The definition of zero temperature (ground state) retarded Green-Function:

$$G_{AB}^{r}(t) = -i\langle\Omega|A(t)B \pm BA(t)|\Omega\rangle$$

where $t > 0$.

In Heisenberg picture

$$A(t) = e^{iHt}Ae^{-iHt}$$

$$G_{AB}^{r}(t) = -i\langle\Omega\left|e^{iHt}Ae^{-iHt}B \pm Be^{iHt}A\mathrm{e}^{-iHt}\right|\Omega\rangle$$

$$G_{AB}^{r}(t) = -i\langle\Omega\left|e^{iE_0 t}Ae^{-iHt}B \pm Be^{iHt}A\mathrm{e}^{-iE_0 t}\right|\Omega\rangle$$

where E_0 is the ground state energy of H.

Fourier transformation of retarded Green Function:

$$G_{AB}^{r}(\omega) = \int_{-\infty}^{\infty} dt G_{AB}^{r}(t)e^{i(\omega+i\eta)t}$$

$$G_{AB}^{r}(\omega) = G_{AB}^{r1}(\omega) \pm G_{AB}^{r2}(\omega)$$

$$G_{AB}^{r1}(\omega) = -i\langle\Omega \mid A\int_{0}^{\infty} dte^{i[(\omega+i\eta)-(H-E_0)]t}\,B \mid \Omega\rangle$$

$$G_{AB}^{r2}(\omega) = -i\langle\Omega \mid B\int_{0}^{\infty} dte^{i[(\omega+i\eta)+(H-E_0)]t}\,A \mid \Omega\rangle$$

$$G_{AB}^{r1}(\omega) = \left\langle\Omega \,\middle|\, A\frac{1}{(\omega+i\eta)-(H-E_0)}B \,\middle|\, \Omega\right\rangle$$

$$G_{AB}^{r2}(\omega) = \left\langle\Omega \,\middle|\, B\frac{1}{(\omega+i\eta)+(H-E_0)}A \,\middle|\, \Omega\right\rangle$$

where $\eta = 0^{+}$.