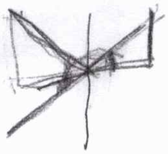


MATH K7.

5-7.1

(a) $y=x^2-x$

$y'=2x-1$

$0=2x-1$

$1=2x$

$x=\frac{1}{2}$

$y'(0)=-1<0 \quad y'(1)=1>0$

$\searrow \quad \frac{1}{2} \quad \nearrow$

$\therefore$ relative minimum point

$x=\frac{1}{2}$

$y=\frac{1}{2}^2-\frac{1}{2}$

$=-\frac{1}{4}$

$\therefore \left(\frac{1}{2},-\frac{1}{4}\right)$

(j) $f(x)=|x^2-121|$

$f(x)=\begin{cases} x^2-121 & \text{otherwise} \\ 121-x^2 & -11\le x\le 11 \end{cases}$

$f'(x)=\begin{cases} 2x & \text{otherwise} \\ -2x & -11\le x\le 11 \end{cases}$

$0=2x$

$x=0$

$\uparrow$ mix

$f'(-20) \quad f'(-1) \quad f'(-1) \quad f'(20)$

$\searrow \; -11 \; \nearrow \; 0 \; \searrow \; 11 \; \nearrow$

$\downarrow$ min $\qquad \downarrow$ min

(h) $y=\cos 2x-x$

$y'=-2\sin 2x-1$

$0=-2\sin 2x-1$

$1=-2\sin 2x$

$-\frac{1}{2}=\sin 2x$

$2x=\frac{7\pi}{6}$ or $\frac{11\pi}{6}$

$\therefore x=\frac{7\pi}{12}$ or $\frac{11\pi}{12}$

$y'\left(\frac{\pi}{2}\right)<0, \; y'\left(\frac{3\pi}{4}\right)>0, \; y'(\pi)<0$

$\searrow \; \frac{7\pi}{12} \; \nearrow \; \frac{11\pi}{12} \; \searrow$

$\downarrow$ min $\qquad \downarrow$ mix

$y=\cos 2\left(\frac{7\pi}{12}\right)-\frac{7\pi}{12}$

$y=-\frac{\sqrt{3}}{2}-\frac{7\pi}{12}$

$=-2.6986 \longrightarrow$ min $\left(\frac{7\pi}{12}, -2.6986\right)$

$y=\cos 2\left(\frac{11\pi}{12}\right)-\frac{11\pi}{12}$

$=-2.0138 \longrightarrow$ mix $\left(\frac{11\pi}{12}, -2.0138\right)$.

(k) $f(x)=\frac{x^3}{x+1} \quad x\neq -1$

$f'(x)=\frac{3x^2(x+1)+x^3}{(x+1)^2}$

$f'(x)=\frac{3x^3+3x^2+x^3}{(x+1)^2}$

$f'(x)=\frac{2\cancel{4}x^3+3x^2}{(x+1)^2}$

$f'(-2) \quad f'(-1.5) \quad f'(1)$

$\searrow \; -\frac{3}{2} \; \nearrow \; 0 \; \nearrow$

$\downarrow$ min

$0=\frac{2\cancel{4}x^3+3x^2}{(x+1)^2}$

$0=2\cancel{4}x^3+3x^2$

$0=x^2(2x+3)$

$0=2x+3$

$-3=2x$

$x=-\frac{3}{2}$ or 0 and $x\neq -1$

5.7.11

d. $y = x^4 - 2x^2 + 3$

$y' = 4x^3 - 4x + 3$

$y'' = 12x^2 - 4$

$0 = 12x^2 - 4$

$4 = 12x^{\cancel{3}2}$

$\frac{1}{3} = x^{\cancel{3}2}$

$x = \pm\sqrt[\cancel{3}]{\frac{1}{3}}$

$x = \pm\frac{1}{\sqrt{3}}$

$y''(-1), y''(0), y''(1)$

$\nearrow -\frac{1}{\sqrt{3}} \searrow \frac{1}{\sqrt{3}} \nearrow.$

∴ concave up at $(-\infty, -\frac{1}{\sqrt{3}})$
and $(\frac{1}{\sqrt{3}}, \infty)$
concave down and $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

(f) $y = \frac{x^2 - 1}{x}$

$y = x - \frac{1}{x}$

$y' = 1 + \frac{1}{x^2}$

$y'' = -2x^{-3}$

$y'' = \frac{-2}{x^3}$ $x \neq 0$.

$y''(-1) \quad y''(1)$

$\nearrow \quad 0 \quad \searrow$

∴ concave up at $(-\infty, 0)$
concave down at $(0, \infty)$

5.7.20.

(a) $y = x^2 - x$

$y' = 2x - 1$

$y'' = 2$.

