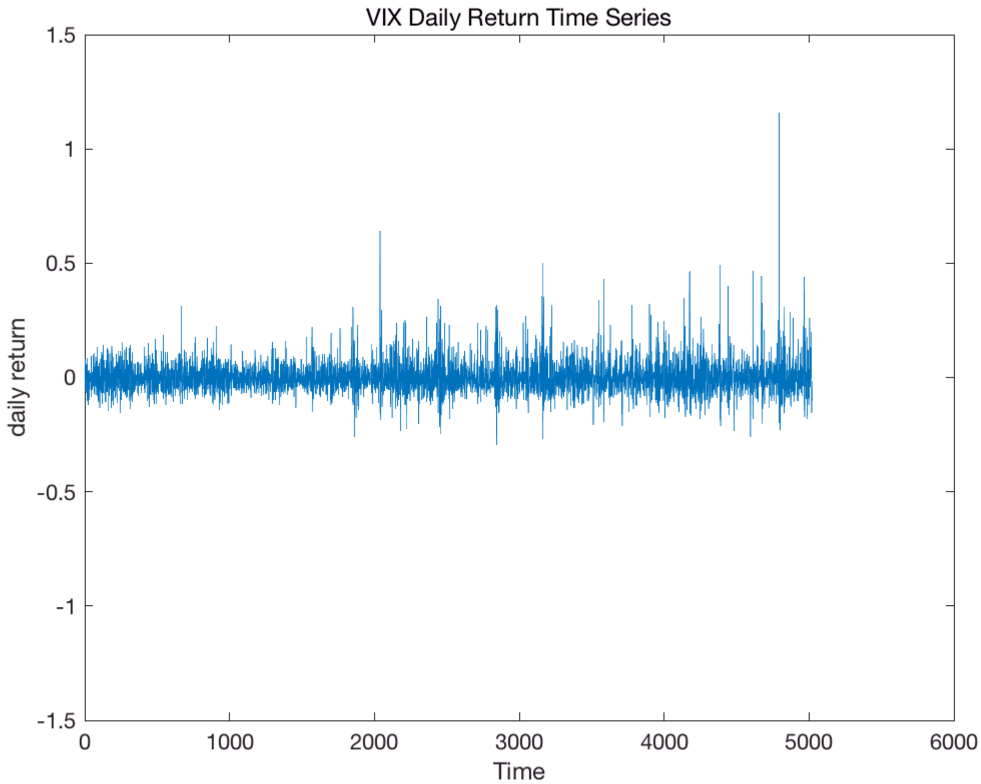
Question 1

```
filename ='HW1_Due20180207.xlsx';
Q1='DataSource';
[num1,text1,raw1]=xlsread(filename,Q1,'A:B','');
vix_price =cell2mat(raw1(3:end,2));
n1=length(vix_price);
m1=length(vix_price)-1;
v_ret =nan(m1,1);
for i= 1:n1-1
   v_ret(i)=(vix_price(i+1)/vix_price(i))-1;
end
v_ret
```

```
v_ret =
         0
   -0.0219
    0.0811
    0.0333
   -0.0257
   -0.0610
    0.0171
   -0.0545
   -0.0662
    0.0541
```

```
%%draw daily return time series
plot(v_ret)
ylim([-1.5,1.5])
xlabel('Time')
ylabel('daily return')
title('VIX Daily Return Time Series')
```




```
%%calculate the sample moments (mean,skewness, and kurtosis) for the VIX daily
returns
vix_mean=nanmean(v_ret)
```

```
vix_mean = 0.0024
```

```
vix_sigma=std(v_ret)
```

```
vix_sigma = 0.0713
```

