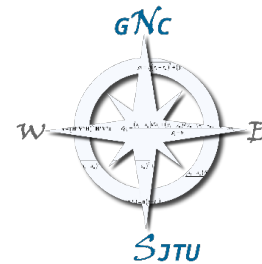




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# Solution Separation-Based Integrity Monitoring for Integer Ambiguity Resolution- Enabled GNSS Positioning

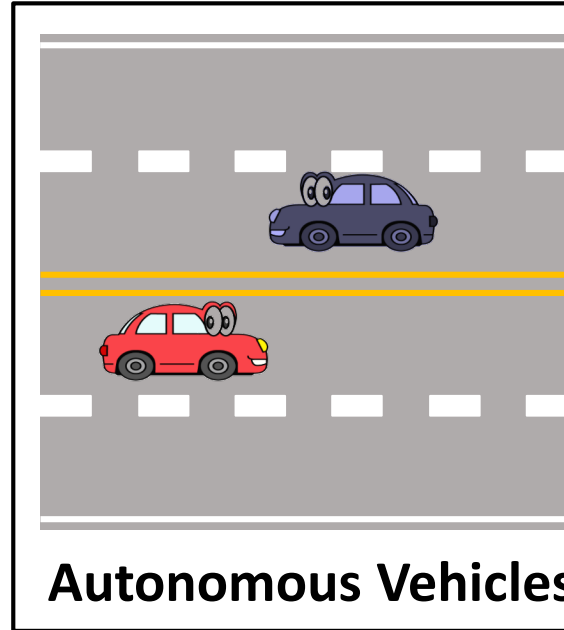
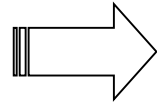
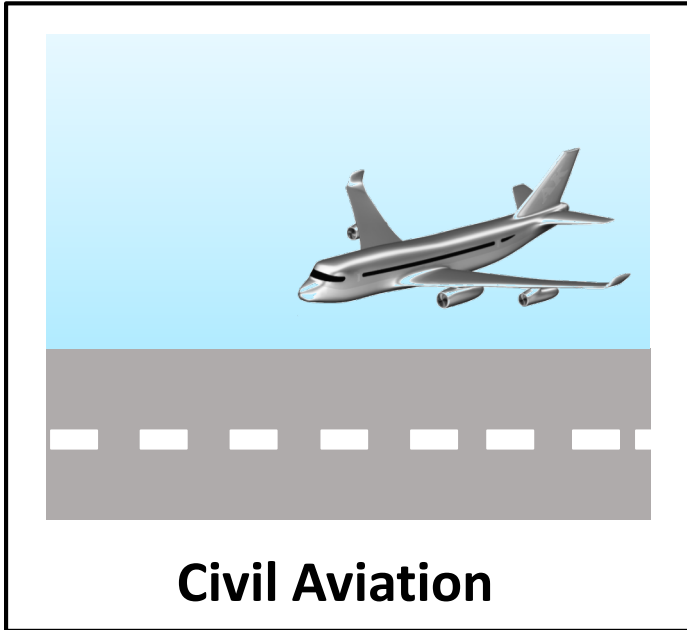
**Shizhuang WANG**, Xingqun ZHAN, Yawei ZHAI, and Zhen GAO

*Shanghai Jiao Tong University, China*

**ION ITM/PTTI 2023**

January 23–26, 2023; Long Beach, CA





*Pictures courtesy of Verdict*

**High-Integrity**

**High-Accuracy** and **High-Integrity**<sup>[1]</sup>

- **Accuracy** describes the uncertainty level of a navigation solution in nominal conditions.
- **Integrity** captures the upper bound of the error while considering the occurrence of faults.<sup>[2]</sup>

[1] Reid TGR, et al. Localization Requirements for Autonomous Vehicles. arXiv preprint, arXiv:1906.01061, 2019.

[2] International Civil Aviation Organization (ICAO). Annex 10, Aeronautical telecommunications, volume 1, amendment 84, Montreal, QC, Canada, 2009.



## ❑ Improving Accuracy

### ✓ GNSS Precise Positioning

- RTK<sup>[3]</sup> and PPP-RTK<sup>[4]</sup>
- ✓ Correction products-augmented
- ✓ Ambiguity Resolution (AR)-enabled
- ✓ Centimeter-level accuracy

## ❑ Ensuring Integrity

### ✓ Integrity Monitoring

- Fault Detection
- Fault Exclusion
- Evaluation of Protection Levels or Integrity Risk

× Integrity solution for GNSS precise positioning with ambiguity resolution?

✓ **Goal:** designing an integrity monitoring algorithm to protect GNSS precise positioning against **all sources of faults** and **incorrect ambiguity fixes**

[3] Odolinski, R., Odijk, D. and Teunissen, P.J.G., Combined GPS and BeiDou Instantaneous RTK Positioning. NAVIGATION, 2014, 61: 135-148.

[4] Zhang B, Chen Y, Yuan Y. PPP-RTK based on undifferenced and uncombined observations: theoretical and practical aspects. Journal of Geodesy, 2019, 93(7): 1011-1024.



1

Ambiguity Resolution (AR)-Enabled GNSS Positioning

2

Integrity Monitoring for AR-Enabled GNSS Positioning



Modified MHSS: Faults & Incorrect Ambiguity Fixes



Closed-Form Protection Level Evaluation

3

Simulations and Results



**1**

## Ambiguity Resolution (AR)-Enabled GNSS Positioning

**2**

## Integrity Monitoring for AR-Enabled GNSS Positioning



Modified MHSS: Faults & Incorrect Ambiguity Fixes



Closed-Form Protection Level Evaluation

**3**

## Simulations and Results



## □ GNSS Measurement Model <sup>[10]</sup>

### Pseudorange:

$$\rho_{r,j}^{s,S} = \|\mathbf{X}^{s,S} - \mathbf{X}_r\| + dt_r - dt^{s,S} + isb_r^S + w_r^s \cdot T_r + \gamma_j^S \cdot I_{r,1}^{s,S} + (d_{r,j} - d_j^{s,S}) + \epsilon_{r,j}^{s,S} + \xi_{r,j}^{s,S}$$

### Carrier Phase:

$$l_{r,j}^{s,S} = \|\mathbf{X}^{s,S} - \mathbf{X}_r\| + dt_r - dt^{s,S} + isb_r^S + w_r^s \cdot T_r - \gamma_j^S \cdot I_{r,1}^{s,S} + \lambda_j^{s,S} \cdot (\mathbf{N}_{r,j}^{s,S} + b_{r,j} - b_j^{s,S}) + \epsilon_{r,j}^{s,S} + \xi_{r,j}^{s,S}$$

$s, r, j, S$  represent satellite, receiver, frequency, and constellation;

$dt$  clock offset;  $isb$  inter-system bias;

$T$  zenith troposphere wet delay;  $I$  slant ionosphere delay;

$d$  code bias;  $b$  phase bias;

$\lambda$  signal wavelength;  $\mathbf{N}$  carrier-phase integer ambiguity;

$\epsilon, \xi$  white noises;  $\xi$  errors that can be modelled, e.g., troposphere dry delay



## □ Precise Products for PPP-RTK

- orbit, clock, troposphere, ionosphere, code bias, phase bias

$$\rho_{r,j}^{s,S} = \| \mathbf{X}^{s,S} - \mathbf{X}_r \| + dt_r - dt^{s,S} + isb_r^S + w_r^s \cdot T_r + \gamma_j^S \cdot I_{r,1}^{s,S} + (d_{r,j} - d_j^{s,S}) + \varepsilon_{r,j}^{s,S} + \xi_{r,j}^{s,S}$$

$$l_{r,j}^{s,S} = \| \mathbf{X}^{s,S} - \mathbf{X}_r \| + dt_r - dt^{s,S} + isb_r^S + w_r^s \cdot T_r - \gamma_j^S \cdot I_{r,1}^{s,S} + \lambda_j^{s,S} \cdot (N_{r,j}^{s,S} + b_{r,j} - b_j^{s,S}) + \epsilon_{r,j}^{s,S} + \xi_{r,j}^{s,S}$$

