

Chapter 11

Inputs and Costs

This chapter is about *firms'* use of marginal analysis to make output decisions.



Vocabulary:

Fixed inputs = inputs whose quantity cannot change easily.

Examples: Factory capacity or the amount of land a farmer has for growing crops



Variable inputs = inputs whose quantity can be changed in the short run.

Examples: The amount of fertilizer or seeds a farmer uses; or the number of workers a factory hires



Long run = the time period over which all inputs can change.

Example: the time it takes to buy more land or build another factory

Short run = the time period over which at least one input is fixed.

Example: in the short run a farmer can change the number of workers but not the size of land.

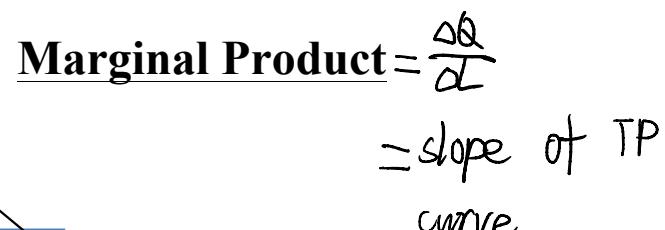
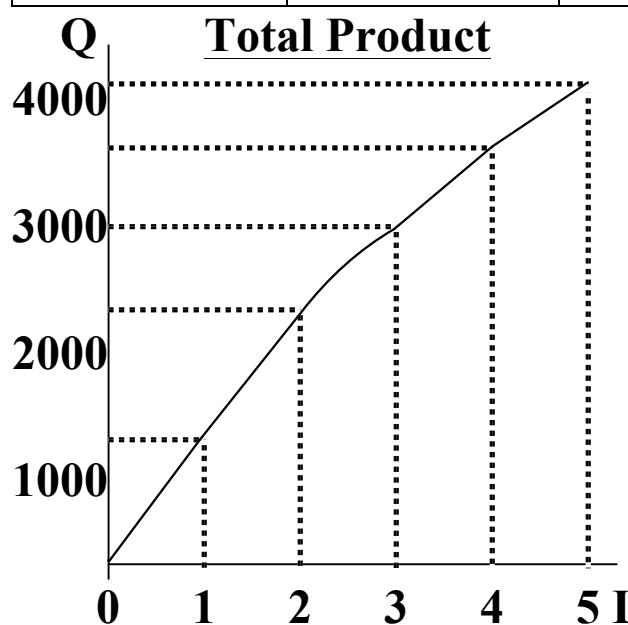
Production Functions

The Total Product Curve shows the relationship between total output and the quantity of the variable input, given a certain quantity of the fixed input.

The marginal product of labour = change in quantity of output which one extra worker, given a certain quantity of the fixed input.

# of Workers	Total Output	MPL
0	0	
1	1200	1200
2	2200	1000
3	3000	800
4	3600	600
5	4000	400

$$MPL = \frac{\Delta Q}{\Delta L}$$



Why is MPL so Important?

- Recall that *rational people think at the margin.*
- When a producer hires an extra worker,
 - costs rise by the wage paid to the worker.
 - Outputs rises by the MPL.
- Comparing the wage to the MPL helps firms decide whether or not it would be profitable to hire the extra worker.



There are diminishing returns to an input when

the MP of the input falls, holding other input fixed.

- In the above example there are *diminishing returns to labour* or *diminishing marginal product of labour*.
- The slope of the TP curve is MP, so a diminishing MP will result in a concave TP curve (decreasing slope).

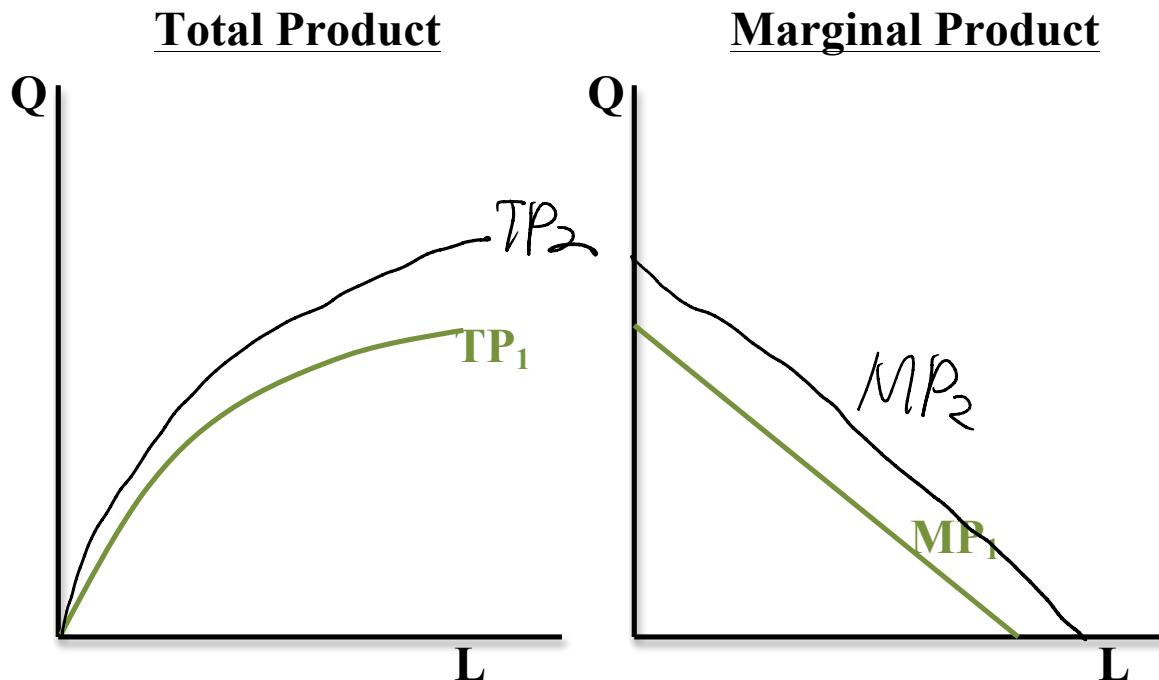
Why MPL Diminishes

The marginal product of an input declines as the quantity of the input increases, for a given amount of fixed input.

E.g. A farmer's output rises by a smaller and smaller amount with each additional worker. Why?

- If a farmer increases workers but not land,

An increase in the amount of the fixed input (e.g. land) will shift the TP and MP curves up:



Technology

Technology is also a fixed input because it takes time to improve. An improvement in technology will have the same effect as more of a fixed input: the TP and MP curves will shift up.



Exercise 1: The Perogy Heaven Food Truck Co.

