$$\frac{x}{\sqrt{x}} \quad \frac{x^2}{x} \quad x$$

3.5.1(a, f, h, p, t), 3.5.2, 3.5.3, 3.5.5, 3.5.6 (a–c), 3.5.8, 3.7.1(a & d)
3.7.2, 3.7.5, 3.7.7, 3.7.9

3.5.1(a) $\lim_{x\to\infty} \left(\sqrt{x^2+x} - \sqrt{x^2-x}\right)$

$$= \frac{\left(\sqrt{x^2+x} - \sqrt{x^2-x}\right)\left(\sqrt{x^2+x} + \sqrt{x^2-x}\right)}{\left(\sqrt{x^2+x} + \sqrt{x^2-x}\right)}$$

$$= \frac{(x^2+x) - (x^2-x)}{\left(\sqrt{x^2+x} + \sqrt{x^2-x}\right)}$$

$$= \frac{2x}{\sqrt{x^2+x} + \sqrt{x^2-x}}$$

$$= \frac{2}{\sqrt{1+\frac{1}{x}} + \sqrt{1-\frac{1}{x}}}$$

$$= \frac{2}{\sqrt{1+0} + \sqrt{1-0}}$$

$$= \frac{2}{2}$$

$$= 1$$

(f) $\lim_{x\to 0^+} \frac{3 + x^{-\frac{1}{2}} + x^{-1}}{2 + 4x^{-\frac{1}{2}}}$

$$= \lim_{x\to 0^+} \frac{3 + \frac{1}{\sqrt{x}} + \frac{1}{x}}{2 + \frac{4}{\sqrt{x}}}$$

$$= \lim_{x\to 0^+} \frac{3\sqrt{x} + 1 + \frac{1}{\sqrt{x}}}{2\sqrt{x} + 4}$$

Does not exist.

$$= \lim_{x\to 0^+} \frac{3\sqrt{x} + 1 + \frac{1}{\sqrt{x}}}{2\sqrt{x} + 4} = \frac{0 + 1 + \infty}{0 + 4} = \infty$$

$$= +\infty$$

$$\cancel{= \lim_{x\to 0^+} \frac{3x + \sqrt{x} + 1}{2x + 4\sqrt{x}}}$$

(h) $\lim_{t\to\infty} \frac{1 - \sqrt{\frac{t}{t+1}}}{2 - \sqrt{\frac{4t+1}{t+2}}}$

$$= \frac{\left(1 - \sqrt{\frac{t}{t+1}}\right)\left(1 + \sqrt{\frac{t}{t+1}}\right)\left(2 + \sqrt{\frac{4t+1}{t+2}}\right)}{\left(2 - \sqrt{\frac{4t+1}{t+2}}\right)\left(2 + \sqrt{\frac{4t+1}{t+2}}\right)\left(1 + \sqrt{\frac{t}{t+1}}\right)}$$

$$= \frac{\left(1 - \frac{t}{t+1}\right)\left(2 + \sqrt{\frac{4t+1}{t+2}}\right)}{\left(4 - \frac{4t+1}{t+2}\right)\left(1 + \sqrt{\frac{t}{t+1}}\right)}$$

$$= \frac{4 - \frac{4t}{t+1}}{8 - \frac{8t+1}{t+2}}$$

$$= \frac{2 - \frac{2t}{t+1}}{4 - \frac{4t+1}{t+2}}$$

$$= \frac{\left(2 - \frac{2t}{t+1}\right)(t+1)(t+2)}{\left(4 - \frac{4t+1}{t+2}\right)(t+1)(t+2)}$$

$$= \frac{(\cancel{2t} + 2 - \cancel{2t})(t+2)}{(\cancel{4t} + 8 - \cancel{4t} - 1)(t+1)}$$

$$= \frac{2(t+2)}{7(t+1)}$$

$$= \frac{2t+4}{7t+7}$$

$$= \frac{2}{7}$$

(p) $\lim_{x \to \infty} (x+5)\left(\frac{1}{2x} + \frac{1}{x+2}\right)$

$= \lim_{x\to\infty} \left(\frac{x}{2x} + \frac{5}{2x} + \frac{5}{x+2} + \frac{x}{x+2}\right)$

$= \lim_{x\to\infty} \left(\frac{1}{2} + \frac{5}{2x} + \frac{5}{x+2} + \frac{x}{x+2}\right)$

$= \lim_{x\to\infty} \frac{1}{2} + \lim_{x\to\infty} \frac{5}{2x} + \lim_{x\to\infty} \frac{5}{x+2} + \lim_{x\to\infty} \frac{x}{x+2}$