## □ State Vector (Using GPS+BDS as an example)

- position, clock, ISB, receiver bias, troposphere, ionosphere, ambiguity

$$\mathbf{x} \triangleq \left[ \underbrace{\mathbf{X}_r}_{\text{pos}} \quad \underbrace{dt^G}_{\text{clk}} \quad \underbrace{isb^C}_{\text{isb}} \quad \underbrace{dcb^G \ b_{r,1}^G \ b_{r,2}^G \ dcb^C \ b_{r,1}^C \ b_{r,2}^C}_{\text{rcv. bias}} \quad \underbrace{T_r \ \mathbf{I}_{r,1}^G \ \mathbf{I}_{r,1}^C}_{\text{trp \& ion}} \quad \underbrace{\mathbf{N}_{r,1}^G \ \mathbf{N}_{r,2}^G \ \mathbf{N}_{r,1}^C \ \mathbf{N}_{r,2}^C}_{\text{amb}} \right]^T$$



State Models  $\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \boldsymbol{\omega}_k$

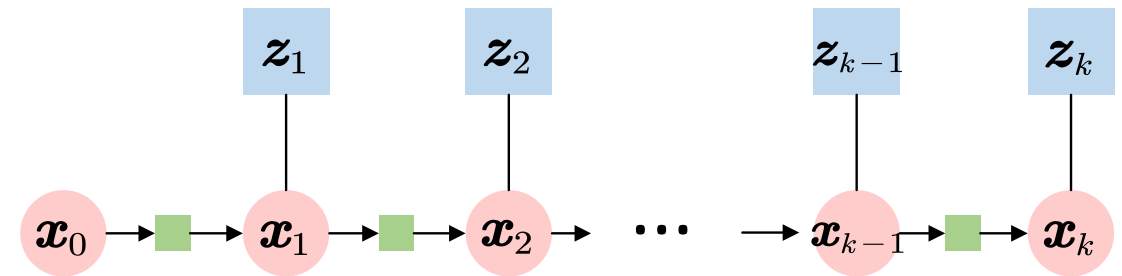
Measurement Models  $\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_{k-1} + \boldsymbol{\nu}_k$

## □ Error Models (Simplified) see Tables 1 ~ 3

- Process noise  $\boldsymbol{\omega}_k$  ; measurement noise  $\boldsymbol{\nu}_k$
- Assumptions: zero-mean white noises & uncorrelated

## □ Kalman Filter

- Recursive Estimation
- Equivalent to Batch Least Squares <sup>[11]</sup>



Ambiguity-Float Solution: State Estimate  $\rightarrow \hat{\mathbf{x}}_k$  Error Covariance  $\rightarrow \hat{\mathbf{P}}_k$





## □ Ambiguity Resolution: Exploiting the integer nature of the ambiguities <sup>[12]</sup>

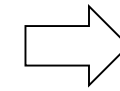
$$(\hat{\mathbf{x}}_k, \hat{\mathbf{P}}_k) \longrightarrow \mathbf{x} \stackrel{\Delta}{=} [\mathbf{x}_{pos} \quad others \quad \mathbf{x}_{amb}]^T \xrightarrow{\Delta} \mathbf{a} = \mathbf{x}_{amb}, \mathbf{b} = \mathbf{x}_{pos}$$

### 1. Fix the ambiguities by Integer Bootstrapping <sup>[11]</sup>

$$\begin{aligned} \check{a}_1 &= [\hat{a}_1] \\ \check{a}_2 &= [\hat{a}_{2|1}] = [\hat{a}_2 - \sigma_{2|1} \sigma_1^{-2} (\hat{a}_1 - \check{a}_1)] \\ &\vdots \\ \check{a}_n &= [\hat{a}_{n|N}] = [\hat{a}_n - \sum_{i=1}^{n-1} \sigma_{n,i|I} \sigma_i^{-2} (\hat{a}_{i|I} - \check{a}_i)] \end{aligned}$$

sequentially fixed!

$$\hat{\mathbf{P}}_{aa} = \mathbf{L}^T \mathbf{D} \mathbf{L}$$



### 2. Adjust the positions

$$\check{\mathbf{b}} = \hat{\mathbf{b}} - \hat{\mathbf{P}}_{ba} \hat{\mathbf{P}}_{aa}^{-1} (\hat{\mathbf{a}} - \check{\mathbf{a}})$$

$$\check{\mathbf{P}}_{bb} = \hat{\mathbf{P}}_{bb} - \hat{\mathbf{P}}_{ba} \hat{\mathbf{P}}_{aa}^{-1} \hat{\mathbf{P}}_{ba}^T \quad \downarrow$$

Improved Accuracy!

## ✓ IB offers a closed-form prior probability of the correct- or incorrect- fix event. <sup>[12]</sup>



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**2**

**Integrity Monitoring for AR-Enabled GNSS Positioning**



**Modified MHSS: Faults & Incorrect Ambiguity Fixes**



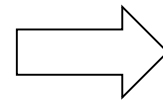
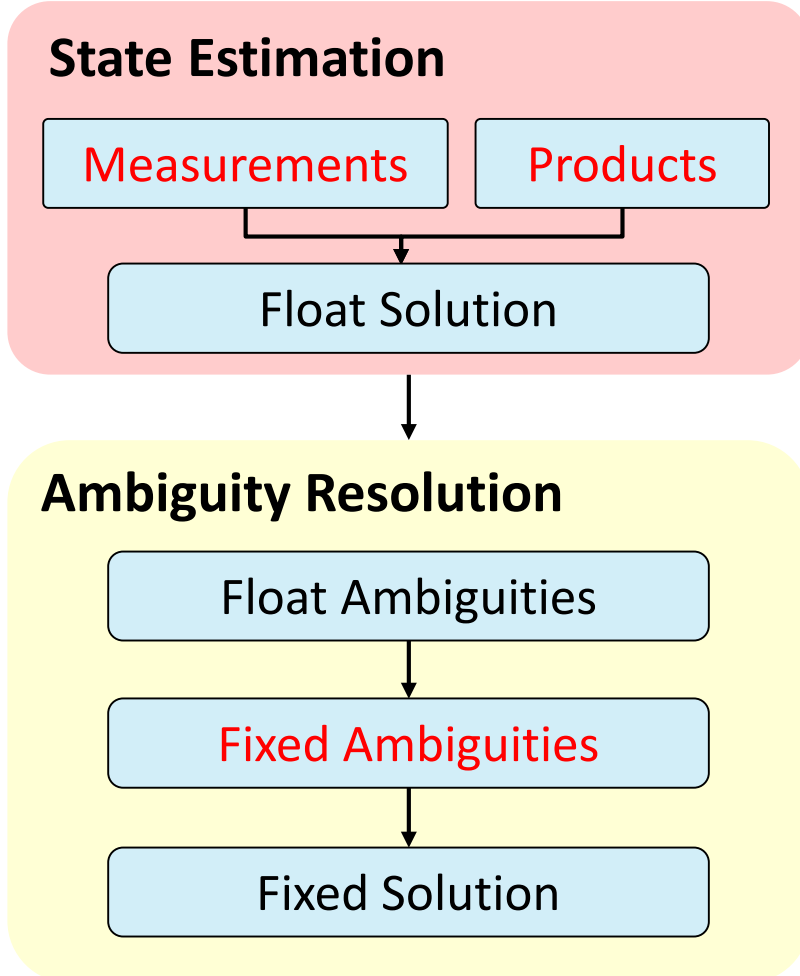
**Closed-Form Protection Level Evaluation**

