7. $T(x) = \begin{cases} 6x & \text{if } 0 \le x \le 50 \\ 11x - \frac{x^2}{10} & \text{if } x > 50. \end{cases}$

$-\frac{b}{2a} = -\frac{11}{2 \cdot -\frac{1}{10}}$

$= \frac{11}{\frac{1}{5}}$

$= 55.$

$T_{max} = \begin{cases} 300 & \text{if } x = 50. \\ 302.5 & \text{if } x = 55. \end{cases}$

$x > 50$

$T(x) = 11x - \frac{x^2}{10}$

$T' = 11 - \frac{x}{5}$

$T'' = -\frac{1}{5}$

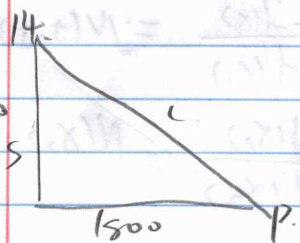
$T' = 0$

$11 - \frac{x}{5} = 0$

$x = 55$

Since $T''(55) < 0$ by SDT. Rel. max & by CPT abs. max when $x = 55$

14.



$t = ?$

$2400 = 400t - 16t^2$

$150 = 25t - t^2$

$t^2 - 25t + 150 = 0$

$(t-10)(t-15) = 0$

$t = 10, 15.$

$S(t) = 400t - 16t^2$

$L^2 = S^2 + 1800^2$

$\cancel{2}L \frac{dL}{dt} = \cancel{2} S \frac{ds}{dt}$

$L \frac{dL}{dt} = (400t - 16t^2)(400 - 32t)$

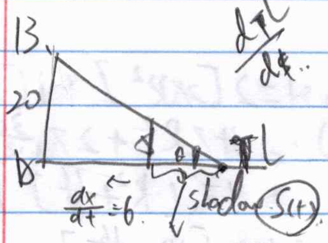
$\frac{dL}{dt} = \frac{(400t - 16t^2)(400 - 32t)}{\sqrt{S^2 + 1800}}$

$\frac{dL}{dt} = \frac{(2400)(80)}{\sqrt{2400^2 + 1800^2}}$

$= 64$ ft/sec

$\tan = \frac{20}{\Delta L} = \frac{5}{S(t)}$

13.



$\tan\theta = \frac{5}{S(t)} = \frac{20}{S(t) + x}$

$5x + 5S(t) = 20S(t)$

$S(t) = \frac{1}{3} x(t)$

$\frac{ds}{dt} = \frac{1}{3} \cdot \frac{dx}{dt} = -2$ ft/s

$T = x + S$

$\frac{dT}{dt} = \frac{dx}{dt} + \frac{ds}{dt}$

$= -6 - 2$

$= -8$ ft/s

4.(a) $f(x) = x + \ln x - 7 = 0.$

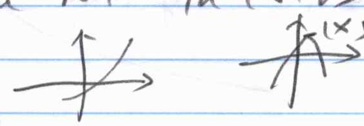
(a) $f(5) = -2 + \ln 5 < 0$

$f(6) = -1 + \ln 6 > 0$

Since $f(5)\,f(6) < 0$

by IVT there is a root in $(5, 6)$

(b) $f' = 1 + \frac{1}{x} > 0.$




$f \uparrow (0, \infty)$

Since $f$ is an increasing function $(0, \infty)$, there is only one root.

(c)

$N(x) = x - \dfrac{f(x)}{f'(x)}$

$= x - \dfrac{[x + \ln x - 7]}{1 + \frac{1}{x}}$

$= \dfrac{x + 1 - x - \ln x + 7}{1 + x^{-1}}$

$= \dfrac{8 - \ln x}{1 + x^{-1}}$

$x_1 = 5.5.$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = N(5.5) = 5.3267816$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = N(x_2) = 5.3271783$

$x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)} = N(x_4) = 5.3271783.$

$r \approx 5.33$

$\therefore r \simeq 5.33$

10.


$V = \pi R^2 H = 32\pi$

$R^2 H = 32$

$H = \frac{32}{R^2}$

Total cost $= 2[2\pi R^2] + 1[2\pi R H]$

$C(R) = 4\pi R^2 + 2\pi \frac{32}{R}$

$= 4\pi[R^2 + \frac{16}{R}] \quad R > 0.$

$C' = 4\pi[2R - \frac{16}{R^2}]$

$C' = 0 \quad R = 2$

Since $C''(2) > 0$ by SDT

Rel. min & by CRT abs. min when $C'' = 4\pi[2 + \frac{32}{R^3}] > 0$

$R = 2 \qquad C(2) = 48\pi$

$H = 8$