

1 a).  $\arg \min_{f(x)} E_{Y|x} \exp(-\frac{1}{k}(Y_1 f_1 + \dots + Y_k f_k))$  subject to  $f_1 + \dots + f_k = 0$

The lagrange of this constrained optimization problem can be written as :

$$\exp(-\frac{f_1(x)}{k-1}) \text{Prob}(C=1|x) + \dots + \exp(-\frac{f_k(x)}{k-1}) \text{Prob}(C=k|x) - \lambda(f_1(x) + \dots + f_k(x)).$$

Where  $\lambda$  is the lagrange multiplier. Taking derivative with respect to

$f_k$  and  $\lambda$  :

$$-\frac{1}{k-1} \exp(-\frac{f_1(x)}{k-1}) \text{Prob}(C=1|x) - \lambda = 0$$

$\vdots$

$$-\frac{1}{k-1} \exp(-\frac{f_k(x)}{k-1}) \text{Prob}(C=k|x) - \lambda = 0$$

$$f_1(x) + \dots + f_k(x) = 0$$

$$\therefore f_k^*(x) = (k-1) \left( \log \text{Prob}(C=k|x) - \frac{1}{k} \sum \log \text{Prob}(C=k|x) \right) \quad k=1, \dots, k$$

$$\begin{cases} \arg \max_k f_k^* = \arg \max_k \text{Prob}(C=k|x) \end{cases}$$

$$\begin{cases} \text{Prob}(C=k|x) = \frac{e^{\frac{1}{k-1} f_k^*(x)}}{e^{\frac{1}{k-1} f_1^*(x)} + \dots + e^{\frac{1}{k-1} f_k^*(x)}} \quad k=1, \dots, k. \end{cases}$$

2. b). Initialize the observation weights  $w_i = 1/n$  from 1 to  $M$  :

$$\text{err}^{(m)} = \frac{\sum_{i=1}^n w_i \mathbb{I}(C_i \neq T^{(m)}(x_i))}{\sum_{i=1}^n w_i}$$

$$\alpha^{(m)} = \log \frac{1 - \text{err}^{(m)}}{\text{err}^{(m)}} + \log(k-1)$$

$$w_i = w_i \cdot \exp(\alpha^{(m)} \mathbb{I}(C_i \neq T^{(m)}(x_i)))$$

$$\text{Output} = C(x) = \arg \max_{k=1, \dots, M} \alpha^{(m)} \cdot \mathbb{I}(T^{(m)}(x) = k)$$

The major difference between Adaboost and this new algorithm is the  $\alpha^{(m)}$ , where in New Algorithm, the  $\alpha^{(m)} = \log \frac{1 - \text{err}^{(m)}}{\text{err}^{(m)}} + \log(k-1)$

In AdaBoost: the  $\alpha^{(m)} = \frac{1}{2} \log \frac{1 - \text{err}^{(m)}}{\text{err}^{(m)}}$

# HW3#2

## Function to Import train data and do pre-processing

In [42]:

```
import numpy as np
import pandas as pd
from sklearn import tree
from sklearn import model_selection
import matplotlib.pyplot as plt
from sklearn.externals.six import StringIO
#import pydotplus
from IPython.display import Image
from sklearn import metrics
%matplotlib inline

def data_processing(file):
    train=file.readlines()

    train_list=[]

    for line in train:
        line_replace=line.replace('?','mode')
        line_list=line_replace.split(",")
        #line_list=line_list.split(" ")
        for i in range(len(line_list)):
            if line_list[i].isdigit()==True:
                line_list[i]=int(line_list[i])

        train_list.append(line_list)
    train_list.pop()
    train_array=np.array(train_list)

    classVotes={}
    for i in range(len(train_list[0])):
        if type(train_list[0][i])==int:
            data=train_array[:,i]
            s=0
            num=1
            for j in data:
                s+=int(j)
                num+=1
            classVotes[i]=s/num
    for i in range(len(train_list[0])):
```

```

        if type(train_list[0][i])==int:
            for j in range(len(train_list)):
                if train_list[j][i]=='mode':
                    train_list[j][i].replace('classVotes[i]','mode')

```

```

    return train_list

```

```

file1=open('/Users/wendy/Documents/2017 Fall/CS 534/HW3/adult-test.csv')
file2=open('/Users/wendy/Documents/2017 Fall/CS 534/HW3/adult-data.csv')

```

```

def data_processing1(file):
    train=file.readlines()

```

```

    train_list=[]

```

```

    for line in train:
        line_replace=line.replace('?','mode')
        line_list=line_replace.split(", ")
        #line_list=line_list.split(" ")
        for i in range(len(line_list)):
            if line_list[i].isdigit()==True:
                line_list[i]=int(line_list[i])

```

```

        train_list.append(line_list)
    train_list.pop()
    train_array=np.array(train_list)