**3**

Simulations and Results



# Threats



## × Measurement Faults

- Heavy multipath / NLOS
- Undetected Cycle Slips

## × Product Faults

- Incorrect precise products

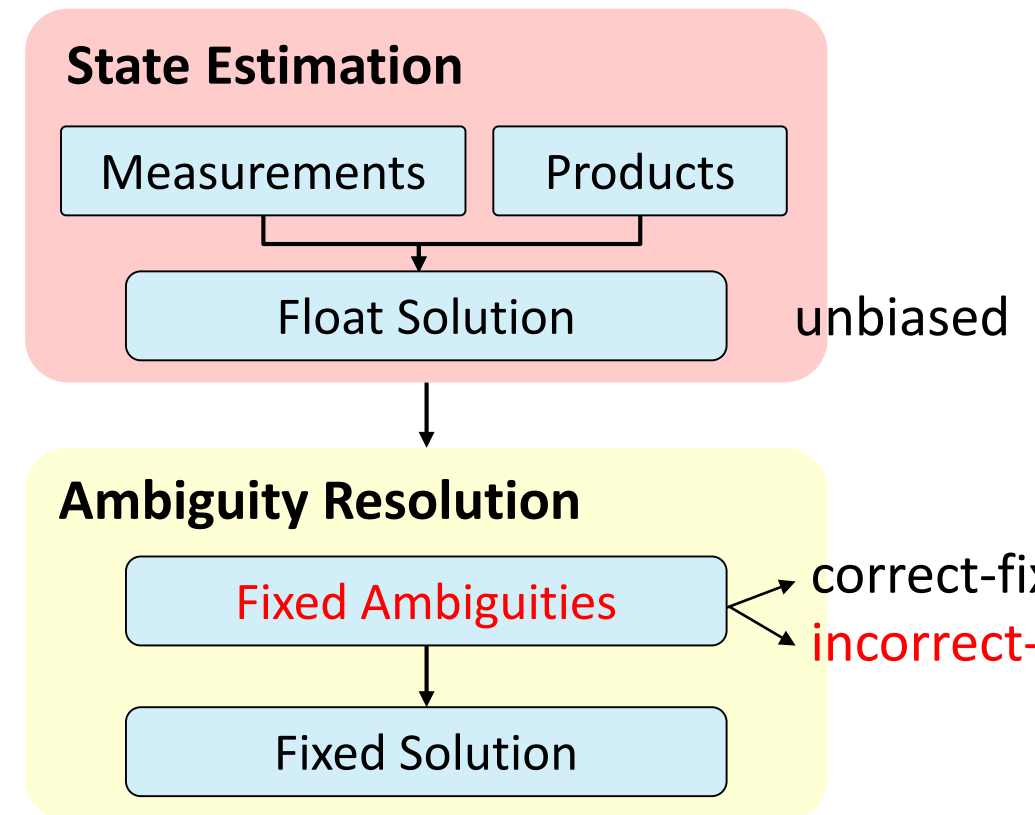
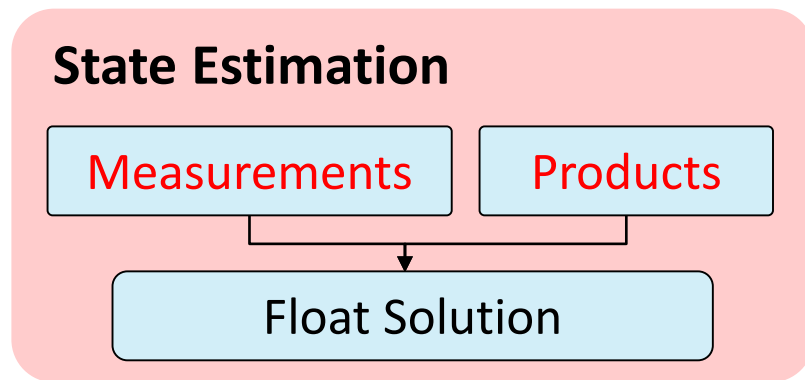
## × Incorrect Ambiguity Resolution

- Incorrect-fix events



## □ Prior Work on PPP/RTK/PPP-RTK Integrity Monitoring:

- evaluated the protection levels for the ***ambiguity-float*** solutions<sup>[5-7]</sup>
- quantified the integrity risk from ***incorrect-fix events*** in a ***fault-free*** condition<sup>[8-9]</sup>



[5] Gunning K., et al. Design and Evaluation of Integrity Algorithms for PPP in Kinematic Applications. ION GNSS+ 2018.

[6] Gunning K. et al. Integrity for Tightly Coupled PPP and IMU. ION GNSS+ 2019.

[7] Blanch J. et al. Reducing Computational Load in Solution Separation for Kalman Filters and an Application to PPP Integrity. ION ITM 2019.

[8] Khanafseh S. and Pervan B. New Approach for Calculating Position Domain Integrity Risk for Cycle Resolution in Carrier Phase Navigation Systems. IEEE TAES, 2010, 46(1): 296-307.

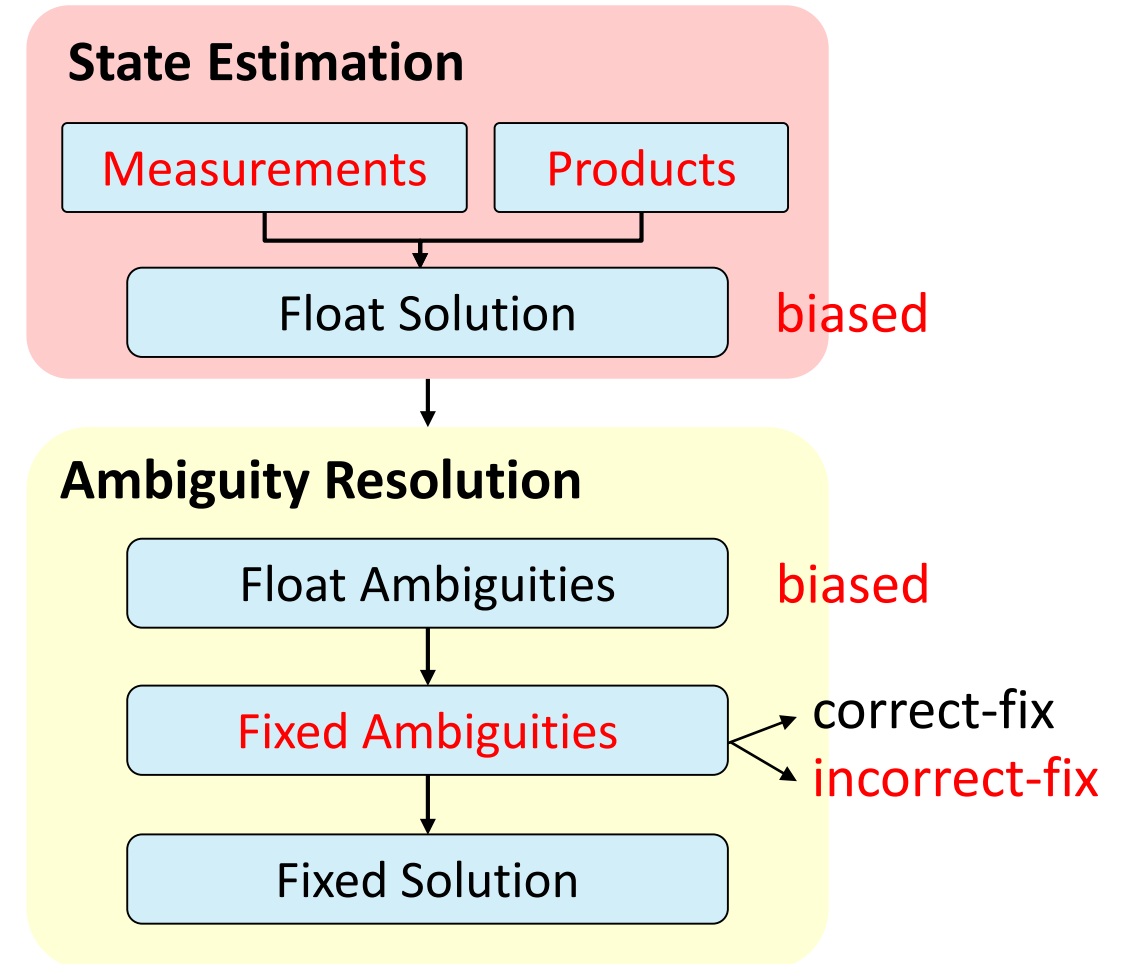
[9] Green G. and Humphreys T. Position-Domain Integrity Analysis for Generalized Integer Aperture Bootstrapping. IEEE TAES, 2019, 55(2): 734-746.



