LECTURE 7
INVENTORY CONTROL

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WHAT IS INVENTORY

The stock of any item or resource used in an organization

- Raw Materials & Component Parts
- Work-in-Process
- Finished Products
- Replacement parts, tools & supplies
- Goods in transit to warehouses or customers

IMPORTANCE OF INVENTORY – 2005 FISCAL YEAR

	WallMart (Billion $)	Boeing	General Motors	Dell
Cash & Short-Term Investment	6.4	5.9	50.4	9.0
Account Receivable	2.6	5.2	180.7	5.4
Inventories	**32.2 (73.5%)**	**7.7 (35%)**	**30.1 (9.6%)**	**0.57 (3.3%)**
Other Current Assets	2.5	2.8	51.7	2.6
Total Current Assets	**43.8 (100%)**	**22 (100%)**	**312.9 (100%)**	**17.7 (100%)**
Other Asset	94.3	38.1	163.1	5.4
Total Assets	138.2	60	476.1	24.1

HOW DOES DELL'S LOW INVENTORY HELP CUT COST?

"The longer you keep it the faster it deteriorates – you can literally see the stuff rot… Because of their short product lifecycles, computer components depreciate anywhere from a half to a full point a week. Cutting inventory is not just a nice thing to do. It's a financial imperative.

-- Kevin Rollins, former Dell's CEO

HOW DOES DELL'S LOW INVENTORY HELP CUT COSTS

Year	Inventory Turnover	Weekly Inventory
1992	4.79	10.856
1993	5.16	10.078
1994	9.4	5.532
1995	9.8	5.306
1996	24.2	2.149
1997	41.7	1.247
1998	52.40	0.992
1999	52.40	0.992
2000	51.4	1.012
2001	63.50	0.819

Assuming 1% depreciation per week)

Depreciation 89%

Depreciation 99%

WHY DO WE NEED INVENTORY

- **To maintain independence of operations**
- **To meet variation in product demand**
- **To allow flexibility in production scheduling**
- **To take advantage of economic of scale**

WHY WE SHOULD NOT HOLD INVENTORY

- **Inventory is costly**
  - Storage cost
  - Opportunity cost of capital
- **Inventory is risky**
  - Price fluctuation and short product life cycle increase risk of product obsolescence
- **Inventory may not be needed**
  - Information substitution

INVENTORY CLASSIFICATION

A classification to help manage inventories better

Type **A** Items

- **Small group of high value items**
- Accounts for 15% by the number of parts, and 70-80% of the total sales of all parts

Type **B** Items

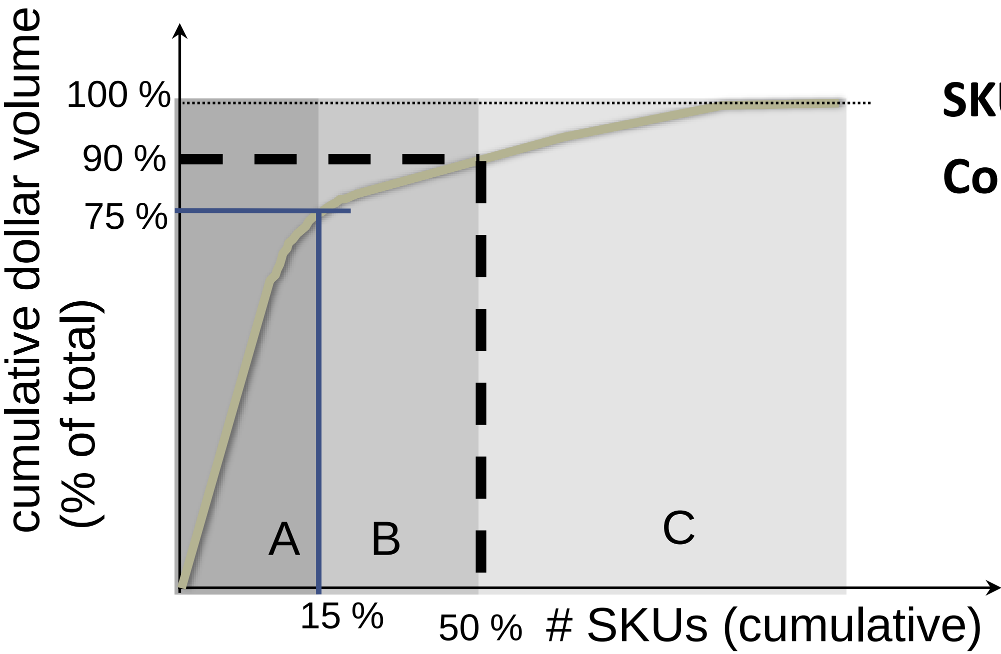
- Accounts for 35% of the number of parts, and 10-15% of the total value

Type **C** Items

- Accounts for 50% of the total number of parts, and for 5-10% of the total value

(The above percentages are approximations)

ABC CLASSIFICATION OF SKUS



SKU = Stock Keeping Unit

Cost of analysis & monitoring

- Focus on greatest impact
- Measure by “significance” of SKU

INVENTORY MANAGEMENT

Hold the right amount of inventory

In the right place

At the right time

THE SINGLE PERIOD INVENTORY CONTROL

Single-period examples:

- Newspapers or other items with rapid obsolescence
- Christmas trees or other seasonal items
- Capacity for short-life products (e.g., capacity for Intel's chip plants)

THE NEWSVENDOR PROBLEM

Assumptions:

- Single period
- Demand is unknown, but distribution is known

Example:

- Ordering cost: \$1
- Selling price: \$3
- Demand:

1	prob. 0.3
2	prob. 0.5
3	prob. 0.2

MARGINAL ANALYSIS

- **Suppose the newsvendor has 1 paper**
- **What is the value of the 2nd paper?**
  - He needs to spend $1 to procure the paper
  - If Demand < 2, then he sells for nothing
  - If Demand >=2, then he sells for $3
- **The value of the 2nd paper**

  = Pr(Demand < 2) * ($0 - $1) + Pr(Demand >=2) * ($3 - $1)

  = 0.3 * (-1) + 0.7 * 2 = 1.1 > 0

MARGINAL ANALYSIS

- **Suppose the newsvendor has 2 paper**
- **What is the value of the 3rd paper?**
  - He needs to spend \$1 to procure the paper
  - If Demand < 3, then he sells for nothing
  - If Demand >=3, then he sells for \$3
- **The value of the 3rd paper**

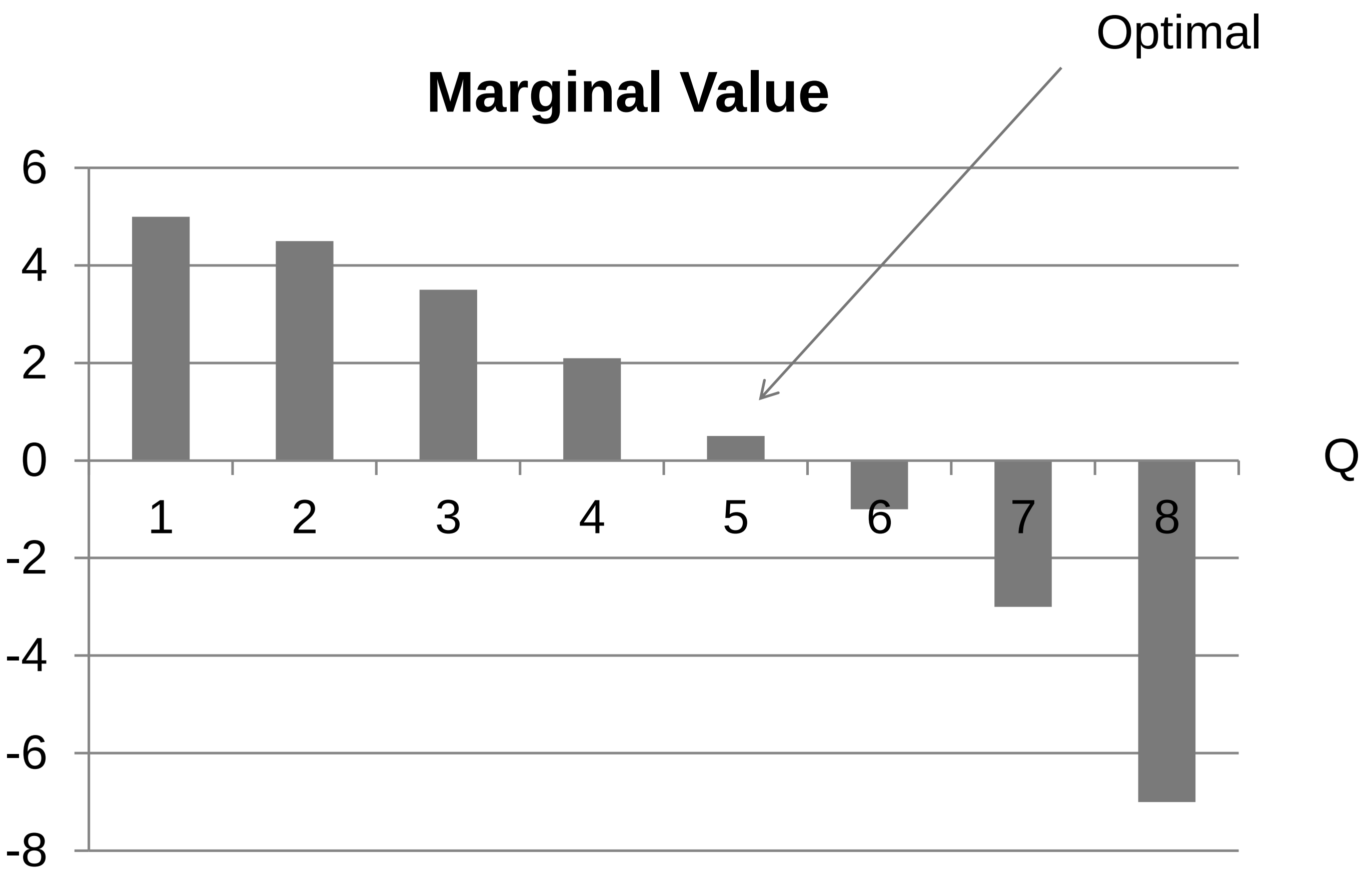
  = Pr(Demand < 3) * (\$0 - \$1) + Pr(Demand >=3) * (\$3 - \$1)

