EP2200 Course Project 2021
Project I - Error Control in Relay Networks

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I. AF Relaying with end-to-end ARQ

Consider the queuing network for AF relaying with end-to-end ARQ shown. Give the queuing model for the RS and MS nodes.

For each RS and MS, the model is **M/M/1**.

1) The end-to-end error probability of packet transmission $(p_{e,e2e})$ and the packet arrival rate at the first relay (λ_1).
The end-to-end error probability of packet transmission $p_{e,e2e}$:

$$p_{e,e2e} = 1 - \prod_{k=1}^{r+1}(1 - p_{(e,k)}) \tag{1}$$

The packet arrival rate at the first delay:

$$\begin{cases} \lambda_1 = \lambda_{r+1} \cdot p_{e,e2e} + \lambda \\ \lambda_2 = \lambda_1 \\ \lambda_3 = \lambda_2 \\ \dots \\ \lambda_{r+1} = \lambda_r \end{cases}$$

According to equation set above,

$$\lambda_{r+1} = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{e,k})}$$

Therefore,

$$\lambda_1 = \lambda_{r+1} \cdot p_{e,e2e} + \lambda = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{e,k})} \tag{2}$$

2) The average queuing delay and the average number of packets (waiting or under processing) at every RS and ms $(\bar{W}_j)$ and $(\bar{N}_j)$.
For RS, the server utilization ρ_i^{AF}:

$$\rho_i^{AF} = \frac{\lambda_i^{AF}}{\mu_i^{AF}} = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{e,k}) \cdot \mu_{AF}}$$

Because for M/M/1 system, the waiting time is $W = \frac{1}{\mu-\lambda} - \frac{1}{\mu}$. Therefore, the average queuing delay for every RS is:

$$W_{RS} = \frac{\lambda}{\mu_{AF}^2 \prod_{k=1}^{r+1}(1 - p_{e,k}) - \lambda \cdot \mu_{AF}} \tag{3}$$

Because for M/M/1 system, the average number in the system is $N = \frac{\rho}{1-\rho}$. Thus, the average number of packets (waiting or under processing) is:

$$N_{RS} = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{e,k}) \cdot \mu_{AF} - \lambda} \tag{4}$$

For MS, the server utilization ρ^{MS}:

$$\rho^{MS} = \frac{\lambda_{r+1}}{\mu_{MS}} = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{e,k}) \cdot \mu_{MS}}$$

Therefore, the average queuing delay for MS is :

$$W_{MS} = \frac{\lambda}{\mu_{MS}^2 \cdot \prod_{k=1}^{r+1}(1 - p_{e,k}) - \lambda \cdot \mu_{MS}} \tag{5}$$

The average number of packets (waiting or under processing) is:

$$N_{MS} = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{e,k}) \cdot \mu_{MS} - \lambda} \tag{6}$$

3) The average end-to-end delay, that is the average time from the generation of a packet until it is received successfully at the MS $(\bar{T})$.
Because, the average time from the generation of a packet until it is received successfully at the MS is the sum of all the time spent in the RS + all the time spent in the MS, i.e.

$$\bar{T} = \sum_{i=1}^{r}(W_{RS} + \frac{1}{\mu_i^{AF}}) + W_{MS} + \frac{1}{\mu_{MS}}$$

Furthermore, since a packet is decoded and checked only at the last hop when the MS receives it, the time spent in each RS is the same. Therefore, the average end-to-end delay is :

$$\begin{aligned}\bar{T} = & \frac{r \cdot \prod_{k=1}^{r+1}(1-p_{e,k})}{\mu_{AF} \cdot \prod_{k=1}^{r+1}(1-p_{e,k}) - \lambda} \\ & + \frac{\prod_{k=1}^{r+1}(1-p_{e,k})}{\mu_{MS} \cdot \prod_{k=1}^{r+1}(1-p_{e,k}) - \lambda}\end{aligned} \tag{7}$$

II. DF RELAYING WITH END-TO-END OR DF RELAYING WITH HOP-BY-HOP ARQ

Consider the queuing network for DF relaying with end-to-end ARQ and with hop-by-hop ARQ.