$0 = 2x - 1$

$x = \frac{1}{2} > 0$

∴ $x = \frac{1}{2}$ is ~~a~~ relative minimum

(e) $y = 3x^4 - 4x^3$

$y' = 12x^3 - 12x^2$

$y'' = 36x^2 - 24x$

$0 = 12x^3 - 12x^2$

$0 = 12x^2(x - 1)$

$x = 0$ or $x = 1$

$y''(0) = 0$. $y''(1) = 36 - 24$
$= 12 > 0$

∴ x=1 is the relative minimum.

(c) $y = x^3 - 9x^2 + 24x$

$y' = 3x^2 - 18x + 24$

$y'' = 6x - 18$

$0 = 3x^2 - 18x + 24$

$x = 4$ or $x = 2$

$y''(4) = 24 - 18$
$= 6 > 0$

$y''(2) = 12 - 18$
$= -6 < 0$

∴ x=4 is the relative minimum.
x=2 is the relative maximum

5.7.26.

(a) $N(t) = -t^3 + 5t^2 + 25t \quad 0 \leq t \leq 8$.

$N'(t) = -3t^2 + 10t + 25$.

$t = 1 \qquad\qquad t = 6$

$N'(1) = -3 + 10 + 25 \qquad N'(6) = 3(6)^2 + 60 + 25$

$= 32 > 0. \qquad\qquad = -23 < 0.$

$\nearrow \qquad\qquad \searrow$

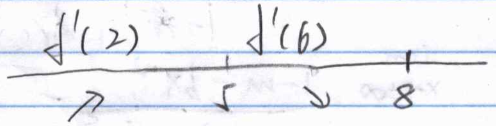
$\therefore$ it is increasing between 6am to 11 am.

it is decreasing between 11am to 2pm.

(b)

$0 = -3t^2 + 10t + 25$

$x = -\frac{5}{3}$ (x) or $x = 5$.



$\therefore$ the maximal at 11 am.

5.7.30.

(a) $f(x) = \frac{1}{x}$

$\lim\limits_{x \to 0} \frac{1}{x} = \infty$

$\lim\limits_{x \to -\infty} \frac{1}{x} = -\infty$

$\therefore$ V.A. is $x = 0$.

$\lim\limits_{x \to +\infty} \frac{1}{x}$

$= 0$

$\lim\limits_{x \to \infty} \frac{1}{x}$

$= 0$.

$\therefore$ H.A. is $y = 0$

(b) $f(s) = \frac{s^2 - 2}{s^2 - 4}$

$\lim\limits_{x \to 2^+} = \infty$

$\lim\limits_{x \to 2^-} = -\infty \quad \therefore$ V.A. is $x = 2$.

$\lim\limits_{x \to \infty^+} = \frac{25}{25}$

$= 1$

$\lim\limits_{x \to \infty^-} = 1 \quad \therefore$ H.A. is $y = 1$.

5.7.31

(a) $f(x) = \frac{5x^2 - 3x + 1}{x + 2}$

$\lim\limits_{x \to \pm\infty} \left[\frac{5x^2 - 3x - 1}{x + 2} - (mx + b) \right] = 0$

$\lim\limits_{x \to \pm\infty} \frac{5x^2 - 3x - 1 - (mx^2 + bx + 2mx + 2b)}{x + 2}$

$= \lim\limits_{x \to \pm\infty} \frac{5x^2 - 3x - 1 - mx^2 - bx - 2mx - 2b}{x + 2}$.

$\lim_{x\to\pm\infty} 10x - 3 - 2mx - b - 2m$

$\lim_{x\to\pm\infty} 10 - 2m$ ~~m~~

$10 - 2m = 0$

$5 - m = 0$

$\lim_{x\to\pm\infty}\left[\frac{5x^2-3x+1}{x+2} - (mx+b)\right]$

$= \lim_{x\to\pm\infty} \frac{5x^2-3x-1-mx^2-bx-2mx-2b}{x+2}$

$= \lim_{x\to\pm\infty} \frac{5x-3-x^{-1}-mx-b-2m-2bx^{-1}}{1+2x^{-1}}$

$= \lim_{x\to\pm\infty} \frac{\cancel{5}+x^{-2}\cancel{-4}+2bx^{-2}}{-2x^{-2}}$

$= \lim_{x\to\pm\infty} \frac{\overset{1}{\cancel{x^{-2}}}+2b\cancel{x^{-2}}}{-2\cancel{x^{-2}}}$