```

```

    classVotes={}
    for i in range(len(train_list[0])):
        if type(train_list[0][i])==int:
            data=train_array[:,i]
            s=0
            num=1
            for j in data:
                s+=int(j)
                num+=1
            classVotes[i]=s/num
    for i in range(len(train_list[0])):
        if type(train_list[0][i])==int:
            for j in range(len(train_list)):
                if train_list[j][i]=='mode':
                    train_list[j][i].replace('classVotes[i]','mode')

```

```

    return train_list

```

```

train=data_processing(file2)
test=data_processing1(file1)
print train[0]
print test[0]
import csv
t=open("train.csv","wb")
writer=csv.writer(t)
writer.writerows(train)

```

```

te=open("test.csv","wb")
writer=csv.writer(te)
writer.writerows(test)
cname=['a','b','c','d',"e",'f','g','h','i','j','k','l','m','n','o']
train_data = pd.read_csv('/Users/wendy/Documents/2017 Fall/CS 534/HW3/train.csv',na
test_data=pd.read_csv('/Users/wendy/Documents/2017 Fall/CS 534/HW3/test.csv',names=c
#print test_data

```

```

[39, ' State-gov', 77516, ' Bachelors', 13, ' Never-married', ' Adm-cl
erical', ' Not-in-family', ' White', ' Male', 2174, 0, 40, ' United-St
ates', ' <=50K\r\n']
[25, 'Private', 226802, '11th', 7, 'Never-married', 'Machine-op-inspct
', 'Own-child', 'Black', 'Male', 0, 0, 40, 'United-States', '<=50K.\n'
]

```

## Data transformation and convert training\_y into numeric categories

In [31]:

```
bDummies = pd.get_dummies(train_data.b, prefix='b').iloc[:, 1:]
dDummies = pd.get_dummies(train_data.d, prefix='d').iloc[:, 1:]
fDummies = pd.get_dummies(train_data.f, prefix='f').iloc[:, 1:]
gDummies = pd.get_dummies(train_data.g, prefix='g').iloc[:, 1:]
hDummies = pd.get_dummies(train_data.g, prefix='h').iloc[:, 1:]
iDummies = pd.get_dummies(train_data.g, prefix='i').iloc[:, 1:]
jDummies = pd.get_dummies(train_data.g, prefix='j').iloc[:, 1:]
nDummies = pd.get_dummies(train_data.g, prefix='n').iloc[:, 1:]

trainDF = pd.concat([train_data, bDummies,dDummies,fDummies,gDummies,hDummies,iDummies,jDummies,nDummies])
print trainDF.columns

y = trainDF.o
trainx = trainDF.drop(trainDF.columns[[1,3,5,6,7,8,9,13,14]], axis=1)
trainy=[]
for i in y:
    if i==' <=50K\r\n':
        trainy.append(0)
    else:
        trainy.append(1)
```

```
Index([u'a', u'b', u'c', u'd', u'e', u'f', u'g', u'h', u'i', u'j',
      ...,
      u'n_ Handlers-cleaners', u'n_ Machine-op-inspct', u'n_ Other-se
rvice',
      u'n_ Priv-house-serv', u'n_ Prof-specialty', u'n_ Protective-se
rv',
      u'n_ Sales', u'n_ Tech-support', u'n_ Transport-moving', u'n_ m
ode'],
      dtype='object', length=114)
```

## Decision\_tree Function

In [34]:

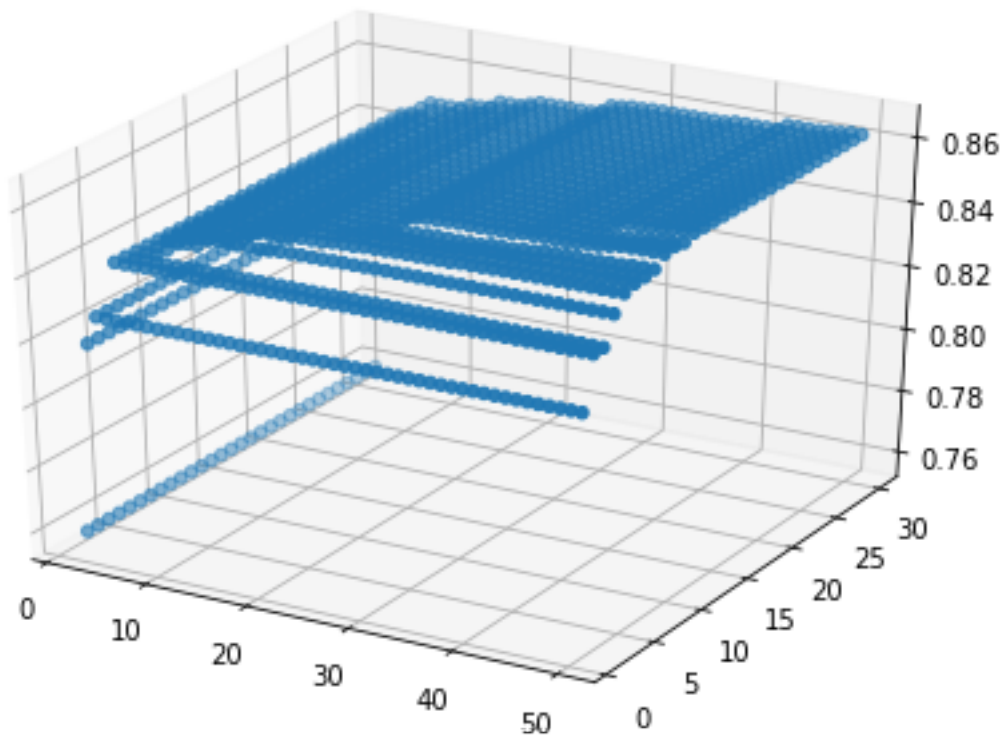
```
def D_Tree(i,j):  
    clf = tree.DecisionTreeClassifier(max_depth=i,max_leaf_nodes=j)  
    clf.fit(trainx, trainy)  
    #score = model_selection.cross_val_score(clf,trainx, trainy, cv=5,scoring='accuracy')  
  
    trainyHat = clf.predict(trainx)  
    err=0  
    for a in range(len(trainyHat)):  
        if trainyHat[a]!=trainy[a]:  
            err+=1  
    return float ((len(trainy)-float(err))/len(trainy))  
  
#print "Train", metrics.classification_report(trainy, trainyHat)
```

