

## Chapter 4: Producer and Consumer Surplus

**Surplus** describes the *net benefit* producers or consumers get from a transaction.

The *net benefit (NB)* is measured as the difference between the benefit and the cost of a transaction



For producers, the net benefit is the difference between the revenue (their benefit) and their cost

For consumers, the net benefit is the difference between their benefit, measured by their willingness to pay, and the price actually paid (their cost).



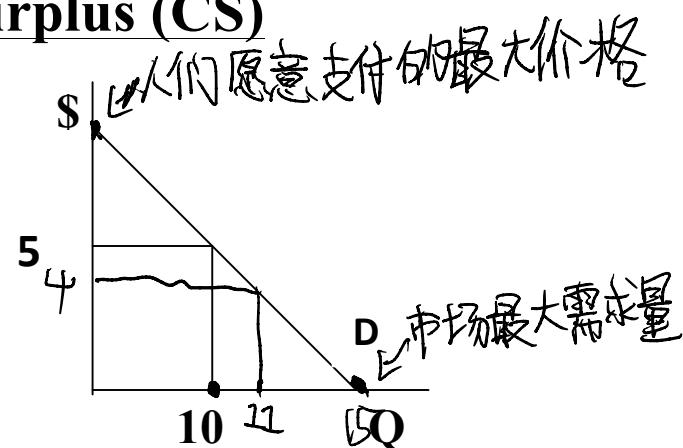
A consumer's *willingness to pay* is the maximum price they would pay for a good.  
→ the demand price.

*Marginal willingness to pay (WTP for one more unit) usually decreases as the quantity of goods purchased increases*



## Consumer Surplus (CS)

The marginal willingness to pay (MWTP) is the height of the demand curve above each quantity, or the price.



E.g. The MWTP for the 10<sup>th</sup> unit is \$5.

MWTP = WTP for one more unit

MWTP = the demand price

Consumer Surplus (CS) = the difference between what a consumer is willing to pay for a good and what he/she actually pays.

CS per unit = MWTP - price

Total CS = sum of CS for each unit

= total willingness to pay - total cost



## Example: Puppies cost \$100 each

MWTP	$Q_D$
\$400	1
\$300	2
\$200	3
\$100	4
\$0	5



A person will buy a good as long as  $MWTP \geq$  the price

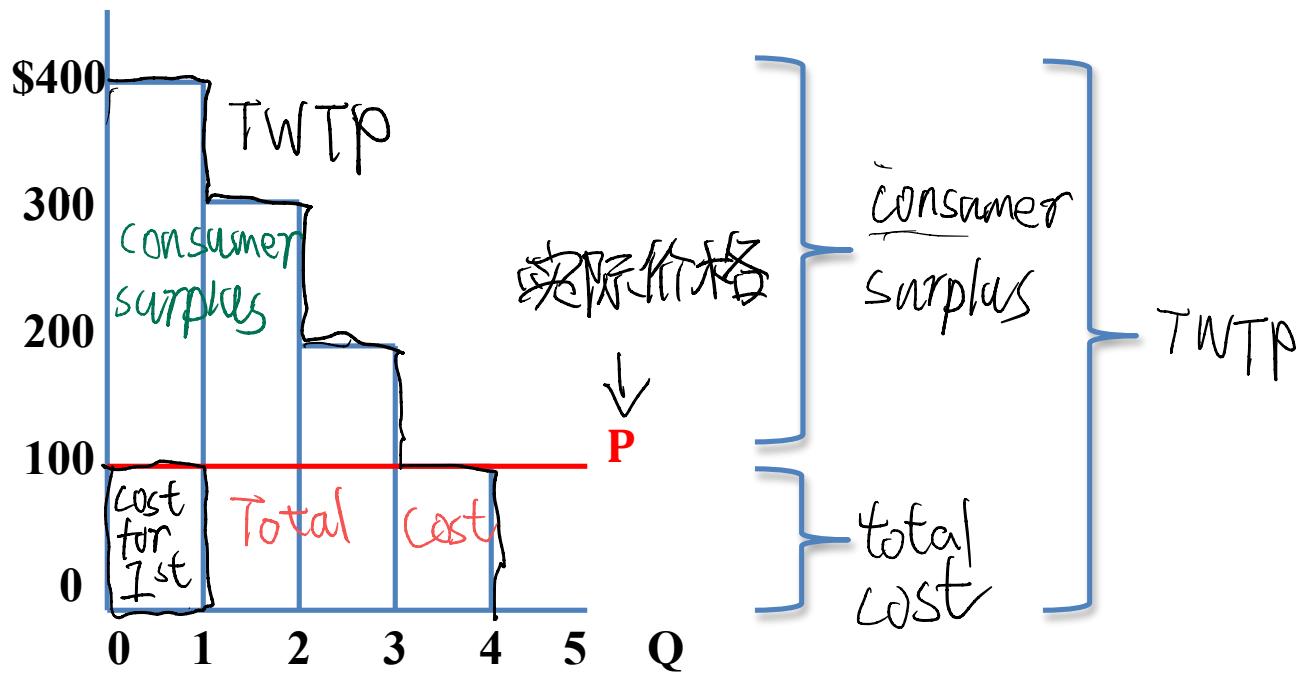
So  $Q^D = 4$  puppies

willingness to pay  $CS = (400 - 100) + (300 - 100) + (200 - 100) + (100 - 100) = 600$

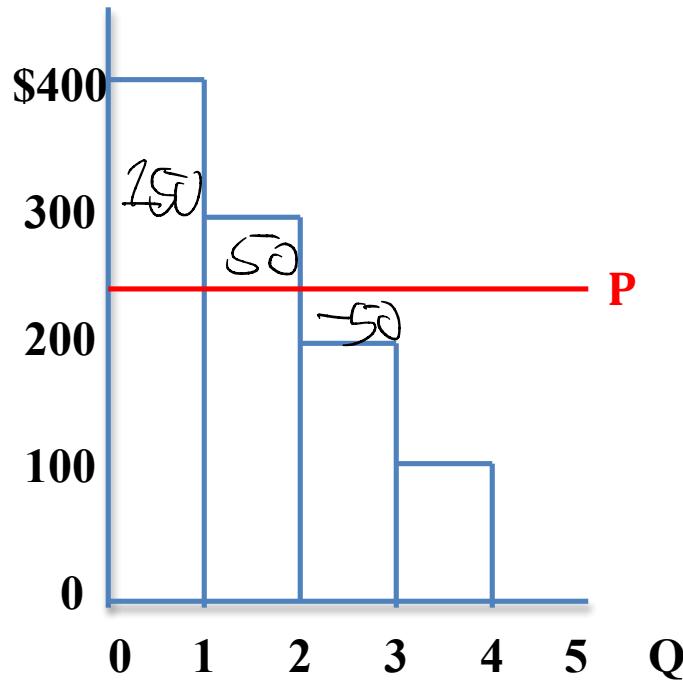
Total WTP =  $400 + 300 + 200 + 100 = 1000$

The total cost is:  $100 \cdot 4 = 400$

$TWTP - TC = \text{Net benefit} = CS = 600$



If the price is \$250  $Q^D$  will be 2 puppies

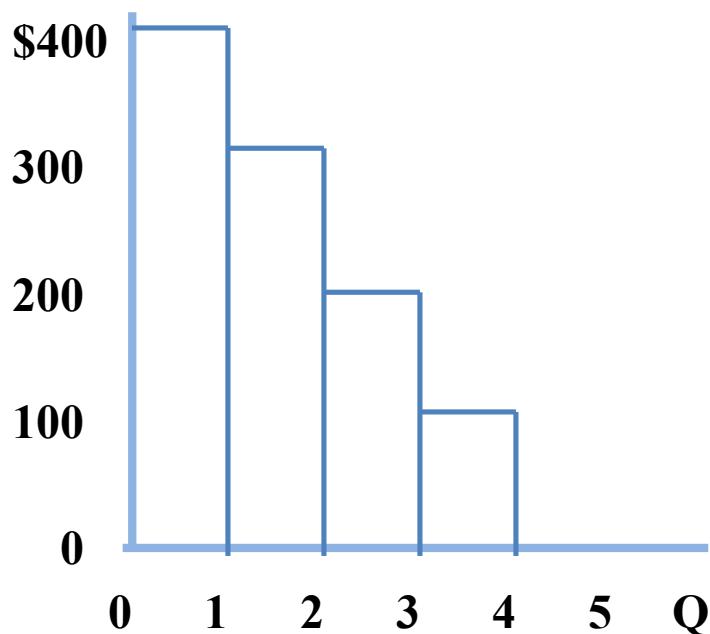


$$CS =$$

$$\begin{aligned} & \underline{150 \text{ for the 1}^{\text{st}} \text{ puppy}} \\ & + \underline{50 \text{ for the 2}^{\text{nd}} \text{ puppy}} \\ & = 200 \text{ total CS} \end{aligned}$$

What if the puppies were free?

