Studies of the $J-K-\Gamma$ Model on Triangular Lattice

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2018年9月11日

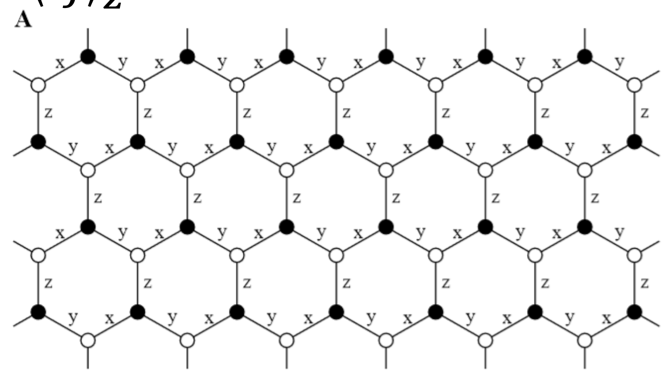
Outline

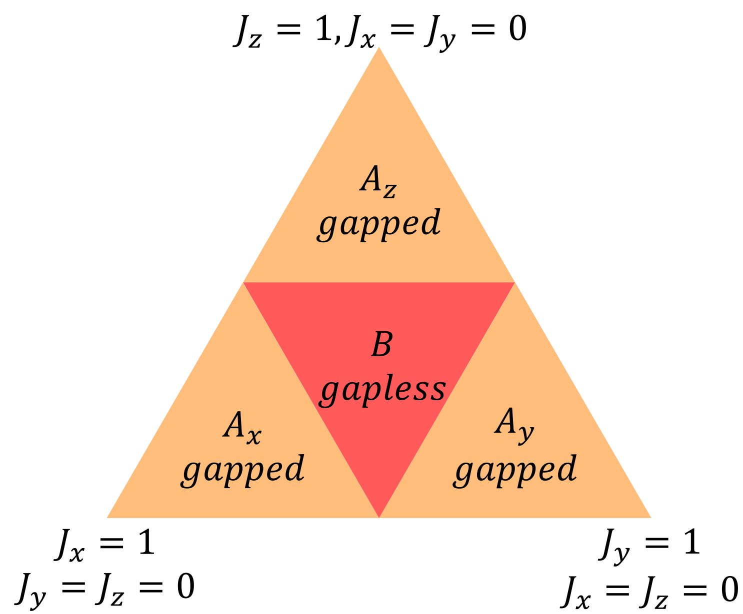
- Background and motivation
- $J-K-\Gamma$ model
  - ED results for undoped $J-K-\Gamma$ model
  - Mean-field analysis of the doped $J-K-\Gamma$ model
- Future work

Kitaev model

$$H = -J_x \sum_{\langle ij \rangle_x} \sigma_i^x \sigma_j^x - J_y \sum_{\langle ij \rangle_y} \sigma_i^y \sigma_j^y - J_z \sum_{\langle ij \rangle_z} \sigma_i^z \sigma_j^z$$

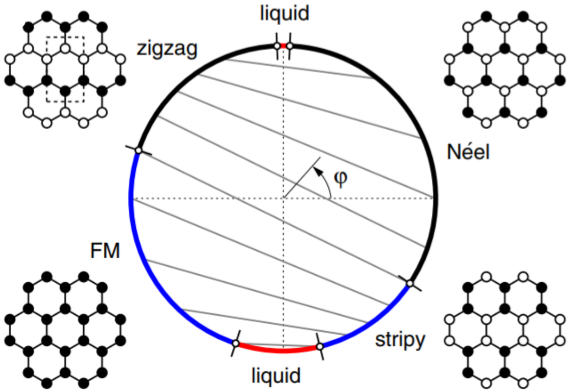
1. Exactly solved 2D quantum model
2. The gapped phase carries excitations that are Abelian anyons
3. For the gapless phase, the excitations are non-Abelian anyons
4. Anyons might be used in fault tolerant quantum computations





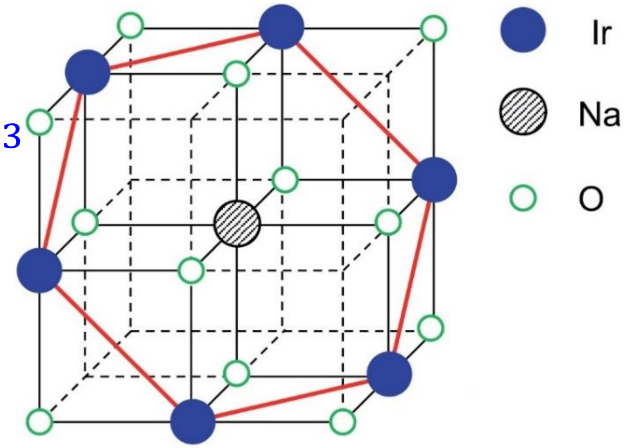
Alexei Kitaev, Annals of Physics **321**, 2 (2006)

Kitaev Heisenberg model



Whether it is possible to realize the Kitaev model with its QSL state in a solid setting?

The necessary interplay of spin-orbit coupling and electron interactions was found to be realized at first in Na_2IrO_3, then in Li_2IrO_3 and more recently in $\alpha RuCl_3$

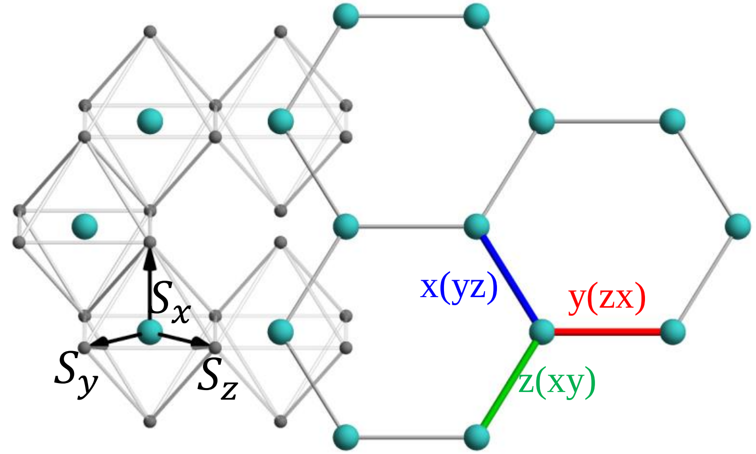


$$H = J\sum_{\langle ij\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + K\sum_{\langle ij\rangle} S_i^{\gamma_{ij}} S_j^{\gamma_{ij}}$$

However, all of these materials were later found to be magnetically ordered at low temperatures, which highlights the importance of further interactions. The symmetric off-diagonal exchange Γ is then proven crucial for explaining some of the observed magnetic orderings