**plot the accuracy as a function of depth and num of leaves**

In [43]:

```
accuracy_list=[]
accuracy=[]
depth_list=[]
#depth_list=list(range(1,3))
leaf_num_list=[]
for i in range(1,31):
    for j in range (2,51):
        depth_list.append(i)
        leaf_num_list.append(j)
        accuracy.append(D_Tree(i,j) )
        accuracy_list.append((i,j,D_Tree(i,j)))

from matplotlib import pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
fig=plt.figure()
ax=Axes3D(fig)
ax.scatter(leaf_num_list,depth_list,accuracy)
plt.show()
```



## Find the optimal depth and leaves

In [44]:

```
from operator import itemgetter
sorted_accuracy=sorted(accuracy_list,key=itemgetter(2),reverse=True)
print "the optimal depth is:",sorted_accuracy[0][0],"the optimal #of leaves is:",sorted_accuracy[0][1]
```

the optimal depth is: 12 the optimal #of leaves is: 49 with the accuracy: 0.862223587224



# Test data transformation

In [51]:

```
bDummies = pd.get_dummies(test_data.b, prefix='b').iloc[:, 1:]
dDummies = pd.get_dummies(test_data.d, prefix='d').iloc[:, 1:]
fDummies = pd.get_dummies(test_data.f, prefix='f').iloc[:, 1:]
gDummies = pd.get_dummies(test_data.g, prefix='g').iloc[:, 1:]
hDummies = pd.get_dummies(test_data.g, prefix='h').iloc[:, 1:]
iDummies = pd.get_dummies(test_data.g, prefix='i').iloc[:, 1:]
jDummies = pd.get_dummies(test_data.g, prefix='j').iloc[:, 1:]
nDummies = pd.get_dummies(test_data.g, prefix='n').iloc[:, 1:]

testDF = pd.concat([test_data, bDummies, dDummies, fDummies, gDummies, hDummies, iDummies, jDummies, nDummies], axis=1)
y = testDF.o
testx = testDF.drop(testDF.columns[[1,3,5,6,7,8,9,13,14]], axis=1)

testy=[]
for i in y:
    if i=='<=50K.\n':
        testy.append(0)
    else:
        testy.append(1)
```

## Accuracy on test data

In [53]:

```
clf = tree.DecisionTreeClassifier(max_depth=12,max_leaf_nodes=49)
clf.fit(trainx, trainy)
testyHat = clf.predict(testx)
err=0
for i in range(len(testyHat)):
    if testyHat[i]!=testy[i]:
        err+=1
print "the accuracy is:",float ((len(testy)-float(err))/len(testy))
print "test", metrics.classification_report(testy, testyHat)
```