  = 0.8 * (-1) + 0.2 * 2 = -0.4 < 0

MARGINAL ANALYSIS

- **The Demand D is random**
- **Cost of overstocking or overage (C_o)**
  - cost of having unit in stock when demand does not materialize
- **Cost of understocking or underage (C_u)**
  - cost of not having unit in stock when demand does materialize
- **The marginal value of the Qth unit:**
  - $- Pr(D < Q) * C_o + Pr(D >= Q) * C_u$
  - The marginal value is decreasing in Q

MARGINAL ANALYSIS



THE OPTIMAL SOLUTION

- **We should choose an Q that satisfy:**
  - $Pr(D < Q)\ C_o \approx Pr(D >= Q)\ C_u$
  - Therefore $Pr(D \leq Q) \approx C_u/(C_u+C_o)$
- **If the demand is continuous**
  - $Q = F^{-1}(C_u/(C_u+C_o))$
- **If the demand is discrete**
  - $Q = F^{-1}(C_u/(C_u+C_o))$ round up to the nearest possible value of Demand

EXAMPLE

- **Ordering cost: \$1, Selling price: \$3, Demand: 100, 200 ~ 500 with equal probability**
  - Overage cost = \$1, Underage cost = \$3-\$1
  - $Q = F^{-1}(C_u/(C_u+C_o)) = F^{-1}(2/3)$ round up = 400
- **What if Demand is a normal distribution with mean 50 and standard deviation 10?**
  - $Q = F^{-1}(2/3) = 50 + 10 * \Phi^{-1}(2/3) = 50 + 10 * 0.43 = 54.3$, where Φ is the cumulative density function of standard normal distribution.
  - If you use Excel, you use NORMINV(2/3, 50, 10)

NEWSVENDOR APPLICATIONS
HOTEL RESERVATION

- **You manage a hotel with 200 rooms near 东方明珠**
- **The summer is coming so you are expecting traveling groups which usually books far in advance**
- **Traveling group usually can get a discount, but individual customers pay full price**
  - Discount price: ￥200 / night
  - Full price: ￥300 / night

- **Should the hotel fill up rooms with traveling groups?**
- **Why?**
- **How many rooms should be set aside for individual customers?**

NEWSVENDOR APPLICATIONS
AIRLINE OVERBOOKING

“I went to the airport but was told that the flight was overbooked...I was paid $50 to take the next flight...Why do airlines think it’s such a great idea to overbook?”

NEWSVENDOR APPLICATIONS
AIRLINE OVERBOOKING

Suppose you are managing the booking of an flight. Based on historical data, you find that the number of no shows is normally distributed with mean 5 and standard deviation 3.

If a customer arrives but there is no seat for him, you have to fully refund the money he has paid plus an additional penalty of $50. The price of the air ticket is $30. How many seats should allow for overbooking?

Underage cost: C_u = $30
Overage cost: C_o = $50
Optimal overbooking number = $F^{-1}(30/80) = 4.04$

FORECAST

- **Why weather forecast constantly overestimate the probability of raining?**
    - Because the cost of not forecasting a rain is larger than the cost of not forecasting a sunny day
- **Why analyst constantly underestimate the company's earnings?**
    - Because the cost of underestimate is larger than overestimate

SELLING TO A NEWSVENDOR

- **Suppose you are a firm printing newspaper**
- **Production cost: $0.1/per unit**
- **Selling price to the newsvendor: $0.5/per unit**
- **Newsvendor sells to customers at $1/per unit**
- **Demand: *N(100,20)***

$0.1

$0.5

$1

N(100, 20)

THE SUPPLY CHAIN PROBLEM

From the newspaper printing firm's point of view, what is the optimal quantity if it directly sells to customers?

- Cu =
- Co =
- Critical ratio =
- Optimal quantity to produce Q_{sc} =

THE NEWSVENDOR PROBLEM

From the newsvendor's point of view, what is the optimal quantity?

- Cu =
- Co =
- Critical ratio =
- Optimal quantity to produce Q_{nv} =

PROFIT COMPARISON

$Q_{nv} = 100$

Expected Profit of the Supply Chain

$= \Pr(Q \geq 100) * 100 * 1 + \int_0^{100} f(q) * q * 1dq - 100 * 0.1$

$= 50 - 10 + 42$

Expected Profit of the Newsvendor $= 50 + 42 - 100 * 0.5 = 42$

$Q_{sc} = 126$

Expected Profit of the Supply Chain

$= \Pr(Q \geq 126) * 126 * 1 + \int_0^{126} f(q) * q * 1dq - 126 * 0.1$

$= 12.6 - 12.6 + 86.89$

Expected Profit of the Newsvendor $= 12.6 + 86.89 - 63 = 36.49$

PROFIT COMPARISON
-- DOUBLE MARGINALIZATION

Q_{nv}	100
Newspaper firm	$40
Newsvendor	$42
Supply Chain	$82

Q_{sc}	126
Newspaper firm	$50.4
Newsvendor	$36.5
Supply Chain	$86.9

A STORY ABOUT BLOCKBUSTER

In the summer of 1997, movie fans flocked to their local Blockbuster, only to find that all ten copies had already been checked out. At $60 a copy, Blockbuster couldn't afford to stock the number of tapes it needed. (At $3 per rental, Blockbuster had to rent a tape at least 20 times!) Its suppliers, the movie studios, had to charge $60 to earn enough revenue themselves. No one – not the suppliers, not the retailer, not the customer – was happy.

REVENUE SHARING

In 1998, Blockbuster agreed to give the studios 50% of the rental fees in return for $9 on tapes. Blockbuster keeps only half of the revenue ($1.5 per rental), but it breaks even after each tape has been rented 6 times. Blockbuster purchases more tapes to satisfy more customers, which means higher profit. The studio also has higher profit from increased tape sales and the revenue share. Customers go home happy.

