1. Evaluate the limits.

3 [4] (a) $\lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right) = \infty \cdot 0$

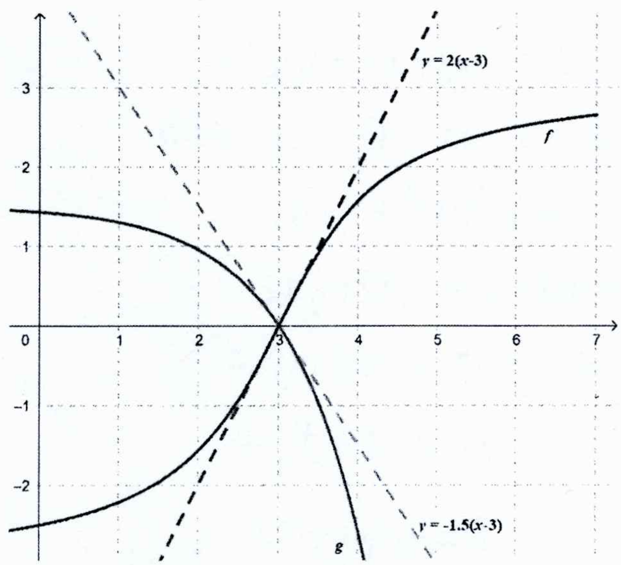
$= \lim_{x\to\infty} \frac{\sin\left(\frac{\pi}{x}\right)}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{\cos\frac{\pi}{x}\left(-\frac{\pi}{x^2}\right)}{-\frac{1}{x^2}}$

$= \lim_{x\to\infty} \cos\frac{\pi}{x} \cdot \pi$

$= \pi$ ✓

5 [6] (b) Use the graphs of f and g and their tangent lines at $(3, 0)$ to find $\lim_{x\to 3} \frac{f(x)}{g(x)} = \frac{0}{0}$



$\lim_{x\to 3} f'(x) = \cancel{\lim_{x\to 3} 2(x-3)}$ 2

$\lim_{x\to 3} g'(x) = \cancel{\lim_{x\to 3} -\frac{3}{2}(x-3)}$ $-\frac{3}{2}$

$\lim_{x\to 3} \frac{f(x)}{g(x)} = \lim_{x\to 3} \frac{f'(x)}{g'(x)} = \cancel{\lim_{x\to 3}} \frac{f'(3)}{g'(3)}$

$= 2 \cdot \left(-\frac{2}{3}\right) = -\frac{4}{3}$ ✓

[8] **2.** Sketch the graph of f and use it to find the absolute and local points of extrema of the given function:

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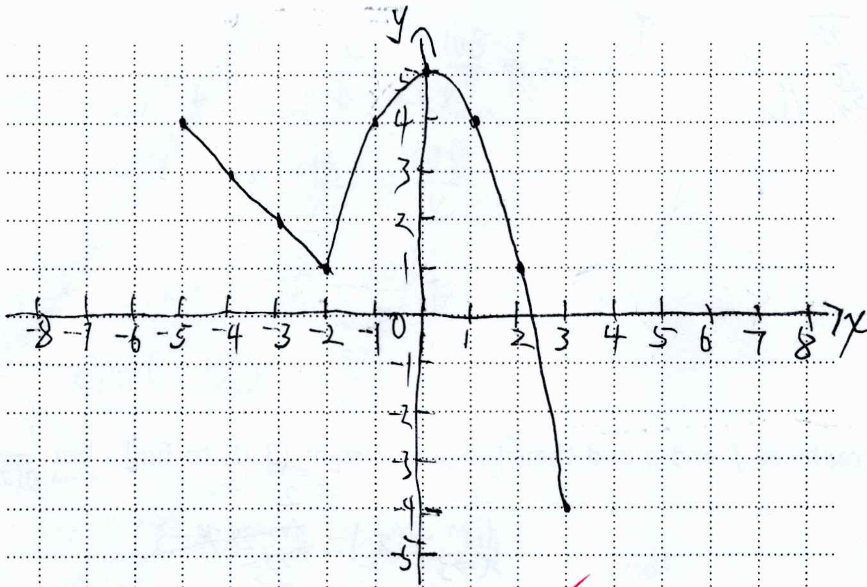
$$f(x) = \begin{cases} -x-1, & -5 \le x \le -2 \\ 5-x^2, & -2 < x \le 3 \end{cases}$$



