MARKS

1. The demand equation relating the quantity demanded q and the per unit price p is given as, (8 pts)

$$q = 192 - p^2, \qquad 0 < p \leq 12$$

(a) Find the elasticity of demand, E.

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{pq'}{q}$$

$$= -\frac{p(-2p)}{192-p^2}$$

$$= \frac{2p^2}{192-p^2}$$

(b) Find the interval of price where the demand is elastic.

$$\frac{2p^2}{192-p^2} > 1$$

$$2p^2 > 192 - p^2$$

$$3p^2 > 192$$

$$p^2 > 64$$

$$12 \geq p > 8$$

(c) What price will maximize the revenue?

the maximize the revenue is 8

(d) Suppose the price is \$6, what is the approximate change in demand if the price is lowered by 1.5%.

$$q = 192 - p^2$$

$$q' = -2p \cdot p'$$

$$q' = 2 \cdot 6 \cdot 0.015$$

$$q' = -0.18$$

$\therefore$ demand change lower 0.18.

2. (a) Evaluate using l'Hospital's Rule: (4 p

$$\lim_{x \to 4} \frac{2x + \sqrt{x}^{\frac{1}{2}} - 10}{2 - \sqrt{x}^{\frac{1}{2}}} = \lim_{x \to 4} \frac{2 + \frac{1}{2\sqrt{x}}}{2 - \frac{1}{2\sqrt{x}}}$$

$$= \frac{2 + \frac{1}{4}}{2 - \frac{1}{4}}$$

$$= \frac{9}{7}$$

(b) Find $f'(x)$. DO NOT SIMPLIFY

i. $f(x) = \dfrac{\log_4 x}{(x+1)^3}$ (4 pts)

$$f'(x) = \frac{\frac{1}{x \ln 4}(x+1)^3 - \log_4 x \cdot 3(x+1)^2}{(x+1)^6}$$

$$= \frac{\frac{(x+1)^3}{x \ln 4} - \log_4 x \cdot 3(x+1)^2}{(x+1)^6}$$

ii. $f(x) = \tan^{-1}(\sin x)$ (2 pts)

$$f'(x) = \frac{\cos x}{1 + \sin^2 x}$$

3. Find the slope of the tangent to the curve defined by the equation (5 pts)

$$x^2 + 2xy^2 - 3y^3 = 5$$

at the point $(2, 1)$.

$2x + 2(y^2 + x2y \cdot y') - 9y^2 \cdot y' = 0$

$2x + 2y^2 + 4xy \cdot y' - 9y^2 \cdot y' = 0$

$4 + 2 + 8y' - 9y' = 0$

$6 - y' = 0$

$y' = 6$

4. Suppose the quantity demanded per week of a certain product is related to the unit price p (6 pts) (in dollars) by the demand equation $q = \frac{1}{5}(225 - p^2)$ where q (measured in units of a hundred) is the quantity demaned each week. Find the price p that will give the revenue a maximum. Use the second derivative test to verify that you have found a local maximum.

5. Let $f(x) = \dfrac{3x}{\sqrt{x^3+4}}$ $\quad f'(x) = \dfrac{3(8-x^3)}{2\left(x^3+4\right)^{3/2}}$ $\quad f''(x) = \dfrac{9x^2\left(x^3-32\right)}{4\left(x^3+4\right)^{5/2}}$

(a) Find any critical point(s) (x, y).

(b) Fill in the blanks below.

i. The function is increasing on the interval(s): ____________________

ii. The function is decreasing on the interval(s): ____________________

Show your work here:

(c) Classify each critical point(s) as either local maximum, local minimum, or neither.

(d) Fill in the blanks below.