$Q^D$  will be 5 puppies



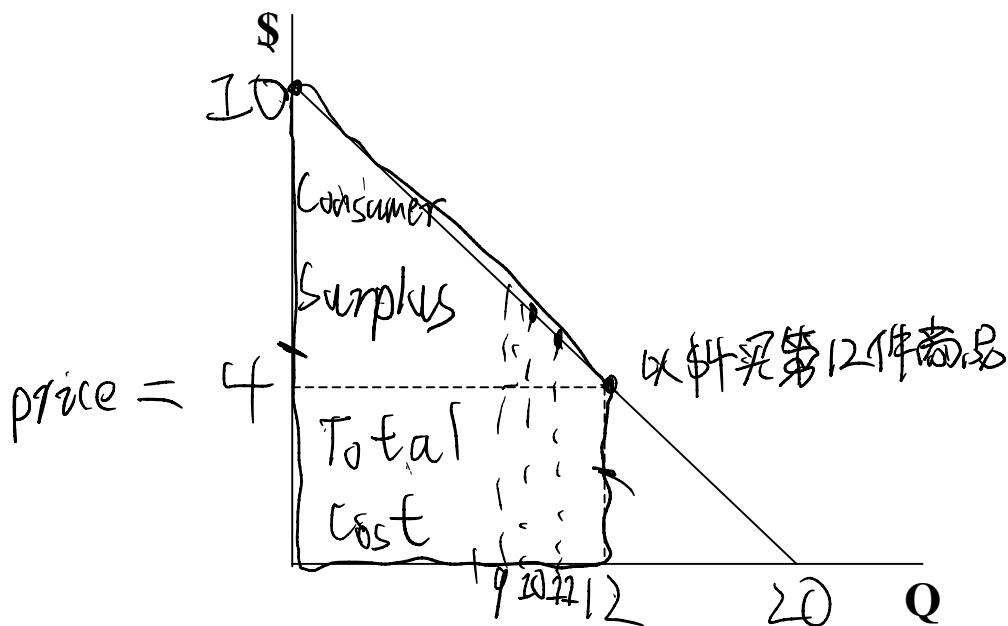
$$CS =$$

$$\begin{aligned} & \underline{400 \text{ for the 1}^{\text{st}} \text{ puppy}} \\ & + \underline{300 \text{ for the 2}^{\text{nd}} \text{ puppy}} \\ & + \underline{200 \text{ for the 3}^{\text{rd}} \text{ puppy}} \\ & + \underline{100 \text{ for the 4}^{\text{th}} \text{ puppy}} \\ & + \underline{0 \text{ for the 5}^{\text{th}} \text{ puppy}} \\ & = 1000 \text{ total CS} \end{aligned}$$

Consumer Surplus: When there is a linear *demand function* the TWTP is the area under the demand curve and the CS is the area between the demand curve and the market price.

Example: Demand function  $P = 10 - 0.5Q$

Market price  $P = \$4$



The *total willingness to pay* for 12 units is area

$$\frac{(10+4)12}{2} = 84$$

The amount paid or *total cost* for the quantity of 12 is area

$$12 \cdot 4 = 48$$

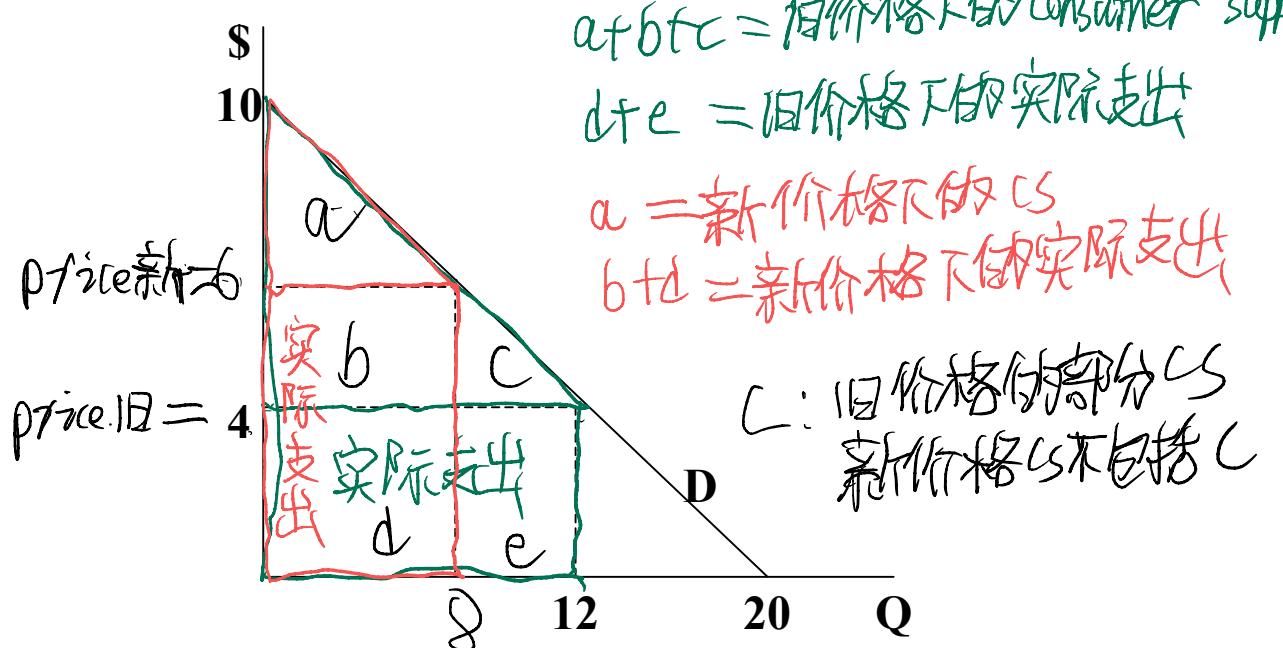
The *net benefit* or *consumer surplus* is area

$$84 - 48 = 36$$

What happens to CS if the price increases from \$4 to \$6?

There is a change in CS because the *price per unit* changes and because the *number of units* changes.

面积 = 每一个需求量上的价格的累加



$$CS_{\text{new}} = a = \frac{8 \cdot (6-4)}{2} = 16$$

$$CS_{\text{new}} - CS_{\text{old}} = 16 - \frac{12 \cdot (6-4)}{2} = 16 - 36 = -20$$

$$\Delta CS = b + c$$

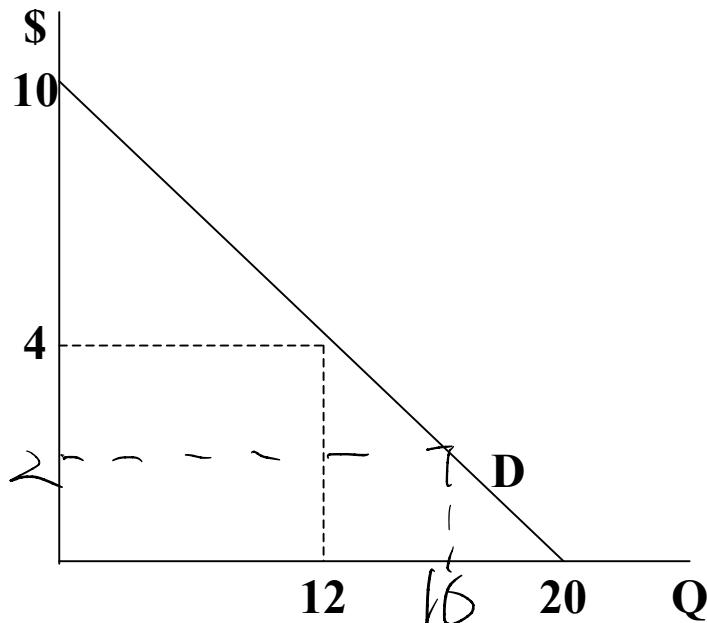
$$b = (6-4) \cdot 8 = 16$$

loss of CS to consumers who remain in the market but now pay a higher price

$$c = \frac{(6-4)(12-8)}{2} = 4$$

loss of CS to consumers who exit the market because the new price is higher than they are willing to pay

**Exercise 1: What happens to CS if the price decreases from \$4 to \$2?**



a) What is the new consumer surplus?

