Consider algebraic expressions such as: $2x^2$, $3a + 2b$ and $x^2 + y^2$. Of these expressions, $2x^2$, $3a$, $2b$, x^2 and y^2 are examples of terms.

Term

A term is a number or a product of a number with one or more variables which can be raised to a power.

Term	$5y$	$-2a^3$	$\frac{1}{2}x^2yz^4$	x	10
Coefficients	5	-2	$\frac{1}{2}$	1	10
Variables	y	a	x, y, z	x	no variable

Polynomial

A polynomial is a term or sum of terms, in which all variables have whole number exponents, and in which variables appear only in the numerator.

Polynomials	Non-Polynomials
5	$x^{\frac{1}{2}}$
$\sqrt{2}\,x$	$2x + \sqrt{y}$
$3a^2 - 2a$	$\frac{1}{2x} + 4$
$\frac{3}{4}y^2 + 3y - 4$	$x^{-3} - 2x$

Classifying Polynomials

Monomial	A polynomial with one term	$3,\ 2x^2y,\ -3a$
Binomial	A polynomial with two terms	$x + 2,\ 2x^2y + 3,\ x^2 - y$
Trinomial	A polynomial with three terms	$3x^2 + 2x - 3,\ \sqrt{2}\,x + y - z$
Polynomial	General term for expressions with more than three terms (can also be used to describe monomial, binomial, or trinomial)	$x^5 - 2x^4 + 3x^3 - 4x^2$

Degree of a Polynomial

The degree of a term of a polynomial is the sum of the exponents of the variables in that term. The degree of a polynomial is the term with the highest degree.

For example: In $3x^2 + 2x - 3$, the term of highest degree is $3x^2$, so the degree is 2.

In $4x^2y^3 + z^4$, the term of highest degree is $4x^2y^3$, so the degree is 5. $(2 + 3 = 5)$

In $-2x^2yz^2 + y^4$, the term of highest degree is $-2x^2yz^2$, so the degree is 5. $(2 + 1 + 2 = 5)$

Leading Term

The term with the highest degree.

For example, consider the expression $3x^2y^3 - 2xy^2 + 2$:

Terms	$3x^2y^3, -2xy^2, 2$
Coefficients	$3, -2, 2$
Degree of each term	$5, 3, 0$
Leading term	$3x^2y^3$
Degree of the polynomial	5

Combining Like Terms

Like terms are either constant terms, or terms that contain the same variable(s) to the same power.

For example: $3x^2,\ 5x^2$ are like terms because they have the same variable and exponent.

$2xy^2,\ 3x^2y$ are not like terms because they have different exponents for each variable.

To combine like terms, add or subtract the coefficients of the terms.

Example 1 Simplify the expression $4x^2 - 3y - x^2 + 5y$.

▶ *Solution*: $4x^2$ and $-x^2$ are like terms, so the coefficients can be added.

$-3y$ and $5y$ are like terms, so the coefficients can be added.

$4x^2 - 3y - x^2 + 5y = 3x^2 + 2y$

Evaluating Polynomials

When a constant is substituted for a variable in a polynomial, the polynomial is evaluated for that constant.

For example: When $3x^2 - 2$ is evaluated for $x = 4$, the result is $3(4)^2 - 2 = 48 - 2 = 46$.

Multiplying a Monomial by a Monomial

To multiply two monomials, first multiply the constant factors, and then multiply the variable factors.

Example 2 Multiply.

a) $(2x^3)(-3x^4)$ **b)** $(-3a^2b^3)(-2a^2b^5)$ **c)** $(-3y^2z^3)(2yz^2)(4y^3z^2)$

▶ *Solution*: **a)** $(2)(-3)(x^3)(x^4) = -6x^{3+4} = -6x^7$

b) $(-3)(-2)(a^2)(a^2)(b^3)(b^5) = 6a^{2+2}b^{3+5} = 6a^4b^8$

c) $(-3)(2)(4)(y^2)(y)(y^3)(z^3)(z^2)(z^2) = -24y^{2+1+3}z^{3+2+2} = -24y^6z^7$

Multiplying a Polynomial by a Binomial

To multiply a polynomial by a binomial, use the distributive property to remove the brackets, then simplify.

The Distributive Property

$a(b + c) = a \times b + a \times c$

Example 3 Multiply.

a) $2(3 + 4)$ **b)** $2x^2(3x^2 - 4y)$ **c)** $-3y(x^4 + 2y^3)$

▶ *Solution*: **a)** $2 \times 3 + 2 \times 4 = 6 + 8 = 14$

b) $2x^2 \times 3x^2 - 2x^2 \times 4y = 6x^4 - 8x^2y$

c) $-3y \times x^4 - 3y \times 2y^3 = -3x^4y - 6y^4$

3.1 Exercise Set

1. Fill in the blank with the correct response.

a) The expression $3x^2$ is called a ____________ .

b) In the term $-4x^3$, the coefficient is ____________ and the exponent is ____________ .

c) The number of terms in the expression $3x^3 - x^2 + 2$ is ____________ .

d) The degree of the polynomial $3x^6$ is ____________ .

e) When $3x^2 - 2x + 1$ is evaluated for $x = -3$, the result is ____________ .

f) $3x^6 + 2x^2 - 5$ is a ____________ of degree 6.

g) ____________ is a example of a monomial with coefficient 4 and variables of degree 7.

