MATH157.

We now recognize(认识) the difference quotient(商) in this fraction(分数)

elasticity of demand $\left\{ \frac{f(p+h)-f(p)}{h} \approx f'(p) = \frac{f'(p)}{\frac{f(p)}{p}} = \frac{pf'(p)}{f(p)} \right.$ $R\begin{cases} <1 \ R\uparrow \\ =1 \ R \ Max \\ >1 \ R\downarrow \end{cases}$

Example 5.2.

The unit price p.

$E(p) = \frac{pf'(p)}{f(p)}$ $\quad f(p)' \quad q = f(p)$

$=$ $\quad \cancel{f(p)} \quad p = -0.02f(p) + 400$

$\frac{p-400}{-0.02} = f(p)$

$p = -0.02q + 400 \quad q = f(p) \quad -0.02f(q) = p - 400$

$f(q) = -50 + 20000$

$p = -0.02f(p) + 400$

$0.02f(p) = -p + 400$

$f(p) = \frac{-p+400}{0.02}$ $\quad f'(p) = \frac{}{-\frac{1}{50}}$

$= -\frac{1}{50}p + 20000$

$E(p) = \cancel{p}\frac{-\frac{1}{50}p}{-\frac{1}{50}p + 20000}$

$E(100) = \frac{-2}{19998}$

Exercise 5.1

5.1.1

(a) $q = -\frac{1}{2}p + 10$, $p = 10$.

$f(p) = -\frac{1}{2}p + 10$. $\quad f'(p) = \frac{1}{2}$

$E(p) = \frac{p \cdot \frac{1}{2}}{-\frac{1}{2}p + 10}$

$= \frac{\frac{1}{2}p}{-\frac{1}{2}p + 10}$

$E(10) = \frac{5}{-5+10}$

$= 1$ $\quad \therefore$ It is unitary

$q = -\frac{3}{2}p + 9$, $p = 1$ $f(p) = -\frac{3}{2}p + 9$

$E(p) = \frac{p \cdot -\frac{3}{2}}{-\frac{3}{2}p + 9}$

$E(1) = \frac{-\frac{3}{2}}{-\frac{3}{2} + 9}$

$= -\frac{1}{5} < 1$ $\therefore$ It is inelastic

?? $q + \frac{1}{3}p - 24 = 0$, $p = 3$

$q = -\frac{1}{3}p + 24$

$E(p) = \frac{-\frac{1}{3}p}{-\frac{1}{3}p + 24}$ $\frac{\frac{1}{3}p}{-\frac{1}{3}p + 24}$ $\frac{-1}{}$ $\frac{1}{23}$

$E(3) = \frac{-1}{23}$ $-\frac{1}{23}$

$= -\frac{1}{23} < 1$ $\therefore$ It is inelastic.

$0.4q + p = 20$ $p = 12$

$0.4q = 20 - p$

$q = 50 - 2.5p$

$= -2.5p + 50$

$E(p) = \frac{-2.5p}{-2.5p + 50}$

$E(p) = \frac{-2.5(12)}{-2.5(12) + 50}$

$= \frac{-30}{20}$

$= -\frac{3}{2}$ $\therefore$ It is elastic.

$\frac{1}{6^{\frac{1}{2}}}$ $\frac{1}{2\sqrt{6}} \cdot 4^{2}$

$\frac{2}{\sqrt{6}} \times \frac{1}{\sqrt{6}}$

$\frac{3}{6}$

(e) $p = 16 - 2q^2$, $p = 4$

$2q^2 = 16 - p$

$q^2 = 8 - \frac{p}{2}$

$q = \sqrt{8 - \frac{p}{2}}$

$q' = \frac{1}{2}(8 - \frac{p}{2})^{-\frac{1}{2}}$

$E(p) = \frac{\frac{1}{2}(8 - \frac{p}{2})^{-\frac{1}{2}} \cdot p}{\cancel{16}\sqrt{8 - \frac{p}{2}}}$ $= \frac{1}{3}$