the accuracy is: 0.861249309011

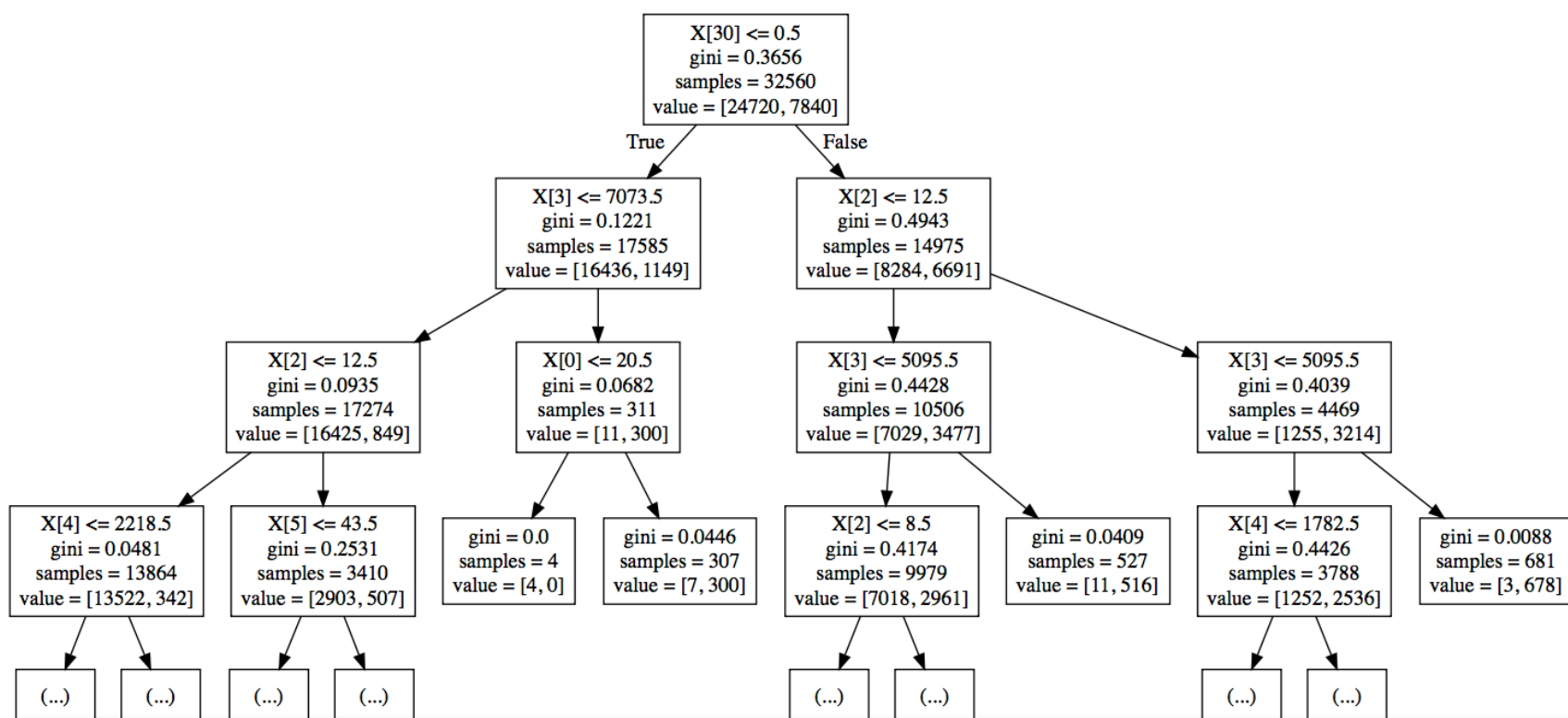
test	precision	recall	f1-score	support
0	0.89	0.94	0.91	12435
1	0.76	0.61	0.67	3846
avg / total	0.86	0.86	0.86	16281



# Plot the tree

In [133]:

```
from sklearn.tree import export_graphviz
tree.export_graphviz(clf, out_file='tree.dot',max_depth=3)
```



# HW3#3 Gradient Boosting

## Import Data

In [1]:

```
import csv
import numpy
import scipy
from sklearn import linear_model
from math import sqrt
import matplotlib.pyplot as plt
training_list=[]
validate_list=[]
testdata_list=[]

tradata=csv.reader(open('/Users/wendy/Documents/2017 Fall/CS 534/homework1/BlogFeedk
for row in tradata:
    training_list.append(row)

valdata=csv.reader(open('/Users/wendy/Documents/2017 Fall/CS 534/homework1/BlogFeedk
for row in valdata:
    validate_list.append(row)

testdata=csv.reader(open('/Users/wendy/Documents/2017 Fall/CS 534/homework1/BlogFeedk
for row in testdata:
    testdata_list.append(row)

training_list=numpy.array(training_list,dtype=float)
validate_list=numpy.array(validate_list,dtype=float)
testdate_list=numpy.array(validate_list,dtype=float)

ytrain=training_list[:,280]
xtrain=training_list[:, 0:279]
yval=validate_list[:,280]
xval=validate_list[:,0:279]
xtest=testdate_list[:,0:279]
ytest=testdate_list[:,280]
```

## Gradient Boosting with mean square loss

In [73]:

```
def GB_squareLoss(v,m,xtrain,ytrain,xval,yval):
    from sklearn import tree
    clf = tree.DecisionTreeRegressor()
    clf = clf.fit(xtrain,ytrain)
    ytrain_hat=clf.predict(xtrain)
    loss=clf.predict(xval)-yval
    #loss_train=clf.predict(xtrain)-ytrain
    for i in range(m):
        r=ytrain_hat-ytrain
        h=tree.DecisionTreeRegressor()
        h = h.fit(xtrain,r)
        ytrain=ytrain_hat
        ytrain_hat=v*h.predict(xtrain)+ytrain_hat
        loss+=v*h.predict(xval)
        #loss_train+=v*h.predict(xtrain)
    return sum(0.5*loss**2)/len(yval)
```