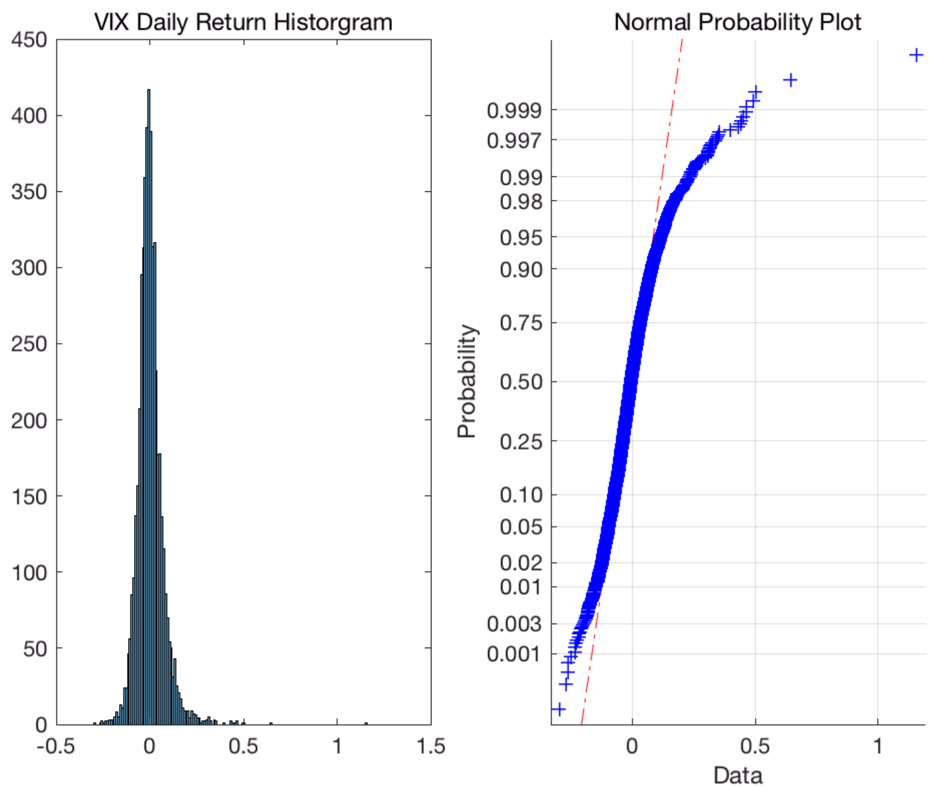
```
vix_skewness=skewness(v_ret)
```

```
vix_skewness = 2.0142
```

```
vix_kurtosis =kurtosis(v_ret)
```

```
vix_kurtosis = 22.2821
```

```
%%test whether the VIX daily returns were normally distributed or not.
 subplot(1,2,1);
 histogram(v_ret);
 title('VIX Daily Return Historgram');
 subplot(1,2,2);
 normplot(v_ret);
```




```
%%% Jarque-Bera Test
  % h=1 if reject normal distribution
 [h,p]=jbtest(v_ret);
```

警告: P is less than the smallest tabulated value, returning 0.001.

```
  fprintf('Jarque-Bera Test: h=%d, p-val=%.4f.\n',h,p);
```

Jarque-Bera Test: h=1, p-val=0.0010.

```
% One-sample Kolmogorov-Smirnov test
% h=1 if reject normal distribution
[h,p]=kstest((v_ret-vix_mean)/vix_sigma);
  fprintf('KS Test: h=%d, p-val=%.4f.\n',h,p);
```

KS Test: h=1, p-val=0.0000.

```
%%Conclution
```

Conclusion: the fat tail of Normal probability plot and the two tests which all have h=1, indicating rejection of normality.Thus, the VIX daily return is not normal distributed.