$$\frac{16 \cdot 8}{2} = 64$$

b) By how much has consumer surplus changed?

$$64 - 3 \cdot 12 = 64 - 36 = 28$$

c) What portion of the increase in CS is for consumers who were already in the market but now pay a lower price?

$$(4-2) \cdot 12 = 24$$

d) What portion of the increase in CS is for new consumers who enter the market because of the new lower price?

$$\frac{(4-2)(16-12)}{2} = 4$$

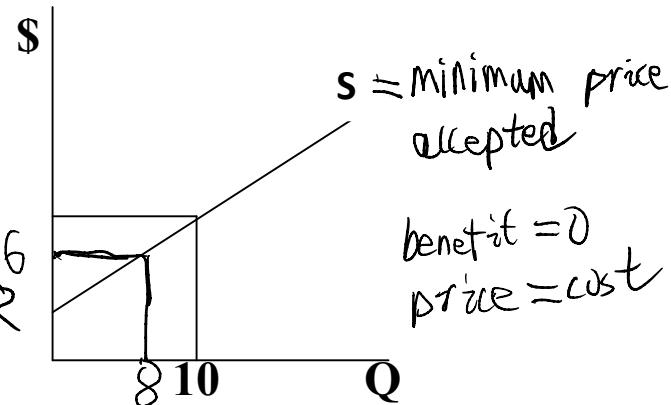
e) What would CS be if the good were free?

$$\frac{20 \cdot 10}{2} = 100$$

## Producer Surplus (PS)

The marginal cost (MC) is the height of the supply curve above each quantity.

MC = cost for one more unit



When the price is 8, produce 10

E.g. The MC for the 10<sup>th</sup> unit is \$8.

The PS or net benefit to sellers is the difference between the price they get and the minimum they would have accepted their cost or their supply price

PS per unit = price - marginal cost

Total PS = total revenue - total cost

Costs Include: Input cost for production; opportunity costs of time; opportunity cost in terms of the value of the good you're giving up when you sell something used (e.g. used textbooks).

The cost, or the minimum suppliers are willing to accept, is given by the supply curve

MC = the supply price at each unit

## Example: Luigi's Used Cars Sales

Quantity	Marginal Cost
1	1,000
2	4,000
3	7,000
4	10,000

Price = \$7500  
per car



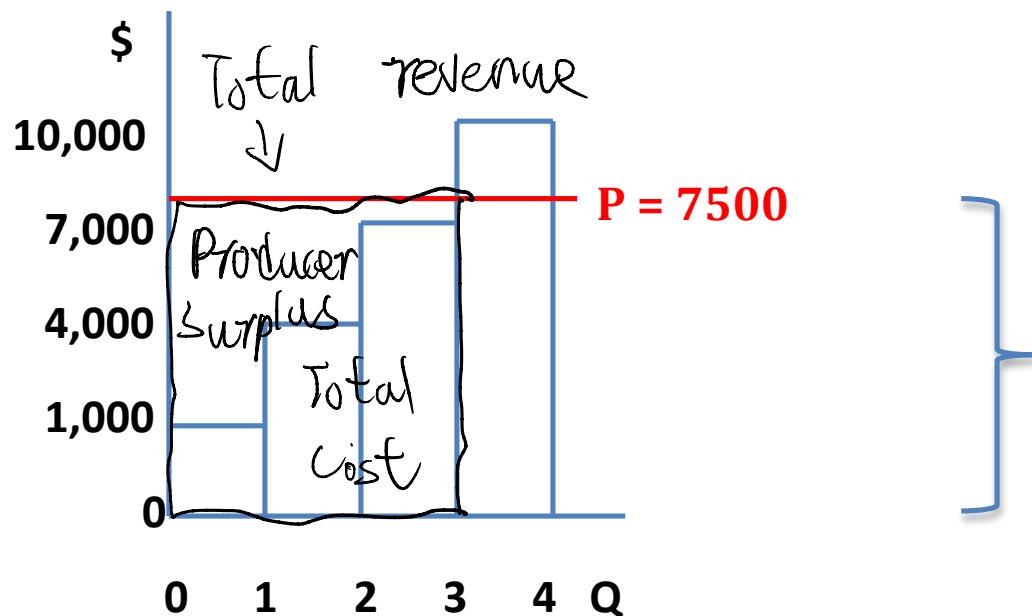
Sellers will sell a good as long as  $P \geq MC$ , so  $Q^s \leq 3$

$$PS = 6500 + 3500 + 500 = 10500$$

$$TR = 7500 \cdot 3 = 22500$$

$$\text{Total Cost } TC = 1000 + 4000 + 7000 = 12000$$

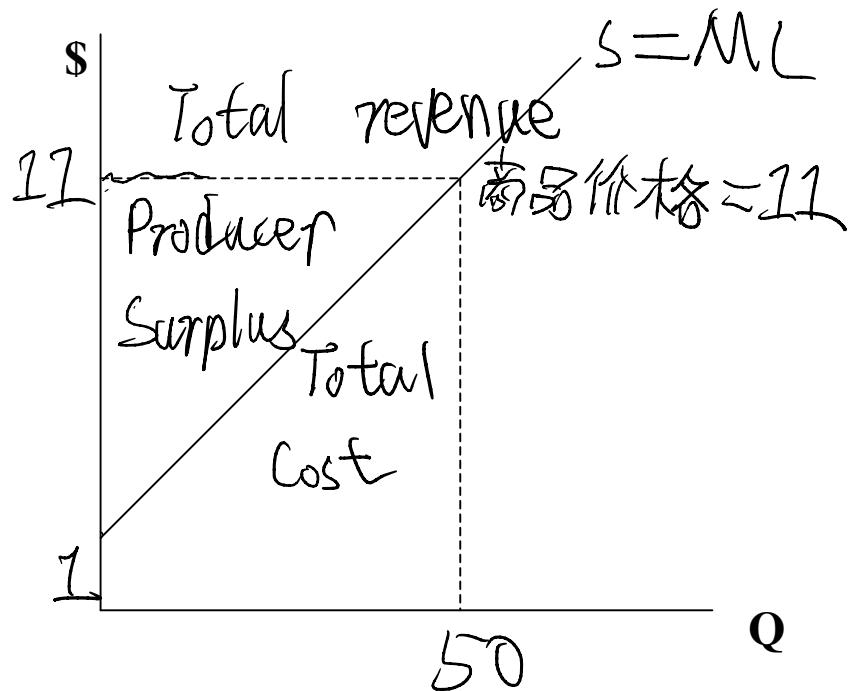
$$TR - TC = NB = PS = 22500 - 12000 = 10500$$



**Producer Surplus:** When there is a linear supply function the total cost is the area below the supply curve and the producer surplus is the area between the supply curve and the market price.

