2. Consider the demand equation

$x = 50[400 - p] = 20000 - 50p$

$$p = -\frac{1}{50}x + 400 \qquad 0 \le x \le 20{,}000$$

Diff → $1 = -\frac{1}{50}\frac{dx}{dp}$

which describes the relationship between the unit price in dollars and the quantity demanded x of lamps.

$E = -\frac{p}{x}\cdot\frac{dx}{dp}$

$= -\frac{p}{x}\cdot -50$

$= \frac{50p}{20000 - 50p}$

$= \frac{p}{400 - p}$

i) Find the elasticity of demand $E(p)$.

$E(100) = \frac{1}{3} < 1$

inelastic demand

if the price is increased the revenue increases,

ii) Compute $E(100)$ and interpret your result.

iii) Compute $E(300)$ and interpret your result.

$E(300) = 3 > 1$

elastic, revenue decreases if the price is increased

iv) Find the price at which the revenue is maximum.

This occurs when $E = 1 \Rightarrow$

$\frac{p}{400 - p} = 1 \Rightarrow p = 200$

v) Suppose the price is increased by 3 % when the price is \$100, find the approximate change in demand.

$E \simeq -\frac{\% \Delta x}{\% \Delta p}$

$E(100) \simeq \frac{-\% \Delta x}{3}$

$\therefore \% \Delta x = -3E(100)$

$= -1\%$

i.e demand decreases by approximately 1%.

Quiz 2 Math 157/1 SOLUTIONS

1. Given $y = \left[\dfrac{(x+1)(x+2)}{(x^2+1)(x^2+2)} \right]^{1/3}$

Use logarithmic differentiation to find $\dfrac{dy}{dx}$. Also find $y'(1)$.

$$\ln y = \frac{1}{3}\left[\ln(x+1) + \ln(x+2) - \ln(x^2+1) - \ln(x^2+2) \right]$$

D_x:

$$\frac{y'}{y} = \frac{1}{3}\left[\frac{1}{x+1} + \frac{1}{x+2} - \frac{2x}{x^2+1} - \frac{2x}{x^2+2} \right]$$

$$\therefore\ y' = \frac{1}{3}\left[\frac{1}{x+1} + \frac{1}{x+2} - \frac{2x}{x^2+1} - \frac{2x}{x^2+2} \right]\left[\frac{(x+1)(x+2)}{(x^2+1)(x^2+2)} \right]^{1/3}$$

put $x=1$

$$\frac{y'(1)}{y(1)} = \frac{1}{3}\left[\frac{1}{2} + \frac{1}{3} - \frac{2}{2} - \frac{2}{3} \right]$$

$$y'(1) = \frac{1}{3}\left[-\frac{1}{2} - \frac{1}{3} \right] = -5/18$$

Note:
$y(1) = 1$