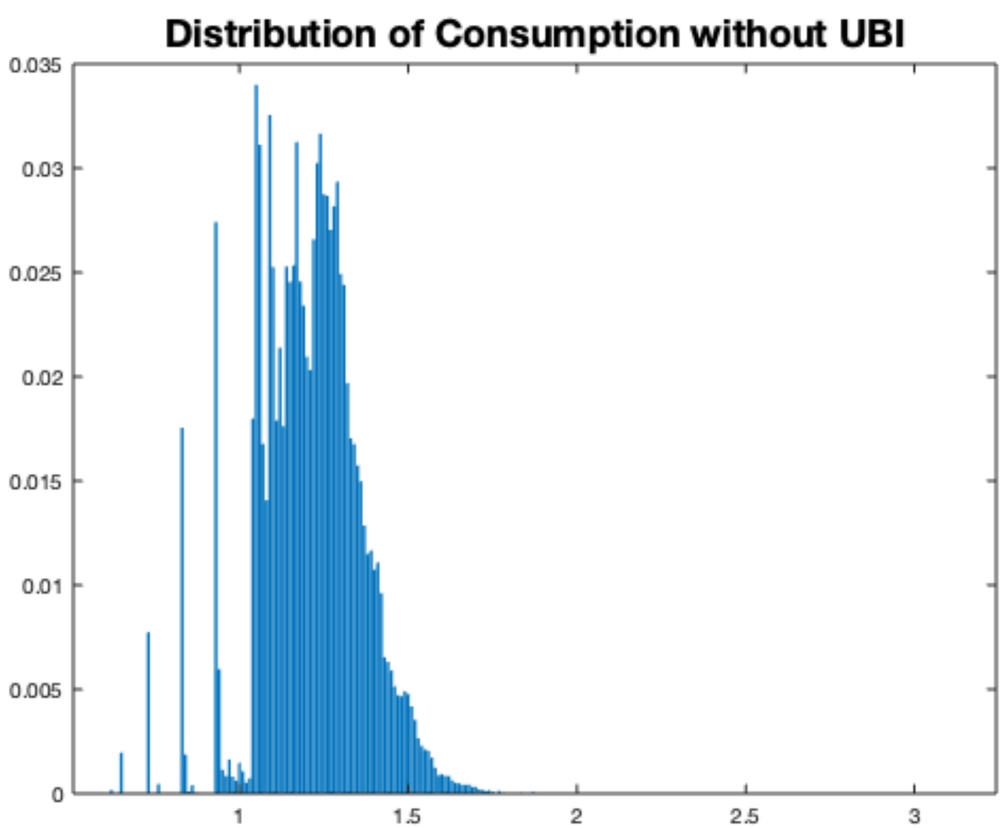

Optimal Choice of Asset with r = 0.02
Highest Income
Lowest Income
45 degree line
Next Period Asset
Current Asset
0
5
10
15
20
25
30