Example: Supply is  $P = 1 + 0.2Q$

Market price:  $P = \$11$



The *total revenue* is area

$$11 \cdot 50 = 550$$

The *total cost* is area

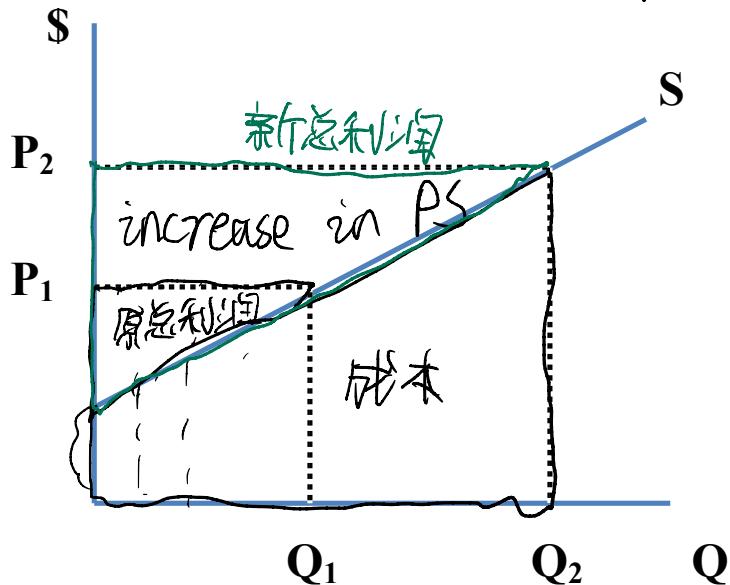
$$\frac{(1+11)50}{2} = 300$$

The *net benefit* or *producer surplus* is area

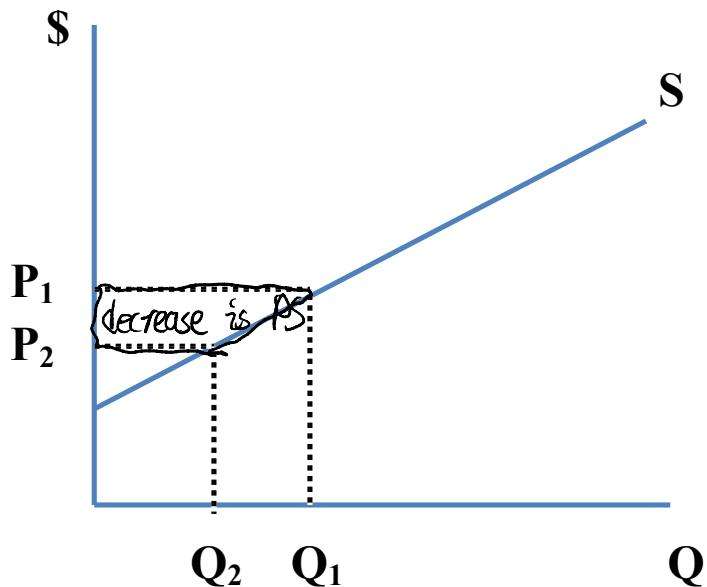
$$\frac{10 \cdot 50}{2} = 250$$

## What happens to PS when the price changes?

When the price increases the seller will get paid more per unit and choose to sell more, so PS will increase



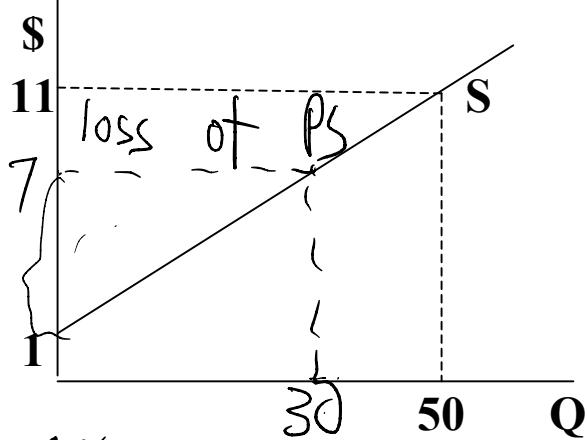
When the price decreases the seller will get paid less per unit and choose to sell less, so PS will decrease



## Exercise 2: Calculate the following changes in PS

$$\text{Supply: } P = 1 + 0.2Q$$

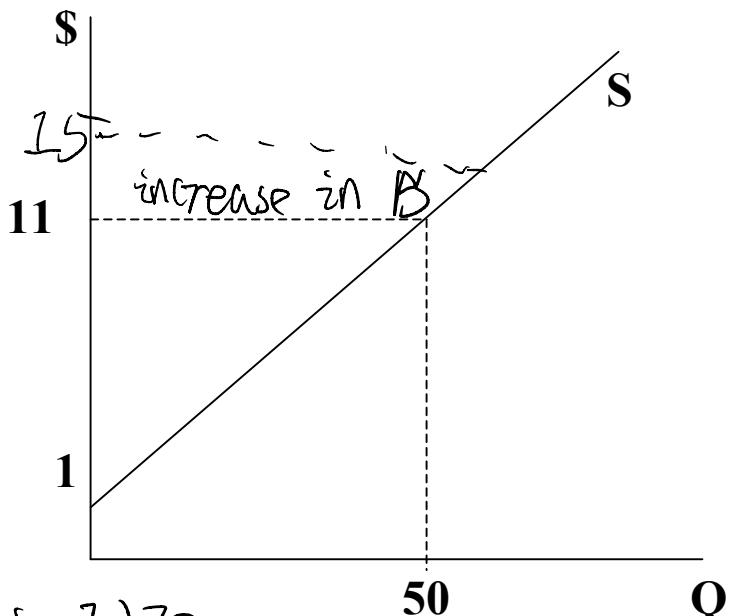
What happens to PS if price decreases from \$11 to \$7?



$$PS_{\text{new}} = \frac{(7-1)30}{2} = 90$$

$$PS_{\text{new}} - PS_{\text{old}} = 90 - 250 = -160$$

What happens to PS if price rises from \$11 to \$15?



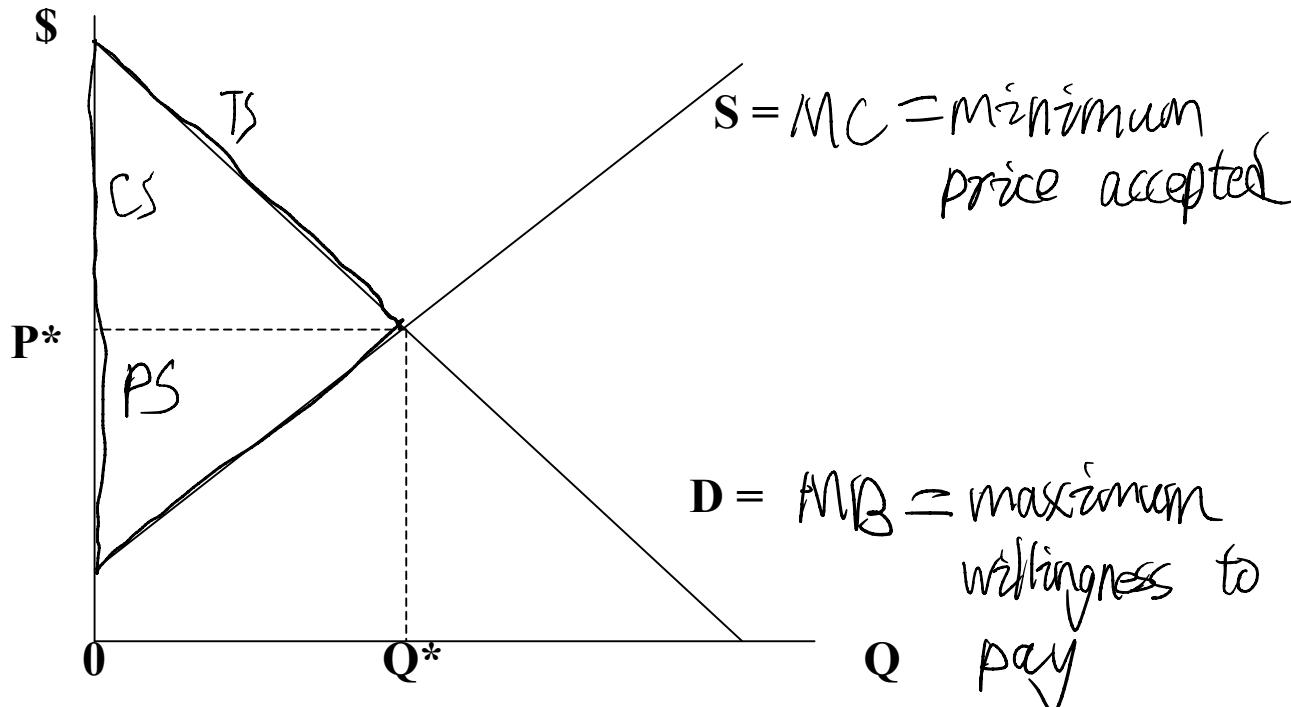
$$PS_{\text{new}} = \frac{(15-1)70}{2} = 490$$

$$PS_{\text{new}} - PS_{\text{old}} = 490 - 250 = 240$$

## Gains from Trade

Possible gains from trade exist whenever the willingness to pay by consumers is higher than the minimum price a seller is willing to accept.

### Looking at Gains from Trade on a Graph



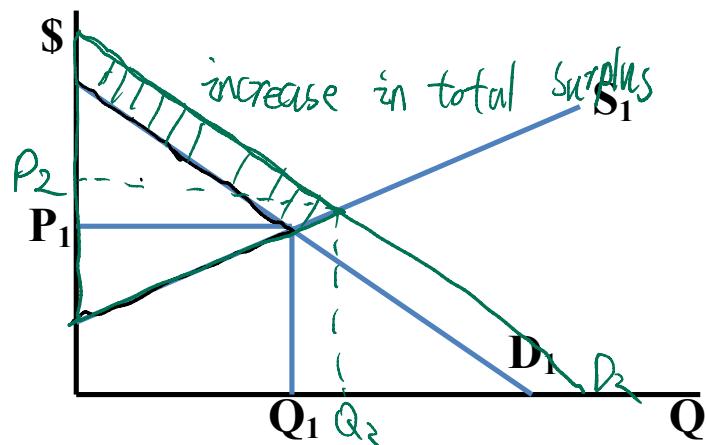
There will be gains from trade for all units for which  $D$  is higher than  $S$ , or from 0 to  $Q^*$

- Gains from trade can also be described as opportunities for buyers and sellers to make themselves better off or make mutually beneficial transactions
- Any transactions for quantities greater than  $Q^*$  will not happen because costs are higher than the benefits ( $S$  is above  $D$ ).

