MIDTERM TWO EXAM

EE 510: Linear Algebra for Engineering — Fall 2023

7 November 2023: 4:00 pm

Directions: This midterm consists of 5 problems. Each problem is worth 6 points. The exam is worth a total of 25 points. The exam is closed book and closed note. Carefully explain your reasoning and clearly state any pertinent lemma or theorem. Do not just state a result. Draw a box around your final answers. You have 90 minutes to complete the exam.

1. Square matrix $A \in \mathbb{C}^{n\times n}$ is diagonalizable with n eigenvalues $\lambda_1, \dots \lambda_n$. Then prove or disprove: All n eigenvalues λ_i lie inside the unit circle of the complex plane ($|\lambda_i| < 1$) if and only if

$$\lim_{r\to\infty} A^r \;=\; \mathbf{0}\,.$$

2. Find the *Jordan canoncial form* (JCF) decomposition of matrix A:

$$A = \begin{bmatrix} 5 & 1 & 0 \\ 1 & 5 & 1 \\ 0 & -1 & 5 \end{bmatrix}.$$

3. Find the *singular value decomposition* (SVD) and the *pseudo-inverse* A^+ of matrix A:

$$A = \begin{bmatrix} 3 & 1 & 3 & 1 \\ 3 & 3 & -3 & -3 \\ 1 & 3 & 1 & 3 \end{bmatrix}.$$

4. Let X be a random n-vector with positive-definite covariance matrix K_{XX}:

$$K_{xx} = \begin{bmatrix} 7 & -3 \\ -3 & 7 \end{bmatrix}.$$

Suppose that the real n-by-n matrix A *filters* X to produce the random n-vector Y: $Y = AX$. Find the *whitening*-filter matrix A^{white} that "whitens" Y so that Y's covariance matrix K_{YY} is the identity matrix I.

5. Matrix A is the a payoff matrix for a two-player *zero-sum* game:

$$A = [a_{ij}] = \begin{bmatrix} -5 & 2 & -7 & 8 & 5 \\ 3 & 6 & 2 & 2 & 10 \\ -1 & 0 & -9 & 0 & 3 \\ 10 & 3 & 8 & 9 & 4 \\ 3 & 2 & 4 & 5 & -3 \end{bmatrix}.$$

Entry a_{ij} is the *payoff* for *Player 1* when Player 1 makes move i and Player 2 makes move j. Find the value $v(A)$ of the game for *stochastic vectors* $\mathbf{x}$ and $\mathbf{y}$:

$$v(A) = \max_{\mathbf{x}} \left(\min_{\mathbf{y}} \mathbf{x}^T A \mathbf{y} \right).$$