An Elegant Stock Investment Strategy

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May 21, 2025

1 Problem Description

In this report, I'd like to discuss a strategy to buy stocks that can earn money with high probability.

Suppose the purchase price of a stock is X USD per share, and the expected price increase is a (e.g., 25% increase means $a = 0.25$). The target profit is Y USD. The number of shares bought is n, and the number of shares sold after the price rises is m, with at least k shares reserved ($k < n$). Our goal is to determine n and m such that selling m shares after the price rises will achieve the expected profit Y while keeping at least k shares.

2 Variable Definitions

- X: Purchase price (USD per share)
- a: Price increase rate (decimal)
- Y: Expected profit (USD, $Y \geq 0$)
- n: Number of shares bought (integer)
- m: Number of shares sold (integer, $m < n$)
- k: Number of shares to hold (integer, $1 \leq k < n$)

2.1 Solution

The amount obtained by selling m shares after price increase is $m \cdot X \cdot (1+a)$ with the total purchase cost $n \cdot X$. Hence the expected profit condition requires:

$$m \cdot X \cdot (1+a) \geq Y + n \cdot X, \tag{1}$$

with the reserve share condition constraint:

$$n - m \geq k \quad . \tag{2}$$

By inequality (1), we have

$$m \geq \frac{n + \frac{Y}{X}}{1+a},$$

and therefore we get the minimum number of shares sold should be

$$m^* = \left\lceil \frac{n + \frac{Y}{X}}{1+a} \right\rceil, \tag{3}$$

where $\lceil \cdot \rceil$ denotes the ceiling function is defined as the smallest integer greater than or equal to x.

Combining inequality (2) and (3), we get an optimization problem

$$\begin{aligned} \min_{n \in \mathbb{N}} \quad & n \\ \text{s.t.} \quad & n - k \geq \left\lceil \frac{n + \frac{Y}{X}}{1+a} \right\rceil \end{aligned} \tag{4}$$

We can design an iterative algorithm to solve optimization problem (4), but it is unnecessary because we can make a relaxation to obtain the following explicitly solvable optimization problem:

$$\begin{aligned} \min_{n \in \mathbb{N}} \quad & n \\ \text{s.t.} \quad & n - k \geq \frac{n + \frac{Y}{X}}{1+a} + 1. \end{aligned} \tag{5}$$

The solution is

$$n^* = \left\lceil \frac{(1+a)(k+1) + \frac{Y}{X}}{a} \right\rceil. \tag{6}$$

3 Conclusion

Under the above assumptions, the solution is

$$n^* = \left\lceil \frac{(1+a)(k+1) + \frac{Y}{X}}{a} \right\rceil \quad \text{and} \quad m^* = \left\lceil \frac{n + \frac{Y}{X}}{1+a} \right\rceil.$$

4 Python Code

4.1 Code

```python
import math

# Parameters
X = 1       # Purchase price (USD per share)
a = 0.25    # Price increase (25%)
Y = 20      # Expected profit (USD)
k = 2       # Minimum shares to hold

# Calculate minimum number of shares to buy n
n = math.ceil((Y / X + (1 + a) * (k + 1)) / a)
# Calculate corresponding number of shares to sell m
m = math.ceil((n + Y / X) / (1 + a))
# Calculate sell amount
sell_amount = m * X * (1 + a)
# Total purchase amount
buy_amount = n * X
# Shares held after selling
hold_amount = n - m

# Output parameters and results
print(f'Purchase price X = {X:.2f} USD/share')
print(f'Price increase a = {a:.2f} ({a*100:.2f}%)')
print(f'Expected profit Y = {Y:.2f} USD')
print(f'Minimum shares to hold k = {k} shares')
print('--------------------------')
print(f'Minimum shares to buy n = {n} shares')
print(f'Corresponding shares to sell m = {m} shares')
print(f'Total purchase amount = {buy_amount:.2f} USD')
print(f'Sell amount = {sell_amount:.2f} USD')
print(f'Shares held after selling = {hold_amount} shares')
print(f'Actual profit = {sell_amount - buy_amount:.2f} USD')
```

```
        # Check conditions
        # Sell amount meets expected profit
        cond1 = (sell_amount >= buy_amount + Y)
        # Hold shares meet minimum requirement
        cond2 = (hold_amount >= k)
        # Shares sold less than shares bought
        cond3 = (m < n)

        if cond1 and cond2 and cond3:
        print('All conditions are met.')
        else:
        print('Some conditions are not met:')
        if not cond1:
        print('- Sell amount is insufficient to achieve expected profit')
        if not cond2:
        print('- Not enough shares held after selling')
        if not cond3:
        print('- Shares sold are not less than shares bought')
```

4.2 Output

```
Purchase price X = 1.00 USD/share
Price increase a = 0.25 (25.00%)
Expected profit Y = 20.00 USD
Minimum shares to hold k = 2 shares
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Minimum shares to buy n = 95 shares
Corresponding shares to sell m = 92 shares
Total purchase amount = 95.00 USD
Sell amount = 115.00 USD
Shares held after selling = 3 shares
Actual profit = 20.00 USD
All conditions are met.
```

5 Numerical Simulation

In this section, we aim to observe the variation curves of n^* and m^* with respect to a and Y.

