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TAX AND EDUCATION POLICY IN A HETEROGENEOUS-AGENT ECONOMY: WHAT LEVELS OF REDISTRIBUTION MAXIMIZE GROWTH AND EFFICIENCY?

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This paper studies the effects of progressive income taxes and education finance in a dynamic heterogeneous-agent economy. Such redistributive policies entail distortions to labor supply and savings, but also serve as partial substitutes for missing credit and insurance markets. The resulting tradeoffs for growth and efficiency are explored, both theoretically and quantitatively, in a model that yields complete analytical solutions. Progressive education finance always leads to higher income growth than taxes and transfers, but at the cost of lower insurance. Overall efficiency is assessed using a new measure that properly reflects aggregate resources and idiosyncratic risks but, unlike a standard social welfare function, does not reward equality per se. Simulations using empirical parameter estimates show that the efficiency costs and benefits of redistribution are generally of the same order of magnitude, resulting in plausible values for the optimal rates. Aggregate income and aggregate welfare provide only crude lower and upper bounds around the true efficiency tradeoff.

KEYWORDS: Heterogeneous agents, income distribution, inequality, growth, education finance, redistribution.

1. INTRODUCTION

ABSENT THE REPRESENTATIVE AGENT assumption, the evolution of macroeconomic aggregates is determined jointly with that of the entire distribution of wealth. In this paper I present a framework where these usually complex dynamics remain analytically tractable, and use it to study fiscal and educational policy. I thus analyze the effects of progressive income taxes and redistributive education finance on aggregate income, inequality, social mobility, individual risk, and intertemporal welfare. For each policy I ask what degree of progressivity is efficient, or simply growth-maximizing; I also compare the relative merits of the two forms of redistribution. For such questions to be posed realistically, two ingredients must be present. Redistributive policies must have costs, due to distortions in agents' effort or savings decisions. They must also have benefits, due to imperfections in asset markets; redistribution then provides both insurance and a means to relax the credit constraints that impede certain investments.

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To analyze this tradeoff theoretically and quantitatively, I develop a stochastic model of human capital accumulation with endogenous effort and missing credit and insurance markets. Fiscal and educational redistributions take place through simple, marginally progressive, schemes. Explicit analytical solutions are obtained for all individual and aggregate variables, under constant or time-varying policies. Given a reasonably broad menu of fiscal instruments, intertemporal distortions are shown to be preventable: to each income tax or education finance policy can be associated a simple combination of consumption taxes and investment subsidies that restores savings to its (constrained) optimal level. The analysis also demonstrates that progressive education finance always leads to higher income growth than taxes and transfers. Both are equally effective at substituting for the missing credit market, but the latter entails smaller distortions to labor supply and (in the absence of corrective measures) to savings, because it redistributes only a fraction of family income. This comes, however, at the cost of lower consumption insurance.

To evaluate more generally the extent to which market distortions and imperfections are worsened or improved by alternative policies, the paper proposes a new measure of overall economic efficiency. This criterion properly reflects (dynamic) variations in the aggregate consumption of goods and leisure and in the idiosyncratic risks that agents face; but, unlike a standard social welfare function, it does not reward interpersonal equality per se. The underlying idea is straightforward. Instead of aggregating individual incomes or consumptions (which washes out all idiosyncratic uncertainty), or individual utilities (which introduces a bias towards egalitarian allocations), one sums up consumption certainty-equivalents, so as to obtain a kind of *risk-adjusted GDP*. Aggregate efficiency is shown to be maximized at some strictly positive rate of redistribution that depends intuitively on parameters like the elasticity of labor supply, the variability of idiosyncratic shocks, and the growth losses from liquidity-constrained investments. Equity concerns can be incorporated as well, but through a separate degree of inequality-aversion that is a priori independent of individuals' attitudes towards risk and intertemporal fluctuations.

Complementing the theoretical analysis, the model is simulated with parameter estimates from the empirical literature. In the baseline specification long run income (or growth) is maximized when the average marginal tax-and-transfer rate equals 21%, which corresponds to a share of redistributive transfers in GDP of 6%. Taking into account the value of insurance and leisure, the maximization of aggregate efficiency raises these numbers to 48% and 14% respectively. Under the alternative policy of progressive education finance, the growth-maximizing equalization rate for school expenditures is 62%, the efficient one 68%. In both cases, the efficient policy results in the top 30% of families subsidizing the bottom 70%, whether through the fiscal or the education system. Maximizing average welfare would always imply much higher rates of redistribution.

Naturally, these numbers should only be taken as providing a rough assessment of the main tradeoffs. More important than the results corresponding to specific parameters are the general lessons emerging from an extensive sensitivity analysis. First, the efficiency costs and benefits of redistribution are typically of the same order of magnitude, so that neither side can be neglected. Second, the effects of redistributive policies on aggregate income and aggregate welfare provide only very crude lower and upper bounds around the true efficiency tradeoff.

This paper relates to two strands of literature. First, and most directly, it relates to the work on growth and distribution with imperfect credit markets (e.g., Loury (1981), Galor and Zeira (1993), Banerjee and Newman (1993), Bénabou (1996a), Durlauf (1996), Aghion and Bolton (1997), Piketty (1997)). This literature has remained essentially theoretical, with the main exception of Fernandez and Rogerson (1998) who study the quantitative impact of redistributing educational expenditures in a model calibrated to US data. The framework proposed here allows new developments on both the theoretical and the quantitative fronts. The second strand of literature deals with the implications of imperfect insurance markets for savings behavior and wealth inequality on the one hand (e.g., Laitner (1992), Aiyagari (1995), Heaton and Lucas (1996), Krusell and Smith (1998)), for public insurance on the other (e.g., Varian (1980), Hansen and Imrohoğlu (1992), Atkeson and Lucas (1995)). The present paper abstracts from precautionary savings, which figure prominently in some of these models. On the other hand, it lets credit constraints bear not just on consumption-smoothing but also on investment, where they have much more substantial aggregate effects.

Section 2 describes the model, then derives agents' optimal labor supply and savings behavior under progressive taxes and transfers. Section 3 does the same for progressive education finance. Section 4 solves for the dynamics and steady-state values (or growth rate) of total income and its cross-sectional distribution. Sections 5 and 6 develop the efficiency criterion, then relate it to aggregate income and social welfare. Section 7 explores the quantitative predictions of the model, and Section 8 concludes. All proofs are gathered in the Appendix.

2. THE MODEL

2.1. *Preferences and Technology*

There is a continuum of infinitely-lived agents or dynasties, $i \in [0, 1]$. In period t, agent i chooses consumption c_t^i and labor supply l_t^i to maximize his intertemporal utility U_t^i. The simplest description of these preferences I will consider is

$$\ln U_t^i \equiv E_t\left[\sum_{k=0}^{\infty} \rho^k (\ln c_{t+k}^i - (l_{t+k}^i)^\eta)\right]. \tag{1}$$

More generally, I use a specification that allows attitudes towards intertemporal substitution and towards risk to be distinguished (Kreps and Porteus (1979), Epstein and Zin (1989), Weil (1990)). Because the latter determines the value placed by agents on the insurance component of redistributive policies, it is important not to constrain it to any particular value. On the other hand, the model's analytical tractability requires a unitary intertemporal elasticity of substitution.[2,3] Agent i's intertemporal utility at time t is thus defined recursively by

$$\text{(2)} \qquad \ln U_t^i \equiv \max_{l_t^i,\, c_t^i} \Big\{ (1-\rho)[\ln c_t^i - (l_t^i)^\eta] + \rho \ln\Big(\big(E_t\big[(U_{t+1}^i)^r \big] \big)^{1/r} \Big) \Big\}.$$

The degree of relative risk aversion to lotteries over U_{t+1}^i is $1-r \geq 0$. When $r=0$ the second term in (2) becomes $\rho E_t[\ln U_{t+1}^i]$ by l'Hospital's rule, and utility reduces to (1). When $r \neq 0$ preferences are not time-separable, and agents care about both the magnitude of uncertainty and the timing of its resolution.[4]

In period or generation t, agent i maximizes his intertemporal utility (2) subject to

$$\text{(3)} \qquad y_t^i = (h_t^i)^\lambda (l_t^i)^\mu,$$

$$\text{(4)} \qquad \hat{y}_t^i = c_t^i(1+\theta_t) + e_t^i,$$

$$\text{(5)} \qquad h_{t+1}^i = \kappa \cdot \xi_{t+1}^i \cdot (h_t^i)^\alpha ((1+a_t)e_t^i)^\beta.$$

Human capital h_t^i is combined with labor l_t^i to produce output.[5] The resulting *pre-tax* income y_t^i is then subject to taxes and transfers, resulting in a *disposable* income denoted $\hat{y}_t^i$. The government also taxes consumption c_t^i at the rate θ_t and subsidizes investment in education e_t^i at the rate a_t. These three dimensions of fiscal policy and the constraints linking them will be examined in more detail later on. Equation (4) reflects the absence of a credit market to finance human capital investment, requiring both consumption and education expenditures to come out of disposable income.[6] Equation (5) describes a child's human capital h_{t+1}^i as the product of three inputs: innate ability ξ_{t+1}^i, the quality of the home or neighborhood environment as measured by parental human capital h_t^i, and purchased educational inputs such as teacher time, classrooms, books or computers,

[2] This last assumption is ubiquitous in the literature on income distribution dynamics—generally in conjunction with a myopic bequest motive rather than the dynastic one assumed here. See, inter alia, Glomm and Ravikumar (1992), Galor and Zeira (1993), Banerjee and Newman (1993), or Aghion and Bolton (1997). In this model, however, it *will not* result in a constant (policy-invariant) savings rate, due to the progressivity of the tax schemes that will be considered.

[3] For labor supply, the intertemporal elasticity of substitution is $1/(\eta-1)$. The model easily extends to the more general specification of felicity $u = \ln c - v(l)$, $v' > 0$, $v'' > 0$.

[4] As explained in Weil (1990), agents prefer early resolution when their aversion to risk is larger than their aversion to intertemporal fluctuations, i.e. when $1-r > 1$. Conversely, they prefer late resolution when $r > 0$.

[5] More generally, h_t^i could be any nontraded asset. Note that y_t^i should be interpreted as income net of the cost of physical capital (which can be rented on a world market), as in Barro, Mankiw, and Sala-i-Martin (1995); see footnote 24 for further details.

[6] The simplest source of such incompleteness is the fact that children cannot be held responsible for the debts incurred by their parents. The human capital accumulation on which the model focuses thus corresponds best to early childhood, elementary and secondary schooling. It is less relevant for higher education, where loans (both public and private) are more readily available, at least in developed countries. Note finally that the education expenditures e_t^i may be incurred directly, as with private school tuitions, or indirectly, in the form of land rents and property taxes conditioning access to a community's public schools (e.g. Bénabou (1996a), Fernandez and Rogerson (1996, 1998)).

$(1+a_t)e_t^i$. The uninsurable ability or productivity shocks ξ_t^i are i.i.d. and log-normally distributed: $\ln \xi_t^i \sim \mathcal{N}(-\omega^2/2, \omega^2)$, hence $E[\xi] = 1$.[7] I shall also assume that $\ln h_0^i \sim \mathcal{N}(m_0, \Delta_0^2)$ and—without prejudging the distribution of $\ln h_t^i$, which is endogenous—denote its mean as m_t and its variance as Δ_t^2. Finally, let $\eta \geq 1 > \rho$ and $\alpha + \beta\lambda < 1$.

The absence of any intertemporal trade is clearly an oversimplified (but quite common) representation of asset market incompleteness; it represents the main price of analytical tractability in the model. Intratemporal linkages between agents, on the other hand, could easily be incorporated—for instance a labor market with different skill levels being complements in production. I shall not pursue this extension here, in order to better focus on the interactions—both intra- and intertemporal—that arise through public policy.

2.2. *Progressive Taxation*

The government redistributes income using *marginally progressive* taxes and transfers, as is the case in most countries.[8] This is represented by a simple scheme where disposable income is a loglinear function of market income,

$$\hat{y}_t^i \equiv (y_t^i)^{1-\tau_t}(\tilde{y}_t)^{\tau_t}, \tag{6}$$

and the break-even level $\tilde{y}_t$ is defined by the balanced-budget constraint

$$\int_0^1 (y_t^i)^{1-\tau_t}(\tilde{y}_t)^{\tau_t}\, di = y_t, \tag{7}$$

where $y_t \equiv \int_0^1 y_t^i\, di$ denotes per-capita income. The elasticity $\tau_t \leq 1$ measures the rate of (residual) *progressivity* in fiscal policy. Denoting $T(y_t^i) \equiv y_t^i - \hat{y}_t^i$ the net tax paid at income level y_t^i, note that when $\tau_t > 0$ both average and marginal rates are rising, and $\tilde{y}_t > y_t$. Furthermore, τ_t is equal to the income-weighted *average marginal tax* (*and transfer*) *rate*:

$$\int_0^1 T'(y_t^i) \cdot (y_t^i/y_t)\, di = \tau_t. \tag{8}$$

The effects of redistribution on growth and efficiency will be analyzed *within this class of policies*, or its analogue for education finance. While its functional

[7] The model can be extended to serially correlated shocks ξ_t^i, say *ar*(1). But it is much simpler, and qualitatively similar, to replace the resulting *ar*(2) process by an *ar*(1) with a higher persistence α.

[8] Implicit here is the standard assumption that the government observes individual incomes y_t^i (and education spending e_t^i, at least at the community level), but not individual productivity or any of its two components, innate ability ξ_t^i and social background h_{t-1}^i. More generally, it is not able to condition its policies on these variables.

form is inevitably restrictive, it succinctly captures the appropriate notion of progressivity.[9]

2.3. *The Shadow Value of Human Capital*

Taking as given the policy sequence $\{\tau_t, \theta_t, a_t\}_{t=0}^{\infty}$, an agent with human wealth h solves the following dynamic programming problem, where l and s denote his effort and savings rate:

$$\text{(9)} \qquad \ln U_t(h) = \max_{l,s} \Big\{ (1-\rho)[\ln((1-s)/(1+\theta_t)) + (1-\tau_t)(\lambda \ln h + \mu \ln l) + \tau_t \ln \tilde{y}_t - l^{\eta}] + \rho \ln\Big((E_t[U_{t+1}(h')^r])^{1/r} \Big) \Big\},$$

$$\text{(10)} \qquad h' = \kappa \xi ((1+a_t)s)^{\beta} (h)^{\alpha+\beta\lambda(1-\tau_t)} (l)^{\beta\mu(1-\tau_t)} (\tilde{y}_t)^{\beta\tau_t}.$$

Clearly, optimal decisions will depend on the private marginal value of human capital, or equivalently on the elasticity $V_t \equiv \partial \ln U_t / \partial \ln h$. This shadow value, in turn, reflects future expected rates of redistribution, both directly and through their impact on the intergenerational persistence of human wealth (see (10)),

$$\text{(11)} \qquad p(\tau_t) \equiv \alpha + \beta\lambda(1-\tau_t).$$

The sequence of $p(\tau_t)$'s, which can also be thought of as inverse measures of social mobility, will play a fundamental role throughout the model. In particular, one shows the following.