## ❑ What if measurement/product faults occur to AR-enabled positioning systems?

### ❑ Motivation:

- Monitoring **measurement/product faults** and **incorrect-fix events** simultaneously.
- The occurrence of **measurement/product faults** will influence the probability of **incorrect-fix events**.
- Therefore, *a simple combination of previous approaches doesn't work!*





## ❑ Approach: Multiple Hypothesis Solution Separation (MHSS) <sup>[13][14]</sup>

### ❑ Satellite (SV) Fault:

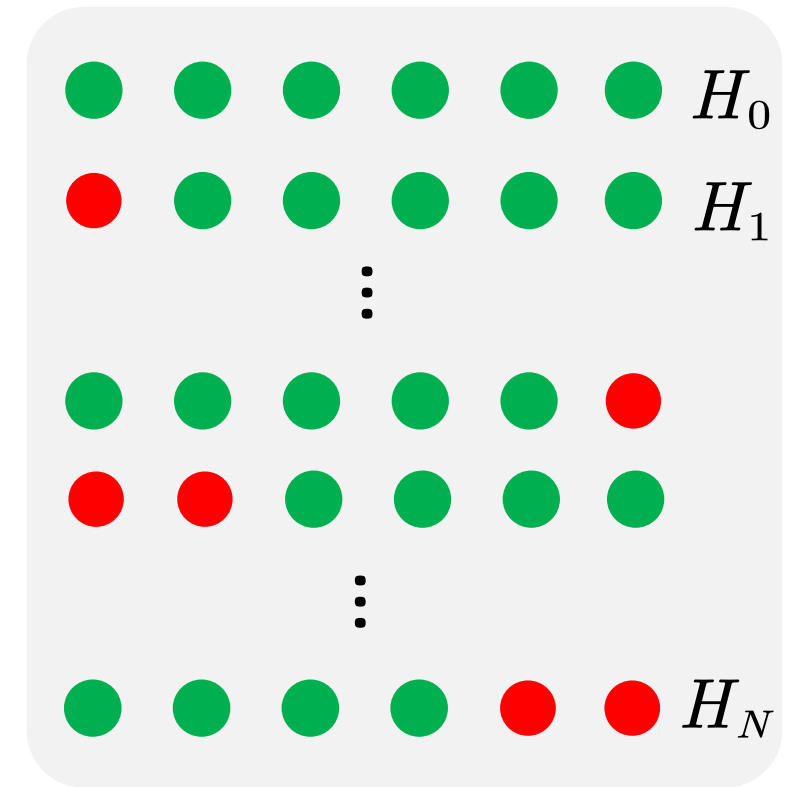
- The measurement or product of this SV is faulted

### ❑ Fault Mode, $H_i$ : <sup>[13][14]</sup>

- An event combination of SV faults

### ❑ Subset solution for a fault mode, $x^{(i)}$ :

- Use the healthy SVs to estimate the states
- The ambiguities of the faulted SVs are not included in  $x^{(i)}$



[13] Blanch J., et al. Baseline Advanced RAIM User Algorithm and Possible Improvements. IEEE Transactions on Aerospace and Electronic Systems, 2015, 51(1): 713-732.

[14] Joerger M., Chan F.C., and Pervan B., Solution separation versus residual-based RAIM. NAVIGATION, 2014, 61(4): 273-291.



## □ Test Statistics [13][14]

For Subset  $i$ :  $\Delta x_q^{(i)} = \check{x}_q^{(i)} - \check{x}_q^{(0)}$ ,  $i = 1, 2, \dots, N_S$ ,  $q = 1, 2, 3$  (east, north, up)

## □ Test Thresholds [13][14]

Threshold:  $T_q^{(i)} = Q^{-1}\left(P_{FA,q}/2N_S\right) \cdot \sigma_{ss,q}^{(i)}$  Solution separation variance:  $\sigma_{ss,q}^{(i)2} = \check{\mathbf{P}}_{q,q}^{(i)} - \check{\mathbf{P}}_{q,q}^{(0)}$  [5]

$P_{FA,q}$ : False alert budget      $Q^{-1}(p)$ : the  $(1 - p)$  quantile of the normal distribution  $N(0, 1)$

## □ Fault Detection

No Fault Alert: *For all  $i$  and  $q$ , we have the following:  $|\Delta x_q^{(i)}| < T_q^{(i)}$*

[5] Gunning K., et al. Design and Evaluation of Integrity Algorithms for PPP in Kinematic Applications. ION GNSS+ 2018.

[13] Blanch J., et al. Baseline Advanced RAIM User Algorithm and Possible Improvements. IEEE Transactions on Aerospace and Electronic Systems, 2015, 51(1): 713-732.

[14] Joerger M., Chan F.C., and Pervan B., Solution separation versus residual-based RAIM. NAVIGATION, 2014, 61(4): 273-291.



□ According to Multiple Hypothesis Solution Separation, [13]

$$P(|\check{x}_q^{(0)} - x_q| > PL_q, \text{ no alert}) < \underbrace{\sum_{i=0}^{N_s} P(H_i) \times P(|\check{x}_q^{(0)} - x_q| > PL_q, \text{ no alert} | H_i)}_{\downarrow} + P_{NM,q}$$

□ Under Fault Mode  $i$ :

$$\begin{aligned} IR^{(i)} &< P(H_i) \times P(|\check{x}_q^{(0)} - x_q| > PL, \text{ no alert} | H_i) \\ &= P(H_i) \times \sum_{j=0}^{\infty} P(|\check{x}_q^{(0)} - x_q| > PL, \text{ no alert} | H_i \cap (\Delta^{(i)} = \Delta_j^{(i)}) ) \times P(\Delta^{(i)} = \Delta_j^{(i)} | H_i) \end{aligned}$$

Ambiguity  
Offset

$$\Delta^{(i)} = \check{\mathbf{a}}^{(i)} - \mathbf{a}^{(i)} \quad (\mathbf{a}^{(i)} : \text{truth for the ambiguity vector in } \mathbf{x}^{(i)})$$

Two-layer  
hypothesis

$$\Delta_0^{(i)} = 0 \quad \textbf{Correct-fix} \qquad \Delta_j^{(i)} \neq 0, j > 0 \quad \textbf{An incorrect-fix event}$$





## □ Integrity Risk Under Hypothesis $i$ :

$$IR^{(i)} = P(H^{(i)}) \times \sum_{j=0}^{\infty} P(|\check{x}_q^{(0)} - x_q| > PL_q, no\ alert | H_i \cap (\Delta^{(i)} = \Delta_j^{(i)})) \times P(\Delta^{(i)} = \Delta_j^{(i)} | H_i)$$

✓ The first term: See the paper for the derivations!

$$P(|\check{x}_q^{(0)} - x_q| > PL, no\ alert | H_i \cap (\Delta^{(i)} = \Delta_j^{(i)})) < Q\left(\frac{PL_q}{\check{\sigma}_q^{(0)}}\right) + Q\left(\frac{PL_q - T_q^{(i)} - \mathbf{A}_q^{(i)} \Delta_j^{(i)}}{\check{\sigma}_q^{(i)}}\right) \quad (i > 0)$$

$$P(|\check{x}_q^{(0)} - x_q| > PL, no\ alert | H_0 \cap (\Delta^{(0)} = \Delta_j^{(0)})) < 2 \times Q\left(\frac{PL_q - \mathbf{A}_q^{(0)} \Delta_j^{(0)}}{\check{\sigma}_q^{(0)}}\right) \quad (i = 0)$$

$Q$ : Tail probability of a zero-mean unit normal distribution

$\mathbf{A}_q^{(i)}$ : Mapping the ambiguity offset  $\Delta^{(i)}$  to the position error  $\mathbf{A}_q^{(i)} = \mathbf{e}_q \cdot \hat{\mathbf{P}}_{ba}^{(i)} \cdot (\hat{\mathbf{P}}_{aa}^{(i)})^{-1}$



## □ Integrity Risk Under Hypothesis $i$ :

$$IR^{(i)} = P(H_i) \times \sum_{j=0}^{\infty} P(|\check{x}_q^{(0)} - x_q| > PL_q, no\ alert | H_i \cap (\Delta^{(i)} = \Delta_j^{(i)})) \times P(\Delta^{(i)} = \Delta_j^{(i)} | H_i)$$

✓ The second term: [8][12]

$$P_{A,j}^{(i)} \triangleq P(\Delta^{(i)} = \Delta_j^{(i)}) = \prod_{\mathfrak{i}=1}^{m_i} \left( \Phi \left( \frac{1 - 2\mathbf{l}_{\mathfrak{i}}^{(i)\top} \cdot \Delta_j^{(i)}}{2\sigma_{\mathfrak{i}|\mathbb{I}}^{(i)}} \right) + \Phi \left( \frac{1 + 2\mathbf{l}_{\mathfrak{i}}^{(i)\top} \cdot \Delta_j^{(i)}}{2\sigma_{\mathfrak{i}|\mathbb{I}}^{(i)}} \right) - 1 \right) \quad \hat{\mathbf{P}}_{aa} = \mathbf{L}^T \mathbf{D} \mathbf{L}$$

✓ Ambiguity resolution success rate (i.e., correct-fix): [12]

$$P_S^{(i)} \triangleq P(\Delta^{(i)} = 0) = \prod_{\mathfrak{i}=1}^{m_i} \left( 2 \times \Phi \left( \frac{1}{2\sigma_{\mathfrak{i}|\mathbb{I}}^{(i)}} \right) - 1 \right)$$