A. *DF Relaying with end-to-end ARQ*

1) The packet arrival rate at each RS and MS (λ_j):

$$\begin{aligned}\lambda_1 &= \lambda + \lambda_1 \cdot p_{e,1} + \lambda_2 \cdot p_{e,2} + \cdots + \lambda_{r+1} \cdot p_{e,r+1} \\ &= \lambda + \lambda_1 \cdot \sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,k})] \\ &= \frac{\lambda}{1-\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{i=1}^{k}(1-p_{e,i})]}\end{aligned}$$

Therefore,

$$\begin{aligned}\lambda_i &= \lambda_1 \prod_{k=0}^{i-1}(1-p_{e,k}) \\ &= \frac{\lambda \cdot \prod_{k=1}^{i-1}(1-p_{e,k})}{1-\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,j})]}\end{aligned} \tag{8}$$

2) The average queuing delay and the average number of packets (waiting or under processing) at every RS and MS ($\bar{W}_j$ and $\bar{N}_j$)

For RS:

The average queuing delay $W_{RS}^i = \frac{1}{\mu_{DF}-\lambda_i} - \frac{1}{\mu_{DF}}$:

$$\frac{\lambda \cdot \prod_{k=1}^{i-1}(1-p_{e,k})}{\mu_{DF}^2[1-\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,j})]] - \lambda\mu_{DF}\prod_{k=1}^{i-1}(1-p_{e,k})} \tag{9}$$

Because the utilization is:

$$\begin{aligned}\rho_{RS}^i &= \frac{\lambda_i}{\mu_{DF}} \\ &= \frac{\lambda \cdot \prod_{k=1}^{i-1}(1-p_{e,k})}{\mu_{DF} - \mu_{DF}\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,j})]}\end{aligned}$$

Therefore, the average number of packets at every RS is $N_{RS}^i = \frac{\rho_{RS}^i}{1-\rho_{RS}^i}$:

$$\frac{\lambda \cdot \prod_{k=1}^{i-1}(1-p_{e,k})}{\mu_{DF} - \mu_{DF}\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,j})] - \lambda \cdot \prod_{k=1}^{i-1}(1-p_{e,k})} \tag{10}$$

For MS:

The average queuing delay $W_{MS} = \frac{1}{\mu_{MS}-\lambda_{r+1}} - \frac{1}{\mu_{MS}}$:

$$\frac{\lambda \cdot \prod_{k=1}^{r}(1-p_{e,k})}{\mu_{MS}^2[1-\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,j})]] - \lambda\mu_{MS}\prod_{k=1}^{r}(1-p_{e,k})} \tag{11}$$

Because the utilization is:

$$\begin{aligned}\rho_{MS} &= \frac{\lambda_{r+1}}{\mu_{MS}} \\ &= \frac{\lambda \cdot \prod_{k=1}^{r}(1-p_{e,k})}{\mu_{MS} - \mu_{MS}\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,j})]}\end{aligned}$$

Therefore, the average number of packets at every MS is $N_{MS} = \frac{\rho_{MS}}{1-\rho_{MS}}$:

$$\frac{\lambda \cdot \prod_{k=1}^{r}(1-p_{e,k})}{\mu_{MS} - \mu_{MS}\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,j})] - \lambda \cdot \prod_{k=1}^{r}(1-p_{e,k})} \tag{12}$$

3) The average end-to-end delay, that is, the average time from the generation of a packet until it is received successfully at the MS ($\bar{T}$).

