Q_1 #3 ①

$$\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & \frac{1}{5} \\ 0 & \frac{4}{5}-\lambda \end{vmatrix} = \lambda^2 - \frac{9}{5}\lambda + \frac{4}{5} = (\lambda - \frac{4}{5})(\lambda - 1) = 0$$

$\lambda_1 = \frac{4}{5}$

$$A - \frac{4}{5}I = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ 0 & 0 \end{bmatrix} \quad x_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\lambda_2 = 1$

$$A - I = \begin{bmatrix} 0 & \frac{1}{5} \\ 0 & -\frac{1}{5} \end{bmatrix} \quad x_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

②

$$\det(A-\lambda I) = \begin{vmatrix} \frac{1}{5}-\lambda & 1 \\ \frac{4}{5} & -\lambda \end{vmatrix} = \lambda^2 - \frac{1}{5}\lambda - \frac{4}{5} = (\lambda + \frac{4}{5})(\lambda - 1) = 0$$

$\lambda_1 = -\frac{4}{5}$

$$A + \frac{4}{5}I = \begin{bmatrix} 1 & 1 \\ \frac{4}{5} & \frac{4}{5} \end{bmatrix} \quad x_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\frac{5}{4} \cdot \frac{4}{9}$

$\frac{5}{4} + 1 = \frac{9}{4}$

$\lambda_2 = 1$

$$A + I = \begin{bmatrix} -\frac{4}{5} & 1 \\ \frac{4}{5} & -1 \end{bmatrix} \quad x_2 = \begin{bmatrix} \frac{5}{4} \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{5}{9} \\ \frac{4}{9} \end{bmatrix}$$

③

$$\det(A-\lambda I) = \begin{vmatrix} \frac{1}{2}-\lambda & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2}-\lambda & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2}-\lambda \end{vmatrix} = -\lambda^3 + \frac{3}{2}\lambda^2 - \frac{9}{16}\lambda + \frac{1}{16}$$

$\lambda_1 = 1$

$$A - I = \begin{bmatrix} -\frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{2} \end{bmatrix} \quad x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

$\lambda_2 = \frac{1}{4}$

$$A - \frac{1}{4}I = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix} \quad x_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda_3 = \frac{1}{4}$ $\quad x_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

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eigenvector is $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$\lambda^4 = 1$

$\lambda^2 = \pm 1$

$\lambda^2 = 1 \quad \lambda^2 = -1$

$\lambda = \pm 1 \quad \lambda = \pm i$

$\lambda_1 = 1 \quad x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{bmatrix}$

$\lambda_2 = -1 \quad x_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

$\lambda_3 = i \quad x_3 = \begin{bmatrix} i \\ -1 \\ -i \\ 1 \end{bmatrix}$

$\lambda_4 = -i \quad x_4 = \begin{bmatrix} -i \\ -1 \\ i \\ 1 \end{bmatrix}$

$u_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$

$u_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$

$u_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$

$u_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$

$\|$

u_0

$m_{11}\times m_{11} + m_{12}\times m_{21} \ \times m_{13}\times m_{31}$

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$$\begin{bmatrix} m_{11} & \cdots & m_{1n} \\ m_{21} & \ddots & \\ \vdots & & \ddots \\ m_{n1} & & m_{nn} \end{bmatrix} \times \begin{bmatrix} m_{11} & \cdots & m_{1n} \\ m_{21} & \ddots & \\ \vdots & & \ddots \\ m_{n1} & & m_{nn} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} m_{1i}\times m_{i1} & \\ \sum_{i=1}^{n} m_{2i}\times m_{i1} & \\ \sum_{i=1}^{n} m_{3i}\times m_{i1} & \\ & \end{bmatrix}$$

$m_{21}\, m_{11} \quad m_{22}\, m_{21} \quad m_{23}\, m_{31}$

$$\sum_{i=1}^{n} m_{i1} = 1$$

for column 1 of M^2.

$\because$ Sum of column 1 $= \sum_{i=1}^{n} m_{1i}\times m_{i1} + \sum_{i=1}^{n} m_{2i}\times m_{i1} + \cdots + \sum_{i=1}^{n} m_{ni}\times m_{i1}$

$$= \sum_{i=1}^{n} (m_{1i}\times m_{i1} + m_{2i}\times m_{i1} + \cdots + m_{ni}\times m_{i1})$$

$$= \sum_{i=1}^{n} [\, m_{i1} \times (m_{1i} + m_{2i} + \cdots + m_{ni})]$$

$$= \sum_{i=1}^{n} m_{i1} \times (\sum_{j=0}^{n} m_{ji})$$

$$= \sum_{i=1}^{n} m_{i1} \times 1$$

$$= 1$$

$\therefore$ silmilar for all columns of M^2. the sum of them are all 1

$\because$ M^2 is non negative

$\therefore$ M^2 is also a Markov matrix

Q2 #2

$$A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$$

$$\det(A^TA - \lambda I) = \lambda^2 - 10\lambda + 16 = (\lambda - 2)(\lambda - 8) = 0$$

