QUIZ 1 Math 157

Name

Section

ID

SOLUTIONS

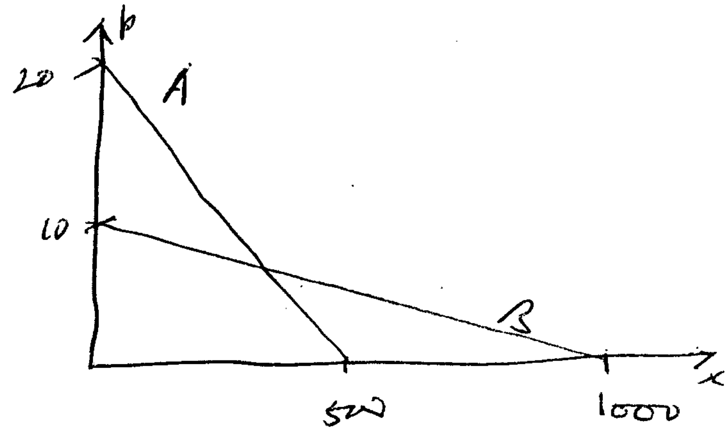
1.

Suppose the demand equation for the version A of cell phones and the version B of cell phones

are respectively

A $\frac{p}{20}+\frac{x}{500}=1 \Rightarrow p = \frac{-20}{500}x + 20$

B $\frac{p}{10}+\frac{x}{1000}=1 \Rightarrow p = -\frac{10}{1000}x + 10$



(a) Sketch the above demand curves.
(b) Which graph has greater slope.
(c) Interpret (b).

$m_A = \frac{-4}{100} = \frac{-1}{25} \quad \therefore \quad m_B > m_A$

$m_B = \frac{-1}{100}$

(c) If price drops by \$1, the demand for A is up by 25 while for B it is up by 100.

2.

You are the manager of a firm. You are considering production of a new product, so you ask the accounting department for cost estimates and the sales department for sales estimate. After you receive the data, you must decide whether to go ahead with production of the new product. Analyze the given data (find a break-even quantity) and then decide what you would do in each case. Include the profit function.

(a) $C(x) = 100x+6000$; $R(x)=250x$; no more than 400 can be sold.

(b) $C(x) = 1000x+5000$; $R(x)=900x$; no more than 400 can be sold.

(a) $P(x) = R(x) - C(x) = 250x - [100x + 6000]$
$= 150x - 6000$

$\therefore$ Break-even when $P=0$ or $R=C$ is $\underline{x=40}$

(b) $P(x) = 900x - [1000x + 5000]$
$= -100x - 5000$

Always loss.

Case (a) Produce more than 40 items

Case (b) Always loss and therefore do not produce any.

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3.Complete the table by computing *f(x)* at the given value of *x*. Use these results to estimate the indicated limit (if it exists)

$f(x) = \frac{|x-1|}{x-1}, \lim_{x \to 1} f(x)$ does not exist

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	-1	-1	-1	1	1	1

Since $\lim_{x \to 1^-} f(x) = -1 \neq \lim_{x \to 1^+} f(x) = 1$

4. Find the following limits, if they exist. State if the limit is ∞ *or* $-\infty$.

a) $\lim_{x \to 4} \frac{x}{4+x} = \frac{4}{4+4} = \frac{1}{2}$

b) $\lim_{x \to 1} \frac{x-1}{x^3+x^2-2x}$. $= \lim_{x \to 1} \frac{x-1}{x(x^2+x-2)} = \lim_{x \to 1} \frac{\cancel{x-1}}{x\cancel{(x-1)}(x+2)}$

$= \lim_{x \to 1} \frac{1}{x(x+2)} = \frac{1}{3}$

Type equation here.

c) $\lim_{h\to 2} \frac{h}{(h-2)}$.

$$\lim_{h\to 2^+} \frac{h}{h-2} = \text{'}\frac{2}{0^+}\text{'} = +\infty$$

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$$\lim_{h\to 2^-} \frac{h}{h-2} = \text{'}\frac{2}{0^-}\text{'} = -\infty$$

d) $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x^2}$ $\cdot \frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2}$

$$\lim_{x\to 0} \frac{x^2+4-4}{x^2\left[\sqrt{x^2+4}+2\right]} = \lim_{x\to 0} \frac{\cancel{x^2}}{\cancel{x^2}} \cdot \frac{1}{\left[\sqrt{x^2+4}+2\right]}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x^2+4}+2}$$

$$= \frac{1}{4}$$

e) $\lim_{x\to\infty} \frac{8x^4-5x^3+x-2}{1-4x^4-x^2}$ $\div x^4$

$$= \lim_{x\to\infty} \frac{8 - \frac{5}{x} + \frac{1}{x^3} - \frac{2}{x^4}}{\frac{1}{x^4} - 4 - \frac{1}{x^2}} = -2$$

f) $\lim_{x\to-\infty} \frac{\sqrt{x^2+4x}}{4x+1}$ $\div x$

Note.

