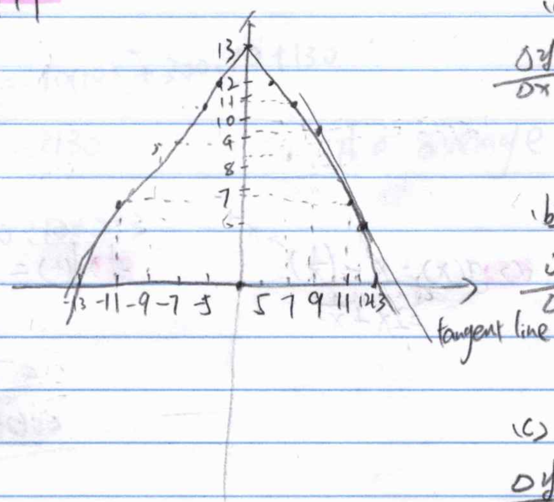
4.1.1



(a) $x=12 \quad x=13$

$$\frac{\Delta y}{\Delta x} = \frac{0-\sqrt{169-(12)^2}}{1}$$

$$= -5$$

(b) $x=12, \ x=12.1$

$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{169-(12.1)^2}-\sqrt{169-(12)^2}}{0.1}$$

$$\approx -2.4711$$

(c) $x=12, \ x=12.01$

$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{169-(12.01)^2}-\sqrt{169-(12)^2}}{0.01}$$

$$\approx -2.4068$$

(d) $x=12 \quad x=12.001$

$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{169-(12.001)^2}-\sqrt{169-(12)^2}}{0.001}$$

$$\approx -2.4007$$

$$f(x) = \sqrt{169-x^2}$$

$$f'(x) = \frac{1}{2}(169-x^2)^{-\frac{1}{2}}$$

$$= \frac{1}{\cancel{2}} \cdot \frac{1}{\sqrt{169-x^2}} \cdot -\cancel{2}x$$

$$= -\frac{x}{\sqrt{169-x^2}}$$

$$f'(12) = -\frac{12}{5}$$

$$= -2.4$$

4.1.13

$$\frac{\Delta y}{\Delta x} = \frac{-0.1(5.05)^2-5.05+125-[-0.1(5)^2-5+125]}{0.05}$$

$$= -2.005$$

$$\frac{\Delta y}{\Delta x} = \frac{-0.1(5.01)^2-5.01+125-[-0.1(5)^2-5+125]}{0.01}$$

$$= -2.001$$

$R'(40) = -200(40)+8000$

$= 0$

$f'(q) = -0.2q - 1$

$f'(5) = -0.2 \cdot 5 - 1$

$= -2$

4.2.1

(a) $f(x) = \sqrt{169 - x^2}$

$= \frac{1}{2(-2x)}$

$= \frac{1}{4x}$

(c) $g(x) = x^2 - \left(\frac{1}{x}\right) \to x^{-1}$

$= 2x + \frac{1}{x^2}$

f) $f(t) = \frac{2}{\sqrt{2t+1}}$

$= \frac{2}{(2t+1)^{\frac{1}{2}}}$

$= \frac{(2t+1)^{\frac{1}{2}} - \frac{\cancel{2}}{\cancel{2}\sqrt{2t+1}}}{2t+1}$

$= \frac{\sqrt{2t+1} - \frac{1}{\sqrt{2t+1}}}{2t+1}$

$= \frac{\frac{2t+1-1}{\sqrt{2t+1}}}{2t+1}$

$= \frac{2}{\sqrt{2t+1}\,(2t+1)}$

4.2.4.

$f(x) = 5 - x - 3x^2$

$f'(x) = -6x - 1$

$f'(2) = -12 - 1$

$= -13.$

$f(2) = 5 - 2 - 3(2)^2$

$= -9$

$y - y_1 = m(x - x_1)$

$y + 9 = -13(x - 2)$

$y + 9 = -13x + 26$

$y = -13x - 17.$

4.2.8.