Jacky runs a food truck business that produces many different flavours of perogies. Jacky inputs include: 3 trucks, insurance, gas, employees, food ingredients, cleaning supplies, and serving supplies (napkins, paper plates, and plastic utensils).

Which of the inputs would be fixed inputs?

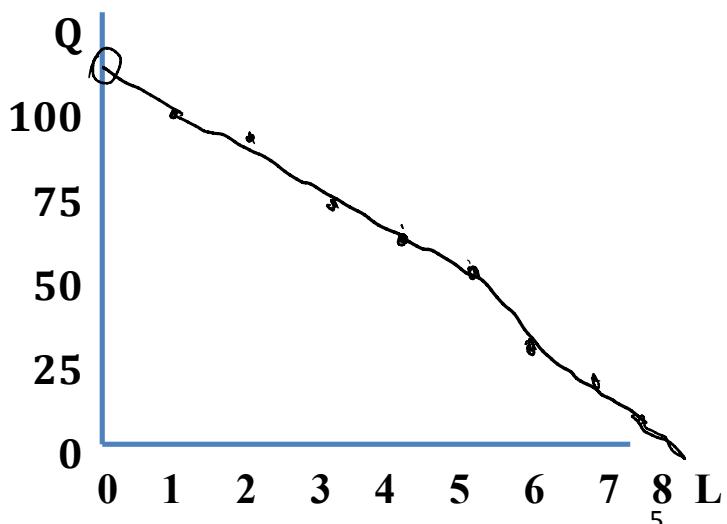
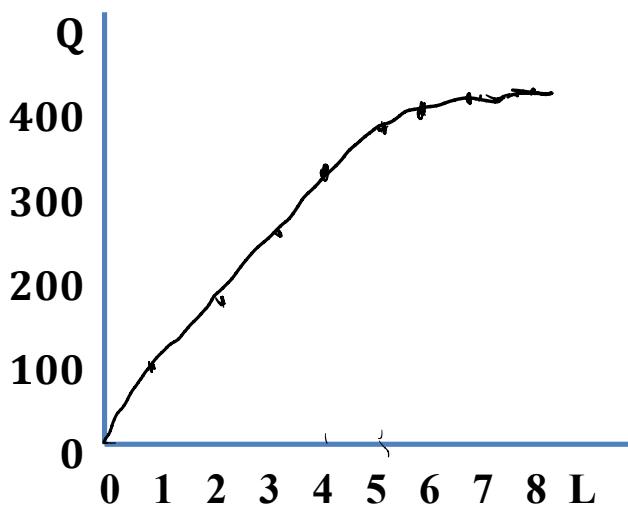
3 trucks, insurance

Which of the inputs would be variable inputs?

gas, employees, food ingredients, cleaning supplies, serving supplies

Fill in the following table and graph the TP and MPL curves:

L	Q	MPL
0	0	
1	100	100
2	185	85
3	255	70
4	310	55
5	350	40
6	375	25
7	385	10
8	385	0



Cost Curves

Cost curves show the relationship between quantity of output and costs of production.



Fixed Costs (FC) = cost of the fixed input

- Fixed costs do not depend on the quantity of output
- They must be paid no matter how much output is produced because the fixed input cannot change.
- Fixed costs are sunk costs
- E.g. Rent, insurance, vehicle lease payments

Variable Costs (VC) = cost of the variable inputs

- Variable costs depend on the quantity of output
- E.g. wages, food in a restaurant, gas for vehicles

Total Costs (TC) = $FC + VC$

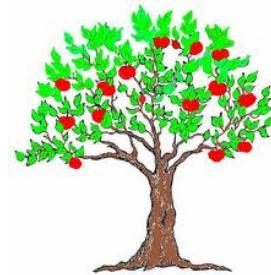
Marginal Cost = the additional cost of making one more unit.

$$MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta VC}{\Delta Q}$$

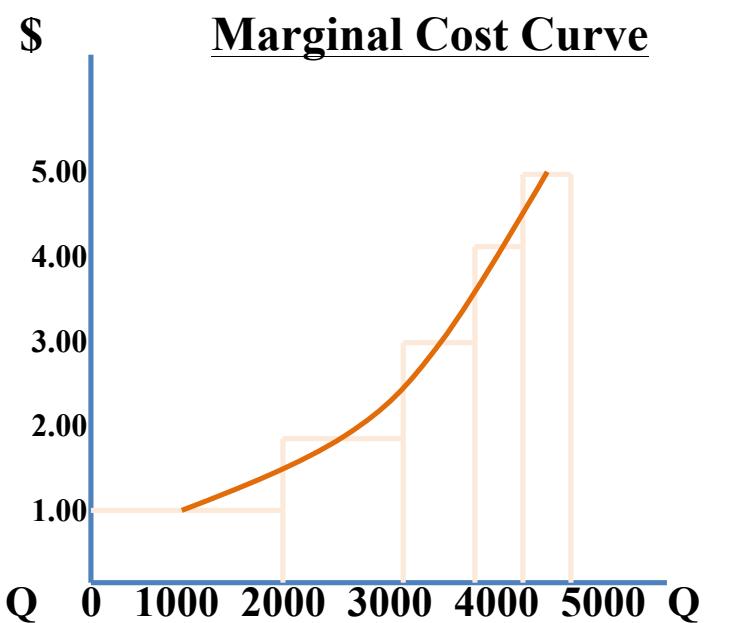
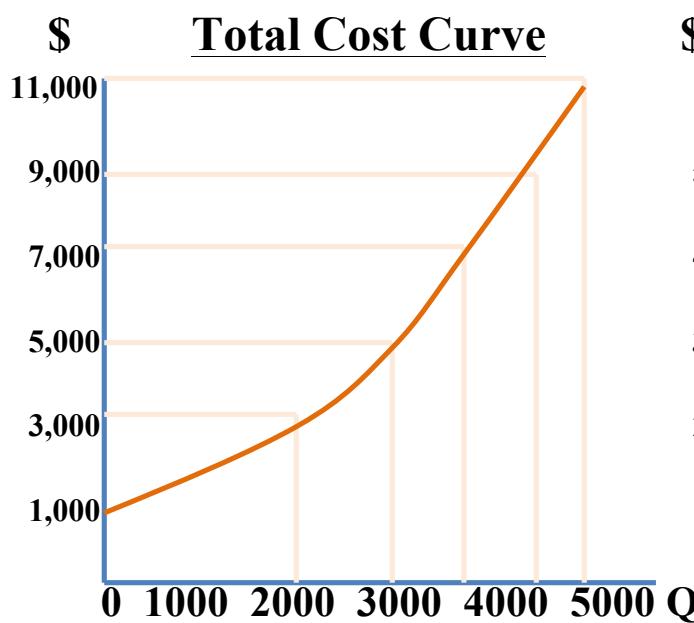
Marginal costs consist of variable costs, the costs that change when output increases by one unit.

