HOMEWORK SET #7

EE 510: Linear Algebra for Engineering

Assigned: 11 October 2023
Due: 21 October 2023

Directions: Please show all work and box answers when appropriate.

1. Introduction to Linear Algebra by Gilbert Strang (5th Edition):

   a) Problem Set 9.2: #8, #9, #11, #16.

2. Introduction to Linear Algebra by Gilbert Strang (5th Edition):

   a) Problem Set 6.4: #4, #5, #7, #13.

3. Let A be a unitary matrix such that $A \in \mathbb{C}^{n \times n}$.

   a) Show that the absolute value of the determinant $|Det(A)| = 1$.

   b) Show that if the eigenvalues of A are distinct then their eigenvectors are orthogonal.

4. Show that if U is unitary an $A = U^H B U$, then B is normal if and only if A is normal.

5. Prove that the eigenvalues of $A^H A$ are nonnegative.

6. Show that if x is an eigenvector of a normal matrix A corresponding to eigenvalue λ, then x is an eigenvector of A^H corresponding to the conjugate λ^*.

7. Find the principal axes, their lengths, and sketch the following ellipsoids:

   a) $2x_1^2 + 4x_2^2 + 4x_3^2 + 2x_1x_2 + 2x_2x_3 - 2x_1x_3 = 1$

   b) $5x_1^2 + 2x_2^2 + 4x_3^2 - 2\sqrt{2}x_1x_3 = 1$.