$= \lim_{x\to\pm\infty} \frac{1+2b}{-2}$

$\frac{1+2b}{-2} = 0$

$1+2b = 0$

$2b = -1$

$b = -\frac{1}{2}$

$5x - \frac{1}{2} = y$

(b) $\frac{x^2}{x-1}$

$\lim_{x\to\pm\infty}\left[\frac{x^2}{x-1} - (mx+b)\right] = 0$

$\lim_{x\to\pm\infty} \frac{x^2-(mx^2+bx-mx-b)}{x-1}$

$\lim_{x\to\pm\infty} \frac{x^2-mx^2+bx+mx+b}{x-1}$

$\lim_{x\to\pm\infty} \frac{2x-2mx-b+m}{1}$

$\lim_{x\to\pm\infty} 2-2m = 0$

$2 = 2m$

$m = 1$

$\lim_{x\to\pm\infty} \frac{x-mx-b+m+bx^{-1}}{1-x^{-1}}$

$\lim_{x\to\pm\infty} \frac{1-m-bx^{-2}}{x^{-2}}$

$\lim_{x\to\pm\infty} \frac{2b\cancel{x^{-3}}}{-2\cancel{x^{-3}}}$

$\frac{2b}{-2} = 0$

$2b = 0$

$b = 0$

$\therefore y = x$

(c) $f(x) = \frac{x^2+3x+2}{x-1}$

$\lim_{x\to\pm\infty}\left[\frac{x^2+3x+2}{x-1} - (mx+b)\right] = 0$

$\lim_{x\to\pm\infty}\left[\frac{x^2+3x+2-(mx^2+bx-mx-b)}{x-1}\right]$

$\lim_{x\to\pm\infty} \frac{x^2+3x+2-mx^2-bx+mx+b}{x-1}$

$\lim_{x\to\pm\infty} \frac{2x+3-2mx-b+m}{1}$

$\lim_{x\to\pm\infty} 2-2m$

$2-2m=0$

$2=2m \quad m=1$

$\lim_{x\to\pm\infty} \frac{x+3-2x^{-1}-mx-b+m+bx^{-1}}{1-x^{-1}}$

$\lim_{x\to\pm\infty} \frac{1+2x^{-2}-m-bx^{-2}}{x^{-2}}$

$\lim_{x\to\pm\infty} \frac{-4\cancel{x^{-3}}+2b\cancel{x^{-3}}}{-2\cancel{x^{-3}}}$

$\lim_{x\to\pm\infty} \frac{\overset{2}{\cancel{-4}}+\overset{-2}{2b}}{\cancel{-2}} = 0$

$2-2b = 0$

$2=2b$

$b=1$

$y = \cancel{x}x+1$

(d) $f(x) = \frac{x^2-5x+4}{x-3}$

$\lim_{x\to\pm\infty} \left[\frac{x^2-5x+4}{x-3} - (mx+b) \right]$

$\lim_{x\to\pm\infty} \frac{x^2-5x+4-(mx^2+bx-3mx-3b)}{x-3}$

$\lim_{x\to\pm\infty} \frac{x^2-5x+4-mx^2-bx+3mx+3b}{x-3}$

$\lim_{x\to\pm\infty} \frac{2x-5-2mx-b+3m}{1}$

$\lim_{x\to\pm\infty} 2-2m$

$2-2m=0$

$2=2m$

$m=1$

$\lim_{x\to\pm\infty} \frac{x-5+4x^{-1}-mx-b+3m+3bx^{-1}}{1-3x^{-1}}$

$\lim_{x\to\pm\infty} \frac{1-4x^{-2}-m-3bx^{-2}}{3x^{-2}}$

$\lim_{x\to\pm\infty} \frac{8\cancel{x^{-3}}+6b\cancel{x^{-3}}}{-6\cancel{x^{-3}}}$

$\frac{8+6b}{-6} = 0$

$8+6b = 0$

$b = -\frac{4}{3}$

$y = x - \frac{4}{3}$

5.7.32

(a) $y = x^5 - 5x^4 + 5x^3$

①

Domain $(-\infty, \infty)$

②

$0 = x^5 - 5x^4 + 5x^3$

$= x^3(x^2-5x+5)$

$) = x^3(x-3.6180)(x-1.3820)$

$\therefore x = 0$ or $x = 3.618$ or $x = 1.382$

$y = 0$.

③. NO V.A.

$\lim_{x\to\infty^+} x^5-5x^4+5x^3$

$= \lim_{x\to\infty^+} 5x^4-20x^3+15x^2$

$= \lim_{x\to\infty^+} 20x^3-60x^2+30x$

$= \lim_{x\to\infty^+} 60x^2-120x+30$

$= \lim_{x\to\infty} 120x-120$

$= {}^{\#}120$.

$\therefore$ H.A $\neq$ 120.