Because, the average time from the generation of a packet until it is received successfully at the MS is the sum of all the time spent in the RS + all the time spent in the MS, i.e.

$$\bar{T} = \sum_{i=1}^{r}(W_{RS}^i + \frac{1}{\mu_i^{DF}}) + W_{MS} + \frac{1}{\mu_{MS}}$$

Therefore, the average end-to-end delay is $\bar{T}$:

$$\begin{aligned}&\sum_{i=1}^{r}\frac{1-\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,j})]}{\mu_{DF} - \mu DF\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,j})] - \lambda\prod_{k=1}^{i-1}(1-p_{e,k})} \\ &+ \frac{1-\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,j})]}{\mu_{MS} - \mu MS\sum_{k=0}^{r}[p_{e,k+1} \cdot \prod_{j=1}^{k}(1-p_{e,j})] - \lambda\prod_{k=1}^{r}(1-p_{e,k})}\end{aligned} \tag{13}$$

B. DF Relaying with hop-by-hop ARQ

1) The packet arrival rate at each RS and MS (λ_j):

$$\begin{cases} \lambda_1 = \lambda + \lambda_1 \cdot p_{e,1} \\ \lambda_2 = \lambda_1 \cdot (1 - p_{e,1}) + \lambda_2 \cdot p_{e,2} \\ \dots \\ \lambda_r = \lambda_{r-1} \cdot (1 - p_{e,r-1}) + \lambda_r \cdot p_{e,r} \\ \lambda_{r+1} = \lambda_r \cdot (1 - p_{e,r}) + \lambda_{r+1} \cdot p_{e,r+1} \end{cases}$$

Therefore, from the equation set above, we can get the equation of λ_i and λ_{r+1}, which are:

$$\begin{cases} \lambda_i = \frac{\lambda}{1-p_{e,i}} \\ \lambda_{r+1} = \frac{\lambda}{1-p_{e,r+1}} \end{cases} \tag{14}$$

2) The average queuing delay and the average number of packets (waiting or under processing) at every RS and MS ($\bar{W}_j$ and $\bar{N}_j$).

For RS:

The average queuing delay is $W^i_{RS} = \frac{1}{\mu_{RS}-\lambda_i} - \frac{1}{\mu_{RS}}$:

$$W^i_{RS} = \frac{\lambda}{\mu_{DF} \cdot [\mu_{DF} \cdot (1 - p_{e,i}) - \lambda]} \tag{15}$$

Because the utilization is :

$$\rho^i_{RS} = \frac{\lambda_i}{\mu_{DF}} = \frac{\lambda}{\mu_{DF} \cdot (1 - p_{e,i})}$$

Therefore, the average number of packet at every RS is $N^i_{RS} = \frac{\rho^i_{RS}}{1-\rho^i_{RS}}$:

$$N^i_{RS} = \frac{\lambda}{\mu_{DF}(1 - p_{e,i}) - \lambda} \tag{16}$$

For MS:

The average queuing delay is $W_{MS} = \frac{1}{\mu_{MS}-\lambda_i} - \frac{1}{\mu_{MS}}$:

$$W_{MS} = \frac{\lambda}{\mu_{MS} \cdot [\mu_{MS} \cdot (1 - p_{e,r+1}) - \lambda]} \tag{17}$$

Because the utilization is:

$$\rho_{MS} = \frac{\lambda_{r+1}}{\mu_{MS}} = \frac{\lambda}{\mu_{MS} \cdot (1 - p_{e,r+1})}$$

Therefore, the average number of packet at MS is $N_{MS} = \frac{\rho_{MS}}{1-\rho_{MS}}$:

$$N_{MS} = \frac{\lambda}{\mu_{MS} \cdot (1 - p_{e,r+1}) - \lambda} \tag{18}$$

3) The average end-to-end delay, that is, the average time from the generation of a packet until it is received successfully at the MS ($\bar{T}$).