Example: An Apple Farmer's Costs

The farmer must pay \$1000 per month for the land, regardless of how much he grows, and the wage for a farm worker is \$2000 per month.



L	Q	MPL	FC	VC	TC	MC
0	0		1000	0	1000	
		2000				$\frac{2000}{2000} = 1$
1	2000	1000	1000	2000	3000	
		1000				$\frac{2000}{1000} = 2$
2	3000	1000	1000	4000	5000	
		667				$\frac{2000}{667} = 3$
3	3667	1000	1000	6000	7000	
		500				$\frac{2000}{500} = 4$
4	4167	1000	1000	8000	9000	
		400				$\frac{2000}{400} = 5$
5	4567	1000	1000	10000	11000	



Explaining the Shape of the Cost Curves:

- The TC curve gets *increases* (slope increases) because marginal costs *increases*
- MC increases because the MP of your variable inputs (labour) diminishes
- The MP of your variable inputs diminishes because of the fixed input (e.g. lands)
- Each additional worker you hire will produce a smaller and smaller additional quantity of output.
- But you have to pay each worker the same → same additional costs.
- So each additional unit of production costs more
Diminishing MP \rightarrow rising MC
Flattening TP curve \rightarrow steepening TC curve.

Exercise 2: The Perogy Heaven Food Truck Co.

Jacky's costs are as follows:

Rent and insurance per day per truck \$100 (there are 3 trucks)

Wages per day for one employee \$80

Ingredients and supplies per perogy serving \$3

Fill in the table:

L	Q	MPL	FC	VC	TC	MC
0	0		300	0	300	
1	100	100	300	380	680	3.80
2	185	85	300	715	1015	3.94
3	255	70	300	1005	1305	4.14
4	310	55	300	1250	1550	4.45
5	350	40	300	1450	1750	5
6	375	25	300	1605	1905	6.2
7	385	10	300	1715	2015	11
8	385	0	300	1795	2095	

Average Cost Curves show how the average costs change as output increases

$$\text{Average Total Cost} = \overline{ATC} = \frac{\overline{TC}}{Q}$$

$$\text{Average Fixed Cost} = \overline{AFC} = \frac{\overline{FC}}{Q}$$

$$\text{Average Variable Cost} = \overline{AVC} = \frac{\overline{VC}}{Q}$$

Exercise 3: The Perogy Heaven Food Truck Co.

Fill in the table:

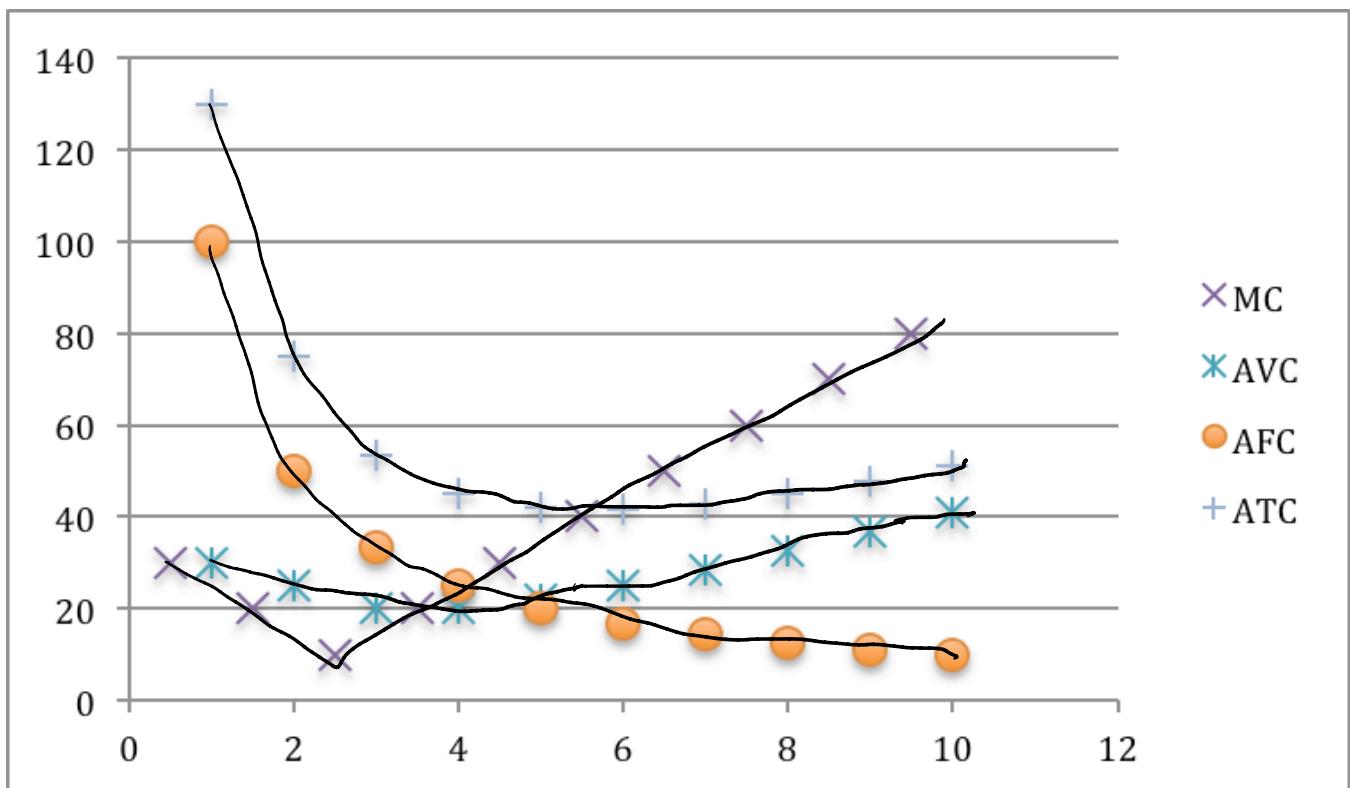
Q	FC	VC	TC	MC	AFC	AVC	ATC
0	300	0	300	-	-	-	-
100	300	380	680	3.80	3.00	3.80	6.80
185	300	715	1015	3.94	1.62	3.86	5.49
255	300	1005	1305	4.14	1.17	3.94	5.11
310	300	1250	1550	4.45	0.97	4.03	5.00
350	300	1450	1750	5.00	0.86	4.14	5.00
375	300	1605	1905	6.20	0.8	4.28	5.08
385	300	1715	2015	11.00	0.77	4.45	5.23
385	300	1795	2095	-	0.77	4.66	5.44