- ❑ Integrity Risk Under Hypothesis  $i$ : **Cannot monitor all the incorrect-fix events**

$$IR^{(i)} = P(H_i) \times \sum_{j=0}^{\infty} P(|\tilde{x}_q^{(0)} - x_q| > PL_q, no\ alert | H_i \cap (\Delta^{(i)} = \Delta_j^{(i)})) \times P(\Delta^{(i)} = \Delta_j^{(i)} | H_i)$$

- ✓ 1. Determine the ambiguity candidates that need monitoring

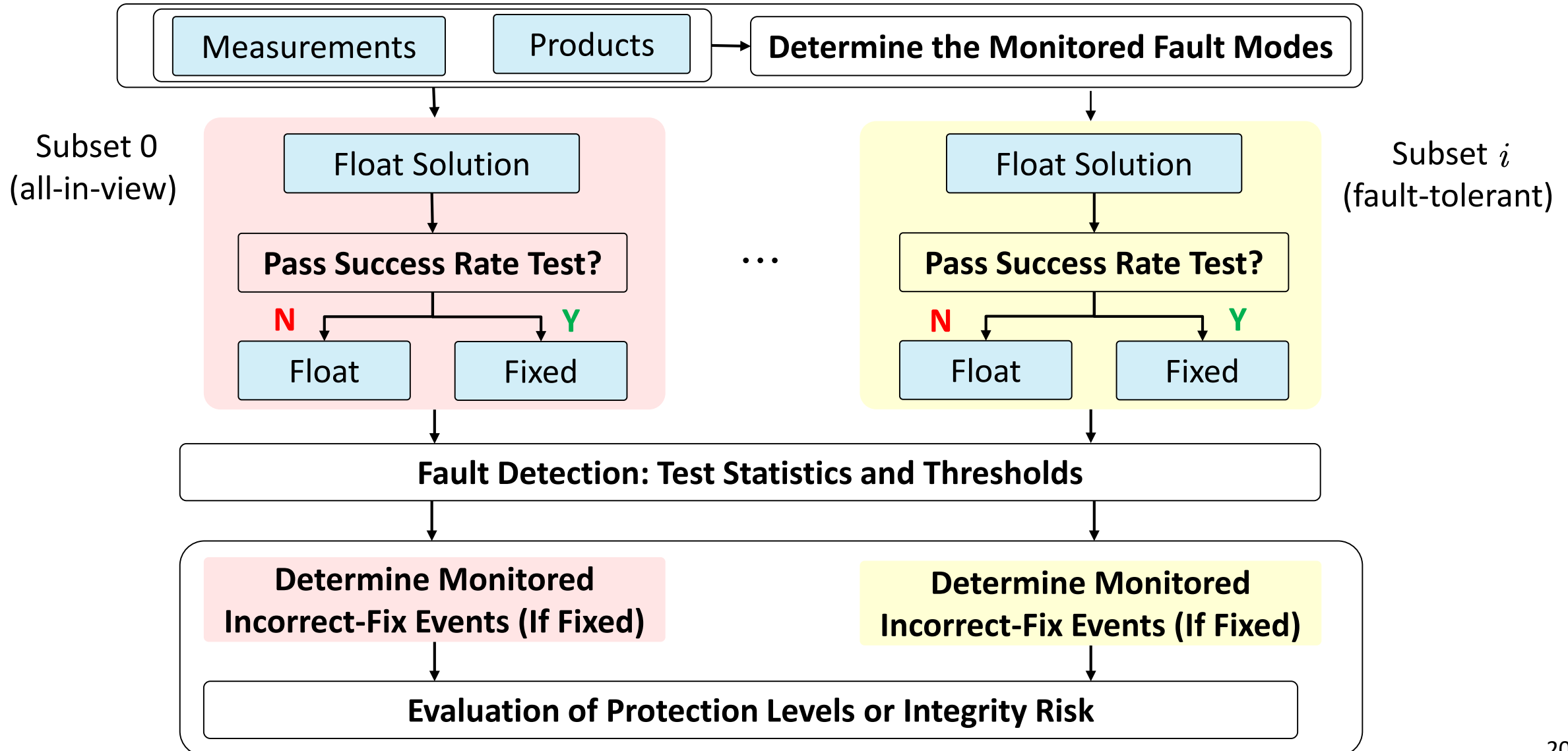
Monitor  $\Delta^{(i)} = \{0, \Delta_1^{(i)}, -\Delta_1^{(i)}, \dots, \Delta_{n_a^{(i)}}^{(i)}, -\Delta_{n_a^{(i)}}^{(i)}\}$  such that  $1 - P_S^{(i)} - 2 \times \sum_{j=1}^{n_a^{(i)}} P_{A,j}^{(i)} < P_{A,NM,thr}^{(i)}$

- ✓ 2. Set the threshold for ambiguity resolution success rate

- Subset  $i$ :  $P_S^{(i)} > P_{S,thr}^{(i)}$  **To Fix**       $P_S^{(i)} < P_{S,thr}^{(i)}$  **Not to Fix**
- If the all-in-view solution is not fixed, then all the subset solutions keep float.