Question 2

```
filename ='HW1_Due20180207.xlsx';
Q2='DataSource';
[num,text,raw]=xlsread(filename,Q2,'D:E','');
Date = cell2mat(raw(3:end,1));
Date =Date +693960;
Date =datestr(Date);

price = cell2mat(raw(3:end,2));
price0 =price(2:end,1);
n= length(price);
m=n-1;
dret=zeros(m,1);
price1 = price(1:n-1,1);
for i =1:n-1
    dret(i)=(price(i+1)/price(i))-1;
end
dret
```

```
dret =
    0.0065
    0.0080
   -0.0265
   -0.0023
    0.0102
    0.0182
   -0.0117
    0.0168
    0.0076
   -0.0059
```

```
%%daily volatility
spy_sigma = std(dret)
```

```
spy_sigma = 0.0121
```

```
%%annual volatility
spy_ansigma= spy_sigma*sqrt(252)
```

```
spy_ansigma = 0.1924
```

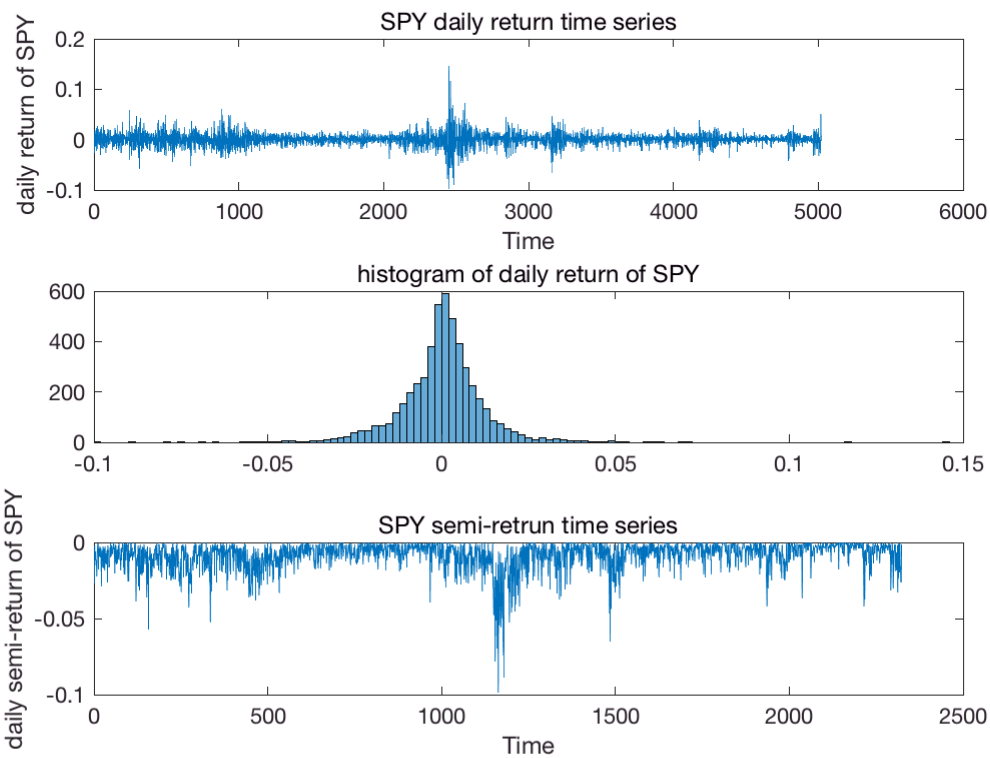
```
%%plot of dret
x = (1:m)';
y= dret;
subplot(3,1,1)
plot(x,y);
title('SPY daily return time series');
xlabel('Time');
ylabel('daily return of SPY');
%%semi daily volatility
spy_semi=std(dret(dret<0))
```

```
spy_semi = 0.0093
```

```
%%semi annual volatility
spy_ansemi=spy_semi*sqrt(252)
```

```
spy_ansemi = 0.1471
```

```
%%plot semi-retrun time series
subplot(3,1,3)
plot(dret(dret<0))
title('SPY semi-retrun time series');
xlabel('Time');
ylabel('daily semi-return of SPY');
%% histogram of daily return
subplot(3,1,2);
histogram(dret);
title('histogram of daily return of SPY');
```




```
%   set a target amount, e.g., the maximum loss in a month in return
%   percentage form, say, -2%
daily_shortfall=-0.02;
%   caculate the shortfall probability (realized)
shortfall_prob_realized=sum(dret<daily_shortfall)/length(dret)
```

```
shortfall_prob_realized = 0.0464
```

```
%   what is the theoretical shortfall probability assuming monthly returns
%   follow normal distribution with mean estimated by sample mean, and
%   variance estimated by sample variance.
z_value=(daily_shortfall-nanmean(dret))/spy_sigma
```

```
z_value = -1.6680
```

```
%   the theoretical short fall probability
```

```
shortfall_prob_theoretical=normcdf(z_value)
```

```
shortfall_prob_theoretical = 0.0477
```

```
%   Value At Risk - 95%
%   i.e., choose the bottom 5% percentile threshold value
VaR=prctile(dret,5)
```

```
VaR = -0.0194
```

```
%   CVaR: the mean return for all returns below the VaR threhold
CVaR=mean(dret(dret<VaR))
```

