Lecture 14: Miscellaneous Topics

Week 14

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This is the last lecture, and we'll touch upon some additional topics in machine learning and related fields.

1 Matrix Factorization

Consider a matrix $M \in \mathbb{R}^{n\times m}$ of rank r ($r < \min\{n, m\}$). We can find $U \in \mathbb{R}^{n\times r}$ and $V \in \mathbb{R}^{m\times r}$ such that $UV^\top = M$ in two steps.

The following two steps solve find a factorization with probability 1.

- Sampling a matrix $U^0 \in \mathbb{R}^{n\times r}$ such that $U^0_{i,j} \overset{i.i.d.}{\sim} \mathcal{N}(0, 1)$.
- Let $V \leftarrow M^\top U^0$, and let $U \leftarrow MV(V^\top V)^{-1}$.

Proposition 1.1. *With probability 1,* $UV^\top = M$.

Proof. Firstly, note that with probability 1, the column space of V equals the column space of $M^\top$. Thus V is of full rank. This means the matrix $V(V^\top V)^{-1}V^\top$ is the projection matrix onto the column space of V, or equivalently, the column space of $M^\top$. Thus we have

$$UV^\top = MV(V^\top V)^{-1}V^\top = M.$$

$\square$

2 Tiling by Random Hyperplanes

Hand-written notes only.

3 Stochastic Gradient/Hessian Estimation

Hand-written notes only.