Because, the average time from the generation of a packet until it is received successfully at the MS is the sum of all the time spent in the RS + all the time spent in the MS, i.e.

$$\bar{T} = \sum_{i=1}^{r} (W^i_{RS} + \frac{1}{\mu_i^{DF}}) + W_{MS} + \frac{1}{\mu_{MS}}$$

Therefore, the average end-to-end delay is $\bar{T}$:

$$\bar{T} = \sum_{i=1}^{r} \frac{1 - p_{e,i}}{\mu_{DF}(1 - p_{e,i}) - \lambda} + \frac{1 - p_{e,r+1}}{\mu_{MS}(1 - p_{e,r+1}) - \lambda} \tag{19}$$

III. Stability Region

A. Stability Region for AF relaying with end-to-end ARQ

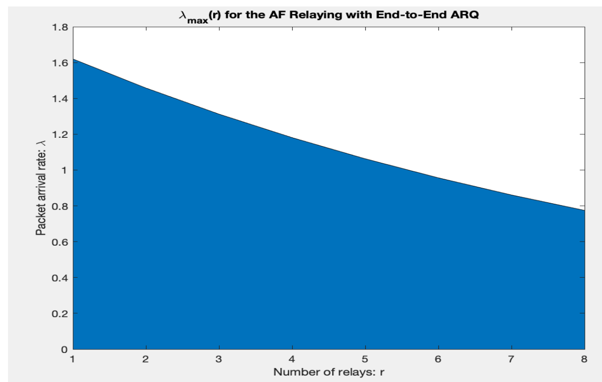

Fig. 1. The maximal allowed arrival rate of packets $\lambda_{max}(r)$ for the AF relaying with end-to-end ARQ.

As $\lambda \leq 2 \cdot 0.9^{r+1}$, when r goes to infinity, $2 \cdot 0.9^{r+1}$ will go to 0. Therefore, $\lambda_{max}(r)$ will go to 0 as well.

B. Stability Region for DF relaying with end-to-end ARQ

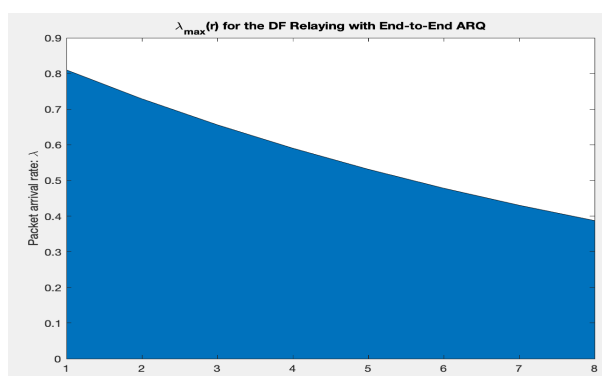

Fig. 2. The maximal allowed arrival rate of packets $\lambda_{max}(r)$ for the DF relaying with end-to-end ARQ.

As $(\lambda \leq 0.9^{r+1}) \cap (\lambda \leq 1.8)$, when r goes to infinity, $2 \cdot 0.9^{r+1}$ will go to 0. Therefore, $\lambda_{max}(r)$ will go to 0 as well.

C. Stability Region for DF relaying with hop-by-hop ARQ

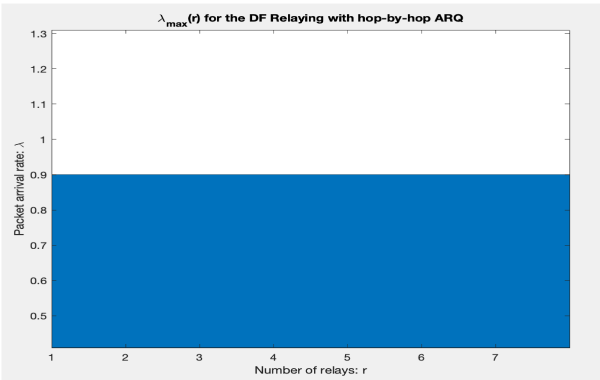

Fig. 3. The maximal allowed arrival rate of packets $\lambda_{max}(r)$ for the DF relaying with hop-by-hop ARQ.