BENEFIT OF REVENUE SHARING

FOR THE RETAILER

	Traditional Pricing	Revenue Sharing
A. Number of tapes purchased	10	30
B. Price per tape	$60	$9
C. Purchase cost	$600	$270
D. Number of rentals	300	500
E. Total rental revenue (D × $3/rental)	$900	$1,500
F. Retailer's share of rental revenue	$900 (100%)	$750 (50%)
G. Retailer's profit	$300	$480
H. Profit per dollar of inventory	$0.50	$1.78

FOR THE SUPPLIER

	Traditional Pricing	Revenue Sharing
I. Number of tapes purchased	10	30
J. Price per tape	$60	$9
K. Revenue from selling tapes	$600	$270
L. Number of rentals	300	500
M. Total rental revenue (L × $3/rental)	$900	$1,500
N. Supplier's share of rental revenue	$0 (0%)	$750 (50%)
O. Supplier's total revenues	$600	$1,020
P. Supplier's production and distribution cost (I × $10/tape)	$100	$300
Q. Supplier's profit	$500	$720

SUMMARY

- Newsvendor model is for the single period situation in which the demand is random.
- The optimal order quantity is obtained by

  $Q = F^{-1}(C_u/(C_u+C_o))$
- Newsvendor model deals with variability in inventory management
- In a two tier supply chain, a wholesale contract will cause double marginalization
- A revenue sharing contract can mitigate the effect of double marginalization

EOQ MODEL (ECONOMIC ORDER QUANTITY)

- **Multi-period model**
- ***D*: annual demand, constant**
- ***Q*: order quantity**
- ***S*: setup cost (regardless of *Q*), or transportation cost**
- ***C*: unit cost**
- ***H*: holding cost per unit per year**
- ***i*: holding cost rate per unit per year, *H* = *iC***

ECONOMY OF SCALE 规模经济

- **The cost per unit decreases with the increase of scale**

INVENTORY MANAGEMENT FOR A RETAILER

Here are some facts about The North Face retail shop:

- D: 1200 jackets / year
- S: \$2,000
- C: \$200 per jacket

What order size (Q) would you recommend for The North Face ?

THE SOUTH FACE

- *D* = annual demand
- *Q* = order size
- Number of orders in a year

  $= D / Q$

- Average Inventory level

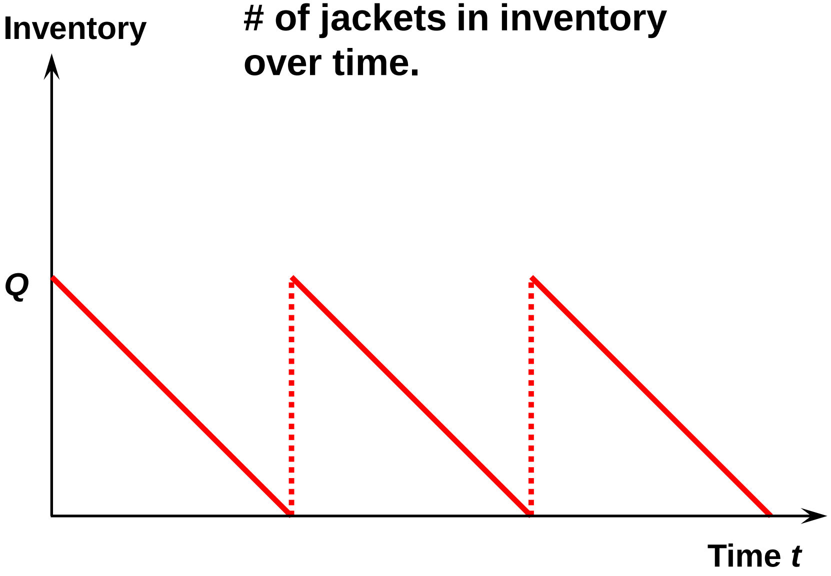
  $= Q / 2$

- Total cost per year

  = setup cost + holding cost + unit cost

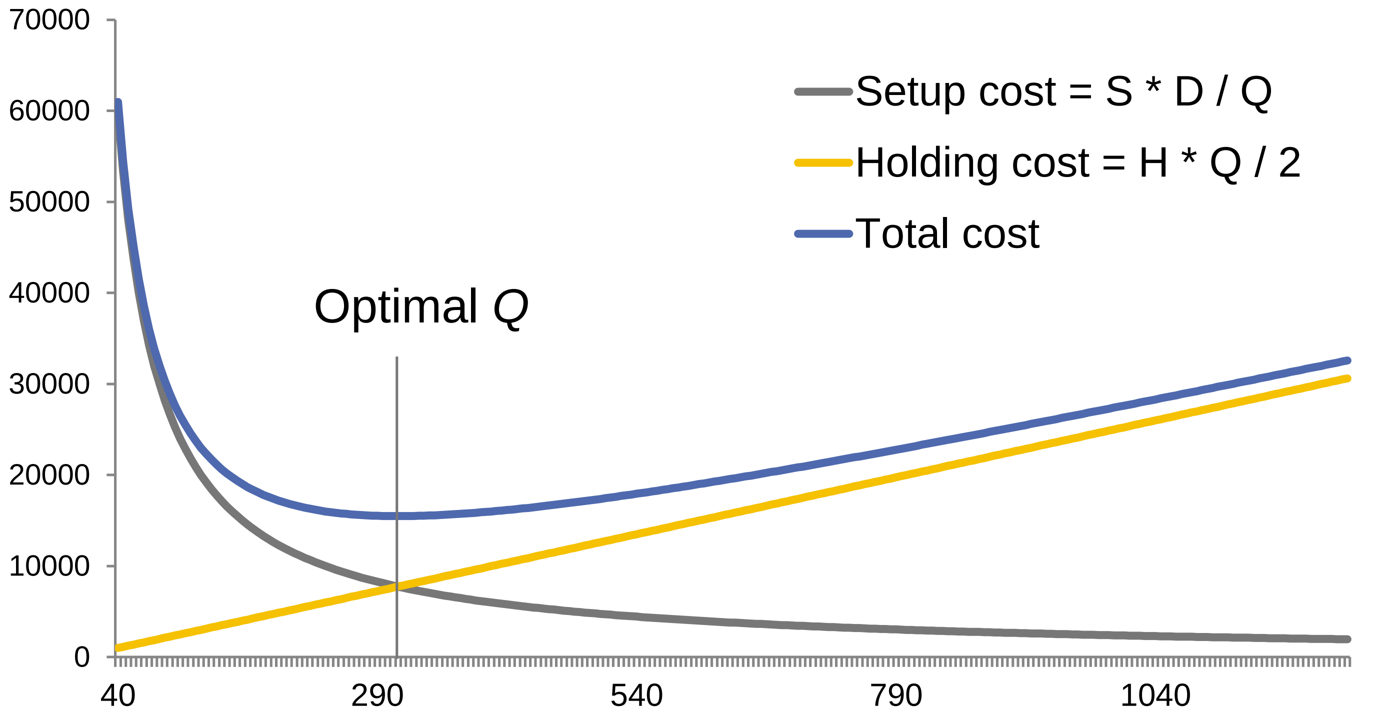
  $= S * D / Q + H\,Q / 2 + D\,C$

Inventory Profile:
# of jackets in inventory over time.



COST MINIMIZATION GOAL

- **The total cost is U-shaped**



ECONOMIC ORDER QUANTITY

- $Q_{opt} = \sqrt{\frac{2\ D\ S}{H}} = \sqrt{\frac{2\ (\text{Annual demand})\ (\text{Setup cost})}{\text{Holding cost}}}$
- **Optimal Cost = $\sqrt{2\ DHS}$ (ignoring the unit cost)**

What happens to cycle inventory if the demand rate increases?

What happens to lot sizes if setup costs decrease?

What happens to lot sizes if interest rates drop?

TRUCKASAURUS AND THE PARKING LOT

The parking lot outside the MGM Grand Detroit casino costs $3 per hour (1 night = 12 hours). Being an expert gambler, you can make 22% per night at the casino. Every time you leave the casino you have to tip the doorman, which costs you $1. Letting the meter run out is not an option – Truckasaurus devours cars with expired parking meters.

How much $ should you plug the meter per trip?

TRUCKASAURUS AND THE PARKING LOT

Twelve hours of parking will cost you $36

- D= $36
- H = $0.22
- S = $1

Optimal $ to feed meter =?

