SOLUTIONS

Name & ID

Math 157 Quiz 4 July 28,2022

$$f' = \frac{1\cdot(x^2-9) - x\cdot 2x}{(x^2-9)^2} = \frac{-9-x^2}{(x^2-9)^2} = -\frac{9+x^2}{(x^2-9)^2} < 0$$

1. **Consider the function $f(x) = \frac{x}{x^2-9}$.**

$$f''(x) = -\left[\frac{2x(x^2-9)^2 - (9+x^2)\cdot 2(x^2-9)\cdot 2x}{(x^2-9)^4}\right]$$

a)Find the intervals where the function is increasing or decreasing and the relative extrema.

$$= \frac{-2x(x^2-9) + 4x(9+x^2)}{(x^2-9)^3}$$

b)Find the intervals where the function is concave upward or concave downward , and the inflection points.

$$= \frac{-2x^3 + 18x + 36x + 4x^3}{(x^2-9)^3}$$

c)Graph the function .

$$= \frac{2x^3 + 54x}{(x^2-9)^3} = \frac{2x(x^2+27)}{(x^2-9)^3}$$

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a) $f'(x) < 0$ for the D_f: $D_f = (-\infty,-3) \cup (-3,3) \cup (3,\infty)$

$\therefore$ f decreasing $(-\infty,-3) \cup (-3,3) \cup (3,\infty)$

No critical numbers and therefore no relative extrema.

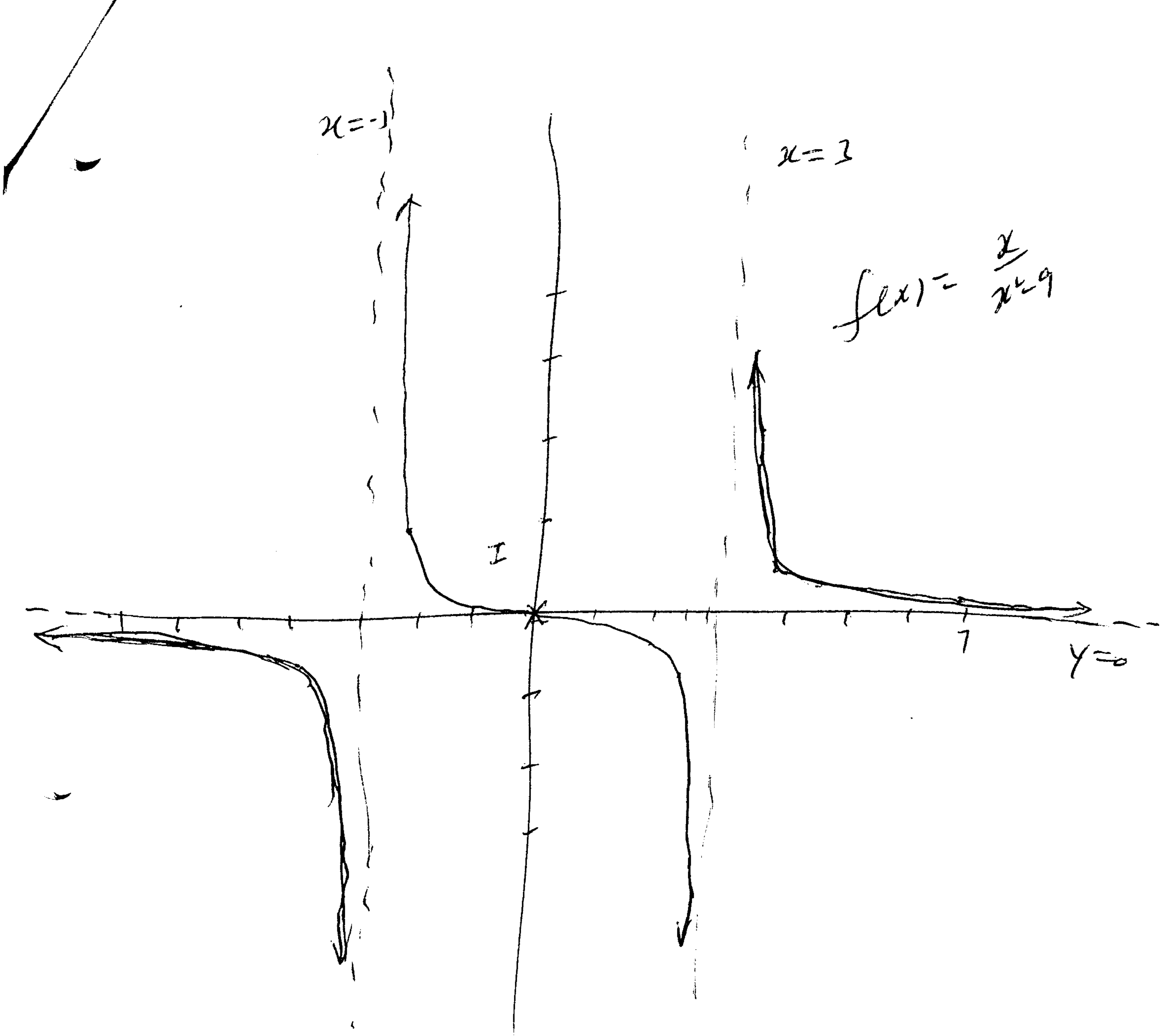
b) $f'' = 0 \Rightarrow x = 0$ possible inflection point $(0,0)$