PROPOSITION 1: *The value function under fiscal redistributions is* $\ln U_t^i = V_t \cdot (\ln h_t^i - m_t) + W_t$, *where* $m_t \equiv \int_0^1 \ln h_t^i \, di$,

$$\text{(12)} \qquad V_t \equiv \lambda(1-\rho) \sum_{k=0}^{\infty} \rho^k (1-\tau_{t+k}) \prod_{j=0}^{k-1} p(\tau_{t+j}),$$

and aggregate welfare W_t *is given in the Appendix as a function of* $\{\tau_{t+k}, \theta_{t+k}, a_{t+k}\}_{k=0}^{\infty}$.[10]

Note how the marginal value reflects current and future rates of redistribution, but is invariant to proportional consumption taxes and investment subsidies (a feature of logarithmic utility).

[9] Jakobsson (1976) and Kakwani (1977) showed that the post-tax distribution resulting from one fiscal scheme will Lorenz-dominate that of another—for *all* pre-tax distributions—if and only if the former's residual elasticity is smaller at every income level. For further discussion of (6), see Bénabou (2000); a similar "constant residual progression" was used in some earlier but static models, e.g. Feldstein (1969) or Persson (1983). Englund and Persson (1982) find that it describes the progressivity of the Swedish tax system (over the relevant income range) fairly well.

[10] Note from (12) that $W_t = \int_0^1 \ln U_t^i \, di$. Throughout that paper I shall use the convention that $\prod_{k=j}^{j'} x_k \equiv 1$ and $\sum_{k=j}^{j'} x_k \equiv 0$ whenever $j' < j$, for any sequence of x_k's.

2.4. *Labor Supply and Savings Decisions*

The complete solution to the agent's problem is easily obtained from (12) and the first-order conditions associated to (9)–(10). I first consider labor supply.

PROPOSITION 2: *Agents choose in every period a common level of effort l_t, which decreases with current and expected future tax rates $\{\tau_{t+k}\}_{k=0}^{\infty}$:*

$$l_t = [(\mu/\eta)(1-\tau_t)(1+\rho(1-\rho)^{-1}\beta V_{t+1})]^{1/\eta}, \tag{13}$$

where V_{t+1} is defined by (12). *Under a constant tax profile $\tau_t = \tau$, in particular,*

$$l = \left(\frac{(\mu/\eta)(1-\tau)(1-\rho\alpha)}{1-\rho(\alpha+\beta\lambda(1-\tau))}\right)^{1/\eta}. \tag{14}$$

Recall that $1/(\eta-1)$ is the intertemporal elasticity of substitution in labor supply, with respect to variations in the real wage. The first result shows that the uncompensated elasticity with respect to $1-\tau_t$, the net-of-tax (progressivity) rate, equals $1/\eta$. The second result makes transparent the role of the other parameters, and will be useful when focusing on steady-states later on. I now turn to agents' propensity to save out of disposable income.

PROPOSITION 3: *Agents choose in every period a common savings rate, $s_t \equiv e_t^i/\hat{y}_t^i$, which decreases with expected future tax rates $\{\tau_{t+k}\}_{k=0}^{\infty}$:*

$$s_t = \frac{\rho\beta V_{t+1}}{1-\rho+\rho\beta V_{t+1}}, \tag{15}$$

where V_{t+1} is defined as before. Under a constant tax profile, $\tau_t = \tau$, in particular,

$$s_t = \frac{(1-\tau)\rho\beta\lambda}{1-\rho\alpha} \equiv (1-\tau)\bar{s}, \tag{16}$$

where $\bar{s}$ is the laissez-faire savings rate.

These results, and in particular the second one, show clearly how the progressive taxation of income distorts human capital accumulation. Income taxes, however, are not the only fiscal instrument available to policy-makers and the voters who elect them.

2.5. *Consumption Taxes and Investment Subsidies*

Recall that the government taxes consumption at the rate θ_t and subsidizes education at the rate a_t, subject to its budget constraint $\theta_t \int_0^1 c_t^i di = a_t \int_0^1 e_t^i di$, or by Proposition 3,

$$\frac{\theta_t(1-s_t)}{1+\theta_t} = a_t s_t. \tag{17}$$

Given a savings rate s_t and a relative price of education or similarly credit-constrained investment goods $1/(1+a_t)$, each agent's effective investment rate is $(1+a_t)s_t$. It can thus be restored to its (credit-constrained) optimal level $\bar{s} \equiv \rho\beta\lambda/(1-\rho\alpha)$ by a consumption tax of

$$\theta_t = \frac{\bar{s} - s_t}{1 - \bar{s}}, \tag{18}$$

whose proceeds are used to subsidize education.[11]

PROPOSITION 4: *For any sequence of current and future rates of redistribution* $\{\tau_{t+k}\}_{k=0}^{\infty}$, *let* $\{\theta_{t+k}, a_{t+k}\}_{k=0}^{\infty}$ *be the unique corresponding sequence of consumption tax rates and investment subsidies such that, in every period* $t+k$:

(i) *the government budget is balanced, as described by* (17);

(ii) *agents' common investment rate is restored to its first-best level* $\bar{s}$, *as described by* (18).

Every agent i *of every generation* t *prefers the policy sequence* $\{\tau_{t+k}, \theta_{t+k}, a_{t+k}\}_{k=0}^{\infty}$ *to any feasible alternative* $\{\tau_{t+k}, \theta'_{t+k}, a'_{t+k}\}_{k=0}^{\infty}$.

This unanimity result means that while the policy space is two-dimensional (taking into account the budget constraint), the *Pareto set* is one-dimensional: distributional conflict concerns only the degree of progressivity $\{\tau_{t+k}\}_{k=0}^{\infty}$. Accordingly, I will from here on restrict attention to undominated policy mixes, setting $(1+a_t)s_t = \bar{s}$ for all t.

Given a reasonably broad menu of fiscal instruments, redistributive taxation thus causes only intratemporal distortions, namely those to labor supply.[12] This result is consistent with the evidence from cross-country regressions, surveyed in Bénabou (1996b). There is no sign of a negative effect of redistribution (shares of various transfers in GDP, average and marginal tax rates) on national investment rates. In fact, the regression coefficient is more often positive than not. By contrast, there is a clear positive association between labor tax rates and national unemployment rates (Daveri and Tabellini (2000)).

If one wanted nonetheless to maintain intertemporal distortions in the model, one would simply constrain a_t and θ_t to zero. This could reflect the underdeveloped fiscal system of a poor country, or some informational constraints that

[11] Imposing budget balance separately on the income tax-and-transfer system and on the consumption-tax/investment-subsidy scheme is only a normalization that allows τ_t to be interpreted as in (8), and involves no loss of generality. Since s_t is independent of $\hat{y}_t^i$, scaling the right-hand side of (7) by any positive number and imposing the overall budget constraint $\int_0^1 (\hat{y}_t^i - y_t^i + a_t e_t^i - \theta_t c_t^i)\,di = 0$ instead of (17) leads to the exact same results.

[12] The policy mix described in Proposition 4 is ultimately equivalent to using a progressive consumption tax to finance a program of progressive education subsidies (with the same rate τ_t) of the type studied in the next section. Because it expropriates only labor and existing human wealth, such a policy generates only effort distortions. Proposition 4 is thus related—but in an incomplete markets setting and with progressivity—to the classical public finance results about the superiority of consumption taxes over (capital plus labor) income taxes. Relatedly, Stokey and Rebello (1995) show that taxes cause only small growth losses when the sector producing human capital is lightly taxed.

make subsidies to human capital investment impractical. Throughout the rest of the paper, all one would need to do is to replace $\bar{s}$ by s_t. As shown by (16) this is particularly easy in steady-state, which will be our ultimate focus.

3. EDUCATION FINANCE

As an alternative to progressive income taxes and transfers, I consider the redistribution of education expenditures (from preschool to the secondary level). This may correspond to a policy of school funding equalization across local communities, such as those mandated by constitutional courts in several U.S. states, or more generally of *subsidizing differentially* the education of rich and poor children. Formally, let income itself remain untaxed, $\hat{y}_t^i = y_t$, while in (5) educational investment $(1+a_t)e_t^i$ is replaced by

$$(19) \qquad \hat{e}_t^i \equiv (1+a_t)(\tilde{y}_t/y_t^i)^{\tau_t} e_t^i.$$

This means that a family or community with income y_t^i faces the price $p_t^i = (1+a_t)^{-1}\,(y_t^i/\tilde{y}_t)^{\tau_t}$ for education. The progressivity rate τ_t can also be thought of as the *degree of equalization* of school resources, while a_t still represents the average rate of education subsidization. Indeed, with all agents saving a fraction s_t of their income (as shown below), $\hat{e}_t^i = (1+a_t)s_t(y_t^i)^{1-\tau_t}(\tilde{y}_t)^{\tau_t}$; the polar cases $\tau_t = 0$ and $\tau_t = 1$ thus correspond respectively to decentralized and egalitarian school funding. Summing (19) across agents, the net subsidy is $a_t s_t y_t$, to be funded, as previously, by a consumption tax.

The Bellman equation is now

$$(20) \qquad \ln U_t(h) = \max_{l,s}\{(1-\rho)[\ln((1-s)/(1+\theta_t)) + \lambda \ln h + \mu \ln l - l^\eta] + \rho \ln\big((E_t[U_{t+1}(h')^r])^{1/r}\big)\},$$

with h' still given by (10). The solution has a similar structure to the one obtained for fiscal redistributions.

PROPOSITION 5: *The value function under progressive school finance is* $\ln U_t^i = V_t \cdot (\ln h_t^i - m_t) + W_t$, *where* $m_t \equiv \int_0^1 \ln h_t^i \, di$,

$$(21) \qquad V_t \equiv \lambda(1-\rho)\sum_{k=0}^{\infty} \rho^k \prod_{j=0}^{k-1} p(\tau_{t+j}),$$

and aggregate welfare W_t *is given in the Appendix as a function of* $\{\tau_{t+k}, \theta_{t+k}, a_{t+k}\}_{k=0}^{\infty}$.

The only difference with (12) is the absence of the factors $1-\tau_{t+k}$ multiplying each dicounted product term. As a result, human capital is more valuable (V_t is higher), for all nonnegative sequences $\{\tau_{t+k}\}_{k=0}^{\infty}$. This reflects the fact that progressive redistribution now applies only to the fraction of income that is saved, and not to that which is consumed. As one would expect, this lessens both inter- and intratemporal distortions.

PROPOSITION 6: *For any expected sequence of education finance equalization rates* $\{\tau_{t+k}\}_{k=0}^{\infty}$, *agents' common savings rate* s_t *is still given by* (15), *but with* V_t *now defined by* (21). *In particular, under a constant education finance policy* $\tau_t = \tau$,

$$s = \frac{\rho\beta\lambda}{1-\rho\alpha+\rho\beta\lambda\tau} \equiv \frac{\bar{s}}{1+\tau\bar{s}}. \tag{22}$$

As to agents' labor supply, it is now

$$l_t = [(\mu/\eta)(1+\rho(1-\rho)^{-1}(1-\tau_t)\beta V_{t+1})]^{1/\eta}, \tag{23}$$

so that under a constant τ,

$$l = \left(\frac{(\mu/\eta)(1-\rho\alpha)}{1-\rho(\alpha+\beta\lambda(1-\tau))}\right)^{1/\eta}. \tag{24}$$

Note that s_t and l_t remain positive even for $\tau_{t+k} \equiv 1$, which corresponds in equilibrium to uniform funding of education, $\hat{e}_t^i = (1+a_t)s_t y_t$. In particular, steady-state effort remains bounded below even as the intertemporal elasticity of substitution $\epsilon \equiv 1/(\eta-1)$ becomes infinite.

As in the case of income taxes, the decline in savings can be fully offset by taxing consumption at the rate θ_t given by (18), and using the proceeds to finance the net (or average) education subsidy a_t that restores the investment rate to $\bar{s}$. Since the distortion to s_t is now smaller, the required rates of θ_t and a_t are lower. Moreover, conditional on any $\{\tau_{t+k}\}_{k=0}^{\infty}$ this policy will once again be supported unanimously, both within and across generations. As explained earlier, the remainder of the paper will incorporate this Pareto-improving policy mix, but if it were for some reason infeasible, one would simply replace $\bar{s}$ by s_t everywhere.

4. THE DYNAMICS OF HUMAN WEALTH AND INCOME

4.1. *Laws of Motion*

Let us now take logarithms in (3) and (5) in the case of income taxes, or in (3) and (19) in that of progressive education finance. Under either redistributive scheme agent i's net investment is $\bar{s}(y_t^i)^{1-\tau_t}(\tilde{y}_t)^{\tau_t}$, so the law of motion of human capital takes the form

$$\begin{aligned}\ln h_{t+1}^i = {} & \ln \xi_{t+1}^i + \ln\kappa + \beta\ln\bar{s} + \beta\mu(1-\tau_t)\ln l_t \\ & + (\alpha+\beta\lambda(1-\tau_t))\ln h_t^i + \beta\tau_t \ln\tilde{y}_t.\end{aligned} \tag{25}$$

This implies that both human wealth h_t^i and income $y_t^i = (h_t^i)^{\lambda}(l_t)^{\mu}$ remain log-normally distributed over time. Thus if $\ln h_t^i \sim \mathcal{N}(m_t, \Delta_t^2)$, then

$$m_{t+1} = (\alpha+\beta\lambda)m_t + \beta\mu\ln l_t + \beta\tau_t(2-\tau_t)\lambda^2\Delta_t^2/2 + \beta\ln\bar{s} + \ln\kappa - \omega^2/2, \tag{26}$$

$$\Delta_{t+1}^2 = (\alpha+\beta\lambda(1-\tau_t))^2\Delta_t^2 + \omega^2, \tag{27}$$

where the first equation is obtained by substituting into (25) the break-even level of income

$$(28) \qquad \ln \tilde{y}_t = \ln y_t + (1-\tau_t)\lambda^2\Delta_t^2/2 = \lambda m_t + \mu \ln l_t + (2-\tau_t)\lambda^2\Delta_t^2/2,$$

defined by the budget constraint (7). Finally, (26)–(27) easily yield the following results.

