Exercises for Section 5.4

Exercise 5.4.1 *Compute the following limits.*

(a) $\displaystyle\lim_{x\to 0}\frac{\cos x-1}{\sin x}$

(b) $\displaystyle\lim_{x\to\infty}\frac{e^x}{x^3}$

(c) $\displaystyle\lim_{x\to\infty}\frac{\ln x}{\sqrt{x}}$

(d) $\displaystyle\lim_{x\to\infty}\frac{\ln x}{x}$

(e) $\displaystyle\lim_{x\to 0}\frac{\sqrt{9+x}-3}{x}$

(f) $\displaystyle\lim_{x\to 2}\frac{2-\sqrt{x+2}}{4-x^2}$

(g) $\displaystyle\lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$

(h) $\displaystyle\lim_{x\to 0}\frac{x^2}{\sqrt{2x+1}-1}$

(i) $\displaystyle\lim_{u\to 1}\frac{(u-1)^3}{(1/u)-u^2+3/u-3}$

(j) $\displaystyle\lim_{x\to 0}\frac{2+(1/x)}{3-(2/x)}$

(k) $\displaystyle\lim_{x\to 0^+}\frac{1+5/\sqrt{x}}{2+1/\sqrt{x}}$

(l) $\displaystyle\lim_{x\to\pi/2}\frac{\cos x}{(\pi/2)-x}$ $\to \sin x$ 90° $\sin\frac{\pi}{2}$ $\sin 90^\circ$

(m) $\displaystyle\lim_{x\to 0}\frac{x^2}{e^x-x-1}$

(n) $\displaystyle\lim_{x\to 1}\frac{\ln x}{x-1}$

(o) $\displaystyle\lim_{x\to 0}\frac{\ln(x^2+1)}{x}$

(p) $\displaystyle\lim_{x\to 1}\frac{x\ln x}{x^2-1}$

(q) $\displaystyle\lim_{x\to 0}\frac{\sin(2x)}{\ln(x+1)}$ $=2$

(r) $\displaystyle\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}$

(s) $\displaystyle\lim_{x\to 0}\frac{\sqrt{x+1}-1}{\sqrt{x+4}-2}$

(t) $\displaystyle\lim_{x\to 0}\frac{\sqrt{x^2+1}-1}{\sqrt{x+1}-1}$ $=0.$

Exercise 5.4.2 *Compute the following limits.*

(a) $\displaystyle\lim_{x\to 0^+}\sqrt{x}\ln x$ *[Hint: Let $t=1/x$]* $=0.$

(b) $\displaystyle\lim_{x\to 0}\frac{(1-x)^{1/4}-1}{x}$ $=\frac{1}{4}$

(c) $\displaystyle\lim_{t\to 0}\left(t+\frac{1}{t}\right)\left((4-t)^{3/2}-8\right)$

(d) $\displaystyle\lim_{t\to 0^+}\left(\frac{1}{t}+\frac{1}{\sqrt{t}}\right)\left(\sqrt{t+1}-1\right)$

(e) $\displaystyle\lim_{x\to 0}\frac{e^x-1}{x}$

(f) $\displaystyle\lim_{x\to 1}\frac{x^{1/4}-1}{x}$

(g) $\displaystyle\lim_{x\to 0}\frac{3x^2+x+2}{x-4}$

(h) $\displaystyle\lim_{x\to 0^+}\frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}$

(i) $\displaystyle\lim_{x\to 1}(x+5)\left(\frac{1}{2x}+\frac{1}{x+2}\right)$

(j) $\displaystyle\lim_{x\to 2}\frac{x^3-6x-2}{x^3+4}$

$\displaystyle\lim_{x\to\pi/2}\frac{-\sin x}{-1}=1$

Exercise 5.4.3 *Discuss what happens if we try to use L'Hôpital's Rule to find the limit* $\displaystyle\lim_{x\to\infty}\frac{x+\sin x}{x+1}$.

$x^{\frac12}\ln x$

$\dfrac{1}{2\sqrt{x}\cdot x}=\dfrac{1}{2x^{\frac32}}=\dfrac{1}{3x^{\frac12}}=\dfrac{1}{\frac32 x^{-\frac12}}$

5.4.2 (c).

$\lim_{t \to 0} (1+\frac{1}{t})[(4-t)^{\frac{3}{2}}-8]$.

① linearization $L(x)=f'(a)(x-a)+f(a)$

$L(x)=f'(a)(x-a)+f(a)$

② differentiate

$dy=f'(x)dx$.

$dy=f'(x)dx$

③ Newton's method.

$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

3

dx.

$\cancel{4}$ $18+2\cancel{0}+20=40$.