MATH.

1. a) $R(100) = 40\cdot 100 - \frac{100^2}{100}$ 100.

$= 40\cdot 100 - 100$

$= 3900$

$R(99) = 40\cdot 99 - \frac{99^2}{100}$

$= 3861.99$

marginal revenue $3900 - 3861.99$

$= 38.01$.

(b) $R'(x) = 40 - \frac{x}{200}$

$R'(100) = 40 - \frac{100}{200}$

$= 39.5$.

(c) $R'(101) = 40 - \frac{101}{200}$

$= 39.495$.

(d)

c will be more accurate than b.

2. (a) $x^5 - 4xy + 4 = 89$

$x^5 - 4xy - 85 = 0$

$\frac{dy}{dx}: 5x^4 - 4(y + xy') = 0$

$5x^4 - 4y - 4xy' = 0$

$5x^4 - 4y = 4xy'$

$\frac{5x^4 - 4y}{4x} = y'$

(b) $\tan^{-1}(x^2y) = x + xy^2$

$\tan^{-1}(x^2y) - x - xy^2 = 0$

$\frac{1}{1+(x^2y)^2} - 1 - (y^2 + 2xy\cdot y') = 0$

$\frac{2xy + x^2\cdot y'}{1+(x^2y)^2} - 1 - y^2 - 2xy\cdot y' = 0$

$\frac{2xy}{1+(x^2y)^2} + \frac{x^2y'}{1+(x^2y)^2} - 1 - y^2 - 2xy\cdot y' = 0$

$\frac{x^2y'}{1+(x^2y)^2} - 2xy\cdot y' = y^2 + 1 - \frac{2xy}{1+(x^2y)^2}$

$y'\left(\frac{x^2}{1+(x^2y)^2} - 2xy\right) = y^2 + 1 - \frac{2xy}{1+(x^2y)^2}$

$$y' = \frac{y^2 + 1 - \frac{2xy}{1+x^4y^2}}{\frac{x^2}{1+x^4y^2} - 2xy}.$$

3. (a) $y = \sin^{-1}(x + \sqrt{1-x^2})$

$$= \sin^{-1}(x + (1-x^2)^{\frac{1}{2}})$$

$$y' = \frac{1}{\sqrt{1-[x+(1-x^2)^{\frac{1}{2}}]^2}}$$

$$y' = \frac{1}{\sqrt{1-[x+(1-x^2)^{\frac{1}{2}}]^2}} \cdot \left[1 + \tfrac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x)\right]$$

$$y' = \frac{1+\frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x)}{\sqrt{1+[x+(1-x^2)^{\frac{1}{2}}]^2}}$$

(b) $y = \sec^{-1}(e^{-2x} + \ln 5x)$

$$y' = \frac{1}{|e^{-2x} + \ln 5x|\sqrt{(e^{-2}+\ln 5x)^2 - 1}}$$

$$= \frac{1}{|e^{-2x} + \ln 5x|\sqrt{(e^{-2}+\ln 5x)^2 - 1}} \cdot \left(-2e^{-2x} + \frac{5}{5x}\right)$$

$$= \frac{-2e^{-2x} + \frac{5}{5x}}{|e^{-2x} + \ln 5x|\sqrt{(e^{-2}+\ln 5x)^2 - 1}}$$

4. $y = (x+1)^{(\sin x + \cos x)}$

(a) $\ln y = (\sin x + \cos x)\ln(x+1)$

$$\frac{y'}{y} = (\cos x - \sin x)\ln(x+1) + \frac{(\sin x + \cos x)}{x+1}$$

$$y' = \cancel{y}\left[(\cos x - \sin x)\ln(x+1) + \frac{(\sin x + \cos x)}{x+1}\right](x+1)^{(\sin x + \cos x)}$$

(b) $y = \dfrac{e^{-2x}\sin^3(x+1)}{x^4\sqrt{5x-1}}$

$$\ln y = (\ln e^{-2x}) + \ln \sin^3(x+1)) - (\ln x^4 + \ln(5x-1)^{\frac{1}{2}})$$

$$\ln y = -2x + 3\ln\sin(x+1) - 4\ln x - \tfrac{1}{2}\ln(5x-1)$$

$\frac{d}{dx}$: $\dfrac{y'}{y} = -2 + \dfrac{3}{\sin(x+1)} \cdot \cos(x+1) - \dfrac{4}{x} - \dfrac{5}{2(5x-1)}$

$$\frac{y'}{y} = -2 + \frac{3\cos(x+1)}{\sin(x+1)} - \frac{4}{x} - \frac{5}{2(5x-1)}$$

$$y' = y\left(-2 + \frac{3\cos(x+1)}{\sin(x+1)} - \frac{4}{x} - \frac{5}{2(5x-1)}\right)$$

$$y' = \frac{e^{-2}\sin^3(x+1)}{x^4\sqrt{5x-1}}\left(-2 + \frac{3\cos(x+1)}{\sin(x+1)} - \frac{4}{x} - \frac{5}{2(5x-1)}\right)$$