PROPOSITION 7: *The distribution of income at time t is $\ln y_t^i \sim \mathcal{N}(\lambda m_t + \mu \ln l_t, \lambda^2\Delta_t^2)$, where m_t and Δ_t^2 evolve according to the linear difference equations* (26)–(27) *and $l_t = l(\tau_t)$ is given by Proposition* 2 *or* 6. *The growth rate of per capita income equals*

$$(29) \qquad \ln y_{t+1} - \ln y_t = \ln\bar{\kappa} - (1-\alpha-\beta\lambda)\ln y_t + \mu(\ln l_{t+1} - \alpha \ln l_t) - \mathfrak{L}(\tau_t)\lambda^2\Delta_t^2/2,$$

where $\ln \bar{\kappa} \equiv \lambda(\ln\kappa + \beta\ln\bar{s}) - \lambda(1-\lambda)\omega^2/2$ is a constant and

$$(30) \qquad \mathfrak{L}(\tau) \equiv \alpha + \beta\lambda(1-\tau)^2 - (\alpha + \beta\lambda(1-\tau))^2 > 0$$

measures the extent to which income inequality slows down growth, given a policy τ.

Whereas Propositions 2 to 6 dealt with the costs of redistribution, the above results bring to light some of the benefits. With respect to equity, (25) and (27) show that a higher τ_t reduces both the persistence $p(\tau_t)$ and the magnitude Δ_{t+1}^2 of disparities in human capital and income. With respect to efficiency, redistribution provides a partial substitute for the missing credit market. This *investment reallocation* effect is reflected in the last term of (29), which measures the shortfall in aggregate growth compared to a representative agent economy. Because decreasing returns and credit constraints imply that poorer families have a higher marginal return than wealthier ones, redistributing education resources directly or indirectly (through income taxes) reduces this loss, but only up to a point: $\mathfrak{L}(\tau)$ is minimized for $\tau = 1 - \alpha/(1-\beta\lambda)$. The more important the *complementary inputs* provided by families and communities, i.e. the greater is α, the less redistribution is called for—at least in the short run, where Δ_t^2 is given.

Having solved for the economy's aggregate and distributional dynamics under any policy profile $\{\tau_t\}_{t=0}^{\infty}$, I now examine steady-states in more detail.

4.2. *Steady-State Income, Inequality, and Redistribution*

• *Aggregate income and its distribution.* Given a constant rate of fiscal progressivity or school finance equalization τ, income inequality converges to

$$(31) \qquad \lambda^2\Delta^2(\tau) \equiv \frac{\lambda^2\omega^2}{1-(\alpha+\beta\lambda(1-\tau))^2},$$

and per capita income to

$$(32) \qquad \ln y(\tau) \equiv \frac{\ln\bar{\kappa} + \mu(1-\alpha)\ln l(\tau) - \mathfrak{L}(\tau)\lambda^2\Delta^2(\tau)/2}{1-\alpha-\beta\lambda}.$$

Redistribution has two opposing effects on steady-state income. First, it reduces labor supply $l(\tau)$; it would also depress savings, were it not for the offsetting effect of consumption taxes and investment subsidies. If those are for some reason infeasible, one simply replaces $\bar{s}$ by $\bar{s}(1-\tau)$ or $\bar{s}/(1+\tau\bar{s})$ in the term $\ln\bar{\kappa}$. At the same time, redistribution alleviates the misallocation of education resources due to credit constraints: by reducing $\mathfrak{L}(\tau)$, up to some point, as well as $\Delta^2(\tau)$, a positive τ tends to raise $y(\tau)$. The degree of progressivity τ_Y^* that maximizes long-run output is easily seen to: (a) decrease with the labor supply elasticity $1/\eta$, the share of labor in production μ, and the discount factor ρ; (b) increase with the variability of shocks to ability or human wealth, ω^2.

• *Endogenous growth.* The above results are easily transposed from long-run levels to *long-run growth rates*, by allowing for spillovers in the accumulation of human capital. This can be done in a "heterogeneity-neutral" manner (that is, without introducing additional costs or benefits of redistribution), by replacing the constant κ in (5) with the human capital index $\kappa_t \equiv \left(\int_0^1 (h_t^i)^\lambda di\right)^{\gamma/\lambda}$.[13] All previous results remain unchanged, with κ_t simply substituted for κ everywhere. In steady-state, the only difference is that the denominator in (32) is now $1-\alpha-\beta\lambda-\gamma$. For $\alpha+\beta\lambda+\gamma=1$ the numerator gives the economy's asymptotic growth rate, which behaves with respect to τ exactly as $\ln y(\tau)$ did in the original specification.

• *Redistribution.* A tax rate τ yields inequality $\lambda\Delta(\tau)$ in earnings, but only $(1-\tau)\,\lambda\Delta(\tau)$ in disposable incomes. When τ is a rate of school finance progressivity, as in Section 3, this narrowing operates only on education spending. To assess the extent and incidence of redistribution implied by either policy, recall from (28) that the threshold $\tilde{y}$ separating losers and gainers is always given by $\ln(\tilde{y}/y) = (1-\tau)\lambda^2\Delta^2(\tau)/2$.[14] This corresponds to a rank in the income distribution of

$$q(\tau) \equiv \Phi[(2-\tau)\lambda\Delta(\tau)/2)], \tag{33}$$

where Φ denotes the c.d.f. of a standard normal. The share of net transfers in national income that results from a rate of fiscal progressivity τ is then[15]

$$b(\tau) \equiv 2\Phi(\tau\lambda\Delta(\tau)/2) - 1. \tag{34}$$

[13] The reason why this specification is heterogeneity-neutral is that it aggregates individual human capital contributions with the same elasticity of substitution as total output. Spillovers embodying social costs or benefits of heterogeneity in human capital interactions can easily be dealt with by letting the elasticity of substitution in κ_t differ from $1/(1-\lambda)$; see Bénabou (1996a).

[14] By losers and gainers I mean: (i) families paying a net tax and those receiving a net transfer, when τ describes fiscal policy; (ii) families whose education expenditures are subsidized beyond the average rate $a(\tau)$ and those whose expenditures are taxed relative to $a(\tau)$, when τ describes school finance policy.

[15] For a lognormal distribution with variance ν^2, the Lorenz curve is: $\Gamma(q;\nu) \equiv \Phi(\Phi^{-1}(q)-\nu)$, for all $q \in [0,1]$. Therefore, the $1-q(\tau)$ richest households earn $1-\Gamma(q(\tau);\lambda\Delta(\tau)) = \Phi(\tau\lambda\Delta(\tau)/2)$ of total pretax income. After redistribution their share falls to $1-\Gamma(q(\tau);(1-\tau)\lambda\Delta(\tau)) = \Phi(-\tau\lambda\Delta(\tau)/2))$. The difference yields (34).

When redistribution occurs only in school expenditures, (34) describes the share of the total educational budget that is transferred from those above $\tilde{y}$ to those below it. Multiplying this number by $\bar{s}$ translates it into a percentage of total income.

5. A CRITERION OF AGGREGATE ECONOMIC EFFICIENCY

Aggregate income or growth provides of course only an incomplete picture of the efficiency implications of redistributive policies. First, it omits the opportunity cost of production, but this can be remedied by looking at the aggregate consumption-leisure bundle. More fundamentally, it fails to reflect redistribution's role as social insurance: by the law of large numbers, individual shocks cancel out when computing macroeconomic aggregates.

Policies are most often evaluated according to some social welfare criterion such as $W_0 = \int_0^1 \ln U_0^i di$, or $\mathcal{U}_0 \equiv \int_0^1 U_0^i di$. Risk and effort concerns are now properly embodied, and I shall indeed compute such utility aggregates. But the problem is that whereas *aggregate income underestimates* the efficiency value of redistribution, *aggregate welfare overestimates* it. Because of the concavity of individual utility, any such utilitarian criterion rises (keeping labor supply and savings fixed) with all current and future redistributions, even when there are no shocks requiring insurance and no credit-constrained investments in need of reallocation.[16]

I shall therefore propose an alternative measure of pure *aggregate economic efficiency*, which puts *zero value* on equity of consumption or income per se, and is affected by redistributions only to the (full) extent that such policies: (i) distort effort and savings decisions; (ii) relax the liquidity constraints that impede growth; (iii) reduce the idiosyncratic risk faced by individuals. The basic idea is very simple:

- First, replace agents' stochastic consumption sequences with appropriate *certainty-equivalents*. In this aggregation over states, the relevant parameter is the degree of risk-aversion.
- Second, *aggregate linearly* individuals' certainty-equivalent consumptions, which are thus treated as perfect substitutes. More generally, when aggregating over individuals the relevant parameter should be society's degree of inequality-aversion, which here is set to zero.
- Finally, aggregate *over time* using agents' common discount rate and intertemporal elasticity of substitution.

The point of this construction is not that this is "the right" social welfare function, nor that society should not care about equity. It is instead that one should be able to *separate* efficiency concerns—namely, the extent to which market distortions and imperfections are worsened or improved by policy—from pure equity concerns (which will be incorporated in the next section). For instance, inequality of initial endowments should not affect a measure of pure efficiency, unless

[16] For instance, in our model, even when $\omega = 0$ and either $(\alpha, \beta\lambda) = (0, 0)$ (no accumulation) or $\alpha + \beta\lambda = 1$ (accumulation with constant returns to investment).

wealth determines an agent's ability to invest or bear risk. Conversely, redistributions should affect such a measure only to the extent that they change the "size of the pie" (the path of aggregate consumption) or the riskiness of individual "slices." We shall see that the index proposed above has these properties, whereas none of the usual (utilitarian) social welfare functions do.[17] These points will be established in the context of the model, but the underlying idea is clearly more general; it can be thought of, intuitively, as adjusting GDP for individual risk.[18]

Let us start with redistribution through fiscal policy. By Proposition 1, the intertemporal utility of agent i can be written as

$$\ln U_0^i = (1-\rho)\left(\sum_{t=0}^{\infty}\rho^t(1-\tau_t)\prod_{k=0}^{t-1}p(\tau_k)\right)\lambda(\ln h_0^i - m_0) + \sum_{t=0}^{\infty}\rho^t(W_t - \rho W_{t+1}). \tag{35}$$

The first term captures the lasting effects of differences in initial endowments. The second represents the part that is common to all agents, including labor supply and risk concerns. The same level of individual welfare would clearly result from the deterministic, or *certainty-equivalent* consumption sequence $\{\bar{c}_t^i\}_{t=0}^{\infty}$ defined by

$$\ln \bar{c}_t^i - (l_t)^\eta \equiv (1-\tau_t)\left(\prod_{k=0}^{t-1}p(\tau_k)\right)\lambda(\ln h_0^i - m_0) + (W_t - \rho W_{t+1})/(1-\rho), \tag{36}$$

with unchanged efforts $\{l_t\}_{t=0}^{\infty}$. Moreover, it is shown in the Appendix that $\bar{c}_t^i$ simplifies to

$$\ln \bar{c}_t^i = E_0[\ln c_t^i \mid h_0^i] + r\left(\frac{\rho}{1-\rho}\right)\left(\frac{V_{t+1}^2\omega^2}{2}\right). \tag{37}$$

In the absence of shocks, $\bar{c}_t^i = c_t^i$. When $r=0$, $\bar{c}_t^i$ is the standard certainty-equivalent consumption given time-separable, logarithmic preferences. In the more general case there is an extra term that might be called (minus) the "resolution premium" for the shock ξ_{t+1}^i.[19] It is negative when tastes favor early resolution of uncertainty ($r<0$), positive in the reverse case.

I next compute total certainty-equivalent consumption, then the efficiency criterion.

DEFINITION 1: Let $\overline{C}_t \equiv \int_0^1 \bar{c}_t^i di$. The *aggregate efficiency* of a tax sequence $\{\tau_t\}_{t=0}^{\infty}$ is defined as

$$\mathscr{E}_0 \equiv (1-\rho)\sum_{t=0}^{\infty}\rho^t[\ln \overline{C}_t - (l_t)^\eta].$$

[17] Neither does the usual total compensating variation, which is easily seen to coincide here with $\mathscr{U}_0 \equiv \int_0^1 U_0^i di$.

[18] See Flodén (2001) and Sheshadri and Yuki (2000) for recent applications of the methodology proposed here.

[19] That shock has variance ω^2 and affects the agent through h_{t+1}^i, which enters the intertemporal utility $\ln U_{t+1}^i$ with a coefficient equal to V_{t+1}; see Proposition 1.

Given two policies $\{\tau_t\}_{t=0}^{\infty}$ and $\{\tau'_t\}_{t=0}^{\infty}$, $\mathscr{E}_0 - \mathscr{E}'_0$ will be interpretable as a percentage difference in total consumption. The lognormality of the $\bar{c}_t^i$'s makes it easy to compute $\overline{C}_t$ and obtain

$$\text{(38)} \qquad \mathscr{E}_0 = W_0 + (1-\rho)\left(\sum_{t=0}^{\infty} \rho^t (1-\tau_t)^2 \prod_{k=0}^{t-1} p(\tau_k)^2\right)\left(\frac{\lambda^2 \Delta_0^2}{2}\right),$$

where $W_0 \equiv \int_0^1 \ln U_0^i di$.[20] Thus $\mathscr{E}_0$ differs from aggregate welfare by a term that increases with Δ_0 and declines with all present and future rates of redistribution. This adjustment eliminates all effects of inequality except those relating to efficiency via market incompleteness, so that $\mathscr{E}_0$ indeed satisfies properties (i) to (iii) postulated earlier. These results appear most clearly with time-invariant policies, but are established more generally in the Appendix.

PROPOSITION 8: (a) *The aggregate efficiency of a constant rate of progressive taxation τ equals*

$$\text{(39)} \qquad \mathscr{E}_0(\tau) = (1-\rho)\sum_{t=0}^{\infty} \rho^t [\ln y_t - (l_t)^\eta] + \ln(1-\bar{s})$$
$$- \rho(1-\tau)^2 \left(\frac{1}{1-\rho p(\tau)^2} - \frac{r(1-\rho)}{(1-\rho p(\tau))^2}\right)\left(\frac{\lambda^2 \omega^2}{2}\right).$$

(b) *For any initial conditions (m_0, Δ_0^2), $\mathscr{E}_0(\tau)$ is maximized at a strictly positive $\tau_{E,0}^*$.*

The interpretation is simple. The first term, which reduces to $\ln y(\tau) - l(\tau)^\eta$ in steady-state, captures the effects of redistribution on the path of total output, net of effort. These operate through its influence on the allocation of investment and on labor supply (plus possibly on savings, if $\bar{s}$ is replaced by $s(\tau)$), and were discussed earlier. Adding the second term yields the utility derived by a fictitious representative agent from aggregate consumption and leisure. More novel is the last term, which measures the disutility that agents suffer from uninsured idiosyncratic shocks—or conversely, the *insurance and uncertainty-resolution value* of redistribution. This risk premium is always positive, hence minimized for $\tau = 1$; it is larger, the greater is agents' risk-aversion $1 - r$. Part (b) of the proposition is also quite intuitive. Starting from $\tau = 0$, a small tax increase generates only second-order welfare losses from labor (and possibly savings) distortions; but due to market frictions it yields a first-order gain in insurance and in the allocation of investment resources across differentially credit-constrained families.