$$\lambda_1 = 2 \qquad x_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \Rightarrow \bar{x}_1 = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \qquad (\sigma_1)^2 = 2 \quad \sigma_1 = \sqrt{2}$$

$$\lambda_2 = 8 \qquad x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \bar{x}_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \qquad (\sigma_2)^2 = 8 \quad \sigma_2 = 2\sqrt{2}$$

$$V = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 2\sqrt{2} \end{bmatrix}$$

$$U_1 = A \cdot V_1 / \sigma_1 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \times \frac{1}{2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$U_2 = A V_2 / \sigma_2 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} \times \frac{1}{\sqrt{2}} \times \frac{1}{2\sqrt{2}} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \times \frac{1}{4} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A = U \Sigma V^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} \sqrt{2} & \\ & 2\sqrt{2} \end{bmatrix} \times \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

#4 $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$

$3\times2 * 2\times3$

$$A^TA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad \det(A^TA - \lambda I) = -\lambda^3 + 4\lambda^2 - 3\lambda = -\lambda(\lambda-1)(\lambda-3) = 0$$

$$\lambda_1 = 0 \quad V_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \bar{V}_1 = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \quad \cancel{U_1 = AV_1/\sigma_1}$$

$$\lambda_2 = 1 \quad V_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \bar{V}_2 = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} \quad U_2 = AV_2/\sigma_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$\lambda_3 = 3 \quad V_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad \bar{V}_3 = \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix} \quad = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$U_3 = AV_3/\sigma_3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix} \times \frac{1}{\sqrt{3}}$$

$AV =$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ -1/\sqrt{3} & 0 & 2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix} = \begin{bmatrix} 0 & -\sqrt{2}/2 & \sqrt{6}/2 \\ 0 & \sqrt{2}/2 & \sqrt{6}/2 \end{bmatrix} \qquad = \begin{bmatrix} 3/\sqrt{6} \\ 3/\sqrt{6} \end{bmatrix} \times \frac{1}{\sqrt{3}} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$U\Sigma =$

$$\begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix} = \begin{bmatrix} 0 & -\sqrt{2}/2 & \sqrt{6}/2 \\ 0 & \sqrt{2}/2 & \sqrt{6}/2 \end{bmatrix} \qquad AV = U\Sigma$$

$AA^T = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $\det(AA^T - \lambda I) = \lambda^2 - 4\lambda + 3 = (\lambda-1)(\lambda-3) = 0$

$\lambda_1 = 1$ $x_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\bar{x}_1 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = u_2$

$\lambda_2 = 3$ $x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\bar{x}_2 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = u_3.$

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$\det(AA^T - \lambda I) = \lambda^2 - 50\lambda = \lambda(\lambda - 50) = 0$

$9+1$

$\lambda_1 = 0$ $u_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ $\bar{u}_1 = \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$

$(\frac{1}{3})^2 + 1$

$\lambda_2 = 50$ $u_2 = \begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix}$ $\bar{u}_2 = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$

$\frac{1}{9} + 1$

$\sqrt{\frac{10}{9}}$

$\frac{1}{3} \cdot \frac{3}{\sqrt{10}}$

$\det(A^TA - \lambda I) = \lambda^2 - 50\lambda = \lambda(\lambda - 50) = 0$

$\lambda_1 = 0$ $V_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ $\bar{V}_1 = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

$\lambda_2 = 50$ $V_2 = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$ $\bar{V}_2 = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$

$\frac{1}{\sqrt{(\frac{1}{2})^2 + 1}}$ $= \frac{1}{4} + \frac{4}{4} = \frac{5}{4}$

$\frac{1}{2} \cdot \frac{2}{\sqrt{5}}$

$u_1 = A\bar{V}_1 / \sigma_1$ X

$u_2 = A\bar{V}_2 / \sigma_2 = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \times \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix} \times \frac{1}{\sqrt{50}} = \begin{bmatrix} \sqrt{5} \\ 3\sqrt{5} \end{bmatrix} \times \frac{1}{\sqrt{5}\sqrt{10}} = \begin{bmatrix} \frac{1}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} \end{bmatrix}$

$$\overset{U}{\begin{bmatrix} 0 & \frac{1}{\sqrt{10}} \\ 0 & \frac{3}{\sqrt{10}} \end{bmatrix}} \times \overset{\Sigma}{\begin{bmatrix} 0 & \\ & \sqrt{50} \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

Q3.