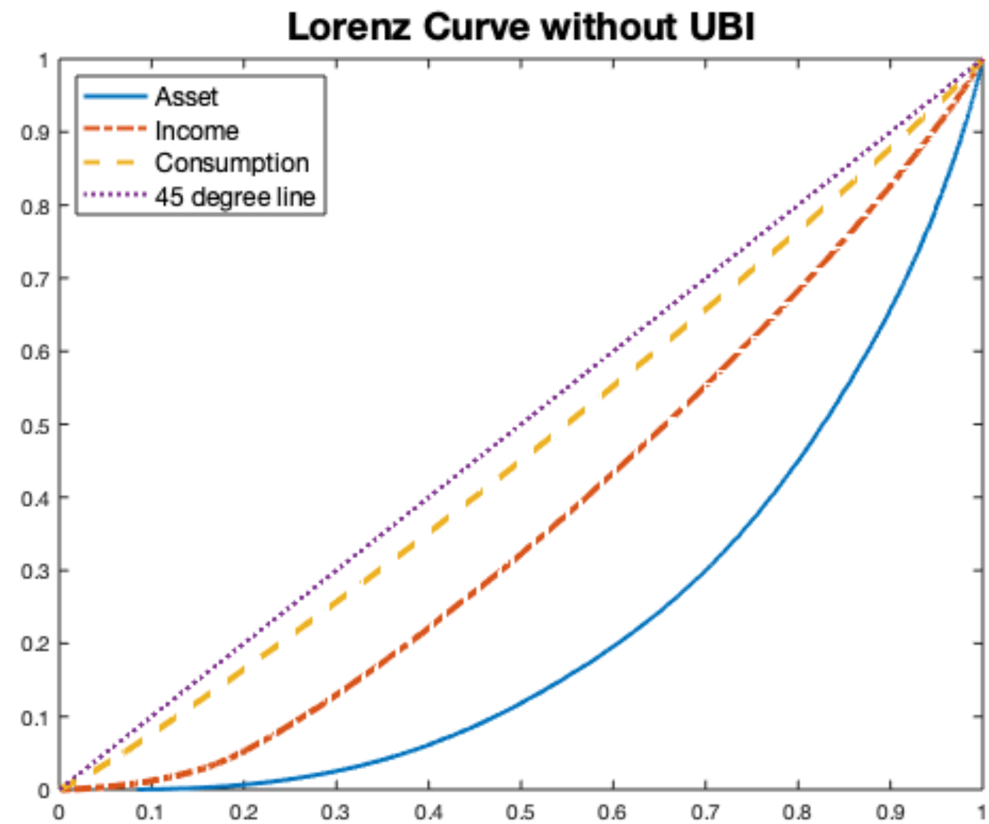

Capital Demand and Expected Saving
Capital Demand
Expected Saving
Interest Rate
Capital
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0
2
4
6
8
10
12

