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# Capital–skill complementarity and inequality over the business cycle

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## Abstract

When capital–skill complementarity is present in the production process, changes in the skill premium are driven not only by changes in the ratio of unskilled to skilled labor inputs (as they are in the case with Cobb–Douglas production), but also by changes in the capital–skill ratio. A simple regression analysis demonstrates that the capital–skill ratio has a positive and significant relation to the skill premium at business cycle frequencies as predicted by the capital–skill complementarity hypothesis. This finding motivates the construction of a stochastic dynamic general equilibrium model that allows for capital–skill complementarity in production. The model with capital–skill complementarity can account for the cyclical behavior of the skill premium and much of its volatility. The model without capital–skill complementarity cannot. These results, together with the available empirical evidence, suggest that capital–skill complementarity is an important determinant of wage inequality over the business cycle.

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## 1. Introduction

Keane and Prasad (1993) report that the aggregate skill premium in the United States is essentially uncorrelated with contemporaneous measures of the business cycle. This study confirms their finding and demonstrates that the aggregate skill premium also lags the

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business cycle and exhibits a volatility that is roughly the same as that of output. The purpose of this study is to test whether or not these empirical observations regarding the cyclical behavior of the skill premium can be explained by the presence of capital–skill complementarity in production.

When capital–skill complementarity is present in the production process, changes in the skill premium are driven not only by changes in the ratio of unskilled to skilled labor inputs (as they are in the case with Cobb–Douglas production), but also by changes in the capital–skill ratio. In US data, both of these factor input ratios have distinct cyclical patterns. Relative labor inputs are procyclical, while the capital–skill ratio has a negative correlation with contemporaneous output and exhibits a strong, positive lag.

A simple regression analysis demonstrates that the capital–skill ratio has a positive and significant relation to the skill premium at business cycle frequencies as predicted by the capital–skill complementarity hypothesis. This finding motivates the construction of a stochastic dynamic general equilibrium model which is used to investigate whether or not a model with capital–skill complementarity can account both qualitatively and quantitatively for the business cycle behavior of the skill premium together with a number of standard business cycle facts.

The model used in this paper extends the standard real business cycle model in three ways. First, the representative agent is replaced by two agent types, skilled and unskilled. Second, the standard, two-factor Cobb–Douglas production function is replaced by a more general, four-factor production function that allows for capital–skill complementarity. Third, the model includes both neutral and investment-specific technological change. The model is a synthesis of the models used in Greenwood et al. (1997) and Krusell et al. (2000).

The model with capital–skill complementarity can account for the cyclical behavior of the skill premium, as well as a large share of its volatility. The model without capital–skill complementarity cannot. These results, together with the empirical evidence mentioned above, suggest that capital–skill complementarity is an important determinant of wage inequality over the business cycle.

The capital–skill complementarity hypothesis, which states that increases (decreases) in the capital–skill ratio increase (decrease) the relative demand for skilled labor and, hence, increase (decrease) relative wages, is by no means new.<sup>1</sup> The importance of capital–skill complementarity for explaining long run trends in wage inequality has already been demonstrated by Krusell et al. (2000) and Lindquist (2001). Other researchers have illustrated the connection between capital–skill complementarity and wage dispersion (e.g. Caselli, 1999). This study makes a contribution to this recent literature by investigating the implications of capital–skill complementarity for the cyclical behavior of wage inequality. This study could also be viewed as a complement to the work of Castañeda et al. (1998) on income inequality over the business cycle.

There are a number of implicit contract models that have specific implications for movements in the skill premium over the business cycle.<sup>2</sup> These models are well motivated

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<sup>1</sup> See Hamermesh (1993) for a review of this literature.

<sup>2</sup> The implicit contract theories of Azariadis (1975, 1976), Bailey (1974), and Gordon (1974), for example, support the existence of wage smoothing arrangements provided by firms for skilled workers. This arrangement

by the empirical observations that wages tend to be uncorrelated with contemporaneous output and that hours worked fluctuate more than wages over the business cycle. Walrasian real business cycle models, on the other hand, always produce wages that are procyclical. The model in this paper is no exception. But, this does not mean that the results presented here can be disregarded by the contracting literature. On the contrary, since preferences and technology matter for the optimal contract (Rosen, 1985), the demand effect, which arises in the presence of capital–skill complementarity, will influence the design of such a contract.

The main benefit of the more naive, Walrasian modeling approach used in this paper is one of tractability. It can account for a more complicated lag structure in the response of relative wages over the business cycle in a manageable fashion and it demonstrates the capital–skill complementarity mechanism in a straightforward manner. It provides us with a parsimonious model that fits neatly into the recent literature on wage inequality.

The remainder of this paper is outlined as follows. In Section 2, the facts regarding the cyclical behavior of the skill premium are presented. An initial analysis of the importance of capital–skill complementarity is carried out in Section 3. This is done in two parts. First, a qualitative example is presented which demonstrates the potential impact of capital–skill complementarity on the cyclical behavior of the skill premium. Then, the empirical relevance of the capital–skill complementarity hypothesis is tested.

In Section 4, a stochastic dynamic general equilibrium model which allows for capital–skill complementarity in production is constructed and its business cycle properties are studied. A sensitivity analysis is then carried out which demonstrates that the results which speak in favor of the model with capital–skill complementarity are robust to substantial parameter changes. Following this, the data concerning wages, employment and hours worked are reexamined and the potential impact of the capital–skill complementarity mechanism in an environment with contracts is discussed. Section 5 concludes.

## 2. The cyclical behavior of the skill premium

Until recently, it had been widely accepted that the aggregate skill premium in the United States moved countercyclically. This belief was based mainly on work done by Reder (1955, 1962) and was introduced into the dynamic macroeconomic literature by Kydland (1984). Since then, it has been recognized that estimates of the skill premium using aggregate data may be biased due to the changing composition of the workforce over the business cycle (Raisian, 1983; Keane and Prasad, 1993; Ziliak et al., 1999). Using microeconomic panel data and controlling for this type of bias, as well as correcting for potential selection bias, Keane and Prasad (1993) found the aggregate skill premium

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implies a countercyclical skill premium. In the implicit contract models of Hashimoto (1981) and Raisian (1983), skilled workers accept higher cyclical fluctuations in wages in return for higher job security. This results in a procyclical skill premium. Both types of models rely on the existence of firm-specific capital (Becker, 1962) to create incentives to avoid costly separations of skilled workers from firms during recessions.

to be uncorrelated with contemporaneous measures of the business cycle.<sup>3</sup> The data presented in this section confirm their result and show that the aggregate skill premium also lags the business cycle and exhibits a volatility which is roughly the same as that of output.

The skill premium used in this paper is defined as the ratio of the hourly wage of a worker with 14 or more years of schooling to the hourly wage of a worker with less than 14 years of schooling. Both the mean and the median skill premiums are calculated. They are calculated using data from the Monthly Outgoing Rotation Group (MORG) of the US Census Bureau's Current Population Survey. Monthly data concerning total hours worked last week, weekly earnings before deductions, the appropriate earnings weights, and educational attainment have been collected for the years 1979 to 2002.<sup>4</sup> Deseasonalized quarterly means and medians were constructed from this monthly data. The quarterly data were then detrended using a Hodrick–Prescott filter with  $\lambda$  set to 1600.<sup>5</sup>

Cross-correlations between output and the skill premium are reported in Table 1 along with a volatility measure for each series. Table 1 tells us several things about the cyclical behavior of the skill premium. First, we see that the mean skill premium is weakly, positively correlated with contemporaneous output,<sup>6</sup> while the median skill premium is uncorrelated with contemporaneous output. Second, we see that both the mean and the median skill premiums lag output. The mean skill premium peaks four quarters after output, while the median peaks five quarters after output. Both lags are statistically significant and positively correlated with output at time  $t$ . Third, we see that the volatility

Table 1  
The cyclical behavior of the skill premium

$x$	Volatility <sup>a</sup>	Cross-correlations of output ( $t$ ) and $x(t+i)$				
		$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+4)$	$x(t+5)$
Output <sup>b</sup>	1.33 (0.121) <sup>c</sup>	0.87 (0.027)	1.00 –	0.87 (0.027)	0.23 (0.103)	0.02 (0.101)
Mean skill premium <sup>d</sup>	1.31 (0.090)	0.06 (0.118)	0.19 (0.106)	0.21 (0.091)	0.36 (0.079)	0.32 (0.090)
Median skill premium <sup>d</sup>	1.56 (0.117)	–0.08 (0.107)	–0.02 (0.100)	0.01 (0.096)	0.33 (0.084)	0.38 (0.081)

<sup>a</sup> Volatility is measured as the standard deviation of perceptual fluctuations around trend.