2. For each polynomial, find the number of terms, and name the coefficient of each term.

a) $3x^5$

b) $-2y^4$

c) $4x^3 - 2x^2$

d) $-3a^3 + 3a - 4^0$

e) $\sqrt{2}y^3 - \frac{2}{3}y - 7$

f) $\sqrt[3]{5}b^2 - \frac{b}{3} + 1$

g) $2x^3y^2 - 3x^2$

h) $2^3b^3 - 3^2$

i) $-x^3y^2z + \sqrt{2}xyz + 4z^3$

j) $\sqrt[3]{2}x^4y^3z^2p + p$

k) x^3

l) $\frac{x}{2} - 2^{-2}$

3. Determine whether the expression is a polynomial.

a) 3^{-4} y / n

b) x^{-2} y / n

c) $\frac{1}{3x}+2$ y / n

d) $\sqrt{2}x^{3}$ y / n

e) $\frac{\sqrt{2}}{x^{3}}$ y / n

f) $\frac{xy}{z}$ y / n

g) $\frac{xy}{\sqrt{2}}$ y / n

h) $\frac{1}{x^{2}+2x+3}$ y / n

i) $\frac{x}{3}+2^{-3}$ y / n

j) $\sqrt[3]{8}x^{2}+\sqrt{9}$ y / n

4. Classify each polynomial as a monomial, binomial or trinomial; if none of these, classify as polynomial.

a) $2x-5$ ____________

b) $-2x+2$ ____________

c) $a^{2}-2a+3$ ____________

d) $4ab$ ____________

e) $3y^{2}$ ____________

f) $2x^{3}-3x^{2}+x-4$ ____________

g) $3x^{3}y^{4}z^{5}$ ____________

h) $2x^{4}-2^{-5}$ ____________

i) $\frac{2}{3}x^{2}-\frac{1}{4}y^{2}$ ____________

j) $3x^{5}-x^{3}+2x-7$ ____________

6. Find the value of the polynomial when $x = -2$.

a) $-3x^2 + 2x - 1$

b) $-3x^2 - 2x + 1$

c) $2x^2 - 3x + 4$

d) $-2x^2 - 3x - 4$

e) $-x^4 + 2x^2 - 3$

f) $x^4 - 2x^2 + 3$

g) $-x^5 - 3x^3$

h) $-x^4 - 3x^2$

i) $0.8x^3 - \frac{x^2}{4}$

j) $-0.8x^3 + \frac{x^2}{4}$

7. Find each product.

a) $3x^3(2x^4)$

b) $-2a^2b^4(4ab^2)$

c) $(3xy)(-4x^2y^3)$

d) $(2ab)(-2ab)(2ab)$

e) $(5x^3)(-2y^3)$

f) $(-4a^4b^3)(2a^3b^2)(3ab)$

g) $(a^2b^4)(a^3b)(-3b^2)$

h) $(-r^4s^2t)(r^3st^2)(-rst)$

i) $(-3ab^2)(2a^3b)(-a^2b^2)(-2a^3b^2)$

j) $(-5a^3b^3c^2d^3)(-2ab^2cd^2)(-4a^2bc^3d)$

8. Find each product. Leave answer in descending order of power.

a) $3x(x-4)$

b) $-2x^2(x+3)$

c) $4y(-2y^2+3y)$

d) $-5y(2y+3y^2-y^4)$

e) $(3a^2)(2a)(-4+2a^2-a^4)$

f) $-2mn^4(-2mn+3m^2n^2-4)$

g) $a^2bc(ab^2c^2-a^2bc^3-2a^3b^3c)$

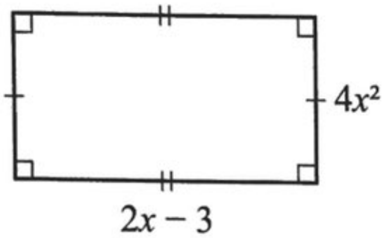
h) $-abc^2(-a^2bc^3+ab^2c-a^3c^2)$

i) $(-x^2y)(xy^3)(xy-xy^2+x^2y^2)$

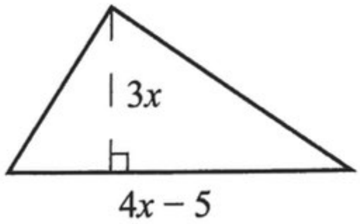
j) $(-a^3b^2)(-a^2b)(-a^2b-a^2b^3+a^3b)$