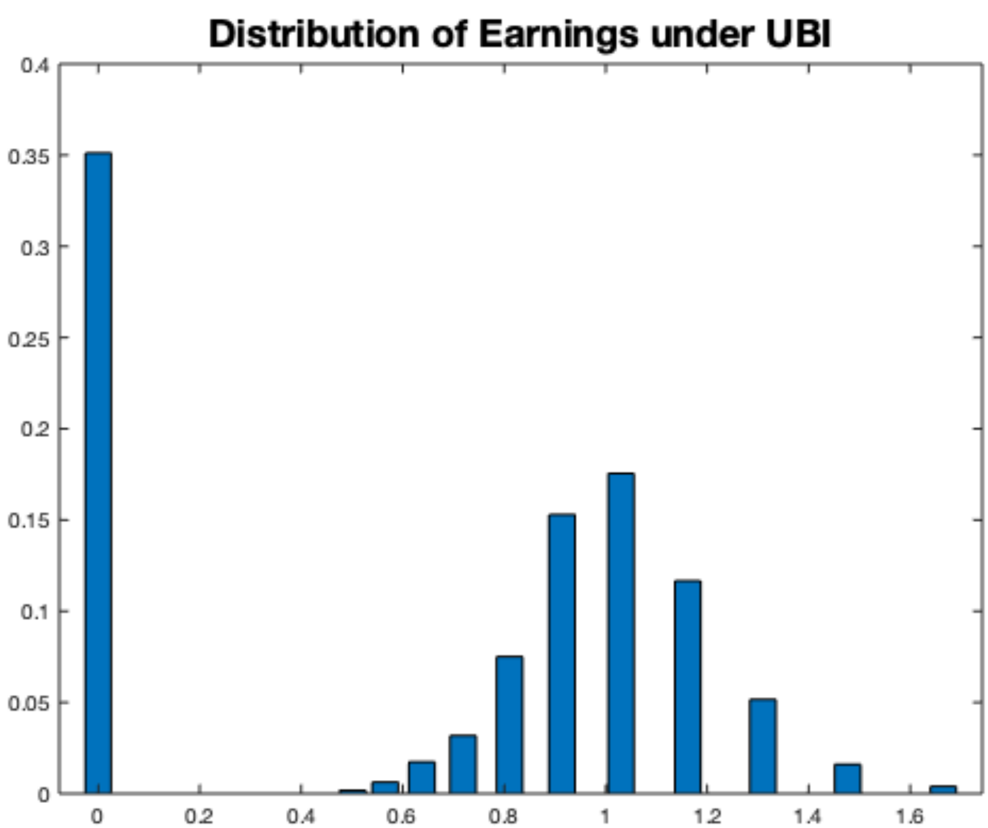

Distribution of Earnings without UBI
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2


Distribution of Income without UBI
0.03
0.025
0.02
0.015
0.01
0.005
0
0.5
1
1.5
2
2.5
3

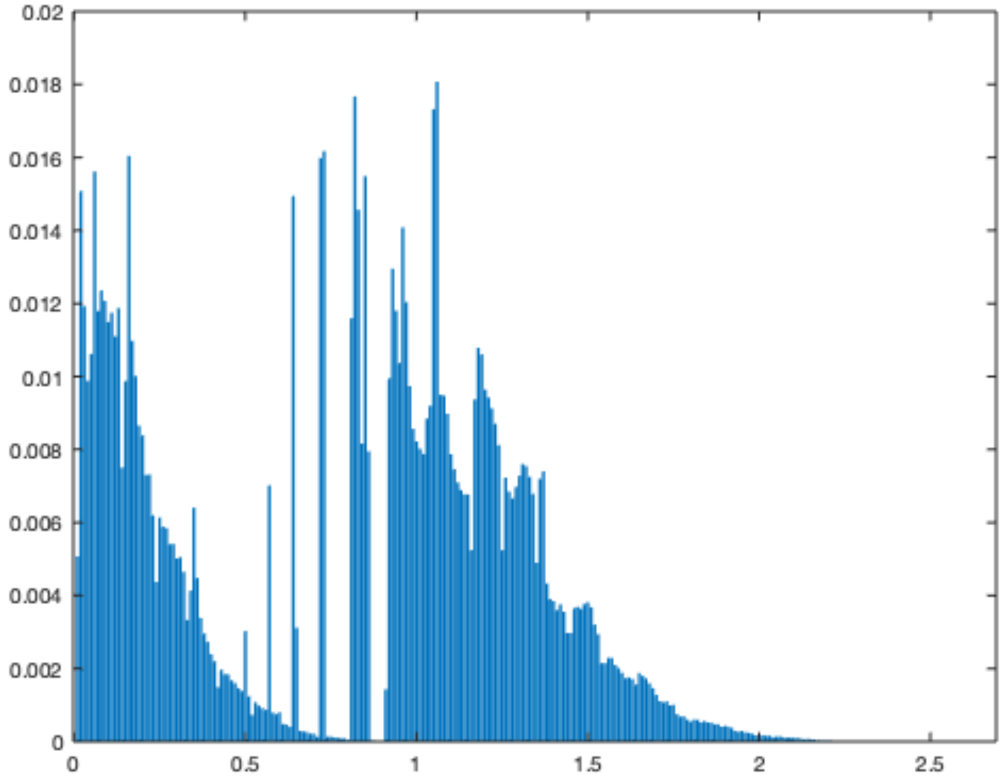

Distribution of Asset without UBI
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
5
10
15
20
25
30


Distribution of Consumption without UBI
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
1
1.5
2
2.5
3

