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Wenzhi Wang [*]

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1. Model

1.1. Households

- The economy is populated by a continuum of workers who value the consumption of a final good.
- Every period they are endowed with a unit of time. Workers do not value leisure, supplying all of their time in a labor market described in detail below.
- They live for S periods, financing consumption using labor earnings.

1.2. The Labor Market

The labor market is divided into submarkets, one for each occupation. There are J occupations available labeled by an index j from 1 to J. Occupations are mutually exclusive; workers can work in only one occupation during any given period. However, they may switch occupations between periods.

During their tenure in occupation j, workers receive a wage w_j per unit of their human capital. Human capital comes in two varieties. The first variety is an occupation-specific ability. At birth, each worker is characterized by a vector $\{\theta_j\}_{j=1}^{J}$. Each θ_j is drawn from a distribution $G_j(\theta_j)$ with variance $\sigma^2_{\theta,j}$, but prior to entering the labor market, the elements of the vector $\{\theta_j\}_{j=1}^{J}$ are unknown. Its values are discovered sequentially as workers experiment and sample different occupations. For a given occupation j, the value of θ_j is revealed to the worker the first time occupation j is tried. Once discovered, the worker retains the specific θ_j, even if he eventually switches to other

[*]This note is written in my MPhil period at the University of Oxford.

occupations. In what follows, it is convenient to define the set $J(s)$ as the set of occupations tried by (the beginning of) age s, and $\{\widetilde{\theta}\}_{j\in J(s)}$ as the set of abilities for those occupations already tried.

The second type of human capital is general and therefore transferable across occupations. The stock of this type of human capital, denoted by z, evolves over a worker's career. Despite its generality, the evolution of this type of human capital depends on the worker's current occupation. To be more specific, while working in a given occupation, z changes randomly, and the shocks that affect it are occupation-specific. Shocks to z are an additional source of occupational mobility and are denoted by ϵ. Formally, although an individual works in occupation j, his general human capital evolves according to $z' = z + \epsilon_j$, where ϵ_j is drawn from a distribution $F_j(\epsilon_j)$ with variance σ_j^2. When workers make an occupational choice, they know the value ϵ_j in their current occupation. They do not know that value in a prospective occupation. We are agnostic about the exact nature of these shocks. They capture, for example, the interaction between a worker's skills and an occupation's response to technological innovation. In other words, occupations react differently to changes in technology, and given such a reaction, a worker's human capital may suffer more or less depending on his portfolio of skills. At any rate, occupation-specific shocks to earnings are a feature of the data

1.3. Technology

There is a set of J intermediate service producers indexed by j. We associate such services with occupations. The quantity of intermediate service j each produces is X_j using a linear technology in labor N_j, that is, $X_j = N_j$. The producer faces prices for her service p_j and wages w_j. Both intermediate services and labor markets are competitive.

The producer of intermediate service j solves the following maximization problem:

$$\max_{N_j} p_j X_j - N_j w_j \tag{1}$$

subject to $X_j = N_j$. Intermediate service producers sell to a final goods producer. To produce Y units of the final good, a Cobb-Douglas technology aggregates intermediate services $\{X_1, \ldots, X_J\}$,

$$Y = \prod_{j=1}^{J} \left\{ X_j^{\alpha_j} \right\}. \tag{2}$$

The final goods producer faces purchase prices $\{p_j\}_{j=1}^{J}$ for the different occupations. The final good is the numeraire and its price is 1. Formally, it producer solves,

$$\max_{\{X_1,\ldots,X_J\}} \prod_{j=1}^{J} \left\{ X_j^{\alpha_j} \right\} - p_j X_j \tag{3}$$

Note that in equilibrium, $X_j = N_j$ and $p_j = w_j$, so the solution to this maximization problem implicitly defines labor demand functions $\left\{ N_j = N_j^d\left(w_j, N_{-j}\right) \right\}_{j=1}^{J}$.

1.4. Worker Optimization

At the beginning of the period, the worker faces an occupational choice decision. The worker knows her current level of general human capital z and the shock in the current occupation ϵ_j. She can remain in her current occupation with total general human capital to $z+\epsilon_j$ and known ability θ_j. Alternatively, she can try another occupation. Some of the alternatives have never been tried before and for those, the ability θ is unknown. Define by $W_s(\Omega_s, z, \epsilon, j)$ the maximum value on age s agent obtains by choosing among J mutually exclusive occupations. This choice depends on the set of occupations the worker has visited before $J(s-1)$, as well as her associated abilities $\{\widetilde{\theta}\}_{j\in J(s-1)}$. These two elements make up Ω_s. The choice also depends on the current stock of general human capital z, its current innovation ϵ, and the current occupation j.