## Gradient Boosting with mean absolute loss

In [74]:

```
def GB_absLoss(v,m,xtrain,ytrain,xval,yval):
    from sklearn import tree
    clf = tree.DecisionTreeRegressor()
    clf = clf.fit(xtrain,ytrain)
    ytrain_hat=clf.predict(xtrain)
    loss=clf.predict(xval)-yval
    #loss_train=clf.predict(xtrain)-ytrain
    for i in range(m):
        r=[]
        for i in range(len(ytrain)):
            if ytrain_hat[i]!=ytrain[i]:
                r.append(1)
            else:
                r.append(0)
        h=tree.DecisionTreeRegressor()
        h = h.fit(xtrain,r)
        ytrain=ytrain_hat
        ytrain_hat=v*h.predict(xtrain)+ytrain_hat
        loss+=v*h.predict(xval)
        #loss_train+=v*h.predict(xtrain)
    return sum(0.5*loss**2)/len(yval)
```

**For number of boosting iterations between 5, 10, 15, 25, plot the validation error as a function of the parameter  $v \in [0,1]$  for square loss.**

In [95]:

```
b=[5,10,15,25]
a=[0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9]
loss_list=[]
m_list=[]
v_list=[]
final_list=[]
for j in range(len(b)):
    for i in range(len(a)):
        loss=GB_squareLoss(a[i],b[j],xtrain,ytrain,xval,yval)
        loss_list.append(loss)
        m_list.append(b[j])
        v_list.append(a[i])
        final_list.append((a[i],b[j],loss))
    print "iteration=",b[j],"shrinkage parameter=",a[i],"loss=",loss
#plt.plot(a,loss_list)
#plt.show()
```

```
iteration= 5 shrinkage parameter= 0.1 loss= 580.144855996
iteration= 5 shrinkage parameter= 0.2 loss= 567.586909633
iteration= 5 shrinkage parameter= 0.3 loss= 594.463417269
iteration= 5 shrinkage parameter= 0.4 loss= 597.615686554
iteration= 5 shrinkage parameter= 0.5 loss= 563.962425064
iteration= 5 shrinkage parameter= 0.6 loss= 586.20157722
iteration= 5 shrinkage parameter= 0.7 loss= 490.082786183
iteration= 5 shrinkage parameter= 0.8 loss= 523.969169283
iteration= 5 shrinkage parameter= 0.9 loss= 530.721345989
iteration= 10 shrinkage parameter= 0.1 loss= 550.576336355
iteration= 10 shrinkage parameter= 0.2 loss= 590.216992578
iteration= 10 shrinkage parameter= 0.3 loss= 553.161433057
iteration= 10 shrinkage parameter= 0.4 loss= 531.571050211
iteration= 10 shrinkage parameter= 0.5 loss= 553.90037111
iteration= 10 shrinkage parameter= 0.6 loss= 542.758901101
iteration= 10 shrinkage parameter= 0.7 loss= 528.496395946
iteration= 10 shrinkage parameter= 0.8 loss= 586.699159649
iteration= 10 shrinkage parameter= 0.9 loss= 590.445409649
iteration= 15 shrinkage parameter= 0.1 loss= 565.734049386
iteration= 15 shrinkage parameter= 0.2 loss= 578.293558894
iteration= 15 shrinkage parameter= 0.3 loss= 607.514487582
iteration= 15 shrinkage parameter= 0.4 loss= 560.120289752
iteration= 15 shrinkage parameter= 0.5 loss= 549.031369937
iteration= 15 shrinkage parameter= 0.6 loss= 593.372466457
iteration= 15 shrinkage parameter= 0.7 loss= 630.716220422
iteration= 15 shrinkage parameter= 0.8 loss= 598.597916072
iteration= 15 shrinkage parameter= 0.9 loss= 523.800064231
```