(a) $f'(x) = 2x - 2.$

(b) $2x - 2 = 0$

$2x = 2$

$x = 1$

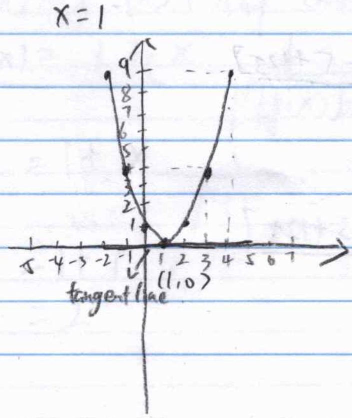
$\therefore f(1) = (1)^2 - 2(1) + 1$

$= 1 - 2 + 1$

$= 0.$

$\therefore$ the point is $(1, 0).$

(c).



$f'(1) = 2(1) - 2$

$= 0.$

$\therefore$ the rate of change is $0.$

4.2.10.

(a) $C'(q) = -20q + 300$.

(b) $C'(10) = -200 + 300$

$= 100$. $\therefore$ the rate of change is 100.

$\bar{C}(10) = -100 + 300 + 13$

$= 213$.

$\bar{C}(q) = \frac{C}{q}$

$= -10q + 300 + \frac{130}{q}$.

* (c)

$C(10) = -10(10)^2 + 300 \cdot 10 + 130$

$= 2130$. $\therefore$ the average cost is 213

$2130 \div 10 = 213$.

4.2.11.

when $h = 1$

$\frac{C(100+1) - C(100)}{1}$

$= \frac{0.000002(101)^3 + 5(101) + 400 - [0.000002(100)^3 + 5(100) + 400]}{1}$

$= 5.06$.

when $h = 0.1$

$\frac{C(100+0.1) - C(100)}{0.1}$

$= \frac{0.000002(100.1)^3 + 5(100.1) + 400 - [0.000002(100)^3 + 5(100) + 400]}{0.1}$

$= 5.06$.

when $h = 0.001$

$C(100+0.001) - C(100)$

$= \frac{0.000002(100.001)^3 + 5(100.001) + 400 - [0.000002(100)^3 + 5(100) + 400]}{0.001}$

$= 5.06$.

when $h = 0.0001$

$C(100+0.0001) - C(100)$

$= \frac{0.000002(100.0001)^3 + 5(100.0001) + 400 - [0.000002(100)^3 + 5(100) + 400]}{0.0001}$

$= 5.06$.

$$\lim_{h \to 0} \frac{0.000002(100+h)^3 + 5(100+h) + 400 - [0.000002(100)^3 + 5(100) + 400]}{h}$$

$= 5.06$.

4.2.17.

(a) $R'(q) = -200q + 8000$

(b) $R'(39) = -200(39) + 8000$

$= 200$.

$R'(40) = -200(40) + 8000$

$= 0$

$R'(41) = -200(41) + 8000$

$= -200$

(C) The prize is 40.

4.3.12

$$\frac{x^3}{x^3-5x+10}$$

$$\frac{dy}{dx}=\frac{3x^2(x^3-5x+10)-x^3(3x^2-5)}{(x^3-5x+10)(x^3-5x+10)}$$

$$=\frac{\cancel{3x^5}-15x^3+30x^2-[\cancel{3x^5}-5x^3]}{x^6-5x^4+10x^3-5x^4+25x^2-50x+10x^3-50x+100}$$

