MATH 157.

1. Differentiate

(a) $h(x) = 3^{-x^2} + \ln|-5+4x| - \sin(\pi x)$

$h'(x) = \ln 3 \cdot 3^{-x^2} \cdot -2x + \frac{4}{-5+4x} - \pi\cos(\pi x)$

$h'(0) = \cancel{\ln 3 \cdot 1 \cdot 0} + \frac{4}{-5} - \pi$

$= -\frac{4}{5} - \pi$.

(b) $y = (3x+1)^{\cos x}$, $\frac{dy}{dx}$ $x=0$.

$\ln y = \cos x \ln(3x+1)$

$\frac{y'}{y} = -\sin x \ln(3x+1) + \frac{3\cos x}{3x+1}$

$y' = \left[-\sin x \ln(3x+1) + \frac{3\cos x}{3x+1}\right]\left[(3x+1)^{\cos x}\right]$

$y'(0) = \left[-\sin 0 \ln(1) + \frac{3\cos 0}{3(0)+1}\right]\left[(0+1)^{\cos 0}\right]$

$y'(0) = \left[0 + \frac{3}{1}\right][1]$

$= 3$

$\frac{1}{2^2}$ $4^{-\frac{1}{2}}$ $\frac{1}{\sqrt{4}}$ $\frac{1}{4}$

(c) $f(x) = (x^5 - 4x^{-1.5})(\sqrt{3x^2+1})$ $f'(1)$.

$f'(x) = x^5\sqrt{3x^2+1} - 4x^{-1.5}\sqrt{3x^2+1}$

$f'(x) = x^5(3x^2+1)^{\frac{1}{2}} - 4x^{-1.5}(3x^2+1)^{\frac{1}{2}}$

$f'(x) = 5x^4(3x^2+1)^{\frac{1}{2}} + \frac{1}{2}x^5(3x^2+1)^{-\frac{1}{2}} \cdot 6x - [-6x^{-2.5}(3x^2+1)^{\frac{1}{2}} + \frac{1}{2}4x^{-1.5}(3x^2+1)^{-\frac{1}{2}} \cdot 6x]$

$f'(x) = \cancel{5x^4(3x^2+1)^{\frac{1}{2}} + \frac{1}{2}x^5(3x^2}$

$f'(x) = 5x^4(3x^2+1)^{\frac{1}{2}} + 3x^6(3x^2+1)^{-\frac{1}{2}} + 6x^{-2.5}(3x^2+1)^{\frac{1}{2}} - 12x^{-0.5}(3x^2+1)^{-\frac{1}{2}}$

$f'(1) = 5(4)^{\frac{1}{2}} + 3(3+1)^{-\frac{1}{2}} + 6(3+1)^{\frac{1}{2}} - 12(3+1)^{-\frac{1}{2}}$

$f'(1) = 10 + \frac{3}{2} + 12 - \frac{12}{2}$

$= 10 + \frac{3}{2} + 12 - 6$

$= 16 + \frac{3}{2}$

$= \frac{35}{2}$

-4 $\sqrt[3]{8}^4$ 2^4 $\frac{1}{4}$ $= \frac{-4}{6^{\frac{2}{3}}} =$

(d) $g(x) = \frac{6}{\sqrt[3]{7-2x}}$ $(7-2x)^{-\frac{1}{3}}$ $g'(-\frac{1}{2})$

$g'(x) = \frac{0 - 6 \cdot -\frac{1}{3}(7-2x)^{-\frac{4}{3}} \cdot -2}{(\sqrt[3]{7-2x})^2}$ $7+1$

$= \frac{3}{(\sqrt[3]{8})^2}$

1. $g(x) = \frac{6}{\sqrt[3]{7-2x}}$ $\qquad g'(-\frac{1}{2})$

$= 6(7-2x)^{-\frac{1}{3}}$

$= 6 \cdot -\frac{1}{3}(7-2x)^{-\frac{4}{3}} \cdot -2$

$= 4(7-2x)^{-\frac{4}{3}}$

$g'(\frac{1}{2}) = 4(7+1)^{-\frac{4}{3}}$

$= 4(8)^{-\frac{4}{3}}$

$= 4 \frac{1}{\sqrt[3]{8^4}}$

$= \cancel{4} \frac{1}{2^4}$

$= \frac{1}{\cancel{8}4}$

2. $x\cos y - \sin y = 5x$

$x\cos y - \sin y - 5x = 0$

$\cos y - x\sin y \cdot y' - \cos y \cdot y' - 5 = 0$

$\cos y - x\sin y \cdot y' - \cos y \cdot y' - 5 = 0$

$-x\sin y \cdot y' - \cos y \cdot y' = 5 - \cos y$

$(x\sin y + \cos y)y' = \cos y - 5$

$y' = \frac{\cos y - 5}{x\sin y + \cos y}$

3. $f(x) = -2xe^{-x}$ the tangent line in ~~H.A.~~ Horizontal. ?

~~$\lim_{x\to\infty} -2xe^{-x}$~~

$\boxed{\text{extreme point}}$

$f'(x) = -2 \cdot e^{-x} + (-2x)e^{-x} \cdot -1$

$= -2e^{-x} - (-2x)e^{-x}$

$= e^{-x}(-2+2x)$

$0 = e^{-x}(-2+2x)$ $\qquad 0 = -2+2x$ $\qquad \therefore f(x) = -2e^{-1}$

~~$e^{-x} = 0$ or~~ $\qquad 2 = 2x$ $\qquad = \frac{-2}{e}$

$x = 1$ $\qquad = -0.7358$

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~~4~~. $f(x)=\frac{1}{3}x^3-3x^2+5x-4$

parallel $\longrightarrow$ $3x+y=5$.

$y=-3x+5$.

$m=-3$.

$f'(x)=x^2-6x+5$.

$-3=x^2-6x+5$

$0=x^2-6x+8$

$x=4$ or $x=2$

$\cancel{f(4)=\frac{1}{3}(4)^3-3(4)^2+5(4)-4}$

$\cancel{=-10.6667}$

$\cancel{f(2)=\frac{1}{3}(2)^3-3(2)^2+5(2)-4}$

$=-3.3333$

Tips: The answer of $f'(x)$ is the 斜率 (slope).
平行，斜率一样

5. $f'(-3)$ $\quad f(x)=1-\sqrt{4-7x}$

$\cancel{\lim_{x\to 3} 1-\sqrt{4-7x}}$

$\cancel{=\lim_{x\to 3} (1-\sqrt{4-7x})(1+\sqrt{4-7x})}$

$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ If the limit exist.

$f'(-3)=\lim_{h\to 0}\frac{f(-3+h)-f(3)}{h}$

$=\frac{1-\sqrt{4-7(-3+h)}-(1-\sqrt{4+21})}{h}$

$=\frac{1-\sqrt{4+21-7h}+4}{h}$

$=\frac{5-\sqrt{25-7h}}{h}$

$=\frac{25-(25-7h)}{\cancel{h(25+7h)}\ h(5+\sqrt{25-7h})}$

$=\frac{7h}{h(5+\sqrt{25-7h})}$

$=\frac{7}{5+\sqrt{25-7h}}$

$=\frac{7}{10}$

$f(x) \begin{cases} 5-2x & x \le -1 \\ x^2+2 & x > -1 \end{cases}$

$x = -1$

① $5-2x=0$

$5 = 2x$

$x = \cancel{0.5}\ \frac{5}{2}$ $(\frac{5}{2}, 0)$

$f(-1) = 5-2(-1)$

$= 5+2$

$= 7$ $(-1, 7)$

$f'(x) = -2$