Quiz 2 Math 157/1

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1. Given $y = \left[\dfrac{(x+1)(x+2)}{(x^2+1)(x^2+2)} \right]^{1/3}$

Use logarithmic differentiation to find $\dfrac{dy}{dx}$. Also find $y'(1)$.

$$\ln y = \frac{1}{3} \ln \left[\frac{(x+1)\ (x+2)}{(x^2+1)\ (x^2+2)} \right]$$

$$\ln y = \frac{1}{3}\left[\ln(x+1) + \ln(x+2) - \ln(x^2+1) - \ln(x^2+2)\right)$$

$$\frac{y'}{y} = \frac{1}{3(x+1)} + \frac{1}{3(x+2)} - \frac{2x}{3(x^2+1)} - \frac{2x}{3(x^2+2)}$$

$$y' = \left[\frac{(x+1)(x+2)}{(x^2+1)(x^2+2)}\right]^{\frac{1}{3}} \cdot \left[\frac{1}{3(x+1)} + \frac{1}{3(x+2)} - \frac{2x}{3(x^2+1)} - \frac{2x}{3(x^2+2)}\right]$$

$$y'(1) = \left[\frac{2(3)}{2(3)}\right]^{\frac{1}{3}} \cdot \left[\frac{1}{3(2)} + \frac{1}{3(3)} - \frac{2}{3\cancel{(2)}} - \frac{2}{3(3)}\right]$$

$$y'(1) = 1 \cdot \left[\frac{1}{6} + \frac{1}{9} - \frac{1}{3} - \frac{2}{9}\right]$$

$$= -\frac{5}{18}$$

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2. Consider the demand equation

$1 = -\frac{1}{50} \cdot x' \qquad -50 = x'$

$$p = -\frac{1}{50}x + 400 \qquad 0 \le x \le 20{,}000$$

which describes the relationship between the unit price in dollars and the quantity demanded x of lamps.

i) Find the elasticity of demand $E(p)$.

ii) Compute $E(100)$ and interpret your result.

iii) Compute $E(300)$ and interpret your result.

iv) Find the price at which the revenue is maximum.

v) Suppose the price is increased by 3% when the price is \$100, find the approximate change in demand.

$p = -\frac{1}{50}x + 400$

(i) $E(p) = \frac{-p f'(p)}{f(p)}$

$= \frac{-p \cdot -50}{-50p + 20000} = \frac{50p}{-50p + 20000}$

$p - 400 = -\frac{1}{50}x$

$-50(p-400) = x$

$x = -50p + 20000$

$dx = -50.$

(ii) $E(100) = \frac{5000}{-5000 + 20000}$

$= \frac{1}{3} < 1$

$\therefore$ It is inelastic.

(iii) $E(300) = \frac{15000}{-15000 + 20000}$

$= 3 > 1$

$\therefore$ It is elastic.

ok

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(iv)

$1 = \frac{50p}{-50p + 20000}$

$-50p + 20000 = 50p$

$20000 = 100p$

$p = 200$

$\therefore$ when the price is 200.

(V) $E(p) = -\frac{p}{q}\frac{dq}{dp}$

$1 = -\frac{100}{15000} \cdot \frac{dq}{3}$

$100 = -\frac{1}{50}x + 400$

$-300 = -\frac{1}{50}x$

$x = 15000$

$\frac{15000 \cdot 3}{-100} = dq$

$-450. = dq$

$\therefore$ the rate is -450. per day.