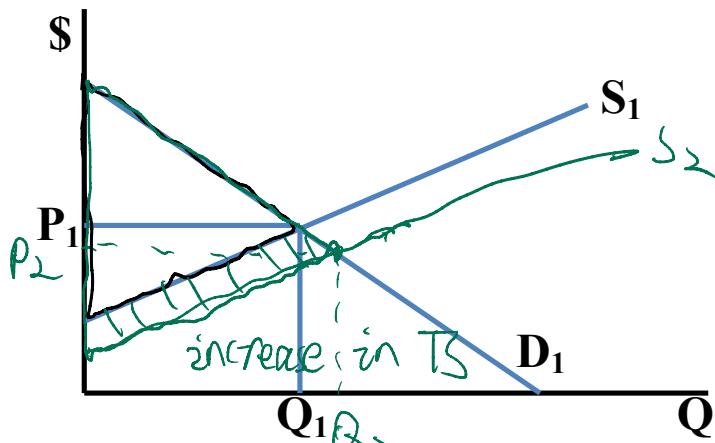
$$\text{Total Surplus} = CS + PS$$

When the curves shift there will be a change in price and quantity, and therefore a change in total surplus.

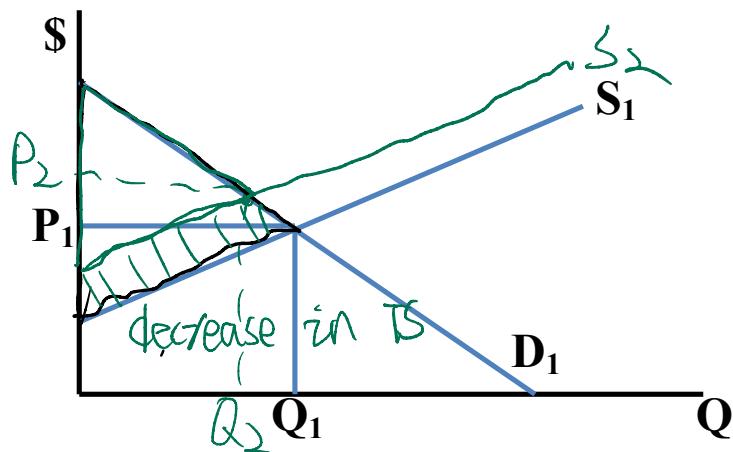
There is an increase in demand (e.g. population growth).



There is an increase in supply (e.g. lower input costs).



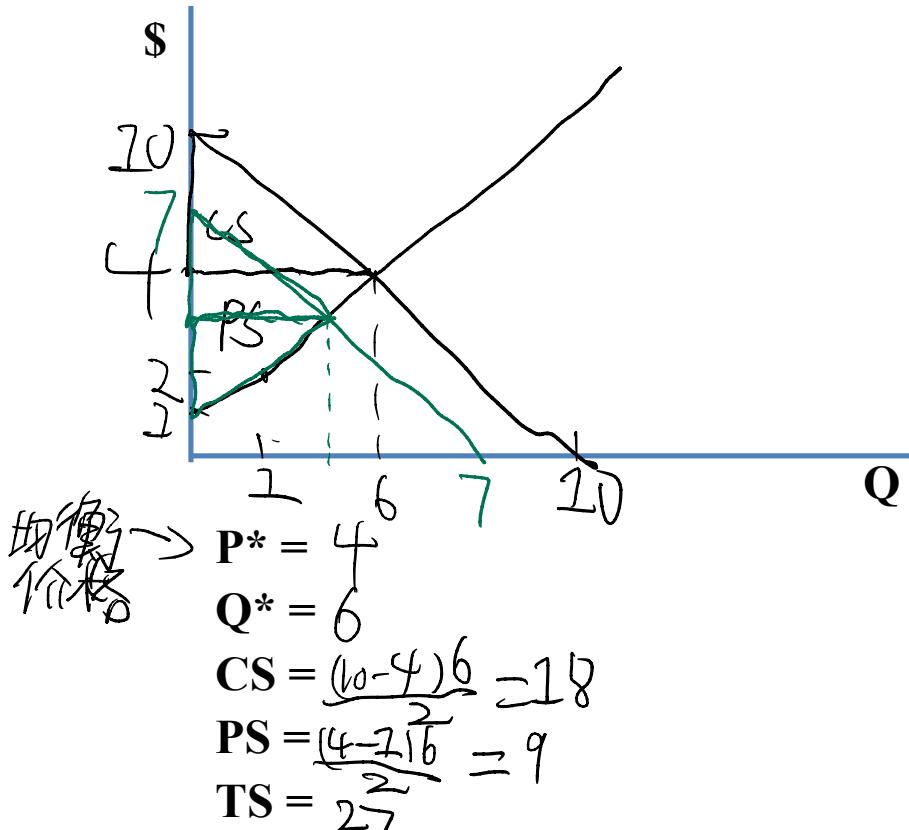
There is a decrease in supply (e.g. higher input costs).



### Exercise 3: Calculate the following changes in surplus

Suppose demand is  $P = 10 - Q$  and supply is  $1 + 0.5Q$ .

Calculate the equilibrium and the CS, PS, and TS at the equilibrium.



If demand decreases to  $P = 7 - Q$  calculate the new equilibrium and the new CS, PS, and TS.

$$P^* = 3$$

$$Q^* = 4$$

$$CS = \frac{(7-3)4}{2} = 8$$

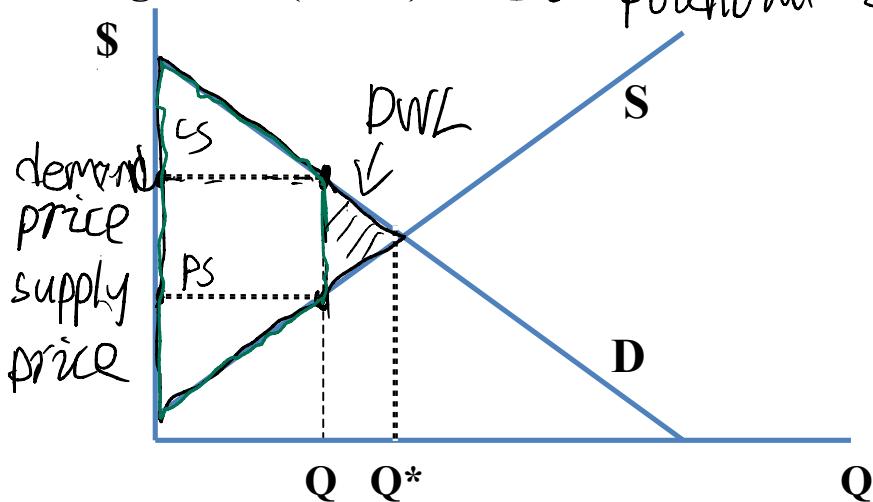
$$PS = \frac{(3-1)4}{2} = 4$$

$$TS = 12$$

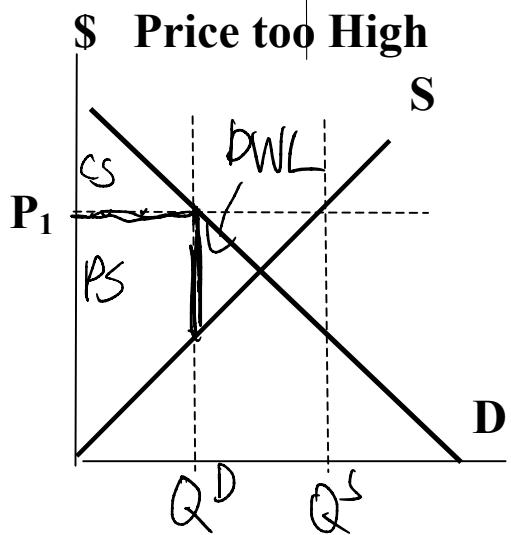
By how much has TS changed?  $\Delta TS = -15$

TS is maximized at  $Q^*$  → maximum gains from trade at  $Q^*$

- If the quantity produced is  $< Q^*$  there are missed opportunities and TS is not maximized.
- There is inefficiency and lost surplus or deadweight loss
- Deadweight loss (DWL) = lost potential surplus

