1

Elasticity 弹性 Of Demand 5.1

This measures how a change in price affects the change in demand.

Demand can be classified as elastic, inelastic or unitary.

Let $q = f(p)$ where q is demand at a price p. The elasticity of demand is

$$E = -\frac{\frac{dq}{q}}{\frac{dp}{p}} = -\frac{\%\text{ change in demand}}{\%\text{ change in price}}$$

$$= -\frac{\frac{dq}{dp}}{q/p} = -\frac{p}{q}\frac{dq}{dp} = -\frac{q'(p)}{q/p}$$

$$E = \frac{p}{q} \cdot \frac{dq}{dp}$$

$$= -\frac{D_p[\ln q(p)]}{D_p[\ln p]} = -\frac{(\ln q)'}{(\ln p)'}$$

when prime indicates differentiation with respect to p.

$$\approx \left| \frac{q_1 - q_2}{q_1 + q_2} \div \frac{p_1 - p_2}{p_1 + p_2} \right|$$

Demand is inelastic if $E < 1$

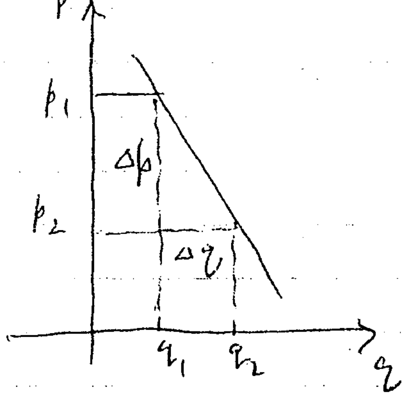
Demand is elastic if $E > 1$

Demand is unitary or has unit elasticity if $E = 1$.

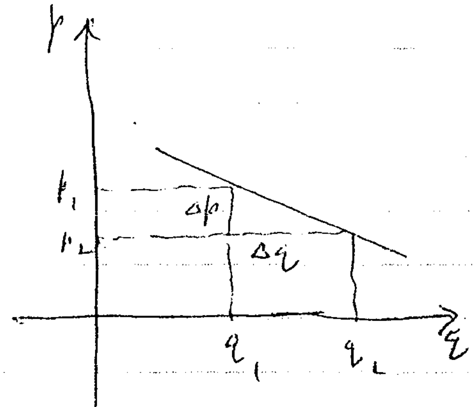
Note. E is a function of p.

#2

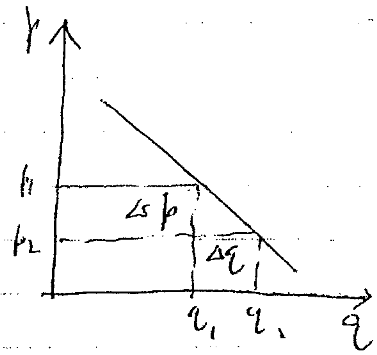
These graphs show different types of elasticity.



$E < 1$
Inelastic Demand



$E > 1$
Elastic Demand



$E = 1$
Unit Elastic Demand

Note. $|\%\text{ change in demand}| = E \cdot |\%\text{ change in price}|$

$$|\Delta q/q| = E \cdot |\Delta p/p|$$

$E < 1$: $\quad |\Delta q/q| < \left|\frac{\Delta p}{p}\right|$

For inelastic demand, a percentage change in price will produce a smaller percentage change in quantity.

$E = 1$ $\quad |\Delta q/q| = |\Delta p/p|$

For unit elastic demand, a percentage change in price will produce an equal percentage change in quantity.

$E > 1$ $\quad |\Delta q/q| > |\Delta p/p|$

For elastic demand, a percentage change in price will produce a greater percentage change in quantity.

2

Examples

Find the elasticity of demand for the following demand functions.

A. $q = D(p) = 1000 - p^{1.5}$, $p = 50$

$$E(50) = -\left.\frac{\frac{dq}{dp}}{q/p}\right|_{p=50} = -\left.\frac{(-1.5\,p^{.5})}{\frac{1000 - p^{1.5}}{p}}\right|_{p=50}$$

$$= \left.1.5\,\frac{p^{1.5}}{1000 - p^{1.5}}\right|_{p=50} = 1.5\,\frac{(50)^{1.5}}{1000 - (50)^{1.5}}$$

$$= 0.82$$

B. $q = D(p) = \frac{800}{p^{1.5}}$

$$E = -\frac{\frac{dq}{dp}}{q/p} = \frac{-\left(-1.5\,\frac{800}{p^{2.5}}\right)}{\left(800/p^{2.5}\right)} = 1.5$$

C. $q = D(q) = \frac{100}{p}$

$$E = -\frac{\frac{dq}{dp}}{q/p} = \frac{-\left(-\frac{100}{p^2}\right)}{\frac{100}{p^2}} = 1$$

