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Math 157 Quiz 4 July 28,2022

$\frac{x}{(x+3)(x-3)}$ $\frac{x}{x+3} \cdot \frac{1}{x-3}$ $\frac{(x+3)-x}{(x+3)^2}$ $\frac{3}{(x+3)^2}$

$\frac{((x^2-9)-2x(x)}{(x^2-9)^2}$ $\lim \frac{-9-x^2}{\#(}$ x^2-9-2x^2

1. Consider the function $f(x) = \frac{x}{x^2-9}$.

a)Find the intervals where the function is increasing or decreasing and the relative extrema.

b)Find the intervals where the function is concave upward or concave downward , and the inflection points.

c)Graph the function .

∴ the function is always decreasing.

(a) $f'(x) = \frac{(x^2-9)-x(2x)}{(x^2-9)^2}$

$= \frac{x^2-9-2x^2}{(x-9)^2}$

$= \frac{-9-x^2}{(x-9)^2}$

$0 = (x-9)^2$

$0 = (x-3)(x+3)$

$x = 3$ or $x = -3$

$f'(-4)$	$f'(0)$	$f'(4)$
$\searrow$	-3 $\searrow$	3 $\searrow$

(b) $f''(x) = \frac{-2x(x-9)^2-2(-9-x^2)(1)}{(x-9)^4}$

$= \frac{-2x(x-9)^2+2(9+x^2)}{(x-9)^4}$

$0 = -2x(x-9)^2+2(9+x^2)$

$x(x-9)^2 = (9+x^2)$

$x(x^2-18x+81) = 9+x^2$

$x^3-18x^2+81x = 9+x^2$

$x^3-19x^2+81x-9 = 0$

$x_1 = 12.0561$

$x_2 = 0.1141$

$x_3 = 6.2297$

$f''(0)$	$f''(1)$	$f''(8)$	$f''(20)$
$\cup$	0.1141 $\cap$	6.2297 $\cup$	12.0561 $\cap$

(c)

① the domain

$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

②

V.A. is -3 and 3.

H.A.

$$\lim_{x \to \infty} \frac{x}{x^2 - 9}$$

$$= \lim_{x \to \infty} \frac{1}{2x}$$

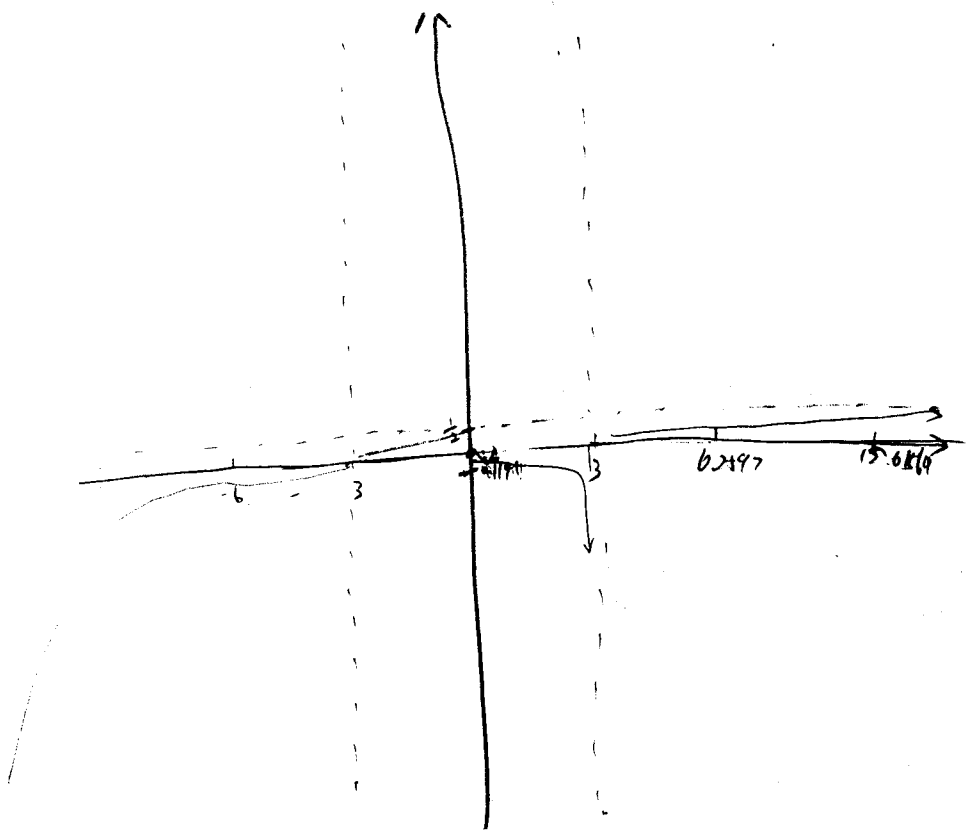
$$= \frac{1}{2}$$

$$\lim_{x \to -\infty} = \frac{1}{2}$$

③ $0 = \frac{x}{x^2 - 9}$

$x = 0$

$y = 0$



2. A company needs to design a metal box container with a volume of 128 m^3 and that is twice as long as it is wide. The top and bottom will be made of a sturdy material that costs \$12 /m^2, while the material for the sides costs \$8/m^2. Find the dimensions and cost of the least expensive container. [8 marks]

$V = r^2\pi \cdot h$

$128 = r^2\pi h$

$h = \frac{128}{r^2\pi}$

The top and bottom: $2\pi r^2 \cdot 12$

The side: $2\pi r \cdot \frac{128}{r^2\pi} \cdot 8$

total is: $24\pi r^2 + \frac{2048}{r}$

$d'(x) = 48\pi r - 2048 r^{-2}$

$0 = 48\pi r - 2048 r^{-2}$

$2048 r^{-2} = 48\pi r$

$\frac{2048}{r^2} = 48\pi r$

$2048 = 48\pi r^3$

$\frac{2048}{48\pi} = r^3$

$r = 2.3859$

$d'(x) = 1$	$d(x) = 3$
↘	2.3859 ↑

$\therefore 24\pi \cdot 2.3859 + \frac{2048}{2.3859}$

$= \$1038.2689$