Finally, note that $\mathscr{E}_0$ is independent of the distribution of initial endowments Δ_0^2, except to the extent that it affects the path of total output, through accumulation. Equation (38) makes clear that such is not the case of W_0. It also shows that the *median voter*, whose intertemporal utility is W_0, would always choose taxes exceeding the efficient level.

[20] By (36), $\ln \overline{C}_t = \int_0^1 \ln \bar{c}_t^i di + (1-\tau_t)^2 \prod_{k=0}^{t-1} p(\tau_k)^2 \lambda^2 \Delta_0^2/2$, and $W_0 = \sum_{t=0}^{\infty} \rho^t \left(\int_0^1 \ln \bar{c}_t^i di - (l_t)^\eta\right)$.

6. EFFICIENCY, EQUALITY, AND SOCIAL WELFARE

I now extend the analysis to incorporate both efficiency and equity concerns. The procedure is the same as for constructing $\mathscr{E}_0$, except that individual certainty-equivalents are aggregated with an *interpersonal* elasticity of substitution $\sigma \gtrless 0$, whose inverse $1/\sigma$ measures society's degree of inequality-aversion. Thus:

$$\overline{C}_{t,\sigma} \equiv \left(\int_0^1 (\bar{c}_t^i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}. \tag{40}$$

I will also compute more standard social welfare functions, which are aggregates of (intertemporal) utilities rather than risk-adjusted consumptions. These have the clearly desirable property that maximizing such a criterion ensures Pareto efficiency. On the other hand, it will be seen that they cannot distinguish between the effects of policy that operate through its role as a substitute for missing markets, and those that reflect an implicit equity concern.

DEFINITION 2: For any $\sigma \in \mathbb{R}$, define the two *social welfare* indices:

$$\mathscr{E}_{0,\sigma} \equiv (1-\rho) \sum_{t=0}^{\infty} \rho^t [\ln \overline{C}_{t,\sigma} - (l_t)^\eta],$$

$$\mathscr{U}_{0,\sigma} \equiv \ln \left(\int_0^1 (U_0^i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}.$$

The following proposition focuses again on the more transparent case of a time-invariant policy, but similar results are established in the Appendix for arbitrary tax sequences $\{\tau_t\}_{t=0}^{\infty}$.

PROPOSITION 9: *The social welfare levels resulting from a constant tax policy τ are equal to*

$$\mathscr{E}_{0,\sigma}(\tau) \equiv \mathscr{E}_0(\tau) - A(\tau)\left(\frac{\lambda^2 \Delta_0^2}{2\sigma} \right), \tag{41}$$

$$\mathscr{U}_{0,\sigma}(\tau) \equiv \mathscr{E}_0(\tau) - B(\tau)\left(\frac{\lambda^2 \Delta_0^2}{2\sigma} \right) - (A(\tau) - B(\tau))\left(\frac{\lambda^2 \Delta_0^2}{2} \right), \tag{42}$$

where

$$A(\tau) \equiv \frac{(1-\rho)(1-\tau)^2}{1-\rho p(\tau)^2} > \left(\frac{(1-\tau)(1-\rho)}{1-\rho p(\tau)} \right)^2 \equiv B(\tau) \quad \textit{for all } \tau < 1.$$

Thus $\mathscr{E}_{0,\sigma}$ can be naturally *decomposed into efficiency and equity* concerns, with the latter's intensity being parameterized by $1/\sigma$.[21] By contrast, there is *no value*

[21] Note that the efficiency index $\mathscr{E}_0$ defined earlier is simply $\mathscr{E}_{0,\infty}$, which corresponds to $1/\sigma = 0$. Similarly, $\mathscr{U}_0 \equiv \int_0^1 U_0^i di = \mathscr{U}_{0,\infty}$.

of σ for which the utility-based criterion $\mathcal{U}_{0,\sigma}$ does not reward equity per se. Indeed, the pure cost of inequality that it embodies, $\mathcal{U}_{0,\sigma} - \mathcal{E}_0$, has two components. The first, proportional to $(1-\tau)^2 \Delta_0^2/\sigma$, arises naturally from society's aversion to disparities in welfare. The second, proportional to $(1-\tau)^2 \Delta_0^2$, arises mechanically but inevitably from the concavity of individual preferences.

A final observation is that aggregate welfare $W_0 = \mathcal{E}_{0,1} = \mathcal{U}_{0,1}$ belongs to both families of indices, and therefore combines their two defining properties: exact decomposability and Pareto-compatibility. But it arbitrarily equates society's degree of *inequality aversion* $1/\sigma$, not even to individuals' risk aversion $1-r$, which might perhaps make sense in an ex-ante "veil-of-ignorance" perspective, but to the inverse of their (unitary) *intertemporal elasticity* of substitution.[22]

In studying efficiency and its relation to aggregate output and social welfare, I have so far concentrated on policies of income redistribution. The case of progressive education finance is treated similarly in the Appendix, leading to the following proposition.

PROPOSITION 10: *The efficiency and social welfare indices* $\mathcal{E}_0$, $\mathcal{E}_{0,\sigma}$, *and* $\mathcal{U}_{0,\sigma}$ *resulting from a sequence of education finance progressivity rates* $\{\tau_t\}_{t=0}^{\infty}$ *are given by the same expressions as in* (38) *and Propositions* 8–9, *except that all terms in* $(1-\tau_t)^2$ *and* $(1-\tau)^2$ *are replaced by* 1.

These results reflect again the fact that parental consumption is not expropriated. This was shown to reduce both intra- and intertemporal distortions, compared to the case of income taxes; the counterpart, made clear by Proposition 10, is that redistributive education finance offers *less risk-sharing*. For instance, with income taxation $\tau = 1$ yields full consumption insurance (hence $\mathcal{E}_0 = (1-\rho)\sum_{t=0}^{\infty} \rho^t [\ln y_t - (l_t)^\eta] + \ln(1-\bar{s})$); but when only educational inputs are equalized, families remain exposed to significant risk.

7. QUANTITATIVE ANALYSIS

That redistribution may generate efficiency and even output gains when insurance and credit markets are incomplete has been understood for some time. Yet there has been relatively little attempt to evaluate these gains and compare them to the losses from distortionary taxation. The framework developed here incorporates policies' effects on labor supply, savings, individual risk, and the allocation of resources across investment projects that are both differentially productive (due to family or neighborhood inputs) and differentially credit-constrained. It therefore allows for a fairly comprehensive quantitative analysis of the *efficiency costs and benefits of redistribution*, be it fiscal or educational.

Naturally, given the model's simplicity, its reliance on specific functional forms, and the lack of empirical consensus over certain parameters, the aim of this exercise can only be a broad assessment of the main tradeoffs. In particular, credit

[22] The desirability of distinguishing between aversion to inequality, to risk and to intertemporal fluctuations also arises in the measurement of social mobility; see Gottshalk and Spolaore (2000).

and insurance markets are completely absent from the model, rather than simply imperfect; there is also no precautionary savings. Since these factors tend to overstate the benefits of redistribution, I will compensate by using conservative values for risk-aversion and the returns to educational spending. On the other hand, by focusing on the efficiency criterion $\mathscr{E}_0$ one abstracts from the equity concerns embodied in standard social welfare functions, as well as from the costs of inequality that arise from production complementarities, crime, or political instability. Another factor that will tend to understate the benefits of redistribution is that I shall focus (for simplicity) on comparisons between steady-states.[23]

7.1. *Parameter Values*

• *Production.* The shares λ and μ of human capital and labor are determined by "maximizing out" physical capital from a three-factor production function.[24] Following Barro, Mankiw, and Sala-i-Martin (1995) I use shares of .5 for human capital, .3 for physical capital, and .2 for labor. This yields $\lambda = .5/.8 = .625$ and $\mu = .3/.8 = .375$.

• *Accumulation.* Most estimates of intergenerational persistence $p(\tau) \equiv \alpha + \beta\lambda(1-\tau)$ range from .3 to .6 (e.g., Solon (1992), Mulligan (1997)). I set $\alpha = .35$ and $\beta = .4$, which allows $p(\tau)$ to range from .35 to .60. The elasticity of children's income to education spending is then $\beta\lambda = .25$, which is slightly above the value of .19 used by Fernandez and Rogerson (1998), but well below those of .35 and .45 used by Hendricks (1999) and Jones, Manuelli, and Rossi (1993), respectively. Given its critical role in human capital accumulation and the lack of empirical consensus, $\beta\lambda$ will be allowed to vary from 0 to .4 in the sensitivity analysis.[25]

• *Inequality.* Given α and $\beta\lambda$, the variability of idiosyncratic shocks determines the feasible range for steady-state inequality, $\lambda\Delta(\tau) = \lambda\omega/\sqrt{1-(\alpha+\beta\lambda(1-\tau))^2}$. Approximating the US distribution of family incomes as a lognormal, the mean-to-median ratio implies a standard deviation of log-incomes of about .61 in the 1990 Census and .69 in that of 1995. Computing the variance directly from the decile income distribution leads to higher values, between .75 (1990 Census) and 1.1 (fiscal data of Bishop, Formby, and Smith (1991)). I set $\omega = 1.0$, so that the feasible range for $\lambda\Delta(\tau)$ is [.67, .78].

• *Labor Supply.* Microeconomic estimates of the intertemporal elasticity of substitution $\varepsilon \equiv 1/(\eta - 1)$ vary between 0 and .4 for males, with a median of about .20.[26] I also consider cross-sectional elasticites of labor supply, which are not

[23] Recall from Proposition 8 that $\tau^*_{E,0} \equiv \arg\max_\tau[\mathscr{E}_0(\tau; m_0, \Delta_0^2)] > 0$, for any initial conditions (m_0, Δ_0^2); this takes into account the full transition path from (m_0, Δ_0^2) to the steady-state $(m(\tau), \Delta^2(\tau))$. On the other hand, $\arg\max_\tau[\mathscr{E}_0(\tau; m(\tau), \Delta^2(\tau))]$ need not be positive a priori.

[24] Thus if gross output is $Y_t^i = (h_t^i)^\phi (k_t^i)^{\phi'} (l_t^i)^{\phi''}$, equating $\partial Y_t^i/\partial k_t^i$ to a fixed world interest rate r makes $y_t^i \equiv Y_t^i - rk_t^i$ (and Y_t^i) proportional to $(h_t^i)^\lambda (l_t^i)^\mu$, with $\lambda \equiv \phi/(1-\phi')$ and $\mu \equiv \phi''/(1-\phi')$.

[25] Fernandez and Rogerson base their choice on cross-sectional studies such as Card and Krueger (1992). Hendricks appeals to estimates from life-cycle earnings profiles such as Heckman (1976). In a study using historical data on US States, Tamura (2001) finds the elasticity to be at least .4.

[26] See Browning, Hansen, and Heckman (1999) for a survey; the sole study reported on female participation yields a value of 1.6.

predicated on the assumption of a frictionless credit market, and more consistent with the present focus on lifetime outcomes and steady-states. For men, the compensated elasticity is typically around .10 (e.g., Pencavel (1986)). For women, Killingsworth and Heckman's (1986) survey includes both high and insignificant values, with a median of about .4. In view of both inter- and intra-temporal estimates, I set $\varepsilon = .20$, but in the sensitivity analysis I explore the full range from $\varepsilon = 0$ to $\varepsilon = \infty$.[27]

- *Discounting.* A typical discount factor in macroeconomic models is around .96 per year, which compounds to $(.96)^{25} = .36$ per generation (25 years). I set $\rho = .4$ in the reference case.
- *Risk-Aversion.* The natural benchmark I choose for $1 - r$ is 1, as it corresponds to intertemporally separable (logarithmic) preferences, and is on the low side of standard estimates. In the sensitivity analysis $1 - r$ will vary from zero (risk-neutrality) to 3.

Figures 1 and 5 display the main simulation results for steady-state output $\ln y(\tau)$, utility from aggregate consumption and leisure $\ln z(\tau) \equiv \ln C(\tau) - l(\tau)^{\eta}$, efficiency $\mathscr{E}(\tau)$, and welfare $W(\tau)$; all are measured as deviations from their values under laissez-faire ($\tau = 0$). The four functions are always single-peaked, with maxima at $\tau_Y^* < \tau_Z^* < \tau_E^* < \tau_W^*$; Tables I and II show how these four optimal redistribution rates vary with the key parameters ε and $\beta\lambda$.

7.2. *Fiscal Redistribution: Benchmark Case*

- *Income.* Figures 1 and 2a show that total income is maximal at $\tau_Y^* = 20.9\%$, which corresponds to a share of transfers in GDP of $b(\tau_Y^*) = 6.2\%$. The output gain with respect to laissez-faire is 1.3%, representing the balance of a 2.7% shortfall due to the fact that agents reduce hours by 4.4%, and a 4.0% increase from relaxing the liquidity constraints of poorer families. The savings distortion is fully offset by a consumption tax of 2.7%, used to finance a 26.4% subsidy for human capital investment.[28] In the endogenous growth version of the model, the 1.3% gain in steady-state income becomes a 0.5 percentage point rise in the long run growth rate.
- *Efficiency.* Aggregate efficiency is maximized at $\tau_E^* = 48.5\%$, or a transfer share $b(\tau_E^*) = 13.2\%$. The gain relative to $\tau = 0$ is the same as would result from a 9.6% increase in every agent's consumption. The actual effect of τ_E^* on aggregate resources, however, is a decline of 0.9%. But this is more than offset by the

[27] Given the utility function (2), the uncompensated elasticity for a wage w and nonwage income R is $\zeta^u = R/((\eta - 1)R + \eta wl)$, and the marginal propensity to earn out of nonwage income is $mpe = -wl/((\eta - 1)R + \eta wl)$. Conversely, we can write $\varepsilon \equiv 1/(\eta - 1) = \zeta^c/(1 + mpe)$, where $\zeta^c = \zeta^u + mpe$ is the compensated elasticity. Thus, typical values such as $\zeta^c \approx -mpe \in [.10, .20]$ yield ε in the range $[.11, .25]$.

[28] The economy's investment rate then remains at $\bar{s} = 11.6\%$ of aggregate income net of physical capital's remuneration, which corresponds to $\bar{s}(1 - \phi') = 8.1\%$ of total factor income; ϕ' was defined in footnote 24. Absent the corrective policy mix, the decline in savings would reduce aggregate output by $(\lambda\beta)(1 - \alpha - \beta\lambda)^{-1} \ln(\bar{s}/s(\tau_Y^*)) = 14.6\%$.

