Occupation Mobility, Human Capital and the Aggregate Consequences of Task-Biased Innovations, Working Paper, 2019

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1. Introduction

In this paper, we use a dynamic general equilibrium model with occupation mobility and endogenous assignment of workers to tasks to quantitatively assess the impact of automation and other task-biased technological innovations. We extend recent quantitative general equilibrium Roy models to a setting with dynamic occupational choices and human capital accumulation. Analytically and quantitatively, we show that these are crucial aspects for the determination of a country's earnings distribution and level of aggregate output, as well as the responses to economy-wide technological advances. In our model, workers have long stochastic lifetimes, are forward-looking and can switch occupations in any period during their participation in labor markets. Workers' occupation mobility choices maximize their expected lifetime utility, and, along with idiosyncratic occupation-specific productivity shocks and the costs of switching occupations, drive their accumulation of human capital over the life cycle. With respect to the demand for labor, the production of final goods is the result of assigning different types of workers and machines to tasks generated a (nested) CES aggregate production function that exhibits cross-occupation differences in productivity levels and the complementarity or substitutability between machines and workers.

It is extremely challenging to solve for a dynamic general equilibrium Roy model with a nontrivial number of occupations and human capital that endogenously evolves with the workers' occupational choices. In this vein, the paper has a number of expansive methodological contributions. First, we extend the static models used in recent general equilibrium quantitative papers to a recursive setting and fully characterize the solution for the problem of a worker with labor market opportunity shocks that every period may affect his comparative advantage across occupations. We use standard dynamic preferences with constant relative risk-aversion preferences, bridging recent quantitative assignment Roy models with the standard dynamic household model for households used in macroeconomics. Interestingly, by combining CRRA preferences and Frechet distributed labor market opportunities, we show that the resulting distribution for the continuation values in the

[*]This note is written in my MPhil period at the University of Oxford.

Bellman equation of workers follows one of the three extreme value distributions, Frechet, Gumbel, or Weibull, depending on whether the coefficient of relative risk aversion is lower than, equal to, or higher than 1, respectively. In all these cases, conditions for existence and uniqueness are provided and the simple recursion formulas that come out from the Bellman equation make for trivial computations. In doing so, the dynamic problem of workers generates occupation mobility probabilities, not just unconditional occupation choices.

Second, we fully characterize the limiting behavior for the employment and human capital of workers as implied by their dynamic occupation choices. From the worker's individual problems, we derive the law of motion for the employment shares of workers across occupations. Associated with any positive vector of skill prices, we show that there exists a unique invariant distribution of workers. For aggregate human capital, we also show that a simple aggregation property holds, which allows us to write down the transition matrix for the vector of aggregate human capital across occupations. We show that the human capital for each of the cohorts inside a country does not settle down to an invariant state. Instead, each cohort's average grows over time. Using the fact that the dominant eigenvalue of the aggregate human capital transition matrix is always unique, positive, and real, we derive simple formulas for the aggregate human capital of the country as a whole. Thus, our dynamic Roy model explicitly uncovers the simple mechanics by which the life-cycle gradient of earnings of workers is determined by occupation mobility and how it affects the human capital of a country and its long run income inequality within and across cohorts of workers.

Third, by embedding our dynamic Roy model in a fairly rich general equilibrium environment, we develop a framework in which different tasks across occupations are endogenously allocated to workers or machines. In the model, there are two forms of physical capital. The first is the traditional one in neoclassical models, and hence complementary to all workers. The second capital is in the form of "machines" which compete with–and substitute for–workers. In our setting, output is produced by performing a large set of tasks which are assigned to either workers or machines depending on their relative productivity and relative costs. The productivities of workers relative to their market price and how they compare with those of machines determine the set of tasks they perform within each occupation. From the equilibrium tasks-workers assignment we show that a transparent and tractable nested CES aggregate production function emerges. Importantly, the output labor-share of the economy is an endogenous function of the wages of workers in different occupations, the rental rates of capital, and the labor and machine productivity terms.

Fourth, we show the existence of a competitive-equilibrium balanced-growth path for the production economy. Fifth, we extend recent dynamic-hat-algebra methods to models with general CRRA preferences and with human capital accumulation. The advantage of this approach is to substantially reduce the set of parameters needed to be calibrated for the quantitative application of the model. Seventh, we discuss multiple relevant extensions of our baseline model.

2. A Canonical Worker's Problem

We consider an infinite horizon maximization problem for a worker with standard preferences. At any time $t = 0, 1, 2, \ldots$, the utility of the worker is given by

$$U_t = \frac{(c_t)^{1-\gamma}}{1-\gamma} + E\left[\sum_{s=1}^{\infty} \beta^s \frac{(c_{t+s})^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

where $0 < \gamma < 1$ is a discount factor (which accounts for a constant survival probability) and $\gamma \geq 0$ is the coefficient of relative risk aversion.