G. Jackeli and G. Khaliullin, Phys. Rev. Lett. **102**, 017205(2009)
J. Chaloupka, G. Jackeli and G. Khaliullin, Phys. Rev. Lett. **110**, 097204(2013)

Extended KH Model on Honeycomb Lattice



$$\boldsymbol{H} = \boldsymbol{H_k} + \boldsymbol{H_{JK\Gamma}}$$

$$\boldsymbol{H_k} = -\tilde{t}\sum_{\langle ij\rangle,\sigma}\left(f^{\dagger}_{i,\sigma,a} f_{i,\sigma,b} + H.c.\right) + \tilde{\mu}\sum_{i,\sigma,o} f^{\dagger}_{i,\sigma,o} f_{i,\sigma,o}$$

$$\boldsymbol{H_{JK\Gamma}} = \Gamma\sum_{\langle ij\rangle}\left(S_i^{\alpha(ij)} S_j^{\beta(ij)} + S_i^{\beta(ij)} S_j^{\alpha(ij)}\right) + J\sum_{\langle ij\rangle}\left(S_i \cdot S_j - \frac{1}{4} n_i n_j\right) + K\sum_{\langle ij\rangle} S_i^{\gamma(ij)} S_j^{\gamma(ij)}$$

1. Another line of previous works proceeded to look at the superconducting phases produced by the Kitaev interaction.
2. At half-filling, the spin model is assumed to be the effective interaction in a (spin-orbit coupled) Mott insulator.
3. Finite doping might induce superconductivity in some Mott insulator.
4. What superconducting phase might produced by Γ interaction?

Johann Schmidt et al., Phys. Rev. B **97,** 014504 (2018)

Schwinger Fermion Representation

- The general approach for understanding its physical properties is mean-field approximation.
- For ordered spin states, we can simply replace the spin operator S_i^α by its quantum averages $\langle S_i^\alpha \rangle$ and obtain a mean-field Hamiltonian.
- For spin-liquid states, this approach is invalid, because the local moment is zero. To obtain the mean-field ground state of the spin-liquids the spin-1/2 charge-neutral spin operators were introduced:

$$S_i^\alpha = \frac{1}{2}\begin{bmatrix} f_{i\uparrow}^\dagger & f_{i\downarrow}^\dagger \end{bmatrix} \sigma^\alpha \begin{bmatrix} f_{i\uparrow} \\ f_{i\downarrow} \end{bmatrix} \quad \alpha = x, y, z \quad \left\{ f_{i\sigma}, f_{j\sigma'}^\dagger \right\} = \delta_{ij}\delta_{\sigma\sigma'} \quad \left\{ f_{i\sigma}, f_{j\sigma'} \right\} = 0$$

For the original spin model, there are two spin states per site, i.e. $|\uparrow\rangle$ and $|\downarrow\rangle$.
Adopting the above representation, there are four states per site, i.e. $|\,0\rangle$, $|\uparrow\rangle$, $|\downarrow\rangle$ and $|\uparrow\downarrow\rangle$.
The equivalence between the original and the rewritten Hamiltonian is valid only in the subspace where there is exactly one fermion per site:

$$f_{i\uparrow}^\dagger f_{i\uparrow} + f_{i\downarrow}^\dagger f_{i\downarrow} = 1$$

Spin-singlet and triplet pairing operators

To study the possible superconducting pairing in the spin model, we introducing spin-singlet and triplet pairing operators defined on the nearest-neighbor bonds:

$$s_{ij}^{\dagger} = \frac{1}{\sqrt{2}}\left[f_{i\uparrow}^{\dagger} \quad f_{i\downarrow}^{\dagger}\right] i\sigma_y\sigma_0 \begin{bmatrix} f_{j\uparrow}^{\dagger} \\ f_{j\downarrow}^{\dagger} \end{bmatrix} \qquad t_{ij}^{\alpha\dagger} = \frac{1}{\sqrt{2}}\left[f_{i\uparrow}^{\dagger} \quad f_{i\downarrow}^{\dagger}\right] i\sigma_y\sigma_\alpha \begin{bmatrix} f_{j\uparrow}^{\dagger} \\ f_{j\downarrow}^{\dagger} \end{bmatrix}$$

where $\alpha = x, y, z$, and σ are the Pauli matrices acting on spin space, with σ_0 sbeing the 2×2 identity matrix.

$$c_{i\uparrow}^{\dagger}c_{j\uparrow}^{\dagger} = \frac{1}{\sqrt{2}}\left(t_{ij}^{x\dagger} - i t_{ij}^{y\dagger}\right) \qquad c_{i\downarrow}^{\dagger}c_{j\downarrow}^{\dagger} = -\frac{1}{\sqrt{2}}\left(t_{ij}^{x\dagger} + i t_{ij}^{y\dagger}\right)$$

$$c_{i\uparrow}^{\dagger}c_{j\downarrow}^{\dagger} = \frac{1}{\sqrt{2}}\left(s_{ij}^{\dagger} - t_{ij}^{z\dagger}\right) \qquad c_{i\downarrow}^{\dagger}c_{j\uparrow}^{\dagger} = -\frac{1}{\sqrt{2}}\left(s_{ij}^{\dagger} + t_{ij}^{z\dagger}\right)$$

Decoupling Scheme

$$\boldsymbol{S}_i \cdot \boldsymbol{S}_j - \frac{1}{4}n_i n_j = -s_{ij}^{\dagger}s_{ij} \qquad S_i^{\alpha}S_j^{\beta} + S_i^{\beta}S_j^{\alpha} = \frac{1}{2}\left(t_{ij}^{\alpha\dagger}t_{ij}^{\beta} + t_{ij}^{\beta\dagger}t_{ij}^{\alpha}\right)$$

$$S_i^x S_j^x = \frac{1}{4}\left(-s_{ij}^{\dagger}s_{ij} - t_{ij}^{x\dagger}t_{ij}^{x} + t_{ij}^{y\dagger}t_{ij}^{y} + t_{ij}^{z\dagger}t_{ij}^{z}\right)$$

$$S_i^y S_j^y = \frac{1}{4}\left(-s_{ij}^{\dagger}s_{ij} + t_{ij}^{x\dagger}t_{ij}^{x} - t_{ij}^{y\dagger}t_{ij}^{y} + t_{ij}^{z\dagger}t_{ij}^{z}\right)$$

$$S_i^z S_j^z = \frac{1}{4}\left(-s_{ij}^{\dagger}s_{ij} + t_{ij}^{x\dagger}t_{ij}^{x} + t_{ij}^{y\dagger}t_{ij}^{y} - t_{ij}^{z\dagger}t_{ij}^{z}\right)$$