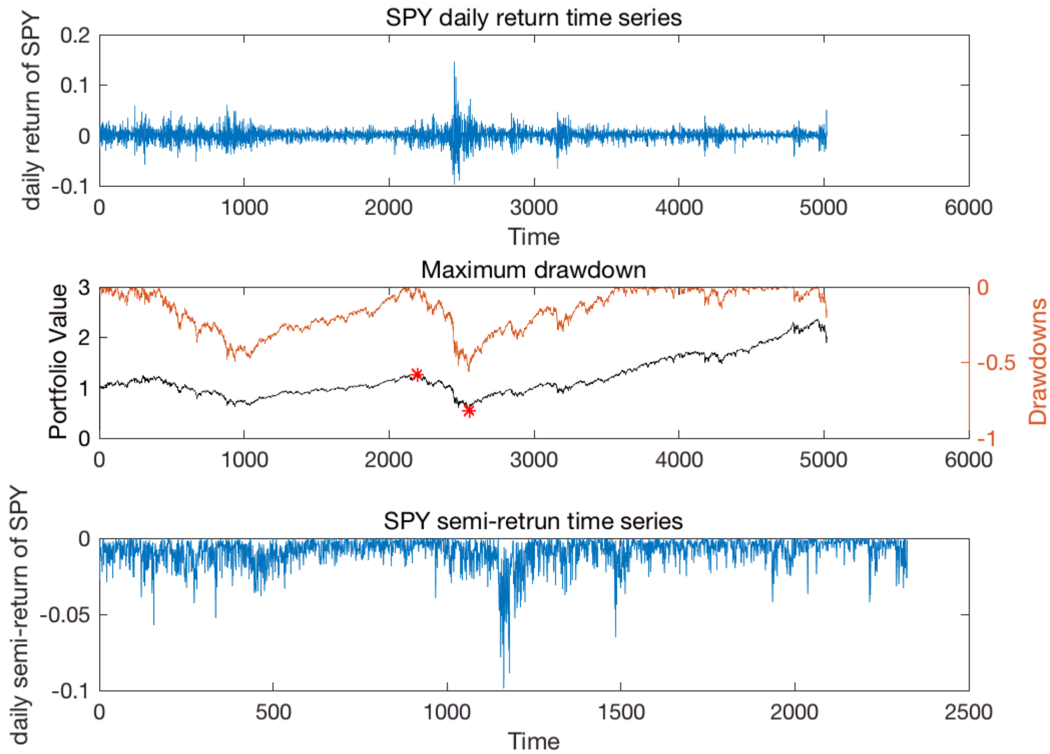
```
CVaR = -0.0288
```

Maximum drawdown

```
%   plot the portfolio value curve for the Emerging Market Equity
%   Benchmark
portval_vec=[1;cumprod(1+dret)];
yyaxis left ;
plot(portval_vec);
title('Maximum drawdown');
ylabel('Portfolio Value');

%   calculate drawdown at each time point
drawdown_vec=nan(length(portval_vec),1);
for i=1:length(portval_vec)
   drawdown_vec(i)=portval_vec(i)/max(portval_vec(1:i))-1;
end
yyaxis right
hold on
plot(drawdown_vec);
ylabel('Drawdowns');

%   locate the maximum drawdown
[max_drawdown,pos_right]=min(drawdown_vec);
[blah,pos_left]=max(portval_vec(1:pos_right));
hold on
yyaxis left
plot([pos_left,pos_right],portval_vec([pos_left,pos_right]),'r*');
```




```
%   output the maximum drawdown
fprintf('Max Drawdown = %.1f%%.\n',max_drawdown*100);
```

```
Max Drawdown = -56.5%.
```

Question 3

```
%%Problem 3
%% Data:    Multi-Asset Class Monthly Returns on Datasource_CAPMAssetClasses

%%Assumptions   1)- no risk-free asset in the investable universe and all the
risky assets are given on Datasource_CAPMAssetClasses;
    %%2)- use historical covariance matrix and historical returns to estimate
covariance matrix and expected return vector (see on lecture 2 notes)
    %%3)- no short-sell allowed. (before we have no such constraint)

    %%1. Run an optimization to locate the minimum-variance portfolio on the new
frontier.
    %%2A. Run optimizations to locate two other efficient portfolios different
than the minimum-variance portfolio
    %%2B. Prove or disprove the minimum-variance portfolio can be represented as
a combination of the two portfolios found on 2A.
    %%3. Plot both the old efficient frontier without the short-sell constraint
and the new efficient frontier with the short-sell constraint on the same
graph.

%   the investable universe
sheet2='DataSource_CAPMAssetClasses'
```

```
sheet2 = 'DataSource_CAPMAssetClasses'
```

```
[num3,text3,raw3]=xlsread(filename,sheet2,'A:M','');

ac_name =text3(1,2:end)
```

```
ac_name = 1×12 cell 数组
    'Fixed-Income: Treasury Inflation Protected Securities (TIPS)'    'Fixed-Income: U.S. Treasury Bonds and Investment Grade Corp Bonds'    'Fixed-Income: U.S. High Yield Corp Bonds'    'Fixed-Income: Non U.S. Sovereign Bonds'
'Equities: U.S. Large Cap Growth'    'Equities: U.S. Large Cap Value'
'Equities: U.S. Small Cap Growth'    'Equities: U.S. Small Cap Value'
'Equities: Developed Countries Non-US '    'Equities: Emerging Markets'
'Commodities'    'Real Estate'
```

```
ac_mret=cell2mat(raw3(3:end,2:end));
%   calculate the covariance matrix based on their historical monthly
%   return data
%   note: we annualize the covariance matrix by multiplying the matrix with 12
Sigma=nancov(ac_mret)*12;

%   let's use the historical annualized  as our estimates of asset
%   expected returns
ExpRet=nan(length(ac_name),1);
for ii=1:length(ac_name)
    ExpRet(ii)=geomean(1+0.01*ac_mret(:,ii))^12*100-100;
    fprintf('Annualized Return=%.2f%% for %s.\n',ExpRet(ii),ac_name{ii});