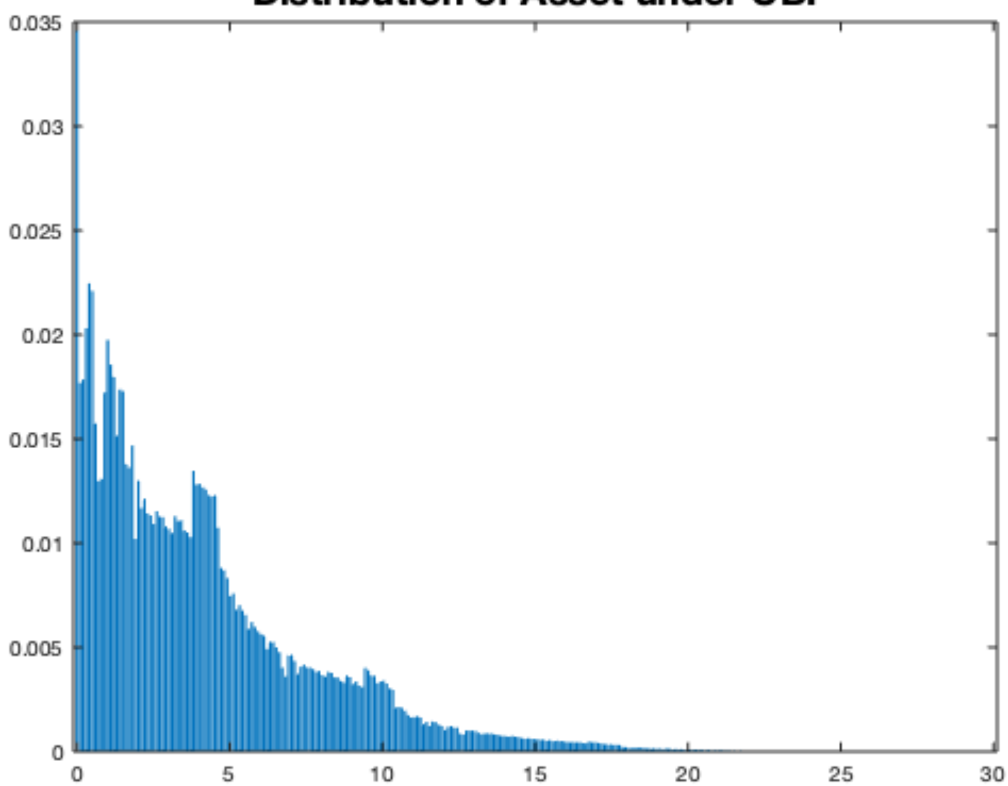

Lorenz Curve without UBI
Asset
Income
Consumption
45 degree line
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