# Summary





1

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2

Integrity Monitoring for AR-Enabled GNSS Positioning



Modified MHSS: Faults & Incorrect Ambiguity Fixes



Closed-Form Protection Level Evaluation

3

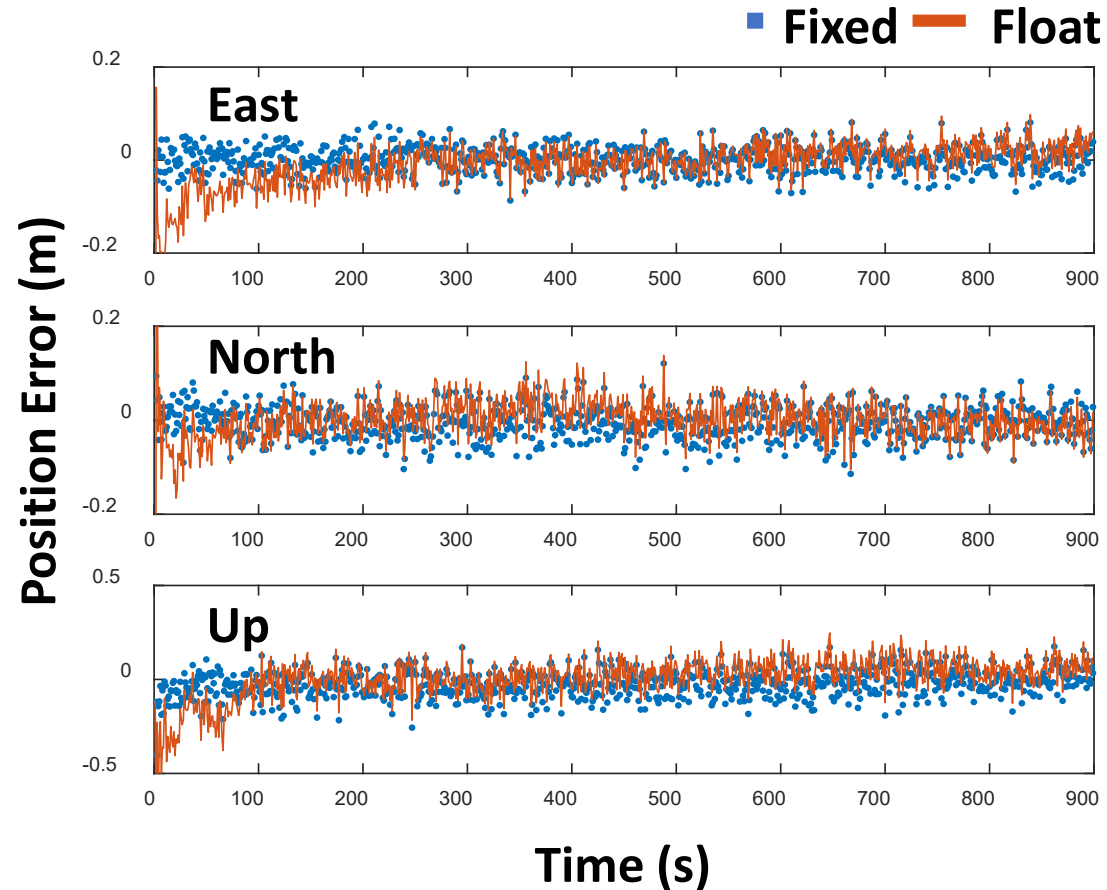
Simulations and Results



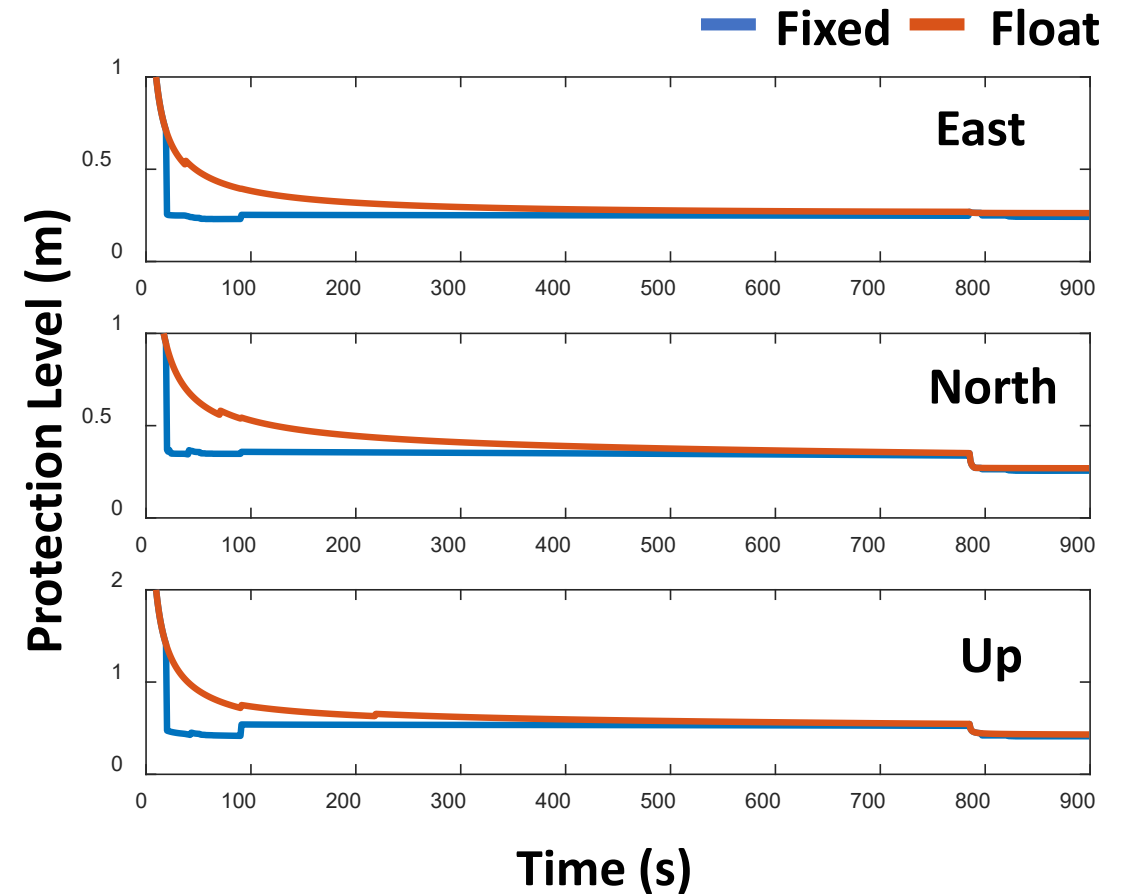
# Simulation Set-up



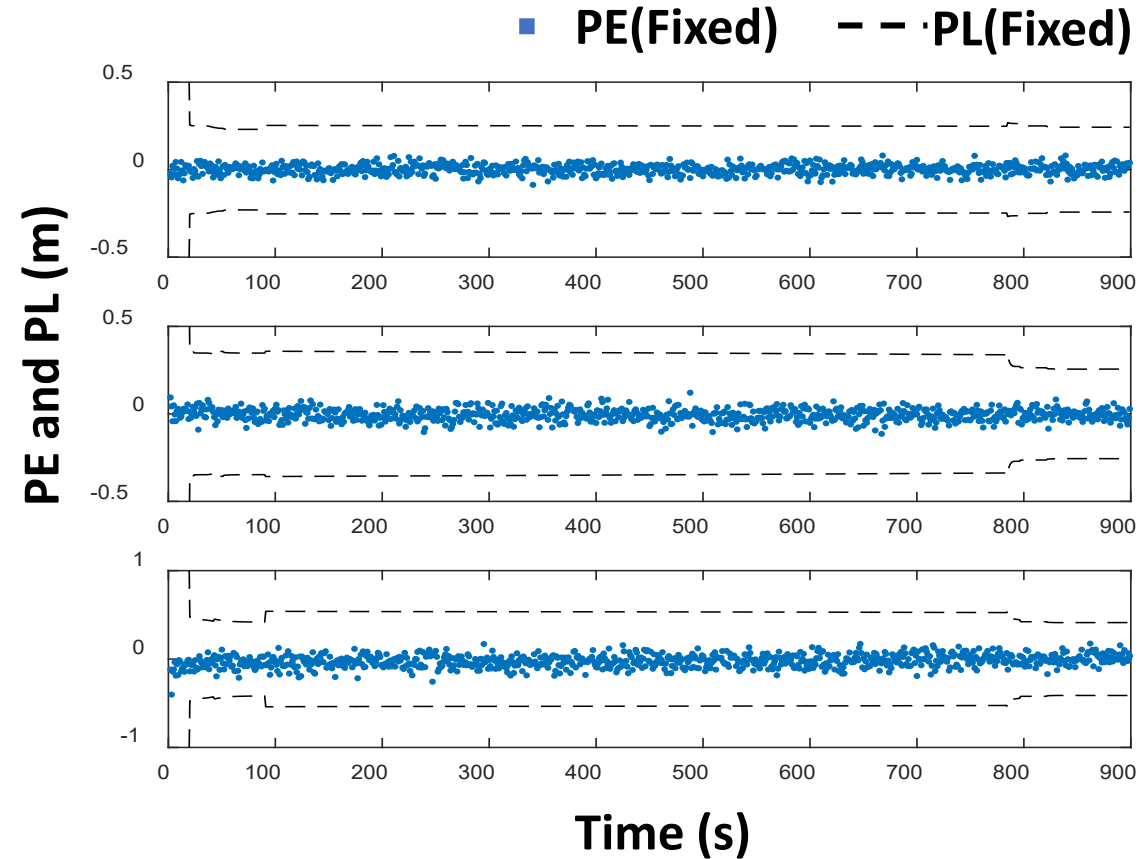
Category	Parameters	Values
Constellations	Constellations in use	GPS + BDS-3
Probabilities	Satellite fault probabilities	$P_{sat} = 10^{-5}$
Performance Requirements	Integrity risk allocations for east, north, and up positions	$P_{HMI,1} = P_{HMI,2} = 1 \times 10^{-9}$ $P_{HMI,3} = 9.8 \times 10^{-8}$
	False alert budgets for east, north, and up positions	$P_{FA,1} = P_{FA,2} = 4.5 \times 10^{-8}$ $P_{FA,3} = 3.9 \times 10^{-6}$
Algorithm Settings	Success rate thresholds for the all-in-view solution and fault-tolerant solutions	$P_{S,thr}^{(0)} = 0.9999$ $P_{S,thr}^{(i)} = 0.99$
	Threshold for the integrity risk from unmonitored fault modes	$P_{THRES} = 6 \times 10^{-8}$
	Thresholds for the integrity risk from unmonitored ambiguity candidates	$P_{A,NM,thr}^{(0)} = 1 \times 10^{-9}$ $P_{A,NM,thr}^{(i)} = 1 \times 10^{-5}$



