

$$\begin{aligned}
 & 1b) \lim_{x \rightarrow 0} \frac{(1-x)^{\frac{1}{4}} - 1}{x} \\
 &= \lim_{x \rightarrow 0} \frac{\frac{1}{4}(1-x) \cdot -1}{1} \\
 &= \lim_{x \rightarrow 0} -\frac{1-x}{4} \\
 &= -\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 & (c) \lim_{t \rightarrow 0} \left(t + \frac{1}{t} \right) \left((4-t)^{\frac{3}{2}} - 8 \right) \\
 &= \lim_{t \rightarrow 0} t \left((4-t)^{\frac{3}{2}} - 8 \right) + \lim_{t \rightarrow 0} \frac{1}{t} \left((4-t)^{\frac{3}{2}} - 8 \right) \\
 &= \lim_{t \rightarrow 0} \frac{(4-t)^{\frac{3}{2}} - 8}{t} \\
 &= \lim_{t \rightarrow 0} \frac{-\frac{3}{2}(4-t)^{\frac{1}{2}}}{1} \\
 &= -3
 \end{aligned}$$

S.4.3

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{x + \sin x}{x+1} \\
 &= \lim_{x \rightarrow 0} \frac{1 + \cos x}{1} \\
 &= \lim_{x \rightarrow 0} \cos x \\
 &= \text{DNE}
 \end{aligned}$$

$$\begin{aligned}
 & d. \lim_{t \rightarrow 0^+} \left(\frac{1}{t} + \frac{1}{t} \right) (\sqrt{t+1} - 1) \\
 &= \lim_{t \rightarrow 0^+} \frac{\sqrt{t+1} - 1}{t} + \lim_{t \rightarrow 0^+} \frac{\sqrt{t+1} - 1}{t} \\
 &= \lim_{t \rightarrow 0^+} \frac{1}{2\sqrt{t+1}} + \lim_{t \rightarrow 0^+} \frac{\frac{1}{2\sqrt{t+1}}}{\frac{1}{2t}} \\
 &= \lim_{t \rightarrow 0^+} \frac{1}{2\sqrt{t+1}} + \lim_{t \rightarrow 0^+} \frac{2t}{2\sqrt{t+1}} \\
 &= \frac{1}{2} + 0 \\
 &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 & (e) \lim_{x \rightarrow 0} \frac{e^x - 1}{x} \\
 &= \lim_{x \rightarrow 0} \frac{e^x}{1} \\
 &= \lim_{x \rightarrow 0} e^x \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 & f) \lim_{x \rightarrow 1} \frac{x^{\frac{1}{4}} - 1}{x} \\
 &= \lim_{x \rightarrow 1} \frac{1-1}{1} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 & (g) \lim_{x \rightarrow 0} \frac{3x^2 + x + 2}{x - 4} \\
 &= \lim_{x \rightarrow 0} \frac{2}{0 - 4} \\
 &= -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 & (h) \lim_{x \rightarrow 0^+} \frac{\sqrt{x+1} + 1}{\sqrt{x+1} - 1} \\
 &= \lim_{x \rightarrow 0^+} \frac{(\sqrt{x+1} + 1)^2}{x+1 - 1} \\
 &= \lim_{x \rightarrow 0^+} \frac{(\sqrt{x+1} + 1)^2}{x} \\
 &= \lim_{x \rightarrow 0^+} \frac{2\sqrt{x+1} \cdot 1}{1} \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 & (i) \lim_{x \rightarrow 1} (x+5) \left(\frac{1}{2x} + \frac{1}{x+2} \right) \\
 &= \lim_{x \rightarrow 1} \frac{x}{2x} + \lim_{x \rightarrow 1} \frac{x}{x+2} + \lim_{x \rightarrow 1} \frac{5}{2x} + \lim_{x \rightarrow 1} \frac{5}{x+2} \\
 &= \frac{1}{2} + \frac{1}{3} + \frac{5}{2} + \frac{5}{3} \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 & (j) \lim_{x \rightarrow 2} \frac{x^3 - 6x - 2}{x^2 + 4} \\
 &= \lim_{x \rightarrow 2} \frac{3x^2 - 6}{3x^2} \\
 &= \lim_{x \rightarrow 2} \frac{12 - 6}{12} \\
 &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 (i) \lim_{u \rightarrow 1} \frac{(u-1)^3}{\frac{1}{u} - u^2 + \frac{3}{u} - 3} \\
 = \lim_{u \rightarrow 1} \frac{(u-1)^3}{u^{-1} - u^2 + 3u^{-1} - 3} \\
 = \lim_{u \rightarrow 1} \frac{3(u-1)^2}{-u^{-2} - 2u - 3u^{-2}} \\
 = \lim_{u \rightarrow 1} \frac{0}{-1 - 2 - 3} \\
 = 0
 \end{aligned}$$

