Idiosyncratic Production Risk, Growth and the Business Cycle - Journal of Monetary Economics - 2006

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1. Introduction

Two strands of literature:

- Following Bewley (1977), an extensive literature has investigated the macroeconomic impact of labor-income risk in the neoclassical growth model. In such settings, incomplete markets lead to a lower interest rate and a higher capital stock in the steady state and tend to have small effects on business-cycle dynamics.
- The development literature, on the other hand, has proposed that financial innovation promotes productivity and growth by helping the reallocation of savings to more productive activities.

This paper builds a bridge between the two literatures by investigating how idiosyncratic production risk affects the steady state and the transitional dynamics of a neoclassical growth economy. We follow the Bewley paradigm by assuming diminishing returns to capital accumulation and allowing a credit market, but extend the standard framework by introducing idiosyncratic risk in private production and investment. The economy is populated by a large number of infinitely-lived agents, or entrepreneurs, each of whom operates her own neoclassical technology with her own labor and capital. Production is subject to firm-specific uncertainty, which generates idiosyncratic risk in entrepreneurial income and investment returns.

Incomplete markets generally imply that aggregate dynamics depend on the wealth distribution. We overcome this "curse of dimensionality" by adopting a CARA-normal specification for preferences and risks, which ensures that risk-taking and therefore investment are independent of wealth. We are thus able to characterize the general equilibrium by a tractable closed-form recursion, which, to be the best of our knowledge, is a methodological innovation.

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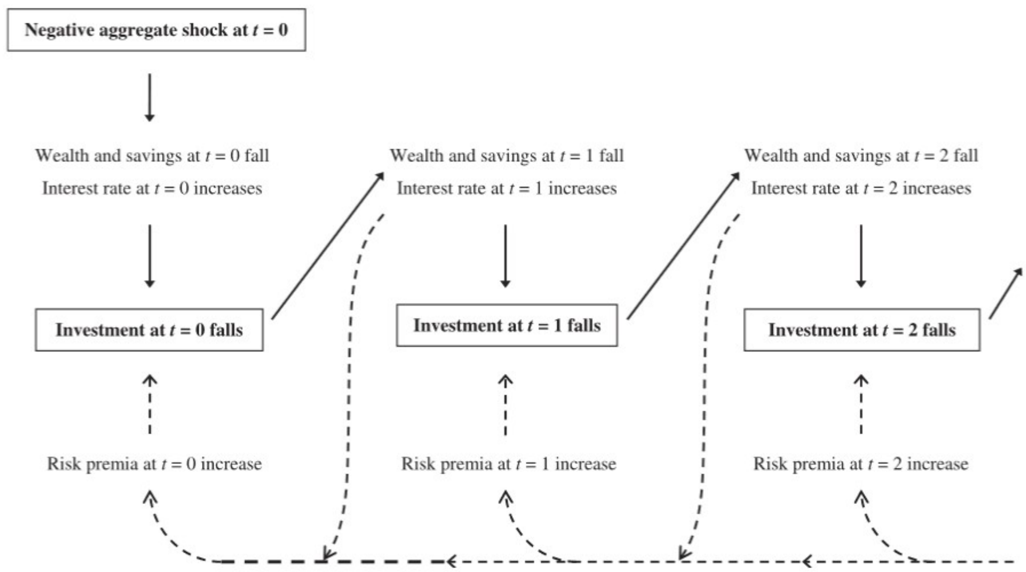

Fig. 1. The figure illustrates how countercyclicality in private risk premia introduces amplification and persistence in the business cycle. The solid arrows represent propagation of the shock under complete markets, whereas the dashed arrows represent the additional feedback through risk premia.

Idiosyncratic production risks introduce a risk premium on private capital, which reduces the demand for investment at any given interest rate. Uninsurable income risks also encourage the precautionary supply of savings, implying a lower interest rate as compared to complete markets. The overall effect of incomplete markets on capital accumulation is therefore ambiguous in general. Nevertheless, the reduction in investment demand dominates the reduction in the interest rate unless the interest elasticity of savings is sufficiently low, thus resulting in under-accumulation of capital in the steady state as compared to complete markets. Hence, improvements in entrepreneurial risk sharing induced, for instance, by financial liberalization or the introduction of new hedging instruments-are likely to have a positive effect on savings and medium run growth. This result holds even though there is no margin of reallocation towards more productive activities. Our findings thus extend and reinforce the findings of the development literature in the context of the neoclassical growth model.

Our framework also allows us to derive novel implications for the transitional dynamics. The expectation of high real interest rates in the near future implies a low willingness to take risk in the present and therefore discourages current investment. This feedback slows down convergence to the steady state and generates a dynamic macroeconomic complementarity that may increase the persistence and magnitude of the business cycle.

