# HUMAN CAPITAL RISK AND ECONOMIC GROWTH*

TOM KREBS

This paper develops a tractable incomplete-markets model of economic growth in which households invest in risk-free physical capital and risky human capital. The paper shows that a reduction in uninsurable idiosyncratic labor income risk decreases physical capital investment, but increases human capital investment, growth, and welfare. A quantitative analysis based on a calibrated version of the model reveals that these effects are substantial and of the same order of magnitude as the effects of distortionary income taxation. The analysis further suggests that government-sponsored severance payments to displaced workers increase growth and welfare even if these payments have to be financed through distortionary income taxation.

## I. INTRODUCTION

There is considerable evidence that individual households face a substantial amount of uninsurable labor income risk. This observation has important economic implications. For example, labor income risk generates precautionary saving if households are prudent. Moreover, labor income risk affects the incentives to invest in human capital if households are risk-averse. Although the first effect on saving and physical capital accumulation has been analyzed extensively,[1] the second effect on human capital accumulation has received surprisingly little attention.[2] This paper takes a first step toward the systematic study of idiosyncratic labor income risk in human capital models of economic growth. More specifically, this paper develops a tractable incomplete-markets model of economic growth and uses the model to analyze the qualitative and quantitative effects of uninsurable idiosyn-

* This paper previously circulated under the title "Idiosyncratic Risk, Aggregate Saving, and Economic Growth." I would like to thank the editor (Alberto Alesina) and an anonymous referee for many insightful comments. Section IV and Appendix 2 are based on their suggestions. I also wish to thank for helpful comments Oded Galor, Peter Howitt, Mark Huggett, Pravin Krishna, David Laibson, Tomo Nakajima, Herakles Polemarchakis, David Weil, and seminar participants at several universities and conferences. All remaining errors are mine. Financial support from the Salomon Research Grant, Brown University, is gratefully acknowledged.

1. See, for example, Carroll [1997], Deaton [1991], and Zeldes [1989] for quantitative micro models and Aiyagari [1994], Caballero [1991], and Huggett [1996] for quantitative macro models. See Huggett and Ospina [2001] for a recent survey.

2. The large literature on human capital investment has typically ignored risk-related considerations. For notable exceptions, see the early contributions by Eaton and Rosen [1980] and Levhari and Weiss [1974] and the more recent work by Benabou [2002] and Pries [2001].



*The Quarterly Journal of Economics,* May 2003

cratic labor income risk on investment in physical and human capital, growth, and welfare.

The model developed in this paper is an incomplete-markets version of the type of human capital model that has been popular in the recent endogenous growth literature.[3] The model assumes that production displays constant-returns-to-scale with respect to physical and human capital, and that there are many ex ante identical, infinitely lived households with log-utility preferences. These households make a consumption/saving decision and a decision regarding the allocation of total saving between physical capital and human capital. Investment in physical capital is risk-free, but investment in human capital is risky due to idiosyncratic shocks to the stock of human capital. In other words, there is no capital income risk, but there is labor income risk because human capital investment is risky. Markets are incomplete in the sense that there are no explicit insurance markets for human capital risk.

The theoretical analysis shows that a reduction in human capital risk decreases investment in physical capital, but increases investment in human capital, economic growth, and welfare. Thus, whereas the literature on precautionary saving suggests that any government policy aimed at reducing labor income risk is bad for (transitional) growth, the current paper shows that this conclusion is likely to be reversed once the human capital choice is endogenized. The intuition for this result is as follows. A reduction in human capital risk induces households to invest a larger fraction of total saving in human capital and a smaller fraction in physical capital. As a consequence of this change in the composition of households' assets, the total return on investment in physical and human capital increases since the expected return on risky human capital investment exceeds the return on risk-free physical capital investment. This increase in total investment return raises economic growth if total saving in physical and human capital does not decrease. For the log-utility case considered in this paper, total saving as a fraction of total wealth is independent of risk, and a reduction in idiosyncratic labor income risk is therefore growth-enhancing.