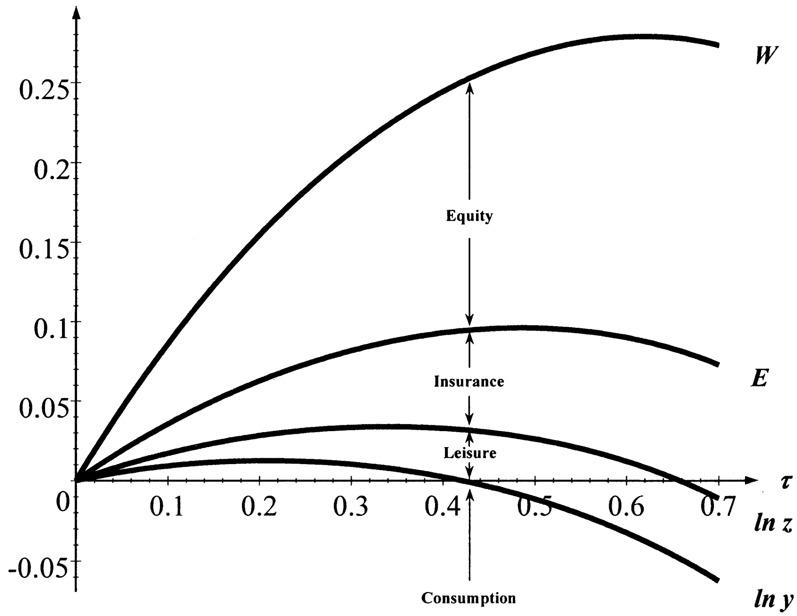


FIGURE 1.—Aggregate income, efficiency, and welfare under fiscal redistribution.

12.1% reduction in effort, worth a 3.6% increase in consumption, and especially by the value of insurance, equivalent to another 6.8% of aggregate consumption. Neutralizing the savings distortion now requires a 6.4% consumption tax and a 94.3% subsidy for human capital investment.

Figure 1 also illustrates one of the most important results, namely the relationship between *aggregate income, economic efficiency*, and a utilitarian *social welfare* criterion. In addition to the utility from leisure, the vertical distance between $\ln y$ and $\mathcal{E}$ measures the *insurance value* of taxation, which increases with risk aversion $1-r$. The additional distance between $\mathcal{E}$ and W reflects the pure *equity value* of redistribution, for a degree of inequality aversion $1/\sigma = 1$. More generally, each value of $1/\sigma > 0$ defines a social welfare function $\mathcal{U}_\sigma$ above $\mathcal{E}$.

- *Mobility and Inequality.* Figure 2b shows how going from laissez-faire to $\tau_Y^* = 20.9\%$ reduces intergenerational persistence $p(\tau)$ from .60 to .55, and a further increase to $\tau_E^* = 48.5\%$ brings it down to .48. The effect on cross-sectional inequality is similar: as τ rises from zero to τ_Y^* and then to τ_E^*, $\lambda\Delta(\tau)$ first falls from .78 to .75, then to .71.
- *Redistribution.* Under the efficient policy $\tau_E^* = 48.5\%$, the cutoff $\tilde{y}$ between losers and gainers occurs at the 70th percentile (1.43 times median income), and the transfer share is $b(\tau_E^*) = 13.7\%$.

To assess the extent of redistribution that is implied, recall that τ is also the (income-weighted) average marginal tax-and-transfer rate. For taxes alone, the

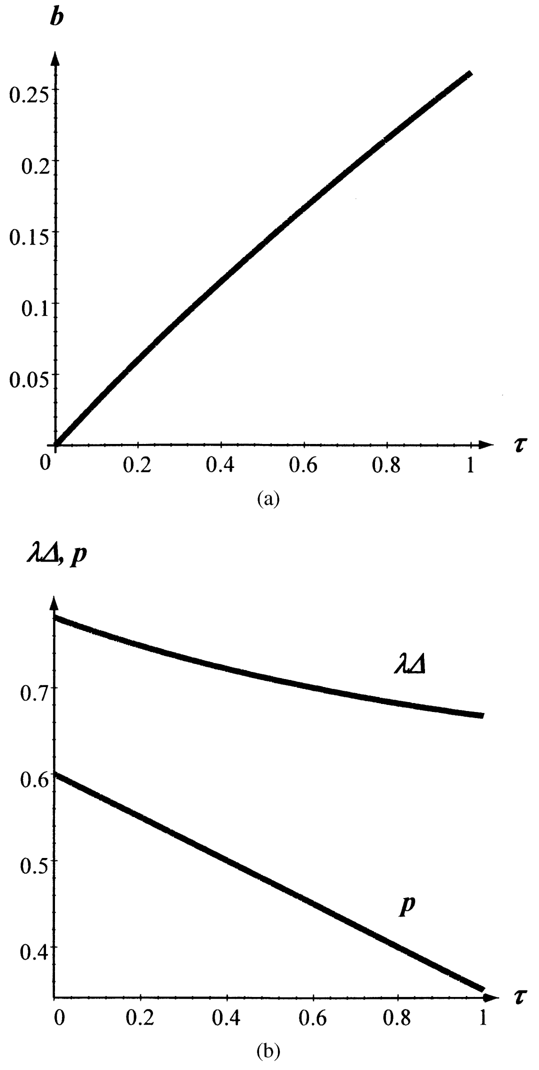


FIGURE 2.— (a) Share of net transfers in total income. (b) Income inequality and intergenerational persistence under fiscal redistribution.

weighted marginal rate in the United States is about 20%.[29] There is no readily available data on the incidence of transfers, but they typically contribute more to progressivity than taxes; a more relevant estimate of τ could thus be 30 to

[29] See Easterly and Rebello (1993) and Gouveia and Strauss (1994). Using the income and tax data reported by Bishop, Formby and Smith (1991), I obtained very similar values. As a comparison, Englund and Persson (1982) estimate (by fitting a scheme like (6) to data on income taxes and housing allowances) that, in Sweden, τ rose from 47% in 1971 to 63% in 1979.

40%, implying $\tau_Y^* < \tau \precsim \tau_E^*$, and certainly $\tau < \tau_W^* = 61.2\%$. Alternatively, let us consider transfers; these represent 16% to 18% of GDP, but only about half are genuinely redistributive (the rest being social security and medicare payments). Thus $b \approx 9\%$, which falls between $b(\tau_Y^*) = 6.2\%$ and $b(\tau_E^*) = 13.7\%$, and in any case well below $b(\tau_W^*) = 17.0\%$. In summary, looking at either the average marginal tax rate or the transfer share suggests (quite tentatively, or course) that fiscal redistribution in the United States exceeds the income or growth-maximizing level, could be somewhat below the efficient level, and is markedly lower than the median family's preferred outcome.[30]

7.3. *Fiscal Redistribution: Sensitivity Analysis*

Figures 3a–3b show how the tradeoff between the efficiency costs and benefits of redistribution worsens as the *elasticity of labor supply* $\varepsilon = 1/(\eta - 1)$ rises. Table I provides more detail on how the optima $\tau_Y^*, \tau_Z^*, \tau_E^*$, and τ_W^* decline with ε. Looking for instance at the row that contains the benchmark case, we see that in the absence of distortions, τ_Y^* would be 66.8%, whereas for $\varepsilon > 0.5$ redistribution only reduces total income. The efficient tax rate τ_E^* and transfer share $b(\tau_E^*)$, on the other hand, remain above 14.5% and 4.4% even as ε tends to infinity.

The effect of *education return* parameter $\beta\lambda$ is more complex. Table I shows a significant positive impact on τ_Y^*, but only a small negative one on τ_E^*. The intuition is as follows. When $\beta\lambda = 0$ there is no investment, so redistribution can only reduce output and deteriorate the consumption-leisure tradeoff; thus $\tau_Y^* = \tau_Z^* = 0$, but $\tau_E^* > 0$ due to the insurance motive. As $\beta\lambda$ rises the investment-reallocation effect gradually replaces insurance as the predominant benefit in the efficiency tradeoff with the labor supply effect (which, for $\varepsilon > 0$, is now also stronger; see (14)). Consequently, τ_Y^* and τ_E^* move towards each another.

Last but not least, Figure 4 shows the role of *risk-aversion*. The efficient rate of redistribution τ_E^* starts at 36.3% for $1 - r = 0$, then rises to 48.5%, 55.6%, and 60.4% for $1 - r = 1, 2$, and 3 respectively. The corresponding transfer shares are $b(\tau_E^*) = 10.5\%$, 13.7%, 15.5%, and 16.7%. These large variations make clear the value of working with the general preferences (2) rather than restricting attention to the separable case, $r = 0$. In all cases τ_E^* remains well below τ_W^*, and well above τ_Y^* (or even τ_Z^*), as was the case throughout Table I.

Beyond the results obtained for specific parameter values, it is the general message from these simulations that is most important. They consistently show that the efficiency costs and benefits of redistribution are *both* quantitatively important, and that per capita income and aggregate welfare provide only very imperfect measures of the resulting tradeoff.

[30] Note, however, that we analyze fiscal and educational redistribution separately. In reality both are present simultaneously, and their effects cumulative.

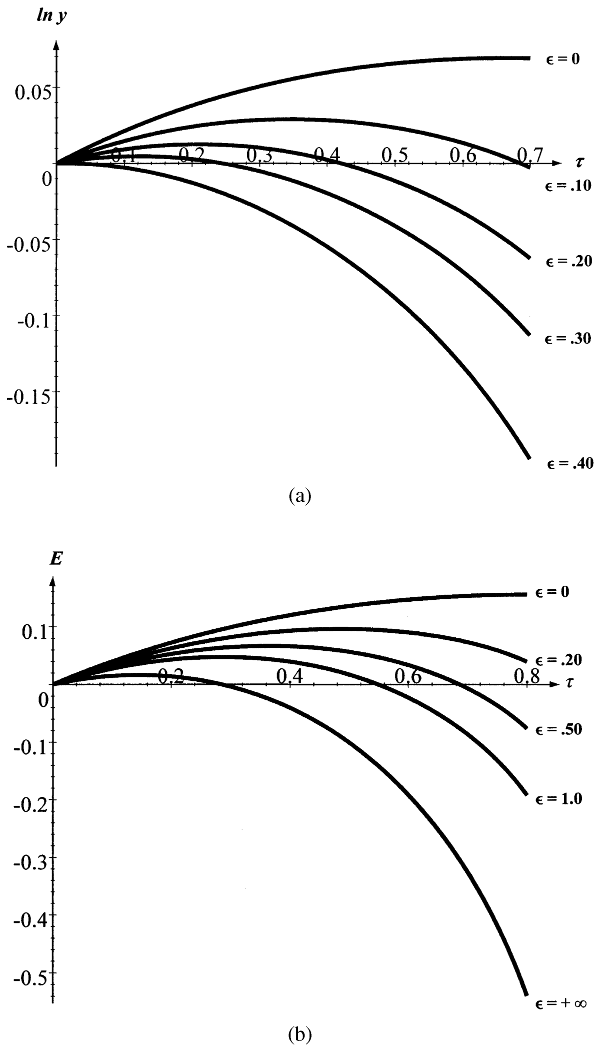


FIGURE 3.— (a) Aggregate income under fiscal redistribution, for different labor supply elasticities. (b) Efficiency under fiscal redistribution, for different labor supply elasticities.

7.4. *Education Finance: Benchmark Case*

• *Income.* The distortions from equalizing educational investments are considerably smaller than from equalizing family incomes. Figure 5 shows that τ_Y^* is now 61.9%, leading to a sizeable 6.1% output gain with respect to laissez-faire (or, in the constant-returns version of the model, a 2.4 percentage point rise in long-run growth). The fall in labor supply is only 1.3%. On the savings side, a

TABLE I
FISCAL REDISTRIBUTION

	Elasticity ε[b]							
Elasticity $\beta\lambda$[a]	0	.10	**.20**	.30	.50	1	∞	Optima[c]
	–	0	0	0	0	0	0	τ_Y^*
	–	0	0	0	0	0	0	τ_Z^*
0	100	63.7	54.5	49.1	42.9	35.9	24.8	τ_E^*
	100	75.9	68.9	64.6	59.2	52.8	41.1	τ_W^*
	71.7	13.6	0	0	0	0	0	τ_Y^*
	71.7	36.1	26.5	21.3	15.6	9.8	1.6	τ_Z^*
.10	91.2	61.7	52.7	47.3	40.9	33.5	20.9	τ_E^*
	96.0	73.6	66.5	62.1	56.5	49.9	37.4	τ_W^*
	68.6	31.0	15.3	5.2	0	0	0	τ_Y^*
	68.6	42.0	32.8	27.3	20.9	13.7	2.0	τ_Z^*
.20	84.2	59.0	50.2	44.7	38.1	30.4	16.6	τ_E^*
	91.6	70.8	63.5	59.0	53.3	46.4	33.3	τ_W^*
	66.8	34.5	**20.9**	12.1	1.2	0	0	τ_Y^*
	66.8	42.9	**34.0**	28.6	22.2	14.9	0	τ_Z^*
.25	80.8	57.3	**48.5**	43.1	36.5	28.7	14.5	τ_E^*
	89.1	69.0	**61.7**	57.1	51.4	44.4	31.1	τ_W^*
	64.8	36.4	24.4	16.8	7.3	0	0	τ_Y^*
	64.8	43.0	34.4	29.2	22.9	15.6	2.7	τ_Z^*
.30	77.4	55.2	46.6	41.3	34.7	26.8	12.6	τ_E^*
	86.4	67.0	59.6	55.0	49.2	42.2	28.9	τ_W^*
	62.3	37.4	26.6	19.9	11.7	2.0	0	τ_Y^*
	62.3	42.4	34.2	29.2	23.1	16.0	0	τ_Z^*
.35	73.6	52.8	44.4	39.1	32.6	24.9	10.9	τ_E^*
	83.3	64.6	57.2	52.6	46.8	39.8	26.7	τ_W^*
	59.4	37.1	27.8	21.9	14.6	6.1	0	τ_Y^*
	59.4	41.7	33.8	29.1	23.5	16.8	4.9	τ_Z^*
.40	68.6	50.4	42.6	37.6	31.5	24.2	10.8	τ_E^*
	79.7	62.8	55.8	51.4	45.8	39.1	26.4	τ_W^*

[a] $\beta\lambda$: elasticity of earnings to educational expenditures; benchmark value in bold.
[b] ε: intertemporal elasticity of substitution in labor supply; benchmark value in bold.
[c] $\tau_Y^*, \tau_Z^*, \tau_E^*, \tau_W^*$: values of τ (in %) that respectively maximize aggregate income, utility from aggregate consumption and leisure, overall efficiency, and aggregate welfare.

consumption tax of 0.9% and a 7.2% average subsidy for education are now sufficient to maintain accumulation at its first-best level.[31]

• *Efficiency.* The efficient rate of education finance equalization is $\tau_E^* = 68.2\%$. In other words, about *two-thirds* of the variations in per-pupil expenditures reflecting differences in family incomes should be offset. The corresponding efficiency gain is 7.3%, of which 6.0% is due to increased aggregate income and

[31] As before, $\bar{s} = 11.6\%$. Absent the corrective scheme, the decline in savings would reduce from 6.6% to 2.3% the gain in long-run income resulting from $\tau_Y^* = 61.9\%$.