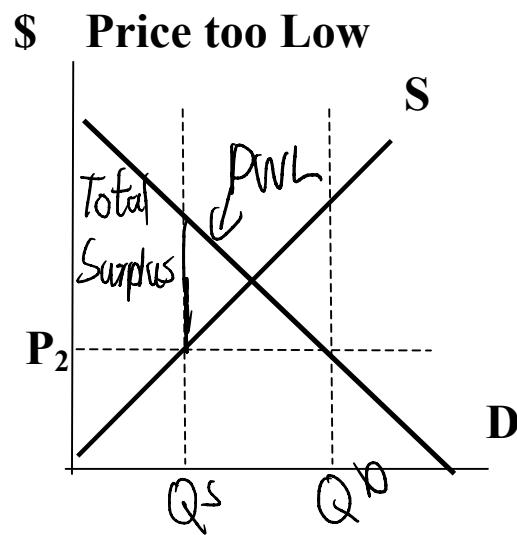


### Examples:



$$\text{Surplus} = Q_S - Q_D$$

$$\text{Amount traded} = Q_D$$



$$\text{Shortage} = Q_D - Q_S$$

$$\text{Amount traded} = Q_S$$

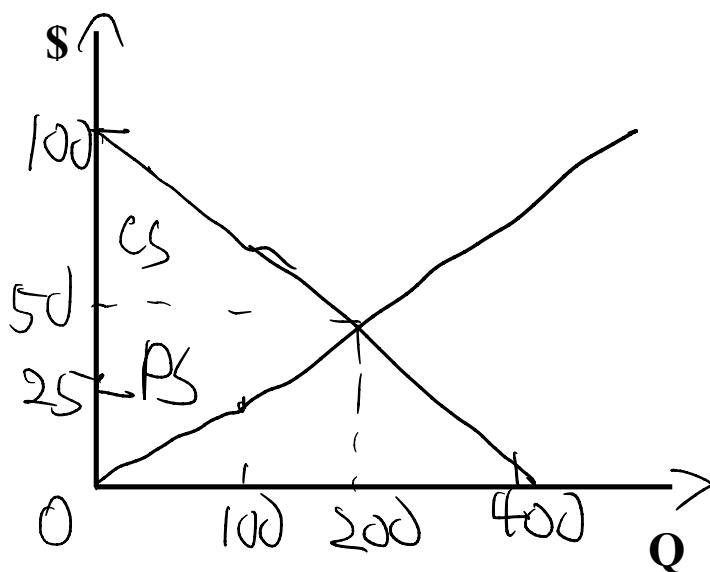
**Example:** The market has the following demand and supply curves:  $P = 100 - 0.25Q$  and  $P = 0.25Q$

Find the Equilibrium and TS at the equilibrium:

$$100 - 0.25Q = 0.25Q$$

$$Q = 200$$

$$P = 50$$

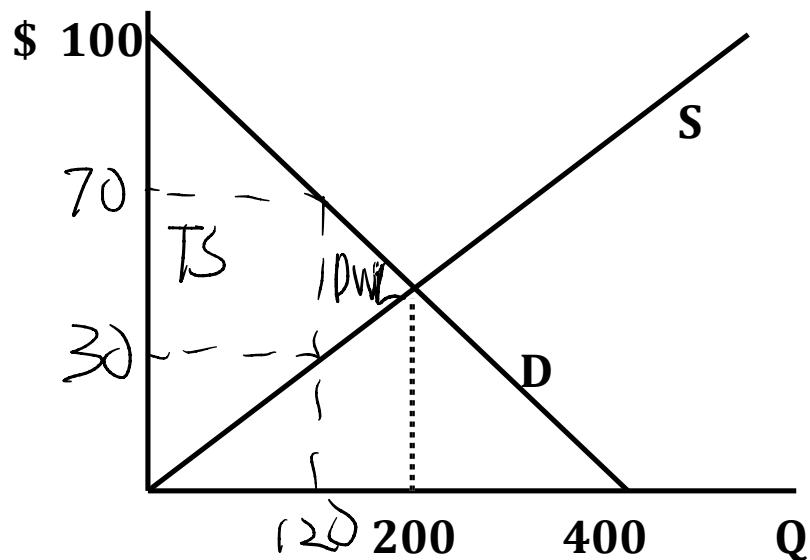


$$TS : \frac{200(100-50)}{2} + \frac{200 \cdot 50}{2} = 10000$$

Suppose that the price is limited to \$30.

Find  $Q_s$ :  $30 = 0.25Q \quad Q = 120$

Find the demand price:  $100 - 0.50(120) = 70$



What would be the *deadweight loss* and the new total surplus?

$$TS = \frac{(100+30)(120)}{2} = 8400$$

$$DWL = \frac{(70-30)(200-120)}{2} = 1600$$

## Application: Taxes

*Taxes can help create equity → provide resources to those with less or support social programs...*

*BUT they create inefficiency.*

→ Taxes discourage production and consumption by making goods and services more expensive.

Examples:

- Income taxes discourage some workers from working more.
- Sales taxes discourage spending.



Taxes interfere with the market equilibrium because they change the price.

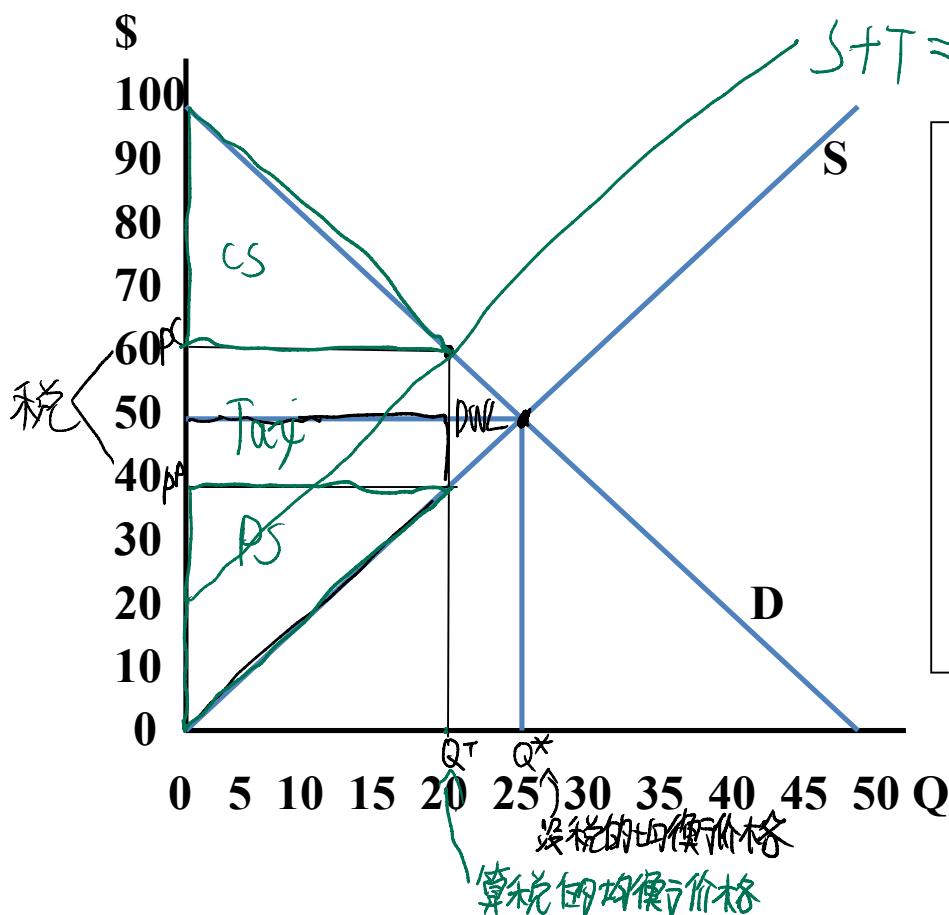
- Excise tax = tax on a specific good
  - E.g. *gas tax, liquor tax, cigarette taxes*
- Sales tax = tax on total goods
  - E.g. *PST and GST*

A tax adds to the supply price because sellers must collect the tax for the government.

Example: A Running Shoe tax of \$20 per pair



Price	QD	QS	P <sup>T</sup>
100	0	50	120
90	5	45	110
80	10	40	100
70	15	35	90
60	20	30	80
50	25	25	70
40	30	20	60
30	35	15	50
20	40	10	40
10	45	5	30
0	50	0	20



The supply curve shift up by the amount of the tax

The new equilibrium:

$$Q^T = 20$$

$$P = P^C = 60$$

Consumers pay the new equilibrium price of  $P^c = 60$   
 and purchase the quantity after tax  $Q^T = 20$   
 The suppliers keep the supply price of  $P^P = 40$   
 and the government keeps the difference (the tax) = 20  
 Total Tax Revenue in this market is:

$$(P^c - P^P)(Q^T) = (60 - 40) \cdot 20 = 400$$

### Example with equations:

Demand:  $P = 100 - 2Q$  and Supply:  $P = 2Q$ .