Revenue and Elasticity

Elasticity can be related to the total revenue R considered as a function of price.

$$\text{Revenue} = (\text{price}) \times (\text{items sold i.e. demand})$$

$$R(p) = p \cdot q(p)$$

Differentiating this equation with respect to p we have,

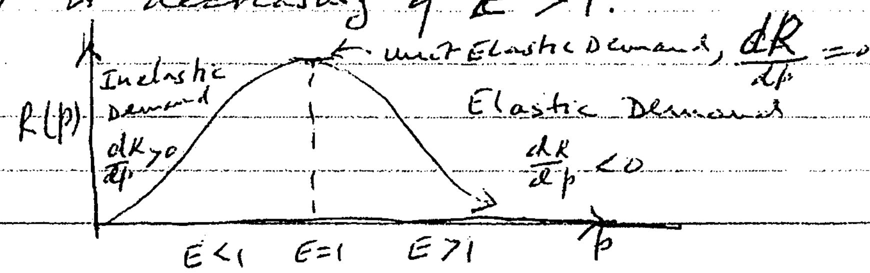
$$\frac{dR}{dp} = p \cdot \frac{dq}{dp} + q = q(1-E)$$

since

$$E = -\frac{dq}{dp} \cdot \frac{p}{q} \quad \text{or} \quad \frac{dq}{dp} = -\frac{q}{p} E.$$

$$\therefore \quad \frac{dR}{dp} = \begin{cases} > 0 & \text{if } E < 1 \\ = 0 & \text{if } E = 1 \\ < 0 & \text{if } E > 1 \end{cases}$$

Hence, $R(p)$ is increasing if $E<1$, $R(p)$ is maximized if $E=1$, and $R(p)$ is decreasing if $E>1$.



Summary: Revenue and Elasticity

1. For inelastic demand, total revenue increases as prices increase
2. For elastic demand, total revenue decreases as price increases.
3. For unit elastic demand, total revenue is maximized at price at which

~~#~~ 1

Related Rates ~~#.#~~ 5.2

Guidelines for Solving ~~a~~ Related Rate Problem

1. Identify all given quantities, as well as quantities to be found.
   Draw a diagram if needed.

2. Write an equation relating the variables of the problem.

3. Use implicit differentiation to find the derivative of both sides of the equation in step 2 with respect to time.

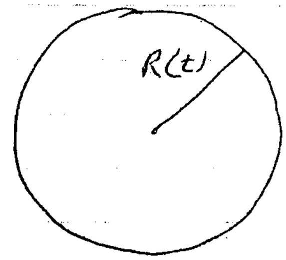
4. Solve for the derivative giving the unknown rate of change and substitute the given values.

#2

Example (Area) A small rock is dropped into a lake. Circular ripples spread over the surface of the water, with the radius of each circle increasing at the $\boxed{\text{rate}}$ of 2 m/sec. Find the rate of change of the area inside the 瞬间. circle formed by a ripple at the instant when the radius is 5 m.

Area $= \pi [R(t)]^2$

$$A(t) = \pi [R(t)]^2$$



$\dfrac{dR}{dt} = 2$ m/sec.

Differentiating this w.r.to t:

$$\frac{dA(t)}{dt} = 2\pi R(t) \cdot \frac{dR(t)}{dt}$$

Area of circle radius R $= \pi R^2$

$A(t) = \pi [R(t)]^2$

$$\frac{dA}{dt} = 4\pi R(t)$$

$$\therefore \left.\frac{dA}{dt}\right|_{R=5} = 20\pi \ m^2/s \approx 62.83 \ m^2/s$$

Thus the rate of change of the area of the circle when its radius is 5 m is approximately 62.83 m^2/s.

3.

Example (Revenue) A company is increasing production of peanuts at the rate of 50 cases per day. All cases produced can be sold. The daily demand function is given by

$$p = 50 - \frac{q}{200}$$

where q is the number of units produced (and sold) and p is price in dollars. Find the rate of change of revenue with respect to time (in days) when the daily production is 200 units.

$$\frac{dq}{dt} = 50 \text{ cases/day}$$

Revenue = quantity × price or $R(q) = qp = 50q - \frac{q^2}{200}$

$R = R(t) = 50\, q(t) - \frac{q^2(t)}{200}$

Differentiating this equation w.r.t. time (in days), we have:

$$\frac{dR}{dt} = 50\frac{dq}{dt} - \frac{2}{200} q(t) \cdot \frac{dq}{dt} = \left(50 - \frac{q}{100}\right)\frac{dq}{dt}$$

$$\left.\frac{dR}{dt}\right|_{\substack{q=200 \\ \frac{dq}{dt}=50}} = \left(50 - \frac{200}{100}\right)(50)$$

$$= \frac{4800}{2}$$

Thus, the revenue is increasing at the rate of \$2400 per day.