<sup>b</sup> Source: US National Income and Product Accounts.

<sup>c</sup> The numbers in parentheses are standard errors which have been calculated by bootstrapping.

<sup>d</sup> Source: Monthly Outgoing Rotation Group of the US Census Bureau's Current Population Survey.

<sup>3</sup> Ziliak et al. (1999) found that the returns to education (in the aggregate economy) for those with above average education levels were, in fact, weakly procyclical. But, their estimate was not statistically significant and, hence, their study cannot reject the hypothesis that the aggregate skill premium is uncorrelated with contemporaneous output.

<sup>4</sup> The data was downloaded from Unicon Research at [www.unicon.com](http://www.unicon.com).

<sup>5</sup> All of the data used in this paper are treated in a similar fashion.

<sup>6</sup> It is significantly different from zero at the 10 percent level.

of the mean skill premium is equal to that of output, while the volatility of the median skill premium is somewhat higher.

The contemporaneous correlation between the mean skill premium and output (reported in Table 1) may suffer from a procyclical bias due to changes in the composition of the workforce over the business cycle.<sup>7</sup> Unfortunately, the MORG data set does not contain enough of the necessary longitudinal information needed to control for this type of bias. But, if skills are normally distributed throughout the population, then changes in the composition of the labor force due to unemployment will probably not be large enough to have a significant impact on the median skill level of the unskilled group. Thus, the procyclical bias which may affect the correlation between the mean skill premium and output will most likely not affect the correlation between the median skill premium and output. Furthermore, median earnings are less susceptible to the problems associated with top coding (see Lerman, 1997). The median skill premium (which is uncorrelated with contemporaneous output) may, therefore, be a more suitable measure of the contemporaneous correlation between output and the skill premium than the mean skill premium (which is weakly, positively correlated with contemporaneous output).<sup>8</sup>

To examine whether or not the cross-correlations reported in Table 1 are stable over time, one can calculate a series of correlation coefficients starting with the correlation coefficient associated with the first 10 observations (1979:I–1981:II), then the first 11 observations (1979:I–1981:III), etc., and continue in this manner until reaching the last correlation, which is calculated using the entire sample (and is reported in Table 1). The mean and standard deviation of such a series of cross-correlation coefficients should give us some information on the stability of the results reported in Table 1.

Examining a series of contemporaneous correlations between output and the mean skill premium constructed in this manner, we find that the mean is 0.18 (0.09).<sup>9</sup> The mean skill premium lags output and peaks three periods after output. The mean of this lagged correlation is 0.24 (0.11). The mean of the series of contemporaneous correlations between output and the median skill premium is 0.12 (0.11). The median skill premium lags output and peaks four periods after output with a mean of 0.33 (0.13). The results from this experiment demonstrate the stability of the relationships reported in Table 1.<sup>10</sup>

In summary, the cyclical behavior of the skill premium can be characterized by the following three stylized facts:

- (i) the skill premium is essentially uncorrelated with contemporaneous output,
- (ii) the skill premium lags the business cycle, and

<sup>7</sup> This procyclical bias is due to the fact that employment variability is greatest among workers at the low end of the productivity distribution. Thus, the average quality of low skilled workers rises when unemployment is high and falls when unemployment is low, while the average productivity of high skilled workers remains roughly constant over the business cycle. This induces a procyclical movement in the relative productivities of these two groups and, subsequently, in the skill premium as well.

<sup>8</sup> It is becoming more common for researchers to report information concerning earnings inequality in terms of median earnings (see e.g. Bradbury, 2002).

<sup>9</sup> Numbers in parentheses are standard deviations.

<sup>10</sup> Similar results are obtained if one calculates the series of cross-correlation coefficients in the reversed time order, i.e. starting with the last ten observations instead of the first ten.

(iii) the volatility of the skill premium is roughly equal to that of output.

The purpose of this study is to test whether or not these three facts regarding the cyclical behavior of the skill premium can be explained by the presence of capital–skill complementarity in production.

### 3. An initial analysis of the importance of capital–skill complementarity

The potential effect of capital–skill complementarity upon the cyclical behavior of the skill premium can be demonstrated with a simple, qualitative example. First, assume that capital,  $k_t$ , and skilled hours worked,  $h_{s,t}$ , are complements, while capital and unskilled hours worked,  $h_{u,t}$ , are substitutes. For ease of exposition, assume also that capital and unskilled hours are perfect substitutes and that they have an elasticity of substitution with skilled hours equal to one. Given these assumptions, the production function of a firm can be written as

$$f(k_t, h_{s,t}, h_{u,t}) = (k_t + h_{u,t})^\theta h_{s,t}^{1-\theta}. \quad (1)$$

The skill premium can be derived from this function using the first order, profit maximizing conditions of the firm under perfect competition

$$\frac{f_{h_{s,t}}}{f_{h_{u,t}}} = \frac{w_{s,t}}{w_{u,t}} = \left( \frac{1-\theta}{\theta} \right) \frac{k_t + h_{u,t}}{h_{s,t}} \quad (2)$$

where  $f_x$  is the derivative of the production function with respect to  $x$  and where  $w_{s,t}$  and  $w_{u,t}$  are the wages of skilled and unskilled workers, respectively.

Taking the derivatives of the skill premium with respect to relative hours worked and to the capital–skill ratio gives us the following partial derivatives:

$$\frac{\partial(w_{s,t}/w_{u,t})}{\partial(h_{u,t}/h_{s,t})} > 0, \quad \frac{\partial(w_{s,t}/w_{u,t})}{\partial(k_t/h_{s,t})} > 0.$$

Thus, when production is characterized by capital–skill complementarity, an increase in the ratio of capital to skilled hours worked raises *ceteris paribus* the skill premium. An increase in the ratio of unskilled to skilled hours worked also raises *ceteris paribus* the skill premium. In contrast to this, the skill premium associated with a Cobb–Douglas production function is a function of relative hours only.<sup>11</sup>

The question then is: Do relative factor inputs move in a particular fashion over the business cycle? In Table 2, we see that they do. The relative labor ratio moves procyclically, while the capital–skill ratio is negatively correlated with contemporaneous output and peaks six quarters after output with a strong, positive correlation.

The empirical relevance of the capital–skill complementarity hypothesis can be examined using a simple autoregressive distributed lag (ADL) model of the mean skill

<sup>11</sup> If the production function is Cobb–Douglas, then  $f_{h_{s,t}}/f_{h_{u,t}} = w_{s,t}/w_{u,t} = \theta_s h_{u,t}/\theta_u h_{s,t}$ , where  $\theta_s$  and  $\theta_u$  are the income shares of skilled and unskilled labor, respectively.