$$\begin{aligned}
 (j) \lim_{x \rightarrow 0} \frac{2 + \frac{1}{x}}{3 - \frac{2}{x}} \\
 = \lim_{x \rightarrow 0} \frac{2 + x^{-1}}{3 - 2x^{-1}} \\
 = \lim_{x \rightarrow 0} \frac{-x^{-2}}{2x^{-2}} \\
 = \lim_{x \rightarrow 0} \frac{-\frac{1}{x^2}}{\frac{2}{x^2}} \\
 = -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 (k) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x} \\
 = \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\sin x}{-1} \\
 = 1
 \end{aligned}$$

$$\begin{aligned}
 (l) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x} \\
 = \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\sin x}{-1} \\
 = 1
 \end{aligned}$$

$$\begin{aligned}
 (m) \lim_{x \rightarrow 0} \frac{x^2}{e^x - x - 1} \\
 = \lim_{x \rightarrow 0} \frac{2x}{e^x - 1} \\
 = \lim_{x \rightarrow 0} \frac{2}{e^x} \\
 = 2
 \end{aligned}$$

$$\begin{aligned}
 (n) \lim_{x \rightarrow 1} \frac{\ln x}{x-1} \\
 = \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{1} \\
 = 1
 \end{aligned}$$

$$\begin{aligned}
 (o) \lim_{x \rightarrow 0} \frac{\ln(x^2+1)}{x} \\
 = \lim_{x \rightarrow 0} \frac{\frac{2x}{x^2+1}}{1} \\
 = \lim_{x \rightarrow 0} \frac{2x}{x^2+1} \\
 = 0
 \end{aligned}$$

$$\begin{aligned}
 (p) \lim_{x \rightarrow 1} \frac{x \ln x}{x^2 - 1} \\
 = \lim_{x \rightarrow 1} \frac{\frac{x}{2x}}{\frac{x-1}{x+1}} \\
 = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 (q) \lim_{x \rightarrow 0} \frac{\sin(2x)}{\ln(x+1)} \\
 = \lim_{x \rightarrow 0} \frac{\cos 2x \cdot 2}{\frac{1}{x+1}} \\
 = 2
 \end{aligned}$$

$$\begin{aligned}
 (r) \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1} \\
 = \lim_{x \rightarrow 1} \frac{\frac{1}{2\sqrt{x}}}{1} \\
 = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 (s) \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{\sqrt{x+4} - 2} \\
 = \lim_{x \rightarrow 0} \frac{\frac{1}{2\sqrt{x+1}}}{\frac{1}{2\sqrt{x+4}}} \\
 = \lim_{x \rightarrow 0} \frac{\frac{1}{2}}{\frac{1}{4}} \\
 = 2
 \end{aligned}$$

$$\begin{aligned}
 (t) \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1} - 1}{\sqrt{x+1} - 1} \\
 = \lim_{x \rightarrow 0} \frac{\frac{2x}{2\sqrt{x^2+1}}}{\frac{1}{2\sqrt{x+1}}} \\
 = \lim_{x \rightarrow 0} \frac{0}{\frac{1}{2}} \\
 = 0
 \end{aligned}$$

$$\begin{aligned}
 (u) \lim_{x \rightarrow 0^+} \frac{\sqrt{x} \ln x}{\frac{1}{\sqrt{x}}} \\
 = \lim_{x \rightarrow 0^+} \frac{\ln \frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}} \\
 = \lim_{x \rightarrow 0^+} \frac{\ln \frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}} \\
 = \lim_{x \rightarrow 0^+} \frac{-\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{x}}} \\
 = -\frac{1}{2}
 \end{aligned}$$

$t = \frac{1}{\sqrt{x}}$
 $x = \frac{1}{t^2}$
 $\frac{1}{\sqrt{x}} = t$
 $\frac{1}{2\sqrt{x}} = \frac{t}{2}$