The government adds a tax of \$20 per pair of shoes.

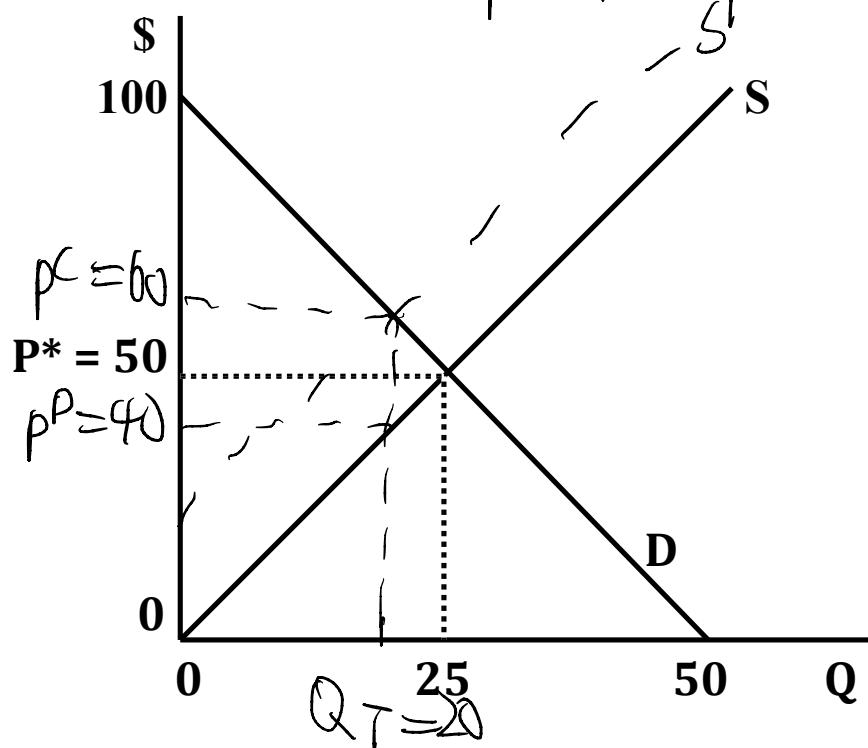
The tax is added to the supply price:

$$P = 2Q + \text{tax} \rightarrow P = 2Q + 20$$

$$\text{Solve for } Q^T: 100 - 2Q = 20 + 2Q \rightarrow Q^T = 20$$

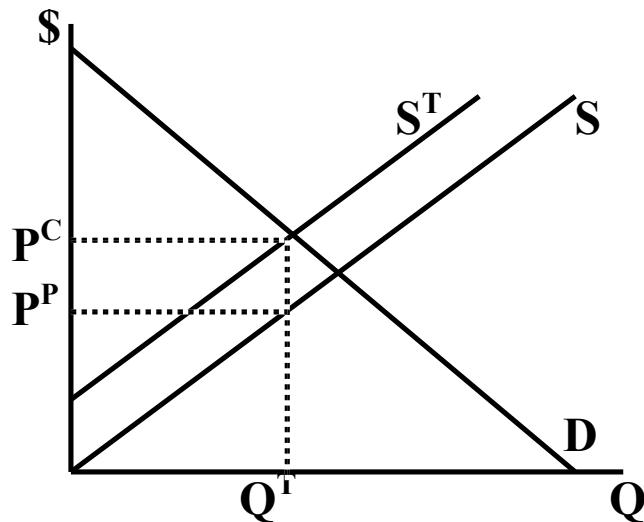
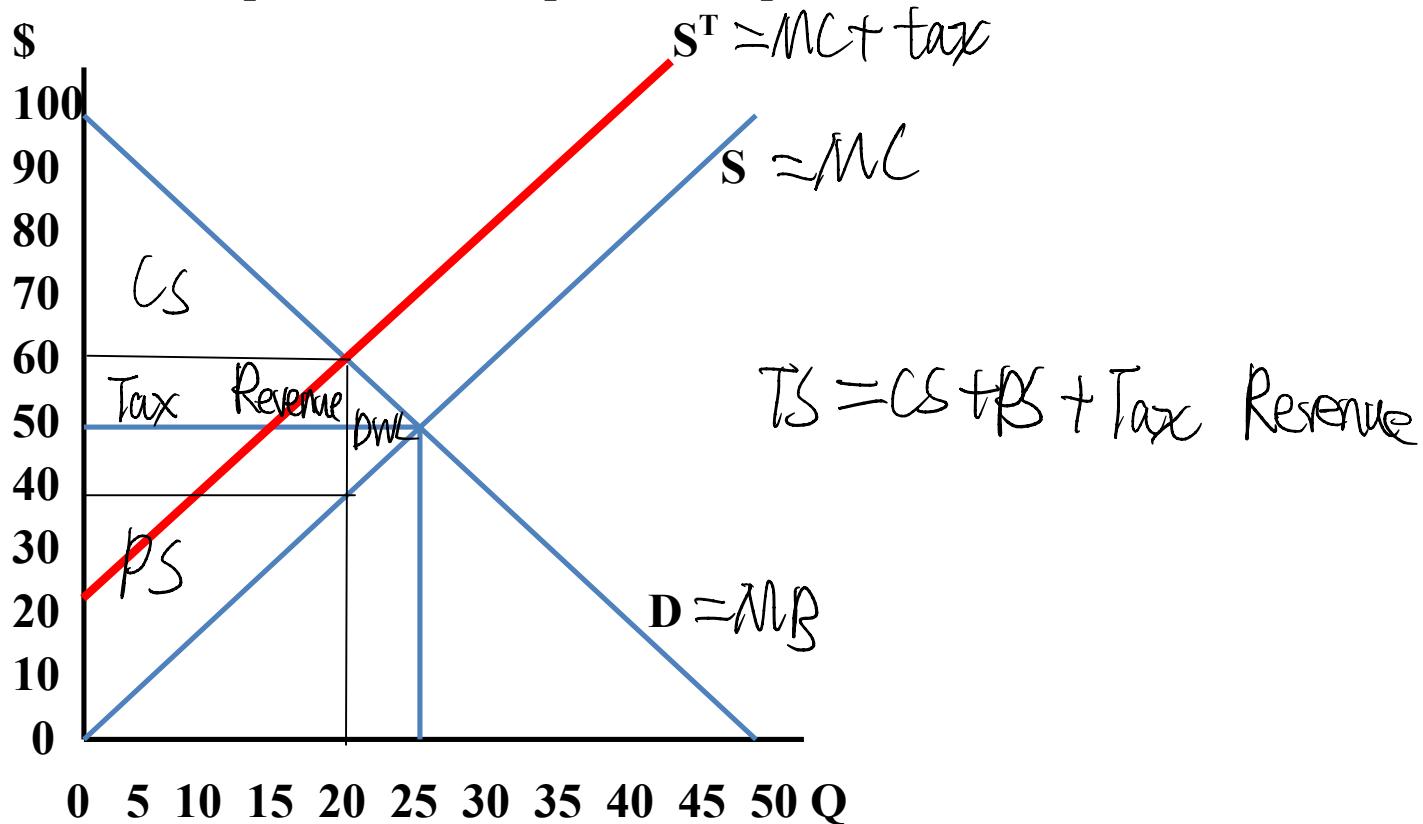
$$\text{Solve for the demand price: } P^c = 100 - 2 \cdot 20 = 60$$

$$\text{Solve for the supply price: } P^P = 2 \cdot 20 = 40$$



The tax is *inefficient* because it prevents mutually beneficial transactions.

Between  $Q^T$  and  $Q^*$  the  $MWTP > MC$  but those units are not purchased because the tax puts a wedge between the consumer price and the producer price.



The total surplus includes consumer surplus, producer surplus, and tax revenue.