Example: Costs of producing chairs

Q	FC	VC	TC	MC	AVC	AFC	ATC
0	100	0	100		-	-	-
				30			
1	100	30	130		30	100	130
				20			
2	100	50	150		25	50	75
				10			
3	100	60	160		20	33.3	53.3
				20			
4	100	80	180		20	25	45.0
				30			
5	100	110	210		22	20	42.0
				40			
6	100	150	250		25	16.7	41.7
				50			
7	100	200	300		28.6	14.3	42.9
				60			
8	100	260	360		32.5	12.5	45.0
				70			
9	100	330	430		36.7	11.1	47.8
				80			
10	100	410	510		41	10	51

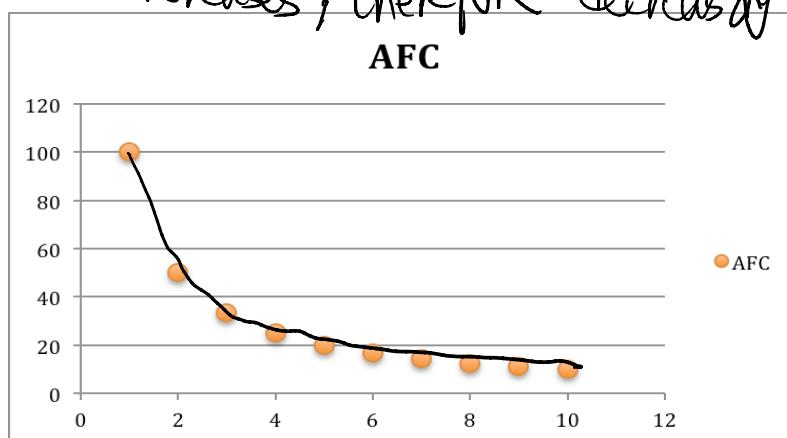


Graphing the Marginal and Average Cost Curves



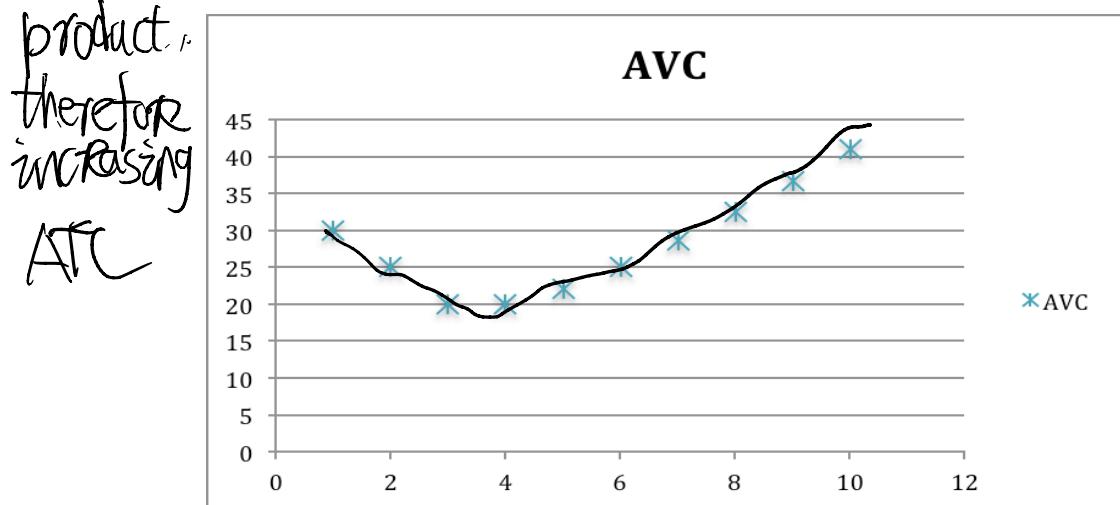
Explaining the shapes of the curves:

Spreading Effect: Average fixed costs decrease as output increases, therefore decreasing ATC.



$$AFC = \frac{FC}{Q}$$

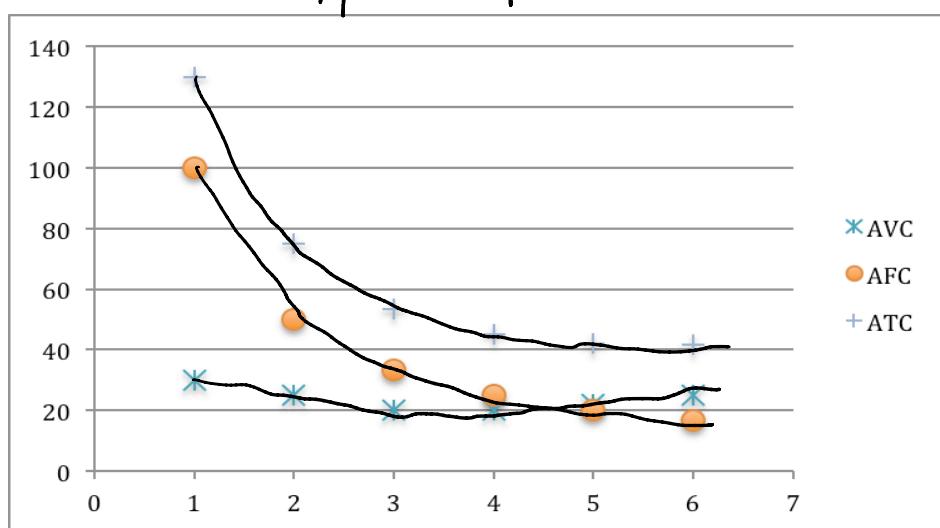
The Diminishing Returns Effect: Average variable costs increases as output increases because of diminishing marginal product, therefore increasing ATC.



Whichever effect is stronger (has a steeper slope) determines what happens to average total cost.

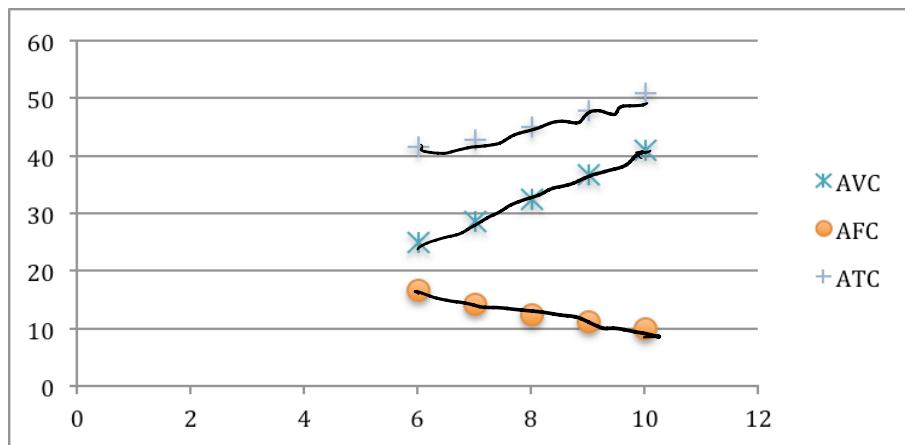
When the spreading effect is stronger than the diminishing returns effect:

- The decrease in AFC is greater than the increase in AVC.
- The ATC curve will slope down

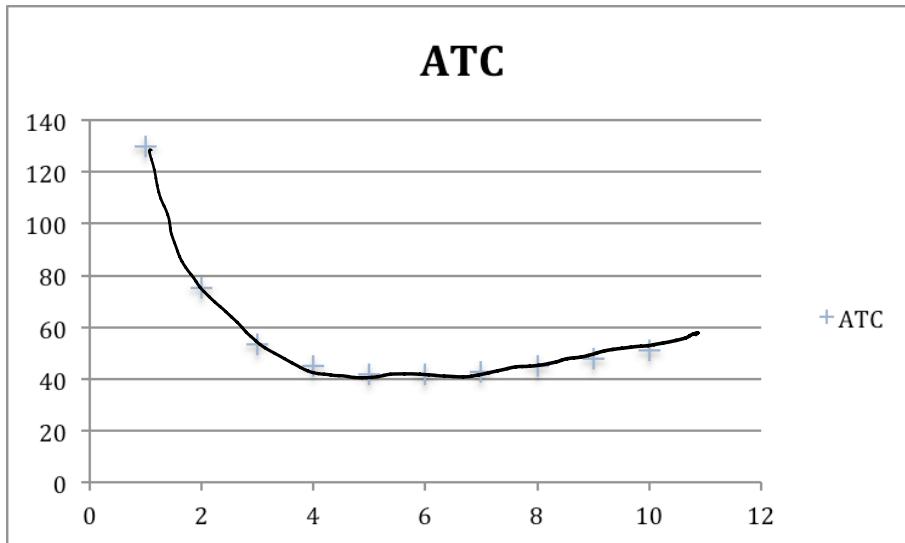


When the diminishing returns effect is stronger than the spreading effect:

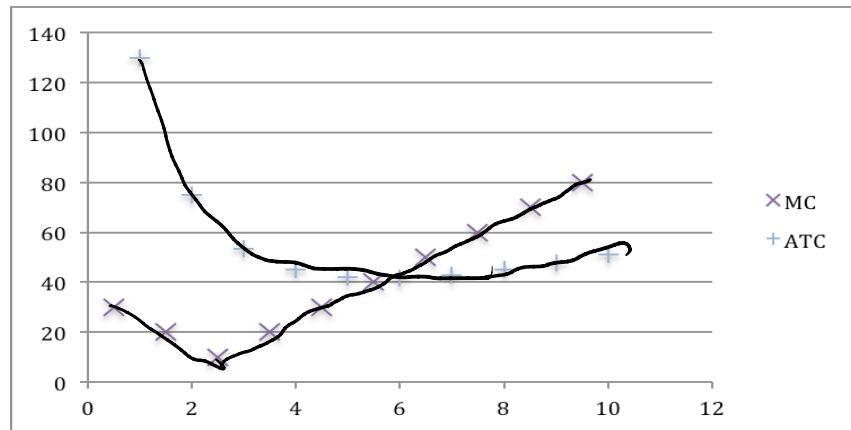
- The decrease in AFC is less than the increase in AVC.
- The ATC curve will



Together the two effects make the ATC curve U-shaped:



Marginal Cost and ATC Curves:



Recall that when calculating an average number:

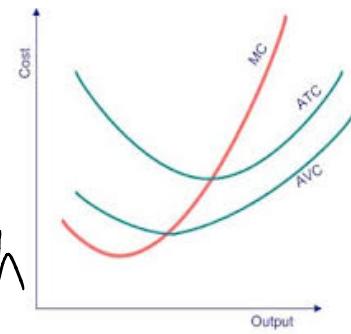
- An additional big number will increase the average.
- An additional small number will decrease the average.

$$\text{Average of } (3, 4, 5) = (3+4+5)/3 = 4$$

$$\text{Average of } (3, 4, 5, 6) = (3+4+5+6)/4 = 4.5$$

$$\text{Average of } (2, 3, 4, 5) = (2+3+4+5)/4 = 3.5$$

- The ATC curve will be above the MC curve as long as the additional marginal cost of a unit of output brings down the average $\rightarrow \text{ATC decreases}$
- When the MC curve is above the ATC curve additional marginal costs are increasing the average $\rightarrow \text{ATC increases}$
- The ATC and AVC curves are normally U-shaped and the MC curve crosses each at their lowest point.
- The quantity of output where ATC is lowest is the minimum-cost output
 - At this point $MC = ATC$



Exercise 4: The Perogy Heaven Food Truck Co.

Over what quantity range of perogy servings is the spreading effect stronger than the diminishing returns effect? 0-310

Over what quantity range of perogy servings is the diminishing returns effect stronger than the spreading effect? 350-385

What is the *minimum cost output* for Jacky? 350

At this point what is the ATC? 5

At this point what is the MC? 5

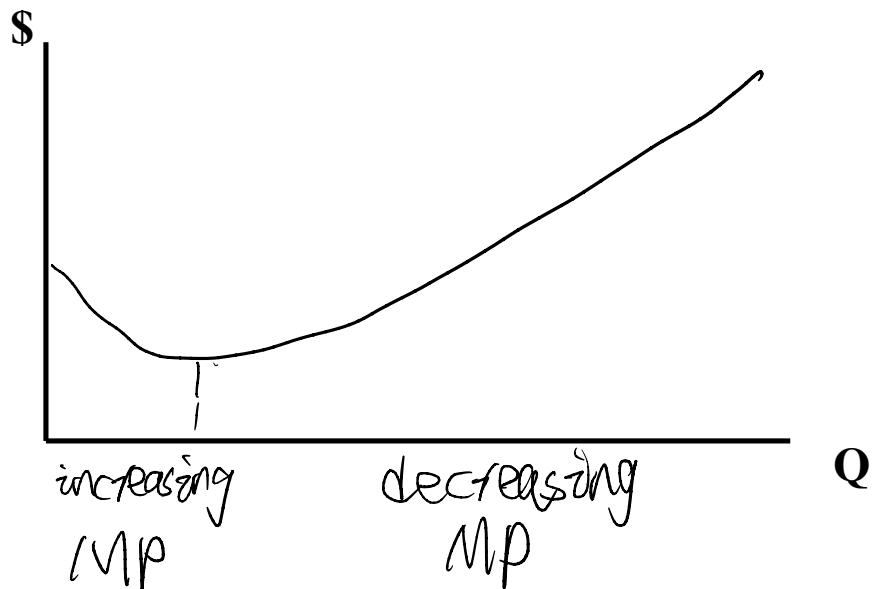
More on the MC Curve:

MC curves may slope down at very low levels of production if the marginal product of labour increases.

