SOLUTIONS PRACTICE QUIZ

1. $$f(x) = \frac{x \sec x}{x^3 + 4x - 10}$$

$$(uv)' = u'v + uv'$$
$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$f'(x) = \frac{(x \sec x)'(x^3+4x-10) - x \sec x \cdot (x^3+4x-1)'}{(x^3+4x-10)^2}$$

$$= \frac{(1 \cdot \sec x + x \cdot \sec x \tan x)(x^3+4x-10) - x \sec x\,(3x^2+4)}{(x^3+4x-10)^2}$$

$$f'(0) = \frac{-10}{100} = -\frac{1}{10}$$

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2. $$f(x) = \frac{x \ln x}{x-9} - x^{-3/2} - e^{\pi}$$

$$f'(x) = \frac{(x\ln x)'(x-9) - (x \ln x)(x-9)'}{(x-9)^2} + \frac{3}{2} x^{-5/2}$$

$$= \frac{\left(1 \cdot \ln x + x \cdot \frac{1}{x}\right)(x-9) - x \ln x}{(x-9)^2} + \frac{3}{2x^2\sqrt{x}}$$

$$f'(1) = \frac{-8}{64} + \frac{3}{2} = \frac{3}{2} - \frac{1}{8} = \frac{11}{8}$$

3. $f(x) = 3\, e^{(x^2+1)^{1/2}}$

$f'(x) = 3 \cdot e^{(x^2+1)^{1/2}} \cdot \left[(x^2+1)^{1/2}\right]'$

$= 3 \cdot e^{\sqrt{x^2+1}} \cdot \frac{1}{2} \cdot (x^2+1)^{-1/2} \cdot 2x$

$= \frac{3x}{\sqrt{x^2+1}}\, e^{\sqrt{x^2+1}}$

$\left[\left[\frac{d}{dx} e^x = e^x\right]\right]$

$f'(\sqrt{8}) = \sqrt{8}\, e^3$

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4. $f(x) = \log_2 \left[\sin^{-1}(x^2-3x)\right]$

$\left[\left[\frac{d}{dx}\log_a |x| = \frac{1}{\ln a}\cdot\frac{1}{x}\right]\right]$

$f'(x) = \frac{1}{\ln 2} \cdot \frac{1}{\sin^{-1}(x^2-3x)} \cdot \frac{d}{dx}\left[\sin^{-1}(x^2-3x)\right]$

$= \frac{1}{\ln 2} \cdot \frac{1}{\sin^{-1}(x^2-3x)} \cdot \frac{1}{\sqrt{1-(x^2-3x)^2}} \cdot \frac{d}{dx}(x^2-3x)$

$= \frac{1}{\ln 2} \cdot \frac{1}{\sin^{-1}(x^2-3x)} \cdot \frac{1}{\sqrt{1-(x^2-3x)^2}} \cdot (2x-3)$

$= \frac{1}{\ln 2} \cdot \frac{1}{\sin^{-1}(x^2-3x)} \cdot \frac{(2x-3)}{\sqrt{1-(x^2-3x)^2}}$

$\left[\left[\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}\right]\right]$

5. $f(x) = \ln\left(\tan^{-1}(2x+1)\right)$

$$f'(x) = \frac{1}{\tan^{-1}(2x+1)} \cdot \frac{d}{dx} \tan^{-1}(2x+1)$$

$$= \frac{1}{\tan^{-1}(2x+1)} \cdot \frac{1}{1+(2x+1)^2} \cdot \frac{d}{dx}(2x+1)$$

$$= \frac{1}{\tan^{-1}(2x+1)} \cdot \frac{1}{1+(2x+1)^2} \cdot 2$$

$\left[\left[\frac{d}{dx}\ln|x| = \frac{1}{x}\right]\right]$ $\left[\left[\frac{d}{dx}\tan^{-1}x\right]\right] = \frac{1}{1+x^2}$

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6. $f(x) = \sec^{-1}(x^{3/2})$

$\left[\left[\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|} \cdot \frac{1}{\sqrt{x^2-1}}, \text{ for } |x|>1\right]\right]$

$$f'(x) = \frac{1}{|x^{3/2}|} \cdot \frac{1}{\sqrt{(x^{3/2})^2-1}} \cdot \frac{d}{dx} x^{3/2}$$

$$\overset{*}{=} \frac{1}{x^{3/2}} \cdot \frac{1}{\sqrt{x^3-1}} \cdot \frac{3}{2} \cdot x^{1/2}$$

* Here $x>1$?

$$= \frac{3}{x^2} \cdot \frac{1}{\sqrt{x^3-1}}$$

7. $$x^3 + xy^2 = y^3 + 4$$

APPLY $\frac{d}{dx}$:

$$3x^2 + [1 \cdot y^2 + x \cdot 2y \cdot y'] = 3y^2 y'$$

Put $x = 2$

$y = 1$

$m = y'(P)$

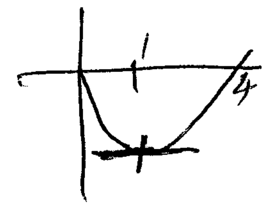
$$12 + [1 + 4m] = 3m \Rightarrow m = -13$$

TP:

$$y - 1 = -13(x - 2)$$

$$y = -13x + 27$$

$$f(x) = -1 + \left(\frac{x}{4}\right)^{\frac{\sqrt{x}}{2}}$$



$$f'(x) = \frac{d}{dx}\left[\left(\frac{x}{4}\right)^{\frac{\sqrt{x}}{2}}\right] = 0\ ?$$

Let $g(x) = \left(\frac{x}{4}\right)^{\sqrt{x}/2}$

$$\ln g = \frac{\sqrt{x}}{2} \cdot \ln\left(x/4\right) = \frac{\sqrt{x}}{2}\left[\ln x - \ln 4\right]$$

d/dx

$$\frac{g'}{g} = \frac{\frac{1}{2}\cdot\frac{1}{\sqrt{x}}}{2}\cdot\left[\ln x - \ln 4\right] + \frac{\sqrt{x}}{2}\left[\frac{1}{x}\right]$$

$$= \frac{1}{4}\frac{1}{\sqrt{x}}\left[\ln\left(\frac{x}{4}\right)\right] + \frac{1}{2}\frac{1}{\sqrt{x}}$$

Note from the graph that tangent is horizontal at $x = 1$

Check
$$\frac{g'(1)}{g(1)} = \left(\frac{1}{4}\ln(1/4) + \frac{1}{2}\right)$$

$$g'(1) = \left(\frac{1}{4}\ln(1/4) + \frac{1}{2}\right)\cdot(0) = 0!$$

Answer $x = 0$, the tangent is horizontal.