

# The Vector problems

Coding the Matrix, 2015

For auto-graded problems, edit the file `The_Vector_problems.py` to include your solution.

## Vector Addition Practice

**Problem 1:** For vectors  $v = [-1, 3]$  and  $u = [0, 4]$ , find the vectors  $v + u$ ,  $v - u$ , and  $3v - 2u$ .

**Problem 2:** Given the vectors  $v = [2, -1, 5]$  and  $u = [-1, 1, 1]$ , find the vectors  $v + u$ ,  $v - u$ ,  $2v - u$ , and  $v + 2u$ .

**Problem 3:** For the vectors  $v = [0, one, one]$  and  $u = [one, one, one]$  over  $GF(2)$ , find  $v + u$  and  $v + u + u$ .

## Expressing one $GF(2)$ vector as a sum of others

**Problem 4:** Here are six 7-vectors over  $GF(2)$ :

|            |         |            |         |
|------------|---------|------------|---------|
| <b>a</b> = | 1100000 | <b>d</b> = | 0001100 |
| <b>b</b> = | 0110000 | <b>e</b> = | 0000110 |
| <b>c</b> = | 0011000 | <b>f</b> = | 0000011 |

For each of the following vectors  $u$ , find a subset of the above vectors whose sum is  $u$ , or report that no such subset exists. You should be able to do this without the help of a computer.

1.  $u = 0010010$
2.  $u = 0100010$

**Problem 5:** Here are six 7-vectors over  $GF(2)$ :

|            |         |            |         |
|------------|---------|------------|---------|
| <b>a</b> = | 1110000 | <b>d</b> = | 0001110 |
| <b>b</b> = | 0111000 | <b>e</b> = | 0000111 |
| <b>c</b> = | 0011100 | <b>f</b> = | 0000011 |

For each of the following vectors  $u$ , find a subset of the above vectors whose sum is  $u$ , or report that no such subset exists.

1.  $u = 0010010$
2.  $u = 0100010$

**Problem 6:** (You should be able to solve this problem without using a computer.) Find a vector  $\mathbf{x} = [x_1, x_2, x_3, x_4]$  over  $GF(2)$  satisfying the following linear equations:

$$1100 \cdot \mathbf{x} = 1$$

$$1010 \cdot \mathbf{x} = 1$$

$$1111 \cdot \mathbf{x} = 1$$

Verify for yourself that  $\mathbf{x} + 1111$  also satisfies the equations.

**Problem 7:** Consider the equations

$$2x_0 + 3x_1 - 4x_2 + x_3 = 10$$

$$x_0 - 5x_1 + 2x_2 + 0x_3 = 35$$

$$4x_0 + x_1 - x_2 - x_3 = 8$$

Your job is not to solve these equations but to formulate them using dot-product. In particular, come up with three vectors  $\mathbf{v}_1$ ,  $\mathbf{v}_2$ , and  $\mathbf{v}_3$  represented as lists so that the above equations are equivalent to

$$\mathbf{v}_1 \cdot \mathbf{x} = 10$$

$$\mathbf{v}_2 \cdot \mathbf{x} = 35$$

$$\mathbf{v}_3 \cdot \mathbf{x} = 8$$

where  $\mathbf{x}$  is a 4-vector over  $\mathbb{R}$ .

## Practice with Dot-Product

**Problem 8:** For each of the following pairs of vectors  $\mathbf{u}$  and  $\mathbf{v}$  over  $\mathbb{R}$ , evaluate the expression  $\mathbf{u} \cdot \mathbf{v}$ :

(a)  $\mathbf{u} = [1, 0], \mathbf{v} = [5, 4321]$

(b)  $\mathbf{u} = [0, 1], \mathbf{v} = [12345, 6]$

(c)  $\mathbf{u} = [-1, 3], \mathbf{v} = [5, 7]$

(d)  $\mathbf{u} = \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right], \mathbf{v} = \left[\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]$