Table 2  
The cyclical behavior of relative factor inputs

$x$	Volatility <sup>a</sup>	Cross-correlations of output ( $t$ ) and $x(t+i)$				
		$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+4)$	$x(t+6)$
Capital–skill ratio <sup>b,d</sup>	2.00 (0.161)	−0.38 (0.076)	−0.33 (0.084)	−0.17 (0.091)	0.44 (0.082)	0.59 (0.071)
Relative labor ratio <sup>d</sup>	2.70 (0.227)	0.23 (0.105)	0.32 (0.103)	0.36 (0.103)	0.40 (0.079)	0.33 (0.092)

<sup>a</sup> Volatility is measured as the standard deviation of perceptual fluctuations around trend.

<sup>b</sup> Source: US National Income and Product Accounts.

<sup>c</sup> The numbers in parentheses are standard errors which have been calculated by bootstrapping.

<sup>d</sup> Source: Monthly Outgoing Rotation Group of the US Census Bureau's Current Population Survey.

premium, which includes the ratio of aggregate relative labor inputs,  $u_t h_{u,t} / s_t h_{s,t}$ ,<sup>12</sup> and the aggregate capital–skill ratio,  $k_{e,t} / s_t h_{s,t}$ ,<sup>13</sup> as well as lagged values of the skill premium. Starting with an ADL(2) model and then reducing the model through a series of  $F$ -tests, gives us the following OLS regression equation

$$\text{mean}\left(\frac{w_{s,t}}{w_{u,t}}\right) = 0.152 \frac{u_t h_{u,t}}{(0.066) s_t h_{s,t}} + 50357 \frac{k_{e,t-1}}{(19441) s_{t-1} h_{s,t-1}} \quad (3)$$

with an  $\bar{R}^2 = 0.12$  and  $DW = 1.90$ .<sup>14</sup> Repeating this exercise using the median skill premium results in the following OLS regression equation

$$\text{median}\left(\frac{w_{s,t}}{w_{u,t}}\right) = 74348 \frac{k_{e,t-1}}{(23091) s_{t-1} h_{s,t-1}} \quad (4)$$

with an  $\bar{R}^2 = 0.10$  and  $DW = 1.58$ .

These regressions demonstrate that the (deseasonalized and detrended) capital–skill ratio has a positive and significant relation to the (deseasonalized and detrended) skill premium.<sup>15</sup> This empirical finding speaks in favor of the use of a model in which production is characterized by capital–skill complementarity and against the use of a model with a standard, Cobb–Douglas production function when studying the cyclical behavior of relative wages.

<sup>12</sup> The variables  $u$  and  $s$  are the numbers of unskilled and skilled workers in the economy, respectively. The variables  $h_u$  and  $h_s$  are the means of hours worked by unskilled and skilled workers, respectively.

<sup>13</sup> The variable for capital equipment,  $k_e$ , was constructed using investment data from the US National Income and Product Accounts. The motivation for examining the impact of capital equipment on the skill premium, as opposed to total capital, will be made clear in Section 4.

<sup>14</sup> The numbers in parentheses are standard errors.

<sup>15</sup> Using output, lagged output, unemployment or lagged unemployment as control variables has little or no impact on the size or significance of the coefficient of the capital–skill variable.

#### 4. An extended analysis

In this section, a stochastic dynamic general equilibrium model is constructed that allows for capital–skill complementarity in production. This model will be used to test whether or not the capital–skill complementarity mechanism can account both qualitatively and quantitatively for the cyclical behavior of the skill premium together with a number of standard business cycle facts.

##### 4.1. The model

Consider an economy inhabited by two types of infinitely lived agents: skilled and unskilled. Let  $s$  and  $u$  represent the measures of skilled and unskilled agents, respectively. These measures sum to the total population which is normalized to one,  $u + s = 1$ .

Agents are born at time zero and acquire their respective skill endowments at birth. Agents have CRRA utility functions

$$U_{i,t}(c_{i,t}, l_{i,t}) = \frac{(c_{i,t}^\alpha l_{i,t}^{1-\alpha})^{1-\gamma} - 1}{1-\gamma}, \quad 0 < \alpha < 1, \quad \gamma \geq 0. \quad (5)$$

They derive utility from consumption,  $c_{i,t}$ , and leisure,  $l_{i,t}$ . Subscript  $i \in \{u, s\}$  identifies an agent's type and  $t$  is a time subscript. Each agent is endowed with one unit of time, so that hours worked by an agent,  $h_{i,t}$ , equal

$$h_{i,t} = 1 - l_{i,t}. \quad (6)$$

There is a continuum of firms in the economy which have identical production functions. Firms use capital structures, capital equipment, skilled labor and unskilled labor to produce output and they behave competitively in product and factor markets.

Assuming constant returns to scale in production allows us to aggregate across firms without loss of generality. Let  $K_{s,t}$  and  $K_{e,t}$  denote the aggregate stock of structures and equipment, respectively. Let  $H_{s,t} = sh_{s,t}$  denote aggregate hours worked by skilled agents and let  $H_{u,t} = uh_{u,t}$  denote aggregate hours worked by unskilled agents. Aggregate production,  $Y_t$ , is then given by<sup>16</sup>

$$Y_t = e^{(z_t)} K_{s,t}^\theta [\mu H_{u,t}^\nu + (1-\mu) [\lambda K_{e,t}^\varphi + (1-\lambda) H_{s,t}^\varphi]^{v/\varphi}]^{(1-\theta)/v}, \quad (7)$$

$$\theta, \mu, \lambda \in (0, 1); \quad \nu, \varphi \in (-\infty, 1); \quad \nu, \varphi \neq 0,$$

where  $z_t$  is a random technology shock.

The parameters  $\nu$  and  $\varphi$  are the two key substitution parameters. The income share of capital structures is given by  $\theta$ , while  $\mu$  and  $\lambda$  determine the income shares of skilled and unskilled labor and of capital equipment. The elasticity of substitution between capital equipment and skilled labor is equal to  $1/(1-\varphi)$ . The elasticity of substitution between capital equipment and unskilled labor and the elasticity of substitution between skilled and unskilled labor are both equal to  $1/(1-\nu)$ . If  $\nu > \varphi$ , then the production function is said to exhibit capital–skill complementarity.

<sup>16</sup> The production function used in this paper is similar to the one estimated in Krusell et al. (2000).



The skill premium associated with the above production function is

$$\frac{w_{s,t}}{w_{u,t}} = \frac{(1-\mu)(1-\lambda)}{\mu} \left[ \lambda \left( \frac{K_{e,t}}{H_{s,t}} \right)^\varphi + (1-\lambda) \right]^{(v-\varphi)/\varphi} \left( \frac{H_{u,t}}{H_{s,t}} \right)^{1-v}.$$

Examining this equation, we see that when  $\nu > \varphi$ , a rise in the stock of capital equipment will ceteris paribus raise the skill premium. Krusell et al. (2000) call this the *capital–skill complementarity effect*. We also see that a rise in the ratio of unskilled to skilled hours worked will ceteris paribus raise the skill premium for any acceptable values of  $\nu$  and  $\varphi$ . Krusell et al. (2000) call this the *relative supply effect*.

The law of motion for aggregate capital structures is

$$K_{s,t+1} = (1 - \delta_s)K_{s,t} + X_{s,t}, \quad 0 \leq \delta_s \leq 1 \quad (8)$$

where  $\delta_s$  is the depreciation rate of capital structures and  $X_{s,t}$  is aggregate investment in new capital structures. The law of motion for aggregate capital equipment is

$$K_{e,t+1} = (1 - \delta_e)K_{e,t} + X_{e,t}e^{(q_t)}, \quad 0 \leq \delta_e \leq 1 \quad (9)$$

where  $\delta_e$  is the depreciation rate of capital equipment and  $X_{e,t}$  is aggregate investment in new capital equipment.

