PRACTICE Oct. 26, 2021

$\frac{3}{4}-1=-\frac{1}{4}$

Problem 6 (24pts). *Compute the derivative* $y' = \dfrac{dy}{dx}$. ~~*Do not simplify. Show all work!*~~

(a) $y = \dfrac{x^5}{3} - 4x^{3/4} + 3x + 8 + 15x^{-1/3}$, Find $y'(1)$.

$y'(x) = \frac{5}{3}x^4 - \frac{3}{\sqrt[4]{x}} + 3$

$y'(1) = \frac{5}{3} - 3 + 3$

$= \frac{5}{3}$

$7x^{-\frac{1}{3}}$ $6x^{\frac{9}{2}}$ $12x^{-1}$ $4x^{-7}$

(b) $y = \dfrac{7}{\sqrt[3]{x}} - 6\sqrt{x^9} + \dfrac{12}{x} + \dfrac{4}{x^7}$ Find $y'(1)$.

$y'(x) = -\frac{7}{3}x^{-\frac{4}{3}} - 27x^{\frac{7}{2}} - 12x^{-2} - 28x^{-8}$

$= -\frac{7}{3\sqrt[3]{x^4}} - 27\sqrt{x^7} - \frac{12}{x^2} - \frac{28}{x^8}$

$y'(1) = -\frac{7}{3} - 27 - 12 - 28$

$= -\frac{208}{3}$

(c) $y = \sqrt[3]{2x^5 - 3x^2 - 2}$ Find $y'(2)$

$y = (2x^5 - 3x^2 - 2)^{\frac{1}{3}}$

$y'(x) = \frac{1}{3}(10x^4 - 6x)^{\frac{2}{3}}$

$= \frac{1}{3\sqrt[3]{(10x^4-6x)^2}}$ $y'(2) = \frac{1}{83.94}$

(d) $y = \dfrac{6x^4 + 5x^3}{x^6 - 2}$ Find $y'(1)$

$y'(x) = \frac{(24x^3+15x^2)(x^6-2) - (6x^4+5x^3)(6x^5)}{(x^6-2)^2}$

$y'(1) = \frac{39(-1) - 11(6)}{1}$

$= -105$

(e) $y = (3x^5 + 4x^4 - 2)(4x^7 - 18)$ Find $y'(2)$

$y'(x) = (15x^4 + 16x^3)(4x^7 - 18) + (3x^5 + 4x^4 - 2)(28x^6 - 18)$

$y'(2) = (368)(494) + (158)(1774)$

$= 462084$

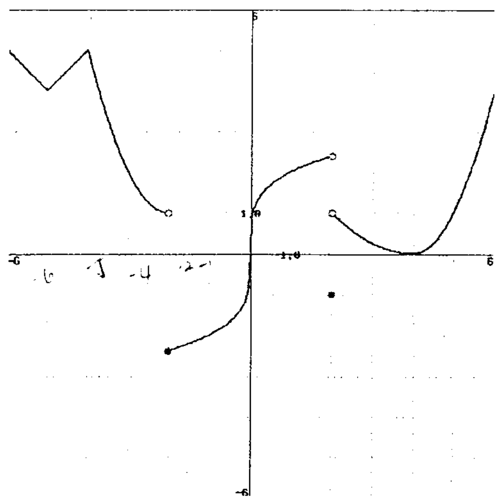
(f) $y = \sqrt{(3x+2)^4 - 15x}^{\frac{1}{2}}$ Find $y'(0)$

$y'(x) = \frac{1}{2}[(3x+4)^4 - 15x]^{-\frac{1}{2}} \cdot [4(3x+2)^3 - 15]$

$y'(0) = \frac{1}{2}[(4)^4 - 0]^{-\frac{1}{2}} \cdot [4(2)^3 - 15]$

$= \frac{1}{2}[16] \cdot [17]$

$= 136$



Problem 7 (8pts). *The graph of $y = f(x)$ is shown above for $-6 < x < 6$.*

(a) *For which x values is $f(x)$ not continuous?*

∴ The values is 2 and -2.

(b) *For which x values is $f(x)$ not differentiable?*

. when the $x = -5, -6,$

(c) *For which x values is the derivative $f'(x) = 0$?*

∴ when the $x = 4$.

Problem 8 (7pts). *Let $F(x) = 3x^3 - 2x^2 - 10$. Find the equation of the tangent line to the graph of $F(x)$ at $x = 1$. Leave your answer in the form $y = mx + b$.*

$F'(x) = 9x^2 - 4x$

$F'(1) = 9 - 4$

$= 5$.

$F(1) = 3(1)^3 - 2(1)^2 - 10$

$= 3 - 2 - 10$

$= -9$

∴ $y + 9 = 5(x - 1)$

$y + 9 = 5x - 5$

$y = 5x - 14$

10

Problem 9 (8pts). *Let $g(x) = (2x-1)^6$.*

(a) *Find $g'(0)$.*

$6(2x-1)$

$g'(x) = 6(2x-1)^5 \cdot 2$

$g'(0) = 6(-1)^5 \cdot 2$

$= -6 \cdot 2$

$= -12$

(b) *Find $g''(0)$.*

$g'(x) = 12(2x-1)^5$

$g''(x) = 60(2x-1)^4 \cdot 2$

$= 120(2x-1)^4$

$g''(0) = 120(-1)^4$

$= 120$

Problem 10 (12pts). *For x units sold, the total revenue function is $R(x) = 30x + 100$. The total cost function is $C(x) = 500 + 8x + \frac{1}{8}x^2$.*

(a) *Find the profit function $P(x)$.*

$P(x) = 30x + 100 - (500 + 8x + \frac{1}{8}x^2)$

$= -\frac{1}{8}x^2 + 22x - 400$

(b) *Find the marginal profit when 100 units are sold.*

$P'(x) = -\frac{1}{4}x + 22$

$P'(100) = -3$

(c) *If $P(100) = 550$, use your part (b) answer to estimate the total profit if 101 units sold.*

$P'(101) = -\frac{1}{4}(101) + 22$

$= -3.25$

(d) *Should the company sell the 101st unit? Explain using your answers above.*

∴ the company should not sell 101st unit, because the profit are in the loss.

(a) State the definition of a function $f(x)$ at $x = a$.

$f(a)$

(b) Use (a) to find $f'(-1)$ where

$f(x) = x^2 + \frac{1}{x}$.

$f'(x) = 2x - \frac{1}{x^2}$

$f'(-1) = -2 - \frac{1}{1}$

$= -2 - 1$

$= -3$

Determine an equation of the tangent line to the curve

$$x^4 - x^2 y + y^4 = 1$$

at the point $P(-1,1)$.

$$4x^3 - (2xy + x^2 \cdot y') + 4y^3 \cdot y' = 1$$

$$-4 - (-2 + y') + 4y' = 1$$

$$-4 + 2 - y' + 4y' = 1$$

$$-2 + 3y' = 1$$

$$3y' = 3$$

$$y' = 1$$