$x = \sqrt{x^2}$ if $x \geq 0$

$= -\sqrt{x^2}$ if $x < 0$

$$= \lim_{x\to-\infty} \frac{-\sqrt{\frac{x^2+4x}{x^2}}}{4+\frac{1}{x}}$$

$$= \lim_{x\to-\infty} -\frac{\sqrt{1+\frac{4}{x}}}{4+1/x} = -1/4$$

5. Use the Intermediate Value theorem to show that $x^4 + 5x^3 + 5x - 1 = 0$ has at least one solution in the interval (-6,-5) and approximate it correct to one decimal place.

$f(x) = x^4 + 5x^3 + 5x - 1$

Since $f(-6) \cdot f(-5) < 0$ by IVT there is a root in $(-6,-5)$.

x	$f(x)$
-6	+185
-5	-26

x	$f(x)$
-5.10	-13.2 < 0
-5.20	+1.1

x	$f(x)$
-5.11	-11.8 < 0
-5.12	-10.4
-5.13	-9.04
-5.14	-7.6
-5.15	< 0
-5.16	< 0

5.1 < root < 5.2

Since $f(-5.19) f(-5.20) < 0$

root lies in $(-5.20, -5.19)$

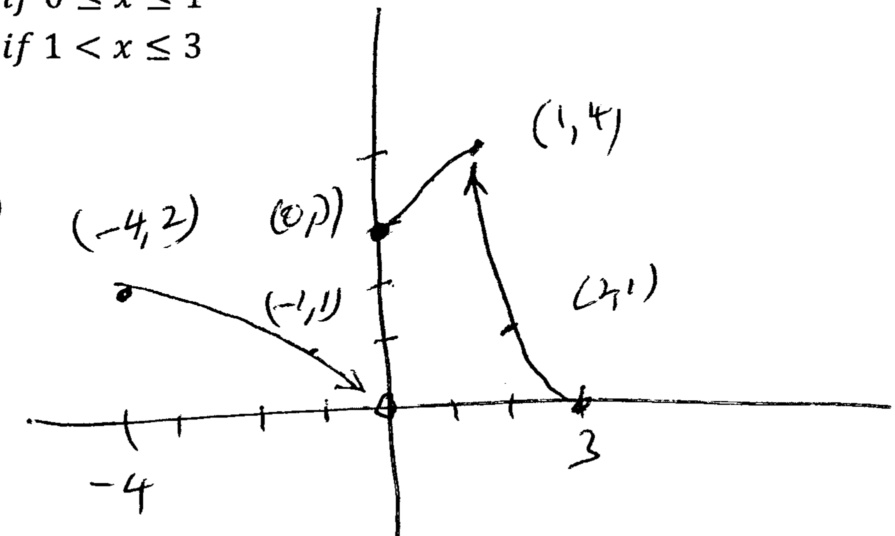
x	$f(x)$
-5.20	+1.1
~~-5.21~~	

x	$f(x)$
-5.17	< 0
-5.18	< 0
-5.19	< 0

i.e. root $x \approx -5.2$

6. Let $f(x) = \begin{cases} \sqrt{-x} & if\ -4 \le x < 0 \\ 3 + x, & if\ 0 \le x \le 1 \\ (x-3)^2 & if\ 1 < x \le 3 \end{cases}$

a) Sketch the graph of $y=f(x)$



b) Is f continuous at $x=0$? Justify your answer.

1. $f(0) = 3$
2. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sqrt{-x} = 0 \neq \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 3+x = 3$

$\therefore$ Discontinuous at 0.

c) Is f continuous at $x=2$? Justify your answer.

1. $f(2) = (x-3)^4\big|_{x=2} = 1$

2. $\lim_{x\to 2} f(x) = \lim_{x\to 2} (x-3)^2 = 1$

3. $\lim_{x\to 2} f(x) = f(2)$

f continuous at $x=2$