$E(4) = \frac{\frac{1}{2}(6)^{-\frac{1}{2}} \cdot 4}{\sqrt{6}}$

$$q' = \frac{1}{2}\cdot\frac{1}{2\sqrt{8-\frac{p}{2}}} \cdot \frac{1}{2}$$

(e) $p = 16 - 2q^2 \quad p = 4.$

$2q^2 = 16 - p$

$q^2 = 8 - \frac{p}{2}$

$q = \sqrt{8 - \frac{p}{2}}$

$$E(p) = -\frac{p \cdot \frac{\cancel{4}\ 8}{\cancel{8}\sqrt{6}}}{\cancel{\sqrt{8}}\sqrt{6}}$$

$$= -\frac{\cancel{4}\ \cancel{8}\ 2}{12}$$

$= -\frac{1}{3} \quad = \frac{1}{-6}$ ineles.

(f) $2p = 144 - q^2 \ , \ P = 48$ $\quad q' = \frac{1}{2}(144-2p)^{-\frac{1}{2}} \cdot 2$

$q^2 = 144 - 2p$

$q = \sqrt{144 - 2p}.$

$$E(p) = -\frac{p\left(\frac{1}{2}(144-2p)^{-\frac{1}{2}}\right)\cdot 2}{\sqrt{144-2p}}$$

$E(48) = -1$ $\quad\therefore$ It is inelastic

5.1.2

(a)

$q = 45 - \frac{1}{5}p^2$

$q' = -\frac{2}{5}p$

$$E(p) = -\frac{p f'(p)}{f(p)}$$

$$= -\frac{p \cdot -\frac{2}{5}p}{45 - \frac{1}{5}p^2}$$

$$E(8) = \frac{-8 - \frac{2}{5}\cdot 8}{48 - \frac{1}{5}\cdot 8^2}$$

$= -\frac{3}{22}$ $\quad\therefore$ It is inelastic

$$E(10) = \frac{-10 - \frac{2}{5}\cdot 10^2}{48 - \frac{1}{5}\cdot 10^2}$$

$$= -\frac{6}{28}$$ $\quad\therefore$ It is inelastic.

$= -\frac{3}{14}$

(b) $E(p) = \frac{-p \cdot -\frac{2}{5}p}{45 - \frac{1}{5}p^2}$ $\quad 225 - p^2 = -5p \cdot -2p \quad P = \frac{15}{\sqrt{11}}$

$1 = \frac{-p \cdot -\frac{2}{5}p}{45 - \frac{1}{5}p^2}$ $\quad 225 - p^2 = 10p^2$

$225 = 11p^2$

$45 - \frac{1}{5}p^2 = -p \cdot -\frac{2}{5}p$ $\quad \frac{225}{11} = p^2$

(c) Increase
(d) Increase

5.1.3.

a) $q' = \frac{1}{3}(36-p^2)^{-\frac{1}{2}} \cdot 2p$

$E(p) = \frac{-p \cdot \frac{2}{3}p(36-p^2)^{-\frac{1}{2}}}{\frac{2}{3}\sqrt{36-p^2}}$

$E(2) = -\frac{1}{8}$

$\therefore$ inelastic

(b) decrease

5.1.4.

(a) $p = \sqrt{q-0.02q}$

$p^2 = q-0.02q$

$p^2-q = -0.02q$

$\frac{p^2-q}{-0.02} = q$

$q = 50p^2-450$

$q' = \cancel{50}\,-100p$

$E(p) = \frac{-pf'(p)}{f(p)}$

$= \frac{-p \cdot -100p}{-50p^2-450}$

$= \frac{100p^2}{-50p^2-450}$

(b)

5.2. Related Rates.

5.3 Linear Approximation.

Differentiacs linearieation Newturn-Raphsm Method