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SIGNATURE: SOLUTIONS

Fraser International College
Test 2 Version A
Math 157
July 21, 2022
Time 70 Minutes
Instructor: Dr. N. Tariq

- **Please ensure that you sign your exam above to certify your identity. Unsigned exams will not be marked.**
- **Use only calculators that do not have any graphing, differentiation or integration capabilities.**
- **The duration of the exam is 70 minutes.**
- **DO NOT OPEN this test booklet until you are told to do so.**
- **Please check that you have all 5 questions of the exam.**
- **Do ALL your work in this test booklet. You may use the backside of each page for scrap work.**
- **The value of each question is shown at the end of each question.**

Question	Score	Maximum
1	5	10
2	6	6
3	4	8
4	6	6
5	11	15
Total	32	45

1. Differentiate the following functions as indicated. [10 marks]

a) $f(x) = sin(3x) + \tan(2x) + e^x + ln|1 - 2x|, find\ f'(0).$

$f'(x) = 3\cos 3x + \sec^2(2x) \cdot 2 + e^x + \frac{1}{1-2x} \cdot (-2)$

$f'(0) = 3 + 2 + 1 - 2 = 4$

b) $f(x) = (\sin \pi x + \cos \pi x)^{(2x+1)}$, find $f'(0)$.

$\ln f = (2x+1) \ln(\sin \pi x + \cos \pi x)$

D_x : $\frac{f'}{f} = 2 \ln(\sin \pi x + \cos \pi x) + (2x+1) \cdot \frac{\pi \cos \pi x - \pi \sin \pi x}{\sin \pi x + \cos \pi x}$

$\frac{f'(0)}{f(0)} = 0 + \pi$

$f(0) = 1$

$f'(0) = \pi$

3. The price p (in dollars) and the demand q for a product are related by $25p^2 + 4q^2 = 20000$, $0 < p < 28$. 5+3 =8 marks]

(a) Find an expression for $E(p)$ (the elasticity of demand).

$\frac{d}{dp}$: $50p + 8q\frac{dq}{dp} = 0 \Rightarrow \frac{dq}{dp} = -\frac{50p}{8q}$

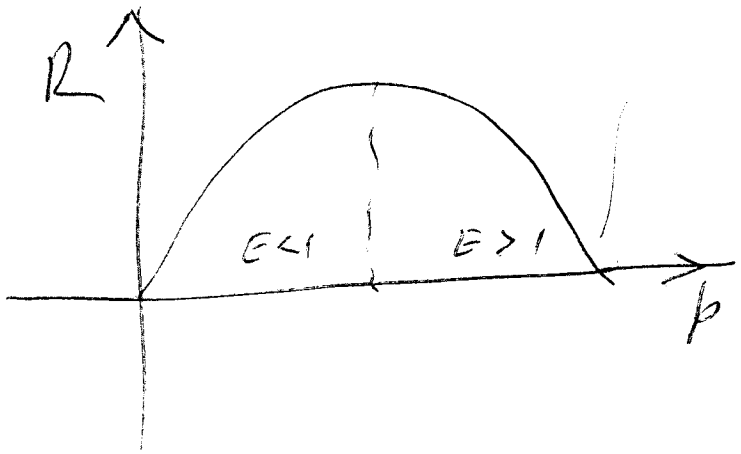
$E = -\frac{p}{q}\cdot\frac{dq}{dp} = +\frac{50p^2}{8q^2} = \frac{25p^2}{20000-25p^2}$

$E(p) = \frac{p^2}{800-p^2}$.

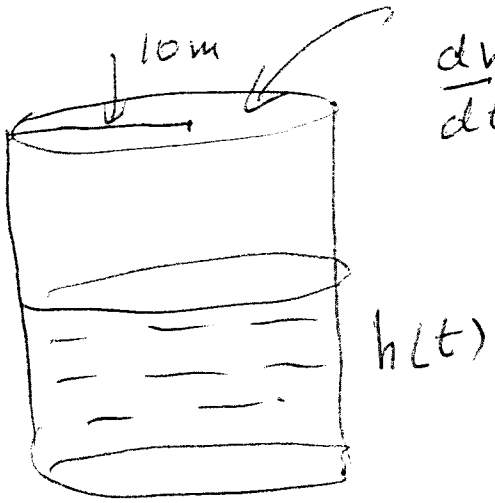
(b) If the current price per unit is \$8, will revenue increase or decrease if the price is raised slightly? Explain.

$E(8) = \frac{64}{800-64} < 1$

Since the demand is inelastic the revenue will increase.



2. A cylindrical tank of radius 10 metres is being filled with wheat at the rate of 100π metres per minute. How fast is the depth of the wheat increasing? [6 marks]



$\frac{dV}{dt} = 100\pi \text{ m/min}$

$V = \pi r^2 h$

$V(t) = \pi \cdot 10^2 \cdot h$

$\frac{d}{dt}$: $\frac{dV}{dt} = 100\pi \frac{dh}{dt}$

$100\pi = 100\pi \frac{dh}{dt}$

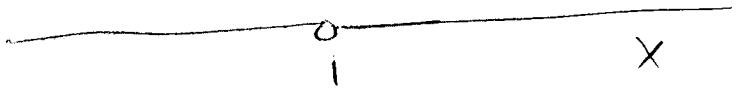
$\therefore \frac{dh}{dt} = 1 \text{ m/min}$

i.e. depth is increasing at the rate of 1 m/min.

5. Let $f(x) = \frac{2x^2+1}{(x-1)^2}, f'(x) = \frac{-2(2x+1)}{(x-1)^3}$ and $f''(x) = \frac{2(4x+5)}{(x-1)^4}$ [13 marks]

a) State the domain of f.