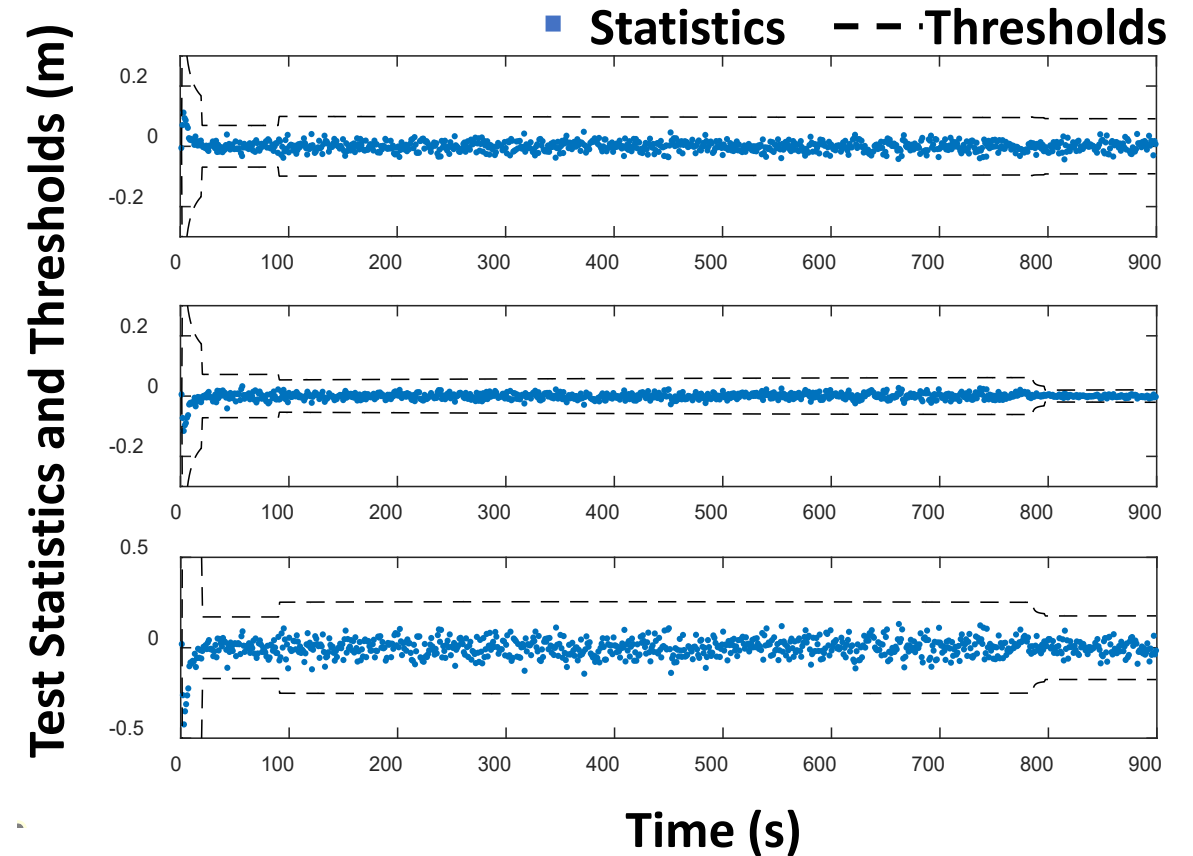
**Fig. 1.** Position error comparison between fixed and float solutions