Since $(\lambda \leq 0.9) \cap (\lambda \leq 1.8)$, when r goes to infinity, there is no influence on $\lambda_{max}(r)$.

IV. Arrival rate-delay characteristics

A. Arrival rate-delay for AF relaying with end-to-end ARQ

$$T = \frac{5 \cdot 0.9^5}{2 \cdot 0.9^5 - \lambda}$$

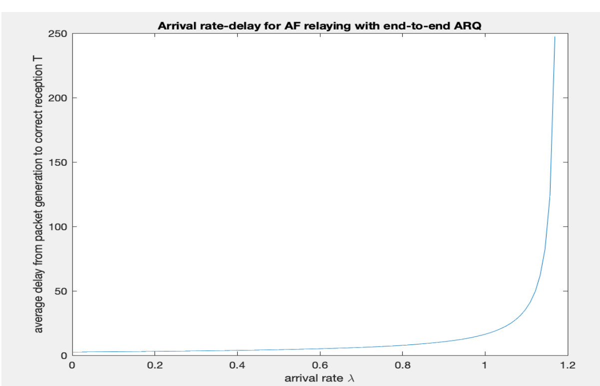

Fig. 4. arrival rate-delay for AF relaying with end-to-end ARQ.

When $p_{e,j}$ increases, the arrival rate-delay curves moves to the left.

B. Arrival rate-delay for DF relaying with end-to-end ARQ

$$T = \sum_{i=1}^{4} \frac{0.9^5}{0.9^5 - \lambda \cdot 0.9^{i-1}} + \frac{0.9^5}{2 * 0.9^5 - 0.9^4 \cdot \lambda}$$

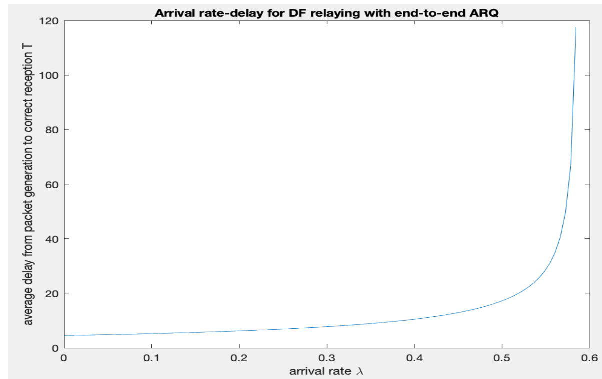

Fig. 5. arrival rate-delay for DF relaying with end-to-end ARQ.

When $p_{e,j}$ increases, the arrival rate-delay curves moves to the left.

C. Arrival rate-delay for DF relaying with hop-by-hop ARQ

$$T = \frac{3.6}{0.9 - \lambda} + \frac{0.9}{1.8 - \lambda}$$

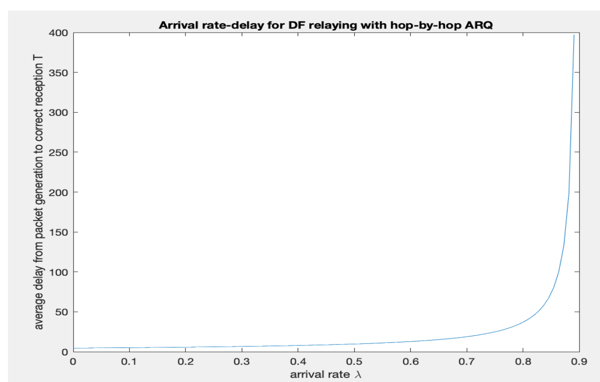

Fig. 6. arrival rate-delay for DF relaying with hop-by-hop ARQ.

When $p_{e,j}$ increases, the arrival rate-delay curves moves to the left.