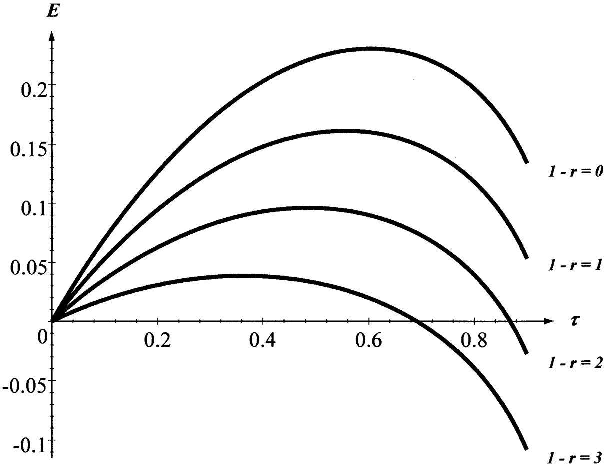


FIGURE 4.—Efficiency under fiscal redistribution, for different degrees of risk aversion.

consumption, 0.6% to lower effort, and only 0.7% to better risk-sharing. Contrasting Figures 1 and 5 clearly shows the reduced value of redistributive education finance as a social insurance scheme, compared to that of taxes and transfers.

• *Mobility and Inequality.* Since the optimal rate of progressivity is much higher than for fiscal policy, so is social mobility: as τ rises from zero to τ_Y^*

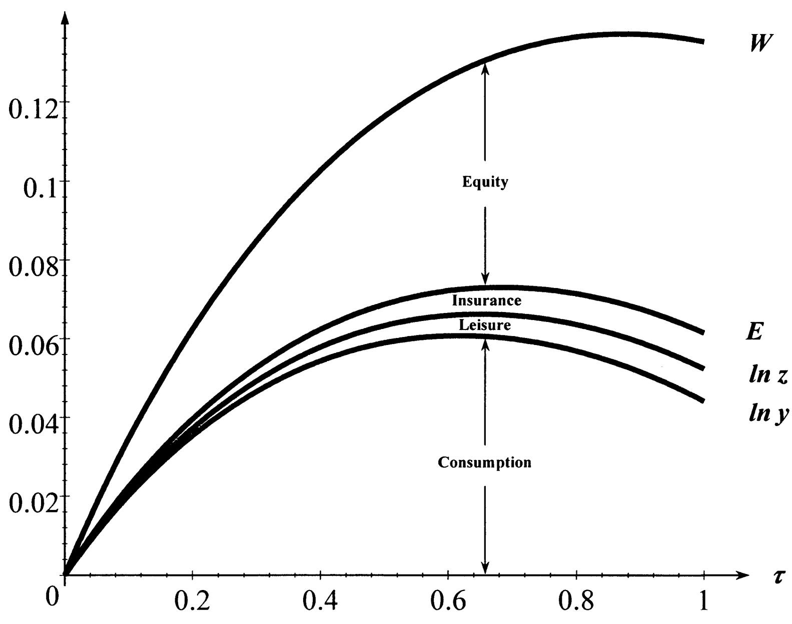
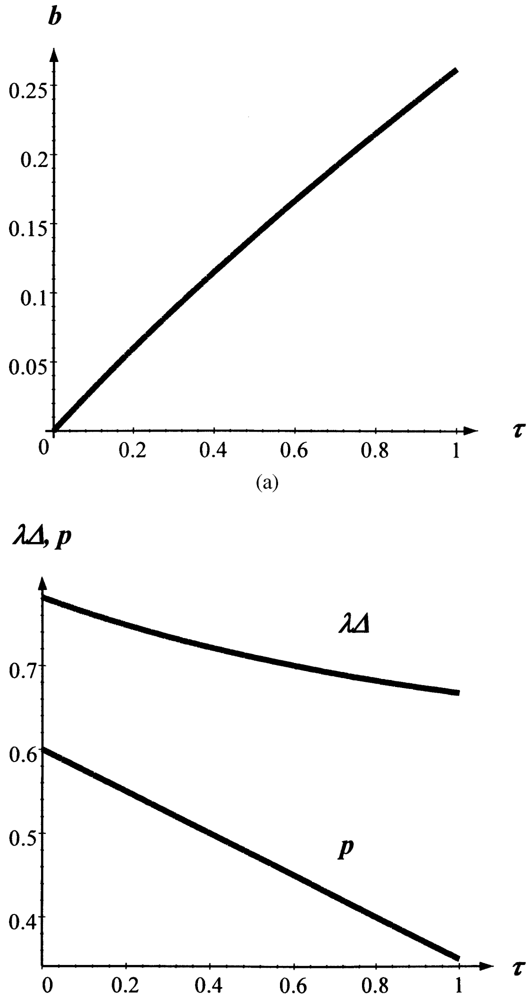


FIGURE 5.—Aggregate income, efficiency, and welfare under redistributive education finance.



(a)

(b)

FIGURE 6.— (a) Share of education expenditures redistributed. (b) Income inequality and intergenerational persistence under redistributive education finance.

and then to τ_E^*, $p(\tau)$ falls from .68 to .45 and then to .43 (Figure 6b). Cooper (1998) finds evidence in the PSID that state-level redistribution of educational expenditures across school districts lowers intergenerational income persistence. Inequality of incomes falls only slightly more than before, but for educational spending it is reduced much further: $\lambda\Delta(\tau)$ and $(1-\tau)\lambda\Delta(\tau)$ go from (.78, .78) under laissez-faire to (.70, .27) under τ_Y^*, (.69, .22) under τ_E^*, and (.68, .08) under $\tau_W^* = 87.4\%$. The expenditure numbers may be compared to Hoxby's (1998) estimates of the coefficient of variation in per-pupil spending among school districts in California, Illinois, and Massachusetts: respectively .16, .25, and .28 for recent years. One can also relate $1-\tau$ to the elasticity of education expenditures to community income. Estimates for the 1950's to the 1970's surveyed by Bergstrom, Rubinfeld, and Shapiro (1982), vary between .46 and .76, and cluster around 2/3. Hoxby (1998) documents for Illinois and Michigan a steep decline in recent decades, from a range of [.4, .6] in the 1960's to [.1, .4] in the 1990's.[32] These comparisons suggest (again very tentatively) that the extent of school finance equalization has risen from below τ_Y^* to levels closer to the efficient value τ_E^*, but remains smaller than the τ_W^* that would maximize median and average welfare.

- *Redistribution.* Under the policy τ_E^*, the break-even point $\tilde{y}$ that separates families whose education is subsidized beyond the average rate $a(\tau)$ from those whose expenditures are taxed relative to $a(\tau)$ occurs at the 68th percentile (1.37 times median income). The 32% richest families earn 59.3% of total income, but their share of total educational expenditures is reduced to 40.7%; the share reallocated to poorer households or communities is thus $b(\tau_Y^*) = 18.7\%$. It rises to 23.2% under the policy τ_W^* that maximizes average and median welfare.

7.5. *Education Finance: Sensitivity Analysis*

The most striking feature of the results reported in Figures 7a–7b and Table II is that *high rates of education finance equalization* remain optimal, no matter *how large* the intertemporal elasticity of labor supply ε. Even when preferences become linear in effort, $\varepsilon = +\infty$, the income-maximizing and efficient rates of equalization are still $\tau_Y^* = 39.2\%$ and $\tau_E^* = 59.6\%$. The corresponding shares of educational resources being redistributed are 11.3% and 16.5% respectively, and the resulting gains still amount to 2.8% for output and 5.9% for efficiency.

A higher *return to education* $\beta\lambda$ now lowers both τ_Y^* and τ_E^*, while significantly increasing the gains resulting from these optimal policies. The intuition is as follows. With labor supply distortions and insurance now both playing relatively modest roles, the key impact of policy is through the reallocation of investment towards the poor. This effect, measured by $\mathcal{Q}(\tau)\Delta^2(\tau)$ in (32), becomes more important as $\beta\lambda$ rises, i.e. as resources matter more for accumulation. Yet the optimal τ declines, because the accumulation technology gets closer to constant private returns; recall that $\mathcal{Q}(\tau)$ is minimized at $\tau = (1-\alpha-\beta\lambda)/(1-\beta\lambda)$.

The role of *family or peer effects* in reducing the optimal degree of school finance equalization (as explained earlier) is also very important. The decline of τ_Y^* (and τ_E^*) with α is best illustrated by starting from a case similar to that of Fernandez and Rogerson (1998): $\alpha = 0$ (no family or peer effect), $\varepsilon = 0$ (inelastic labor), and $\beta\lambda = .20$. The optimal policy is then complete equalization ($\tau_Y^* = \tau_E^* = 1$), and brings a 3.8% gain in long run output over decentralized funding—a

[32] Since Hoxby's regression also includes property values (whose coefficient rises over time), the income elasticity is not directly comparable to previous estimates. It can be seen as a lower bound on $1-\tau$, and the sum of the two coefficients as an upper bound.

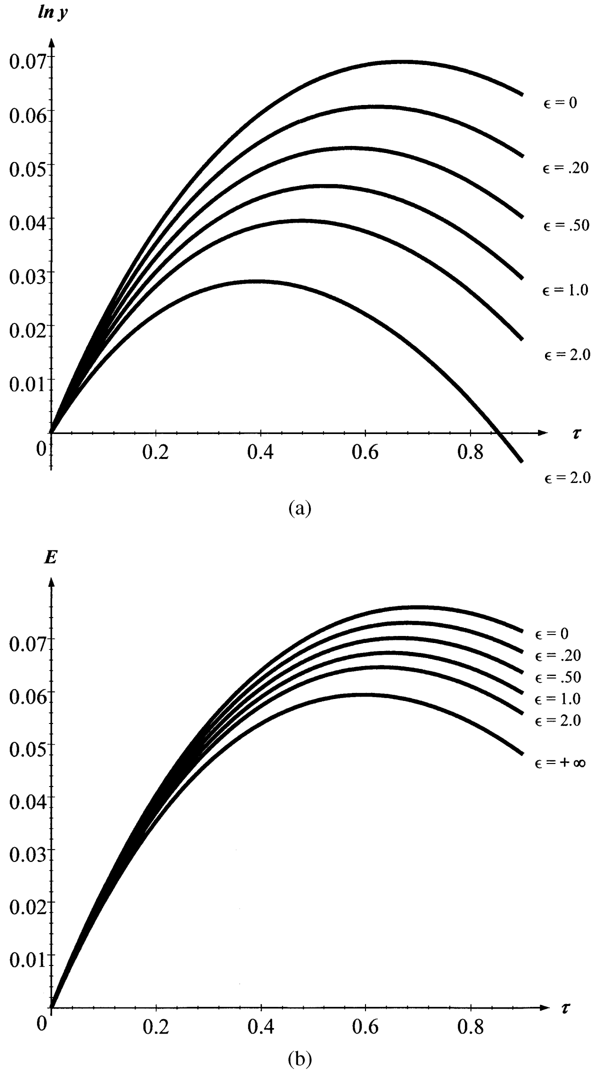


FIGURE 7.— (a) Aggregate income under redistributive education finance, for different labor supply elasticities. (b) Efficiency under redistributive education finance, for different labor supply elasticities.

figure close to Fernandez and Rogerson's 3.2%. Once we take into account the role of social background, ($\alpha = .35$), however, τ_Y^* falls significantly, to 69%.[33]

As explained earlier, *risk-aversion* plays much less of a role than under fiscal redistribution. Figure 8 shows that the efficient progressivity rate τ_E^* starts at

[33] On the other hand, long-run income gains are magnified by the higher total return, $\alpha + \beta\lambda$: 4.6% under τ_Y^*, but only 3.9% under $\tau = 1$.

TABLE II
EDUCATION FINANCE REDISTRIBUTION

Elasticity $\beta\lambda$[a]	Elasticity ε[b] .10	**.20**	.30	.50	1	∞	Optima[c]
0	70.5	68.5	66.8	64.2	59.8	46.9	τ_Y^*
	72.8	72.6	72.5	72.3	72.0	71.1	τ_Z^*
	76.2	76.1	76.0	75.8	75.4	74.5	τ_E^*
	97.8	97.7	97.5	97.4	97.0	96.1	τ_W^*
.10	69.2	67.1	65.4	62.6	58.2	45.1	τ_Y^*
	71.3	71.0	70.8	70.4	69.7	67.8	τ_Z^*
	74.7	74.5	74.2	73.8	73.2	71.3	τ_E^*
	95.6	94.3	95.1	94.7	94.0	92.1	τ_W^*
.20	66.0	63.8	62.0	59.1	54.5	41.2	τ_Y^*
	67.9	67.3	66.8	65.9	64.5	60.6	τ_Z^*
	71.2	70.6	70.0	69.2	67.8	63.8	τ_E^*
	90.9	90.3	89.7	88.9	87.6	83.8	τ_W^*
.25	64.1	**61.9**	60.0	57.0	52.3	39.2	τ_Y^*
	65.8	**65.0**	64.3	63.3	61.5	56.5	τ_Z^*
	69.1	**68.2**	67.6	66.5	64.7	59.6	τ_E^*
	88.2	**87.4**	86.8	85.8	84.1	79.3	τ_W^*
.30	61.9	59.6	57.7	54.7	50.0	37.1	τ_Y^*
	63.5	62.5	61.7	60.3	58.2	52.1	τ_Z^*
	66.6	65.6	64.7	63.4	61.2	55.1	τ_E^*
	85.3	83.3	83.5	82.3	80.3	74.2	τ_W^*
.35	59.4	57.0	55.1	52.0	47.3	34.9	τ_Y^*
	60.8	59.6	56.8	57.0	54.5	47.4	τ_Z^*
	63.8	62.5	61.5	59.9	57.3	50.2	τ_E^*
	81.9	80.8	79.9	78.4	76.0	69.2	τ_W^*
.40	56.4	54.0	52.0	48.9	44.3	32.5	τ_Y^*
	57.7	56.2	55.0	53.2	50.3	42.5	τ_Z^*
	60.4	58.9	57.7	55.8	52.9	44.9	τ_E^*
	78.0	76.7	75.6	73.8	71.1	63.4	τ_W^*

[a] $\beta\lambda$: elasticity of earnings to educational expenditures; benchmark value in bold.
[b] ε: intertemporal elasticity of substitution in labor supply; benchmark value in bold.
[c] $\tau_Y^*, \tau_Z^*, \tau_E^*, \tau_W^*$: values of τ (in %) that respectively maximize aggregate income, utility from aggregate consumption and leisure, overall efficiency, and aggregate welfare.

