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Fraser International College
Test 2 Version A
Math 157
July 21, 2022
Time 70 Minutes
Instructor: Dr. N. Tariq

- **Please ensure that you sign your exam above to certify your identity. Unsigned exams will not be marked.**
- **Use only calculators that do not have any graphing, differentiation or integration capabilities.**
- **The duration of the exam is 70 minutes.**
- **DO NOT OPEN this test booklet until you are told to do so.**
- **Please check that you have all 5 questions of the exam.**
- **Do ALL your work in this test booklet. You may use the backside of each page for scrap work.**
- **The value of each question is shown at the end of each question.**

Question	Score	Maximum
1	9	10
2	6	6
3	7	8
4	6	6
5	14	15
Total	42	45

CRIM101
STAT S203
CA135 ~~203~~
PSYC280
PSYC109

1. Differentiate the following functions as indicated. [10 marks]

a) $f(x) = sin(3x) + \tan(2x) + e^x + ln|1-2x|, find\ f'(0).$

$f'(x) = 3\cos(3x) + 2\sec^2(2x) + e^x + \frac{-2}{1-2x}$

$f'(0) = 3 + 2 + 1 + \frac{-2}{1}$

$f'(0) = 3 + \cancel{2} + 1 - \cancel{2}$

$= 4$

5

b) $f(x) = (\sin \pi x + \cos \pi x)^{(2x+1)}$, find $f'(0)$.

$\ln f(x) = (2x+1)\ln(\sin\pi x + \cos\pi x)$

$\frac{f'(x)}{f(x)} = 2\ln(\sin\pi x + \cos\pi x) + \frac{(2x+1)\cdot(\cos\pi x - \sin\pi x)\cdot\pi^2}{\sin\pi x + \cos\pi x}$

$f'(x) = 2\ln(\sin\pi x + \cos\pi x) + \frac{(2x+1)(\cos\pi x - \sin\pi x)\pi^2}{\sin\pi x + \cos\pi x} \cdot (\sin\pi x + \cos\pi x)^{(2x+1)}$

$f'(0) = 2\ln(0+1) + \frac{(0+1)(1-0)\pi^2}{0+1} \cdot (0+1)^{(1)}$

$f'(0) = 0 + \frac{1\pi^2}{1} \cdot 1$

$f'(0) = \pi^{\cancel{2}}$

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2. A cylindrical tank of radius 10 metres is being filled with wheat at the rate of 100 π metres per minute. How fast is the depth of the wheat increasing? [6 marks]

$r = 10$ m.

$\frac{dv}{dt} = 100\pi$

$V = r^2\pi \cdot h$

$V = 100\pi h$

$\frac{dv}{dt} = 100\pi \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = 1$ m.

$\therefore$ the depth is 1m/minute //

✓

3. The price p (in dollars) and the demand q for a product are related by

$$25p^2 + 4q^2 = 20000\,, \quad 0 < p < 28. \text{ 5+3 =8 marks]}$$

(a) Find an expression for $E(p)$ (the elasticity of demand).

$E(p) = \frac{pq'}{q}$

$50p + 8q \cdot q' = 0$

$8q \cdot q' = -50p$

$q' = \frac{-50p}{8q}$

$q' = \frac{-4p}{q}$

$25p^2 + 4q^2 = 20000$

$4q^2 = 20000 - 25p^2$

$q^2 = 5000 - \frac{25p^2}{4}$

$E(p) = \frac{p}{q} \cdot \frac{-4p}{q}$

$= \frac{-4p^2}{q^2}$

$= \frac{4p^2}{5000 - \frac{25p^2}{4}}$

$= \frac{16p^2}{20000 - 25p^2}$

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(b) If the current price per unit is \$8, will revenue increase or decrease if the price is raised slightly? Explain.

$E(p) = \frac{16p^2}{20000 - 25p^2}$

$E(8) = \frac{16(8)^2}{20000 - 25(8)^2}$

$= \frac{32}{575} < 0$

$\therefore$ the revenue will increasing if the price is raised slightly.

4. (a) Find the linearization $L(x)$ of $f(x) = 2x^3 - 7x^2 + 9x + 6$ at $a = 2$. [4 marks]

$L(x) = f(a) + f'(a)(x-a)$

$f(2) = 2(2)^3 - 7(2)^2 + 9(2) + 6$

$= 12$

$f'(x) = 6x^2 - 14x + 9$

$f'(2) = 6(2)^2 - 14(2) + 9$

$= 5$

$L(x) = 12 + 5(x-2)$

$= 12 + 5x - 10$

$= 5x + 2$

$L(x) = 5x + 2$

$a = 2$

(b) Use $L(x)$ to approximate $f(2.02)$. [2 marks]

$a = 2$

$L(x) = 12 + 5(x-2)$

$L(x) = 12 + 5(2.02 - 2)$

$= 12.1$

5. Let $f(x) = \frac{2x^2+1}{(x-1)^2}, f'(x) = \frac{-2(2x+1)}{(x-1)^3}$ and $f''(x) = \frac{2(4x+5)}{(x-1)^4}$ [13 marks]

a) State the domain of f.

domain $(-\infty, \infty)$

X

b) Find the $\boldsymbol{x}$ −intercept(s) of f, if any.