V. AF or DF?

A. Condition I

Denote $p_{e,j} = p, j = 1, 2, \cdots, r+1$. In this condition, the average system delay for the three models is shown as followed:

The average delay for AF relaying with end-to-end ARQ:

$$T_1 = 5$$

The average delay for DF relaying with end-to-end ARQ:

$$T_2 = \sum_{i=1}^{4} \frac{(1-p)^5}{(1-p)^5 - 0.5 \cdot (1-p)^{i+4}} + \frac{(1-p)^5}{2 \cdot (1-p)^5 - 0.5 \cdot (1-p)^9}$$

The average delay for DF relaying with hop-by-hop ARQ:

$$T_3 = \frac{26}{3}$$

Therefore, the figure can be drawn with $0.05 \leq j \leq 0.95$.

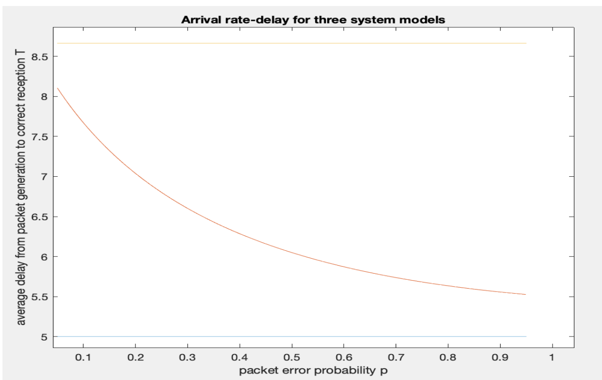

Fig. 7. arrival rate-delay for three system models.

As can be seen from Fig.7 above, choosing AF relaying with end-to-end ARQ will be the optimal choice.

B. Condition II

In this scenario, the average delay for AF relaying with end-to-end ARQ:

$$T = \frac{4}{k-1} + 1$$

The average delay for DF relaying with end-to-end ARQ:

$$T = \sum_{i=1}^{4} \frac{0.9^5}{0.9^5 - 0.5 \cdot 0.9^{i+4}} + \frac{0.9^5}{2 \cdot 0.9^5 - 0.5 \cdot 0.9^9}$$

The average delay for DF relaying with hop-by-hop ARQ:

$$T = \frac{26}{3}$$

Therefore, the figure can be drawn when k increases from 1 to 4.

As can be observed from the graph that, when $1 \leq k \leq 1.6$, DF relaying with end-to-end ARQ will be the optimal choice. However, when $1.6 \leq k \leq 4$, AF relaying with end-to-end will shows its advantage in system average delay.

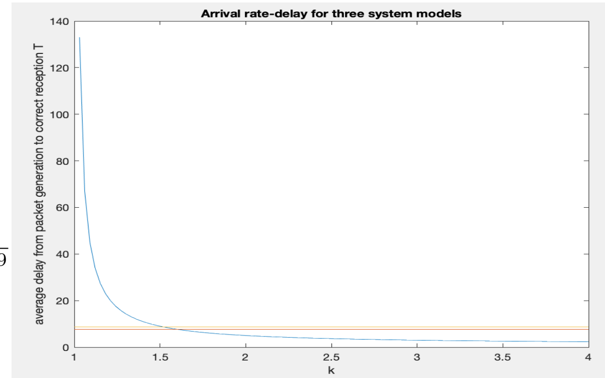

Fig. 8. arrival rate-delay for three system models.

VI. Probabilistic ARQ - Extra Exercise for the Best

A. The P-ARQ queuing network mode

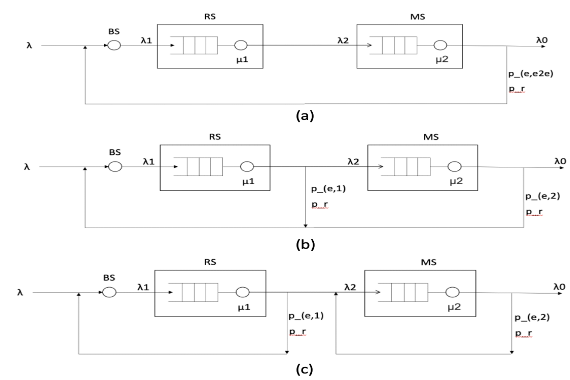

Fig. 9. P-ARQ queuing network model for (a)AF relaying with end-to-end ARQ, (b)DF relaying with end-to-end ARQ, and (c)DF relaying with hop-by-hop ARQ