$(-\infty, 1) \cup (1, \infty)$



b) Find the x −intercept(s) of f, if any.

None

c) Find the y − intercept of f, if any.

$f(0) = 1$

d) Find the equations of all horizontal asymptote(s) of f.

$\lim_{x \to \pm\infty} f(x) = 2$

$\therefore \quad y = 2 \quad H.A$

$y = 2 = \frac{2x^2+1}{(x-1)^2}$

$2x^2 - 4x + 2 = 2x^2 + 1$

$x = 1/4$

e) Find the equations of all vertical asymptote(s) of f.

$\lim_{x \to 1^{\pm}} f(x) = \infty$

$\therefore \quad x = 1 \quad V.A$

4. (a) Find the linearization $L(x)$ of $f(x) = 2x^3 - 7x^2 + 9x + 6$ at $a = 2$. [4 marks]

$L(x) = f(a) + f'(a)\cdot(x-a)$

$f'(x) = 6x^2 - 14x + 9 \quad ; \quad f'(2) = 5$

$f(2) = 16 - 28 + 18 + 6 = 12$

$\therefore \quad L(x) = 12 + 5(x-2)$

(b) Use $L(x)$ to approximate $f(2.02)$. [2 marks]

$f(2.02) \approx L(2.02)$

$= 12 + 5(2.02 - 2)$

$= 12 + 0.1$

$= 12.1$

$f'=0 \Rightarrow x=-1/4$ critical point $A(-\frac{1}{2}, \frac{2}{3})$

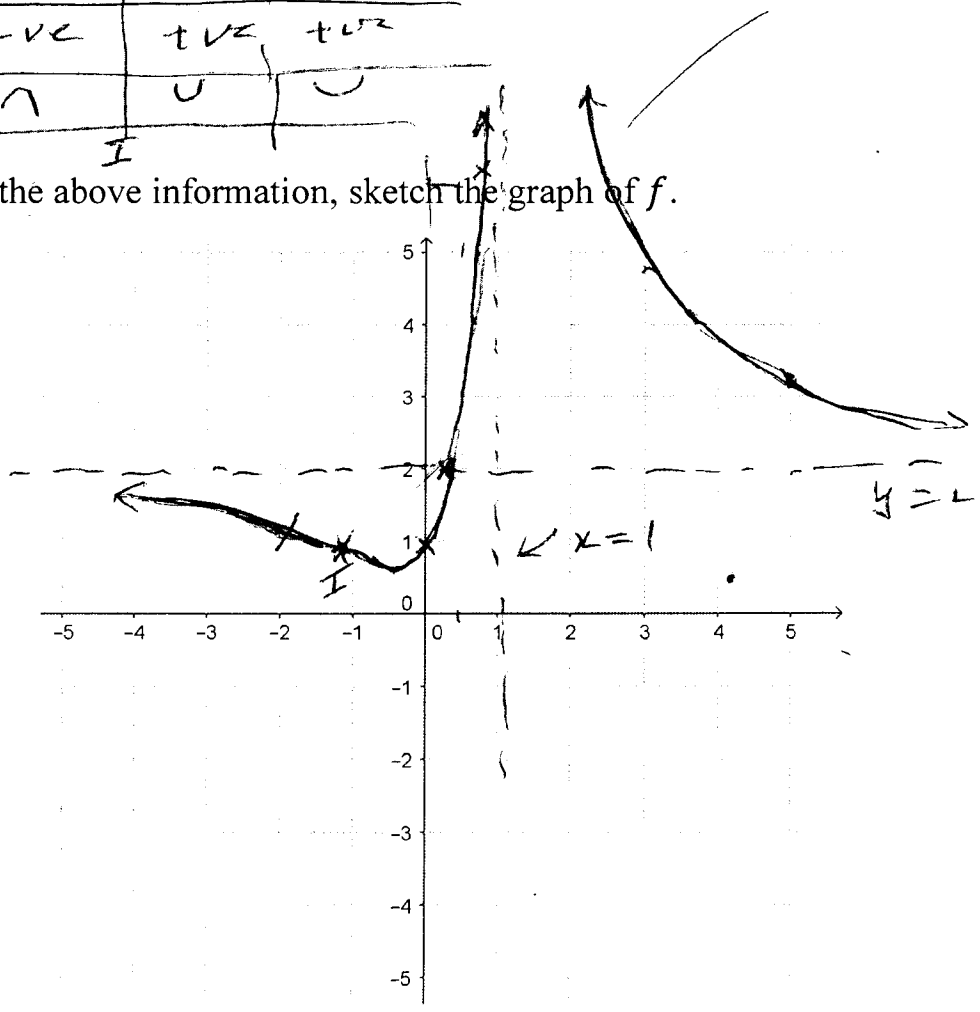
f) Find the intervals where f is increasing or decreasing and the points of relative extrema.

I	$(-\infty, -1/2)$	$(-1/2, 1)$	$(1, \infty)$
z	-1	0	2
$f'(z)$	$-ve$	$+ve$	$-ve$
f	$\searrow$	$\nearrow$	$\searrow$
FDT		Relmax at A	

g) Find the intervals where the function f is concave upward or downward and the points of inflection. $f''=0 \Rightarrow -5/4$ Possible inflection point $I(-5/4, \frac{22}{27})$

I	$(-\infty, -5/4)$	$(-5/4, 1)$	$(1, \infty)$
z	-10	0	10
$f''(z)$	$-ve$	$+ve$	$+ve$
f	$\cap$	$\cup$	$\cup$
Inflection pt		I	

h) Using the above information, sketch the graph of f.



x	$f(x)$
$-1/2$	$2/3$
-1	1
-5	$51/36$

x	$f(x)$
$1/2$	6
$1/4$	2
0	1
2	9
3	$19/4$
5	$51/16$