Figure 1 illustrates the transitional dynamics in an economy hit by an unanticipated negative shock to aggregate wealth. The solid lines represent transmission under complete markets. Agents smooth consumption by reducing current investment, which results in low wealth, low savings and high real interest rates in later periods. This is the fundamental propagation mechanism in the neoclassical growth paradigm. In the presence of idiosyncratic production uncertainty, the traditional channel is complemented by the endogenous countercyclicality of private risk premia, as illustrated by the dashed arrows in the figure. Anticipating higher real interest rates and thus larger self-insurance costs in the near future, agents become less willing to take risk in the present, which further reduces investment and hinders the recovery. This mechanism is shown to increase persistence in the transitional dynamics. More generally, we expect that macroeconomic fluctuations are further amplified by additional sources of countercyclicality in private premia, such as business-cycle variation in firm-specific volatility or risk-sharing opportunities.

The effeccts exhibited in this paper originate in the non-marketability of idiosyncratic risks. Our approach thus complements, but also differs from, the literature examining the effects of credit imperfections on entrepreneurial activity (e.g.,). This earlier research has focused on the effect of wealth and borrowing constraints on the individual ability to invest. We show that incomplete markets also affect the willingness to invest, which has novel implications for capital accumulation and the business cycle. We illustrate these effects in their sharpest version by using a model where agents face no borrowing constraints, individual investment does not depend on own wealth, and wealth heterogeneity has no impact on aggregate dynamics.

2. A Ramsey Economy with Idiosyncratic Production Risk

Time is discrete and infinite, indexed by $t \in \{0;1,...\}$. The economy is populated by a continuum of agents, indexed by $j \in [0,1]$, who are born at $t = 0$ and live forever. Each individual is an "entrepreneur" who operates her own production scheme using her own labor and capital.

2.1. Technology and Idiosyncratic Risks

The gross output of entrepreneur j at time t is given by $A_t^j f(k_t^j)$, where k_t^j is her capital stock at the beginning of the period., A_t^j is her random total factor productivity (TFP), and f is a neoclassical production function. The function f is common across households. The individual controls k_t^j through her investment choice at date $t-1$, but only observes the idiosyncratic productivity A_t^j at date t. The return on investment is thus subject to idiosyncratic uncertainty.

For comparison with production risk, we find it useful to also introduce endowment risk. The individual receives an exogenous idiosyncratic income e_t^j, which does not affect investment or production opportunities. The overall non-financial income of household j is

$$y_t^j = A_t^j f\left(k_t^j\right) + e_t^j. \tag{1}$$

Variation in A_t^j capture idiosyncratic entrepreneurial or production risk, whereas variation in e_t^j captures endowment risk.

We assume that idiosyncratic production and endowment risks are Gaussian, mutually independent, and iid across time and individuals:

$$A_t^j \sim \mathcal{N}\left(1, \sigma_A^2\right) \quad \text{and} \quad e_t^j \sim \mathcal{N}\left(0, \sigma_e^2\right).$$

The averages $\mathbb{E} A_t^j = 1$ and $\mathbb{E} e_t^j = 0$ are simple normalizations. The standard deviations σ_A and σ_e parsimoniously parameterize the magnitude of the uninsurable production and endowment shocks. Under complete markets, σ_A and σ_e are both equal to zero. Traditional Bewley economies only consider idiosyncratic labor-income risk, which corresponds to $\sigma_e > 0$ but $\sigma_A = 0$.

2.2. Financial Markets

Agents can buy and sell a riskless short-term bond. One unit of the bond purchased at date t yields $1 + r_t$ units of the good with certainty at date $t + 1$. In equilibrium, the interest rate r_t clears the period-t bond market. We rule out default, borrowing constraints, and any other credit-market imperfections. Without loss of generality, the riskless bond is in zero net supply [1].

Let c_t^j, i_t^j, and θ_t^j denote the consumption, capital investment, and bond purchases of agent j in period t. The budget constraint in period t is given by

$$c_t^j + i_t^j + \theta_t^j = y_t^j + (1 + r_{t-1})\,\theta_{t-1}^j, \tag{2}$$

where y_t^j is the non-financial income defined in (1). The law of capital accumulation, on the other hand, is

$$k_{t+1}^j = (1 - \delta)\,k_t^j + i_t^j, \tag{3}$$

where $\delta \in [0, 1]$ is the depreciation rate. To simplify notation, we combine (2) and (3) and conveniently rewrite the budget set in terms of stock variables:

$$c_t^j + k_{t+1}^j + \theta_t^j = w_t^j, \tag{4}$$

where

$$w_t^j \equiv A_t^j f(k_t) + (1 - \delta)\,k_t^j + e_t^j + (1 + r_{t-1})\,\theta_{t-1}^j \tag{5}$$

represents the agent's total wealth at date t.

