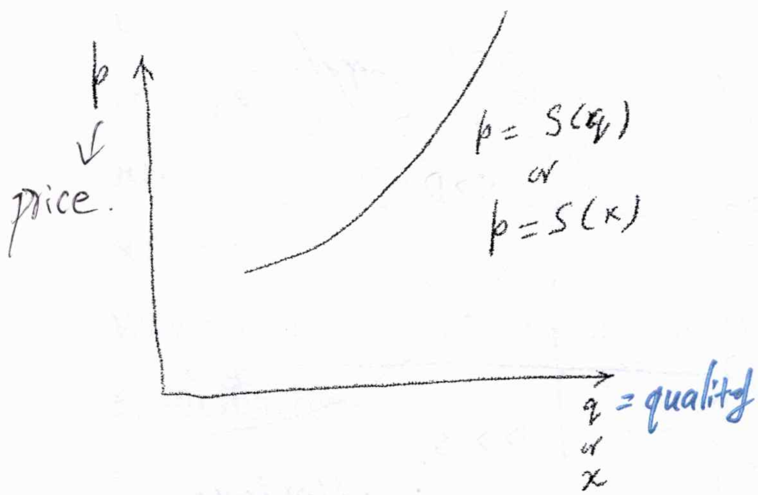
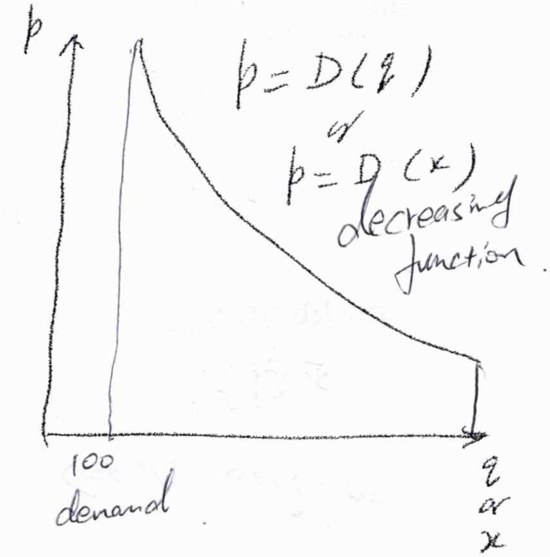
SOME FUNCTIONS OF BUSINESS & ECONOMICS

供给 需求
Supply and Demand curves



Supply curve



Demand curve

In many situations there is a relationship between prices and quantities demanded by consumers and prices and quantities supplied by producers.

For example: as gas prices increase people buy (demand) less gas.

As rents or house prices go up builders tend to build (supply) more houses.

Note.

Although p is an independent variable, the economists following the English economist Alfred Marshall (1842 – 1924), plot it along the vertical axis. We will follow this practice. This is contrary to the standard method used in mathematics of plotting it along the horizontal axis.

Here, we will follow the economists in their practice of plotting price, p, along the vertical axis. and q, (q for quantity) or x for quantity along the horizontal axis.

or $p = S(x)$ or $p = D(x)$

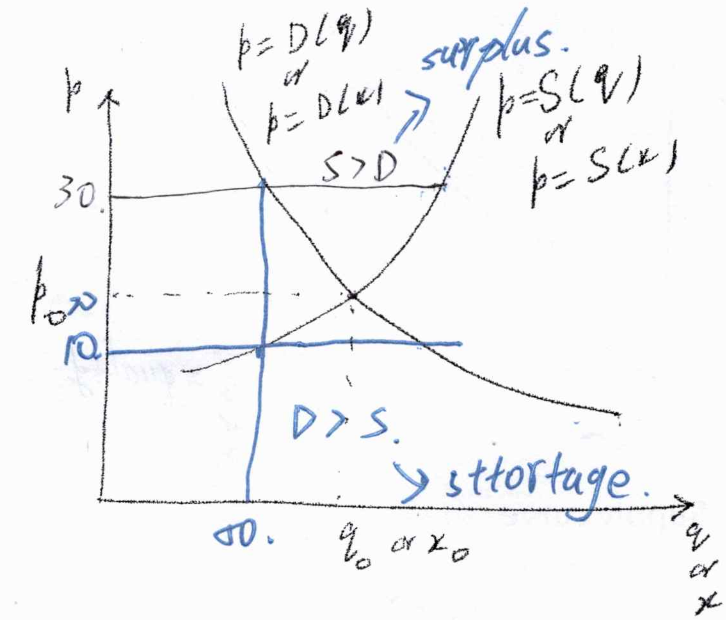
We write p = S(q) for the supply curve and p = D(q) for the demand curve, although it is p that is really the independent variable.

In many situations, supply and demand curves are represented by straight lines.

Equilibrium point

平衡点.

$20 = S(20) = D(20)$



The intersection of supply and demand curves or lines is called the equilibrium point. The price at this point is called the equilibrium price and the quantity is called the equilibrium quantity.

费用 价格.

Cost Function

Cost Function C (x) is the total cost of producing x times.

C (x) = Fixed Cost + Variable Cost

不变成本 可变成本

Fixed Cost is the cost that is incurred before any items are produced; i.e. it is the cost of producing zero items.

Fixed Cost = C $(\boldsymbol{0})$

Variable cost is the cost that is incurred when x items are produced.

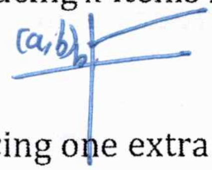
每一个的成本

边际成本

If the cost function is linear and if the fixed costs are b and the marginal cost is m, then the cost function for producing x items is given by

Marginal cost
$= $ cost making $(x+1)^{th}$ item

$$C(x) = mx + b$$



$C(x)=mx+b$
$C(0)=b.$

Marginal cost is the cost of producing one extra item when the level of production is x.

$= C(x+1) - C(x)$
$= [m(x+1)+b] - (mx+b)$
$= m.$

i.e. $\quad m = C(x+1) - C(x) = [C(x+1) - C(x)] / [(x+1) - (x)]$

收入.

Revenue function

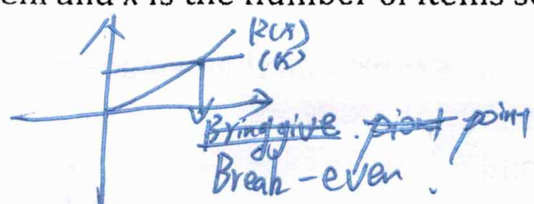
$P(x) = R(x) - C(x).$

$R(x) = px$

Where p is the price per item and x is the number of items sold.

利润

Profit function



$P(x) = R(x) - C(x)$

不赚不赔

Break-even point is the point of intersection of cost and revenue curves, this is where the number of items x_b sold produce zero profit. (I.e. revenue equals cost). "x_b" is called the break-even quantity. The corresponding ordered pair gives the break-even point:

$(x_b, R(x_b)) = (x_b, C(x_b))$

Example

制造 成本.

Q1 : Manufacturing Costs

不变成本.

TMI, a manufacturer of blank audio-cassette tapes, has a monthly fixed cost of \$12,100 and a variable cost of \$0.60/tape. Find a function C that gives the total cost incurred by TMI in the manufacturer of x tapes/month.

总的 平均数

Suppose $C(x)$ is a total cost function. Then the average cost function, denoted by $\bar{C}(x)$ (read "C bar of x") is:

每一个的平均成本.

$$\bar{C}(x) = \frac{C(x)}{x}, x > 0.$$

$C(x) = 12100 + 0.6x.$

$\bar{C}(x) = \dfrac{12100 + 0.6x}{x}$

Examples 2 Use above to find $\bar{C}(x)$.

Q1. <u>*Supply and demand*</u>

蟹肉

The supply and demand for crabmeat in a local fish store are related by the equations,

Supply: $P = S(q) = 6q + 3$

Demand: $P = D(q) = 19 - 2q$

磅(重量) 量

Where p is the price in dollars per pound and q is the quantity of crabmeat in pounds per day. Find the supply and demand at each of the following prices.

a. \$10
b. \$15
c. \$18
d. Graph both the supply and the demand functions on the same axes.
e. Find the equilibrium price.
f. Find the equilibrium quantity.

Solution:

a. $P = \$10$

Supply function $10 = 6q + 3, \quad q = 7/6$

Demand function $10 = 19 - 2q, \quad q = 9/2$

Thus at price of \$10 the supply is 1.16 pounds and the demand is 4.5 pounds.

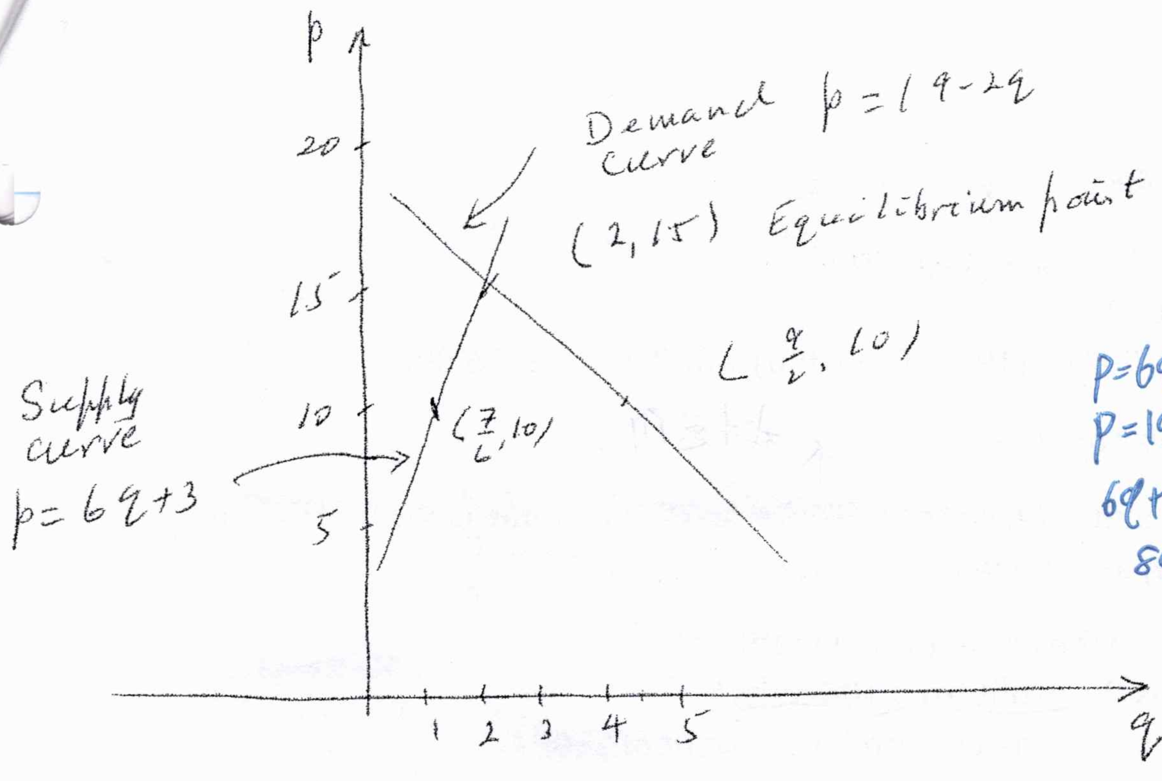
b. $P = \$15$

Supply Function $15 = 6q + 3, \quad q = 2$

Demand Function $15 = 19 - 2q, q = 2$

Thus at price \$15 the supply and demand are both 2 pounds. Hence, (q, p) = (2, 10) is the equilibrium point.

c. $P = \$18$ Supply: $18 = 6q + 3, q = 2.5$; Demand $18 = 19 - 2q, q = 0.5$
d.



$p = 6q + 3$
$p = 19 - 2q$
$6q + 3 = 19 - 2q$
$8q = 16$
$q = 2.$
$p = 12 + 3$
$= 15$
$= \$15.$

e. The equilibrium price is $15.

f. The equilibrium quantity is 2 pounds.

Q2. *Marginal cost of a new plant* 设施.

In deciding whether to set up a new manufacturing plant, company analysts 分析师. have decided that a linear function is a reasonable estimate 估价 for the total cost C(*x*) in dollars to produce *x* items. They estimate the cost to produce 10,000 items is $547,000, and the cost to produce 50,000 items is $737,000.

b. $C(x) - \cancel{\ \ } m(x) = \text{Fixed cost}$

$547000 - 4.75 \times 10000 =$
$= 500,000$

a. Find a formula for C (*x*)
b. Find the fixed cost.
c. Find the total cost to produce 100,000 items.
d. Find the marginal cost of the items to be produced in this plant and what does this mean to the manager?

Solution:

~~Since C(x)~~
~~The~~ ~~[illegible]~~ ~~manager~~

$C(x) = mx + b$

$m = (737{,}000 - 547{,}000)/(50{,}000 - 10{,}000) = 190{,}000/40{,}000 = 19/4 = 4.75$

$C(x) - 737{,}000 = (19/4)(x - 50{,}000)$

$C(x) = mx + b.$
$m = C(x+1) - C(x)$

$y - y_1 = m(x - x_1)$

{*point-slope form* : $y - y_1 = m(x - x_1)$}

a. C (*x*) = 19/(4x + 500,000)
b. $500,000
c. C (100,000) = (19/4)(100,000) + 500,000 = $975,000

Q3. *Break-even analysis*

成本总价

Producing *x* units of tacos costs C(*x*) = 5x +20; revenue is R(*x*) = 15x, when C(*x*) and R(*x*) are in dollars.

a. What are the break-even quantities?
b. What is the profit from 100 units?
c. How many units will produce a profit of $500?

Solution:

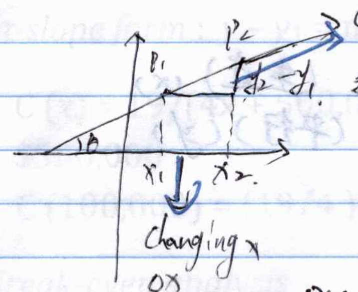
a. We solve

$$C(x) = R(x)$$
$$5x + 20 = 15x$$
$$10x = 20$$
$$x = 2$$

Break-even quantity in 2 units

b. P (*x*) = R(*x*) – C(*x*) = [15x] – [5x +20]
   P (*x*) = 10x -20
   P (100) = 1000 – 20 = $980
c. P = 500, *x*=? ., 500 = 10x – 20, *x*= 52

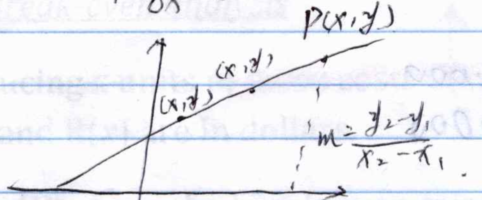
MATH.



斜率: slope $P_1P_2 = \tan\theta$

$$= \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{RISE}{RUN} = \frac{\Delta y}{\Delta x} =$$ RATE OF change y with respect to x

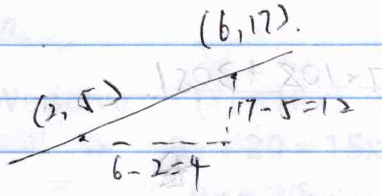


$$m = \frac{y_2-y_1}{x_2-x_1}$$

$$y_2 - y_1 = m(x - x_1)$$

$$y - y_2 = m(x - x_2)$$

$$= \frac{y-y_1}{x-x_1} = \frac{y-y_2}{x-x_2}$$



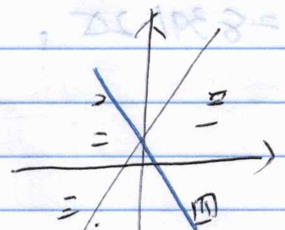
$m = \frac{12}{4} = 3 = \frac{3}{1}$ $\rightarrow$ $y - 17 = 3(x - 6)$

$y - 5 = 3(x - 2)$ $\quad y = 3x - 1$

$= y = 3x - 1$

m = constant

How to define slope a curve?



When line in first and third is positive. $m > 0$.
increasing
rising line $m > 0$.
When line in second and forth is negative $m < 0$
decreasing or falling line $m < 0$.

verticle line. m is undefinde. (垂直) (x)

Horizontal line. m = 0. (平行) (y)

Q_2:

c. $C(x) = mx + b$

$= 4.75(100,000) + 500,000$

$= \cancel{1145000,000}$ 975,000

Exerkise.

1. $C(x) = 17x + 3081$

$P = -0.25x + 615$ 108^{th}

$P = -0.25 \times 108 + 615$ $C(x) = 17 \times 108 + 3081$

$= 588$

$= 170x + 3081$

$C(x) = 21441$

$\frac{2A}{4} =$

$C(x) = 170x + 3081$

$R(x) = -\frac{1}{4}x^2 + 615x$

$R(x) = 588 \times 108 =$

$= 63504$

$P(x) = 63504 - 21441$

$= 42063$

$P = R - C = -\frac{1}{4}x^2 + 445x - 3081$

$P(108) - P(107)$

$= -\frac{1}{4}(108^2 - 107^2) + 445$

$= -\frac{1}{4}(215 + 445)$

$= 8391.25$

~~Exercise 2.7.1~~