Quiz #3 MATH 157 WATTE 2 21) Jiaxuan Xu (Vivy).

1. Use the L'Hospital's Rule to evaluate the following limits.

a) $\lim\limits_{x\to 1} \frac{x^2-1}{x^2+3x-4}$ b) $\lim\limits_{x\to 0} \frac{\sin.(x^2)}{\tan(5x)}$

c) $\lim\limits_{x\to 4} \frac{\sqrt{x}-2}{x-4}$ d) $\lim\limits_{x\to 0} \frac{3^x-2^x}{x^2-x}$

$a^x = \frac{1}{\ln x} \cdot \frac{1}{a}.$

10/15

cos sec

cos

(a) $\lim\limits_{x\to 1} \frac{x^2-1}{x^2+3x-4}$

$\cancel{\lim\limits_{x\to 1} \frac{2x}{2x+3}}$

$\cancel{= \lim\limits_{x\to 1} \frac{(x+1)(x-1)}{(x-1)(x+4)}}$

$\cancel{= \lim\limits_{x\to 1} \frac{x+1}{x+4}}$

$\cancel{= \frac{2}{5}}$

$= \lim\limits_{x\to 1} \frac{2x}{2x+3}$

$= \lim\limits_{x\to 1} \frac{2}{2+3}$

$= \frac{2}{5}$ ✓

(b) $\lim\limits_{x\to 0} \frac{\sin(x^2)}{\tan(5x)}$

$= \lim\limits_{x\to 0} \frac{\cos x^2 \cdot 2x}{\sec \cancel{\csc} 5x \cdot 5}$

$= \cancel{\lim\limits_{x\to 0}} \lim\limits_{x\to 0} \frac{1 \cdot 0}{1 \cdot 5}$

$= 0$ ✓

(c) $\lim\limits_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

$= \lim\limits_{x\to 4} \frac{\frac{1}{2\sqrt{x}}}{1}$

$= \lim\limits_{x\to 4} \frac{1}{2\sqrt{x}}$

$= \frac{1}{4}$ ✓

(d) $\lim\limits_{x\to 0} \frac{3^x-2^x}{x^2-x}$

$= \lim\limits_{x\to 0} \frac{\frac{1}{3\ln x} - \frac{1}{2\ln x}}{2x-1}$

$= \frac{-\frac{1}{6\ln x}}{2x-1}$

$= \cancel{\frac{-1}{-6(\ln x)(2x-1)}}$

$= \frac{\ln x^{-1}}{-6(2x-1)}$

$= \frac{\ln(x^{-1})}{-12x+6}$

$= \frac{x}{-12}$

$= 0.$

✗

3/4

2. For a particular product, the revenue and cost functions are:

$$R(x) = 10x^3 \; and \; C(x) = 300x + 5000$$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Use Newton's method to approximate the break-even point to the nearest hundredth.[6 marks]

$P = 10x^3 - 300x - 5000$

$P' = 30x^2 - 300$

~~1) x=0~~ 2) $x = 10$

$f(x) = 2000$

$x_2 = 10 - \frac{10(10)^3 - 300(10) - 5000}{30(10)^2 - 300}$

$x_2 = 10 - \frac{2000}{2700}$

$x_2 = 9.26$

$x_3 = 9.26 - \frac{162.23}{2272.43}$

$x_3 = 9.189$

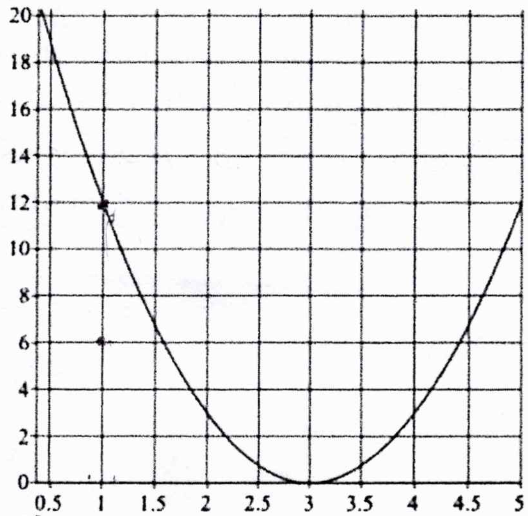
$x_4 = \cancel{9.189}\ 9.188990$

$x_5 = 9.18887.17$

≈ 9.19

3/3

3. Suppose you are told that $f(1) = 5$ and given a graph of the derivative, $f'(x)$,



$L(x) = f'(a)(x-a) + f(a)$

$f(1.1)$

(a) Use linear approximation to estimate the value of $\cancel{f'(1.1)}$.

(b) Is your estimate too large or too small? Justify your answer.

(a) $(1, 12)$ $(3, 0)$.

$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$\frac{y-12}{x-1} = \frac{0-12}{3-1}$

$\frac{y-12}{x-1} = \cancel{\frac{-12}{2}}\ 6$

$y-12 = -6(x-1)$

$y-12 = -6x+6$

$y' = -6x + \cancel{6} + 18$

$\cancel{L(x) = 12(x-1) + 5}$

$\cancel{L(x) = 12x - 7}$

$\cancel{L(1) = 6.2}$

(b) ∴ too small

The $x = 1.1$ in the graph is 11.4 ~~11.6~~, but my answer is 6.2.

3

$L(x) = 12\cancel{18}(x-1) + 5$

$= \cancel{6.8}\ 6.2$

4. Suppose the wholesale price of a certain brand of medium-sized eggs p (dollars/carton) is related to the weekly supply x (in thousands of cartons) by the equation $7255p^2 - x^2 = 100$. If 25,000 cartons of eggs are available at the beginning of a certain week and the price is falling at the rate of \$0.02 carton/week, at what rate is the supply falling?

$7255p^2 - x^2 = 100$

$7255p^2 - x^2 - 100 = 0$

$\cancel{7255}\ 14510p \cdot \frac{dp}{dt} - 2x \cdot \frac{dx}{dt} = 0.$

$7255\ p^2 - (\cancel{25})^2 = 100$ (25000)

$p = 293.51.$

$14510 \cdot 293.51 \cdot 0.02 - 2(25000)\frac{dx}{dt} = 0$

$\frac{85176.602}{12500} = \frac{dx}{dt}$

$\frac{dx}{dt} \approx 7$

~~Sa~~

$\therefore$ supply is falling 7 carton per day.