end
```

```
Annualized Return=5.20% for Fixed-Income: Treasury Inflation Protected Securities (TIPS).

Annualized Return=4.55% for Fixed-Income: U.S. Treasury Bonds and Investment Grade Corp Bonds.

Annualized Return=6.30% for Fixed-Income: U.S. High Yield Corp Bonds.

Annualized Return=3.40% for Fixed-Income: Non U.S. Sovereign Bonds.

Annualized Return=5.05% for Equities: U.S. Large Cap Growth.

Annualized Return=6.15% for Equities: U.S. Large Cap Value.

Annualized Return=6.14% for Equities: U.S. Small Cap Growth.

Annualized Return=8.23% for Equities: U.S. Small Cap Value.

Annualized Return=3.52% for Equities: Developed Countries Non-US .

Annualized Return=8.52% for Equities: Emerging Markets.

Annualized Return=0.57% for Commodities.

Annualized Return=9.65% for Real Estate.
```

```
%  unit vector
unit_vec=ones(length(ac_name),1);

%  calculate the minimum variance portfolio and the pseudo-maximum sharpe
%  ratio portfolio
Sigma_inv=inv(Sigma);
w_minv=Sigma_inv*unit_vec/(unit_vec'*Sigma_inv*unit_vec);
w_msr=Sigma_inv*ExpRet/(unit_vec'*Sigma_inv*ExpRet);
fprintf('minv and msr portfolio respectively:(short-sell allowed) ')
```

```
minv and msr portfolio respectively:(short-sell allowed)
```

```
[w_minv,w_msr]
```

```
ans =
   -0.2648   -0.1828
    1.2018    1.1267
    0.0495    0.0833
   -0.0480   -0.0695
    0.0311    0.0653
    0.0757    0.0430
   -0.0032   -0.0586
    0.0372    0.1238
   -0.0335   -0.1366
   -0.0242    0.0446
```

```
% min variance portfolio without shortsell

 %  we'll formulate a portfolio by adding consrtraints to the min-variance
portfolio(no shortselling)
%  put return target on constraint
target_ret=0.7;

%  objective function
fun=@(w) w'*Sigma*w;

%  initial guess
w0=ones(length(ac_name));
w0=w0/sum(w0);

%  constraint 1: target return
%  Ax<=b
A=-ExpRet';
b=-target_ret;

% constraint 2: ub and lb for no shortsell
ub=ones(12,1);
lb=zeros(12,1);

%  constraint 3: fully invested
Aeq=ones(1,length(ac_name));
beq=1;

%  run the optimization
w_new_minv=fmincon(fun,w0,A,b,Aeq,beq,lb,ub);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the optimality tolerance,
and constraints are satisfied to within the default value of the constraint
```

```
tolerance.

<stopping criteria details>
```

```
[w_new_minv,w_minv]
```

```
ans =
    0.0000   -0.2648
    0.9265    1.2018
    0.0000    0.0495
    0.0000   -0.0480
    0.0130    0.0311
    0.0388    0.0757
    0.0081   -0.0032
    0.0000    0.0372
    0.0000   -0.0335
    0.0000   -0.0242
```

```
%   build the efficient frontier

%   define a function for weight vector in terms a combination of two known
%   portfolios
wt_func=@(c) (1-c)*w_minv+c*w_msr;
%   portfolio return and volatility function
pret_func=@(c) ExpRet'*wt_func(c);
prisk_func=@(c) sqrt(wt_func(c)'*Sigma*wt_func(c));

%without short sell, efficient frontier
%%2A. Run optimizations to locate two other efficient portfolios different than
the minimum-variance portfolio
Numassets =length(ac_name)
```

```
Numassets = 12
```