- Xiao-Gang Wen, Phys. Rev. B **65,** 165113 (2002)
- S. Okamoto, Phys. Rev. Lett. **110,** 066403 (2013)
- Kai Li, Shun-Li Yu, Jian-Xin Li, New J. Phys. **17,** 043032 (2015)

Mean-Field Approximation

$$H_{JK\Gamma} = J\sum_{\langle ij\rangle} -s_{ij}^{\dagger}s_{ij} + \frac{\Gamma}{2}\sum_{\langle ij\rangle}\left(t_{ij}^{\alpha_{ij}\dagger}t_{ij}^{\beta_{ij}} + t_{ij}^{\beta_{ij}\dagger}t_{ij}^{\alpha_{ij}}\right)$$
$$+ \frac{K}{4}\sum_{\langle ij\rangle}\left(-s_{ij}^{\dagger}s_{ij} + t_{ij}^{\alpha_{ij}\dagger}t_{ij}^{\alpha_{ij}} + t_{ij}^{\beta_{ij}\dagger}t_{ij}^{\beta_{ij}} - t_{ij}^{\gamma_{ij}\dagger}t_{ij}^{\gamma_{ij}}\right)$$

$$\boldsymbol{d} = \begin{bmatrix} d_1^x & d_2^x & d_3^x \\ d_1^y & d_2^y & d_3^y \\ d_1^z & d_2^z & d_3^z \end{bmatrix}$$

Replacing the singlet and triplet pairing operators by their expectation values. For complete generality, there are in total 12 different order parameters. Three of these are singlet order parameters, one for each nearest-neighbor bond. The remaining nine are triplet order parameters which make up the usual $\boldsymbol{d}$ vector.

The resulting mean-field Hamiltonian is:

$$H_{JK\Gamma}^{mean} = \sum_{\langle ij\rangle}\left(\Delta_{ij}s_{ij}^{\dagger} + \sum_{\alpha} d_{ij}^{\alpha}t_{ij}^{\alpha\dagger} + h.c\right)$$

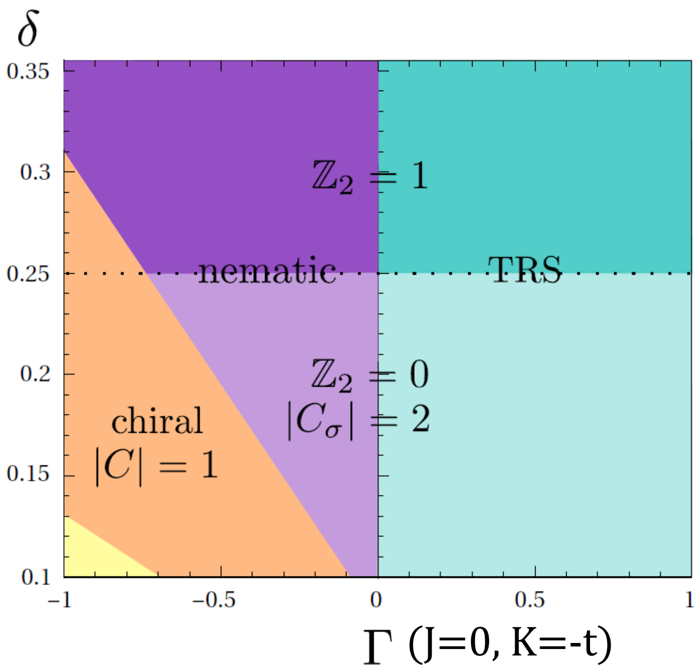
$$\Delta = \frac{1}{\sqrt{2}}\left(-J - \frac{K}{4}\right)\left(\langle s_{i\delta_1}\rangle, \langle s_{i\delta_2}\rangle, \langle s_{i\delta_3}\rangle\right)$$

$$\boldsymbol{d}^x = \frac{1}{\sqrt{2}}\left(-\frac{K}{4}\langle t_{i\delta_1}^x\rangle, \frac{K}{4}\langle t_{i\delta_2}^x\rangle + \frac{\Gamma}{2}\langle t_{i\delta_2}^z\rangle, \frac{K}{4}\langle t_{i\delta_3}^x\rangle + \frac{\Gamma}{2}\langle t_{i\delta_3}^y\rangle\right)$$

$$\boldsymbol{d}^y = \frac{1}{\sqrt{2}}\left(\frac{K}{4}\langle t_{i\delta_1}^y\rangle + \frac{\Gamma}{2}\langle t_{i\delta_1}^z\rangle, -\frac{K}{4}\langle t_{i\delta_2}^y\rangle, \frac{K}{4}\langle t_{i\delta_3}^y\rangle + \frac{\Gamma}{2}\langle t_{i\delta_3}^x\rangle\right)$$

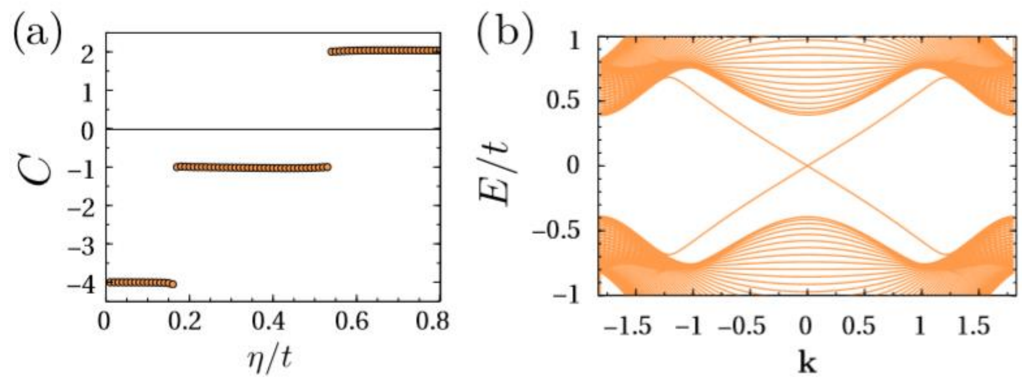
$$\boldsymbol{d}^z = \frac{1}{\sqrt{2}}\left(\frac{K}{4}\langle t_{i\delta_1}^z\rangle + \frac{\Gamma}{2}\langle t_{i\delta_1}^y\rangle, \frac{K}{4}\langle t_{i\delta_2}^z\rangle + \frac{\Gamma}{2}\langle t_{i\delta_2}^x\rangle, -\frac{K}{4}\langle t_{i\delta_3}^z\rangle\right)$$