The accumulation equation for capital equipment differs from that of capital structures, since it includes a factor,  $q_t$ , which represents the current state of technology for producing equipment. It determines the amount of equipment that can be purchased for one unit of output. Changes in  $q_t$  formalize the notion of investment-specific technological change, while changes in  $z_t$  represent neutral technological change.

The inclusion of investment-specific technological change in the model is motivated by the studies of Greenwood et al. (2000) and Dejong et al. (2000). Both studies show that investment-specific technological change is an important source of business cycle fluctuations. In this paper, we shall see that investment-specific technological change plays an important role in determining the cyclical behavior of the skill premium. It affects the skill premium through the capital–skill complementarity mechanism.

To complete the model, both  $z_t$  and  $q_t$  are assumed to follow AR(1) processes

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \quad (10)$$

$$q_t = \rho_q q_{t-1} + \varepsilon_{q,t}. \quad (11)$$

The vector  $[\varepsilon_{z,t}, \varepsilon_{q,t}]$  is drawn from a bivariate normal distribution with

$$\varepsilon_z \sim N(\mu_z, \sigma_z^2)$$

where  $E[\mu_z] = 0$  and with

$$(\varepsilon_q | \varepsilon_z) \sim N[(\mu_q - \eta\mu_z) + \eta\varepsilon_z, \varpi^2]$$

where  $\eta = \sigma_{zq}/\sigma_z^2$ ,  $\varpi^2 = \sigma_q^2 - \sigma_{zq}^2/\sigma_z^2$  and  $E[\mu_q] = 0$ . Shocks to investment-specific technology and shocks to neutral technology are allowed to covary ex ante. The correlation coefficient between the two types of shocks,  $\rho_{zq}$ , is given by  $|\sigma_{zq}/\sigma_z\sigma_q| \leq 1$ .

#### 4.1.1. The social planner's problem

If we imagine that this economy is governed by a benevolent social planner, the problem faced by the planner is to choose sequences for consumption, labor supply, and capital, that, given  $K_{e,0}$ ,  $K_{s,0}$ ,  $s$ , and  $u$ , maximizes the weighted sum of the expected utilities of the two groups of agents

$$\max_{\substack{\{c_{s,t}, c_{u,t}, h_{u,t}, h_{s,t}, \\ K_{s,t+1}, K_{e,t+1}\}}} E_0 \sum_{t=0}^{\infty} \beta^t \{ (1 - \Psi) u U_{u,t}(c_{u,t}, l_{u,t}) + \Psi s U_{s,t}(c_{s,t}, l_{s,t}) \} \quad (12)$$

subject to the aggregate resource constraint

$$C_{s,t} + C_{u,t} + X_{s,t} + X_{e,t} = Y_t \quad (13)$$

and to Eqs. (5)–(11), where  $C_{s,t} = s c_{s,t}$ ,  $C_{u,t} = u c_{u,t}$ ,  $\beta$  is the discount factor of the social planner and  $\Psi$  is the social planner's weight on skilled utility.

#### 4.1.2. Model equilibrium and solution

An equilibrium for this economy consists of a set of decision rules  $h_i(K_e, K_s, z, q)$  and  $c_i(K_e, K_s, z, q)$  for  $i \in \{u, s\}$ , such that (i) the decision rules solve the social planner's welfare maximization problem and (ii) prices are equal to marginal products.

The decision rules  $h_i(K_e, K_s, z, q)$  and  $c_i(K_e, K_s, z, q)$  can be found in the following manner. First, the Lagrangian function,  $\mathcal{L}$ , associated with the social planner's problem is constructed:

$$\begin{aligned} \mathcal{L}(K_{e,t}, K_{s,t}, c_{i,t}, h_{i,t}, \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}) \\ = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1 - \Psi) u \frac{[c_{u,t}^\alpha (1 - h_{u,t})^{1-\alpha}]^{1-\gamma} - 1}{1 - \gamma} \right. \\ \left. + \Psi s \frac{[c_{s,t}^\alpha (1 - h_{s,t})^{1-\alpha}]^{1-\gamma} - 1}{1 - \gamma} \right\} \\ - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_{1,t} [C_{s,t} + C_{u,t} + K_{s,t+1} - (1 - \delta_s) K_{s,t} + \dots + K_{e,t+1} / e^{(q_t)} \\ - (1 - \delta_e) K_{e,t} / e^{(q_t)} - e^{(z_t)} F(K_{e,t}, K_{s,t}, H_{s,t}, H_{u,t})] \\ - E_0 \sum_{t=0}^{\infty} \beta^t \{ \lambda_{2,t} [z_t - \rho_z z_{t-1} - \varepsilon_{z,t}] \} - E_0 \sum_{t=0}^{\infty} \beta^t \{ \lambda_{3,t} [q_t - \rho_q q_{t-1} - \varepsilon_{q,t}] \} \end{aligned} \quad (14)$$

where  $\lambda_{1,t}$ ,  $\lambda_{2,t}$  and  $\lambda_{3,t}$  are the Lagrange multipliers associated with the aggregate resource constraint, the law of motion for  $z_t$  and the law of motion for  $q_t$ , respectively, and where  $e^{(z_t)} F(\cdot)$  is aggregate production. The agents' time constraints have been substituted directly into their utility functions and the laws of motion for aggregate capital structures and equipment have been substituted into the aggregate resource constraint.

From this problem we can derive the necessary first-order conditions for an optimal solution to the social planner's problem. For convenience sake,  $\lambda_{1,t}$  is eliminated from the

set of equations. We are then left with a system of eight equations in eight unknowns which characterizes the equilibrium processes of the model.

The next step is to solve this system for its steady state values. The entire system of equations can then be log linearized around these steady state values. The resulting system of linear expectational difference equations is solved using the generalized Schur method as described in Klein (2000). The solution to this problem delivers the decision rules,  $h_i(K_e, K_s, z, q)$  and  $c_i(K_e, K_s, z, q)$ .

#### 4.2. Model calibration

The model is calibrated as follows. First, the proportion of skilled workers in the economy,  $s$ , is set equal to 47 percent. This is equal to the average share of workers in the MORG data set with 14 or more years of schooling.

Following Greenwood et al. (1997), agents are assumed to be risk averse with  $\gamma$  equal to 1 and the income share of capital structures,  $\theta$ , is set equal to 0.13. The discount rate,  $\beta = 0.9875$ , the depreciation rate of capital structures,  $\delta_s = 0.014$ , and the depreciation rate of capital equipment,  $\delta_e = 0.031$ , are all set to match the annualized parameter values used in Greenwood et al. (1997). The key substitution parameters,  $\nu = 0.401$  and  $\varphi = -0.495$ , were estimated in Krusell et al. (2000).

The income share parameters,  $\mu = 0.413$  and  $\lambda = 0.553$ , the social planner's weight on skilled utility,  $\Psi = 0.589$ , and the consumption share in utility,  $\alpha = 0.611$ , are calibrated simultaneously. They are calibrated so that in the steady state agents work (on average) 40 hours a week, skilled workers work 7 percent more than unskilled workers, the skilled wage is 50 percent higher than the unskilled wage, and the income share of capital is 30 percent (as in Greenwood et al., 1997). The skill premium and the ratio of relative hours worked in the steady state are both averages calculated from the MORG data set.