This paper also conducts a quantitative analysis of the mac-

3. More precisely, the model is an incomplete-markets version of the class of convex growth models first analyzed by Jones and Manuelli [1990] and Rebelo [1991]. For textbook treatments, see Aghion and Howitt [1998] and Barro and Sala-i-Martin [1995].

roeconomic effects of uninsurable idiosyncratic labor income risk based on a calibrated version of the model. The analysis reveals that the effects on investment, growth, and welfare are substantial and of the same order of magnitude as the effects of distortionary income taxation.[4] Indeed, the analysis suggests that the gains from additional labor income insurance outweigh the cost of financing the insurance payments through distortionary income taxation. More specifically, a government program that makes severance payments to displaced workers would increase both growth and welfare by a significant amount even if these payments had to be financed through distortionary income taxation.

As noted before, an important assumption underlying the present analysis is that human capital investment is riskier than physical capital investment. This assumption seems plausible given that many types of financial investments are close to risk-free (Treasury Bills) or free of nondiversifiable idiosyncratic risk (diversified stock portfolio), whereas human capital investment has an inherently nondiversifiable idiosyncratic risk component. However, the stronger assumption that all physical capital investment is free of nondiversifiable idiosyncratic risk is clearly at odds with reality. Appendix 2 therefore contains a discussion of a two-sector version of the model in which households can invest their savings of physical capital either in a production technology without nondiversifiable idiosyncratic risk (corporate sector) or in a production technology with nondiversifiable idiosyncratic risk (entrepreneurial sector). A quantitative analysis of the calibrated model economy shows that the introduction of this second production sector reduces the effects of uninsurable idiosyncratic labor income risk on investment, growth, and welfare, but that the main results of the paper still remain valid: uninsurable idiosyncratic labor income risk is likely to be detrimental to growth; its effect on investment, growth, and welfare is quantitatively important; and for realistic levels of idiosyncratic labor income risk the gains from additional labor income insurance outweigh the cost of financing these insurance payments through distortionary income taxation.

4. These distortionary effects have been extensively studied by the quantitative literature on tax reform in representative-agent models of economic growth. See, for example, Jones, Manuelli, and Rossi [1993], Lucas [1990], Rebelo [1991], and Stokey and Rebelo [1995]. Aiyagari [1995] conducts a qualitative analysis of the optimal tax policy in an incomplete-markets model with exogenous human capital.

In contrast to the results reported here, several previous papers have found that the quantitative effects of uninsurable idiosyncratic risk on aggregate saving and asset prices are negligible.[5] This difference arises because the current paper allows for human capital shocks that amount to permanent labor income shocks; that is, in equilibrium the individual labor income process follows (approximately) a logarithmic random walk. Consequently, self-insurance is a very ineffective means to smooth consumption, and the macroeconomic consequences of market incompleteness are therefore substantial. The empirical importance of permanent income shocks has been well documented,[6] and this paper uses the estimates obtained by the empirical literature on labor income risk as a guideline for calibrating the model economy. The model's emphasis on permanent income shocks, however, has one drawback: the cross-sectional distributions of income (consumption) and wealth are diverging (random walk!). Thus, whereas most previous macro models predict too little cross-sectional dispersion, the current model predicts too much cross-sectional dispersion. This, however, is not a serious shortcoming since there are plausible extensions of the model that retain the assumption of permanent income shocks but lead to equilibria with stationary cross-sectional distributions of income and wealth.[7]

The current paper is closely related to the work on entrepreneurial risk or country risk and economic growth.[8] In all these models, the focus is on the allocation of capital between a low risk-low return investment opportunity and a high risk-high return investment opportunity. Despite this similarity, there are also important differences. From an economic point of view, the main difference is a difference in interpretation: the current

5. See, for example, Aiyagari [1994], Heaton and Lucas [1996], and Krusell and Smith [1998].

6. See, for example, Carroll and Samwick [1997], Hubbard, Skinner, and Zeldes [1994], Meghir and Pistaferri [2001], and Storesletten, Telmer, and Yaron [2001].