- Q = sqrt(2 D S / H) = sqrt(2 * 36 * 1 / 0.22) = $18

ROBUSTNESS OF THE EOQ SOLUTION

- What if we make an order quantity $Q = aQ_{opt}$?
- The corresponding cost = $\frac{a+\frac{1}{a}}{2}\sqrt{2\,DHS}$
- If the order quantity is either 50% higher than the EOQ (a=3/2) or the EOQ is 50% higher than the order quantity (a=2/3), then the resulting cost is (13/12)C*, which is about 8% higher than optimal.
- large errors in the implemented order quantities lead to small cost penalties

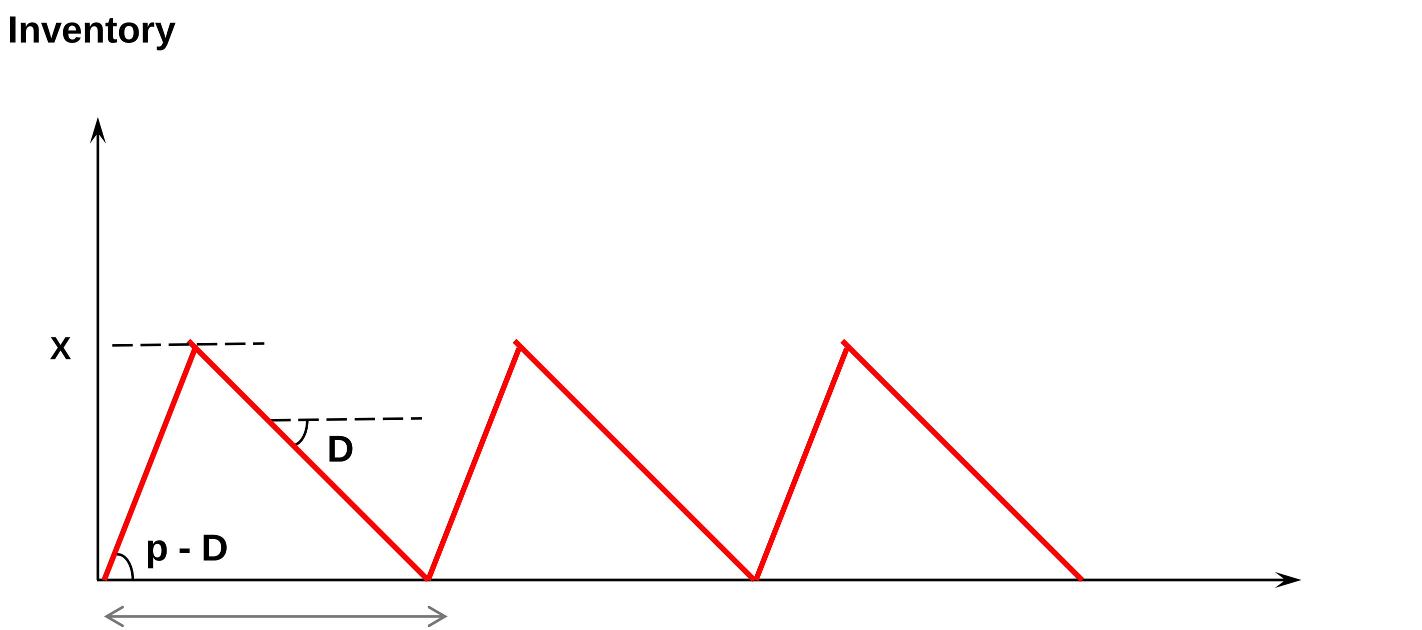
TWO EXTENSIONS OF THE EOQ MODEL

- **EOQ Model with constant production rate**
- **EOQ Model with quantity discount**

EOQ WITH CONSTANT PRODUCTION RATE

- D: annual demand, constant
- p: production rate, units / year
- S: setup cost for starting the machine
- C: unit cost
- H: holding cost per unit per year
- i: holding cost rate per unit per year, $H = iC$

EOQ WITH CONSTANT PRODUCTION RATE



T = X / (p-D) + X / D

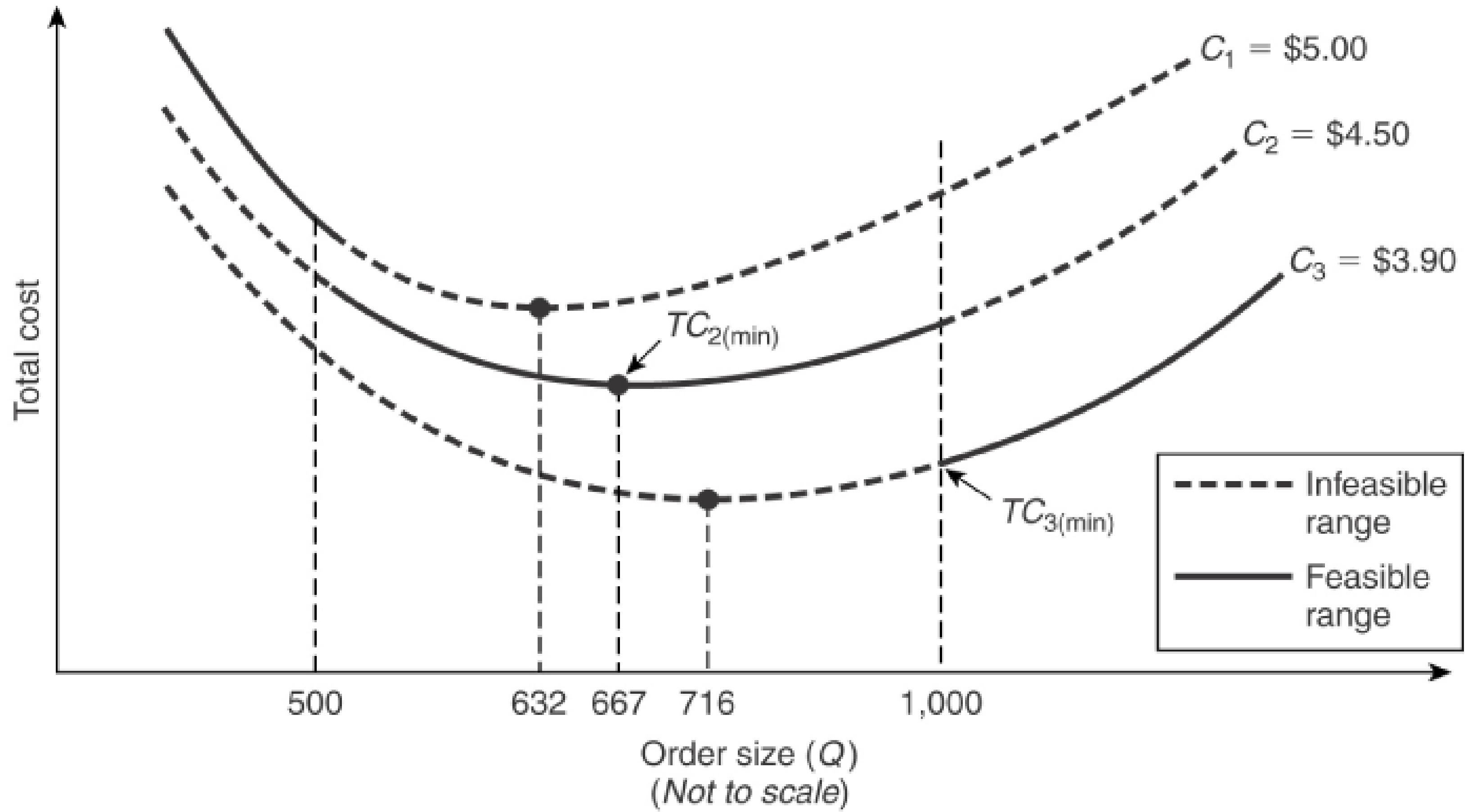
Total Cost = X H / 2 + S / T

Optimal X = $\sqrt{\frac{2SD}{H}\frac{p-D}{p}}$, i.e. the production stop when the inventory level hit X

EOQ MODEL WITH QUANTITY DISCOUNT

- **Two types of quantity discounts**
  - Incremental discounts which apply increasing discounted unit prices as orders reach or exceed certain quantity levels of units.
  - In the all-unit model, discounts are applied to all units with the unit cost determined by the size of the purchase order.

TOTAL COST CURVE FOR ALL-UNIT DISCOUNT MODEL



STEPS FOR SOLVING THE ALL-UNIT DISCOUNT MODEL

- **Step 1. Sort the unit prices from lowest to highest and then calculate the economic order quantity for each price until a feasible economic order quantity is found. By feasible, we mean that the price is in the correct corresponding range.**
- **Step 2. If the first feasible economic order quantity is for the lowest unit price, then stop. Otherwise, calculate the total cost for the first feasible economic order quantity (you did these from lowest to highest price) and also calculate the total cost at each price break lower than the price associated with the first feasible economic order quantity. This the lowest order quantity at which you can take advantage of the price break. The optimal Q is the one with the lowest cost.**

EXAMPLE

- **D = 10,000**
- **S = $20**
- **i = 20%**
- **C = $5.00, if orders of 0 to 499 units,**
- **$4.50, if orders of 500 to 999 units,**
- **$3.90, if orders of 1000 and up**

SOLUTION

- **Step 1, Solve for the EOQ for c = \$3.90, \$4.50, \$5.00:**
  - c = \$3.90, EOQ = 716 Not feasible
  - c = \$4.50, EOQ = 666 Feasible
  - c = \$5.00, EOQ = 633 Not feasible