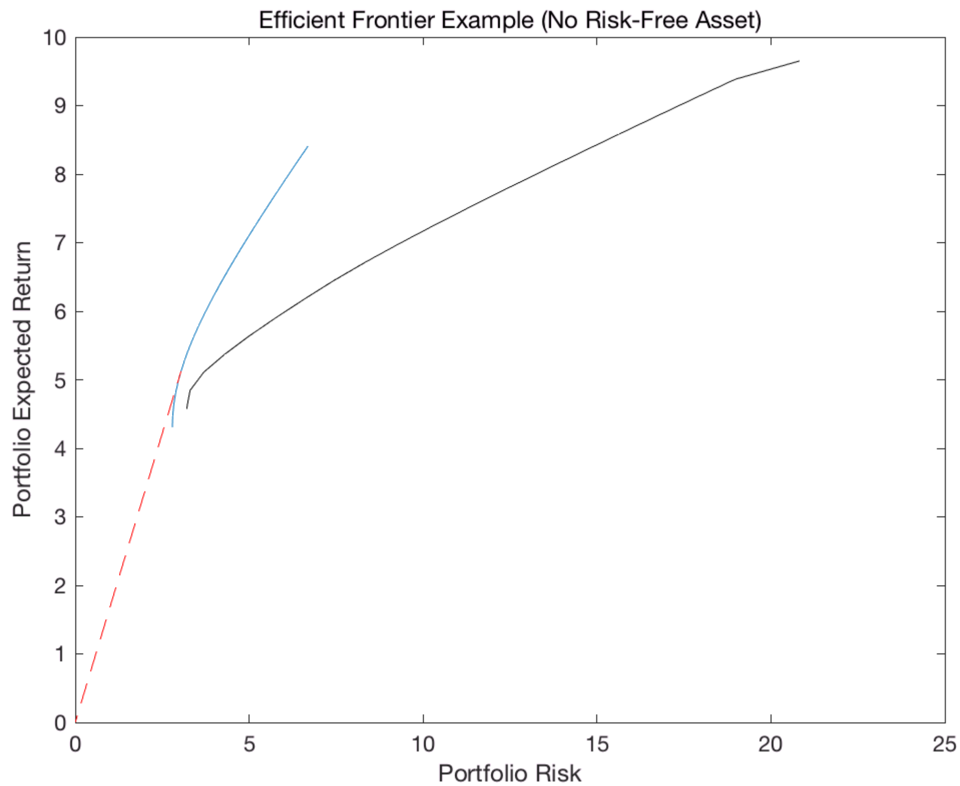
```
p = Portfolio();
p = setDefaultConstraints(p,Numassets);
p = setInitPort(p,1/12);
p = setAssetMoments(p,ExpRet,Sigma);

pwgt = estimateFrontier(p,20);
[prsk,pret] = estimatePortMoments(p,pwgt);

% Plot efficient frontier


%   efficient frontier with short selling
c=0:0.2:5;
close();
plot(arrayfun(prisk_func,c),arrayfun(pret_func,c));%% draw EF with short
selling
title('Efficient Frontier Example (No Risk-Free Asset)')
xlabel('Portfolio Risk')
ylabel('Portfolio Expected Return')
hold on
plot([0,prisk_func(1)],[0,pret_func(1)],'--','Color','r');%%get tangent
portfolio
```

```
hold on
plot(prsk,pret,'Color','black') %%draw EF without shorsell
```