The following expression formally describes the choice between an known occupation j and a set of alternative occupations j':

$$W_s\left(\Omega_s, z, \epsilon, j\right) = \max\left\{V_s\left(\Omega_s, z, \epsilon, j\right), \left\{M_s\left(\Omega_s, z, j'\right)\right\}_{j'\neq j}\right\}, \tag{4}$$

where $V_s(\cdot)$ represents the value of staying in the current occupation and $M_s(\cdot)$ the value of an alternative occupation. We describe them in detail in what follows.

The value of remaining in the current occupation j, $V_s(\Omega_s, z, \epsilon, j)$, is conditional on a particular value of the random variable ϵ (the shock to general capital z). In other words, workers know the contemporaneous productivity shock in their current occupation, but take expectations over possible values of productivity in prospective occupations, hence the dependence on ϵ_j of the value of staying in the current occupation. This assumption reflects workers' better information about their performance in their current job. Alternative occupations–those labeled j'–never depend on $\epsilon_{j'}$ and depend on $\theta_{j'}$, only if it is already known–that is, if the worker has worked in j' at some point in his past.

The value of staying is given by the maximum value attained by working in occupation j:

$$V_s\left(\Omega_s, z, \epsilon, j\right) = \left\{u(c) + \beta \int W_{s+1}\left(\Omega_{s+1}, z', \epsilon', j\right) dF_j\left(\epsilon'\right)\right\}, \text{ subject to,} \tag{5}$$

$$c = T\left(w_j e^{\theta_j} e^z e^\epsilon\right), \tag{6}$$

$$z' = z + \epsilon, \tag{7}$$

$$\Omega_{s+1} = \Omega_s. \tag{8}$$

The consumption value is the maximum among J occupations, knowing that productivity in occupation j will experience a shock ϵ'. The flow budget cconstraint (6) equates consumption to total income, which is simply after-tax earnings $T\left(w_j e^{\theta_j} e^z e^\epsilon\right)$. Pretax earnings are equal to the product of a wage rate w_j and the amount of efficiency units $e^{\theta_j} e^z e^\epsilon$. A progressive tax function $T(\cdot)$ applied to pretax earnings gives the after-tax amoung available to finance expenditures. We work with the following class of tax functions: $y_a = T\left(y_p\right) = \phi_0 y_p^{1-\phi}$, where y_p and y_a are pre- and after-tax earnings, respectively. Any revenue collected by the governemnt is wasted.

The (log of) general human capital z evolves according to (7). The current shock ϵ is added to the shock z to update it to its new value z'. Finally, remaining in the same occupation adds no new information to Ω_s, and as a result $\Omega_{s+1} = \Omega_s$.

By switching occupations, a worker bets that his performance will improve as a result of the change. If the worker has chosen that occupation for the first time, the outcome is uncertain because both ϵ and θ in that prospective occupation are unknown. The worker takes expectations with respect to both distributions to compute the value of the alternative occupation. If at some point, the worker has tried occupation j', only the value of ϵ is uncertain.

Recall that Ω includes the set $J(s-1)$, the set of inspected occupations. If j' is not an element of $J(s-1)$, the value of the alternative occupation is

$$M_s\left(\Omega_s, z, j'\right) = \int H_s\left(\Omega_s, \theta, z, \epsilon, j'\right) dG_{j'}(\theta) dF_{j'}(\epsilon). \tag{9}$$

Conditional on a particular θ and ϵ, the value of the alternative occupation is the maximum attained by adding the utility flow from earnings plus the continuation value:

$$H_s\left(\Omega_s, \theta_{j'}, z, \epsilon, j'\right) = \left\{u(c) + \beta \int W_{s+1}\left(\Omega_{s+1}, z', \epsilon', j'\right) dF_{j'}\left(\epsilon'\right)\right\}, \text{ subject to }, \tag{10}$$

$$c = T\left(w_{j'} e^{z} e^{\theta_{j'}} e^{\epsilon_{j'}} e^{-c(s,\kappa)}\right), \tag{11}$$

$$z' = z + \epsilon, \tag{12}$$

$$\Omega_{s+1} = \left\{\Omega_s, j', \theta_{j'}\right\}. \tag{13}$$

This value is similar to that of remaining in the same occupation. There are two differences. First, according to (13), the set Ω_s grows, because the worker obtains new information about his ability in the new occupation j'. The second difference is the term $\exp(-c(s,\kappa))$, affecting the amount of efficiency units and reflecting a (temporary) human capital loss. This cost is borne by all switchers, regardless of whether the new occupation has been tried before. The function $c(s,\kappa)$ reflects mobility costs; it depends on age and on a vector of parameters κ. This specification permits modeling in a flexible way the mobility costs facing workers as they are.