**Fig. 2.** Protection level comparison between fixed and float solutions

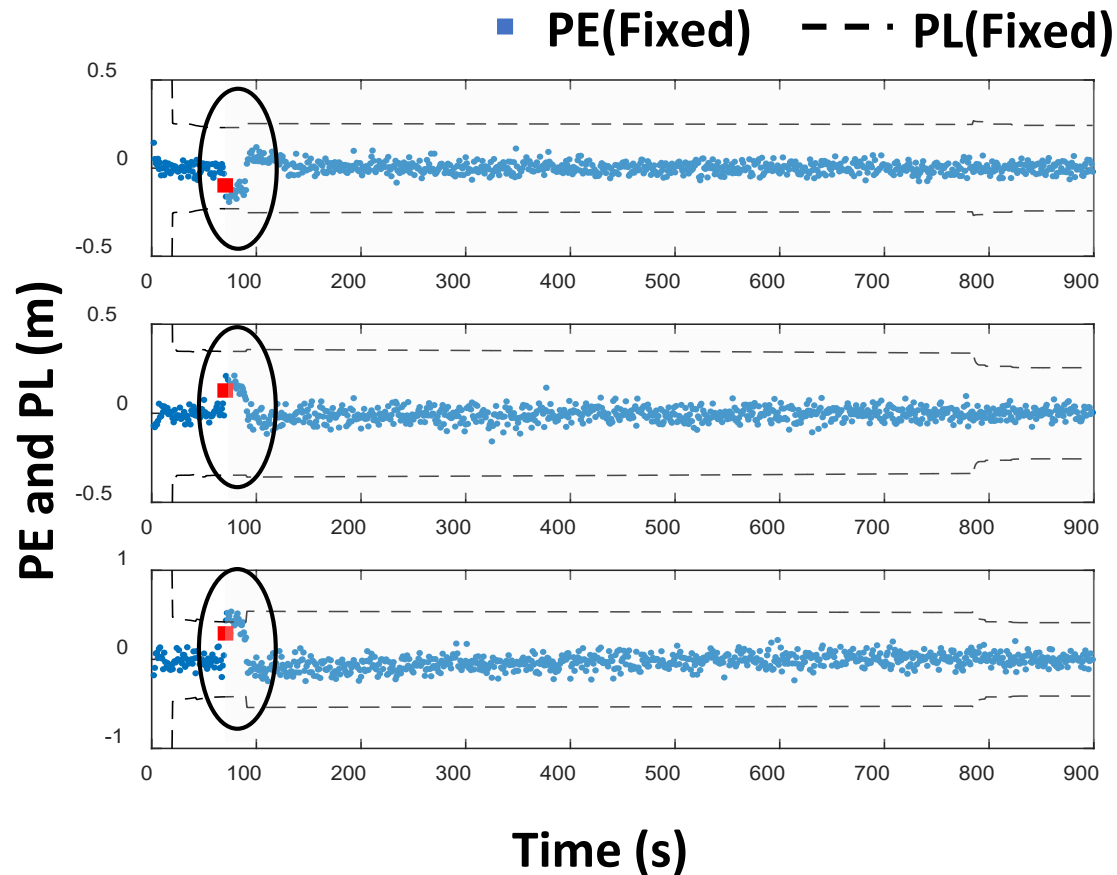


**Fig. 3.** Protection levels (PL) and position errors (PE) in a fault-free condition

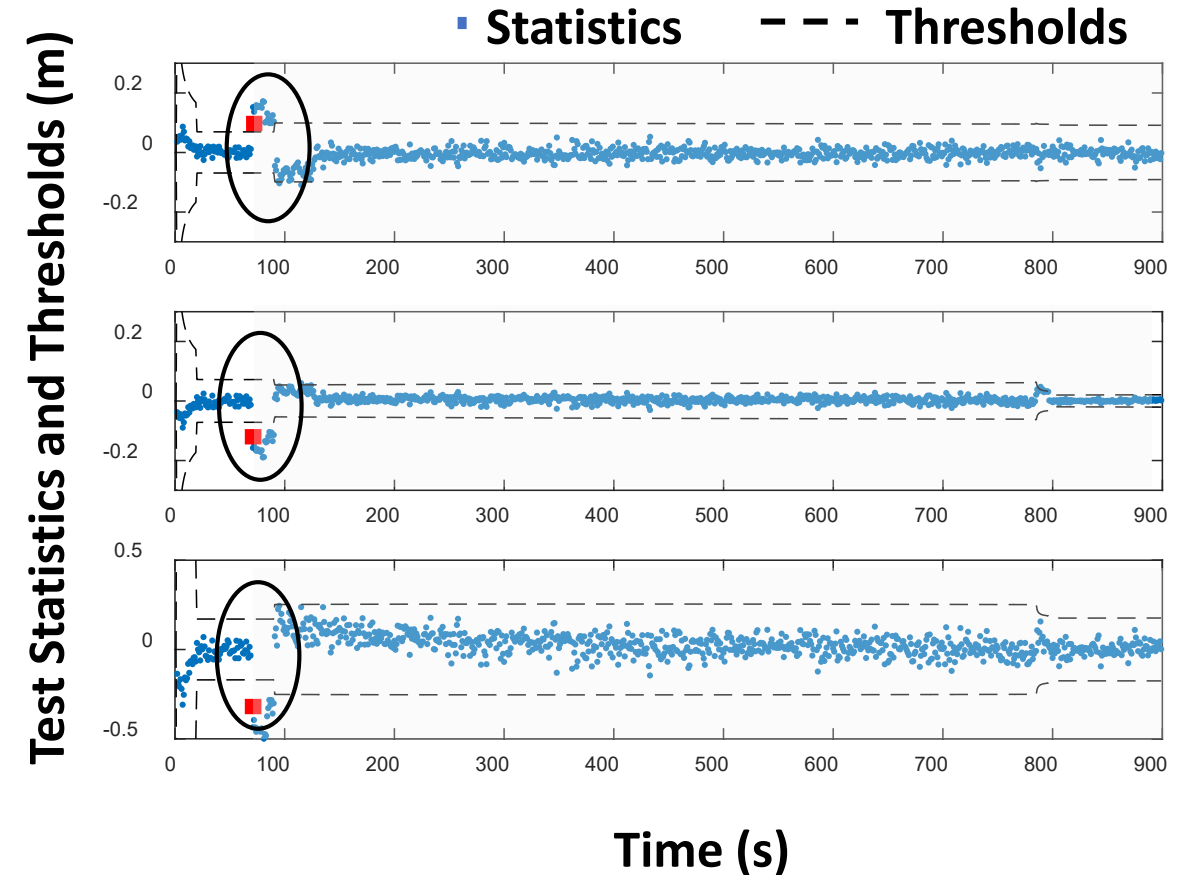


**Fig. 4.** Test statistics and thresholds (for subset 1) in a fault-free condition

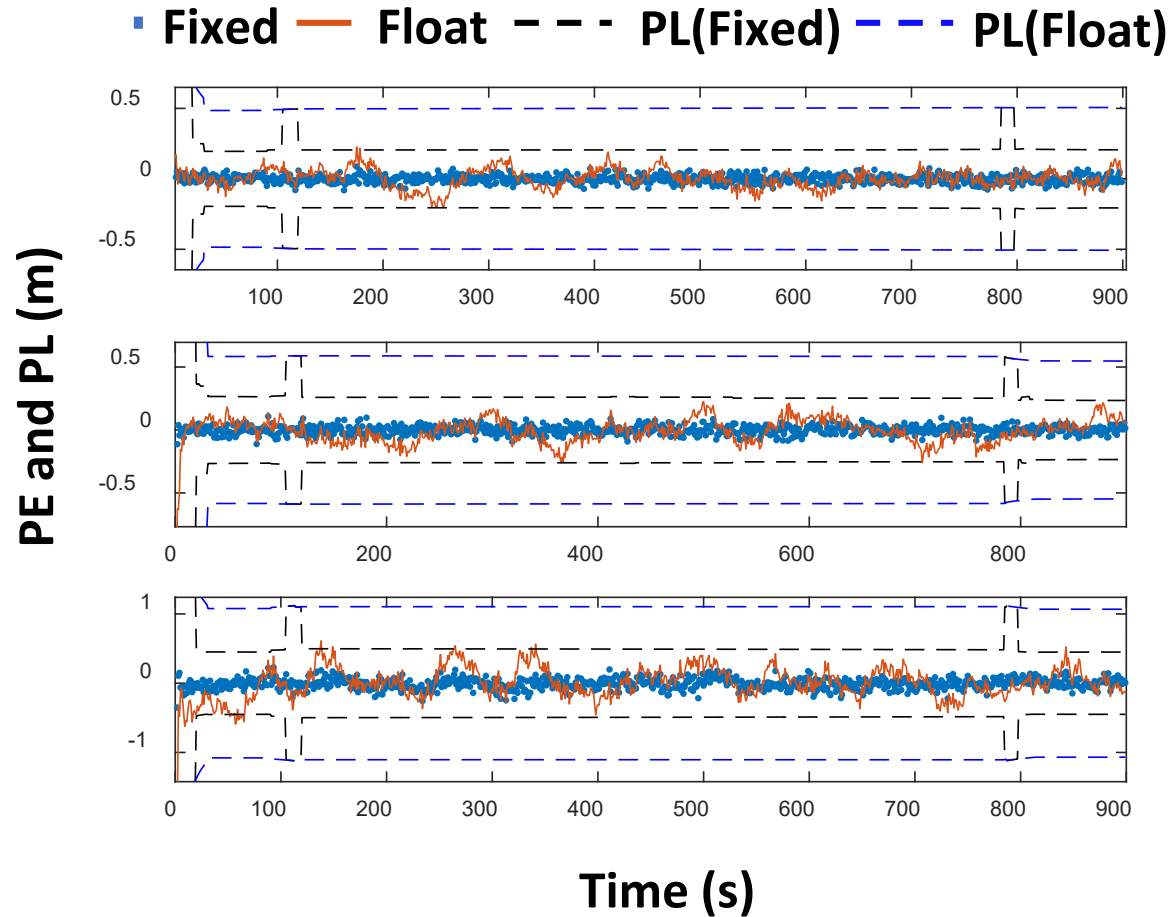




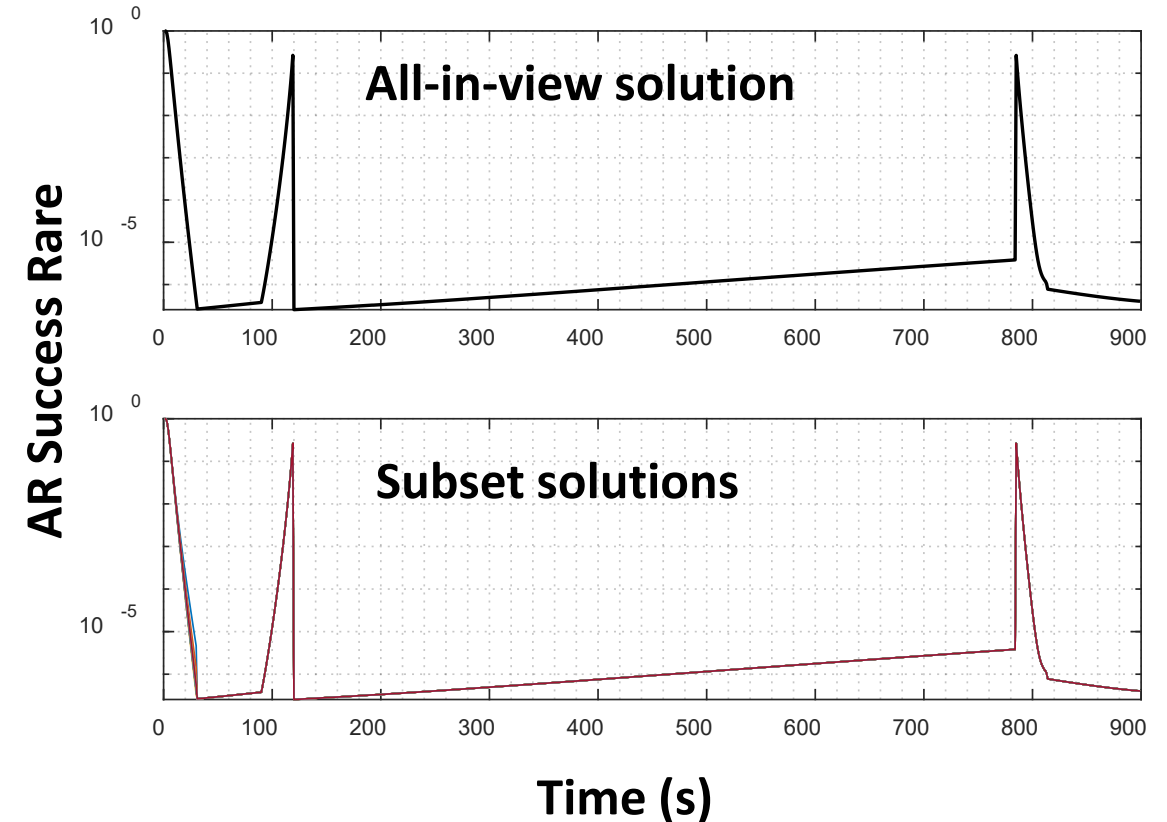
**Fig. 5.** Protection levels and position errors in the presence of an undetected cycle slip



**Fig. 6.** Test statistics and thresholds for subset 1 in the presence of an undetected cycle slip



**Fig. 7.** Protection levels and position errors with a sliding-window least-squares estimator



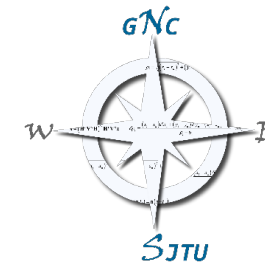
**Fig. 8.** Ambiguity resolution success rate with a sliding-window least-squares estimator



- **We develop an integrity monitoring algorithm for GNSS positioning with ambiguity resolution, based on a modified MHSS architecture:**
  - We derive the closed-form protection level evaluation equation
  - Monitor measurement/product faults and incorrect ambiguity resolution simultaneously.
  - Simulation results suggest the effectiveness of this algorithm and imply that enabling ambiguity resolution can benefit navigation accuracy and integrity.
  
- **Our future work will focus on:**
  - (a) Fault exclusion; (b) support other AR methods; (c) time correlation



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# Thank you for your attention!

## Questions? More Information?

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Full-text

