SOLUTIONS TEST 2 . Nov. 2021

1. **Differentiate the following functions as indicated. [6 marks]**

   a) $f(x) = \cos(3x) + \tan(2x) + e^x + \ln|1-2x|$, find $f'(0)$.

   b) $f(x) = \sqrt{5x-5}^{\sqrt{(5x-5)}}$, find $f'(6)$.

a) $f'(x) = -\sin 3x \cdot 3 + \sec^2 2x \cdot 2 + e^x + \frac{1}{1-2x} \cdot (-2)$

$f'(0) = 0 + 2 + 1 - 2 = 1$

b) $\ln f(x) = \frac{1}{2}(5x-5)^{\frac{1}{2}} \ln(5x-5)$

Dx:

$\frac{f'(x)}{f(x)} = \frac{1}{2}\left[\frac{1}{2}\frac{1}{\sqrt{5x-5}} \cdot 5 \cdot \ln(5x-5) + \sqrt{5x-5} \cdot \frac{1}{5x-5} \cdot 5\right]$

$= \frac{5}{4}\frac{1}{\sqrt{5x-5}} \cdot \ln(5x-5) + \frac{5}{2}\frac{1}{\sqrt{5x-5}}$

$\frac{f'(6)}{f(6)} = \frac{1}{2}(1 + \ln 5)$; $f(6) = 5^5$

$\therefore f'(6) = \frac{3125}{2}(1 + \ln 5) \approx 4077.25$

2. The price p (in dollars) and the demand q for a product are related by

$$25p^2 + 4q^2 = 20000 \qquad 0 < p < 28.$$ [5 marks]

(a) Find an expression for $E(p)$ (the elasticity of demand). [3 marks]

$\frac{p f'(p)}{f(p)}$ $\frac{pq'}{q}$

(b) If the current price per unit is \$8, will revenue increase or decrease if the price is raised slightly? Explain. [2 marks]

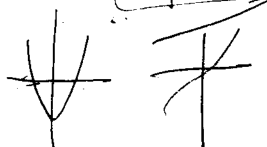
(a) Apply $\frac{d}{dp}$! $\quad 50p + 8q\frac{dq}{dp} = 0 \quad \frac{dq}{dp} = -\frac{25}{4}\frac{p}{q}$

$$E = -\frac{p}{q} \cdot -\frac{25}{4}\frac{p}{q} = \frac{25p^2}{4q^2} = \frac{25p^2}{20000 - 25p^2} = \frac{p^2}{800 - p^2}$$

(b) $E(8) = \frac{64}{800-64} = \frac{2}{23} < 1$ i.e. inelastic demand

Revenue increases if the price is raised slightly.

$f(x) = \sqrt{1-x^2}$



a) Find the linearization $L(x)$ of $f(x) = 2x^3 - 7x^2 + 9x + 6$ at $= 2$. [3 marks]

b) Use $L(x)$ to approximate $f(1.8)$. [2 marks]

" linearization of f at $x = a$: "

$L(x) = f(a) + f'(a) \cdot (x - a)$

$f(2) = 16 - 28 + 18 + 6 = 12$

$f'(x) = 6x^2 - 14x + 9$

$f'(2) = 24 - 28 + 9 = 5$

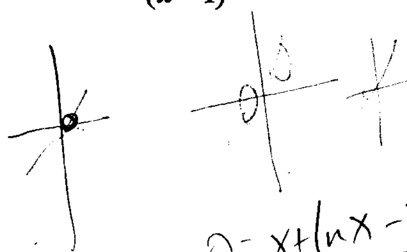
$\therefore \; L(x) = 12 + 5(x-2) = 5x + 2$

$f'(1.8) \approx L(1.8) = 12 + 5(-0.2) = 11$

4. Let $f(x)=\dfrac{2x^2+1}{(x-1)^2}$, $f'(x)=\dfrac{-2(2x+1)}{(x-1)^3}$ and $f''(x)=\dfrac{2(4x+5)}{(x-1)^4}$. [10 marks]

a) State the domain of f.

$(-\infty, 1) \cup (1, \infty)$



b) Find the x −intercept(s) of f, if any.

NONE

$0 = x + \ln x - 7$

$7 = x + \ln x$

$0 = x + \ln x - 7$

$7 = x + \ln x$

c) Find the y − intercept of f, if any.

$f(0) = 1$

The only number equal to. $7 = x + \ln x$ is between $[5, 6]$ So there is no other root.

d) Find the equations of all horizontal asymptote(s) of f.

$f(x) = \dfrac{2+\frac{1}{x^2}}{1-\frac{2}{x}+\frac{1}{x^2}}$; $\lim_{x\to\infty} f(x) = 2^+$ $\lim_{x\to-\infty} f(x) = 2^-$ $y = 2$

[e) Find the equations of all vertical asymptote(s) of f.

$\lim_{x\to 1^+} f(x) = \infty$

$\lim_{x\to 1^-} f(x) = \infty$

$x = 1$

f) Find the intervals where f is increasing or decreasing and the points of relative extrema.

$f' = 0 \Rightarrow x = -\frac{1}{2}$ $P \equiv (-\frac{1}{2}, \frac{2}{3})$ 0.67

I	$(-\infty, -\frac{1}{2})$	$(-\frac{1}{2}, 1)$	$(1, \infty)$
z	-10	0	10
f'	$-ve$	$+ve$	$-ve$
f	↘	↗	↘

FDT

Rel min P

g) Find the intervals where the function f is concave upward or downward and the points of inflection.

0.81(

(g) ∴ $f''=0 \Rightarrow x=-5/4$ Possible Inflection point $I=\left(-\frac{5}{4}, \frac{22}{27}\right)$

I	$(-\infty, -5/4)$	$(-5/4, 1)$	$(1, \infty)$
Z	-10	0	10
f''	$-ve$	$+ve$	$+ve$
f	CD	CU	CU
Inflection pt		I	

[h) Using the above information, sketch the graph of f .