$$=\frac{-10x^3+30x^2}{x^6-10x^4+20x^3+25x^2-100x+100}$$

$$=\frac{-10x^3+30x^2}{(x^3-5x+10)}$$

4.3.16

$$f(x)=\frac{x^2-4}{5-x}$$

$$f'(x)=\frac{2x(5-x)-(-1)(x^2-4)}{(5-x)^2}$$

$$=\frac{10x-2x^2+x^2-4}{(5-x)^2}$$

$$=\frac{10x-x^2-4}{(5-x)^2}$$

$$f'(3)=\frac{30-9-4}{4}$$

$$=\frac{17}{4}$$

$$=4.25$$

4.3.18

$f'(4)=5$

$g'(4)=12$

$(fg)(4)=f(4)g(4)=2$

$g(4)=6$

$$\therefore f(4)=\frac{2}{g(4)}$$

$$=\frac{2}{6}$$

$$=\frac{1}{3}$$

$$\frac{d}{dx}\frac{f(4)}{g(4)}$$

$$=\frac{f'(4)g(4)-g'(4)f(4)}{g(4)^2}$$

$$=\frac{5\cdot 6-\cancel{12}^{4}\cdot\frac{1}{3}}{36}$$

$$=\frac{30-4}{36}$$

$$=\frac{13}{18}$$

4.3.31

(a) $C(x) = 100x + 200{,}000$

$\bar{C} = \frac{100x + 200{,}000}{x}$

(b) $\bar{C}' = \frac{100(x) - 1)(100x + 200{,}000)}{x^2}$

$= \frac{100x - \cancel{100x} - 200{,}000}{x^2}$

$= \frac{-200{,}000}{x^2}$

(c) If x is very large, $\bar{C}(x)$ will be 100 because according to $\bar{C}'$ the x will be large, and $\bar{C}'$ will be ~~zero~~ 100, thus the $\bar{C}(x)$ will be ~~les~~ 100.

4.3.32

(a) $R = \frac{50}{0.001x^2+1} \cdot x$

$R = \frac{50x}{0.001x^2+1}$

$= 50 \cdot \frac{x}{0.001x^2+1}$

(b) $R' = \frac{50(0.001x^2+1) - (0.002x) \cdot 50x}{(0.001x^2+1)^2}$

$R' = \frac{0.05x^2 + 50 - 0.1x^2}{(0.001x^2+1)^2}$

$= \frac{0.15x^2 + 50}{(0.001x^2+1)^2}$

$50 \frac{(0.001x^2+1) - x(0.002x)}{(0.001x^2+1)^2}$

$= 50 \frac{0.001x^2 + 1 - 0.002x^2}{(0.001x^2+1)^2}$

$= 50 \frac{-0.001x^2+1}{(0.001x^2+1)^2}$

(c) $R'(2) = \frac{0.15(2)^2 + 50}{(0.001 \cdot 2^2+1)^2}$

$= \frac{50.6}{1.008016}$

$= 50.20.$

$R'(2) = 50 \frac{-0.001(2)^2+1}{(0.001 \cdot 2^2+1)^2}$

$= 49.4.$

4.4.3

$f(x) = \frac{(x-2)^{\frac{1}{3}}}{(x^3+4x-1)^2}$

$f'(x) = \frac{\frac{1}{3}(x-2)^{-\frac{2}{3}}(x^3+4x-1)^2 - (x-2)^{\frac{1}{3}} \cdot 2(x^3+4x-1) \cdot (3x^2+4)}{(x^3+4x-1)^4}$

$f(1) = \frac{\frac{1}{3}(-1)^{-\frac{2}{3}}(\cancel{1}+4\cancel{-1})^2 - (1-2)^{\frac{1}{3}} \cdot 2(1+\cancel{4}\cancel{-1}) \cdot (3+4)}{(\cancel{1}+4\cancel{-1})^4}$

$= \frac{\frac{1}{3}(4)^2 - (-1) \cdot 2 \cdot 7}{4^4}$

$= \frac{\frac{16}{3} + 14}{256}$

$= 0.07$

4.4.5

$\frac{dy}{dx} = (2x - 4 \cancel{+5})\sqrt{25-x^2} + (x^2 - 4x + 5) \cdot \frac{1}{2}(25-x^2)^{-\frac{1}{2}} \cdot -2x$

$= \cancel{4}\ 2\sqrt{\cancel{16}}\ \cancel{2 \cdot 16} + 2 \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot -6$