The AR(1) process governing investment specific technological shocks is calibrated following Greenwood et al. (2000), with  $\rho_q$  equal to 0.64 and  $\sigma_z$  equal to 0.0038.<sup>17</sup> The persistence parameter of the neutral technological process,  $\rho_z$ , is set equal to 0.95, while  $\sigma_z$  is scaled so that the volatility of output in the model is always the same as that found in the NIPA data. In the benchmark case,  $\sigma_z$  is equal to 0.0076. The correlation coefficient between neutral and investment-specific technological shocks was estimated in Lindquist (2002) using the Krusell et al. (2000) data set. That estimate,  $\rho_{zq} = -0.31$ , will be used here as well. Support for this negative correlation can be found in Dejong et al. (2000).<sup>18</sup>

<sup>17</sup> The actual estimate of  $\sigma_z$  used in Greenwood et al. (2000) is 0.035. Here, a value of 0.0038 reproduces their result that investment specific shocks account for 30 percent of the fluctuations in output over the business cycle.

<sup>18</sup> While not reported in their paper, David Dejong was kind enough to supply an estimate of this negative correlation. He reported a value of  $-0.15$ , which is based on posterior modes of smoothed shock values. The performance of the model calibrated using parameters from Dejong et al. (2000) will be examined as part of the sensitivity analysis.

### 4.3. Business cycle properties of the model

In this section, the business cycle behavior of the model is evaluated and discussed. The main conclusion is that the model with capital–skill complementarity matches the data concerning movements in the skill premium over the business cycle, while a model without capital–skill complementarity does not. The model also performs well along a number of standard measures of the business cycle.

The business cycle properties of the aggregate real variables in the US economy are well known. All of them are highly procyclical, except the capital stock which is uncorrelated with contemporaneous output. Investment is much more volatile than output and hours worked, which are, in turn, more volatile than consumption and the capital stock. Productivity leads the business cycle and the capital stock lags the business cycle. All other real variables peak at the same time as output. Table 3 summarizes some of the basic statistics from the data. Model statistics are presented in Table 4.

In the model, all aggregate variables are highly procyclical with the exception of the capital stock which is uncorrelated with contemporaneous output and lags the business cycle (peaking in period  $t + 4$ ). This lag (which we also observe in the data) will play an important role in explaining movements of the skill premium over the business cycle. Overall, the model does as well as a standard real business cycle model at matching the cyclical behavior of the aggregate US economy (see e.g. Cooley and Prescott, 1995).

The dynamic behavior of the aggregate variables can also be illustrated using the impulse response functions generated by the model. Examining the impulse response of the aggregate variables to a positive neutral technology shock (see Fig. 1), we see that all

Table 3  
Summary statistics from the data

$x$	Volatility <sup>a</sup>	Correlations of output ( $t$ ) with $x(t + i)$		
		$x(t - 1)$	$x(t)$	$x(t + 1)$
Output <sup>b</sup>	1.33 (0.121) <sup>c</sup>	0.87 (0.027)	1 –	0.87 (0.027)
Consumption <sup>b</sup>	1.09 (0.088)	0.79 (0.040)	0.83 (0.031)	0.73 (0.052)
Investment <sup>b</sup>	6.34 (0.503)	0.79 (0.036)	0.91 (0.015)	0.79 (0.042)
Labor hours <sup>d</sup>	1.39 (0.108)	0.65 (0.069)	0.78 (0.044)	0.74 (0.044)
Total capital <sup>e</sup>	0.53 (0.030)	–0.15 (0.091)	0.02 (0.099)	0.21 (0.098)
Productivity <sup>f</sup>	0.87 (0.098)	0.25 (0.087)	0.23 (0.091)	0.08 (0.088)

<sup>a</sup> Volatility is measured as the standard deviation of perceptual fluctuations around trend.

<sup>b</sup> Source: US National Income and Product Accounts.

<sup>c</sup> The numbers in parentheses are standard errors which have been calculated by bootstrapping.

<sup>d</sup> Source: Monthly Outgoing Rotation Group of the US Census Bureau's Current Population Survey.

<sup>e</sup> The capital stock is calculated using investment data from the US National Income and Product Accounts.

<sup>f</sup> Productivity = output/labor hours.

Table 4  
Summary statistics from the model

$x$	Volatility <sup>a</sup>	Correlations of output ( $t$ ) with $x(t+i)$		
		$x(t-1)$	$x(t)$	$x(t+1)$
Output	1.33	0.67	1	0.67
Consumption	0.56	0.45	0.81	0.64
Investment	5.31	0.69	0.96	0.61
Labor hours	0.57	0.67	0.93	0.57
Total capital	0.39	−0.11	0.09	0.39
Productivity <sup>b</sup>	0.84	0.62	0.95	0.69

<sup>a</sup> Volatility is measured as the standard deviation of perceptual fluctuations around trend.

<sup>b</sup> Productivity = output/labor hours.

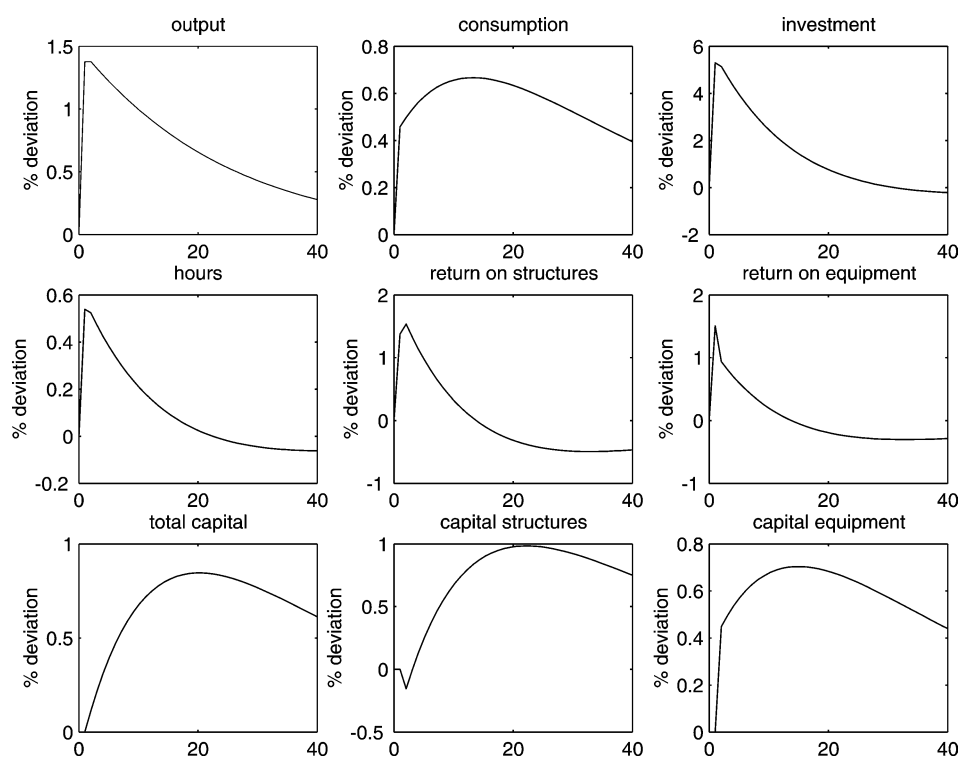


Fig. 1. Impulse response of aggregate variables to a neutral productivity shock.

of the aggregate variable have an immediate and positive response, with the exception of the capital stock which shows a pronounced lag.