2.1. Dynamic Roy Models: Challenges

The worker starts each period $t = 1, 2, \ldots$ attached to one of $j = 1, \ldots, J$ occupations, carrying over from the previous period a vector of human capital x of size J which describes the efficiency units of labor of this worker in each of the J occupations. To set up our framework, we first specify the general dynamic problem of the worker given a time-invariant vector of strictly positive (and finite) wages per unit of human capital $w = \left[w^1, w^2, \ldots, w^J\right]$. Therefore, the worker's earnings for the period given her current occupation j are $w^j x_t^j$.

In our model, workers are dynamic optimizers, with their human capital returns as their sole source of income in every period. The worker's consumption in each period is simply the current earnings $w^j x_t^j$. For simplicity, we assume that the evolution of human capital depends on worker's occupational choices, the current level of human capital in all occupations and some random idiosyncratic forces. This minimal set of assumptions allows us to write the problem recursively in the following way,

$$V\left(j, x_1, x_2, \ldots, x_J\right) = \frac{\left(w^j x_j\right)^{1-\gamma}}{1-\gamma} + \beta \max_{\ell} \left\{E\left[V\left(\ell, x_1', x_2', \ldots, x_J'\right) \mid j, x_1, x_2, \ldots, x_J\right]\right\}. \tag{2}$$

This dynamic Roy problem is quite general. It only assumes a first order Markovian dependence of human capital. However, solving for a dynamic general equilibrium model with this level of generality and for a non-trivial number of occupations is extremely challenging. On the one hand, the problem of the worker has the whole vector of human capital as a state variable. With a medium to large number of occupations, this is a high-dimensional object, rendering the problem intractable. On the other hand, solving for the general equilibrium requires an aggregation of individual labor supplies for all markets, which, in a non-stationary environment, requires also the characterization of the dynamic evolution of the aggregate labor supplies in all occupations.

2.2. Main Assumptions

Additional assumptions are needed for a tractable dynamic problem that can handle a large number of occupations and make the model suitable for aggregation and general equilibrium. Our assumptions closely connect to recent works on static Roy models with Frechet distributed shocks, extending this literature to a dynamic general equilibrium context.

The two key aspects in a worker's vector of skills, $x \in \mathbb{R}^J$, are his absolute advantage and the comparative advantage across occupations. The first one is given by the magnitude of the vector, $\|x\|$, and defines a common factor on how productive the worker is across the different occupations. The second aspect is determined by the direction of the vector, $x(j)/x(k), 1 \leq j \neq k \leq J$, and generally determines his best occupation choices.

The first key assumption in our analysis is the homogeneity in the utility function of workers and in the law of motion of their human capital. In particular, we assume standard relative risk aversion (CRRA) preferences and a linear law of motion for the absolute level of skills of workers. Under these assumptions, we can factor out the absolute advantage $\|x_t\|$ of workers and their comparative advantage fully determines the optimal occupation choices for workers.

The second key assumption is a simple Markov structure for the comparative advantage of workers. In particular, conditional on worker characteristics and, possibly other pre-determined variables, the current occupation and idiosyncratic random labor market opportunities determine the comparative advantage of workers across occupations. In our model, a matrix τ determines the transferability of skills from each occupation to all others. The diagonal entries of such a matrix govern the average growth in skills in each occupation while the off-diagonal terms capture the different depreciation rates associated with occupation mobility. Moreover, a matrix χ determines non-pecuniary costs or benefits of moving across occupations. In addition to these pecuniary and non-pecuniary costs, in each period, workers realize random labor market opportunities across all occupations. Thus, in each period, the worker's human capital for the next period, in each occupation, is governed by the current occupation, the transferability matrix τ, the absolute level of skills $\|x_t\|$ and a random component for each of the occupations. The absolute level of skills $\|x_t\|$ of workers will grow over time, encoding the worker's entire history of labor market choices and opportunities.

Our assumptions lead to a recursive formulation in which workers optimally self-select across occupations, on the basis of the random opportunities, the human capital transferability and non-pecuniary costs of moving, τ and χ, and endogenously determined valuations of occupations. These valuations solve a fixed point problem that incorporates not only the current prices of skills, but also the future occupation alternatives for workers. At the individual level, the model generates a simple probabilistic model, in which job transition probabilities capture persistence in occupation choices and internalize self-selection and the current and future returns of jobs. At the aggregate level, the model is very tractable for general equilibrium, since it leads to very simple aggregation of the workers and human capital across the different occupations.