$$A = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 6 \end{bmatrix} \qquad \det(A) = 0$$

$$AA^T = \begin{bmatrix} 12 & 12 & -20 \\ 12 & 12 & -20 \\ -20 & -20 & 44 \end{bmatrix} \qquad \det(AA^T - \lambda I) = -\lambda^3 + 68\lambda^2 - 256\lambda = -\lambda(\lambda^2 - 68\lambda + 256)$$

$$= -\lambda(\lambda - 4)(\lambda - 64) = 0$$

$$\lambda_1 = 0 \qquad u_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \qquad \vec{u}_1 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$\lambda_2 = 4 \qquad u_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \bar{u}_2 = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

$$\lambda_3 = 64 \qquad u_3 = \begin{bmatrix} -1/2 \\ -1/2 \\ 1 \end{bmatrix} \qquad \bar{u}_3 = \begin{bmatrix} -\sqrt{2}/2\sqrt{3} \\ -\sqrt{2}/2\sqrt{3} \\ \sqrt{2}/\sqrt{3} \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 12 & 12 & -20 \\ 12 & 12 & -20 \\ -20 & -20 & 44 \end{bmatrix}$$ get same eigenvalue and eigenvectors.

$$\therefore \quad U = \begin{bmatrix} \sqrt{2}/2\sqrt{3} & 1/\sqrt{3} & 0 \\ \sqrt{2}/2\sqrt{3} & 1/\sqrt{3} & 0 \\ \sqrt{2}/\sqrt{3} & 1/\sqrt{3} & 0 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 8 & & \\ & 2 & \\ & & 0 \end{bmatrix} \qquad V^T = \begin{bmatrix} -\sqrt{2}/2\sqrt{3} & \sqrt{2}/2\sqrt{3} & \sqrt{2}/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

$$A = U \Sigma V^T$$

$$A^{\dagger} = V \Sigma^{+} U^T = \begin{bmatrix} \sqrt{2}/2\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{2} \\ \sqrt{2}/2\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{2} \\ \sqrt{2}/\sqrt{3} & 1/\sqrt{3} & 0 \end{bmatrix} \times \begin{bmatrix} \frac{1}{8} & & \\ & \frac{1}{2} & \\ & & 0 \end{bmatrix} \times \begin{bmatrix} -\sqrt{2}/2\sqrt{3} & -\sqrt{2}/2\sqrt{3} & \sqrt{2}/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{16} & \frac{3}{16} & \frac{1}{8} \\ \frac{3}{16} & \frac{3}{16} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{4} \end{bmatrix}$$

Q4

$$\det(A-\lambda I) = -\lambda^3 + \frac{7}{10}\lambda^2 + \frac{7}{10}\lambda - \frac{2}{5} = 0$$

$$\frac{19}{9} + \frac{17}{9} + \frac{9}{9} = \frac{45}{9} = 5$$

$$\lambda_1 = 1 \quad X_1 = \begin{bmatrix} \frac{19}{9} \\ \frac{17}{9} \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 19/45 \\ 17/45 \\ 1/5 \end{bmatrix}$$

$$\lambda_2 = -4/5 \quad X_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda_3 = \frac{1}{2} \quad X_3 = \begin{bmatrix} -7/13 \\ -6/13 \\ 1 \end{bmatrix}$$

$$\det(B-\lambda I) = -\lambda^3 + \frac{1}{5}\lambda^2 + \lambda - \frac{1}{5}$$

$$\lambda_1 = 1 \quad X_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}$$

$$\lambda_2 = -1 \quad X_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda_3 = \frac{1}{5} \quad X_3 = \begin{bmatrix} \frac{1}{2} \\ \frac{-3}{2} \\ 1 \end{bmatrix}$$

Q5 $\quad AA^T = \begin{bmatrix} 15 & 8 \\ 8 & 18 \end{bmatrix} \qquad A^TA = \begin{bmatrix} 2 & 2 & 1 & 2 & -2 \\ 2 & 4 & 6 & 0 & 2 \\ 1 & 6 & 13 & -4 & 9 \\ 2 & 0 & -4 & 4 & -6 \\ -2 & 2 & 9 & -6 & 10 \end{bmatrix}$

[U, S, V] = svd(A)

B = pinv(A)

[U, S, V] = svd(A)

```
>> [U,S,V] = svd(A)

U =

   -281/440       -1489/1935
  -1489/1935       281/440


S =

   2606/525          0             0             0             0
      0           4982/1723        0             0             0


V =

    363/13768      -487/1000      -471/583       305/6156      -927/2834
  -1099/4271      -2285/4293       206/6627     -738/1421      1174/1905
   -245/352        -887/2487       872/2265      805/3916      -383/861
    324/1045       -671/1519       337/1925      457/618       1739/4800
   -928/1563       4073/10273    -1558/3807      568/1525       898/2093
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$$A = U \Sigma V^T$$

$$A^+ = V \Sigma^+ U^T$$

$$A^+ = \begin{bmatrix} \frac{13}{103} & \frac{-23}{206} \\ \frac{18}{103} & \frac{-8}{103} \\ \frac{19}{103} & \frac{3}{103} \\ \frac{8}{103} & \frac{-15}{103} \\ \frac{-3}{103} & \frac{67}{206} \end{bmatrix}$$