Unlike neutral technological shocks, which enter the production function directly, investment-specific technological shocks can only affect current output by encouraging labor to work more or less in the current period. They do, however, have a direct impact on the quality of current investments in capital equipment. In Fig. 2, we see that a positive

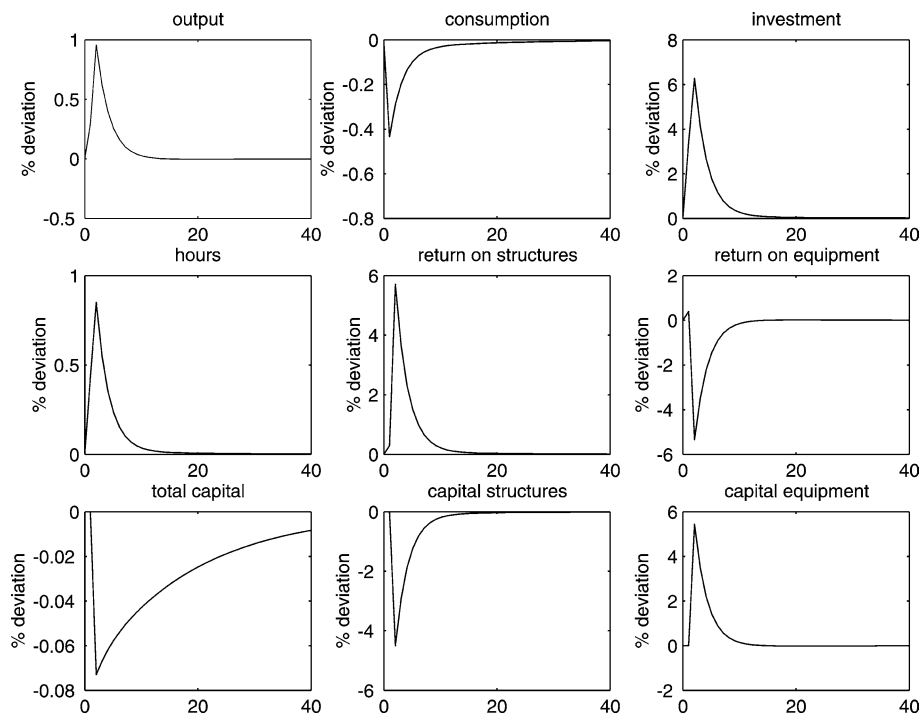


Fig. 2. Impulse response of aggregate variables to an investment-specific productivity shock.

investment-specific technology shock raises investment and lowers consumption in the current period. Hours worked also increase. Higher labor income is used to finance the increase in investment. Increased hours also allows for a rise in the marginal product of capital, despite the fact that capital in the current period is fixed and that the shock does not enter the production function directly. Both output and hours respond immediately to investment-specific shocks, but they do not peak until the period following the shock.

There is almost no response in the total capital stock to an investment-specific shocks. These shocks do, however, affect the composition of the capital stock. The amount of capital equipment in the economy is now greater than it was before the shock (in both absolute terms and relative to structures). It is now more profitable for agents to invest in capital equipment and less in structures.<sup>19</sup> This portfolio shift affects relative wages in the model since capital equipment and skilled labor are assumed to be complementary.

In Table 5, we see that the skill premium in the model is essentially uncorrelated with contemporaneous output and that it lags the business cycle. The skill premium in the model has a correlation coefficient with contemporaneous output equal to 0.06. The same correlation for the mean skill premium in the data is 0.19 (0.106) and for the median

<sup>19</sup> Investments in new structures are still positive. But, they are not large enough to offset depreciation. This is why we observe a negative impulse response of capital structures to investment-specific technology shocks. It is not the case that investors are reversing previous investment decisions by converting old structures into new equipment.

it is  $-0.02$  (0.100).<sup>20</sup> The skill premium in the model lags the business cycle, peaking at time  $t + 4$  with a correlation coefficient with time  $t$  output of 0.18. The mean skill premium in the data also lags the business cycle, peaking at time  $t + 4$  with a correlation coefficient with time  $t$  output of 0.36 (0.079), while the median skill premium peaks at  $t + 5$  with a correlation of 0.38 (0.081). In the model, the capital–skill complementarity effect dominates the relative supply effect, so that the lag in the capital–skill ratio induces a lag in the skill premium.

The dynamic behavior of the per capita and relative variables can also be understood by examining the impulse response functions generated by the model. In Fig. 3, we see that a positive shock to neutral technological change raises the wages of both skilled and unskilled workers. Higher wages induce both types of labor to work more hours. The increase in the supply of unskilled hours, however, is stronger than that of skilled hours, since unskilled workers have a lower marginal disutility of labor. Thus, the ratio of unskilled to skilled hours worked initially rises in response to a neutral technology shock.

The ratio of capital equipment to skilled hours worked falls initially, since skilled hours respond more quickly than the stock of capital equipment to neutral productivity

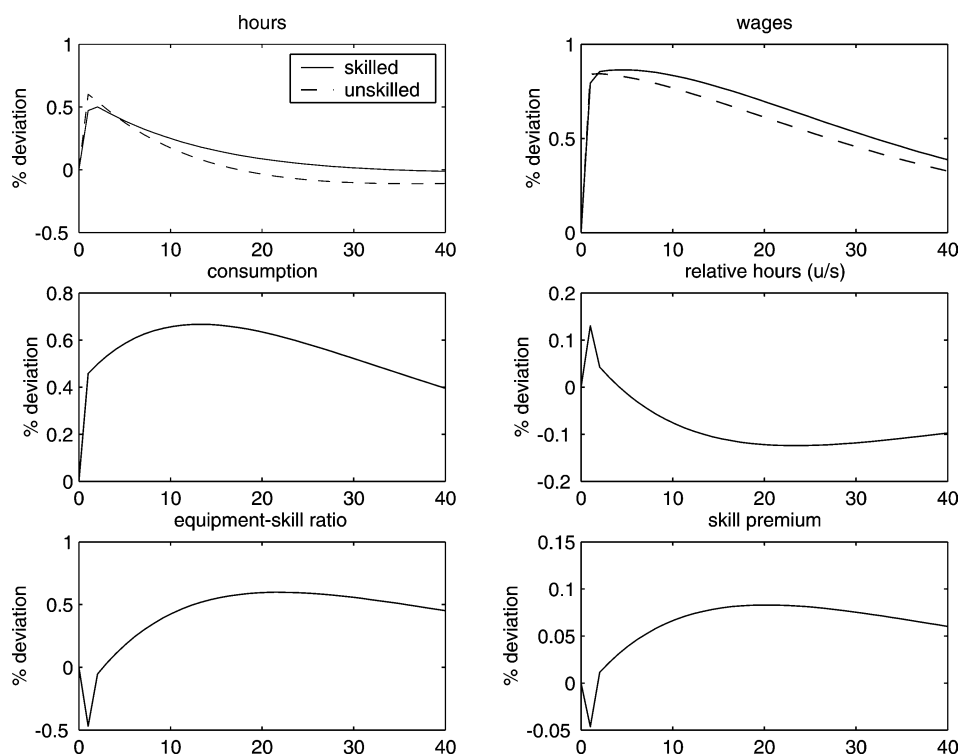


Fig. 3. Impulse response of per capita and relative variables to a neutral productivity shock.

<sup>20</sup> The numbers in parentheses are standard errors.