Mean Field Phase diagram



Time-reversal symmetry-breaking states:

$$\boldsymbol{d}_{chiral} = \eta \begin{bmatrix} 0 & 1 & e^{\pm i2\pi/3} \\ e^{\mp i2\pi/3} & 0 & e^{\pm i2\pi/3} \\ e^{\mp i2\pi/3} & 1 & 0 \end{bmatrix}$$



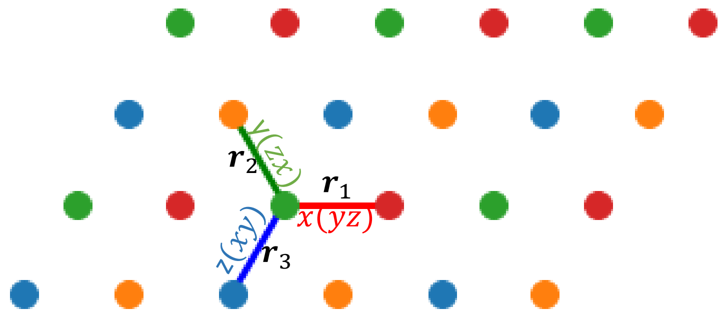
For $\Gamma > 0$, the time-reversal symmetric solution:

$$\boldsymbol{d}_{TRS} = \eta \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

is stable at all doping level. Beneath $\delta = 0.25$, it hosts a symmetry-protected topological phase, while above the stronger $\mathbb{Z}_2$ invariant becomes nontrivial.

For $\Gamma < 0$, at large doping, the superconducting order $\boldsymbol{d}_{nematic}$ breaks the C_3 symmetry but it topologically equivalent to $\boldsymbol{d}_{TRS}$; at intermediate doping, the $\boldsymbol{d}_{chiral}$ breaks time-reversal symmetry and classified by a nonzero Chern number.

$J-K-\Gamma$ model on triangular lattice



$$H_{JK\Gamma} = J\sum_{\langle ij\rangle}\boldsymbol{S}_i\cdot\boldsymbol{S}_j + K\sum_{\langle ij\rangle}S_i^{\gamma(ij)}S_j^{\gamma(ij)} + \Gamma\sum_{\langle ij\rangle}(S_i^{\alpha(ij)}S_j^{\beta(ij)} + S_i^{\beta(ij)}S_j^{\alpha(ij)})$$

$J = A\sin\beta\sin\alpha$ $\quad K = A\sin\beta\cos\alpha$ $\quad \Gamma = A\cos\beta$ $\quad A = \sqrt{J^2+K^2+\Gamma^2} = 1$

In many-body system, the matrix representation of the spin operator:

$$M_{S_i^\alpha} = I_n \otimes \cdots I_{i+1} \otimes S_i^\alpha \otimes I_{i-1} \otimes \cdots \otimes I_1$$

Where S_i^α is sigma matrix and I_j is 2×2 identity matrix

The ground state energy:

$$GSE(\alpha,\beta) = \langle\Omega|HM|\Omega\rangle$$

$\widetilde{\boldsymbol{S}_0} = \boldsymbol{S}_0$

$\widetilde{\boldsymbol{S}_1} = (S_1^x, -S_1^y, -S_1^z)$

$\widetilde{\boldsymbol{S}_2} = (-S_2^x, S_2^y, -S_2^z)$

$\widetilde{\boldsymbol{S}_3} = (-S_3^x, S_3^y, -S_3^z)$

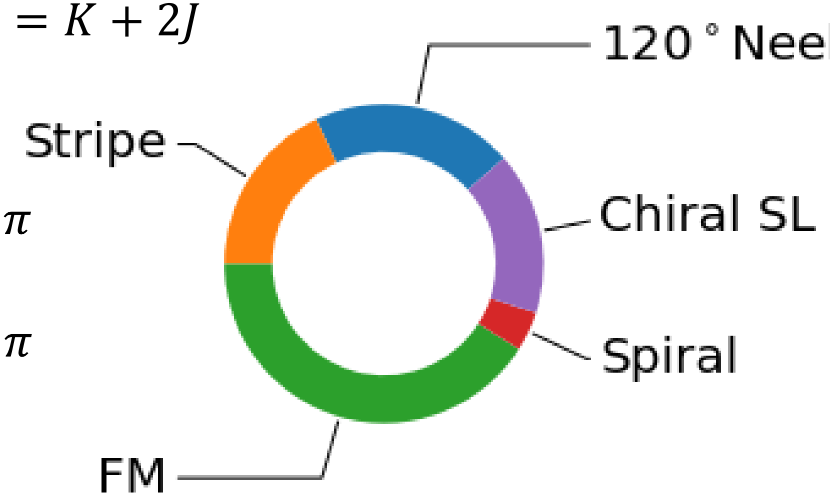
$\tilde{H} = \tilde{J}\,\widetilde{H_J} + \tilde{K}\,\widetilde{H_K} + \tilde{\Gamma}\,\widetilde{H_\Gamma}$ $\quad \tilde{J} = -J$ $\quad \tilde{K} = K + 2J$ $\quad \tilde{\Gamma} = \Gamma$

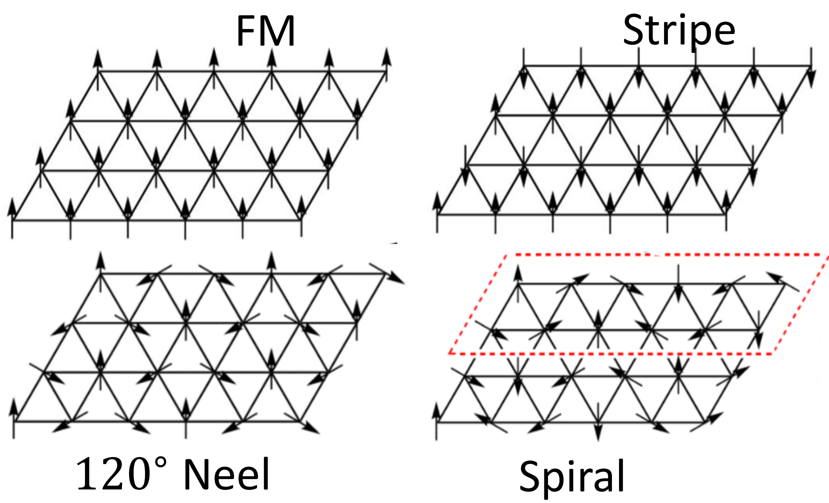
$J = \sin\beta \sin\alpha \qquad K = \sin\beta \cos\alpha \qquad \Gamma = \cos\beta$