$= \frac{53}{2}$ $\quad 8 + \frac{-6}{4}$

$= \frac{13}{2}$

$y - 8 = \frac{13}{2}(x - 3)$

$y = \frac{13}{2}x - \frac{39}{2} + 8$

$y = \frac{13}{2}x - \frac{23}{2}$

4.4.7

$$\sqrt{(x^2+1)^2+\sqrt{1+(x^2+1)^2}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left((x^2+1)^2+\sqrt{1+(x^2+1)^2}\right)^{-\frac{1}{2}} \cdot \left[2(x^2+1)\cdot 2x + \frac{1}{2}(1+(x^2+1)^2)^{-\frac{1}{2}} \cdot 2(x^2+1)\cdot 2x\right]$$

$$= \frac{1}{2}\left((2)^2+\sqrt{1+(2)^2}\right)^{-\frac{1}{2}} \cdot \left[2(2)\cdot 2 + \frac{1}{2}(1+(2)^2)^{-\frac{1}{2}} \cdot 2(2)\cdot 2\right]$$

$$= \frac{1}{2}(4+\sqrt{5})^{-\frac{1}{2}} \cdot \left[8 + \frac{1}{\cancel{2}}(5)^{-\frac{1}{2}} \cdot \overset{4}{\cancel{8}}\right]$$

$$= \frac{1}{2\sqrt{4+\sqrt{5}}} \cdot \left[8 + \frac{4}{\sqrt{5}}\right]$$

$$= \frac{1}{2\sqrt{4+\sqrt{5}}} \cdot \frac{\overset{4}{8}\sqrt{5}+\overset{2}{4}}{\sqrt{5}}$$

$$= \frac{4\sqrt{5}+2}{\sqrt{20+5\sqrt{5}}}$$

4.4.9

(a) $(f\circ g)'(2)$

$= f'(g(2)) \cdot g'(2)$

$= +1 \cdot \frac{1}{2}$

$= +\frac{1}{2}$

(b) $f'(f(1)) \cdot f'(1)$

$= 1 \cdot 1$

$= 1$

(c) $g(f(3)) \cdot f'(3)$

$= 0$

$(1, 4)\ (-3, 2)$

$g(x):\ m_1 = \frac{-1}{-4}$

$= \frac{1}{2}$

$(3, 5)\ (5, 4)$

$g(x)\ m_2 = \frac{-1}{2}$

$= -\frac{1}{2}$

$(-\cancel{2}4, \cancel{-2}\,-1)\ (-3, \cancel{-1}\,-2)$

$f(x):\ m_1 = \cancel{\frac{-1+2}{-3+4}}\ \ \frac{-2+1}{-3+4}$

$= 1\ \ = \frac{-1}{1}$

$= -1$

$(-1, -2)\ (1, 0)$

$f(x):\ m_2 = \frac{+2}{1+1}$

$= +1$

(d) $f'(g(3)) \cdot g'(3)$

$= 0.$ DNE

(e) $g(f(-2)) \cdot f'(-2)$

$= 0$ DNE

(f) $g(f(-4)) \cdot f'(-4)$

$= \frac{1}{2} \cdot 1$

$= \frac{1}{2}$

4.4.16

$N'(x) = \frac{1}{2}(10,000 - 40x - 0.02x^2)^{-\frac{1}{2}} \cdot (-0.04x - 40)$

$N'(10) = \frac{1}{2}(10,000 - 40\cdot 10 - 0.02\cdot 10^2)^{-\frac{1}{2}} \cdot (-0.04\cdot 10 - 40)$

$= -0.82$ -20.6%

$N'(100) = \frac{1}{2}(10,000 - 40\cdot 100 - 0.02\cdot 100^2)^{-\frac{1}{2}} \cdot (-0.04\cdot 100 - 40)$

$= -1.16$ -28.9%

$N'(150) = \frac{1}{2}(10,000 - 40\cdot 150 - 0.02\cdot 150^2)^{-\frac{1}{2}} \cdot (-0.04\cdot 150 - 40)$

$= -1.54$ -38.6%.