9. Find the area of each figure.

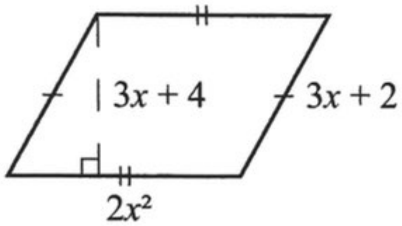
a)



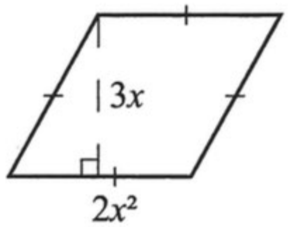
b)



c)



d)

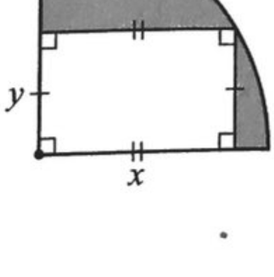


10. Determine the area of the shaded region in terms of x, y and π.

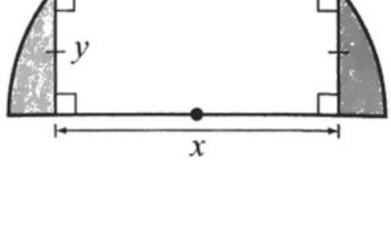
a)

b)

c)



d)



Polynomials - Solutions

3.1 Classifying Polynomials, page 118

1. **a)** term **b)** $-4, 3$ **c)** 3 **d)** 6 **e)** 34 **f)** trinomial **g)** (answers will vary) $4x^7$, $4x^3y^4$

2. **a)** 1; 3 **b)** 1; -2 **c)** 2; 4, -2 **d)** 3; $-3, 3, -1$ **e)** 3; $\sqrt{2}, -\frac{2}{3}, -7$ **f)** 3; $\sqrt[3]{5}, -\frac{1}{3}, 1$

g) 2; 2, -3 **h)** 2; 8, 9 **i)** 3; $-1, \sqrt{2}, 4$ **j)** 2; $\sqrt[3]{2}, 1$ **k)** 1; 1 **l)** 2; $\frac{1}{2}, -\frac{1}{4}$

3. **a)** y **b)** n **c)** n **d)** y **e)** n **f)** n **g)** y **h)** n **i)** y **j)** y

4. **a)** binomial **b)** binomial **c)** trinomial **d)** monomial **e)** monomial **f)** polynomial

g) monomial **h)** binomial **i)** binomial **j)** polynomial

5. **a)** $2x^2+3x$ **b)** $2x^4$ **c)** $-1.8x^2+3x+3$ **d)** $2y^4-2y^2-2y$ **e)** $-4x^3$ **f)** x^3+x^2-x

g) $-\frac{1}{12}x^3+3\sqrt{2}x^2$ **h)** $-\frac{5}{6}y^5+3\sqrt[3]{2}y^3$ **i)** $\frac{4}{3}x^2+\frac{1}{2\sqrt{2}}x$ **j)** $-y^3-\frac{1}{4}y^2-\frac{4\sqrt{6}}{3}y$

6. **a)** -17 **b)** -7 **c)** 18 **d)** -6 **e)** -11 **f)** 11 **g)** 56 **h)** -28 **i)** -7.4 **j)** 7.4

7. **a)** $6x^7$ **b)** $-8a^3b^6$ **c)** $-12x^3y^4$ **d)** $-8a^3b^3$ **e)** $-10x^3y^3$ **f)** $-24a^8b^6$ **g)** $-3a^5b^7$ **h)** $r^8s^4t^4$

i) $-12a^9b^7$ **j)** $-40a^6b^6c^6d^6$

8. **a)** $3x^2-12x$ **b)** $-2x^3-6x^2$ **c)** $-8y^3+12y^2$ **d)** $5y^5-15y^3-10y^2$ **e)** $-6a^7+12a^5-24a^3$

f) $-6m^3n^6+4m^2n^5+8mn^4$ **g)** $-2a^5b^4c^2-a^4b^2c^4+a^3b^3c^3$ **h)** $a^3b^2c^5+a^4bc^4-a^2b^3c^3$

i) $-x^5y^6+x^4y^6-x^4y^5$ **j)** $-a^7b^6+a^8b^4-a^7b^4$

9. **a)** $8x^3-12x^2$ **b)** $6x^2-\frac{15}{2}x$ **c)** $6x^3+8x^2$ **d)** $12x^2-3x$

10. **a)** $\frac{\pi}{4}x^2+\frac{\pi}{4}y^2-\frac{1}{2}xy$ **b)** $\frac{25}{4}\pi x^2-12x^2$ **c)** $\frac{\pi}{4}x^2+\frac{\pi}{4}y^2-xy$ **d)** $\frac{\pi}{8}x^2+\frac{\pi}{2}y^2-xy$