absolute maximum: 5 at $x=0$

absolute minimum: -4 at $x=3$

local minimum: 1 at $x=-2$

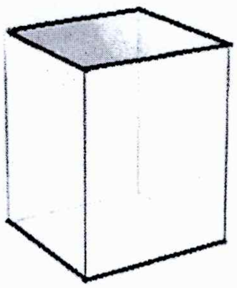
local maximum: 5 at $x=0$

[10] **3.** A jewelry box is to have a rectangular base with a length double the width, and a volume of ~~108~~ 216 cm^3. The material and manual labour for the base will cost \$1 per cm^2, for the top will cost \$2 per cm^2, and for the other four sides will cost \$0.5 per cm^2.

3

Use the method of optimization problems to determine the dimensions of the most economical cost for constructing such a container.

~~Assume~~ $2x^2h = 216$ $h = \frac{108}{x^2}$



Set ~~length~~ $=x$, width $=y$, height $=h$, cost $=c$

$x y h = 216$

~~$(2xy)' = 3$~~

~~$(yh)' = 0.5$~~

the area of bottom & top $2xy$

the area of side $2h(x+y)$

~~$x = 2\sqrt[3]{18}$, $h = -$~~

~~total cost = 2 $c = 4(yh)' + 2xy$~~

Total area $=$ ~~xy~~ $2xy + 2h(x+y)$

$= 2(xy + hx + hy)$

$\frac{\text{Total cost}}{\text{Total area}} =$

$c = xy(1) + xy(2) + 2h(x+y)\cdot\frac{1}{2}$

$= xy + 2xy + hx + hy$

$= 3xy + hx + hy$

when $c' = 0$, the cost is lowest

$c' = (3xy)' + (hx)' + (hy)'$

$= 3x'y + 3xy' + h'x + hy' + h'y + hy'$

$= 3y + 3x + x + h + y + h$

$= 4x + 4y + 2h = 0$

$2x + 2y + h = 0$

$x = 2y$

$2y \cdot y \cdot h = 2y^2 h = 216$

$4y + 2y + h = 0$

$6y + h = 0$

$h = -6y$

~~$-12y^3 = 216$~~

~~$2y^3 = 36$~~ ~~$y^3 = 18$~~ ~~$y = \sqrt[3]{18}$~~

4. Given the function $G(x) = x\sqrt{6-x}$.

5 [5] (a) Find the critical numbers of G.

$$G'(x) = \sqrt{6-x} + x\frac{1}{2\sqrt{6-x}}\cdot(-1)$$

$$= \sqrt{6-x} - \frac{x}{2\sqrt{6-x}}$$

$$\sqrt{6-x} - \frac{x}{2\sqrt{6-x}} = 0$$

$$\sqrt{6-x} = \frac{x}{2\sqrt{6-x}} \qquad x \neq 6$$

$$2(6-x) = x$$

$$12 - 2x = x$$

$$x = 4$$

~~The~~ $6 - x \geq 0$

$$x \leq 6$$

The critical numbers are 4 and 6

3 [3] (b) Find the absolute maximum and minimum values of G over the interval $[2, 6]$.

x	2	4	6
$G'(x)$	+	0	− //

$G(2) = 2\cdot\sqrt{6-2} = 4$

$G(4) = 4\sqrt{2}$

$G(6) = 6\sqrt{0} = 0$

The absolute maximum value is $4\sqrt{2}$,

The absolute minimum value is 0

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[8] **5.** Let f be a function defined on the real domain. Given that $f'(x) = \dfrac{4-x}{\sqrt[3]{x}}$. Do work below (such as a sign chart) and answer the following related to function f:

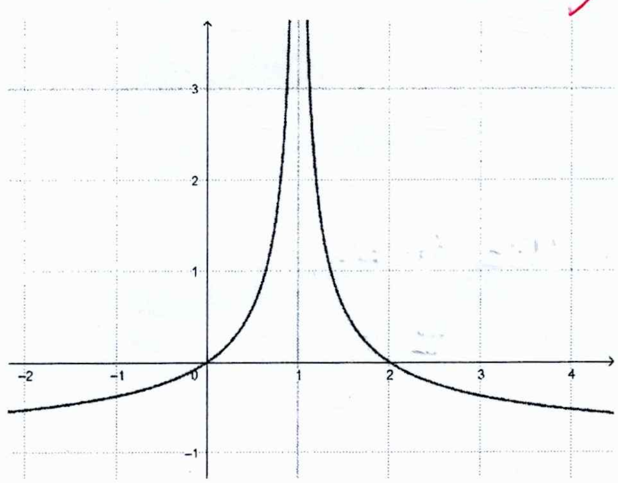
x	$-\infty$	0		4	$+\infty$
$f(x)$	$\searrow$	Lm	$\nearrow$	L.M.	$\searrow$
$f'(x)$	$-$	//	$+$	0	$-$

- The interval(s) where the function is increasing: $(0,4)$
- The interval(s) where the function is decreasing: $(-\infty,0)\cup(4,+\infty)$
- The function has a local maximum, if any, at the point where $x =$ 4
- The function has a local minimum, if any, at the point where $x =$ 0

~~No local minimum~~

5 [6] **6.** The graph of the derivative g' of a continuous function is given below. Use it to answer the following about the given function.

1. g is incresing on: ~~$(-\infty,1)$~~ $(0,1)\cup(1,2)$
2. g has a local max where $x = 2$
3. g is convave down on: ~~$(1,+\infty)$~~ ~~$(-\infty$~~ ~~$(2,+\infty)$~~ ~~$(-\infty,0)\cup(2,4)$~~ $(1,\cancel{4})$
4. g has a point of inflection where $x = 1$



4 [6] **7.** Decoding the sign chart below, answer the following related to function g:

x	$-\infty$		-3		0		4		$+\infty$
$g''(x)$		$-$	0	$+$	0	-	DNE	$+$	
$g(x)$			2		1		0		

- g is concave up on: ~~$(-\infty,-3)\cup(0,$~~ $(-3,0)\cup(4,+\infty)$
- g is concave down on: $(-\infty,-3)\cup(0,4)$
- The inflection point(s) of function g is(are): 4, 0, -3

[14] 8. Given the function $f(x) = x^3 - 3x^2$, find the $x-$ and $y-$ intercepts, the points of extrema, the increasing/decresing behaviour, concavity and the inflection point(s). Use the gathered info to graph the function.

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$x = 0,\ y = 0$

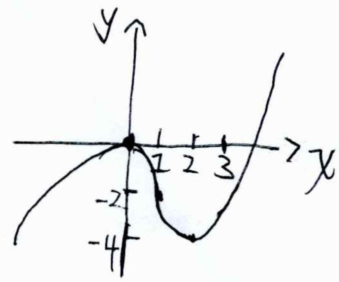
$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$x = 0$ or 3

y- intercepts ~~is~~ are 0 and 3,

x- intercept is 0



$f'(x) = 3x^2 - 6x = 3x(x-2)$

x	$-\infty$		0		2		3	$+\infty$
$f'(x)$		+ +	0	−	0	+		
$f(x)$	$-\infty$	↗	0	↘	−4	↗	0	↗

$\lim_{x \to -\infty} x^2(x-3) = -\infty$

$f''(x) = 6x - 6 = 6(x-1)$

x	$-\infty$	0	1	2	3	$+\infty$
$f''(x)$	$-\infty$	$\cancel{0}$	0	+		$+\infty$
$f(x)$	$-\infty$ ↗	0 ↘ −2	~~HD~~ ↘	−4	↗	$+\infty$

~~No inflect~~