4.4.17

$P(16) = \frac{400}{1 + \frac{\sqrt{16}}{8}} + 200$

$= \frac{400}{1 + \frac{1}{2}} + 200$

$= 666.67$

$f'(p) = \frac{100}{9} \cdot \frac{1}{2}(810,000 - p^2)^{-\frac{1}{2}} \cdot 2p$

$f'(666.67) = \frac{100}{9} \cdot \frac{1}{2}(810,000 - 666.67^2)^{-\frac{1}{2}} \cdot 2 \cdot 666.67$

$= 12.25$.

$\therefore$ the rate is 12.25.

00)+400]

4.5.4

$R'(x) = -100 \cdot -\sin(2\pi x)$.

September is $\frac{\cancel{8}^{4\,2}}{\cancel{12}_{6\,3}} = \frac{2}{3}$

$R'(\frac{2}{3}) = -100 \cdot -\sin(2\pi \cdot \frac{2}{3})$

$= -86.60$.

4.5.5

third quarter $\therefore x = 3$.

$S'(x) = \frac{1}{10} \cdot \cos(\frac{\pi}{2}(x+1)) \cdot \frac{\pi}{2}$

$S'(3) = \frac{1}{10} \cdot \cos(\frac{\pi}{2}(3+1)) \cdot \frac{\pi}{2}$

$= 0.16$.

4.6.3

$y' = e^{2x-3}$ $2\times\frac{3}{2}$ $2x-3$ $2\times\frac{3}{2}-3$

$y'(\frac{3}{2}) = e^{\cancel{2\cdot\frac{3}{2}}}$ 0 2

$= 1$

$y - 1 = 1(x - \frac{3}{2})$

$y = 2x - \frac{3}{2} + 1$

$y = 2x - \cancel{\frac{3}{2}} - 2$

4.6.5

$y' = \frac{1}{x}$

$= \frac{1}{a}$

$y = \ln(a)$

$y - \ln(a) = \frac{1}{a}(x - a)$

put $(0,0)$ in.

$0 - \ln(a) = \frac{1}{a}(0 - a)$

$y = \ln(x)$.

$\ln(a) = -1$

$\ln(a) = -1$

$a = e$

$= 0$

4.6.7

(a) $R(x) = 100e^{-0.0001x} \cdot x$

(b) $R'(x) = 100e^{-0.0001x}$

(c) $R'(x_0) = 100e^{-0.0001 \cdot 10}$

$= 100 \cdot 10$

4.7.8

$x^4 = y^2 + x^2 \quad (2, \sqrt{12})$

$4x^3 = 2y \cdot y' + 2x$

$4 \cdot 8 = 2\sqrt{12}\, y' + 4$

$32 = 2\sqrt{12}\, y' + 4$

$28 = 2\sqrt{12}\, y'$

$y' = \frac{28}{2\sqrt{12}}$

$y' = \frac{28}{4\sqrt{3}}$

$y' = \frac{28\sqrt{3}}{12}$

$y' = \frac{7\sqrt{3}}{3}$

$y - \sqrt{12} = \frac{7\sqrt{3}}{3}(x-2)$

4.7.12

$y = \frac{(x-1)^8 (x-23)^{\frac{1}{2}}}{27x^6 (4x-6)^8}$

$\ln y = \ln \frac{(x-1)^8 (x-23)^{\frac{1}{2}}}{27x^6 (4x-6)^8}$

$\ln y = \ln[(x-1)^8 \cdot (x-23)^{\frac{1}{2}}] - \ln[27x^6 \cdot (4x-6)^8]$

$\ln y = \ln (x-1)^8 + \ln (x-23)^{\frac{1}{2}} - \ln(27x^6) - \ln(4x-6)^8$

$= \ln 8\ln(x-1) + \frac{1}{2}\ln(x-23) - 6\ln\sqrt{3}x - 8\ln(4x-6)$

$\frac{1}{y} \cdot y' = \frac{8}{x-1} + \frac{1}{2} \cdot \frac{1}{x-23} - 6 \cdot \frac{\sqrt{3}}{\sqrt{3}x} - 8 \cdot \frac{\cancel{4}\,2}{\cancel{4x-6}\; 2x-3}$