- **Step 2, the first feasible EOQ is not for the lowest price, we have to compare it with the next price-break quantity that is higher than this feasible economic order quantity: Q = 1000**
  - For Q = 666, Cost = \$45,600
  - For Q = 1000, Cost = \$39,590 (Optimal)

SUMMARY

- **EOQ Model (Economic Order Quantity) is for a multiple-period situation**
- **Demand is assumed to be deterministic**
- **No consideration for backlog**
- **Two extensions**
  - EOQ with constant production rate
  - EOQ with quantity discount
- **EOQ Model deals with Economy of Scale in inventory management**

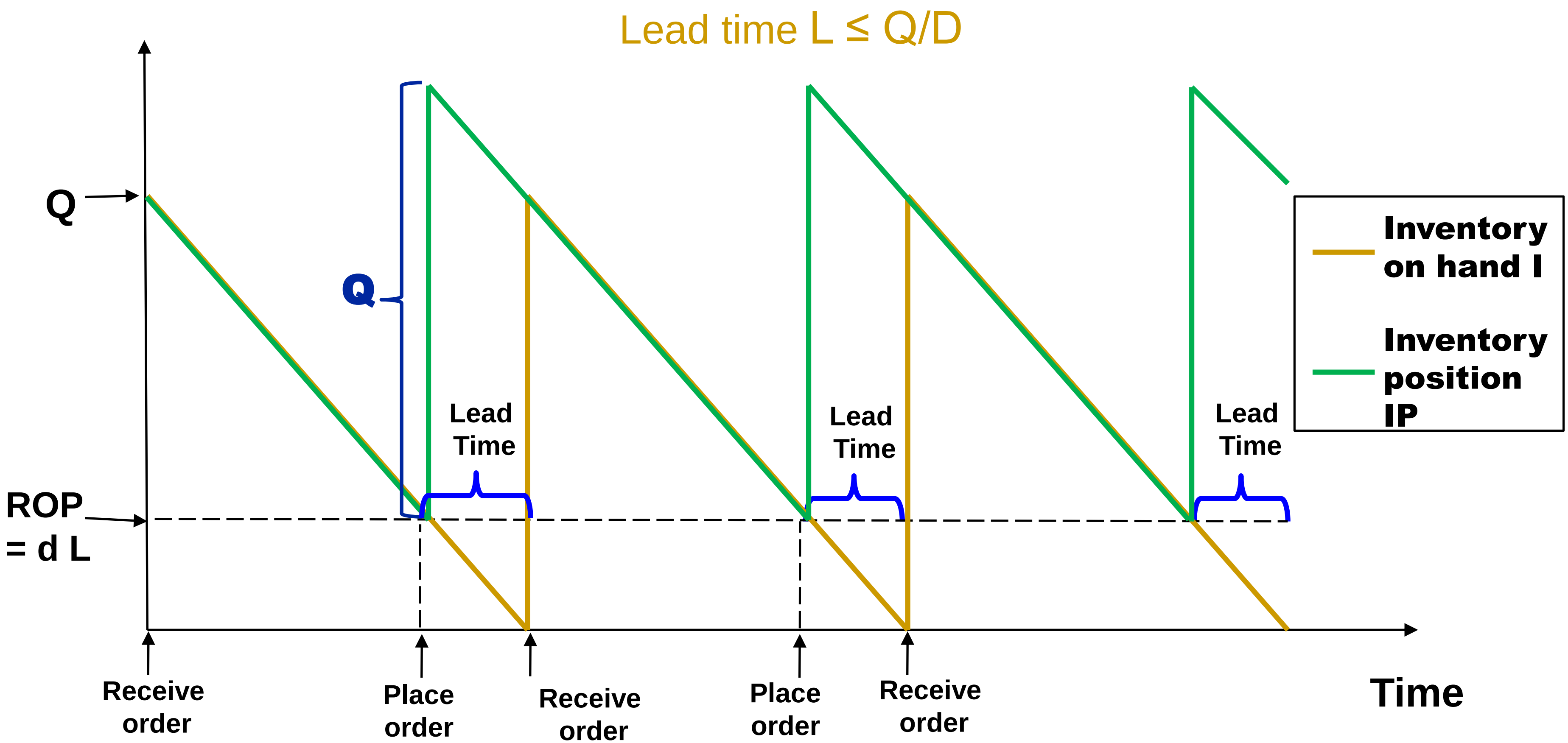
LEAD TIME

What if the order cannot arrive instantaneously, or there is a lead time (订货期，提前期) *L*?

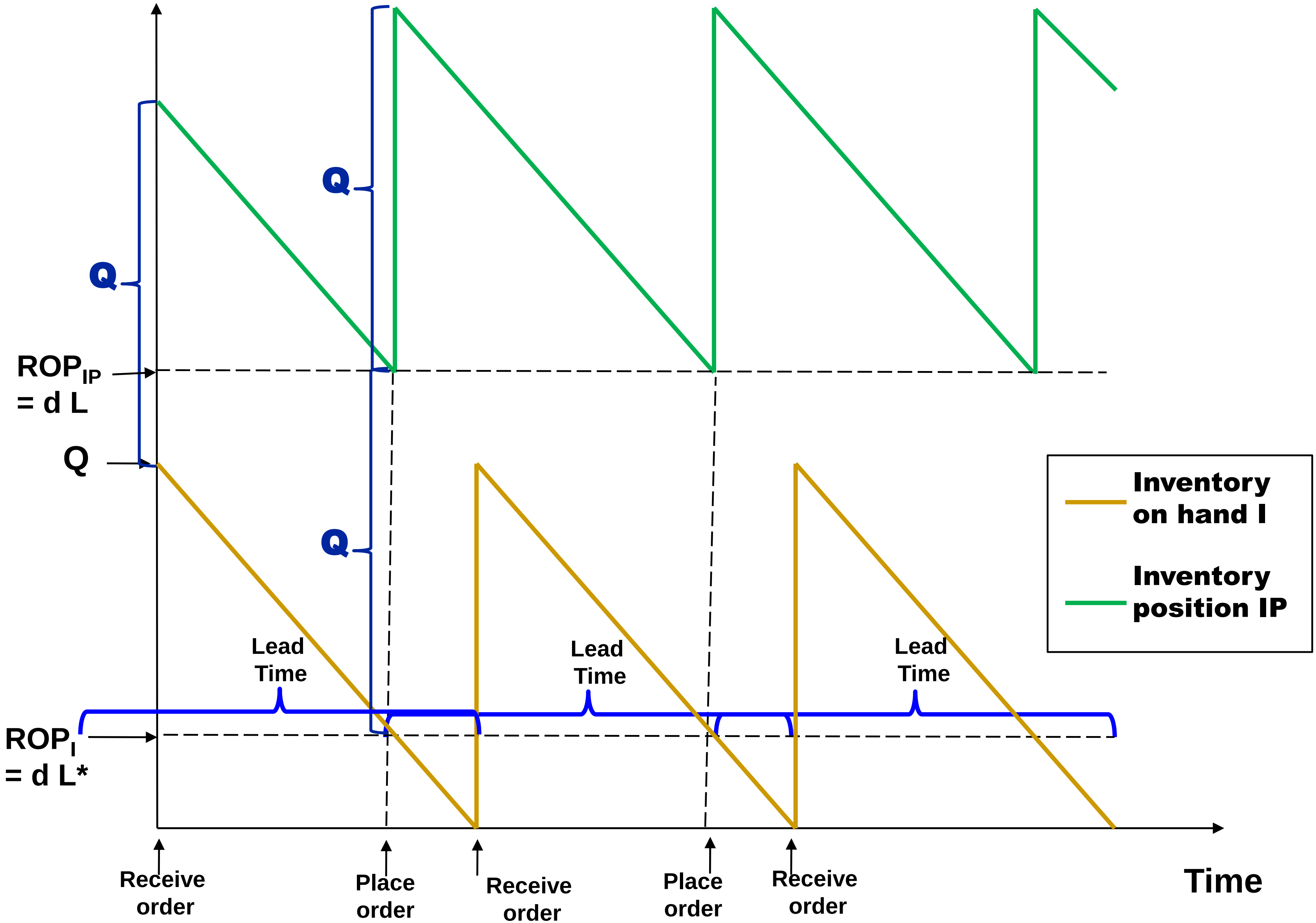
- Optimal order quantity is the same
- Reorder whenever the inventory hit the reorder level = $d\,L$
- d= Average daily demand

WHAT HAPPENS IF THERE IS A "LEAD TIME"?

- We denote the reorder point by ROP



Lead time L > Q/D



EOQ WITH REORDER POINT

$$T = CD + \frac{D}{Q}S + \frac{Q}{2}H$$

$$Q_{OPT} = \sqrt{\frac{2DS}{H}}$$

Reorder point, $ROP_{IP} = d\,L$, $ROP_{I} = d\,L^*$

d = (hourly, daily, weekly) demand rate

L = Lead time [hours, days, weeks]

L^* = Effective time between an order is placed and an order arrives [hours, days, weeks],

I: Inventory on hand

IP: Inventory position

$L^* = (L \bmod Q_{OPT}/D)$

- $L \mod Q/D$ is the remainder after division of L by Q/D (sometimes called modulus).
  - For example $5 \mod 4 = 1$, $12 \mod 10 = 2$, $12 \mod 5 = 2$.

SELF-TEST QUESTION

Suppose the lead time (time between ordering jackets and receiving jackets) is 2 weeks, what is the re-order point with respect to the inventory on hand?

$D = 1200$ jackets / year
$d = 3.3$ jackets / day

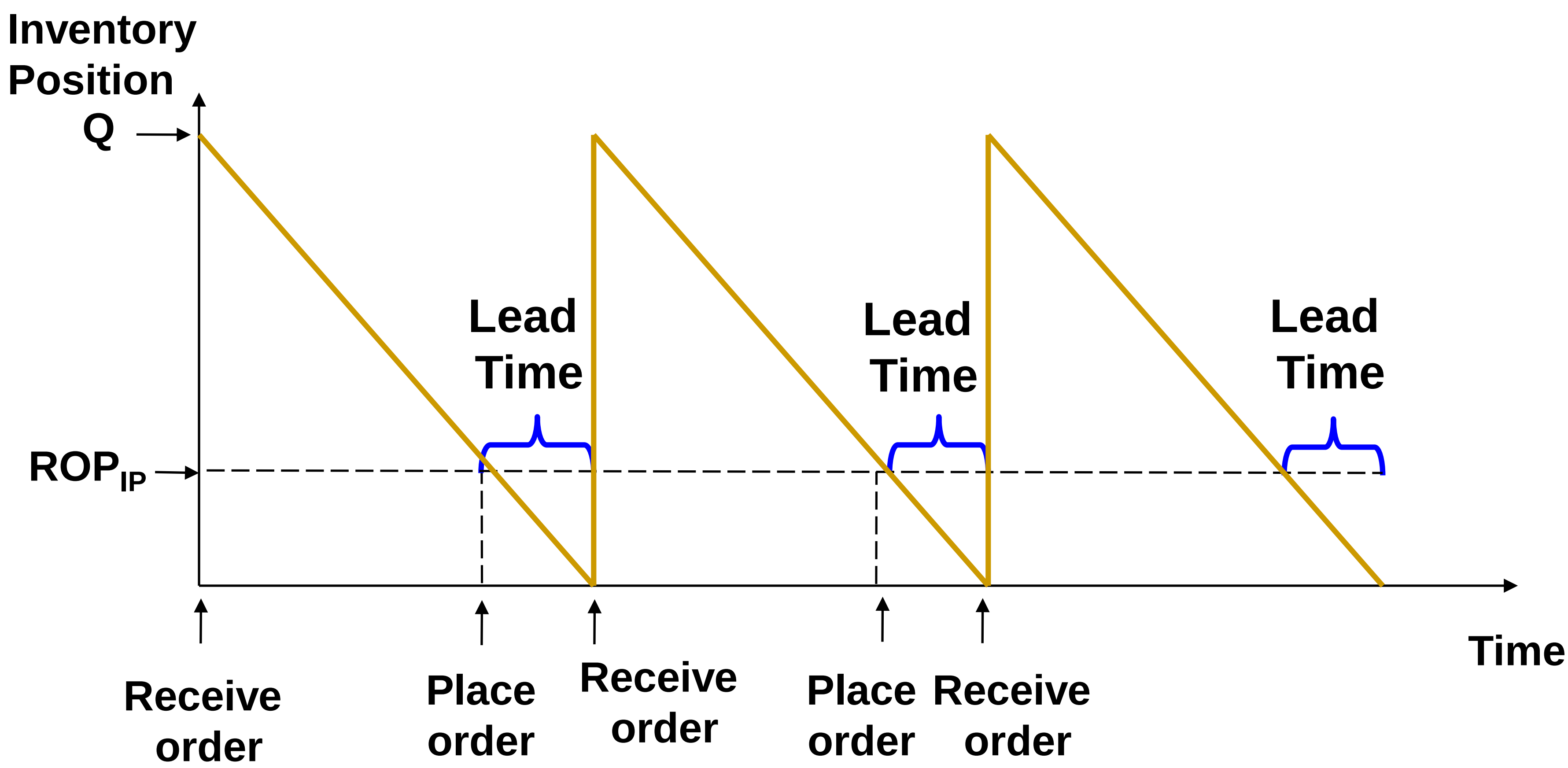
$Q_{OPT}/D \approx 3$ months > 2 weeks = L