2.3. Preferences

The model is most tractable when agents have exponential expected utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$, where $u(c) = -\Psi \exp(-c/\Psi)$. It is useful for the analysis, however, to distinguish between intertemporal substitution and risk aversion. We thus assume more generally that agents have preferences of the Kreps-Porteus/Epstein-Zin type. For every stochastic consumption stream $\{c_t\}_{t=0}^{\infty}$, the utility stream is recursively defined by

$$u_t = u(c_t) + \beta u\left(\mathbb{CE}_t\left[u^{-1}(u_{t+1})\right]\right), \tag{6}$$

where

$$u(c) = -\Psi \exp(-c/\Psi), \; v(c) = -\exp(-\Gamma c)/\Gamma,$$

[1] Ricardian equivalence holds in our model because agents have infinite horizons and can freely trade the riskless bond. Therefore, as long as public debt is financed by lump-sum taxation and the economy is closed, there is no loss of generality in assuming zero net supply for the riskless bond.

and

$$\mathbb{CE}_t \equiv v^{-1}\left[\mathbb{E}_t v(x_{t+1})\right]$$

is the certainty equivalent of x_{t+1} conditional on period t information. A high Ψ corresponds to a strong willingness to substitute consumption through time, while a high Γ implies a high degree of risk aversion. When $\Gamma = 1/\Psi$, the functions u and v coincide and the preference structure (6) reduces to standard expected utility, $u_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$.

The CARA specification entails both costs and benefits. It implies, for instance, that private productive investment is independent of individual wealth; our framework thus cannot be used to analyze the interaction between wealth inequality and aggregate capital formation. The CARA-normal specification, however, is probably not essential for the main insights of the paper and brings the benefits of tractability. As will be shown in the next section, the cross-sectional distribution of wealth–an infinite dimensional object–will not be a relevant state variable for aggregate dynamics, and general equilibrium will be characterized in closed form.

2.4. Equilibrium

Idiosyncratic risks are independent in the population and cancel out in the aggregate. We thus consider equilibria in which the aggregate dynamics are deterministic.

Definition 1. An incomplete-market equilibrium is a deterministic interest rate sequence $\{r_t\}_{t=0}^{\infty}$ and a collection of contingent plans $\left(\left\{c_t^j, k_{t+1}^j, \theta_t^j\right\}_{t=0}^{\infty}\right)_{j\in[0,1]}$ such that: (i) the plan $\left(\left\{c_t^j, k_{t+1}^j, \theta_t^j\right\}_{t=0}^{\infty}\right)$ maximizes the utility of each agent j; and (ii) the bond market clears in every date an event.

3. Decision Theory

In this section we characterize the solution of the individual decision problem. To simplify the exposition, we focus on the special case of expected utility ($\Psi = 1/\Gamma$). We also drop the agent-specific index j from all decision variables.

3.1. Optimal Savings and Investment

Given a deterministic interest rate sequence $\{r_t\}_{t=0}^{\infty}$, the household chooses a contingent plan $\{c_t, k_{t+1}, \theta_t\}_{t=0}^{\infty}$ that maximizes expected lifetime utility subject to (4). Since idiosyncratic risks are uncorrelated over time, individual wealth w_t fully characterizes the state of the household in period t. The value function $V_t(w)$ satisfies the Bellman equation:

$$V_t(w_t) = \max_{c_t, k_{t+1}, \theta_t} \left\{u(c_t) + \beta \mathbb{E}_t\left[V_{t+1}(w_{t+1})\right]\right\},$$

where the maximization is subject to the budget constraint (4). Given teh CARA-normal specification, an educated guess is to consider an exponential value function and a linear consumption rule:

$$V_t(w) = u(a_t w + b_t) \quad \text{and} \quad c_t = \widehat{a}_t w + \widehat{b}_t, \tag{7}$$

where $a_t, \widehat{a}_t \in \mathbb{R}_+$ and $b_t, \widehat{b}_t \in \mathbb{R}$ are non-random coefficients to be determined.

By (5), individual wealth is Gaussion with conditional mean

$$\mathbb{E}_t\left[w_{t+1}\right] = f(k_{t+1}) + (1-\delta)\, k_{t+1} + (1+r_t)\,\theta_t,$$

and variance $\mathrm{var}_t\, w_{t+1} = \sigma_e^2 + \left[f(k_{t+1})\right]^2 \sigma_A^2$. The value function thus satisfies

$$\mathbb{E}_t V_{t+1}(w_{t+1}) = V_{t+1}\left(\mathbb{E}_t w_{t+1} - \frac{\Gamma_t}{2}\mathrm{Var}_t\, w_{t+1}\right),$$

where $\Gamma_t \equiv \Gamma a_{t+1}$ measures absolute risk aversion in period t with respect to wealth variation in period $t+1$. We henceforth call Γ_t the effective risk aversion at date t. We will later see that endogenous variations in Γ_t can generate persistence in the transitional dynamics.