61.6% for $1-r=0$ but rises relatively slowly, to 68.2%, 74.9%, and 84.8% for $1-r=1, 2$, and 3 respectively. Except for very low values of risk-aversion, aggregate income still underestimates the value of redistribution, although less so than with taxes and transfers. The bias in aggregate welfare, on the contrary, is now much more severe: compare Figures 1 and 5. Of course, pure equity considerations (a positive value of $1/\sigma$) may well be more relevant in educational than in tax policy.

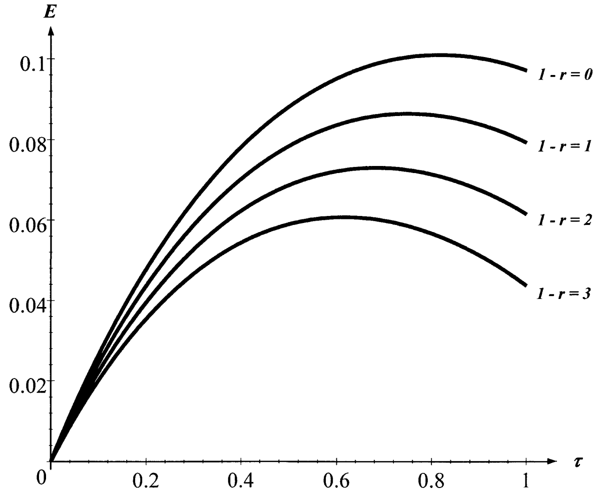


FIGURE 8.— Efficiency under redistributive education finance, for different degrees of risk aversion.

8. CONCLUSION

This paper has studied how progressive income taxation and education finance affect the level and distribution of income in a dynamic heterogeneous-agent economy. The model was first solved analytically; then quantitative policy exercises were performed. A simple combination of consumption taxes and education subsidies can help restore investment to its undistorted level. Whether or not this additional policy instrument is used, redistributive education finance always dominates taxes and transfers from the point of view of growth, but is inferior from that of insurance. Simulating the model with empirical parameter estimates leads to generally plausible results. For the benchmark specification, efficiency is maximized with transfers equal to 14% of GDP, or with a 68% equalization rate for school expenditures. In both cases the richest 30% of families end up subsidizing the education (and possibly the consumption) of the remaining 70%, whether through school finance or through the fiscal system. More generally, the efficiency costs and benefits of redistribution remain of the same order of magnitude over a wide range of parameters values, so that omitting either side can seriously bias the policy analysis. Another robust conclusion is that per capita income and average welfare provide only crude lower and upper bounds around a more proper (risk-adjusted but distribution-free) measure of overall efficiency.

The model's analytical structure has a number of advantages, which hopefully justify the strong simplifying assumptions that lie behind it. One is the transparency of the insights obtained from closed-form solutions. Another is allowing anyone to easily generate alternative policy assessments, by inserting into the formulas their preferred parameter values. Finally, the model can be extended in several interesting directions. One could thus analyze fiscal and educational policy jointly rather than separately, and look for the optimal mix that alleviates the imperfections in the credit and insurance markets with minimal distortions. Another route is to endogenize the degree of redistribution through a political mechanism. This is pursued in Bénabou (2000), which seeks to explain how countries with similar economic and political fundamentals can nonetheless choose very different fiscal and education systems.

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APPENDIX

PROOFS OF PROPOSITIONS 1, 2, AND 3: The first-order conditions for the optimal savings rate and labor supply in (9) are

$$\text{(A.1)} \qquad \frac{s_t}{1-s_t} = \frac{\rho\beta}{1-\rho} \times \frac{E_t[(U_{t+1})^r(\partial \ln U_{t+1}/\partial \ln h')]}{E_t[(U_{t+1})^r]},$$

$$\text{(A.2)} \qquad \eta(l_t)^\eta = \mu(1-\tau_t)\left[1 + \frac{\rho\beta}{1-\rho} \times \left(\frac{E_t[(U_{t+1})^r(\partial \ln U_{t+1}/\partial \ln h')]}{E_t[(U_{t+1})^r]}\right)\right],$$

where we used the fact that $\partial \ln h'/\partial s = \beta h'/s$ and $\partial \ln h'/\partial l = \beta\mu(1-\tau_t)h'/l$. Next, we guess that the value function is of the form: $\ln U_t^i = V_t \ln h_t^i + B_t$. Substituting into (9) yields

$$\begin{aligned} \text{(A.3)} \qquad V_t \ln h_t^i + B_t = {} & \rho(B_{t+1} + V_{t+1} \ln \kappa) + \max_l\{(1-\rho+\rho\beta V_{t+1})(1-\tau_t)\mu \ln l - (1-\rho)l^\eta\} \\ & + \max_s\{(1-\rho)\ln((1-s)/(1+\theta_t)) + \rho\beta V_{t+1} \ln(s(1+a_t))\} \\ & + [(1-\rho)\lambda(1-\tau_t) + \rho V_{t+1}(\alpha + \beta\lambda(1-\tau_t))] \ln h_t^i \\ & + (1-\rho+\rho\beta V_{t+1})\tau_t \ln \tilde{y}_t - (\rho/r)[rV_{t+1}(1-rV_{t+1})\omega^2/2]. \end{aligned}$$

This problem is strictly concave, so (A.1)–(A.2) are sufficient for optimality (and immediately deliver Propositions 2 and 3), provided that (A.3) does hold. This, in turn, requires that V_t satisfy (12). As to B_t, it is given as the solution to the difference equation:

$$\begin{aligned} \text{(A.4)} \qquad B_t - \rho B_{t+1} = {} & (1-\rho+\rho\beta V_{t+1})(1-\tau_t)\mu \ln l_t - (1-\rho)(l_t)^\eta + \rho V_{t+1} \ln \kappa \\ & + (1-\rho)\ln((1-s_t)/(1+\theta_t)) + \rho\beta V_{t+1} \ln(s_t(1+a_t)) \\ & + (1-\rho+\rho\beta V_{t+1})\tau_t \ln \tilde{y}_t - (\rho/r)[rV_{t+1}(1-rV_{t+1})]\omega^2/2, \end{aligned}$$

with the transversality condition $\lim_{t\to\infty}(\rho^t B_t) = 0$. To simplify further, let $\tilde{s}_t \equiv s_t(1+a_t)$ and note that since $l_t^i = l_t$ and $s_t^i = s_t$ for all i, one can rewrite (10) as

$$\text{(A.5)} \qquad \ln h_{t+1}^i = \ln \xi_{t+1}^i + \ln \kappa + \beta \ln \tilde{s}_t + \beta\mu(1-\tau_t)\ln l_t + (\alpha + \beta\lambda(1-\tau_t))\ln h_t^i + \beta\tau_t \ln \tilde{y}_t.$$

Thus h_t^i remains log-normally distributed over time. If $\ln h_t^i \sim \mathcal{N}(m_t, \Delta_t^2)$, then (7) yields for $\tilde{y}_t$ the value given by (28). Substituting into (A.5) yields (27), and a more general version of (26):

$$\text{(A.6)} \qquad m_{t+1} = (\alpha+\beta\lambda)m_t + \beta\mu \ln l_t + \beta\tau_t(2-\tau_t)\lambda^2\Delta_t^2/2 + \beta \ln \tilde{s}_t + \ln \kappa - \omega^2/2.$$

Let us now define $W_t \equiv V_t m_t + B_t$, so that $\ln U_t(h) = V_t(\ln h - m_t) + W_t$. Substituting $B_t = W_t - V_t m_t$ into (A.4), then using (A.6) and (27) to eliminate m_{t+1} and Δ_{t+1}^2, the budget constraint (17) to eliminate θ_t, and (28) to eliminate $\ln \tilde{y}_t$, we have:

$$\begin{aligned}
W_t - \rho W_{t+1} &= (1-\rho+\rho\beta V_{t+1})(1-\tau_t)\mu \ln l_t - (1-\rho)(l_t)^\eta + \rho V_{t+1} \ln \kappa \\
&\quad + (1-\rho)\ln(1-\tilde{s}_t) + \rho\beta V_{t+1} \ln \tilde{s}_t - (\rho/r)[rV_{t+1}(1-rV_{t+1})]\omega^2/2 \\
&\quad + (1-\rho+\rho\beta V_{t+1})\tau_t[\lambda m_t + \mu \ln l_t + (2-\tau_t)\lambda^2\Delta_t^2/2] + V_t m_t \\
&\quad - \rho V_{t+1}[(\alpha+\beta\lambda)m_t + \beta\mu \ln l_t + \beta\tau_t(2-\tau_t)\lambda^2\Delta_t^2/2 + \beta \ln \tilde{s}_t + \ln \kappa - \omega^2/2].
\end{aligned}$$

Grouping terms and using the recursion equation (12) to simplify the coefficient on m_t, we obtain

$$\text{(A.7)} \qquad \begin{aligned}
\frac{W_t - \rho W_{t+1}}{1-\rho} &= \mu \ln l_t - (l_t)^\eta + \ln(1-\tilde{s}_t) + \lambda m_t \\
&\quad + \tau_t(2-\tau_t)\lambda^2\Delta_t^2/2 + r\rho(1-\rho)^{-1}V_{t+1}^2\omega^2/2.
\end{aligned}$$

It remains to solve this difference equation forward to obtain W_t. First, observe that (A.6) implies

$$\begin{aligned}
&\sum_{t=0}^{\infty} \rho^t[\lambda m_t + \mu \ln l_t + \tau_t(2-\tau_t)\lambda^2\Delta_t^2/2] \\
&\quad = (1-\rho(\alpha+\beta\lambda))^{-1}\left(\lambda m_0 + \rho\lambda \sum_{t=0}^{\infty} \rho^t(\beta \ln \kappa - \omega^2/2 + \beta \ln \tilde{s}_t)\right) \\
&\qquad + (1+\rho\beta\lambda(1-\rho(\alpha+\beta\lambda))^{-1})\left(\sum_{t=0}^{\infty} \rho^t[\mu \ln l_t + \tau_t(2-\tau_t)\lambda^2\Delta_t^2/2]\right).
\end{aligned}$$

So, finally

$$\text{(A.8)} \qquad \begin{aligned}
\frac{W_t}{1-\rho} &= \frac{\lambda m_t}{1-\rho(\alpha+\beta\lambda)} + \sum_{k=0}^{\infty} \rho^k\left[\left(\frac{1-\rho\alpha}{1-\rho(\alpha+\beta\lambda)}\right)\mu \ln l_{t+k} - (l_{t+k})^\eta\right] \\
&\quad + \sum_{k=0}^{\infty} \rho^k\left[r(1-\rho)^{-1}V_{t+k+1}^2 - \frac{\lambda}{1-\rho(\alpha+\beta\lambda)}\right]\left(\frac{\rho\omega^2}{2}\right) \\
&\quad + \sum_{k=0}^{\infty} \rho^k\left[\ln(1-\tilde{s}_{t+k}) + \frac{\rho\lambda(\ln \kappa + \beta \ln \tilde{s}_{t+k})}{1-\rho(\alpha+\beta\lambda)}\right] \\
&\quad + \left(\frac{1-\rho\alpha}{1-\rho(\alpha+\beta\lambda)}\right)\sum_{k=0}^{\infty} \rho^k\left[\tau_{t+k}(2-\tau_{t+k})\frac{\lambda^2\Delta_{t+k}^2}{2}\right].
\end{aligned}$$

In particular, under a constant policy $\{\tau_t = \tau\}_{t=0}^{\infty}$ and with $\tilde{s}_{t+k} \equiv \bar{s}$, aggregate welfare simplifies to

$$\text{(A.9)} \qquad W_0 = \bar{u}_0(\tau) + (1-\rho)\Omega_\Delta(\tau)\left(\frac{\lambda^2\Delta_0^2}{2}\right) + \rho\Omega_\omega(\tau)\left(\frac{\lambda^2\omega^2}{2}\right),$$

where

$$\begin{aligned}
\bar{u}_0(\tau) &\equiv \frac{\lambda[(1-\rho)m_0 + \rho(\ln \kappa + \beta \ln \bar{s})]}{1-\rho(\alpha+\beta\lambda)} + \ln(1-\bar{s}) + \left(\frac{1-\rho\alpha}{1-\rho(\alpha+\beta\lambda)}\right)\mu \ln l(\tau) - l(\tau)^\eta, \\
\Omega_\Delta(\tau) &\equiv \left(\frac{1-\rho\alpha}{1-\rho(\alpha+\beta\lambda)}\right)\left(\frac{\tau(2-\tau)}{1-\rho p(\tau)^2}\right), \\
\Omega_\omega(\tau) &\equiv \Omega_\Delta(\tau) - \frac{1/\lambda}{1-\rho(\alpha+\beta\lambda)} + r(1-\rho)\left(\frac{1-\tau}{1-\rho p(\tau)}\right)^2.
\end{aligned}$$

In steady-state it simplifies further, as Δ_0^2 becomes equal to $\Delta_\infty^2(\tau) = \omega^2/(1-p(\tau)^2)$.

PROOF OF PROPOSITION 4: Given the budget constraint (17), choosing $\{\theta_{t+k}, a_{t+k}\}_{k=0}^{\infty}$ is equivalent to choosing $\{\tilde{s}_{t+k} \equiv s_{t+k}(1+a_{t+k})\}_{k=0}^{\infty}$. Since agent i's utility is $\ln U_t^i = V_t(\ln h_t^i - m_t) + W_t$ and V_t is independent of $\{\tilde{s}_{t+k}\}_{t=0}^{\infty}$, all agree on the optimal path, namely the one that maximizes aggregate welfare W_t. By (A.8), this sequence is given by

$$\frac{1-\rho(\alpha+\beta\lambda)}{1-\tilde{s}_{t+k}} = \frac{\rho\beta\lambda}{\tilde{s}_{t+k}}, \quad \text{or} \quad \tilde{s}_{t+k} = \frac{\rho\beta\lambda}{1-\rho\alpha} = \bar{s}.$$

A similar result is easily established for the case of education finance developed in Section 3.

