1. For x units sold, the total revenue function is $R(x) = 30x + 100$.
The total cost function is $C(x) = 500 + 8x + \frac{1}{8}x^2$. [6 marks]

a) Find the profit function $P(x)$.

$R(x) - C(x)$

$= 30x + 100 - 500 - 8x - \frac{1}{8}x^2$

$= -\frac{1}{8}x^2 + 22x - 400.$

b) Find the marginal profit when 100 units are sold.

$P(100) - P(99)$

$= -\frac{1}{8}(100)^2 + 22(100) - 400 - [-\frac{1}{8}(99)^2 + 22(99) - 400]$

$= 550 - 552.875$

$= -2.875.$

-1

c) If $P(100) = 550$, use your part b answer to estimate the total profit if 101 units sold.

$P(101) = 550 - 2.875$

$= 547.125.$

d) Should the company sell the 101^{st} unit? Explain using answers above.

∴ No, because the profit is become less.

2. Find the instantaneous rate of change for $f(x) = 3x^2 - 5x + 1$ at $x = 4$ using the limit definition of the derivative. [4 marks]

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \frac{3(x+h)^2 - 5(x+h) + \cancel{1} - 3x^2 + 5x - \cancel{1}}{h}$$

$$= \frac{3(x+h)^2 - \cancel{5x} - 5h - 3x^2 + \cancel{5x}}{h}$$

$$= \frac{3(x^2 + 2xh + h^2) - 5h - 3x^2}{h}$$

$$= \frac{\cancel{3x^2} + 6xh + 3h^2 - 5h - \cancel{3x^2}}{h}$$

$$= \frac{\cancel{h}(6h + 3h - 5)}{\cancel{h}}$$

$$= 6h + 3h - 5 \quad = \quad 6x - 5$$

-1

$f'(x) = 6x + 3x - 5$

$f'(4) = 24 + 12 - 5$

$= 31$

$f(4) = 3(4)^2 - 5(4) + 1$

$= 29$

$\therefore y - 29 = 31(x - 4)$

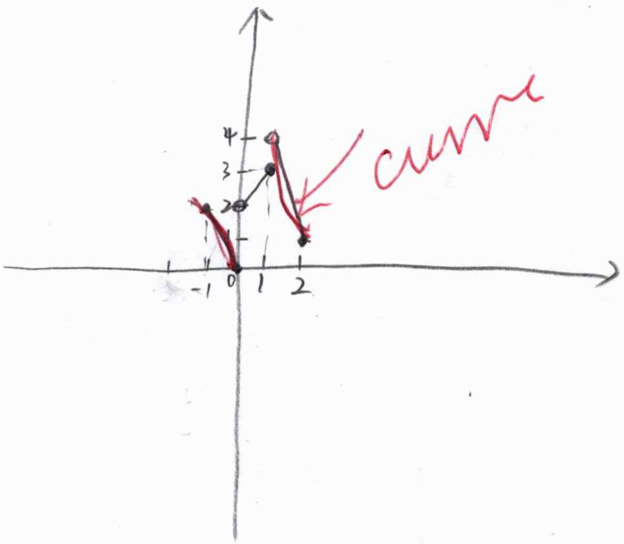
$y - 29 = 31x - 124$

$y = 31x - 95$

3. Let $f(x) = \begin{cases} \sqrt{-4x} & if\ -1 \le x \le 0 \\ 2 + x, & if\ 0 < x \le 1 \\ (3x-5)^2 & if\ 1 < x \le 2 \end{cases}$

[6 marks]

a) Sketch the graph of $y = f(x)$



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b) Is f continuous at $x = 0$? Justify your answer.

Yes, it is continuous, according to the graph it continuous at 0.
and if $\lim_{x \to 0^+} = 2+x$, $\lim_{x \to 0^-} = 2+x$, the right and left side are equal
$= 2$ $\quad = 2$ So, it is continuous.

X

c) Is f continuous at $x = 1$? Justify your answer.

Yes, it is continuous, according to the graph it continuous at 3. X
and if $\lim_{x \to 1^+} (3x-5)^2$, $\lim_{x \to 1^-} (3x-5)^2$, the both side are equal, so,
$= 4$ $\quad = 4$ it is continuous.

