

$$5.2.12.$$

$$V = \pi r^2 \frac{h}{3}$$

$$= \pi \left(\frac{h}{2}\right)^2 \frac{h}{3}$$

$$= \pi \frac{h^3}{12}$$

$$\frac{dV}{dt} = \frac{\pi}{12} 3h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{\pi h^2} \frac{dV}{dt}$$

$$\frac{dh}{dt} = \frac{4}{9\pi} (15)$$

$$= \frac{20}{3\pi} \text{ cm/s}$$

5.2.14.

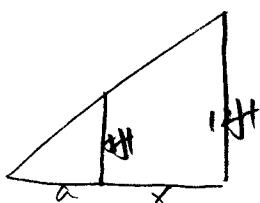


$$\frac{dl}{dt} = \frac{1}{d(t)} \left( h(t) \frac{dh}{dt} + x(t) \frac{dx}{dt} \right)$$

$$\frac{dl}{dt} = \frac{1}{d(2)} \left( h(2) \frac{dh}{dt} + x(2) \frac{dx}{dt} \right)$$

$$\frac{dl}{dt} = \frac{1}{\sqrt{20^2 + 60^2}} (60(2) + 20(10)) = \frac{1}{\sqrt{5}} \frac{d}{dt}$$

5.2.16.



$$\frac{12}{5} = \frac{a}{x+a}$$

$$a = \frac{4}{3}x$$

$$17 \frac{da}{dt} = 5 \frac{dx}{dt}$$

$$\frac{da}{dt} = \frac{5}{17} \times \frac{7}{2} = \frac{35}{34}$$

$$L = x + a$$

$$\frac{dL}{dt} = \frac{dx}{dt} + \frac{da}{dt}$$

$$= \frac{7}{2} + \frac{35}{34}$$

$$= \frac{119}{34} + \frac{35}{34} = \frac{154}{34} = \frac{77}{17}$$

5.3.1.

a)  $f(x) = \sqrt{x}$ ,  $a = 4$ ,  $x = 3$ .

$$f'(x) = \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$= \sqrt{4} + \frac{1}{2\sqrt{4}}(x-4)$$

$$= 2 + \frac{1}{4}(x-4)$$

$$f(3) = 2 + \frac{1}{4}(3-4)$$

$$= \frac{7}{4}$$

b)  $f(x) = \sqrt{x}$ ,  $a = 8$ ,  $x = 9$ .

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$= 2 + \frac{1}{12}(x-8)$$

$$f(9) = 2 + \frac{1}{12}(9-8)$$

$$= \frac{25}{12}$$

c)  $f(x) = x^2 + 3$ ,  $a = 2$ ,  $x = 2.2$

$$f(x) = 2x$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$= 4 + 3 + 4(x-2)$$

$$= 7 + 4(x-2)$$

$$f(2.2) = 7 + (4)(0.2)$$

$$= 7 + 0.8 = 7.8$$

c)  $f(x) = \frac{1}{x}$ ,  $a = 5$ ,  $x = 5.3$ .

$$f'(x) = -\frac{1}{x^2}$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$= \frac{1}{5} - \frac{1}{25}(x-5)$$

$$f(5.3) = \frac{1}{5} - \frac{1}{25}(0.3)$$

$$= \frac{47}{100}$$

d)  $f(x) = \frac{1}{x^2}$ ,  $a = 3$ ,  $x = 2.8$ .

$$f'(x) = -\frac{2}{x^3}$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$= \frac{1}{9} - \frac{2}{27}(x-3)$$

$$f(2.8) = \frac{1}{9} - \frac{2}{27}(0.2)$$

$$= \frac{13}{135}$$

e)  $f(x) = (x-2)^3$ ,  $a = 3$ ,  $x = 3.1$

$$f'(x) = 3(x-2)^2$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$= 1 + 3(x-3)$$

$$f(3.1) = 1 + 3(0.1)$$

$$= 1.3$$

5.3.1.4.

$$f(x) = \sin x$$

$$f'(x) = \cos x$$

$$L(x) = \sin 0 + \cos 0(x-0) = x$$

$$\text{let } f(x) = \cos x$$

$$f'(x) = -\sin x$$

$$\text{when } x = 0$$

$$L(x) = \cos 0 - \sin 0(x-0)$$

$$= 1$$

5.3.1.5.

$$2) f(7) = \sqrt[3]{8} = 2$$

$$L(x) = f(7) + f'(7)(x-7)$$

$$= f(7) + \frac{1}{3}(x+1)^{-\frac{2}{3}} \cdot (x-7)$$

$$= 2 + \frac{1}{3} \cdot 8^{-\frac{2}{3}} (x-7)$$

$$= 2 + \frac{1}{12} (x-7)$$

$$\therefore \sqrt[3]{9} = f(8)$$

$$= 2 + \frac{1}{12} (1)$$

$$= \frac{25}{12}$$

5.3.6.

$$a) f(x) = 2x^2$$

$$df = f'(x) dx$$

$$= 4x dx$$

$$b) g(t) = t^3 - t$$

$$dy = (3t^2 - 1) dx$$

$$c) f(t) = \sqrt{t+1}$$

$$dy = \frac{1}{2\sqrt{t+1}} (dx)$$

$$d) p(q) = 2q^{\frac{3}{2}} + q^{\frac{1}{2}}$$

$$dy = (3q^{\frac{1}{2}} + \frac{1}{2}q^{-\frac{1}{2}}) dx$$

$$(e) h(s) = s + \frac{2}{s}$$

$$= s + 2s^{-1}$$

$$dy = (1^2 - 2s^{-2}) dx$$

$$f) p(q) = \frac{q-1}{q^2+1}$$

$$dy = \frac{1(q^2+1) - 2q(q-1)}{(q^2+1)^2} \cdot dx$$

$$g) f(x) = \sqrt{3x^2 - x}$$

$$dy = \frac{6x-1}{2\sqrt{3x^2-x}} \cdot dx$$

$$5.3.8 f(x) = x^2 - 1$$

$$a) df = 2x dx$$

$$f(x) = \frac{1}{x}$$

$$df = -\frac{1}{x^2} dx$$

$$b) f(x) = x^2 - 1$$

$$2y: dy = f'(x) dx$$

$$= f'(1)(0.01)$$

$$= 2(0.01)$$

$$= 0.02$$

$$f(x) = \frac{1}{x}$$

$$dy: dy = f'(x) dx$$

$$= f'(-1)(-1.01+1)$$

$$= -1(0.01)$$

$$= -0.01$$

$$(c) f(x) = x^2 + 1$$

$$dy = f(0.9) - f(1)$$

$$= (0.9^2 + 1) - (1^2 + 1)$$

$$= -0.19$$

$$f(x) = \frac{1}{x}$$

$$dy = f(1.01) - f(1)$$

$$= \left(\frac{1}{1.01}\right) - (-1)$$

$$= 1.00990$$