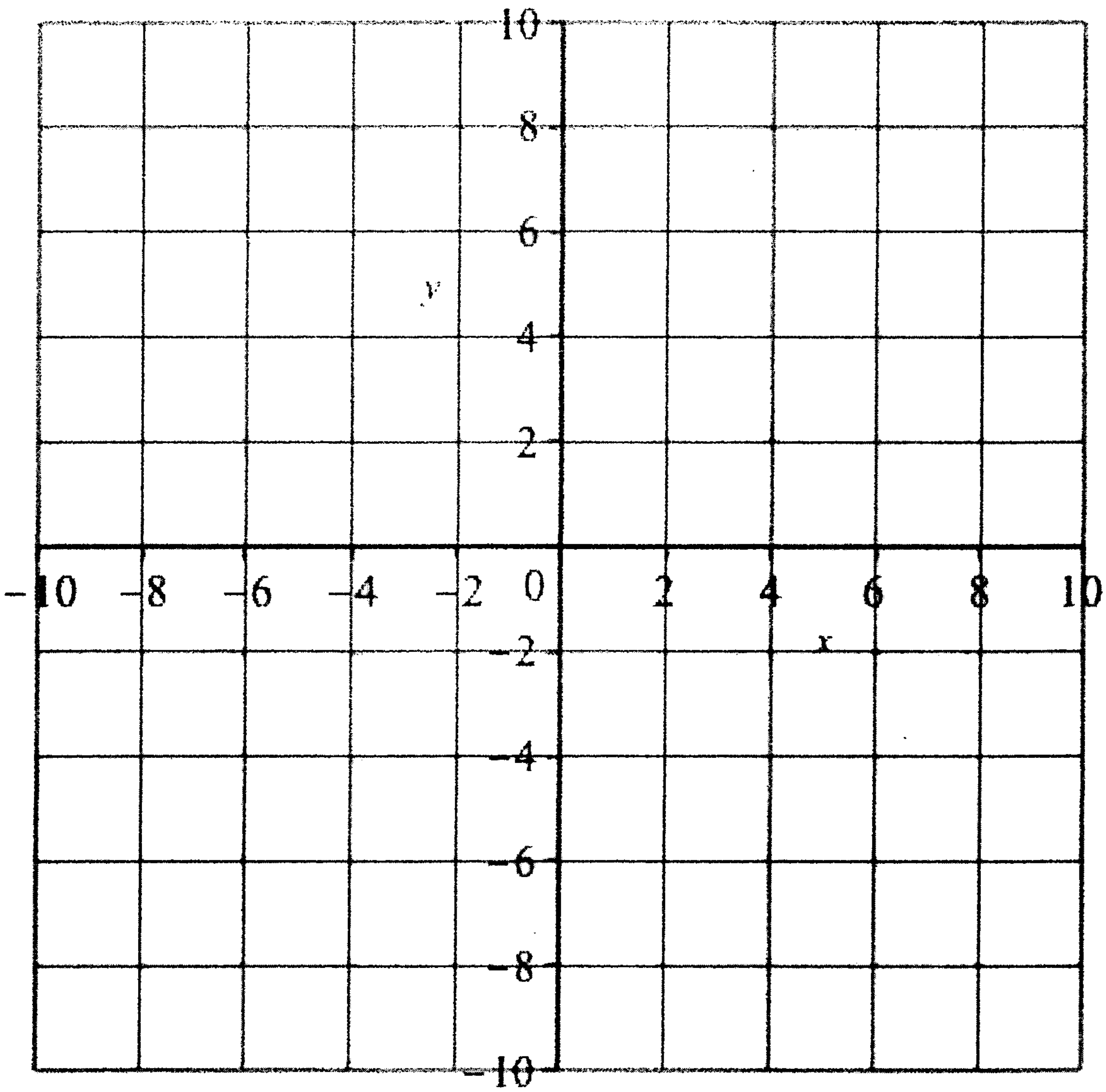
i. The function is concave up on the interval(s): ____________________

ii. The function is concave down on the interval(s): ____________________

Show your work here:

(e) Give the coordinates (x, y) of any inflection points.____________________

(f) Sketch the function using parts (a) - (e) as well as any other useful information. Indicate the approximate location of any local maxima or minima that you find.



6. The volume V of a cube with sides of length x cm is increasing with respect to time at a rate of 2 cm^3/sec. At a certain instant of time, the sides of the cube are 5 cm long, how fast is the total area of the cube changing at that instant of time? (6 pts)