2.3. Individual Problem

The worker starts each period $t = 1, 2, \ldots$ attached to one of $j = 1, \ldots, J$ occupations, carrying over from the previous period an absolute level of human captial $h > 0$. Available for the next period, the worker realizes a vector $\epsilon_t = \left[\epsilon_t^1, \ldots, \epsilon_t^J\right] \in \mathbb{R}_+^J$ of labor market opportunities. Each entry in the vector corresponds to the labor market opportunity in the respective occupation. On the basis of these occupations, the worker chooses to either stay in the current occupation j or to move to an alternative occupation l.

Switching occupations entails costs (or returns) which we specify as follow: A $J \times J$ human capital transferability matrix, with strictly positive entries, τ_{jl}, determines the fraction of the human capital h that can be transferred from the current occupation j to a new occupation l. On average, there is depreciation if $\tau_{jl} < 1$ or positive accumulation if $\tau_{jl} > 1$. The diagonal terms, τ_{jj}, may be higher than one, capturing learning-by-doing, i.e. the accumulated experience capital of a worker as he spends more time in an occupation j. The off-diagonal terms τ_{jl} may be less than one to capture a potential mismatch between the human capital acquired in one occupation and the productivity of the worker in a different occupation. Still, some of the off-diagonal terms could be greater than one, capturing skill transferability and cross-occupation training or upgrading. In our specification, these occupation-switching costs have a permanent impact on the human capital of the worker for all future periods and for all future occupation choices. In addition, a $J \times J$ utility cost matrix, with strictly positive entries, χ_{jl}, captures the non-pecuniary costs (or benefits) of switching from current occupation j to occupation l. These costs shape the occupational decision, but have no direct impact on current or future earnings. We assume these non-pecuniary costs are proportional to the expected lifetime utility of the destination occupation at the time of making the decision.

The human capital of the worker evolves according to the labor market opportunities ϵ_t and the occupation choice of the worker. Given a level of human capital, h, a current occupation j, and a vector $\epsilon_t \in \mathbb{R}^J_+$ of labor opportunities, the vector

$$h_t \tau_{j,\cdot} \odot \epsilon_t \in \mathbb{R}^J_+,$$

describes how many efficient units of labor services, or effective human capital, the worker can provide for each of the alternative occupations $l = 1, \ldots, J$. Here the operator $\odot$ denotes an element-by-element multiplication. After choosing which occupation to take, the scale of the human capital level for the worker for the next period would be

$$h_{t+1} = h_t \tau_{j_t, l_{t+1}} \epsilon_{l,t}, \tag{3}$$

where j_t and l_{t+1} indicate, respectively, the occupations at period t and $t+1$. It is important to highlight that the role of variable h this model. On the one hand, it is an absolute level of general human capital (or an absolute advantage across occupations). On the other hand, all the past history of the worker in terms of different occupation choices and realization of idiosyncratic shocks ϵ, is summarized in a single value for h.

Denote by $V(j, h, \epsilon)$ the expected life-time discounted utility of the worker. The Bellman Equation that defines this value function is,

$$V(j, h, \epsilon) = \frac{\left(w^j h \epsilon^j\right)^{1-\gamma}}{1-\gamma} + \beta \max_l \left\{\chi_{j,l} \mathbb{E}_{\epsilon'}\left[V(l, h', \epsilon')\right]\right\}, \tag{4}$$

where $\mathbb{E}_{\epsilon'}$ is the expectation over the next period's vector of job market opportunities and h' is given by equation (3).

To characterize this BE, we first show that it can be factorized, i.e., its value can be decomposed into a factor that depends only on the current occupation and labor market realizations, (j,ϵ), and another factor that depends only on the absolute level of human capital, h. This can be done for any generic distribution for the labor market shocks ϵ for which an expectation satisfies a boundedness condition. In what follows, we assume that ϵ is distributed independently over time and across workers, and that all the required moments involving ρ are finite and well defined.

First, note that if occupation l is chosen, then, the next period human capital is $h' = h\tau_{jl}\epsilon_l$. Then, observe that the period utility function is homogenous of degree $1-\gamma$ in h, for any pair (j,ϵ), it can be factorized into a real value $v(j,\epsilon)$ and a human capital factor $h^{1-\gamma}$, i.e., $V(j,h,\epsilon) = v(j,\epsilon)h^{1-\gamma}$. Under this hypothesis, the BE (4) becomes

$$v(j,\epsilon)h^{1-\gamma} = \left(\frac{\left(w^j\epsilon^j\right)^{1-\gamma}}{1-\gamma} + \beta\max_l\left\{\chi_{j,l}\left(\tau_{j,l}\epsilon^l\right)^{1-\gamma}\mathbb{E}_{\epsilon'}\left[v(l,\epsilon')\right]\right\}\right)h^{1-\gamma}.$$

