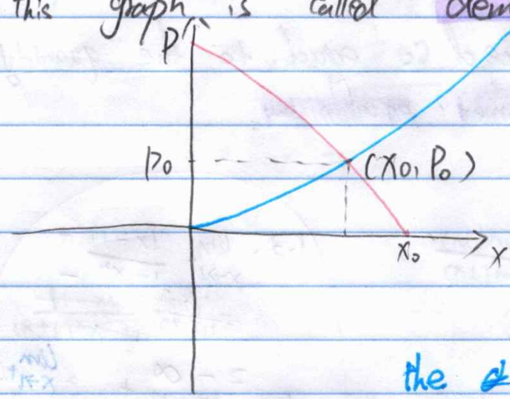
2-7 Economic Models

2-7.1 Demand and supply Functions.

- the consumer demand for a particular commodity is depends on the unit price of the commodity.

- The relationship between unit price and quantity of demand is called demand equation

And this graph is called demand curve



supply curve

demand curve.

※: many people demand the commodity, and the price will get low.

the ~~c~~ commodity are supplying more, and the price will get high.

→ 供不应求

(x_0, P_0) corresponds to market equilibrium.

∴ quantity demanded commodity ↑ unit price ↓.
commodity ↓ unit price ↑.

quantity supply commodity ↑ unit price ↑
commodity ↓ unit price ↓

Definition 2.40: Demand Function.

A demand Function is defined by $p = f(x)$

unit price commodity

always decreasing function, $p = f(x)$ decreases, x increase.

the unit price of commodity is dependent on the commodity's availability in the market.

The relationship between the unit price and the quantity

---

particular. adj. 特定的
commodity n. 商品.
curve. n. 曲线
quantity. n. 量, 数目
articulated. adj. 清晰的.
articulated by. 明确地指出了
so-called 叫做…的
equation.
correspond v. 相当于
equilibrium. (供求的)平衡
vice verse. adj. 反之亦然.
definition. n. 定义
generally. adv. 大概.
characterized. adj. 以…为特点.
assume. v. 假设
nonnegative. adj. 正的

supplied so-called supply equation, the graph is supply curve.
unit price induce the produser increase or decrease the quantity of commodity.

Definition 2.41: Supply function.

supply function defined by $p = f(x)$ → increase function.

unit price; commodity

x increase and p increase.

When the quantity produced is equal to the quantity demanded, so-called market equilibrium.

Exercises 3.1

1.1: $\lim_{x \to 0} \cancel{\tan} \frac{\sin x}{x} = 0$

1.2: $\lim_{x \to 0} \frac{\tan(3x)}{\tan(2x)} = 0$

1.3. $\lim_{x \to 1^+} \frac{(x-1)}{1-x^2} \; {+ \atop -}$
$= \lim_{x \to 1^+} \frac{-\cancel{(x-1)}}{\cancel{(1-x)}(1+x)}$
$= -\infty$ $\;+\;$ $= \lim_{x \to 1^+} -\frac{1}{1+x}$ $= -\frac{1}{2}$

$\lim_{x \to 1^-} \frac{(x-1)}{1-x^2} \; {+ \atop -}$
$= 0$ $\;$ $= \lim_{x \to 1^-} \frac{\cancel{(x-1)}}{\cancel{(1-x)}(1+x)}$ $1-x$
$= \lim_{x \to 1^-} \frac{1}{1+x}$
$= \frac{1}{2}$

Exercises 3.2.

3.3.1

(a) $\lim_{x \to 4} f(x) = 8$ (b) $\lim_{x \to -3} f(x) = 6$ (c) $\lim_{x \to 0} f(x) = -1,$ -2

(d) $\lim_{x \to 0^-} f(x) = -2$ (e) $\lim_{x \to 0^+} f(x) = -1$ (f) $f(-2) = 8$

(g) $\lim_{x \to 2} f(x) = 7$ (h) $\lim_{x \to -2} f(x) = 6$ (i) $\lim_{x \to 0} f(x+1) = 3$

(j) $f(0) = -1.5$ (k) $\lim_{x \to 1^-} f(x-4) = 6$ (l) $\lim_{x \to 0^+} f(x-2) = 2$

3.3.2

(a) $\lim_{x \to -1} f(x) = -2$ (b) $\lim_{x \to 0^+} f(x) = 2$ (c) $\lim_{x \to 1} f(x) = 2$

(d) $f(1) = 0$ (e) $\lim_{x \to 2} f(x) = 2$

3.4

3.4.1

induce
v. 导致
respectively
adj. 分别地.
competitive
adj. 有竞争力的
eventually.
adj. 最终
certain.
adj. 确定的.
namely
adv. 也就是
simply put.
简单地说
conversely
adj. 反过来说
expression.
n. 表达.

what mean it is exist or not?

(a) $\lim_{x\to3}\frac{x^2+x-12}{x-3}$
$=\lim_{x\to3}\frac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}}$
$=7$

(b) $\lim_{x\to1}\frac{x^2+x-12}{x-3}$
$=\lim_{x\to1}\frac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}}$ No
$=5$

(c) $\lim_{x\to-4}\frac{x^2+x-12}{x-3}$
$=\lim_{x\to-4}\frac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}}$ No
$=0$

? (d) $\lim_{x\to2}\frac{x^2+x-12}{x-2}$, No ?
$=\frac{-6}{x-2}$.

No exist.

$x\to2^+$, $x\to2^-$ } Not equal.

(e) $\lim_{x\to0^+}\sqrt{\frac{1}{x}+2}-\sqrt{\frac{1}{x}}$
$=\lim_{x\to0^+}\frac{(\sqrt{\frac{1}{x}+2}-\sqrt{\frac{1}{x}})(\sqrt{\frac{1}{x}+2}+\sqrt{\frac{1}{x}})}{(\sqrt{\frac{1}{x}+2}+\sqrt{\frac{1}{x}})}$
$=\lim_{x\to0^+}\frac{(\sqrt{\frac{1}{x}+2})^2-(\sqrt{\frac{1}{x}})^2}{(\sqrt{\frac{1}{x}+2}+\sqrt{\frac{1}{x}})}$
$=\lim_{x\to0^+}\frac{\cancel{\frac{1}{x}}+2-\cancel{\frac{1}{x}}}{\sqrt{\frac{1}{x}+2}+\sqrt{\frac{1}{x}}}$
$=\lim_{x\to0^+}\frac{2}{+\infty}$
$=0$

(f) $\lim_{x\to1}\frac{\sqrt{x+8}-3}{x-1}$
$=\lim_{x\to1}\frac{(\sqrt{x+8}-3)(\sqrt{x+8}+3)}{(x-1)(\sqrt{x+8}+3)}$
$=\lim_{x\to1}\frac{(x+8)-9}{(x-1)(\sqrt{x+8}+3)}$
$=\lim_{x\to1}\frac{x+8-9}{(x-1)(\sqrt{x+8}+3)}$
$=\lim_{x\to1}\frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x+8}+3)}$
$=\lim_{x\to1}\frac{1}{\sqrt{x+8}+3}$
$=\frac{1}{6}$

(g) $\lim_{x\to2}3$.

(h) $\lim_{x\to4}3x^3-5x$ No
$=\lim_{x\to4}x(3x^2-5)$
$=172$.

(i) $\lim_{x\to0}\frac{4x-5x^2}{x-1}$ No.
$=0$

(j) $\lim_{x\to1}\frac{x^2-1}{x-1}$ No.
$=\lim_{x\to1}\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$
$=\lim_{x\to1}x+1$
$=2$.

(k) $\lim_{x\to0^+}\frac{\sqrt{2-x^2}}{x}$ $+\infty$ / $+0$.
$=+\infty$

(l) $\lim_{x\to0^+}\frac{\sqrt{2-x^2}}{x+1}$
$=\sqrt{2}$.

~~No exist~~ ?

$a^3-b^3=(a-b)^3+3ab(a-b)$
$=(a-b)(a^2+b^2+ab)$

(m) $\lim_{x\to a}\frac{x^3-a^3}{x-a}$
$=\lim_{x\to a}\frac{\cancel{(x-a)}(x^2+ax+a^2)}{\cancel{x-a}}$
$=\lim_{x\to a}x^2+ax+a^2$
$=\lim_{x\to a}a^2+a^2+a^2$
$=3a^2$

(n) $\lim_{x\to2}(x^2+4)^3$
$=512$.

3.4.2.

$L=\lim_{x\to0}g(x)$.
$L=0$.

$f(x)=\begin{cases}1 & \text{if } x\neq0\\ 0 & \text{if } x=0\end{cases}$, $g(x)=0$.

$M=\lim_{x\to L}f(x)$
$M=\lim_{x\to0}f(x)$
$M=1$

$\lim_{x\to0}f(0)$
$=0$

∴ It is not true.

3.4.4

(a) $\lim_{x \to a} [p(x) - q(x)]$
$= -1$

(b) $\lim_{x \to a} \sqrt{q(x)}$
$= 2$

(c) $\lim_{x \to a} \left[\frac{2p(x) - q(x)}{p(x)q(x)} \right]$ $6-4$
$= \frac{1}{6}$

indicated
指 打 od

3.4.5

(a) not exist.

(b) $\lim_{x \to 1} \frac{x^2}{x(x-1)}$
$= \lim_{x \to 1} \frac{x^2}{x^2 - x}$
$\lim_{x \to 1^+} \frac{x^2}{x^2 - x}$
$= +\infty$
$\lim_{x \to 1^-} \frac{x^2}{x^2 - x}$
$= +\infty$

(c) $\lim_{x \to 1} \frac{x-1}{\sqrt{x} - 1}$
$= \lim_{x \to 1} \frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)}$
$= \lim_{x \to 1} \frac{\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{x-1}}$
$= \lim_{x \to 1} \sqrt{x} + 1$
$= 2.$

(d) $\lim_{x \to -1} \frac{x+1}{x^2+1}$
$= 1$

(e) $\lim_{x \to 1} \frac{x^3 - 3x + 2}{x^3 - x^2 - x + 1}$
$= \lim_{x \to 1} \frac{x^3 - 3x + 2}{x^2(x-1) - (x-1)}$
$= \lim_{x \to 1} \frac{x^3 - 3x + 2}{(x^2-1)(x-1)}$
$= \lim_{x \to 1} \frac{x^3 - x - 2x + 2}{(x^2-1)(x-1)}$
$= \lim_{x \to 1} \frac{x(x^2-1) - 2(x-1)}{(x^2-1)(x-1)}$
$= \lim_{x \to 1} \frac{x(x-1)(x+1) - 2(x-1)}{(x^2-1)(x-1)}$
$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x(x+1)-2)}{\cancel{(x-1)}(x^2-1)}$
$= \lim_{x \to 1} \frac{(x^2+x-2)}{x^2-1}$
$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+1)}$
$= \lim_{x \to 1} \frac{x+2}{x+1} = \frac{3}{2}$

(f) $\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1}$
$= \lim_{x \to 0} \frac{x(\sqrt{x+1}+1)}{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}$
$= \lim_{x \to 0} \frac{\cancel{x}(\sqrt{x+1}+1)}{\cancel{x+1-1}}$
$= \lim_{x \to 0} (\sqrt{x+1} + 1)$
$= \lim_{x \to 0^+} (\sqrt{x+1} + 1)$
$= +\infty$
$\lim_{x \to 0^-} (\sqrt{x+1} + 1)$
$= +\infty.$

3.4.6.

(a) $\lim_{x \to 1^-} (2x+4)$
$= 6$

(b) $\lim_{x \to 2^+} \frac{x-3}{x+2}$
$= -\frac{1}{4}$

(c) $\lim_{x \to 0^-} \frac{1}{x}$
$= -\infty$

(d) $\lim_{x \to 0^-} \frac{x-1}{x^2+1}$
$= \frac{-1}{1} = -1$

(e) $\lim_{x \to 1^+} \frac{1+x}{1-x}$
$= -\infty$

(f) $\lim_{x \to 0^+} f(x)$
$\lim_{x \to 0^+} 2x$ $\lim_{x \to 0^-} x^2$
$= 0$ $= 0.$

(g) $\lim_{x \to 0^+} -x+1$
$= 1$

$\lim_{x \to 0^-} 2x+3$
$= 3.$

3.5, 1.

(a) $\lim_{x\to\infty} (\sqrt{x^2+x} - \sqrt{x^2-x})$

$= \lim_{x\to\infty} \frac{(\sqrt{x^2+x} - \sqrt{x^2-x})(\sqrt{x^2+x} + \sqrt{x^2-x})}{(\sqrt{x^2+x} + \sqrt{x^2-x})}$

$= \lim_{x\to\infty} \frac{x^2+x-(x^2-x)}{\sqrt{x^2+x} + \sqrt{x^2-x}}$

$= \lim_{x\to\infty} \frac{\cancel{x^2}+x-\cancel{x^2}+x}{\sqrt{x^2+x} + \sqrt{x^2-x}}$

$= \lim_{x\to\infty} \frac{2x}{\sqrt{x^2+x} + \sqrt{x^2-x}}$

$= \lim_{x\to\infty} \frac{2\cancel{x}}{\sqrt{\frac{x^2+x}{x^2}} + \sqrt{\frac{x^2-x}{x^2}}}$

$= \lim_{x\to\infty} \frac{2}{\sqrt{1+\frac{1}{x}} + \sqrt{1-\frac{1}{x}}}$

$= \lim_{x\to\infty} \frac{2}{\sqrt{1}+\sqrt{1}}$

$= 1$

(c) $\lim_{t\to 1^+} \frac{(\frac{1}{t})-1}{t^2-2t+1}$

$= \lim_{t\to 1^+} \frac{(\frac{1}{t})-1}{(t-1)^2}$

~~$= \lim_{t\to 1^+} \frac{1-t}{(t-1)^2 t}$~~

~~$= \infty$~~

$= \lim_{t\to 1^+} \frac{1-t}{(t-1)^2 t}$

$= \lim_{t\to 1^+} \frac{-1}{(t-1)t}$

$= \lim_{t\to 1^+} \frac{-1}{t^2-t}$

$= -\infty$

(b) $\lim_{x\to\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}}$

$= \lim_{x\to\infty} \frac{e^x+\frac{1}{e^x}}{e^x-\frac{1}{e^x}}$

$= 1$

(d) $\lim_{t\to\infty} \frac{t+5-\frac{2}{t}-\frac{1}{t^3}}{3t+12-\frac{1}{t^2}}$

$\infty + \infty - \infty -$

$= \lim_{t\to\infty} \frac{t^4+5t^3-2t^2-1}{3t^4+12t^3-t}$

$= \frac{1}{3}$

(e) $\lim_{y\to\infty} \frac{\sqrt{y+1}+\sqrt{y-1}}{y}$

$= \lim_{y\to\infty} \frac{}{}$