4. Find an equation of the tangent line to the curve $x^5 - x^2y - y^4 = 27$ at the point $P(2,1)$. [4 marks]

$$x^5 - x^2y - y^4 = 27$$

$$\frac{dy}{dx} \quad 5x^4 - (2xy + x^2y') - 4y^3 \cdot y' = 0$$

$$5x^4 - 2xy - x^2y' - 4y^3y' = 0$$

$$5x^4 - 2xy = (x^2 + 4y^3)y'$$

$$\frac{5x^4 - 2xy}{x^2 + 4y^3} = y'$$

put (2,1) in.

$$\therefore y' = \frac{5(2)^4 - 2(2)}{(2)^2 + 4}$$

$$= 9.5$$

$$y - 1 = 9.5(x - 2)$$

$$y - 1 = 9.5x - 19$$

$$y = 9.5x - 18.$$

5. Find the following limits, if they exist. [9 marks]

a) $\lim_{x\to 5} \frac{x^2-(10-x)^2}{10-2x}$

$= \lim_{x\to 5} \frac{x^2-(100-20x+x^2)}{10-2x}$

$= \lim_{x\to 5} \frac{20x-100}{10-2x}$

$= \lim_{x\to 5} \frac{10\cancel{(2x-10)}}{\cancel{10-2x}}$

$= -10$

b) $\lim_{x\to 4} \frac{-4+\sqrt{4x}}{x-4}$

$= \lim_{x\to 4} \frac{-4+4}{x-4}$

$= 0.$

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c) $\lim_{x\to -\infty} \frac{\sqrt{9x^2+7x-1}}{3x-5}$

$= -\infty$

6. Differentiate the following functions as indicated: [12 marks]

a) $y = f(x) = x^5 + \frac{1}{x^2} - \frac{1}{\sqrt{x}} + 5^{\pi}, find\ f'(1).$

b) $y = g(x) = \frac{2x^2-1}{x^2+1}, find\ g'(1).$

c) $y = f(x) = \log_4[tan^{-1}(x+1)], find\ f'(0).$ $\frac{1}{\ln a} \cdot \frac{1}{x}$ $\tan^{-1} = \frac{1}{1+x^2}$

(a)

$y = x^5 + x^{-2} - x^{-\frac{1}{2}} + 5\pi$

$y' = 5x^4 - 2x^{-3} + \frac{1}{2}x^{-\frac{3}{2}}$

$f'(1) = 5(1)^4 - 2(1)^{-3} + \frac{1}{2}(1)^{-\frac{3}{2}}$

$= 5 - 2 + \frac{1}{2}$

$= 3.5$ ✓

4

(b)

$g'(x) = \frac{4x(x^2+1) - (2x^2-1)(2x)}{(x^2+1)^2}$

$g'(1) = \frac{4(2) - (2-1)(2)}{1+1}$

$= \frac{8-2}{2}$

$= \frac{6}{2}$

$= 3$ ✗

3

(c)

$f'(x) = \frac{1}{\ln 4} \cdot \frac{1}{\tan^{-1}(x+1)} \cdot \frac{1}{1+(x+1)^2} \cdot 1$

$f'(0) = \frac{1}{\ln 4} \cdot \frac{1}{\tan^{-1} 1} \cdot \frac{1}{2}$

$= \frac{1}{2\ln 4 \tan^{-1} 1}$ ✓

$\approx 0.46.$

4

7. Use logarithmic differentiation to find the derivative of $f(x) = (\sin x + \cos x)^{(2x+1)}$. Calculate $f'(0)$. [4 marks]

$$\ln y = (2x+1)\ln(\sin x + \cos x)$$

$$\frac{dy}{dx} \quad \frac{y'}{y} = 2\ln(\sin x + \cos x) + (2x+1) \cdot \frac{1}{\sin x + \cos x} \cdot (\cos x - \sin x)$$

$$y' = \left[2\ln(\sin x + \cos x) + \frac{(2x+1)(\cos x - \sin x)}{\sin x + \cos x}\right] \cdot (\sin x + \cos x)^{(2x+1)}$$

$$f(0) = \left[2\ln(0+1) + \frac{1(1-0)}{0+1}\right] \cdot (0+1)^{(1)}$$

$$f'(0) = [0+1] \cdot 1$$

$$f'(0) = 1$$