$0 = \frac{2x^2+1}{(x-1)^2}$

$0 = 2x^2+1$

$-1 = 2x^2$

$x^2 = -\frac{1}{2}$ (X)

No x- intercept.

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c) Find the $\boldsymbol{y}$ − intercept of f, if any.

$f(0) = \frac{2(0)^2+1}{(0-1)^2}$

$= \frac{1}{1}$

$= 1$

∴ y-intercept is (0, 1)

2

d) Find the equations of all horizontal asymptote(s) of f.

$\lim_{x \to +\infty} \frac{2x^2+1}{(x-1)^2}$

$= \lim_{x \to +\infty} \frac{4x}{2x-2}$

$= \lim_{x \to +\infty} \frac{4}{2}$

$= 2$

$\lim_{x \to -\infty} = 2$

∴ H.A. is y = 2

2

e) Find the equations of all vertical asymptote(s) of f.

$(x-1)^2 \neq 0$

$x-1 \neq 0$

$x \neq 1$

$\lim_{x \to 1} \frac{2x^2+1}{(x-1)^2}$

$= \infty$

∴ V.A. is x = 1

2

f) Find the intervals where f is increasing or decreasing and the points of relative extrema.

$f'(x) = \frac{-2(2x+1)}{(x-1)^3}$

$0 = \frac{-2(2x+1)}{(x-1)^3}$

$0 = -2(2x+1)$

$0 = 2x+1$

$-1 = 2x$

$x = -\frac{1}{2}$

	$(-\infty, -\frac{1}{2})$	$(-\frac{1}{2}, 1)$	$(1, \infty)$
z	-1	0	2
$f'(z)$	$-$ve	$+$ve	$-$ve
	$\rightarrow$	$\nearrow$	$\searrow$

2

g) Find the intervals where the function f is concave upward ~~or downward and the~~ points of inflection.

$f''(x) = \frac{2(4x+5)}{(x-1)^4}$

$0 = \frac{2(4x+5)}{(x-1)^4}$

$0 = 2(4x+5)$

$0 = 4x+5$

$4x = -5$

$x = -\frac{5}{4}$

	$(-\infty, -\frac{5}{4})$	$(-\frac{5}{4}, 1)$	$(1, \infty)$
z	-2	0	2
$f''(z)$	$-$ve	$+$ve	$+$ve
	$\cap$ IVI	$\cup$	$\cup$

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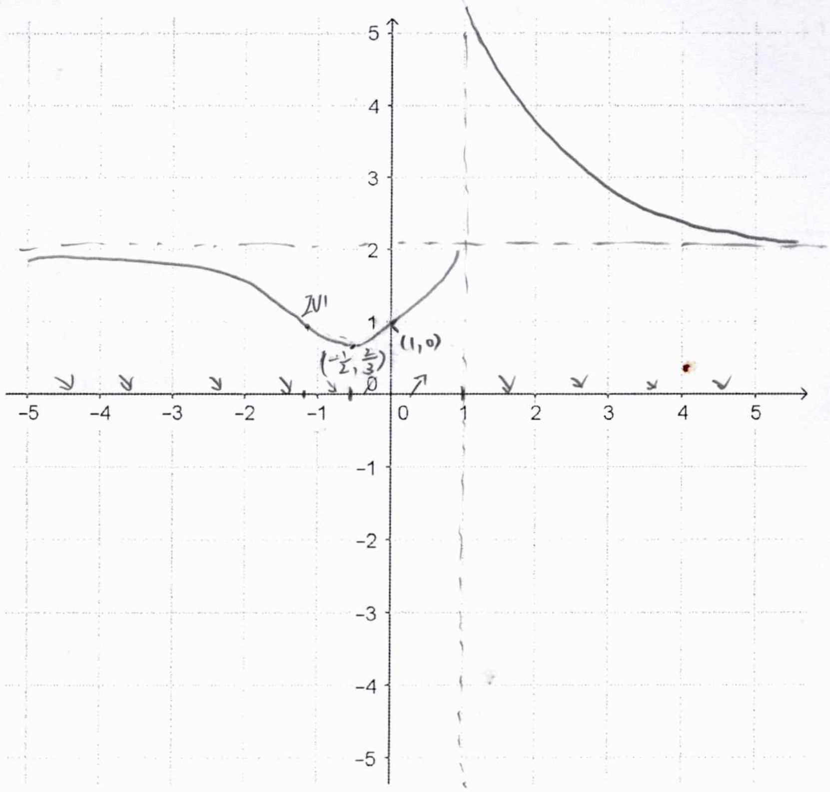
h) Using the above information, sketch the graph of f.

$f(x) = \frac{2x^2+1}{(x-1)^2}$



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