5.1 Python Code

```
import numpy as np
import matplotlib.pyplot as plt
import math

X = 10
k = 10
a_values = np.arange(0.1, 1.1, 0.1)
Y_values = np.arange(100, 1100, 100)

Y_fixed = 50
n_a = np.zeros_like(a_values)
m_a = np.zeros_like(a_values)
for i, a in enumerate(a_values):
n_a[i] = math.ceil((Y_fixed / X + (1 + a) * (k+1)) / a)
m_a[i] = math.ceil((n_a[i] + Y_fixed / X) / (1 + a))

a_fixed = 0.25
n_Y = np.zeros_like(Y_values)
m_Y = np.zeros_like(Y_values)
for i, Y in enumerate(Y_values):
n_Y[i] = math.ceil((Y / X + (1 + a_fixed) * k) / a_fixed)
m_Y[i] = math.ceil((n_Y[i] + Y / X) / (1 + a_fixed))

plt.figure(figsize=(10, 8))
plt.subplot(2, 1, 1)
plt.plot(a_values, n_a, '-o', linewidth=2, label='Buy n')
plt.plot(a_values, m_a, '-s', linewidth=2, label='Sell m')
plt.xlabel('Increase a')
plt.ylabel('Shares')
plt.title(f'Reserve k={k} shares, fixed profit Y={Y_fixed},
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```
    n and m vs a')
    plt.grid(True)
    plt.legend(loc='upper left')

    plt.subplot(2, 1, 2)
    plt.plot(Y_values, n_Y, '-o', linewidth=2, label='Buy n')
    plt.plot(Y_values, m_Y, '-s', linewidth=2, label='Sell m')
    plt.xlabel('Profit Y')
    plt.ylabel('Shares')
    plt.title(f'Reserve k={k} shares, fixed increase a={a_fixed:.2f}, n and m vs Y')
    plt.grid(True)
    plt.legend(loc='upper left')

    plt.tight_layout()
    plt.show()
```

5.2 Results

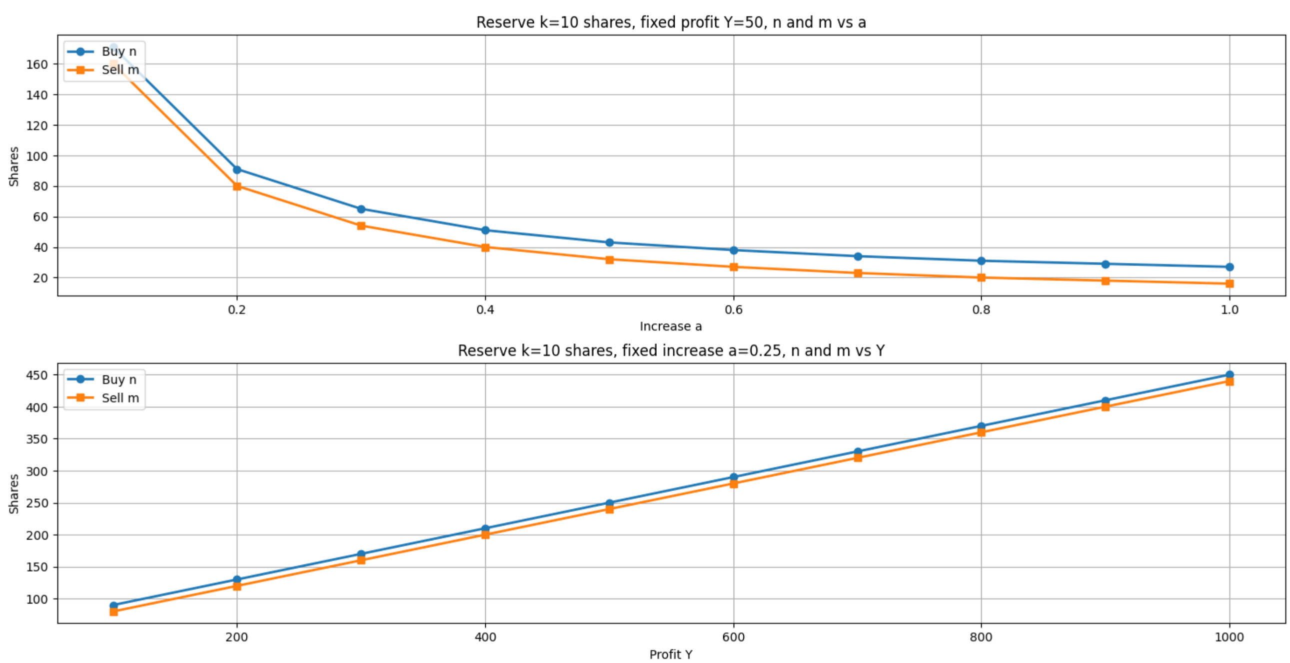

Figure 1: The variation curves of n and m with respect to a and Y.