5-4.

$\ln x = -\infty$

$\lim\limits_{x\to 0^+} x \ln x$

$= \lim\limits_{x\to 0^+} \frac{\ln x}{\frac{1}{x}}$

$= \frac{-\infty}{\infty}$

$\frac{\text{cause}}{\text{L'H}} = \lim\limits_{x\to 0^+} \frac{(\ln x)'}{(\frac{1}{x})'} = \lim\limits_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$

$= \lim\limits_{x\to 0^+} (-x)$

$= 0.$

$63^{\frac{2}{3}} \approx 64^{\frac{2}{3}} = 16.$

① by linearigatin. $\sqrt[3]{x}$ $\sqrt[3]{63^2}$ $= \frac{1}{3\sqrt[3]{x^2}}$

② by differatiate

$\sqrt[3]{x}$

$L(x) = f(a) + f'(a)(x-a).$

$= \sqrt[3]{63^2} + \frac{1}{3\sqrt[3]{63^2}}(x - 63^2).$

$= 15.896.$

same.

① by linearization.

$f(x) = x^{\frac{2}{3}}$ $f'(x) = \frac{2}{3}\frac{1}{x^{\frac{1}{3}}}$, $a = 64.$

linearization of f at 64 in.

$L(x) = f(64) + f'(64).(x-64) = 16 + \frac{1}{6}(x-64)$

$f(6) = 63^{\frac{2}{3}} \approx L(63) = 16 + \frac{1}{6}(-1)$

$= 16 - \frac{1}{6}$

$= \frac{95}{6}$

$= 15.833.$

② by differentiate

$63^{\frac{2}{3}} = (64-1)$

$f(64-1) \approx f(64) + f'(64) - 1$

$= \frac{95}{6}.$

3. Newton's method to solve

let $x = 63^{\frac{2}{3}}$

$f(x) = x^{\frac{3}{2}} - 63 = 0$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}}$

$N(x) = x - \frac{x^{\frac{3}{2}} - 63}{\frac{3}{2}x^{\frac{1}{2}}}$

$= \frac{\frac{3}{2}x^{\frac{3}{2}} - x^{\frac{3}{2}} + 63}{\frac{3}{2}x^{\frac{1}{2}}} = \frac{\frac{1}{2}x^{\frac{3}{2}} + 63}{\frac{3}{2}x^{\frac{1}{2}}} = \frac{x^{\frac{3}{2}} + 126}{3x^{\frac{1}{2}}} = \frac{1}{3}\left(x + \frac{126}{\sqrt{x}}\right)$

Exercises for section 5.4.

(a) $\lim_{x\to 0} \frac{\cos x - 1}{\sin x}$

$\lim_{x\to 0}' \frac{-\sin x}{\cos x}$

$= \lim_{x\to 0} \frac{-0}{1}$

$= 0.$

(b) $\lim_{x\to\infty} \frac{e^x}{x^3}$

$= \lim_{x\to\infty}' \frac{e^x}{3x^2}$

$= \lim_{x\to\infty} \frac{e^x}{6x}$

$= \lim_{x\to\infty} \frac{e^x}{6}$

$= \infty$

(c) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

$= \lim_{x\to\infty}' \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}$

$= \lim_{x\to\infty}' \frac{1}{\sqrt{x}}$

$= 0.$

(d) $\lim_{x\to\infty} \frac{\ln x}{x}$

$= \lim_{x\to\infty}' \frac{\frac{1}{x}}{1}$

$= \lim_{x\to\infty} \frac{1}{x}$

$= 0$

(e) $\lim_{x\to 0} \frac{\sqrt{9+x} - 3}{x}$

$= \lim_{x\to 0}' \frac{\frac{1}{2}(9+x)^{-\frac{1}{2}}}{1}$

$= \lim_{x\to 0}' \frac{1}{2\sqrt{9+x}}$

$= \frac{1}{6}$

(f) $\lim_{x\to 2} \frac{2-\sqrt{x+2}}{4-x^2}$

$= \lim_{x\to 2}' \frac{-\frac{1}{2}(x+2)^{-\frac{1}{2}}}{-2x}$

$= \lim_{x\to 2}' \frac{\frac{1}{2\sqrt{x+2}}}{2x}$

$= \lim_{x\to 2}' \frac{\frac{1}{4}}{4} = \frac{1}{16}$

(g) $\lim_{x\to 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$

$= \lim_{x\to 1} \frac{\sqrt{x} - 1}{x^{\frac{1}{3}} - 1}$

$= \lim_{x\to 1}' \frac{\frac{1}{2\sqrt{x}}}{3x^2}$

$= \frac{\frac{1}{2}}{3} = \frac{1}{6}$

(h) $\lim_{x\to 0} \frac{x^2}{\sqrt{2x+1} - 1}$

$= \lim_{x\to 0} \frac{2x}{\frac{1}{2}(\sqrt{2x+1})^{-\frac{1}{2}} \cdot 2}$

$= \lim_{x\to 0} \frac{2x}{\frac{1}{2\sqrt{2x+1}} \cdot \cancel{2}}$

$= \lim_{x\to 0} \frac{2x}{\frac{1}{\sqrt{2x+1}}}$

$= \lim_{x\to 0} \frac{0}{\frac{1}{\sqrt{0+1}}}$

$= 0$