$ROP_I = d \times L^* = d \times L$

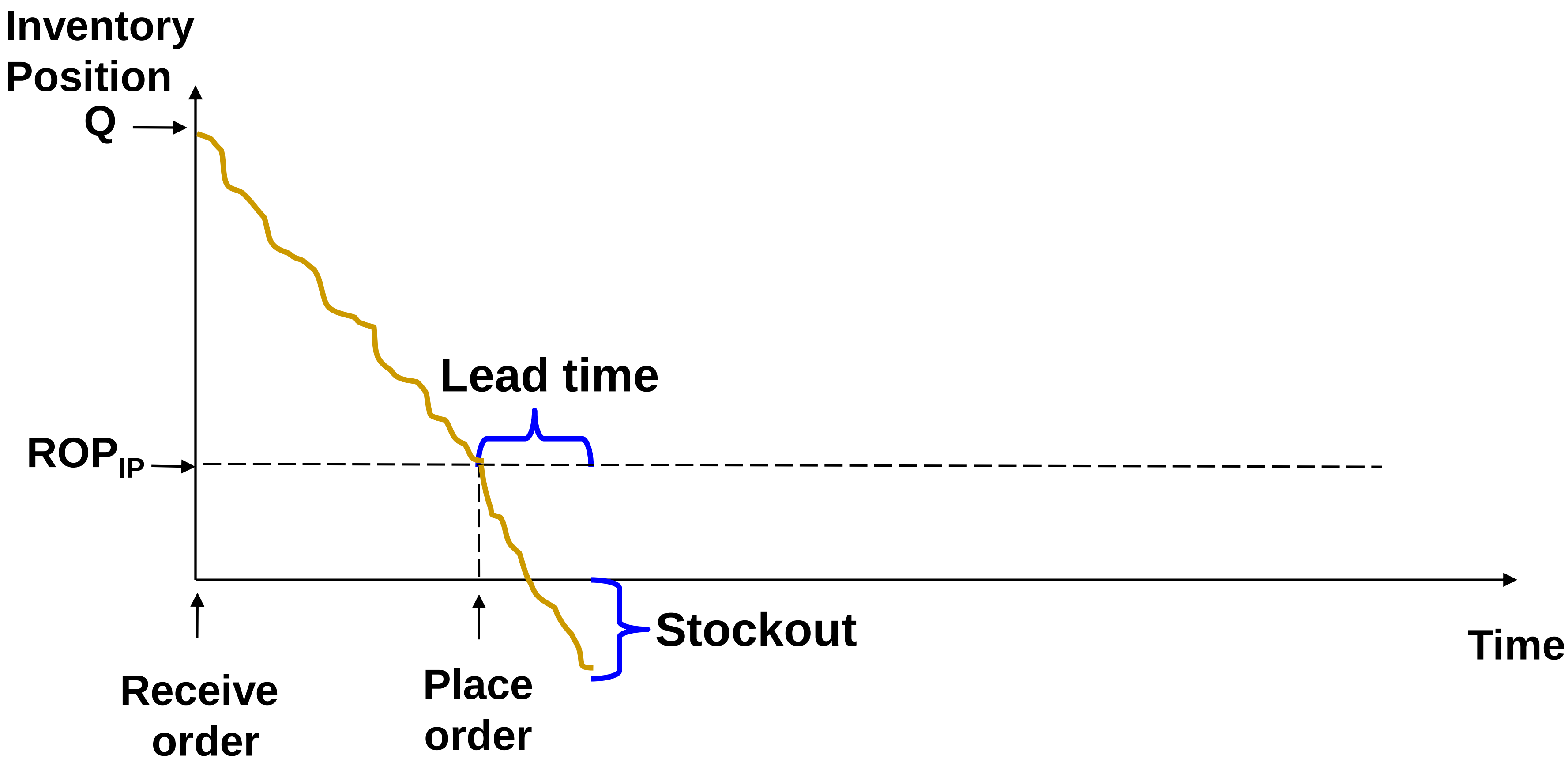
$= 3.3$ jackets / day $\times$ 14 days

$= 46$ jackets

SO FAR WE ASSUMED DEMAND IS CONSTANT

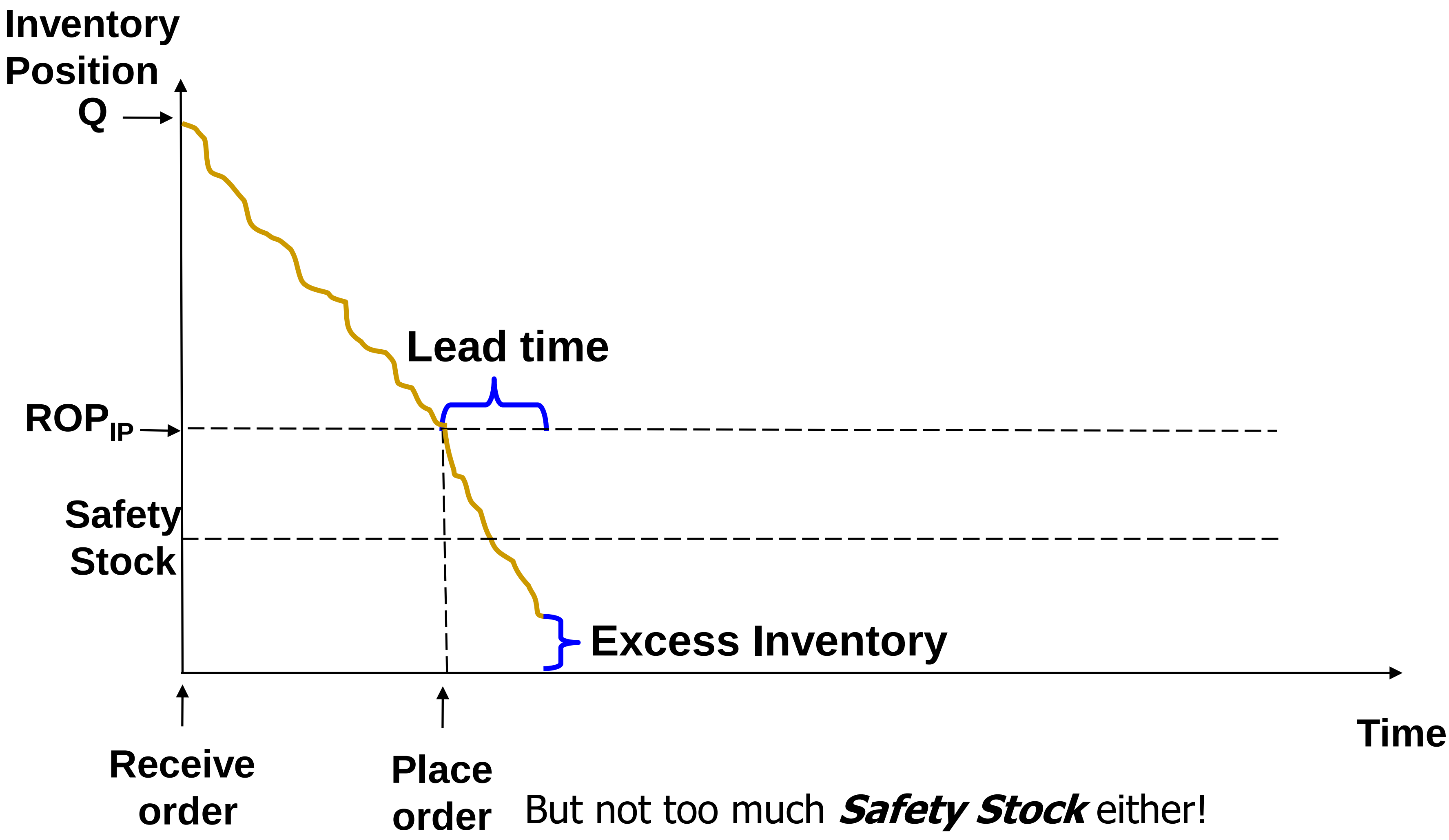


… BUT WHAT IF IT IS UNCERTAIN?



To minimize ***Stockouts*** we must have a ***Safety Stock***

… BUT WHAT IF IT IS UNCERTAIN?



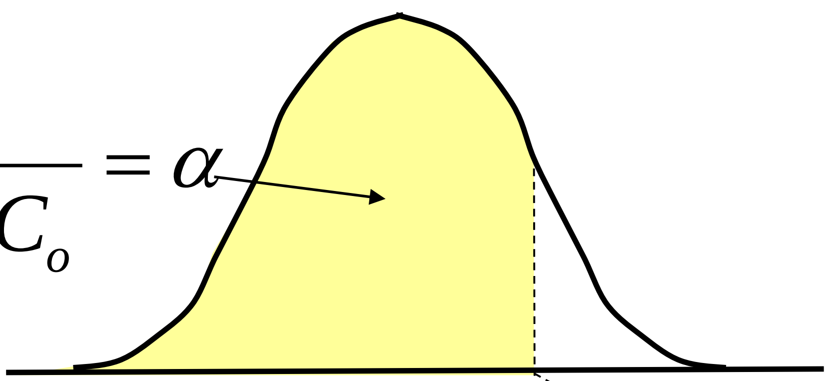
But not too much ***Safety Stock*** either!

REVIEW: NEWSVENDOR MODEL

$$\underbrace{P\left(D \le q^{OPT}\right)}_{\text{In-Stock Probability}} = \frac{C_u}{\underbrace{C_u}_{\substack{\text{cost of}\\ \text{under-}\\ \text{stocking}}} + \underbrace{C_o}_{\substack{\text{cost of}\\ \text{over-}\\ \text{stocking}}}} \Rightarrow \mathrm{q}^{\mathrm{OPT}} = F^{-1}\left(\frac{C_u}{C_u + C_o}\right)$$

$$q^{OPT} - E[D] = \text{safety stock}$$

$$\frac{C_u}{C_u + C_o} = \alpha$$



q*

Demand Distribution

<u>Remark:</u> If D is Normal(μ,σ),

$q^{OPT} = \mu + z^*\sigma$

$\Phi(z) = C_u / (C_u + C_o) = \alpha$

α = 95%	→	k = 1.64
α = 99%	→	k = 2.32
α = 99.9%	→	k = 3.09

REVIEW: SAFETY STOCK AND SERVICE LEVEL

When demand has (unpredictable) uncertainty, safety stock is held to cushion against uncertainties

Shortages occur when demand during lead time exceeds the expected demand + safety stock

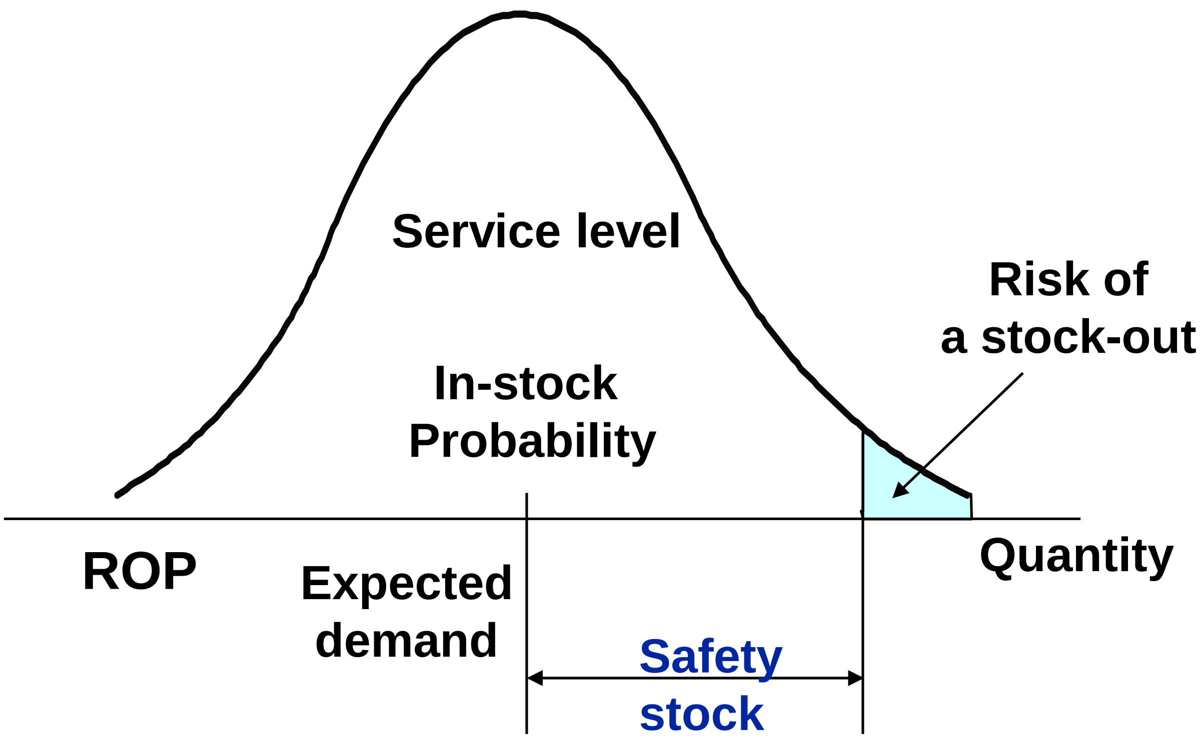
Service level is a measure of reliability of the system:

Service Level = In-Stock Probability

= 1 - Probability of a stock-out

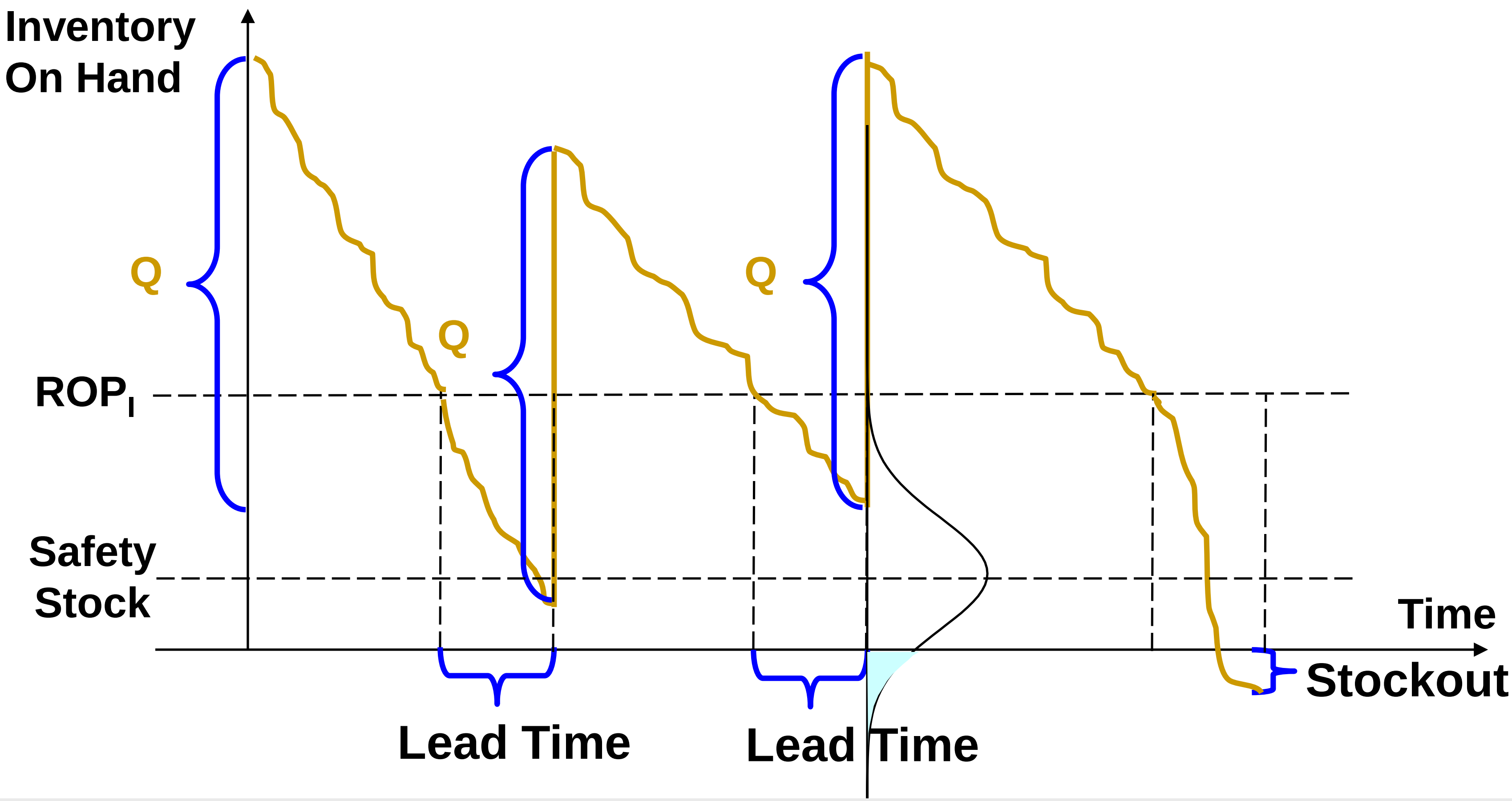
Desired service level, or the probability of shortage / stock-out, determines the amount of safety stock to hold.

REVIEW: SAFETY STOCK AND SERVICE LEVEL



Safety stock is the amount of inventory carried in addition to the expected demand.

REORDER POINT & SAFETY STOCK



What demand determines the Safety Stock (SS)?

To find this point, consider the mean demand during the lead time: $\mu_L = d \times L$

When demand has a normal distribution, safety stock $SS = z\sigma_L$

CALCULATING μ_L AND σ_L: FORMULA

Suppose **daily** demand is normally distributed with mean μ_d and standard deviation σ_d and suppose lead time L (days). What is $\boldsymbol{\mu_L}$ and $\boldsymbol{\sigma_L}$?

Mean Demand During Lead Time:

$\boldsymbol{\mu_L = \mu_d \times L}$

Time units must be consistent!

Variance in Overall Demand During Lead Time:

$$\sigma_d^2 = \sigma_{d_1}^2 + \sigma_{d_2}^2 + \sigma_{d_3}^2 + \ldots + \sigma_{d_L}^2$$

Standard Deviation in Demand During Lead Time:

$$\sigma_d = \sqrt{\sigma_{d_1}^2 + \sigma_{d_2}^2 + \sigma_{d_3}^2 + \ldots + \sigma_{d_L}^2}$$

$$\boldsymbol{\sigma_L = \sigma_d \sqrt{L}}$$

Time units must be consistent!

EOQ MODEL WITH SAFETY STOCK

$$T = CD + \frac{D}{Q}S + \underbrace{\left(\frac{Q}{2} + SS\right)}_{\text{Average Inventory (excluding Pipeline)}}H$$

$$Q_{OPT} = \sqrt{\frac{2DS}{H}}$$

$$P\left(D_L \le ROP_{IP}\right) = \alpha$$

α = Service Level

$$\boxed{\text{Reorder point, } ROP_{IP} = d\,L + SS}$$

Uncertainty!

d = (hourly, daily, weekly) demand rate **IP is the inventory position**

L = Lead time (expressed in hours, days, weeks)

When demand has a normal distribution:

$$\boldsymbol{SS = z \times \sigma_L \quad where \quad \sigma_L = \sigma_d \times \sqrt{L}}$$

SAFETY STOCK PROPERTIES

Safety stock increases with an increase in:

- Demand variability (or forecast error),
- Delivery lead time for the same level of service,
- Delivery lead time variability for the same level of service.

INVENTORY MANAGEMENT SYSTEMS

Fixed-order quantity, Q model

Feature: Continuous review system

- Each order is the exact same size
- The ordering is triggered by an event
- The timing of this event is random
- A typical example:

An order of a fixed quantity *Q* is placed every time the *inventory position* falls below a reorder point *R*

P-MODEL

Fixed-time period model (P-model)

Feature: Periodic review system

- Each order is of a different size
- The ordering is triggered by a moment in time
- The timing is deterministic and predictable

Right amount to order

- Order-up-to policy
- Every **P** days, current inventory position (IP) is obtained and an order quantity **q** is placed such that IP goes up to a level **S**

INVENTORY MANAGEMENT SYSTEMS

Fixed order quantity, Q model

Feature: Continuous review system

- Each order is the exact same size
- Smaller safety stock

Fixed time period, P model

Feature: Periodic review system

- Each order is of a different size
- Larger safety stock
- Ease of coordination