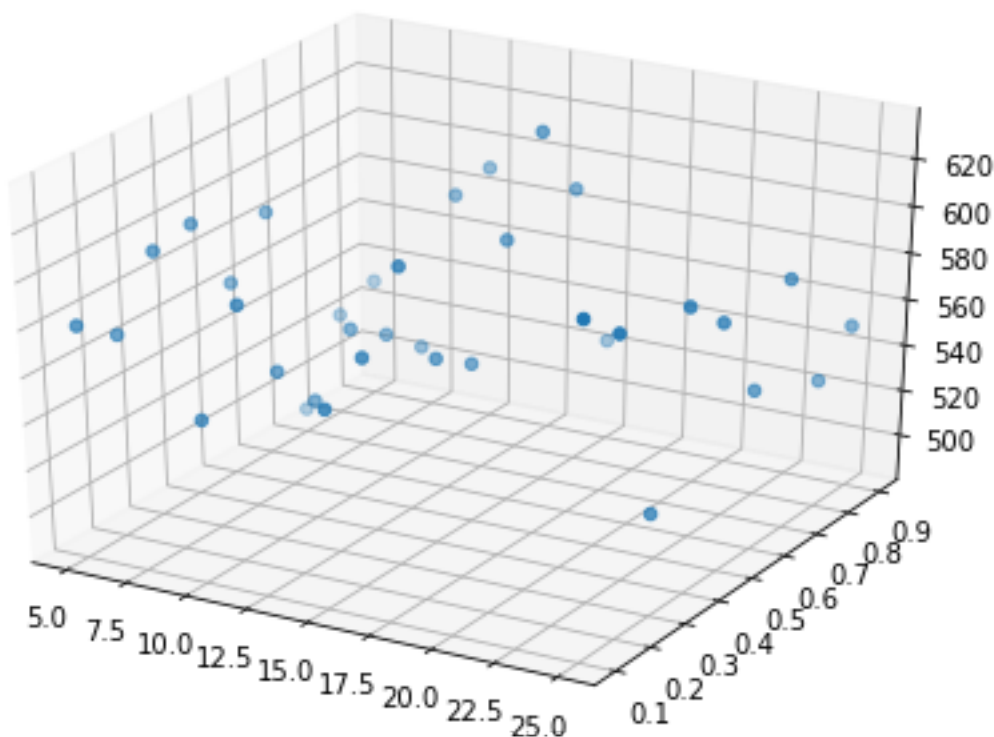
```
iteration= 25 shrinkage parameter= 0.1 loss= 623.161183271
iteration= 25 shrinkage parameter= 0.2 loss= 608.58082358
iteration= 25 shrinkage parameter= 0.3 loss= 524.319803026
iteration= 25 shrinkage parameter= 0.4 loss= 602.050391093
iteration= 25 shrinkage parameter= 0.5 loss= 586.816371681
iteration= 25 shrinkage parameter= 0.6 loss= 549.268983728
iteration= 25 shrinkage parameter= 0.7 loss= 587.886924065
iteration= 25 shrinkage parameter= 0.8 loss= 535.439746091
iteration= 25 shrinkage parameter= 0.9 loss= 550.612633616
```

## Find out the optimal num of iteration and optimal shrinkage parameter

In [96]:

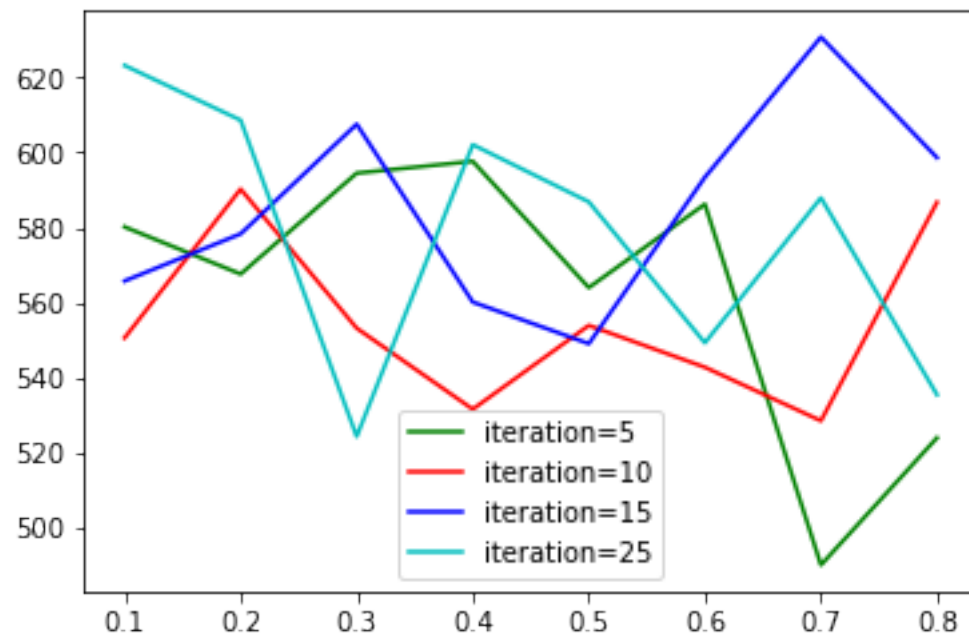
```
from operator import itemgetter
sorted_loss=sorted(final_list,key=itemgetter(2))
print "the optimal #of iteration is:",sorted_loss[0][1],"the optimal v is:",sorted_
from matplotlib import pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
fig=plt.figure()
ax=Axes3D(fig)
ax.scatter(m_list,v_list,loss_list)
plt.show()
```

the optimal #of iteration is: 5 the optimal v is: 0.7 with the loss on validation set: 490.082786183



In [97]:

```
ax=plt.gca()
ax.plot(v_list[0:8],loss_list[0:8],"g",label="iteration=5")
ax.plot(v_list[9:17],loss_list[9:17],"r",label="iteration=10")
ax.plot(v_list[18:26],loss_list[18:26],"b",label="iteration=15")
ax.plot(v_list[27:35],loss_list[27:35],"c",label="iteration=25")
plt.legend()
plt.show()
```