④. $f'(x) = 5x^4-20x^3+15x^2$

$0 = 5x^4-30x^3+15x^2$

$0 = x^2(5x^2-30x+15)$

$0 = 5x^2(x^2-6x+3)$

$x=0$ or $x=5.4495$ or $x=0.5505$

$f'(-1)$, $f'(0.2)$, $f'(1)$, $f'(6)$

↗ 0 ↗ 0.5505 ↘ 5.4495 ↗

⑤ $f''(x) = 20x^3-60x^2+15x$

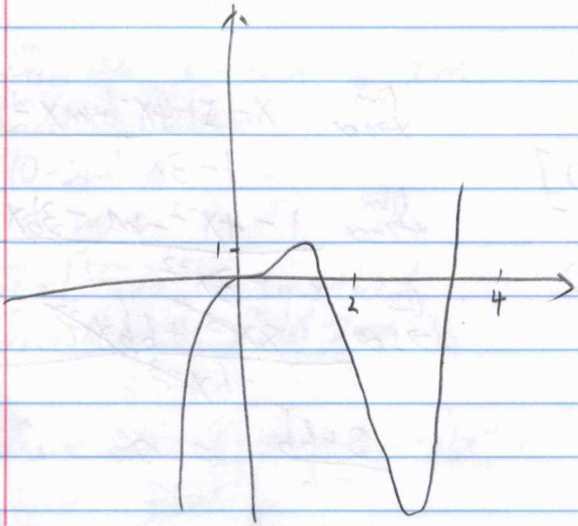
$0 = 20x^3-60x^2+15x$

$0 = 5x(4x^2-12x+3)$

$x=0$ or $x=2.7247$ or $x=0.2753$.

$f''(-1)$, $f''(0.1)$, $f''(1)$, $f''(3)$

$\cap$ 0 $\cup$ 0.2753 $\cap$ 2.724 $\cup$



c) $y = (x-1)^2(x+3)^{\frac{2}{3}}$

① Domain $(-\infty, \infty)$

②

$0 = (x-1)^2(x+3)^{\frac{2}{3}}$

$\therefore x = 1$ or $x = -3$

$y = 1, \sqrt[3]{3^2}$

$y = \sqrt[3]{3^2}$

③ NO V.A.

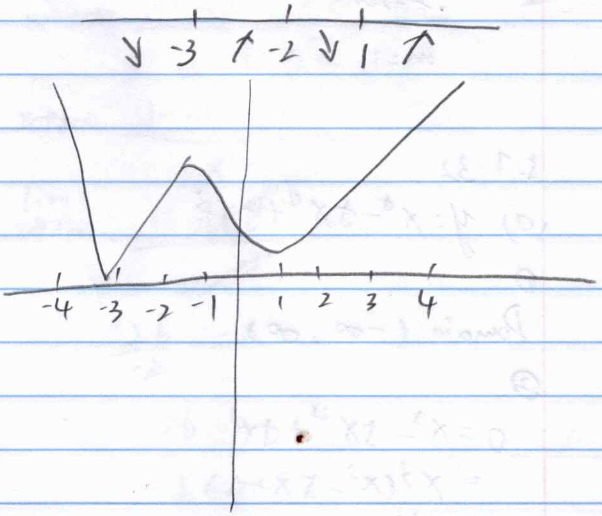
$\lim_{x \to \infty} (x-1)^2(x+3)^{\frac{2}{3}}$

$= 2(x-1)^{\frac{2}{3}}(x+3)^{+\frac{1}{3}}$

$= \infty$

NO H.A.

④ $f'(x) = 2(x-1)(x+3)^{\frac{2}{3}} + (x-1)^2\frac{2}{3}(x+3)^{-\frac{1}{3}}$



e) $y = x^5 - x$

① Domain $(-\infty, \infty)$

②

$0 = x^5 - \cancel{x}$

$x = \cancel{x^5}x^5$

$x = 1$

$y = 0$.

③ $\lim_{x \to \infty} x^5 - x$

$= 5x - 1$

$= 5$.

$\therefore$ H.A is 5.

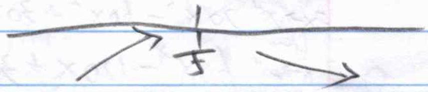
NO V.A.

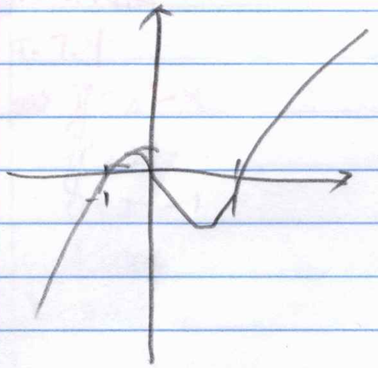
④ $f'(x) = 5x - 1$

$0 = 5x - 1$

$1 = 5x$

$x = \frac{1}{5}$





5.7.33

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