Supplemental File for Non-Iterative Coordination of Interconnected Power Grids via Dimension Decomposition-Based Flexibility Aggregation

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I. Parameters of IEEE 118 Test case

TABLE I
Configuration of the IEEE 118 System

Item	Number
Bus	118
Branch	186
Thermal power generator	32
PV stations	23
Wind farms	18
Tie-lines	2 (AC@Bus17, DC@Bus31)

II. Parameters of 5-Region Interconnected Power Grid

TABLE II
Configuration of Generation Units in Five Regional Power Grids

Index	Case name	Thermal power generators	PV stations	Wind farms
R1	case39	10	10	6
R2	case118	32	12	11
R3	case89pegase	24	10	8
R4	case39	10	6	10
R5	case89pegase	24	8	10

TABLE III
Tie-Lines of 5-Region Interconnected Power Grids

From RPG	From bus	To RPG	To bus	Type
R1	30	R2	25	DC
	33		28	DC
	35		75	DC
	37		81	DC
R1	31	R3	913	AC
	34		2107	AC
	38		2267	AC
R2	82	R3	5097	DC
	108		6233	DC
R2	16	R4	30	AC
	80		32	AC
	100		33	AC
R3	3659	R5	6798	AC
	4586		7960	AC
	6798		8605	DC
	7279		9239	DC
R4	35	R5	2107	AC
	34		3659	AC
	36		4586	AC
	38		6233	AC

III. Solution Method of the Min-Max Problem

To solve the Stackelberg game min-max problem in (23), the internal variables $\boldsymbol{x}^{\text{int}}$ are eliminated via duality and (23) is transformed into a minimization problem:

$$
\begin{aligned}
&\min_{\boldsymbol{x}^{\text{bd}},\boldsymbol{\lambda},\boldsymbol{\pi}} \boldsymbol{\pi}^{\top}\boldsymbol{f}-\boldsymbol{\lambda}^{\top}\boldsymbol{d}+\boldsymbol{\lambda}^{\top}\boldsymbol{x}^{\text{bd}} \\
\text{s.t.}\quad & \boldsymbol{H}_t^{\text{bd}}\boldsymbol{x}^{\text{bd}} \in \mathcal{E}_t^{\text{bd}}\left(\boldsymbol{E}_{t(k)}^{\text{bd}},\boldsymbol{e}_t^{\text{bd}}\right), \forall t \in \mathcal{T} \\
& \boldsymbol{A}_i^{\text{tie}}\boldsymbol{S}_i^{\text{tie}}\boldsymbol{x}^{\text{bd}} \le \boldsymbol{b}_{i(k)}^{\text{tie}}, \forall i \in \mathcal{I}^{\text{tie}} \\
& \boldsymbol{\lambda}^{\top}\boldsymbol{C}+\boldsymbol{\pi}^{\top}\boldsymbol{F}=\boldsymbol{0} \\
& \boldsymbol{\pi} \ge \boldsymbol{0}
\end{aligned}
\tag{A.1}
$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\pi}$ are the dual variables associated with the equality (23d) and inequality constraints (23e), respectively.

The problem in (A.1) is a bilinear programming problem with a bilinear term $\boldsymbol{\lambda}^{\top}\boldsymbol{x}^{\text{bd}}$ in the objective function, which poses challenges for direct solution with commonly used solvers. According to the propriety of dual variables, violation of the equality constraints (23d) will lead to the objective function an infinity value $-\infty$. Therefore, the optimal value of each element in vector $\boldsymbol{\lambda}$ is the 3 possible values of 0, $+\infty$ and $-\infty$. To facilitate computation, we discretize each element in the unbounded vector variable $\boldsymbol{\lambda}$ to three specific values, $-M$, 0, and M, where M is a sufficient large positive constant. By introducing two discrete variables, $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$, $\boldsymbol{\lambda}$ can be expressed as follows:

$$
\begin{aligned}
&\boldsymbol{\lambda}=(\boldsymbol{z}_1-\boldsymbol{z}_2)\cdot M \\
&\boldsymbol{z}_1+\boldsymbol{z}_2 \le \boldsymbol{1}_l, \boldsymbol{z}_1, \boldsymbol{z}_2 \in \{0,1\}^l
\end{aligned}
\tag{A.2}
$$

where $l := \text{len}(\lambda)$. Subsequently, the bilinear term $\boldsymbol{\lambda}^{\top}\boldsymbol{x}^{\text{bd}}$ can be reformulated as products between binary and continuous variables, which can then be transformed into a mixed-integer linear programming problem or directly solved using off-the-shelf solvers, such as Gurobi.