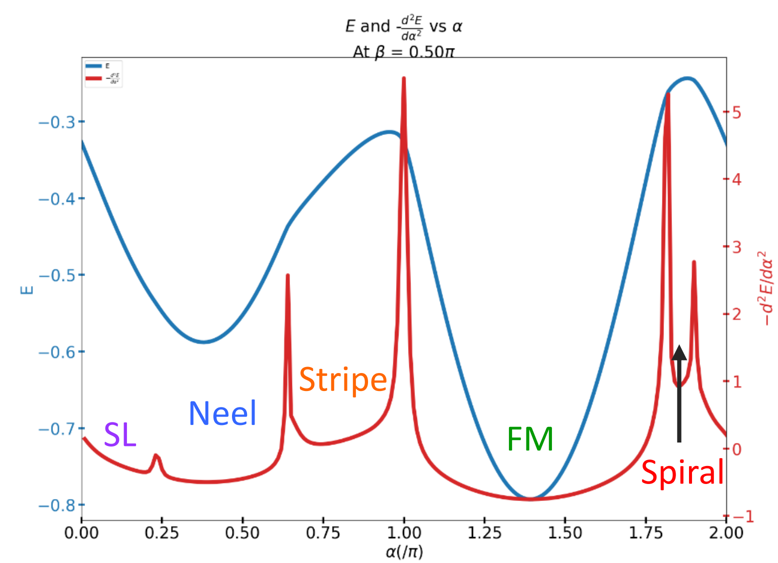
$\widetilde{H} = \tilde{J}\, \widetilde{H_J} + \widetilde{K}\, \widetilde{H_K} + \tilde{\Gamma}\, \widetilde{H_\Gamma} \qquad \tilde{J} = -J \qquad \widetilde{K} = K + 2J$

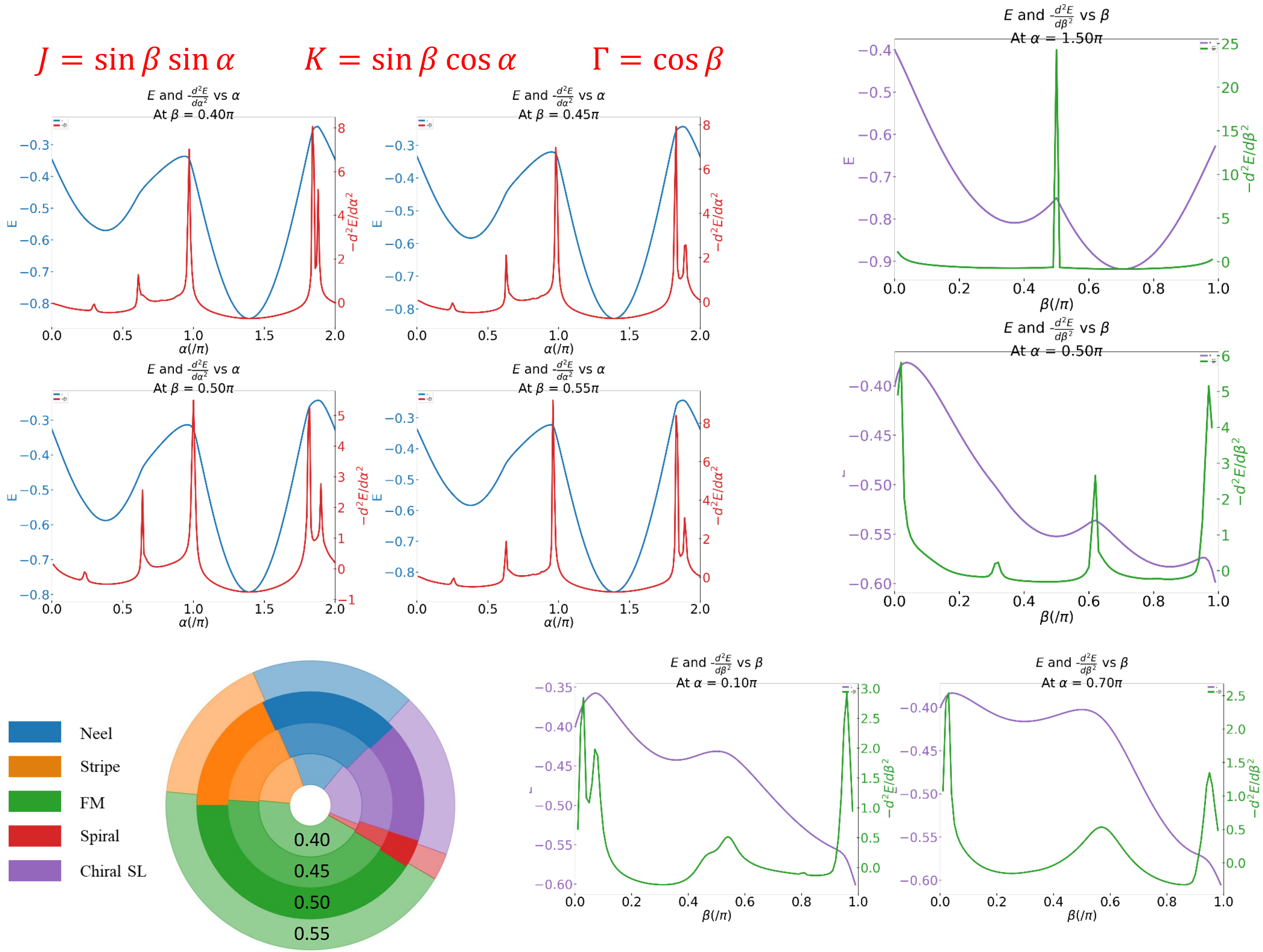
$\tilde{J} = 1, \widetilde{K} = 0 \rightarrow J = -1, K = 2 \rightarrow \alpha = -0.463\pi$

$\tilde{J} = -1, \widetilde{K} = 0 \rightarrow J = 1, K = -2 \rightarrow \alpha = 0.852\pi$