**For number of boosting iterations between 5, 10, 15, 25, plot the validation error as a function of the parameter  $v \in [0,1]$  for absolute loss.**

In [78]:

```
b=[5,10,15,25]
a=[0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9]
loss_list=[]
m_list=[]
v_list=[]
final_list=[]
for j in range(len(b)):
    for i in range(len(a)):
        loss=GB_absLoss(a[i],b[j],xtrain,ytrain,xval,yval)
        loss_list.append(loss)
        m_list.append(b[j])
        v_list.append(a[i])
        final_list.append((a[i],b[j],loss))
    print "iteration=",b[j],"shrinkage parameter=",a[i],"loss=",loss
#plt.plot(a,loss_list)
#plt.show()
```

iteration= 5 shrinkage parameter= 0.1 loss= 597.500071367

iteration= 5 shrinkage parameter= 0.2 loss= 517.212567799



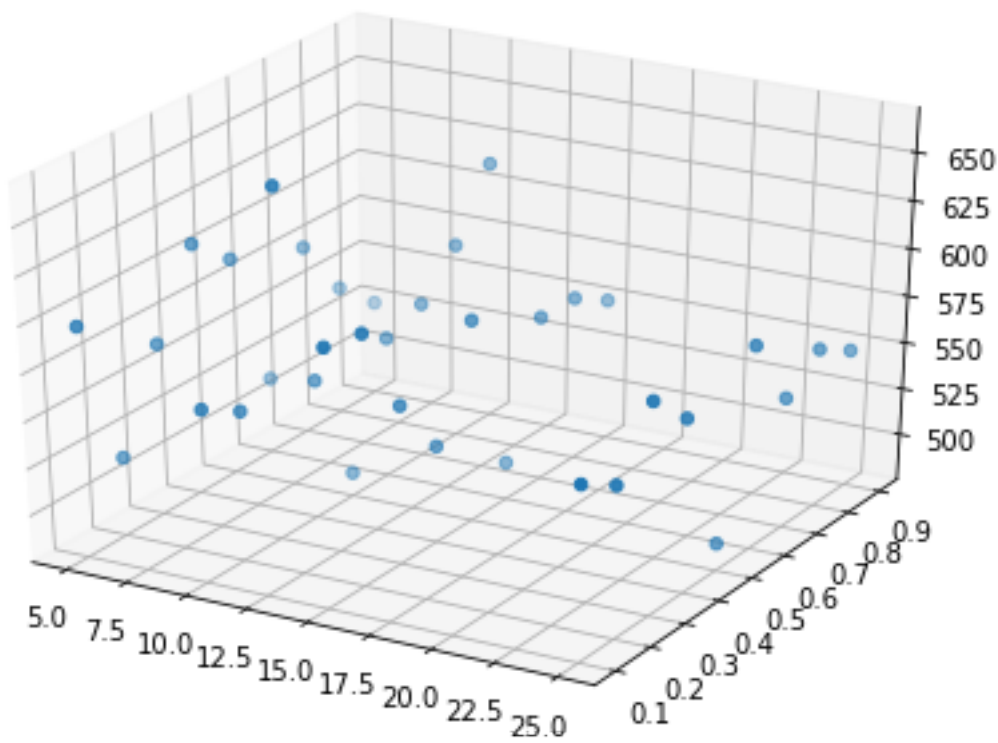
iteration=	5	shrinkage	parameter=	0.3	loss=	566.787421298
iteration=	5	shrinkage	parameter=	0.4	loss=	608.758676848
iteration=	5	shrinkage	parameter=	0.5	loss=	590.609834428
iteration=	5	shrinkage	parameter=	0.6	loss=	515.337353697
iteration=	5	shrinkage	parameter=	0.7	loss=	576.240517057
iteration=	5	shrinkage	parameter=	0.8	loss=	543.795755424
iteration=	5	shrinkage	parameter=	0.9	loss=	525.110690836
iteration=	10	shrinkage	parameter=	0.1	loss=	567.112733728
iteration=	10	shrinkage	parameter=	0.2	loss=	555.032511815
iteration=	10	shrinkage	parameter=	0.3	loss=	660.225750944
iteration=	10	shrinkage	parameter=	0.4	loss=	548.999327719
iteration=	10	shrinkage	parameter=	0.5	loss=	487.615766843
iteration=	10	shrinkage	parameter=	0.6	loss=	549.928133029
iteration=	10	shrinkage	parameter=	0.7	loss=	557.734013703
iteration=	10	shrinkage	parameter=	0.8	loss=	579.0470668
iteration=	10	shrinkage	parameter=	0.9	loss=	612.6331002
iteration=	15	shrinkage	parameter=	0.1	loss=	611.798922352
iteration=	15	shrinkage	parameter=	0.2	loss=	608.007743363
iteration=	15	shrinkage	parameter=	0.3	loss=	559.73668998
iteration=	15	shrinkage	parameter=	0.4	loss=	526.886077806
iteration=	15	shrinkage	parameter=	0.5	loss=	582.628996574
iteration=	15	shrinkage	parameter=	0.6	loss=	494.993683985
iteration=	15	shrinkage	parameter=	0.7	loss=	562.86429489
iteration=	15	shrinkage	parameter=	0.8	loss=	562.509745615
iteration=	15	shrinkage	parameter=	0.9	loss=	550.752114259
iteration=	25	shrinkage	parameter=	0.1	loss=	568.370075649
iteration=	25	shrinkage	parameter=	0.2	loss=	556.160255495
iteration=	25	shrinkage	parameter=	0.3	loss=	587.977412218
iteration=	25	shrinkage	parameter=	0.4	loss=	567.814337711
iteration=	25	shrinkage	parameter=	0.5	loss=	489.695586323
iteration=	25	shrinkage	parameter=	0.6	loss=	583.562232372
iteration=	25	shrinkage	parameter=	0.7	loss=	544.836220739
iteration=	25	shrinkage	parameter=	0.8	loss=	559.896354712
iteration=	25	shrinkage	parameter=	0.9	loss=	548.429025121