I	$(-\infty,-3)$	$(-3,0)$	$(0,3)$	$(3,\infty)$
z	-10	-1	1	10
f''	$-$ve	$+$ve	$-$ve	$+$ve
f	$\cap$	$\cup$	$\cap$	$\cup$

I

Inflection pt.

we note
H.A. $y=0$
V.A $x=\pm 3$
$f(-x) = -f(x)$ odd
symmetric about the origin.



x	$f(x)=\frac{x}{x^2-9}$
-1	0.125
-2	0.4
-2.5	0.90..
4	0.57
7	0.175

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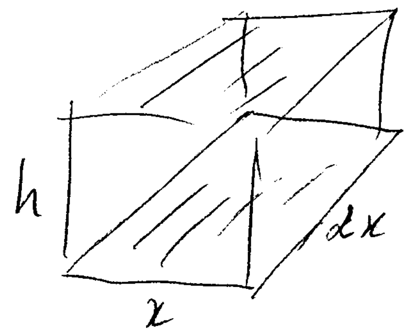
2. A company needs to design a metal box container with a volume of 128 m^3 and that is twice as long as it is wide. The top and bottom will be made of a sturdy material that costs \$ 12 /m^2, while the material for the sides costs 8/m^2. Find the dimensions and cost of the least expensive container. [8 marks]



$$\text{Volume} = 128 = x \cdot 2x \cdot h$$

$$\therefore \quad h = \frac{64}{x^2}$$

$$\text{total Cost} = 2(2x^2)(12) + 8\left[2hx + 2 \cdot 2x h\right]$$

$$= 48x^2 + 48hx$$

$$C(x) = 48x^2 + \frac{(48)(64)}{x} = 48\left[x^2 + \frac{64}{x}\right], \quad x > 0$$

$$C'(x) = 48\left(2x - \frac{64}{x^2}\right) \quad ; \quad C' = 0 \Rightarrow x = 32^{1/3} \approx 3.17480$$

$$h = \frac{(2)(32)}{32^{2/3}} = (2)(4)^{1/3}$$

$$\approx 6.34960$$

$$C''(x) = 48\left(2 + \frac{128}{x^3}\right) > 0$$

Since $C''(32^{1/3}) > 0$, by SDT rel. min, and by CPT the absolute minimum.

$\therefore$ For the least expensive container we have $(32)^{1/3} \times 2(32)^{1/3} \times 2(32)^{1/3}$

and this minimum cost $= (144)(32)^{2/3} \approx \$1451.43$