This will happen if there is a substantial increase in productivity from an additional worker due to the benefits of specialization.

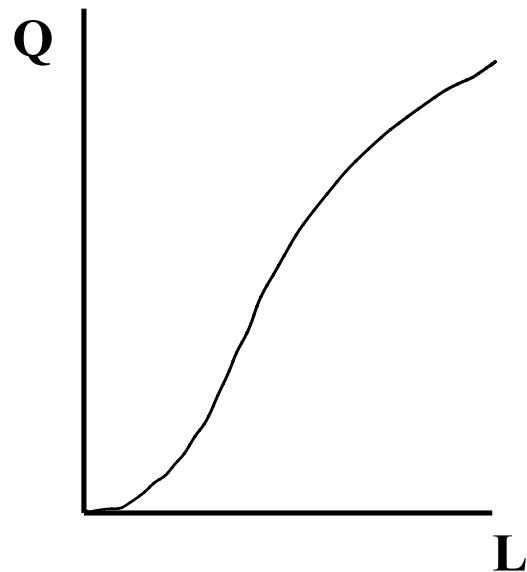
E.g. A restaurant could more than triple production by increasing workers from 1 to 3. One worker can cook, one serve food, and one do the dishes rather than someone trying to do all three tasks.

Typical MC curve:

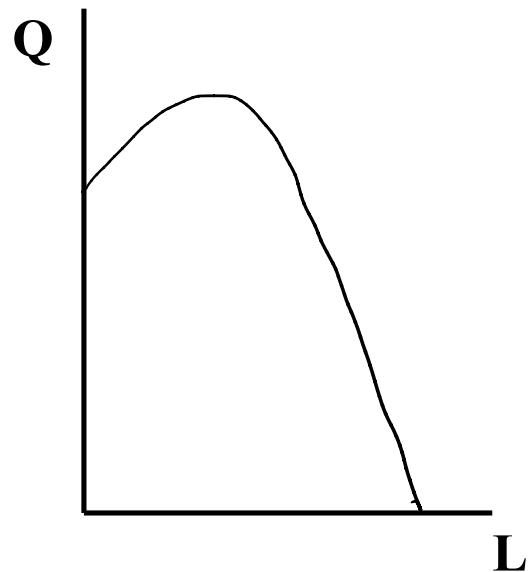


Summary:

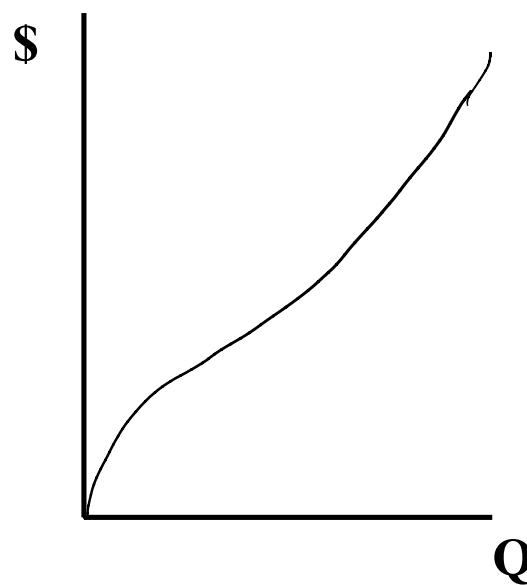
TP



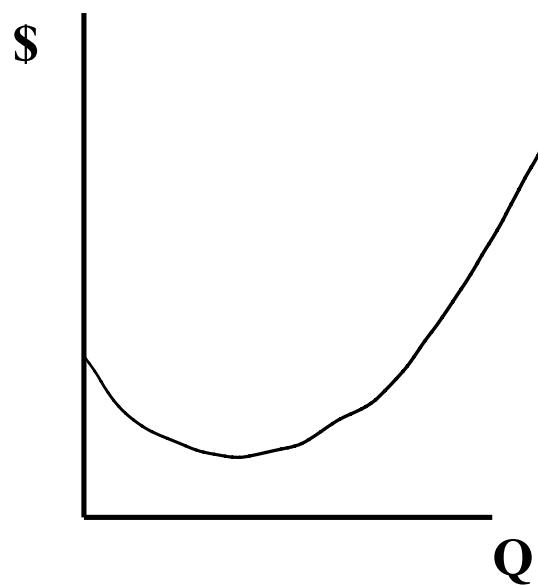
MP



TC



MC



Example with Equations:

A total cost function for producing a good is:

$$TC = 10 + Q + Q^2$$

Variable costs depend on the output produced, so the variable cost function is: $Q^2 + Q$

Fixed costs are not dependent on output. So the fixed costs are:

$$FC = 10$$

The average total cost function is:

$$\frac{Q^2 + Q + 10}{Q} = Q + 1 + \frac{10}{Q}$$

The average fixed cost function is:

$$\frac{10}{Q}$$

The average variable cost function is:

$$Q + 1$$

The average costs of producing 2 units of output are:

$$ATC = \frac{10}{2} + 1 + 2 = 8$$

$$AVC = \frac{10}{2} = 5$$

$$AVC = 1 + 2 = 3$$

Exercise 5: Total Cost Function

The total and marginal cost functions for a firm are:

$$TC = 50 + 5Q + 0.2Q^2$$

$$MC = 5 + 0.4Q$$

What are the average cost functions?

$$ATC = \frac{1}{5}Q + 5 + \frac{50}{Q}$$

$$AVC = \frac{1}{5}Q + 5$$

$$AFC = \frac{50}{Q}$$

At what quantity is ATC = MC?

$$\frac{1}{5}Q + 5 + \frac{50}{Q} = 5 + \frac{2}{5}Q$$

$$\frac{50}{Q} = \frac{Q}{5}$$

$$Q^2 = 250$$

$$Q = 5\sqrt{10}$$

What is this quantity called?

The minimum cost output

Summary of Short Run Cost Calculations

Total fixed costs	FC	Costs that are the same at all output levels
Total variable costs	VC	Costs that vary with output
Total cost	TC	Sum of variable and fixed costs
Marginal costs	MC	Extra cost of producing one more unit
Average total cost	ATC	TC per unit of output (TC/Q)
Average fixed cost	AFC	FC per unit of output (FC/Q)
Average variable costs	AVC	VC per unit of output (VC/Q)

Long Run Costs

In the long run all inputs can be changed, so a firm can change its fixed inputs, which will change the average total costs

- E.g. Buying more land or buildings, renting more office space or machinery.
- This will increase fixed costs but the productivity of workers will increase if they have more of the fixed inputs to work with, so marginal and variable costs will decrease.