Evaluating an occupation j' that has been visited before is simpler. The only uncertainty facing the worker is with respect to the shock ϵ in j'. The alternative value for this case–the analog to Equation (9)–can be written as

$$M_s\left(\Omega_s, z, j'\right) = \int V_s\left(\Omega_s, z, \epsilon, j'\right) dF_{j'}(\epsilon). \tag{14}$$

Note that the ability parameter $\theta_{j'}$ is an element of Ω_s, because the worker has previously visited that occupation. The calculation of the value oof switching is almost identical to (10-13). The exception is (13), which now becomes (8): The set Ω_s does not change because no new information is revealted about the worker's innate abilities.

The previous description of the occupational decision problem holds for all periods except the first one. In the first period, a fraction f_j of workers is exogenously assigned to occupation j. These workers learn their comparative advantage in that occupation but experience no ϵ shocks (i.e., their z is 0). In the second and subsequent periods, they optimally choose their occupation as described above.

1.5. Equilibrium

Let us denote the policy function that describes the occupational decision of an individual of age s characterized by a realization ϵ, a set Ω_s,and productivity z, who is currently in occupation j' and who switches to occupation j by $I_{j,s}\big(j', w, z, \epsilon\big)$.

For aggragation purposes, it is necessary to specify the position of individuals across states. Let $\Psi_{j,s}(\Omega_s, z, \epsilon)$ be the mass of individuals of age s in occupation j, with productivity z, and shock ϵ, who have been in other occupations in the past with their respective ability, represented by Ω_s. The measure Ψ is defined for all the possible values of $\Omega_s, z,$ and ϵ that belong to sets that are Borel subsets of $\mathbb{R}$.

The dynamic evolution of the mass of individuals reads as follows: As described above, the mass of newborns in occupation j is exogenously determined and given by f_j. Thus, for $s = 0$,

$$\Psi_{j,0}\left(\Omega_0, z, \epsilon\right) = \frac{1}{S} f_j \quad \forall j \in \{1, \ldots, J\}. \tag{15}$$

In addition, since individuals live S number of years, we have that for $S+1$,

$$\Psi_{j,S+1}\left(\Omega_{S+1}, z, \epsilon\right) = 0 \quad \forall j \in \{1, \ldots, J\}. \tag{16}$$

For $0 < s < S$, Ψ obeys the following recursion:

$$\Psi_{j,s+1}\left(\Omega_{s+1}, z, \epsilon\right) = \sum_{j'} \Psi_{j',s}\left(\Omega_s, z, \epsilon\right) I_{j,s}\left(j', \omega_s, \epsilon, z\right) \quad \forall \quad j' \in \{1, \ldots, J\}. \tag{17}$$

The aggregate mass of efficiency units in each occupation is thus given by

$$N_j = \frac{1}{S} \sum_{s \in S} \int e^{z} e^{\theta_{j'}} e^{\epsilon_{j'}} d\Psi_{j,s}\left(\Omega_s, z, \epsilon\right) + \frac{1}{S} \sum_{s \in S} \sum_{j \neq j'} \int e^{-c(s,\kappa)} d\Psi_{j',s-1}\left(\Omega_{s-1}, z, \epsilon\right). \tag{18}$$

We can now define a stationary competitive equilibrium that consists of (i) a set of occupation-level wages $\{w_j\}_{j=1}^{J}$, (ii) occupation populations (or masses) $bc\Psi_{j}{}_{j=1}^{J}$, (iii) a set of intermediate goods prices $bcp_{j}{}_{j=1}^{J}$, (iv) occupation-level efficiency-weighted employment levels $bcN_{j}{}_{j=1}^{J}$, and (v) occupation-specific decision rules $bcI_{j,s}{}_{j=1}^{J}$ and associated value functions $bcV_{s}{}_{s=1}^{S}$ that satisfy the following conditions:

1. The occupation decision rules solve the optimization problems described in Subsection 1.4.
2. The labor inputs N_j are the solution to the intermediate producer optimization problem.
3. The intermediate goods quantities X_j solve the final goods produce's problem.
4. Prices $_j$ equate supply and demand of intermediate goods.
5. The wage in occupation j is the marginal product of a unit of efficiency in that occupation:

$$w_j = \alpha_j N_j^{\alpha_j - 1} \prod_{j' \neq j} \left\{ N_j^{\alpha'_j} \right\}. \tag{19}$$

6. Labor markets clear at the occupational level.
7. In a given occupation j, Ψ_j is the stationary distribution.

By Walras's law, the market for the final good also clears.