4.4.17 continued

$$\left.\frac{dx}{dp}\right|_{t=16} = \frac{100}{9} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{810000 - p^2}} \cdot -2p \Bigg|_{p=\frac{1400}{3}}$$

$$= -\frac{100}{9} \frac{p}{\sqrt{810000 - p^2}}$$

$$p(16) = \frac{400}{1+\frac{1}{2}} + 200 = \frac{1400}{3}$$

$$\left.\frac{dx}{dp}\right|_{t=16} = -\frac{100}{9} \cdot \frac{(1400/3)}{\sqrt{810000 - \left(\frac{1400}{3}\right)^2}} \rightarrow (1)$$

$$\left.\frac{dp}{dt}\right|_{t=16} = 400 \cdot -\left(1 + \frac{\sqrt{t}}{8}\right)^{-2} \cdot \frac{1}{8} \cdot \frac{1}{2} \frac{1}{\sqrt{t}} \Bigg|_{t=16}$$

$$= -25 \frac{1}{\left(1+\frac{1}{2}\right)^2} \cdot \frac{1}{4} = -25/9$$

$$\rightarrow (2)$$

(*) becomes

$$\frac{-25}{9} \cdot \frac{-100}{9} \cdot \frac{\frac{1400}{3}}{\sqrt{810000 - \left(\frac{1400}{3}\right)^2}}$$

$$\approx \underline{18.716}$$

Exercise 4.4.17

The quantity demanded per month, $x$, of a certain make of personal computer (PC) is related to the average unit price, $p$ (in dollars), of PCs, by the equation

$$x = x(p) = f(p) = \frac{100}{9}\sqrt{810000 - p^2}.$$

It is estimated that $t$ months now, the average price of a PC will be given by

$$p(t) = \frac{400}{1+\frac{\sqrt{t}}{8}} + 200, \quad 0 \le t \le 60$$

dollars. Find the rate ~~of change~~ at which the quantity demanded per month of PC's will be changed 16 months from now.

Solution

$$\left.\frac{dx}{dt}\right|_{t=16} = \left.\frac{dx}{dp}\cdot\frac{dp}{dt}\right|_{t=16} \qquad *$$