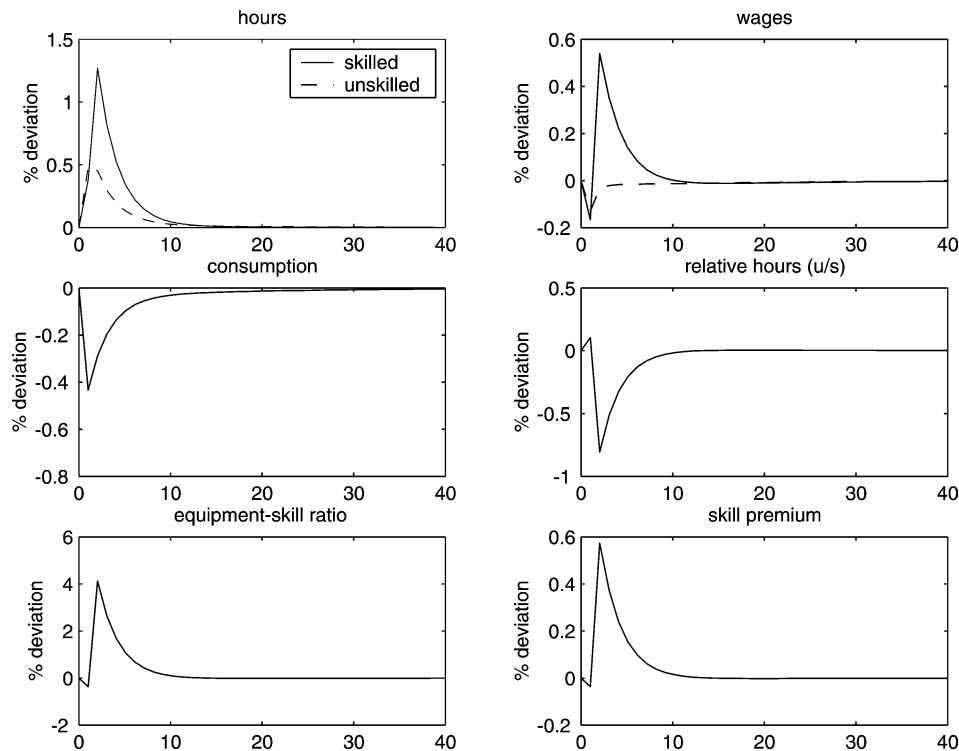


Fig. 4. Impulse response of per capita and relative variables to an investment-specific productivity shock.

shocks. It then increases as the capital stock increases. Thus, the relative supply effect and the capital–skill complementarity effect move in opposite directions in response to neutral productivity shocks, resulting in a skill premium which is weakly, negatively correlated with contemporaneous output and exhibits a significant, positive lag. The shape of the impulse response function of the skill premium shows us that the capital–skill complementarity effect dominates the relative supply effect.

In Fig. 4, we see that wages and consumption initially fall in response to a positive shock to investment-specific technology. Falling wages are due to an increase in the supply of both types of labor. For unlike neutral technology shocks, which enter the production function directly, investment-specific shocks cannot support wages in the face of an increasing supply of labor. But, despite this drop in wages, total labor income still goes up. The added income is used together with savings from lower consumption to invest in new capital equipment, which is now more profitable. Once this new capital equipment is in place, the capital–skill ratio jumps up, as does the skilled wage. This results in a distinctly procyclical movement in the skill premium, since the capital–skill complementarity effect dominates the relative supply effect.

The impulse response functions from the model illustrate how dissimilar the wage dynamics associated with these two different types of shocks are. Investment-specific shocks produce a procyclical skill premium, while neutral shocks produce a skill premium



Table 5  
The skill premium and relative factor inputs in the model

$x$	Volatility <sup>a</sup>	Cross-correlations of output ( $t$ ) with $x(t+i)$			
		$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+4)$
Skill premium	0.26	−0.07	0.06	0.07	0.18
Capital–skill ratio	1.84	−0.13	−0.01	0.02	0.19
Relative labor ratio	0.38	0.19	0.10	0.03	−0.19

<sup>a</sup> Volatility is measured as the standard deviation of perceptual fluctuations around trend.

that is weakly countercyclical and that exhibits a strong positive lag. Together, they produce the results reported in Table 5. The model balances the two types of shocks in order to produce a skill premium that is uncorrelated with contemporaneous output and that lags the business cycle, just as it does in the data.

The volatility of the model skill premium is driven mainly by investment-specific shocks. The model, however, is only able to explain 20 percent of the volatility observed in the data (compare Tables 1 and 5). In the following section, we shall see that this shortcoming arises from a rather conservative calibration of the model. It can be remedied by increasing the degree of capital–skill complementarity in the model and by strengthening investment specific shocks.

#### 4.3.1. Sensitivity analysis

In this section, a sensitivity analysis is carried out which demonstrates that the qualitative results that speak in favor of the model with capital–skill complementarity are robust to substantial parameter changes.<sup>21</sup> It also shows that the skill premium in a model without capital–skill complementarity bears no resemblance to the data. Furthermore, the analysis demonstrates that the model can be calibrated in such a way as to raise the volatility of the model skill premium so that it matches that found in the data. But, this can only be done by shocking the model with arbitrarily large investment-specific shocks.

The first exercise concerns the behavior of the skill premium in a model without capital–skill complementarity. Here, it is assumed that production is given by a four-factor, Cobb–Douglas production function,  $k_{s,t}^{\theta_{ks}} k_{e,t}^{\theta_{ke}} h_{s,t}^{\theta_s} h_{u,t}^{\theta_u}$ , with  $\theta_{ks} = 0.13$ ,  $\theta_{ke} = 0.17$ ,  $\theta_u = 0.29$ , and  $\theta_s = 0.41$ . Everything else in the model is left unchanged. In this case, the skill premium is a function of relative hours worked only.

In Table 6, we see that the volatility of the skill premium in this model is almost zero. The skill premium is also strongly procyclical, which, of course, is not true for the data. In short, the model without capital–skill complementarity is not able to explain the cyclical behavior of the skill premium.<sup>22</sup>

<sup>21</sup> Systematic changes in the degree of risk aversion, the subjective discount rate and the depreciation rates of structures and equipment were also examined. They had no significant impact on the results and are, therefore, not reported here.

<sup>22</sup> In this model, setting  $u = s = 0.5$  results in a constant skill premium. Lowering the proportion of skilled workers in the economy raises the volatility of the skill premium. The skill premium, however, is always strongly procyclical.

Table 6  
Sensitivity analysis<sup>a</sup>

$x$	Volatility <sup>b</sup>	Cross-correlations of output ( $t$ ) with $x(t+i)$				
		$x(t)$	$x(t+1)$	$x(t+2)$	$x(t+4)$	$x(t+5)$
C–D production $f$	0.03	0.94	0.59	0.32	–0.04	–0.15
Dejong et al. (2000)	0.11	0.00	0.27	0.28	0.25	0.22
Strong CSC	0.63	0.04	0.05	0.13	0.18	0.18
strong CSC & strong IS shocks	1.39	0.14	–0.16	0.08	0.25	0.25

<sup>a</sup> C–D = Cobb–Douglas, CSC = capital–skill complementarity, IS = investment-specific.

<sup>b</sup> Volatility is measured as the standard deviation of perceptual fluctuations around trend.

The next experiment involves a recalibration of the model using parameters from Dejong et al. (2000). Their study examines the importance of investment-specific and neutral technology shocks for business cycle fluctuations. But, in contrast to the off-the-shelf calibration method used in this paper, they estimate the parameters of their model directly. This makes it interesting to see how the model in this paper performs using their parameter values, particularly since their estimates of the technological processes are quite different from the calibrated values used in this paper.<sup>23</sup>

Following Dejong et al. (2000),  $\gamma$  is set to 1.428 and  $\beta$  to 0.989. They estimate a capital share of 0.285, which, in this experiment, is split into a capital structures share,  $\theta = 0.12$ , and capital equipment share of 0.165. They estimate a quarterly depreciation rate of approximately 2 percent. Here, the depreciation rate for structures is set to 1.5 percent and the depreciation rate for equipment is set to 2.5 percent. The autoregressive parameter  $\rho_z$  is set equal to 0.979,  $\rho_q$  to 0.949 and  $\rho_{zq}$  is set to zero. Their estimate of  $\sigma_z$  is 0.00346 and  $\sigma_q$  is 0.00219. These are scaled up by a constant until the volatility of output in the model matches that in the data.

The model calibrated using the Dejong et al. (2000) estimates retains the qualitative conclusions from the benchmark model (compare Tables 5 and 6). The skill premium is uncorrelated with contemporaneous output and it has a significant, positive lag. The volatility of the skill premium, however, is reduced due to the change in the  $\rho_q$  from 0.64 to 0.949.