**Find out the optimal num of iteration and optimal shrinkage parameter**

In [86]:

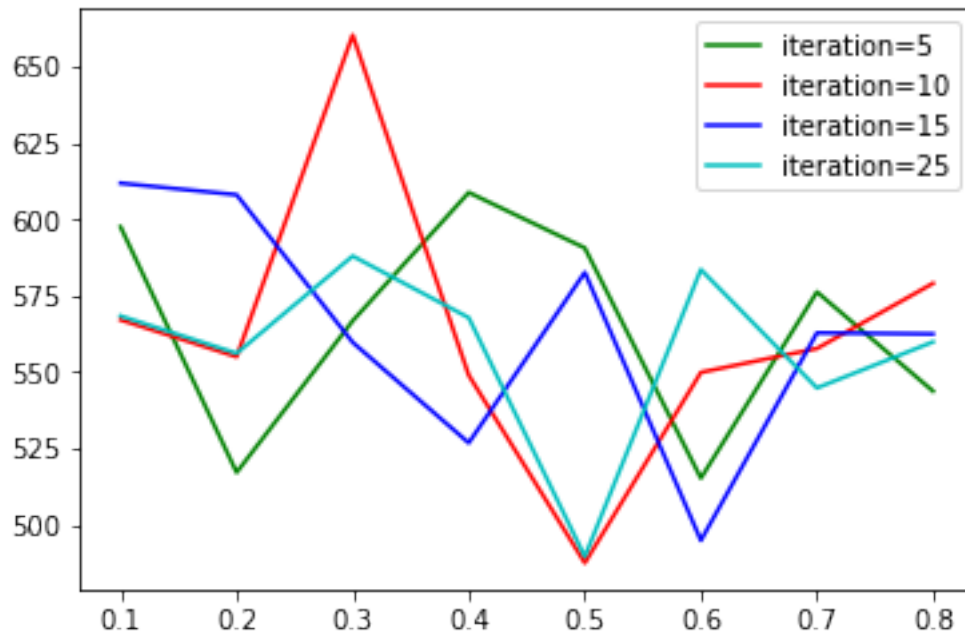
```
from operator import itemgetter
sorted_loss=sorted(final_list,key=itemgetter(2))
print "the optimal #of iteration is:",sorted_loss[0][1],"the optimal v is:",sorted_
from matplotlib import pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
fig=plt.figure()
ax=Axes3D(fig)
ax.scatter(m_list,v_list,loss_list)
plt.show()
```

the optimal #of iteration is: 10 the optimal v is: 0.5 with the loss o  
n validation set: 487.615766843



In [90]:

```
ax=plt.gca()
ax.plot(v_list[0:8],loss_list[0:8],"g",label="iteration=5")
ax.plot(v_list[9:17],loss_list[9:17],"r",label="iteration=10")
ax.plot(v_list[18:26],loss_list[18:26],"b",label="iteration=15")
ax.plot(v_list[27:35],loss_list[27:35],"c",label="iteration=25")
plt.legend()
plt.show()
```



from the plots in terms of optimal parameters and the use of the shrinkage parameter, the increase of num of iteration does not affect the optimal shrinkage parameter

## Calculate the loss on test data with the optimal parameters v and boosting iterations for each of the loss function

In [98]:

```
test_loss1=GB_squareLoss(0.7,5,xtrain,ytrain,xtest,ytest)
print "loss on test data with square loss is :",test_loss1
test_loss2=GB_absLoss(0.5,10,xtrain,ytrain,xtest,ytest)
print "loss on test data with absolute loss is :",test_loss2
```

```
loss on test data with square loss is : 591.876784185
loss on test data with absolute loss is : 597.463644019
```

The result on test data is worse than the result on training data and validation data. this is because gradient boosting may bring about the problem of overfitting, so the model cannot fit well on testing data

In [ ]: