MATH

4. Derivatives

4.1 The Rate of Change of a Function.

slope defined : quotient of the difference in y-values and x-values.

difference between two quantities denote : "Δ" "delta"

delta notation : calculating and interpreting the slope of a line

Calculating and Interpreting the slope of a line.

if we have (x_1, y_1) (x_2, y_2), on $y = f(x)$, so
the slope of the line is calculated by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$
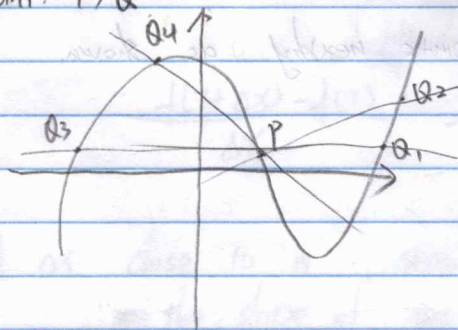
eg. if $y = 100x + 5$, a small change in x correspond,
a 100 times large in y. So y is sensitive to changes
in x.

· the properties of ~~tage~~ tangent line and
            secant line

Secant line.

Secant → cut.

secant line cut arbitrary curve. ⟶ $y = f(x)$ through two
point. P, Q

Definition 4.1   Slope of Secant line — Average (平均值) Rate of change

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derivatives
n.衍生物

embark.
V.着手,从事

groundward
n.基础.

terminology.
n.术语

slope
n.斜度

quotient.
n.程度

notation.
n.注释

interpreting.
n.解释

correspond.
V.相当于

property.
n.性质

pertinent.
adj.相关的

arbitrary.
adj.任意的

curve.
n.曲线

respectively.
adv.分别地.

necessity.
n.必要

readily
adj.轻而易举.

MATH

suppose have two points $(x_1, y_1)(x_2, y_2)$ the curve is $y=f(x)$ suppose.
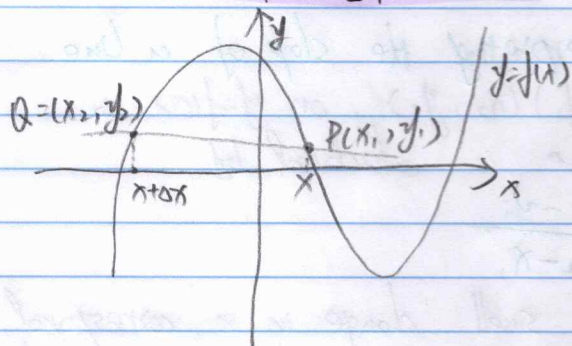Then the slope is calculated by.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

change the x ~~≠~~ . the two point will be $(x, f(x))(x+\Delta x, f(x+\Delta x))$ quotient
so :

$$m = \frac{\Delta y}{\Delta x} = \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$$

V. 假设
referred to.
被提及.
n. 高(数学)
geometry.
n. 几何

Note :

1. In the above figure, Δx must be negative.
2. It is also average rate of change. $D(f) = [x, x+\Delta x]$
3. The expression $\frac{f(x+\Delta x) - f(x)}{\Delta x}$ is referred to as ~~≠~~ the difference quotient.

Tangent Line.

Tangent = touch., tangent line touches an arbitrary curve
$= y = f(x)$
Point P. but not any other points nearby as shown.

tangent line just touch one point, we don't have any solution.
to calculate the slope with slope formula unless we use geometry.

MATH

Suppose $y = f(x)$, y is ~~sensen~~ sensitive, that even small change of x.

Example 4.2 ; Small changes in x.

$$y = f(x) = \sqrt{625 - x^2} \text{ (radius 25) }, x = 7.$$

when $x = 7$.
$$y = \sqrt{625 - 49}$$
$$= 24.$$

when $x = 7$ change to $x = 7.1$

$$\Delta x = 0.1.$$

$$\Delta y = f(7.1) - f(7)$$
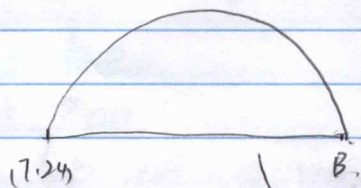$$= \sqrt{625 - 7.1^2} - \sqrt{625 - 7^2}$$

$$\approx -0.0294.$$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{-0.0294}{0.1}$$

$\therefore y$ is not very sensitive.

$$= -0.294.$$

★: It is not in fact at the particular function.
The quantity $\frac{\Delta y}{\Delta x} \approx -0.294$, may be interpreted as the slope of the secant line through $(7, 24)$ and $(7.1, 23.970b)$

$(7,24)$ B.

the slope of this secant line is the so-called
difference quotient

$$\frac{f(7 + \Delta x) - f(7)}{\Delta x} = \frac{\sqrt{625 - (7 + \Delta x)^2} - 24}{\Delta x}$$

若 Δx close to 0 , secant line will close to tangent line.
$\therefore$ ~~se~~ the slope of secant line will close to slope of
tangent line.