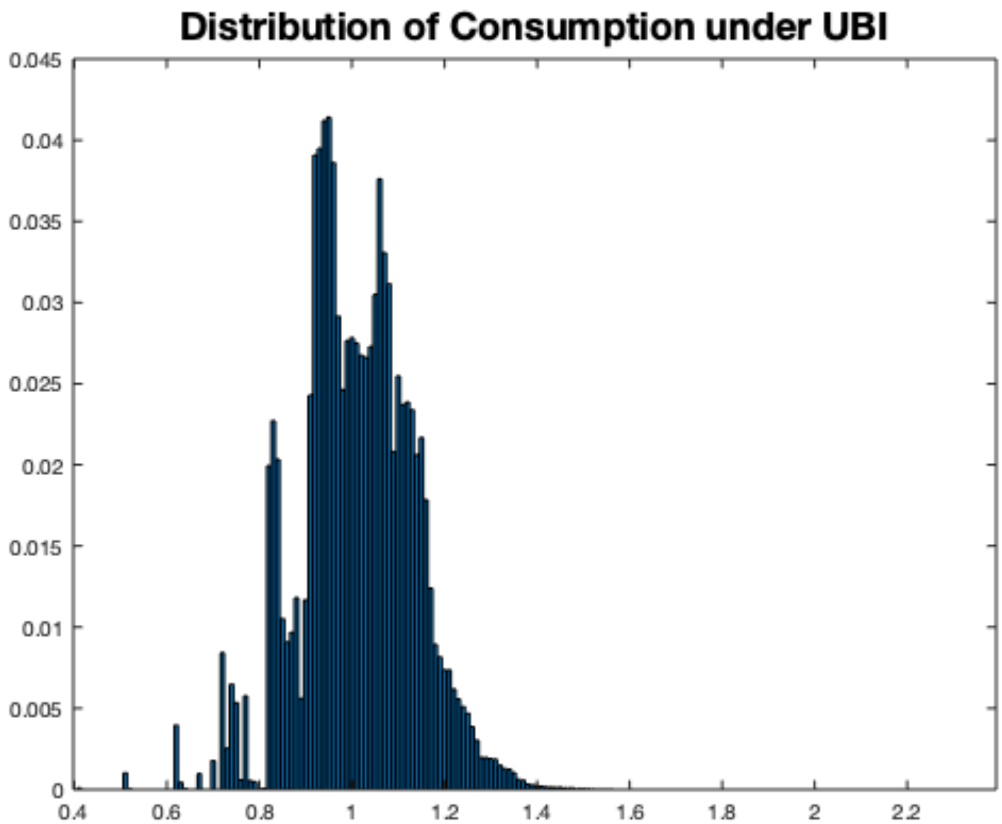

Distribution of Earnings under UBI
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

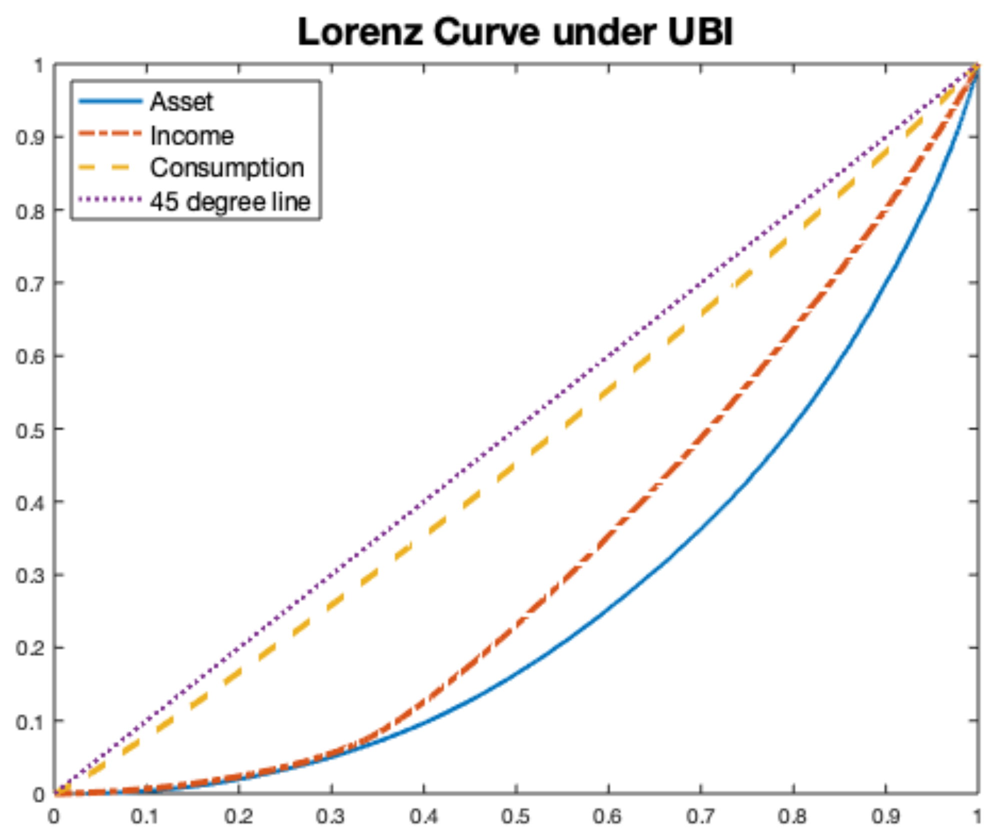
Distribution of Income under UBI



Distribution of Asset under UBI




Distribution of Consumption under UBI
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2


Lorenz Curve under UBI
Asset
Income
Consumption
45 degree line
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1