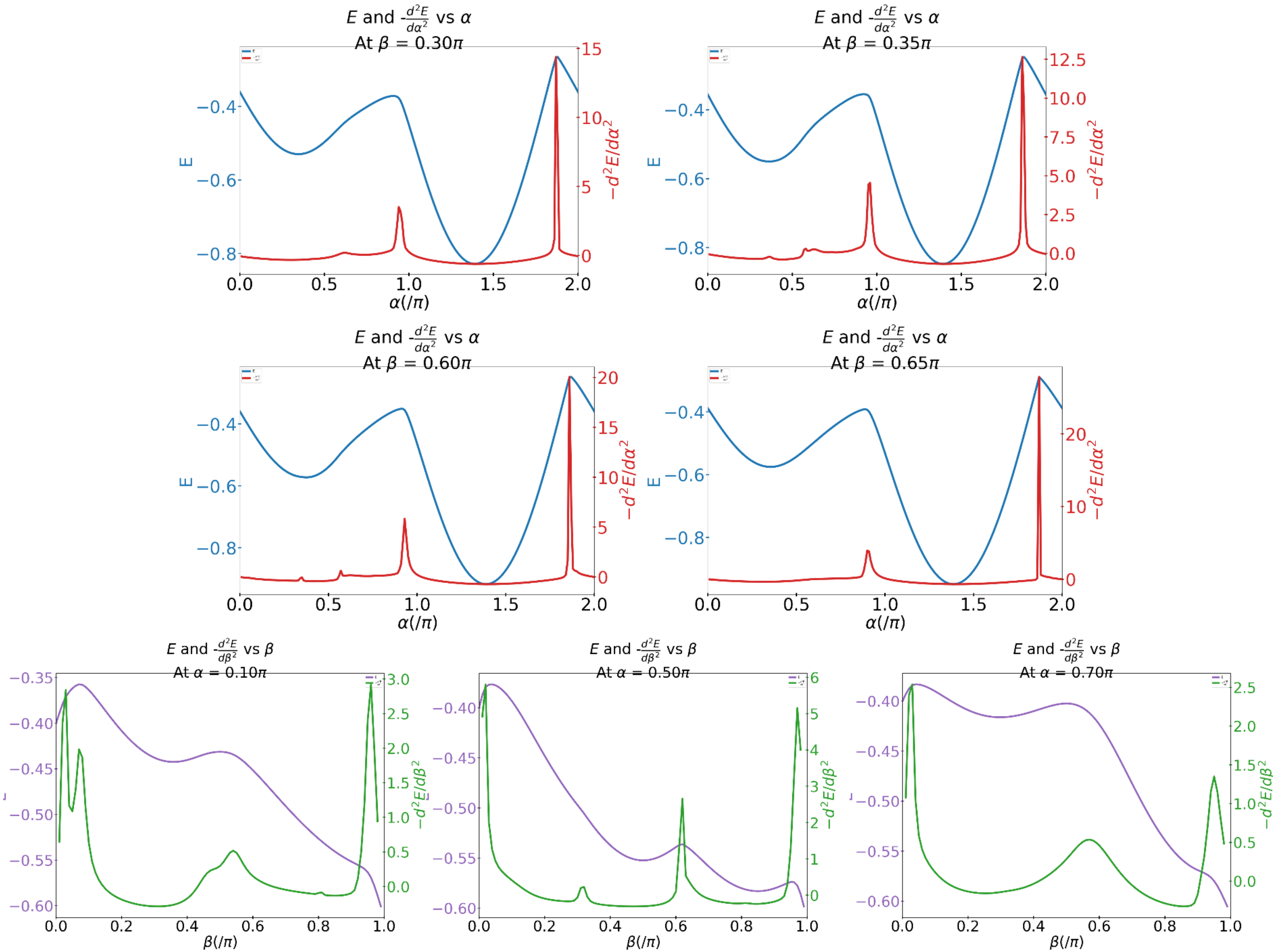




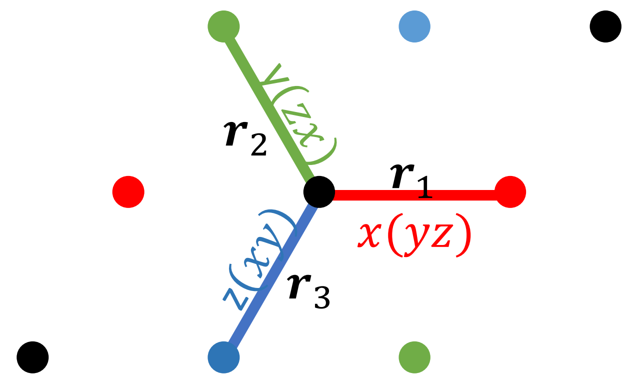


$J = \sin\beta \sin\alpha$
$K = \sin\beta \cos\alpha$
$\Gamma = \cos\beta$
E and -$\frac{d^2E}{d\alpha^2}$ vs α At β = 0.40π
E and -$\frac{d^2E}{d\alpha^2}$ vs α At β = 0.45π
E and -$\frac{d^2E}{d\alpha^2}$ vs α At β = 0.50π
E and -$\frac{d^2E}{d\alpha^2}$ vs α At β = 0.55π
$\alpha(/\pi)$
E
$-d^2E/d\alpha^2$
E and -$\frac{d^2E}{d\beta^2}$ vs β At α = 1.50π
E and -$\frac{d^2E}{d\beta^2}$ vs β At α = 0.50π
E and -$\frac{d^2E}{d\beta^2}$ vs β At α = 0.10π
E and -$\frac{d^2E}{d\beta^2}$ vs β At α = 0.70π
$\beta(/\pi)$
$-d^2E/d\beta^2$
Neel
Stripe
FM
Spiral
Chiral SL
0.40
0.45
0.50
0.55

$J = \sin\beta \sin\alpha \qquad K = \sin\beta \cos\alpha \qquad \Gamma = \cos\beta$



Effect of finite doping



$$H = H_k + H_{JK\Gamma}$$

$$H_k = -t\sum_{\langle ij\rangle\sigma}\left(c_{i\sigma}^\dagger c_{i\sigma} + h.c.\right) + \mu\sum_{i\sigma} c_{i\sigma}^\dagger c_{i\sigma}$$

$$H_{JK\Gamma} = J\sum_{\langle ij\rangle}(\boldsymbol{S}_i\cdot\boldsymbol{S}_j - \frac{1}{4}n_i n_j) + K\sum_{\langle ij\rangle} S_i^{\gamma(ij)} S_j^{\gamma(ij)} + \Gamma\sum_{\langle ij\rangle}(S_i^{\alpha(ij)} S_j^{\beta(ij)} + S_i^{\beta(ij)} S_j^{\alpha(ij)})$$

Considering the $J-K-\Gamma$ mode as the effective interaction in a Mott insulator at half filling. In order to deal with doping effect near a Mott insulating state we assume that the double occupancy is prohibited due to the strong on-site repulsive interactions and adopt the $U(1)$ slave-boson approach:

$$c_{i\sigma}^\dagger = f_{i\sigma}^\dagger b_i$$

with constraint

$$b_i^\dagger b_i + \sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} = 1$$

The bosonic holons b_i are assumed to be condensed, i.e., $b_i^\dagger \approx b_i \approx \sqrt{\langle b_i^\dagger b_i\rangle} = \sqrt{\delta}$

$$H_k' = -\delta t\sum_{\langle ij\rangle\sigma}\left(f_{i\sigma}^\dagger f_{i\sigma} + h.c.\right) + \tilde{\mu}\sum_{i\sigma} f_{i\sigma}^\dagger f_{i\sigma}$$

