Approximate $63^{2/3}$

$$64^{2/3} = (64^{1/3})^2 = 16$$

1. By Linearization "$L(x) = f(a) + f'(a)\cdot(x-a)$"
2. By Differentials $f(x+dx) \approx f(x) + f'(x)\cdot dx$
3. By Newton's Method.

---

1. $f(x) = x^{2/3}$, $f'(x) = \frac{2}{3}\frac{1}{x^{1/3}}$; $a = 64$

Linearization of f at 64 is

$$L(x) = f(64) + f'(64)\cdot(x-64) = 16 + \frac{1}{6}(x-64)$$

$$\therefore f(63) = 63^{2/3} \approx L(63) = 16 + \frac{1}{6}(-1) = 16 - \frac{1}{6} = 95/6$$

$$= 15.8333\ldots$$

---

2. $$63^{2/3} = f(64-1) \approx f(64) + f'(64)\cdot(-1) = 95/6$$

$$f(x+dx) \approx f(x) + f'(x)\,dx$$

---

3. Let $x = 63^{2/3} \longrightarrow f(x) = x^{3/2} - 63$; $f'(x) = \frac{3}{2}\sqrt{x}$,

$$N(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^{3/2} - 63}{3/2\sqrt{x}} = \frac{1}{3}\left(x + \frac{126}{\sqrt{x}}\right)$$

STEP I Since $f(15)f(16) < 0$, by IVT root is in $(15, 16)$

STEP II Let $x_1 = 15.8$, $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = N(x_1) = 15.83297\ldots$

STEP III Repeat: $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = N(x_2) = 15.832896$

$x_4 = \ldots = N(x_4) = 15.832896$

$\therefore 63^{2/3} \approx 15.83$ Correct to two decimal places