## Who really pays, or bears the burden, of the tax?

Consumers are now paying \$10 more, so

**Consumer's Tax Burden (Consumer Incidence) =**

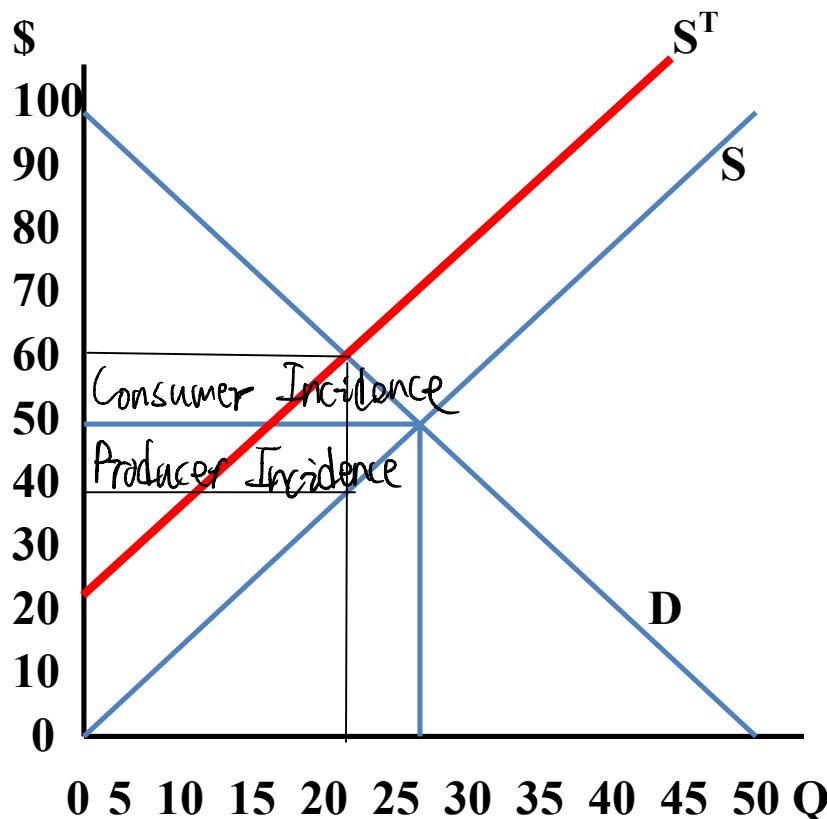
$$(P^c - P^*)Q^T = 10 \cdot 20 = 200$$

Producers are keeping \$10 less after the tax, so

**Producer's Tax Burden (Producer Incidence) =**

$$(P^* - P^P)Q^T = 10 \cdot 20 = 200$$

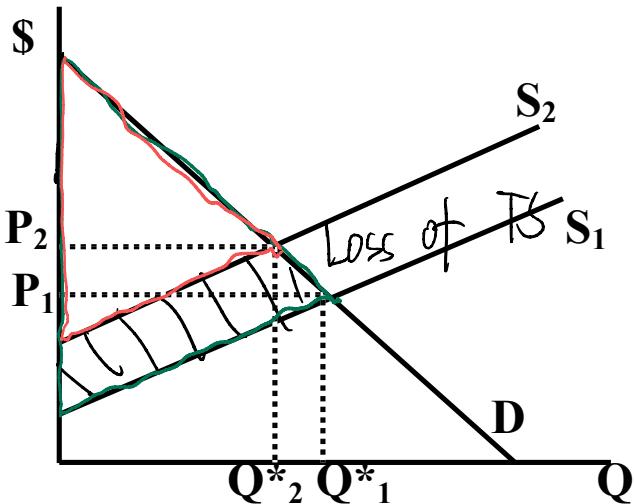
**Consumer incidence + Producer incidence = tax revenue**



## A Tax versus a Decrease in Supply

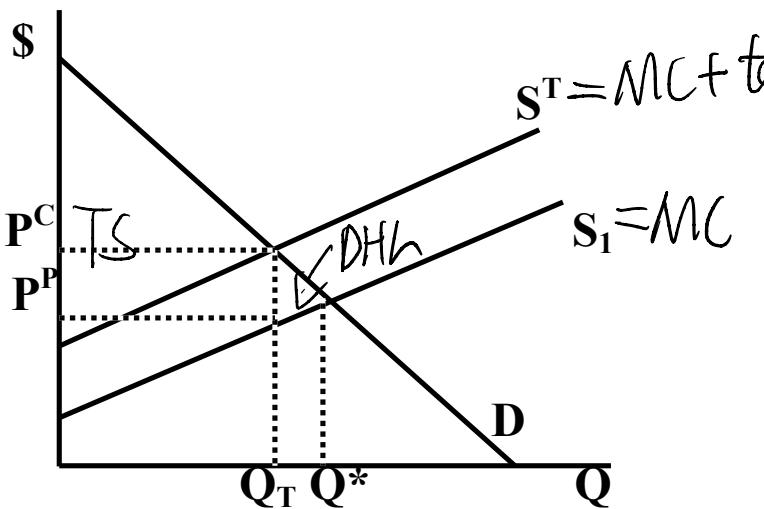
Both a tax and a decrease in supply will decrease the quantity, increase the price for consumers, and decrease total surplus. However only a tax will cause deadweight loss.

### A Decrease in Supply (e.g. the cost of labour increases)



Total surplus decreases but because the market is still at the equilibrium there is no deadweight loss.

### A Tax



Total surplus decreases because the market is no longer at the equilibrium and therefore there is deadweight loss.

## **Exercise 4: Do the following calculations**

**Demand Curve:  $P = 50 - 2.5Q$**

**Supply Curve:  $P = 10 + 1.5Q$**       **Tax = \$10**

**What is the  $S^T$  function?**

$$P = 1.5Q + 20$$

**Graph the D, S, and  $S^T$  curves. Calculate the equilibrium before the tax ( $Q^*$ ,  $P^*$ ) and the tax equilibrium ( $Q^T$ ,  $P^C$ ,  $P^P$ ).**

**Before Tax Equilibrium:**

$$\begin{aligned} 10 + 1.5Q &= 50 - 2.5Q \\ 4Q &\approx 40 \\ Q &= 10 \end{aligned}$$

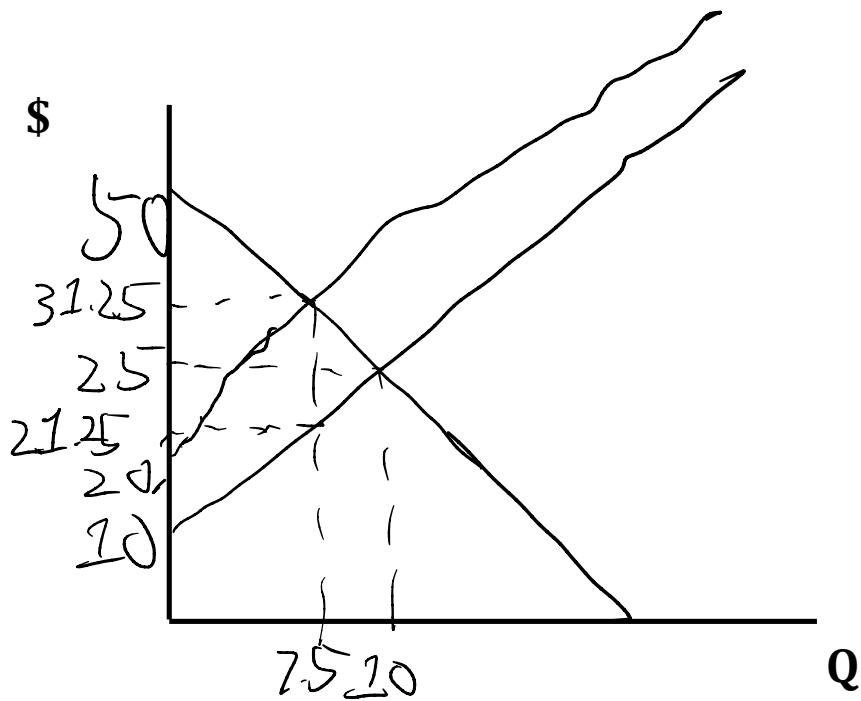
**Tax Equilibrium:**

$$50 - 2.5Q \approx 1.5Q + 20$$

$$70 \approx 4Q$$

$$Q \approx 17.5$$

$$P = 10 + 1.5 \times 17.5 = 10 + 26.25 = 27.25$$



$$CS = \frac{(50 - 31.25)7.5}{2} = 70.3125$$

$$PS = \frac{(25 - 21.25)7.5}{2} = 12.1875$$

$$\text{Tax revenue} = (31.25 - 25)7.5 = 75$$

$$\text{Total surplus} = 70.3125 + 12.1875 + 75 = 187.5$$

$$\text{Or } \frac{(40 + 5)7.5}{2}$$

$$DWL = \frac{10 \cdot (10 - 7.5)}{2} = 12.5$$

$$\text{Consumer incidence} = (31.25 - 25)7.5 = 46.875$$

$$\text{Producer incidence} = (25 - 21.25)7.5 = 31.875$$