Mean-Field Hamiltonian in $\boldsymbol{k}$ space:

$$H_{JK\Gamma}^{mean} = \sum_{\boldsymbol{k}\in BZ} \left[c_{\boldsymbol{k}\uparrow}^{\dagger}, c_{\boldsymbol{k}\downarrow}^{\dagger}\right] M \begin{bmatrix} c_{-\boldsymbol{k}\uparrow}^{\dagger} \\ c_{-\boldsymbol{k}\downarrow}^{\dagger} \end{bmatrix} + H.c.$$

$$M = M_0 + M_x + M_y + M_z$$

$$M_0 = i\sigma_y\sigma_0\left(\Delta_1 e^{i\boldsymbol{k}\cdot\boldsymbol{r}_1} + \Delta_2 e^{i\boldsymbol{k}\cdot\boldsymbol{r}_2} + \Delta_3 e^{i\boldsymbol{k}\cdot\boldsymbol{r}_3}\right)/\sqrt{2}$$

$$M_x = i\sigma_y\sigma_x\left(d_1^x e^{i\boldsymbol{k}\cdot\boldsymbol{r}_1} + d_2^x e^{i\boldsymbol{k}\cdot\boldsymbol{r}_2} + d_3^x e^{i\boldsymbol{k}\cdot\boldsymbol{r}_3}\right)/\sqrt{2}$$

$$M_y = i\sigma_y\sigma_y\left(d_1^y e^{i\boldsymbol{k}\cdot\boldsymbol{r}_1} + d_2^y e^{i\boldsymbol{k}\cdot\boldsymbol{r}_2} + d_3^y e^{i\boldsymbol{k}\cdot\boldsymbol{r}_3}\right)/\sqrt{2}$$

$$M_z = i\sigma_y\sigma_z\left(d_1^z e^{i\boldsymbol{k}\cdot\boldsymbol{r}_1} + d_2^z e^{i\boldsymbol{k}\cdot\boldsymbol{r}_2} + d_3^z e^{i\boldsymbol{k}\cdot\boldsymbol{r}_3}\right)/\sqrt{2}$$

$$\boldsymbol{\Delta} = \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{2}}}\left(-\boldsymbol{J} - \frac{\boldsymbol{K}}{\boldsymbol{4}}\right)\left(\langle \boldsymbol{s}_{i\delta_1}\rangle, \langle \boldsymbol{s}_{i\delta_2}\rangle, \langle \boldsymbol{s}_{i\delta_3}\rangle\right)$$

$$\boldsymbol{d}^x = \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{2}}}\left(-\frac{\boldsymbol{K}}{\boldsymbol{4}}\langle \boldsymbol{t}_{i\delta_1}^x\rangle, \frac{\boldsymbol{K}}{\boldsymbol{4}}\langle \boldsymbol{t}_{i\delta_2}^x\rangle + \frac{\boldsymbol{\Gamma}}{\boldsymbol{2}}\langle \boldsymbol{t}_{i\delta_2}^z\rangle, \frac{\boldsymbol{K}}{\boldsymbol{4}}\langle \boldsymbol{t}_{i\delta_3}^x\rangle + \frac{\boldsymbol{\Gamma}}{\boldsymbol{2}}\langle \boldsymbol{t}_{i\delta_3}^y\rangle\right)$$

$$\boldsymbol{d}^y = \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{2}}}\left(\frac{\boldsymbol{K}}{\boldsymbol{4}}\langle \boldsymbol{t}_{i\delta_1}^y\rangle + \frac{\boldsymbol{\Gamma}}{\boldsymbol{2}}\langle \boldsymbol{t}_{i\delta_1}^z\rangle, -\frac{\boldsymbol{K}}{\boldsymbol{4}}\langle \boldsymbol{t}_{i\delta_2}^y\rangle, \frac{\boldsymbol{K}}{\boldsymbol{4}}\langle \boldsymbol{t}_{i\delta_3}^y\rangle + \frac{\boldsymbol{\Gamma}}{\boldsymbol{2}}\langle \boldsymbol{t}_{i\delta_3}^x\rangle\right)$$

$$\boldsymbol{d}^z = \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{2}}}\left(\frac{\boldsymbol{K}}{\boldsymbol{4}}\langle \boldsymbol{t}_{i\delta_1}^z\rangle + \frac{\boldsymbol{\Gamma}}{\boldsymbol{2}}\langle \boldsymbol{t}_{i\delta_1}^y\rangle, \frac{\boldsymbol{K}}{\boldsymbol{4}}\langle \boldsymbol{t}_{i\delta_2}^z\rangle + \frac{\boldsymbol{\Gamma}}{\boldsymbol{2}}\langle \boldsymbol{t}_{i\delta_2}^x\rangle, -\frac{\boldsymbol{K}}{\boldsymbol{4}}\langle \boldsymbol{t}_{i\delta_3}^z\rangle\right)$$

$$H = \sum_{\boldsymbol{k}} \left[c_{\boldsymbol{k}\uparrow}^{\dagger}, c_{\boldsymbol{k}\downarrow}^{\dagger}, c_{-\boldsymbol{k}\uparrow}, c_{-\boldsymbol{k}\downarrow}\right] H_M(\boldsymbol{k}) \left[c_{\boldsymbol{k}\uparrow}^{\dagger}, c_{\boldsymbol{k}\downarrow}^{\dagger}, c_{-\boldsymbol{k}\uparrow}, c_{-\boldsymbol{k}\downarrow}\right]^{\dagger}$$

$$H_M(\boldsymbol{k}) = \begin{bmatrix} \begin{matrix} -g(\boldsymbol{k},\mu) & 0 \\ 0 & -g(\boldsymbol{k},\boldsymbol{\mu}) \end{matrix} & M \\ M^{\dagger} & \begin{matrix} g(\boldsymbol{k},\boldsymbol{\mu}) & 0 \\ 0 & g(\boldsymbol{k},\mu) \end{matrix} \end{bmatrix}$$

$$g(\boldsymbol{k},\mu) = \tilde{t}(\cos \boldsymbol{k}\cdot\boldsymbol{r_1} + \cos \boldsymbol{k}\cdot\boldsymbol{r_2} + \cos \boldsymbol{k}\cdot\boldsymbol{r_3}) - \mu/2$$