B. Performance evaluation

1) P-ARQ - AF - end-to-end T^P_{AF-ETE}:

$$\begin{aligned} &\frac{1-(1-p_{e,1})[1-(1-p_{e,1})(1-p_{e,2})]p_r}{\mu_{AF} - \mu_{AF}p_r[1-(1-p_{e,1})(1-p_{e,2})] - \lambda} \\ &+ \frac{1-(1-p_{e,1})[1-(1-p_{e,1})(1-p_{e,2})]p_r}{\mu_{MS} - \mu_{MS}p_r[1-(1-p_{e,1})(1-p_{e,2})] - \lambda} \end{aligned} \tag{20}$$

2) P-ARQ - DF - end-to-end T^P_{DF-ETE}:

$$\begin{aligned} &\frac{1-p_{e,1}p_r-(1-p_{e,1})p_{e,2}p_r}{\mu_{DF} - \mu_{DF}p_{e,1}p_r - \mu_{DF}(1-p_{e,1})p_{e,2}p_r - \lambda} \\ &+ \frac{(1-p_{e,1})(1-p_{e,1}p_r-(1-p_{e,1})p_{e,2}p_r)}{\mu_{MS} - \mu_{MS}p_{e,1}p_r - \mu_{MS}(1-p_{e,1})p_{e,2}p_r - \lambda(1-p_{e,1})} \end{aligned} \tag{21}$$

3) P-ARQ - DF - hop-by-hop T^P_{DF-HBH}:

$$\begin{aligned} &\frac{1 - p_{e,1}p_r}{\mu_{DF} - \mu_{DF}p_{e,1}p_r - \lambda} \\ &+ \frac{(1 - p_{e,2}p_r)(1 - p_{e,1}p_r)}{\mu_{MS}(1 - p_{e,2}p_r)(1 - p_{e,1}p_r) - (1 - p_{e,1})\lambda} \end{aligned} \tag{22}$$

4) Plotting the figure with the given parameters, the equations above can be switched into the following:

$$T^P_{AF-ETE} = \frac{2 - 0.342 \cdot p_r}{1.8 - 0.38 \cdot p_r}$$

$$T^P_{DF-ETE} = \frac{1 - 0.19 \cdot p_r}{0.8 - 0.19 \cdot p_r} + \frac{0.9 - 0.171 \cdot p_r}{1.82 - 0.38 \cdot p_r}$$

$$T^P_{DF-HBH} = \frac{1 - 0.1 \cdot p_r}{0.8 - 0.1 \cdot p_r} + \frac{(1 - 0.1 \cdot p_r)^2}{2 \cdot (1 - 0.1 \cdot p_r)^2 - 0.18}$$

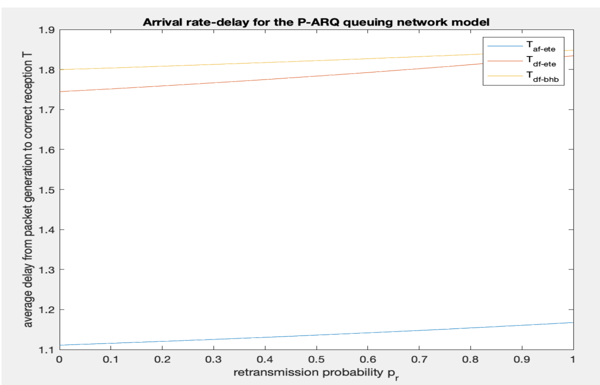

Fig. 10. end-to-end delay for P-ARQ queuing network model.