Simplifying out the term $h^{1-\gamma}$ we end up with

$$v(j,\epsilon) = \left(\frac{\left(w^j\epsilon^j\right)^{1-\gamma}}{1-\gamma} + \beta\max_l\left\{\chi_{j,l}\left(\tau_{j,l}\epsilon^l\right)^{1-\gamma}\mathbb{E}_{\epsilon'}\left[v(l,\epsilon')\right]\right\}\right), \tag{5}$$

which verifies the factorization hypothesis. Therefore, the characterization of $V(j,h,\epsilon)$ boils down to the characterization of $v(j,\epsilon)$, a random variable that depends on each realization ϵ. For all occupations $j = 1,\ldots,J$, denote by v^j the conditional expectation of this random variable, i.e.,

$$v^j \equiv \mathbb{E}_\epsilon\left[v(j,\epsilon)\right].$$

Using this definition, and taking the expectation $\mathbb{E}_\epsilon$ in both the right- and left-hand sides of (5), the expectation reduces to a recursion on v^j:

$$v^j = \begin{cases} \Upsilon^j\frac{\left(w^j\right)^{1-\gamma}}{1-\gamma} + \beta E_\epsilon\left[\max_\ell\left\{\chi_{j,\ell}\left[\tau_{j,\ell}\epsilon^\ell\right]^{1-\gamma}v^\ell\right\}\right], & \text{for } \gamma \neq 1, \\ \Upsilon^j + \ln w^j + \beta E_\epsilon\left[\max_\ell\left\{v^\ell + \frac{\ln\left(\tau_{j,\ell}\epsilon^\ell\right)}{1-\beta}\right\}\right], & \text{for } \gamma = 1, \end{cases} \tag{6}$$

where $Upsilon^j = \mathbb{E}\left[\left(\epsilon^j\right)^{1-\gamma}\right]$ for $\gamma \neq 1$, and $Upsilon^j = \mathbb{E}\left[\log\left(\epsilon^j\right)\right]$ if $\gamma = 1$, are scalars that are specific to occupation j.

For all $\gamma \geq 0$, the following lemma establishes simple conditions on the stochastic behaviour of the labor market opportunities of workers, that guarantee the existence and uniqueness of values $v \in \mathbb{R}^J$ that solove (6). All along, we assume that $\tau_{j,l} > 0$ and $\chi_{jl} > 0$ for all j,l and that the support of ϵ^l is $[0,\infty)$ for all l.

Depending on the value of γ, and for each $j = 1, \ldots, J$, we define the terms, Φ_j as follows:

$$\Phi_j \equiv \begin{cases} \mathbb{E}_\epsilon \left[\max_\ell \left\{ \chi_{j,\ell} \left[\tau_{j,\ell} \epsilon_\ell \right]^{1-\gamma} \right\} \right], & \text{for } 0 \leq \gamma < 1, \\ \mathbb{E}_\epsilon \left[\max_\ell \left\{ \ln \left(\tau_{j,\ell} \epsilon_\ell \right) \right\} \right], & \text{for } \gamma = 1, \\ \mathbb{E}_\epsilon \left[\min_\ell \left\{ \left[\chi_{j,\ell} \tau_{j,\ell} \epsilon_\ell \right]^{1-\gamma} \right\} \right], & \text{for } \gamma > 1. \end{cases}$$

Also, conditioning ont he relevant definition of Ψ_j for each γ, we define

$$\overline{\Psi} = \max_j \Psi_j.$$

The following lemma shows that if the average labor market opportunities available to workers are bounded, as summarized by bounds on $\overline{\Psi}$, then, we can guarantee that the dynamic programming problem (6) has a unique and well-defined solution.

Lemma 1. Let $w \in \mathbb{R}_+^J$ be the vector of unitary wages across all occupations J. Assume that preferences are characterized by a CRRA $\gamma \geq 0$ and that the costs $\tau_{j,l}$ and $\chi_{j,l}$ and labor market opportunity shocks ϵ satisfy the assumptions above. Then: (a) for all $0 < \gamma \neq 1$, if $\beta\overline{\Psi} < 1$, then there exists a unique, finite $v \in \mathbb{R}^J$ that solves $v^j = \Upsilon^j \frac{(w^j)^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_e \left[\max_l \left\{ \chi_{j,l} \left(\tau_{j,l} \epsilon^l \right)^{1-\gamma} v^l \right\} \right]$ for all j. Moreover, if $\gamma < 1$, the fixed point v is positive $(v \in \mathbb{R}_{+J})$ and if $\gamma > 1$, the fixed point v is negative $(v \in \mathbb{R}_-^J)$. (b) For the special case of log preference, $\gamma = 1$, if $-\infty < \Psi_j < +\infty, \forall j$, and $\beta < 1$, then, there exists a unique, finite $v \in \mathbb{R}^J$ such that $v^j = \Psi^j + \ln w^j + \beta \mathbb{E}_\epsilon \left[\max_l \left\{ v^l + \frac{\ln(\tau_{j,l}\epsilon^l)}{1-\beta} \right\} \right]$ for all j.