$$U_{\boldsymbol{k}}^{\dagger} H_M(\boldsymbol{k}) U_{\boldsymbol{k}} = \begin{bmatrix} \begin{matrix} -\epsilon_{0\boldsymbol{k}} & 0 \\ 0 & -\epsilon_{1\boldsymbol{k}} \end{matrix} & 0 \\ 0 & \begin{matrix} \epsilon_{1\boldsymbol{k}} & 0 \\ 0 & \epsilon_{0\boldsymbol{k}} \end{matrix} \end{bmatrix} \qquad \left[\boldsymbol{A}_{\boldsymbol{k}}^{\dagger}, \boldsymbol{B}_{\boldsymbol{k}}^{\dagger}, \boldsymbol{C}_{\boldsymbol{k}}^{\dagger}, \boldsymbol{D}_{\boldsymbol{k}}^{\dagger}\right] = \left[\boldsymbol{c}_{\boldsymbol{k}\uparrow}^{\dagger}, \boldsymbol{c}_{\boldsymbol{k}\downarrow}^{\dagger}, \boldsymbol{c}_{-\boldsymbol{k}\uparrow}, \boldsymbol{c}_{-\boldsymbol{k}\downarrow}\right] \boldsymbol{U}_{\boldsymbol{k}}$$

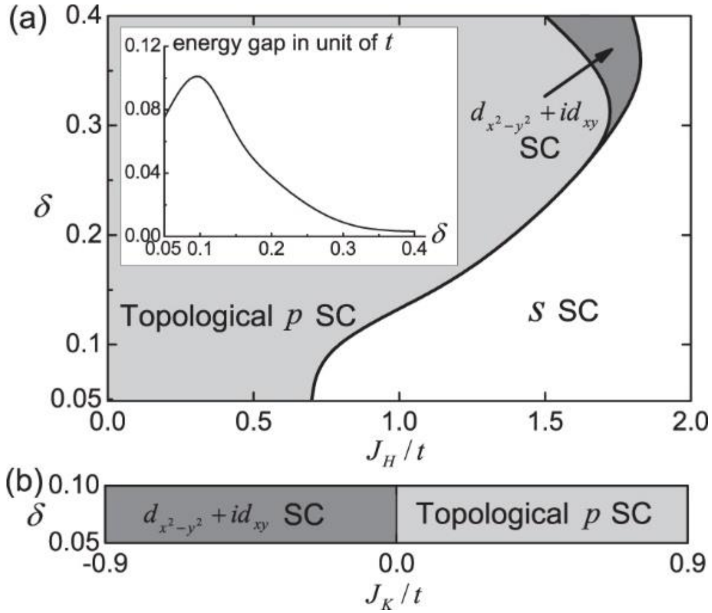
$$H = \sum_{\boldsymbol{k}\in BZ} \left[-\left(\epsilon_{0\boldsymbol{k}} A_{\boldsymbol{k}}^{\dagger} A_{\boldsymbol{k}} + \epsilon_{1\boldsymbol{k}} B_{\boldsymbol{k}}^{\dagger} B_{\boldsymbol{k}}\right) + \left(\epsilon_{\boldsymbol{1k}} C_{\boldsymbol{k}}^{\dagger} C_{\boldsymbol{k}} + \epsilon_{\boldsymbol{0k}} D_{\boldsymbol{k}}^{\dagger} D_{\boldsymbol{k}}\right)\right]$$

$$\left\langle \Omega \,\middle|\, \left[c_{i\uparrow}^{\dagger}, c_{i\downarrow}^{\dagger}\right] \begin{bmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{bmatrix} \,\middle|\, \Omega \right\rangle = {}^{1}\!/_{N} \textstyle\sum_{\boldsymbol{k}\in BZ} (M_{00} + M_{11}) \qquad M = U_0^{\dagger} U_0$$

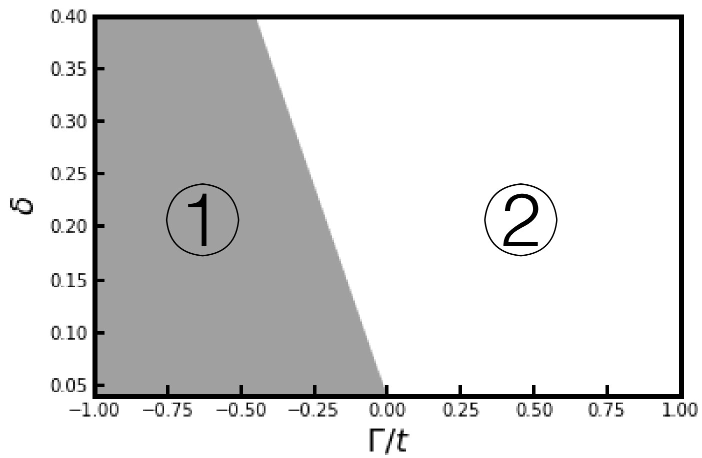
$$\left\langle \Omega \,\middle|\, \left[c_{i\uparrow}^{\dagger}, c_{i\downarrow}^{\dagger}\right] T \begin{bmatrix} c_{j\uparrow} \\ c_{j\downarrow} \end{bmatrix} \,\middle|\, \Omega \right\rangle = {}^{1}\!/_{N} \textstyle\sum_{\boldsymbol{k}\in BZ} e^{i\boldsymbol{k}\cdot(\boldsymbol{r}_j - \boldsymbol{r}_i)} \, (M_{00} + M_{11}) \qquad M = U_0^{\dagger} T U_0$$

$$\langle \Omega \mid \left[c_{i\uparrow}^{\dagger}, c_{i\downarrow}^{\dagger}\right] P \begin{bmatrix} c_{j\uparrow}^{\dagger} \\ c_{j\downarrow}^{\dagger} \end{bmatrix} \mid \Omega = {}^{1}\!/_{N} \textstyle\sum_{\boldsymbol{k}\in BZ} e^{i\boldsymbol{k}\cdot(\boldsymbol{r}_j - \boldsymbol{r}_i)} \, (M_{00} + M_{11}) \qquad M = U_0^{\dagger} P U_1$$

U_0 is the first two rows of $U_{\boldsymbol{k}}$ and U_1 is the last two rows of $U_{\boldsymbol{k}}$



- p-wave pairing:
  All nine triplet pairing order parameters are equal and pure imaginary.
- s-wave pairing:
  $$\Delta_1 = \Delta_2 = \Delta_3$$
  All three singlet pairing order parameters are equal and real. Both the p-wave and s-wave SC are TR invariant.
- $d_{x^2-y^2} + id_{xy}$-wave pairing:
  $$e^{-i\frac{2\pi}{3}}\Delta_1 = e^{i\frac{2\pi}{3}}\Delta_2 = \Delta_3$$



① $$d = \eta \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

② $$d = \eta_1 \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \eta_2 \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} + \eta_3 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

Thank you!