Example: Costs of producing chairs

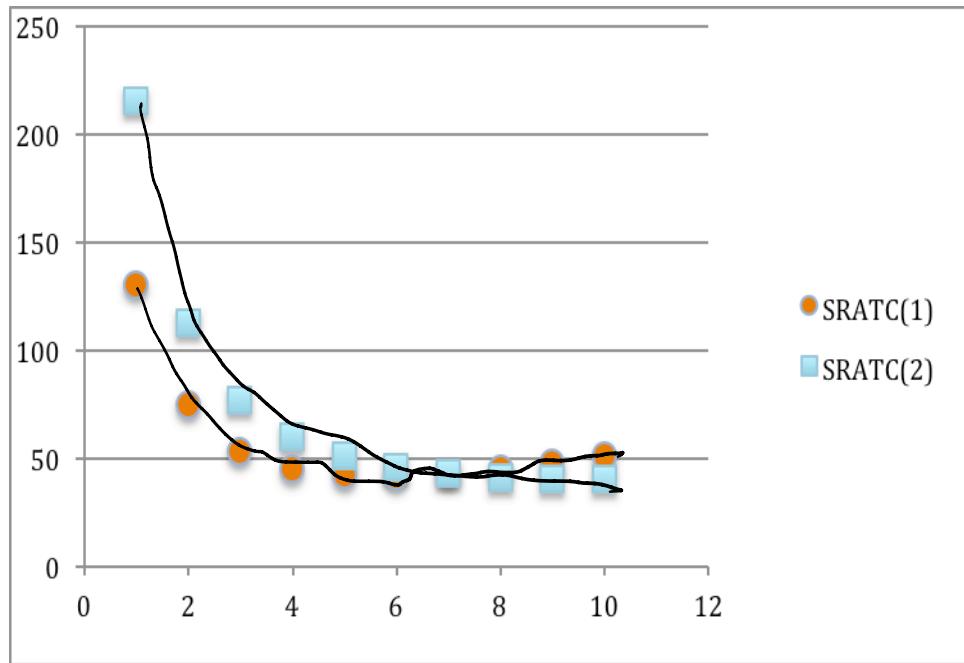
If the firm doubles its fixed inputs (factories) it can decrease its variable costs (wages) by one half.



The ATC for the two different quantities of fixed inputs are now called the *short run ATC curves* (SRATC).

Q	FC(1)	VC(1)	TC(1)	SRATC(1)	FC(2)	VC(2)	TC(2)	SRATC(2)
0	100	0	100		200	0	200	
1	100	30	130	130	200	15	215	215
2	100	50	150	75	200	25	225	112.5
3	100	60	160	53.3	200	30	230	76.7
4	100	80	180	45	200	40	240	60
5	100	110	210	42	200	55	255	51
6	100	150	250	41.7	200	75	275	45.8
7	100	200	300	42.9	200	100	300	42.9
8	100	260	360	45	200	130	330	41.3
9	100	330	430	47.8	200	165	365	40.6
10	100	410	510	51	200	205	405	40.5

Graph of the SRATC curves:



SRATC¹ is lowest when $Q \leq 7$. The lower fixed costs outweigh the higher variable costs.

SRATC² is lowest when $Q \geq 7$. The lower variable costs outweigh the higher fixed costs.

The amount of fixed input the firm chooses will depend on its output:

- If the firm only wants to produce $Q \leq 7$ it is better off with one set of fixed inputs (SRATC¹ → one factory).
- If the firm wants to produce $Q \geq 7$ it is better off with two sets of fixed inputs (SRATC² → two factories).

The long run ATC curve (LRATC) shows the relationship between output level and ATC when fixed costs have been chosen to minimize ATC.

→ All inputs are variable in the long run.