% EF with shortsell allowed(blue curve), EF without shortsell(black curve)

%---%

%%2A. Run optimizations to locate two other efficient portfolios different than the minimum-variance portfolio, the weights in pwgt are all the optimized portfolios on the new efficient frontier, so I choose two as the optimized portfolios other than min-variance portfolio.

```
awgt= pwgt(:,1)
```

```
awgt =
         0
    0.9265
         0
         0
    0.0130
    0.0388
    0.0081
         0
         0
         0
```

```
bwgt =pwgt(:,2)
```

```
bwgt =
         0
    0.8922
    0.0426
         0
         0
    0.0095
         0
    0.0557
         0
         0
```

```
aret= ExpRet'*awgt
```

```
aret = 4.5768
```

```
arsk=awgt'*Sigma*bwgt
```

```
arsk = 10.3420
```

```
bret= ExpRet'*awgt
```

```
bret = 4.5768
```

```
brsk=awgt'*Sigma*bwgt
```

```
brsk = 10.3420
```

```
[w_new_minv,awgt,bwgt]
```

```
ans =
    0.0000         0         0
    0.9265    0.9265    0.8922
    0.0000         0    0.0426
    0.0000         0         0
    0.0130    0.0130         0
    0.0388    0.0388    0.0095
    0.0081    0.0081         0
    0.0000         0    0.0557
    0.0000         0         0
    0.0000         0         0
```

```
%It's easy to see these two portfolios(not the min-variance portfolio) can not
be combined to formulate the min-variance portfolio
```