$= \frac{8}{x-1} + \frac{1}{2x-46} - \frac{6}{x} - \frac{16}{2x-3}$

$y' = y\left(\frac{8}{x-1} + \frac{1}{2x-46} - \frac{6}{x} - \frac{16}{2x-3}\right)$

$= \frac{(x-1)^8 (x-23)^{\frac{1}{2}}}{27x^6 (4x-6)^8} \cdot \left(\frac{8}{x-1} + \frac{1}{2x-46} - \frac{6}{x} - \frac{16}{2x-3}\right)$

4.8.8

$\sin^{-1}(x) + \cos^{-1}(x)$ $(x^2+1)\tan^{-1}(x)$

$\frac{dy}{dx}\sin^{-1}(x) + \frac{dy}{dx}\cos^{-1}(x)$ $\frac{dy}{dx} = 2x\tan^{-1}(x) + (x^2+1)\frac{1}{1+x^2}$

$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}$ $= 2x\tan^{-1}(x) + 1$

$= 0$ $= \cancel{2x \cdot \frac{1}{1+x^2} + 1}$

$\cancel{\frac{2x}{1+x^2} + 1}$

4.8.9

$y = \sin^{-1}(x^2)$ $y = \tan^{-1}(3x)$

~~$y = \sin$~~

$y = \sin^{-1} x^2$ $y = \tan^{-1}(3x)$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-(x^2)^2}} \cdot \frac{dy}{dx} x^2$ $= \frac{1}{1+(3x)^2}$

$= \frac{2x}{\sqrt{1-x^4}}$

$=0$

4.7.13

$f(x) = (x+1)^{\sin x}$

$y = (x+1)^{\sin x}$

$\ln y = \sin x \ln(x+1)$

$\frac{1}{y} \cdot y' = \cos x \ln(x+1) + \sin x \cdot \frac{1}{x+1}$

$y' = (x+1)^{\sin x} \cdot [\cos x \ln(x+1) + \frac{\sin x}{x+1}]$

4.7.14

$g(x) = \frac{e^x (\cos x + 2)^3}{\sqrt{x^2+4}}$

$y = \frac{e^x ((\cos x+2)^3}{\sqrt{x^2+4}}$

$\ln y = \ln e^x + \ln(\cos x+2)^3 - \ln\sqrt{x^2+4}$

$\ln y = x + 3\ln(\cos x+2) - \frac{1}{2}\ln(x^2+4)$

$\frac{1}{y} \cdot y' = 1 + \frac{3 \cdot (-\sin x)}{\cos x+2} - \frac{\cancel{2}x}{\cancel{2}(x^2+4)}$

$y' = \frac{e^x (\cos x+2)^3}{\sqrt{x^2+4}} \left(1 + \frac{3 \cdot (-\sin x)}{\cos x+2} - \frac{x}{x^2+4}\right)$

4.8.~~8~~.7

$f(x) = x^3 + 4x + 2$

$g(x) = x f^{-1}(x)$ $\quad x = 7$ $\quad f(7) = x^3 + 4x + 2$

$x = 1$

$f'(x) = 3x^2 + 4.$ $\quad f'(1) = 3+4$

$= 7$

$g'(x) = f^{-1}(x) + x[f^{-1}{}'(x)]$

$g'(x) = 1 + x \frac{1}{f'[f^{-1}(x)]}$

$= 1 + x \frac{1}{f'(1)}$

$= 1 + 1$

$= 2$