Quiz #3 MATH 157 NAME & ID

Solutions

1. Use the L'Hospital's Rule to evaluate the following limits.

a) $\lim\limits_{x\to 1}\frac{x^2-1}{x^2+3x-4}$ b) $\lim\limits_{x\to 0}\frac{\sin.(x^2)}{\tan(5x)}$

c) $\lim\limits_{x\to 4}\frac{\sqrt{x}-2}{x-4}$ d) $\lim\limits_{x\to 0}\frac{3^x-2^x}{x^2-x}$

a) $\lim\limits_{x\to 1}\frac{x^2-1}{x^2+3x-4} = \text{'}\frac{0}{0}\text{'} \overset{L'H}{=} \lim\limits_{x\to 1}\frac{2x}{2x+3} = 2/5$

b) $\lim\limits_{x\to 0}\frac{\sin x^2}{\tan(5x)} = \text{'}0/0\text{'} \overset{L'H}{=} \lim\limits_{x\to 0}\frac{2x.\cos x^2}{5\sec^2(5x)} = \frac{0}{5} = 0$

c) $\lim\limits_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \text{'}\frac{0}{0}\text{'} \overset{L'H}{=} \lim\limits_{x\to 4}\frac{1}{2\sqrt{x}} = \frac{1}{4}$

d) $\lim\limits_{x\to 0}\frac{3^x-2^x}{x^2-x} = \text{'}\frac{0}{0}\text{'}$.

$\overset{L'H}{=} \lim\limits_{x\to 0}\frac{\ln 3 . 3^x - \ln 2 . 2^x}{2x-1}$

$= \frac{\ln 3 - \ln 2}{-1} = \ln(2/3)$

2. For a particular product, the revenue and cost functions are:

$$R(x) = 10x^3 \ and \ C(x) = 300x + 5000$$

Use Newton's method to approximate the break-even point to the nearest hundredth.[6 marks]

We must have $R = C$

$10x^3 - 300x - 5000 = 0$

$f(x) = x^3 - 30x - 500 = 0$

$f'(x) = 3x^2 - 30$

x	$f(x)$
10	$200 > 0$
9	$-41 < 0$

Since $f(9)f(10) < 0$ by the IVT, there is a root in $(9,10)$

Let $x_1 = 9.1$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = N(x_1) = 9.18899484$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = N(x_2)$

$= 9.1879 \ldots$

$\therefore$ to the nearest hundredth

root ≈ 9.19

break even point $\approx (9.19, 7760)$

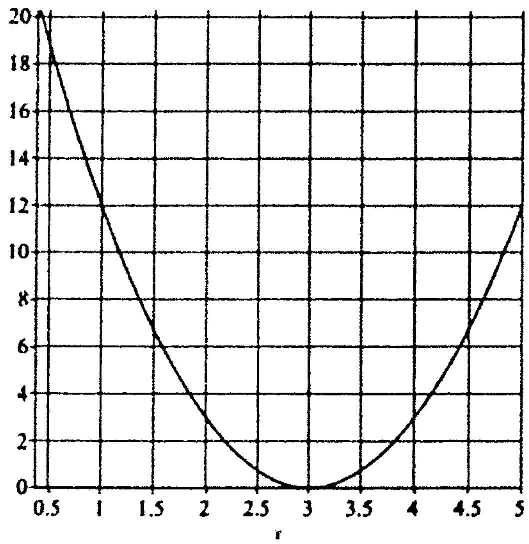
$N(x) = x - \frac{f(x)}{f'(x)}$

$= \frac{xf' - f}{f'}$

$= \frac{3x^3 - \cancel{30x} - x^3 + \cancel{30x} + 500}{3x^2 - 30}$

$= \frac{2x^3 + 500}{3x^2 - 30}$

3. Suppose you are told that $f(1) = 5$ and given a graph of the derivative, $f'(x)$,



(a) Use linear approximation to estimate the value of $\cancel{f'(1.1)}$ $f(1.1)$.

(b) Is your estimate too large or too small? Justify your answer.

a) linearization of f at $x=a$

$L(x) = f(a) + f'(a)\cdot(x-a)$

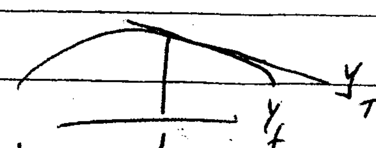
$L(x) = f(1) + f'(1)\cdot(x-1)$

$f(1.1) \approx L(1.1) = 5 + (12)(.1) = 6.2$

b) f' is decreasing near $x=1$,

therefore $f'' < 0$ near 1

$\therefore$ f looks like



i.e. concave downward

$y_T > y_f$ $\therefore$ the approximation is too large.

temc
tencent

批发

4. Suppose the wholesale price of a certain brand of medium-sized eggs p (dollars/carton) is related to the weekly supply x (in thousands of cartons) by the equation $7255p^2 - x^2 = 100$. If 25,000 cartons of eggs are available at the beginning of a certain week and the price is falling at the rate of \$0.02 carton/week, at what rate is the supply falling?

$t=0$

$$x(0) = 25, \quad p(0) = \sqrt{\frac{100+625}{7255}} = 0.3161\ldots$$

$$\frac{dp}{dt} = -\$0.02 \text{ cartons}/\text{week}$$

$$\frac{dx}{dt} = ?$$

Apply $\frac{d}{dt}$ to the equation:

$$(7255)\left(\cancel{2}p\frac{dp}{dt}\right) - \cancel{2}x\frac{dx}{dt} = 0; \quad \text{Put } t=0$$

$$(7255)(p(0)(-0.02)) - x(0) \cdot x'(0) = 0$$

$$\therefore \quad x'(0) = -\frac{(0.02)(7255)p(0)}{x(0)} \simeq -1.837$$

Supply is falling approximately at the rate of 1835 cartons/week.

[redacted]