The Chair producer's Long Run ATC Curve

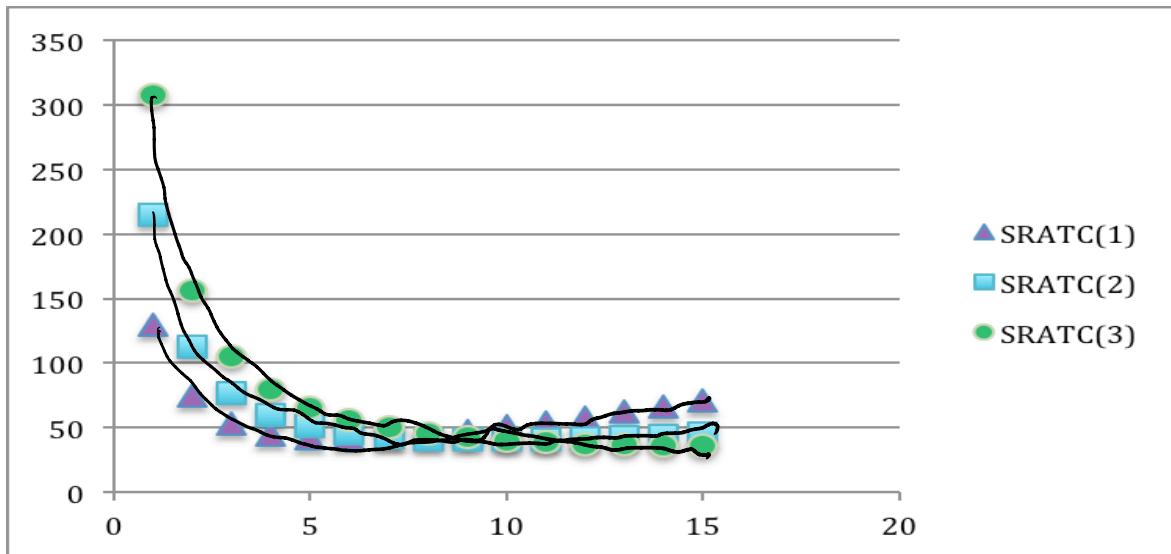
SRATC¹ = 1 factory

SRATC² = 2 factories

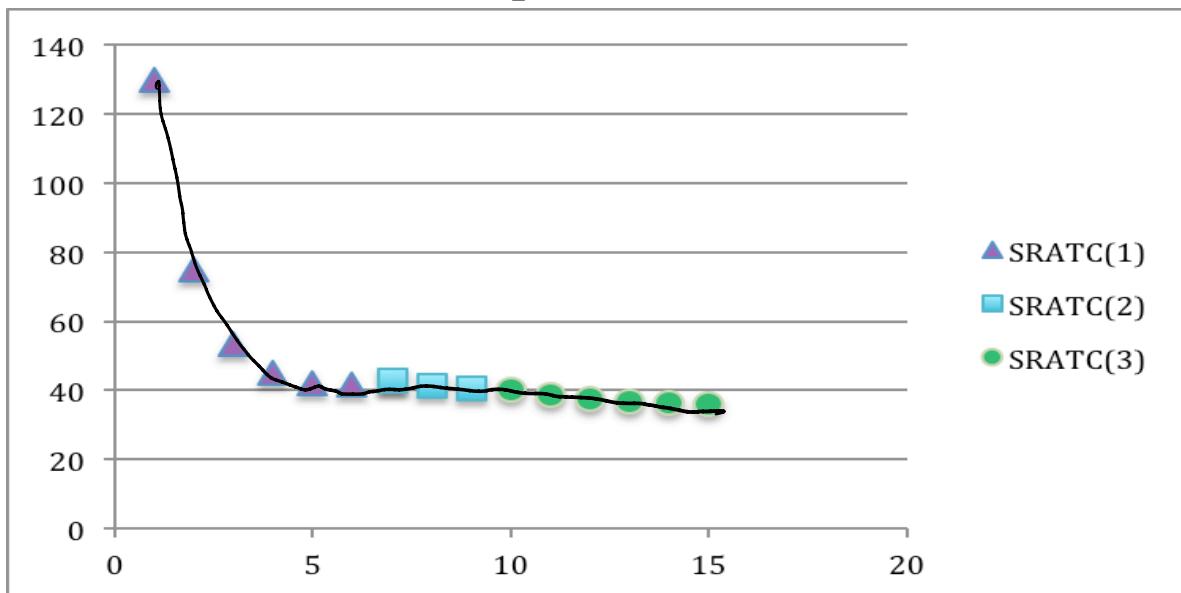
SRATC³ = 3 factories

The firm's LRATC is comprised of all the lowest point of its SRATC curves.

Q	SRATC(1)	SRATC(2)	SRATC(3)
0			
1	130	215	307.5
2	75	112.5	156.3
3	53.3	76.7	105
4	45	60	80
5	42	51	65.5
6	41.7	45.8	56.3
7	42.9	42.9	50
8	45	41.3	45.6
9	47.8	40.6	42.5
10	51	40.5	40.3
11	54.5	40.9	38.6
12	58.3	41.7	37.5
13	62.3	42.7	36.7
14	66.4	43.9	36.3
15	70.7	45.3	36



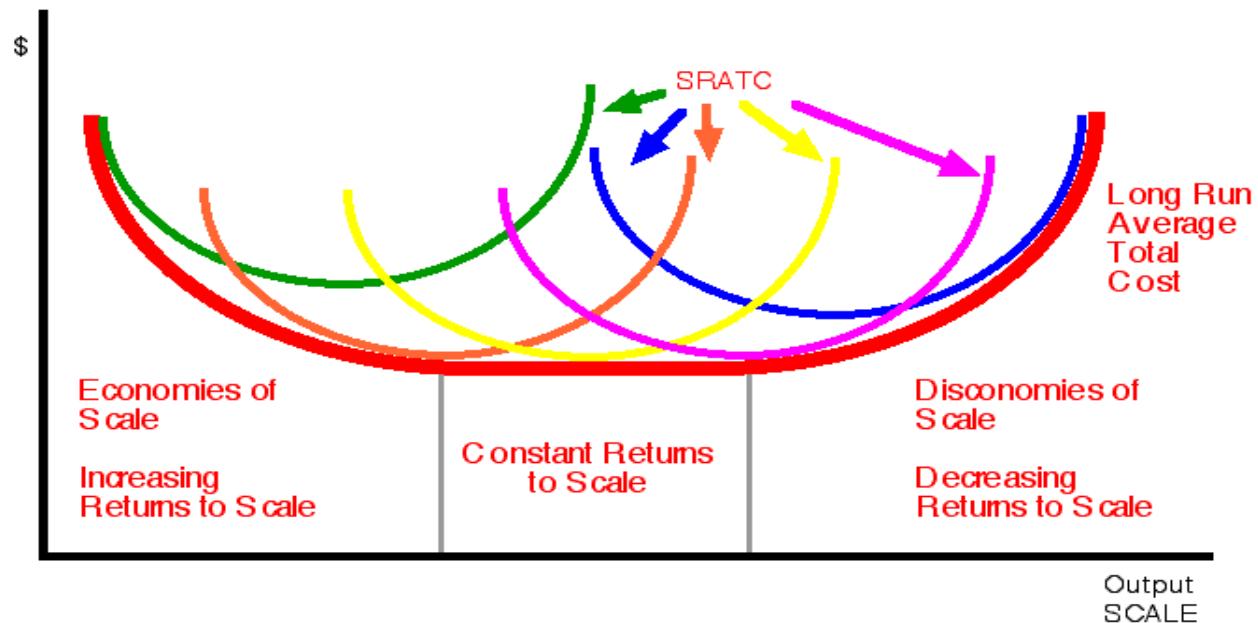
The Chair producer's LRATC:



What if the Firm wants to Change Output?

In the short run, the firm must *move along the same short run curve*, but in the long run the firm can change its fixed inputs and *move to a different short run curve* to minimize costs.

A Typical Long Run ATC Curve:



Why is the LRATC curve U-shaped?

LRATC will decrease as output increases because a larger firm can allow its workers to specialize and become more productive.

- This is called economies of scale or increasing returns to scale.
If, when a firm gets too big, there are coordination and communication problems, then LRATC will increase.
- This is called dis-economies of scale or decreasing returns to scale.

If there are no changes to average total costs as a firm grows this is called Constant returns to scale.

Exercise 6: Long Run Average Total Costs

The following table shows average total costs for a coffee shop when they have a varying number of espresso machines.

Q	ATC ₁	ATC ₂	ATC ₃	ATC ₄
0	-	-	-	-
100	300	350	400	450
200	275	330	370	420
300	250	300	340	380
400	225	270	310	340
500	250	240	280	310
600	275	210	250	280
700	300	240	220	250
800	325	270	190	220
900	350	300	220	190
1000	375	330	250	220

If the coffee shop wants to produce 100 coffees how many espresso machines would be optimal in the long run?

1 machine

If the coffee shop wants to produce 400 coffees how many espresso machines would be optimal in the long run?

1 machine

If the coffee shop wants to produce 800 coffees how many espresso machines would be optimal in the long run?

3 machines

Suppose the coffee shop currently has 1 espresso machine and on average they are selling 100 coffees. The next Saturday is really busy and they sell 500 coffees. What is their ATC for that day? 250

If they continued to be busier every day and sell 500 coffees per day what would they do in the long run?

They would get a second machine.

Draw the coffee shop's *long run average total cost curve*.