$= \frac{1}{2} + 0 + 0 + 1$

$= \frac{3}{2}$

(t) $\lim_{x\to -\infty} \frac{x^4+1}{x^3-1}$

$= -\infty$

$y = kx + b$

$k = \lim_{x\to\infty} \frac{f(x)}{x}$

$b = \lim_{x\to\infty} [f(x) - kx]$

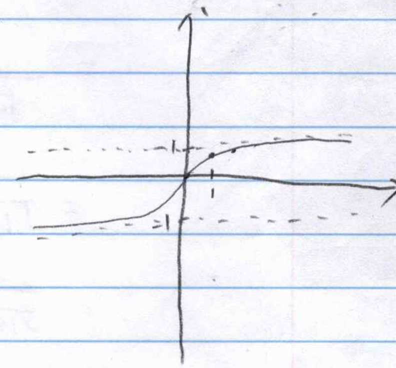
3.5.2

$\lim_{x\to\infty} f(x) = \frac{x}{\sqrt{x^2+1}}$

$\lim_{x\to+\infty} f(x) = \sqrt{x^2+1}$ $\frac{x}{\sqrt{x^2+1}}$ $\qquad \lim_{x\to-\infty} f(x) = \frac{x}{\sqrt{x^2+1}}$

$= 1 \qquad\qquad = -1$

$\therefore$ H.A. $= 1$ & -1



3.5.3

$f(x) = \frac{\ln x}{x-2} \qquad \ln x > 0$

$x \neq 2$

$\therefore$ V.A $= 2$ & 0

3.5.5

$f(x) = \frac{x^2+x+6}{x-3}$

$y = kx + b$

$k = \lim_{x\to\infty} \frac{\frac{x^2+x+6}{x-3}}{x}$

$k = \lim_{x\to\infty} \frac{x^2+x+6}{x^2-3x}$

$k = 1$

$b = \lim_{x\to\infty} \left[\frac{x^2+x+6}{x-3} - x\right]$

$= \lim_{x\to\infty} \left[\frac{x^2+x+6}{x-3} - \frac{x^2-3x}{x-3}\right]$

$= \lim_{x\to\infty} \left[\frac{\cancel{x^2}+x+6-\cancel{x^2}+3x}{x-3}\right]$

$= \lim_{x\to\infty} \left[\frac{4x+6}{x-3}\right]$

$b = 4$

$\therefore y = x + 4$

$\therefore$ slant asymptote is $y = x + 4$.

$\tan\frac{\pi}{2} = \infty$

$\tan^{-1}\infty = \frac{\pi}{2}$?

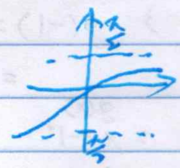
3.5.6

(a) $\lim_{x\to-\infty}(2x^3-x)$
$= \lim_{x\to-\infty} x(2x^2-1)$
$= -\infty$

(b) $\lim_{x\to\infty} \tan^{-1}(e^x)$?

$\because x\to\infty$ if $t=e^x$

$e^x\to\infty$ $\therefore \lim_{t\to\infty}\tan^{-1}t = \frac{\pi}{2}$



(c) $\lim_{x\to-\infty} \tan^{-1}(e^x)$
$= \lim_{x\to-\infty} \frac{1}{\tan e^x}$

$\lim_{x\to-\infty} e^x = \frac{1}{e^{+\infty}}$

$= +\infty$

$\because x\to-\infty$

$\therefore e^x\to 0$

if $t=e^x$

$\lim_{x\to\infty}\tan^{-1}(e^x)=0$

$\lim_{t\to 0}\tan^{-1}t = 0$

3.5.8

(a) $T(1) = \frac{120}{1+4}$
$= \frac{120}{5}$
$= 24$

$T(2) = \frac{120\cdot 2^2}{2^2+4}$
$= 60$

$T(3) = \frac{120\cdot 3^2}{3^2+4}$
$= 83.0769$

(b) $\lim_{x\to+\infty} = \frac{120x^2}{x^2+4}$
$= 120$.

3.7.1

(a)
when to x go to 0 in the two condition and the $f(x)$ is not same, thus it is discontinuous.

(b) continuous.
because the same value.

(c) discontinuous.
It don't have same value when x go to 0.

(d) discontinuous.
It don't have same value.

3.7.2

(a) $f(s) = \frac{2}{s^2+1}$
It is continuous.

(b) $g(t) = \frac{2t+1}{(t-1)(t+2)}$ $\therefore t\neq 1$, $t\neq -2$

$\therefore t=1$, $t=-2$, It is discontinuous.

g continuous for all t except $t=-2$ and 1 (c). $h(u) = \begin{cases} \frac{u^2-1}{u-1} & u \neq 1 \\ 2 & u=1 \end{cases}$

It is continuous.

$\frac{u^2-1}{u-1} = \frac{(u-1)(u+1)}{(u-1)}$

$= u+1$

$h(u) = \begin{cases} u+1 & u \neq 1 \\ 2 & u=1 \end{cases}$

$\lim_{u \to 1^+} h(u) = 2$

$\lim_{u \to 1^-} h(x) = 2$

3.1.5.