Now, let us turn our attention to the substitution parameters  $\nu$  and  $\varphi$ , which are central to the analysis in this paper. Estimates of their values were taken from Krusell et al. (2000) in order to calibrate the model. Their estimates imply elasticities of substitution of 1.67 between skilled and unskilled labor and between unskilled labor and capital, and of 0.67 between skilled labor and capital. These estimates may, however, be somewhat on the conservative side, at least when compared to the existing literature.

If we examine the studies summarized in Hamermesh (1993), and restrict ourselves to those which use aggregate or large industry data for the United States, we find that the average elasticity of substitution between production workers and capital is equal to 1.96 (the highest reported estimate is equal to 2.92 and the lowest is equal to 0.91). The

<sup>23</sup> Keep in mind, however, that the model they estimated is also quite different from the model in this paper.

average elasticity of substitution between production and non-production workers is equal to 3 (the highest reported estimate is equal to 5.51 and the lowest is 0.49).<sup>24</sup> Values of the elasticity of substitution between skilled labor and capital range down to 0.04 in the studies summarized by Hamermesh (1993).

Setting  $\nu$  equal to 0.756 and  $\varphi$  to  $-2$  implies an elasticity of substitution between skilled and unskilled labor and between capital and unskilled labor of 4.1<sup>25</sup> and an elasticity of substitution between skilled labor and capital of 0.33. Using these new values raises the volatility of the skill premium in the model to 0.63 percent, while leaving the two main qualitative results unchanged. The skill premium remains uncorrelated with contemporaneous output and it still lags the business cycle. Thus, strengthening the degree of capital–skill complementarity increases the volatility of the model skill premium.

Retaining the strong degree of capital–skill complementarity, from the previous experiment, while simultaneously strengthening the relative importance of investment-specific shocks by setting  $\sigma_z$  to 0.0093,  $\sigma_q$  to 0.0085 and  $\rho_{zq}$  to  $-0.75$ , results in a volatility of 1.39 percent (see Table 6). These parameters, however, are simply chosen in order to produce the desired results. That is, we can produce a higher volatility by choosing arbitrarily large investment-specific shocks. In turn, these shocks must be balanced against the neutral shocks by making their correlation more strongly negative.<sup>26</sup>

#### 4.3.2. Capital–skill complementarity and implicit contracts

Table 7 shows us that wages in the data are essentially uncorrelated with contemporaneous output. The skilled wage also exhibits a much stronger lag than the unskilled wage. This paper argues that this lag in the skilled wage (and, hence, the skill premium) is due to the lag in the capital–skill ratio and to the presence of capital–skill complementarity in production.

Table 8 shows us that wages in the model are far too procyclical when compared with the data. So, while the model is able to reproduce the cyclical behavior of relative wages, it is not able to reproduce the cyclical behavior of the underlying wage series. This is a common failure of Walrasian real business cycle models.

In the model, skilled hours are more volatile than unskilled hours worked,<sup>27</sup> while wages are equally volatile. In the data, total hours worked are actually less volatile for skilled workers than for unskilled workers. Wages for skilled workers, on the other hand, appear to be more volatile. These observations, together with quite low contemporaneous correlations between wages and output, suggest that the labor market may be better characterized as a market with implicit contracts than as a Walrasian market place.

<sup>24</sup> It is important to note, however, that not all of the elasticities in these studies are directly comparable to the substitution elasticities in this model. The substitution elasticities in the model are direct elasticities, while in Hamermesh's review, there are a good number of Allen partial elasticities as well.

<sup>25</sup> This is the direct elasticity between manual and non-manual workers estimated by Dougherty (1972).

<sup>26</sup> The volatility of the skill premium in the annual data set of Krusell et al. (2000) is only two-thirds that of output. In this case, the model can match the volatility by simply raising  $\nu$  from 0.401 to 0.756 (see Lindquist, 2002).

<sup>27</sup> Keep in mind that  $s$  and  $u$  are held constant in the model (i.e. there is no unemployment). So, movements in  $h_{s,t}$  and  $h_{u,t}$  in the model are equivalent to movements in total hours,  $s_t h_{s,t}$  and  $u_t h_{u,t}$ , in the data.

Table 7  
The cyclical behavior of wages and hours in the data

$x$	Volatility <sup>a</sup>	Cross-correlations of output ( $t$ ) and $x(t+i)$			
		$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+4)$
Skilled wages	1.63 (0.110) <sup>b</sup>	0.08 (0.102)	0.15 (0.094)	0.24 (0.082)	0.43 (0.084)
Unskilled wages	1.21 (0.133)	0.05 (0.095)	0.01 (0.088)	0.10 (0.077)	0.19 (0.090)
Skilled hours ( $s_t h_{s,t}$ )	1.68 (0.144)	0.38 (0.082)	0.42 (0.076)	0.36 (0.081)	-0.08 (0.092)
Unskilled hours ( $u_t h_{u,t}$ )	2.12 (0.150)	0.53 (0.076)	0.66 (0.063)	0.66 (0.062)	0.43 (0.080)

<sup>a</sup> Volatility is measured as the standard deviation of perceptual fluctuations around trend.

<sup>b</sup> The numbers in parentheses are standard errors which have been calculated by bootstrapping.

Table 8  
The cyclical behavior of wages and hours in the model

$x$	Volatility <sup>a</sup>	Cross-correlations of output ( $t$ ) and $x(t+i)$			
		$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+4)$
Skilled wage	0.84	0.60	0.97	0.70	0.17
Unskilled wage	0.84	0.62	0.95	0.67	0.11
Skilled hours	0.63	0.55	0.81	0.51	0.03
Unskilled hours	0.58	0.72	0.95	0.57	-0.09

<sup>a</sup> Volatility is measured as the standard deviation of perceptual fluctuations around trend.

We know, however, that preferences and technology matter for the optimal contract (Rosen, 1985). So, the demand side effect, which arises in the presence of capital–skill complementarity, will influence the design of such a contract. Thus, the main result presented in this paper cannot be disregarded by the contracting literature. In fact, the introduction of capital–skill complementarity, or of the more general concept of skill-biased technological change, into the contracting framework could prove to be a fruitful line of research similar in spirit to the research of Boldrin and Horvath (1995), Danthine and Donaldson (1995), and Gomme and Greenwood (1993).<sup>28</sup>

## 5. Conclusion

When capital–skill complementarity is present in the production process, changes in the skill premium are no longer driven by changes in the ratio of unskilled to skilled labor inputs alone. They are also driven by changes in the capital–skill ratio. In US data, both of these factor input ratios have distinct cyclical patterns. A simple regression analysis

<sup>28</sup> Boldrin and Horvath (1995), for example, have shown that contracts can be introduced successfully into the RBC framework to improve its performance in reproducing labor market statistics.

demonstrates that the capital–skill ratio has a positive and significant relation to the skill premium at business cycle frequencies as predicted by the capital–skill complementarity hypothesis. This empirical finding motivates the construction of a stochastic dynamic general equilibrium model that allows for capital–skill complementarity in production. The model with capital–skill complementarity can account for the cyclical behavior of the skill premium and much of its volatility. The model without capital–skill complementarity cannot. The findings of this study suggest that capital–skill complementarity is an important determinant of wage inequality over the business cycle.

Previous research has shown that the capital–skill complementarity mechanism illustrated in this paper can be used successfully to help us understand increasing trends in wage inequality (Krusell et al., 2000; Lindquist, 2001) and increasing wage dispersion (Caselli, 1999). Together, these and other studies, allow us to conclude that capital–skill complementarity is an important ingredient in a successful, competitive theory of relative wages and that such a theory can, in fact, help us to understand changes in the structure of relative wages.

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