PROOF OF PROPOSITION 8: First, observe that (37) follows directly from (A.7) and (25), given the definition of $\bar{c}_t^i$ in (36). Next, we solve (A.7) forward from $t=0$ to obtain

$$W_0/(1-\rho) = \sum_{t=0}^{\infty} \rho^t [\ln y_t - (1-\tau_t)^2 \lambda^2 \Delta_t^2/2 - (l_t)^\eta + \ln(1-\tilde{s}_t) + r\rho(1-\rho)^{-1} V_{t+1}^2 \omega^2/2].$$

Substituting into (38) yields

$$\mathscr{E}_0/(1-\rho) = \sum_{t=0}^{\infty} \rho^t [\ln y_t - (l_t)^\eta + \ln(1-\tilde{s}_t)]$$
$$+ \sum_{t=0}^{\infty} \rho^t (1-\tau_t)^2 \lambda^2 \left[r\rho(1-\rho) \left(\frac{1}{1-\rho p(\tau_t)} \right)^2 \left(\frac{\omega^2}{2} \right) - \frac{\Delta_t^2}{2} + \left(\prod_{k=0}^{t-1} p(\tau_k)^2 \right) \frac{\Delta_0^2}{2} \right].$$

To simplify this expression, note first that (27) implies

$$\Delta_t^2 = \left(\prod_{k=0}^{t-1} p(\tau_k)^2 \right) \Delta_0^2 + \left(\sum_{k=1}^{t} \prod_{j=k}^{t-1} p(\tau_j)^2 \right) \omega^2.$$

Second, by Proposition 4 we have $\tilde{s}_t = \bar{s}$ when θ_t and a_t are set optimally (when they are not, one just keeps $\tilde{s}_t$ in the formula). Thus

$$\text{(A.10)} \quad \frac{\mathscr{E}_0(\tau)}{1-\rho} = \sum_{t=0}^{\infty} \rho^t [\ln y_t - (l_t)^\eta] + \ln(1-\bar{s}) + \sum_{t=0}^{\infty} \rho^t (1-\tau_t)^2 \left(\frac{r\rho(1-\rho)}{(1-\rho p(\tau_t))^2} - \sum_{k=1}^{t} \prod_{j=k}^{t-1} p(\tau_j)^2 \right) \left(\frac{\lambda^2 \omega^2}{2} \right).$$

With a constant τ the last present value, times $1-\rho$, becomes $-\rho(1-\tau)^2 \varphi(\tau, r) \lambda^2 \omega^2/2$, where

$$\varphi(\tau, r) \equiv \frac{1}{1-\rho p(\tau)^2} - \frac{r(1-\rho)}{(1-\rho p(\tau))^2} \geq \varphi(\tau, 1) = \frac{\rho(1+p(\tau)^2 - 2p(\tau))}{(1-\rho p(\tau)^2)(1-\rho p(\tau))^2} \geq 0,$$

with strict inequality for all $p(\tau) < 1$. This establishes part (a) of the proposition, together with the fact that the risk premium is always positive and minimized at $\tau = 1$.

We now prove part (b) of the proposition. From (A.9) and (38), we have

$$\text{(A.11)} \qquad \mathscr{E}_0 = \bar{u}_0(\tau) + (1-\rho) \left(\Omega_\Delta(\tau) + \frac{(1-\tau)^2}{1-\rho p(\tau)^2} \right) \left(\frac{\lambda^2 \Delta_0^2}{2} \right) + \rho \Omega_\omega(\tau) \left(\frac{\lambda^2 \omega^2}{2} \right).$$

It is easily verified from (14) and (16) that $\bar{u}_0'(0) = 0$; this holds whether the investment rate is $\bar{s}$ or $\tilde{s}(\tau)$. Next, straightforward but somewhat tedious derivations show that $\Omega_\Delta'(0) + (\partial/\partial\tau)_{\tau=0}[(1-\tau)^2/(1-\rho p(\tau)^2)] \geq 0$, with strict inequality unless $\rho\beta\lambda = 1$; and that $\Omega_\omega'(0) \geq 0$, with strict inequality unless $\alpha + \beta\lambda = 1$. Therefore $\mathscr{E}_0'(0) > 0$; hence $\tau_{E,0}^* > 0$.

PROOFS OF PROPOSITIONS 5 AND 6: The first-order condition for the savings rate in (20) is unchanged, that is, still given by equation (A.1). For labor supply, it becomes

$$\text{(A.12)} \qquad \eta(l_t)^\eta = \mu \left[1 + \rho\beta(1-\tau_t) \left(\frac{E_t[(U_{t+1})^r (\partial \ln U_{t+1}/\partial \ln h')]}{E_t[(U_{t+1})^r]} \right) \right].$$

We guess once again that U_t^i takes the form: $\ln U_t^i = V_t \ln h_t^i + B_t$. Substituting into (20) yields

$$\text{(A.13)} \qquad \begin{aligned} V_t \ln h_t^i + B_t &= \max_l \{(1-\rho+\rho\beta(1-\tau_t)V_{t+1})\mu \ln l - (1-\rho)l^\eta\} \\ &\quad + \max_s \{(1-\rho)\ln((1-s)/(1+\theta_t)) + \rho\beta V_{t+1}\ln(s(1+a_t))\} \\ &\quad + [(1-\rho)\lambda + \rho V_{t+1}(\alpha+\beta\lambda(1-\tau_t))]\ln h_t^i + (\rho\beta V_{t+1})\tau_t \ln \tilde{y}_t \\ &\quad - (\rho/r)[rV_{t+1}(1-rV_{t+1})\omega^2/2] + \rho(B_{t+1} + V_{t+1}\ln\kappa). \end{aligned}$$

If this holds, (A.1)–(A.12) are again sufficient and yield Proposition 6. In turn, (A.13) requires that (21) hold, while B_t is given as the solution to the difference equation

$$\text{(A.14)} \qquad \begin{aligned} B_t - \rho B_{t+1} &= (1-\rho+\rho\beta(1-\tau_t)V_{t+1})\mu \ln l_t - (1-\rho)(l_t)^\eta + \rho V_{t+1}\ln\kappa \\ &\quad + (1-\rho)\ln((1-s_t)/(1+\theta_t)) + \rho\beta V_{t+1}\ln(s_t(1+a_t)) \\ &\quad + (\rho\beta V_{t+1})\tau_t \ln\tilde{y}_t - (\rho/r)[rV_{t+1}(1-rV_{t+1})]\omega^2/2, \end{aligned}$$

with $\lim_{t\to\infty}(\rho^t B_t) = 0$. The transition equation for $\ln h_t^i$, the formula for $\ln \tilde{y}_t$, and the dynamics of (m_t, Δ_t^2) remain unchanged from (25), (28), and (A.6)–(27). Defining again $W_t \equiv V_t m_t + B_t$, so that $\ln U_t(h) = V_t(\ln h - m_t) + W_t$, then substituting θ_t from (17) and $\ln \tilde{y}_t$ from (28), we obtain

$$\begin{aligned} W_t - \rho W_{t+1} &= (1-\rho+\rho\beta(1-\tau_t)V_{t+1})\mu \ln l_t - (1-\rho)(l_t)^\eta + \rho V_{t+1}\ln\kappa \\ &\quad + (1-\rho)\ln(1-\tilde{s}_t) + \rho\beta V_{t+1}\ln\tilde{s}_t - (\rho/r)[rV_{t+1}(1-rV_{t+1})]\omega^2/2 \\ &\quad + (\rho\beta V_{t+1})\tau_t[\lambda m_t + \mu\ln l_t + (2-\tau_t)\lambda^2\Delta_t^2/2] + V_t m_t \\ &\quad - \rho V_{t+1}[(\alpha+\beta\lambda)m_t + \beta\mu\ln l_t + \beta\tau_t(2-\tau_t)\lambda^2\Delta_t^2/2 + \beta\ln\tilde{s}_t + \ln\kappa - \omega^2/2]. \end{aligned}$$

Regrouping terms and using (21) to simplify the coefficient on m_t yields

$$\text{(A.15)} \qquad \frac{W_t - \rho W_{t+1}}{1-\rho} = \lambda m_t + \mu \ln l_t - (l_t)^\eta + \ln(1-\tilde{s}_t) + r\rho(1-\rho)^{-1}V_{t+1}^2\omega^2/2.$$

It just remains to solve this difference equation forward to obtain W_t. Now, (A.6) implies

$$\begin{aligned} \sum_{t=0}^{\infty}\rho^t(\lambda m_t + \mu\ln l_t) &= (1+\rho\beta\lambda(1-\rho(\alpha+\beta\lambda))^{-1})\left(\sum_{t=0}^{\infty}\rho^t\mu\ln l_t\right) + (1-\rho(\alpha+\beta\lambda))^{-1} \\ &\quad \times \left(\lambda m_0 + \rho\lambda\sum_{t=0}^{\infty}\rho^t[\ln\kappa - \omega^2/2 + \beta\ln\tilde{s}_t + \beta\tau_t(2-\tau_t)\lambda^2\Delta_t^2/2]\right). \end{aligned}$$

So, ultimately,

$$\text{(A.16)} \qquad \begin{aligned} \frac{W_t}{1-\rho} &= \frac{\lambda m_t}{1-\rho(\alpha+\beta\lambda)} + \sum_{k=0}^{\infty}\rho^k\left[\left(\frac{1-\rho\alpha}{1-\rho(\alpha+\beta\lambda)}\right)\mu\ln l_{t+k} - (l_{t+k})^\eta\right] \\ &\quad + \sum_{k=0}^{\infty}\rho^k\left[r(1-\rho)^{-1}V_{t+k+1}^2 - \frac{\lambda}{1-\rho(\alpha+\beta\lambda)}\right]\left(\frac{\rho\omega^2}{2}\right) \\ &\quad + \sum_{k=0}^{\infty}\rho^k\left(\ln(1-\tilde{s}_{t+k}) + \frac{\rho\lambda(\ln\kappa + \beta\ln\tilde{s}_{t+k})}{1-\rho(\alpha+\beta\lambda)}\right) \\ &\quad + \left(\frac{\rho\beta\lambda}{1-\rho(\alpha+\beta\lambda)}\right)\sum_{k=0}^{\infty}\rho^k\left[\tau_{t+k}(2-\tau_{t+k})\frac{\lambda^2\Delta_{t+k}^2}{2}\right]. \end{aligned}$$

PROOF OF PROPOSITION 9: The log-normality of the $\bar{c}_t^i$'s in (36) makes it again straightforward to compute $\ln \overline{C}_{t,\sigma} = \int_0^1 \ln \bar{c}_t^i \, di + (1 - 1/\sigma) \operatorname{var}[\ln \bar{c}_t^i]/2$, and hence $\mathscr{E}_{0,\sigma}$. As to $\mathscr{U}_{0,\sigma}$, it is easily obtained by recalling that $\ln U_t^i = V_t(\ln h_0^i - m_0) + W_0$. Thus,

$$\text{(A.17)} \qquad \mathscr{E}_{0,\sigma} = W_0 + \left(\frac{\sigma-1}{\sigma}\right)\left((1-\rho)\sum_{t=0}^{\infty}\rho^t(1-\tau_t)^2\prod_{k=0}^{t-1}p(\tau_k)^2\right)\left(\frac{\lambda^2\Delta_0^2}{2}\right),$$

$$\text{(A.18)} \qquad \mathscr{U}_{0,\sigma} = W_0 + \left(\frac{\sigma-1}{\sigma}\right)\left((1-\rho)\sum_{t=0}^{\infty}\rho^t(1-\tau_t)\prod_{k=0}^{t-1}p(\tau_k)\right)^2\left(\frac{\lambda^2\Delta_0^2}{2}\right).$$

Since $\mathscr{E}_{0,\infty} = \mathscr{E}_0$, we have the following decompositions: $\mathscr{E}_{0,\sigma} = \mathscr{E}_0 - (\mathscr{E}_0 - W_0)/\sigma$, and $\mathscr{U}_{0,\sigma} = \mathscr{E}_0 - (\mathscr{U}_{0,\infty} - W_0)/\sigma - (\mathscr{E}_{0,\infty} - \mathscr{U}_{0,\infty})$. In both cases the second term is proportional to $\lambda^2\Delta_0^2/2\sigma$ and increasing in all τ_t's. The third component of $\mathscr{U}_{0,\sigma}$, proportional to $\lambda^2\Delta_0^2/2$, represents an additional welfare cost of inequality, which is always positive unless $\tau_t \equiv 1$ for all t. Indeed:

$$\frac{\mathscr{E}_{0,\infty} - \mathscr{U}_{0,\infty}}{\lambda^2\Delta_0^2/2} = \left((1-\rho)\sum_{t=0}^{\infty}\rho^t(1-\tau_t)^2\prod_{k=0}^{t-1}p(\tau_k)^2\right) - \left((1-\rho)\sum_{t=0}^{\infty}\rho^t(1-\tau_t)\prod_{k=0}^{t-1}p(\tau_k)\right)^2 \geq 0,$$

since

$$(1-\rho)\left(\sum_{t=0}^{\infty}\rho^{2t/2}x_t\right)^2 \leq (1-\rho)\left(\sum_{t=0}^{\infty}\rho^t\right)\left(\sum_{t=0}^{\infty}\rho^t x_t^2\right) = \left(\sum_{t=0}^{\infty}\rho^t x_t^2\right) \quad \text{for all} \quad x_t \geq 0,$$

by Schwartz's inequality. Setting $\tau_t \equiv \tau$ in these results yields Proposition 9 as a special case.

PROOF OF PROPOSITION 10: By Proposition 5, the intertemporal utility of an agent i is

$$\ln U_0^i = (1-\rho)\lambda\left(\sum_{t=0}^{\infty}\rho^t\prod_{k=0}^{t-1}p(\tau_k)\right)(\ln h_0^i - m_0) + \sum_{t=0}^{\infty}\rho^t(\ln W_t - \rho\ln W_{t+1}),$$

which is identical to (35), but without the $1-\tau_t$ factors in each period t. Consequently, the certainty-equivalent consumption sequence $\{\bar{c}_t^i\}_{t=0}^{\infty}$ is now defined just as in (36) but with the same modification. Given (A.7) and (25), this implies once again that $\ln \bar{c}_t^i$ satisfies (37). Summing up the lognormally distributed $\bar{c}_t^i$'s to obtain $\overline{C}_t \equiv \int_0^1 \bar{c}_t^i \, di$ yields for $\mathscr{E}_0 \equiv (1-\rho)\sum_{t=0}^{\infty}\rho^t[\ln \overline{C}_t - (l_t)^\eta]$ the same expression as (38), but with all the $(1-\tau_t)^2$'s replaced by 1. Finally, applying (A.16) to compute W_0, we obtain the claimed analogue to Proposition 8. Similar derivations can be carried out for any welfare index $\mathscr{E}_{0,\sigma}$ or $\mathscr{U}_{0,\sigma}$; hence the stated results.

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