$\lim_{x \to -1^-} g(x) \; 2-2c^2 x$

$= \lim_{x \to -1^-} g(x) \; 2-2c^2(-1)$

$= 2+2c^2$

$g(-1) = 6-7cx^2$

$= 6-7c$

$= \lim_{x \to -1^+} g(x) \; 6-7cx^2$

$= 6-7c$

$2+2c^2 = 6-7c$

$2c^2+7c-4 = 0$

$0 = (2c-1)(c+4)$

$\therefore c = \frac{1}{2}$ or $c=-4$

f is continuous at $x=a$.

1. $f(a)$ is exist.

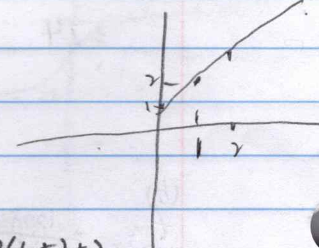
2. $\lim_{x \to a} f(x)$ exist.

3. $\lim_{x \to a} f(x) = f(a)$.

$\frac{u^2-1}{u-1} \quad u \neq 1$

$\frac{\cancel{(u-1)}(u+1)}{\cancel{u-1}} \quad u \neq 1$

$\cancel{u+1} \quad u \neq 1$



3.1.7

$f = x^3-4x^2+2x+2$

If $x=1$

$f(1) = 1-4+2+2$

$= 1$

If $x=2$

$f(2) = 2^3-4(2)^2+2(2)+2$

$= -2$

$f(1) > 0 > f(2)$

$f(1.5) = (1.5)^3-4(1.5)^2+2(1.5)+2$

$= -0.625.$

$f(1.4) = (1.4)^3-4(1.4)^2+2(1.4)+2$

$= -0.296$

$f(1.3) = (1.3)^3-4(1.3)^2+2(1.3)+2$

$= 0.037$

$f(1.3) > 0 > f(1.4)$

$f(1.35) = (1.35)^3-4(1.35)^2+2(1.35)+2$

$= 0.129625$

$f(1.34) = (1.34)^3-4(1.34)^2+2(1.34)+2$

$= 0.096296$

$f(1.33) \doteq (1.33)^3-4(1.33)^2+2(1.33)+2$

$= -0.062963$

$f(1.32) = (1.32)^3-4(1.32)^2+2(1.32)+2$

$= -0.029632.$

$\therefore$ the root is 1.3. $\quad f(1.31) = (1.31)^3-4(1.31)^2+2(1.31)+2$

$= 0.03691 \qquad f(1.31) > 0 > f(1.32)$

3.7.9

$\sqrt[3]{x}+x=1$

establish $y=\sqrt[3]{x}+x-1$

when $x=0$.

$y=-1<0$.

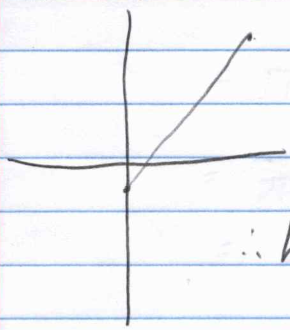
when $x=8$

$y=\sqrt[3]{8}+8-1$

$=2+8-1$

$=9>0$

So have a "a" in $(0,8)$.

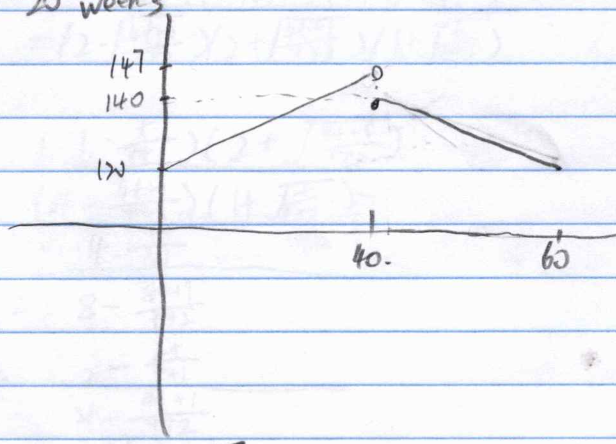


$\therefore$ have a solution in $(0,8)$

additional question

120-lb woman. + 27 lb during the pregnancy delivers 7-lb baby.

(a)(i) 20 weeks



(ii) It is uncontinuous,

$t=40$.

(B) $f(x)=e^{\sqrt{x-9}}$

$D_{f(x)}=x>9$

$\therefore$ when $x>9$, it is a continuous fuction.

$D_{f(x)}=\{x\neq 3,\ \frac{x+2}{x-3}\neq 0$

$\therefore x\neq -2$.

$D_{f(x)}=\{x\neq 3,\ x\neq -2\}$

$\therefore$ when the $x\neq 3$, $x\neq 2$, It is uncontinuous.

$x=3$ $x=2$

(c)

(i) ✓

(ii) ✓

(iii) x It can not ~~close~~ be the point of limit.

(iv) x It is not connect.

(v) x polynomial function is not continuous.