7. For example, Constantinides and Duffie [1996] show that the introduction of uncertain lifetimes generates stationary cross-sectional distributions of income and wealth even if individual income follows a logarithmic random walk, and Acemoglu and Ventura [2002] demonstrate how price (terms-of-trade) effects can produce a stable world income distribution in a world in which individual countries have access to linear technologies. Notice also that there is *some* empirical evidence against the stationarity hypothesis [Deaton and Paxson 1994].

8. For the former, see the papers by Acemoglu and Zilibotti [1997], Angeletos and Calvet [2001], Bencivenga and Smith [1991], Greenwood and Jovanovic [1990], and King and Levine [1993]. For the latter, see the work by Devereux and Smith [1994] and Obstfeld [1994].

model emphasizes the choice between physical and human capital, whereas the previous literature dealt with the choice between two (or more) types of physical capital investment. Besides this difference in economic interpretation, there are also two important formal differences. First, most of the previous work is based on a linear one-factor model, whereas the current paper uses a constant-returns-to-scale technology with two complementary factors of production (capital and effective labor). The two-factor neoclassical production function employed here implies that changes in government policy may affect the interest rate and wages through their effect on the allocation of capital and effective labor. For the type of policy changes analyzed in this paper, these price effects turn out to be quantitatively important.

The second formal difference between the current paper and the literature on entrepreneurial or country risk relates to the financial market structure. More specifically, the previous literature is based on the joint assumption of incomplete insurance markets and imperfect credit markets (ad hoc borrowing constraints), and therefore cannot disentangle the macroeconomic effects of missing insurance markets from the effects of credit market imperfections.[9] In contrast, the model developed in this paper combines the assumption of incomplete insurance markets with the assumption of well-functioning credit markets, and therefore allows for a clear separation between market incompleteness and other asset market imperfections. Consequently, the growth and welfare effects of idiosyncratic human capital risk reported in this paper are due to market incompleteness per se.

## II. Model

### II.A. Economy

Consider a discrete-time, infinite-horizon economy with one nonperishable good that can be consumed or invested. There is one firm that produces the "all-purpose" good. The firm combines physical capital $k$, with human capital $h$, to produce output, $y =$

9. Obstfeld [1994] is a prominent example. Many papers use an OLG-framework with two-period lived agents, which can be reinterpreted as a model with infinitely lived agents and ad hoc borrowing constraints [Woodford 1986]. For a definition and discussion of ad hoc borrowing constraints, see Aiyagari [1994] and Ljungqvist and Sargent [2000]. Notice that the separation of market incompleteness from other asset market imperfections has a long tradition in the incomplete markets literature [Magill and Quinzii 1996].

$F(k,h)$, where all variables are expressed in per-household terms. The function $F$ is a standard neoclassical production function. In particular, it displays constant-returns-to-scale. The firm rents the two input factors (physical and human capital) in competitive markets. We denote the rental rate of physical capital in period $t$ by $\tilde{r}_{kt}$ and the rental rate of human capital (the wage rate per efficiency unit of labor) in period $t$ by $\tilde{r}_{ht}$. In each period, the firm hires capital and labor up to the point where current profit is maximized. Thus, the firm solves the following static maximization problem:

$$\max_{k_t,h_t} \{F(k_t,h_t) - \tilde{r}_{kt}k_t - \tilde{r}_{ht}h_t\}. \tag{1}$$

There are many ex ante identical, infinitely lived households indexed by $i$. We abstract from population growth and normalize the total mass of households to one. Let $k_{it}$ and $h_{it}$ stand for the stock of physical and human capital owned by household $i$, and let $x_{kit}$ and $x_{hit}$ stand for the investment in physical and human capital by household $i$. If we denote household $i$'s consumption by $c_{it}$, then the sequential budget constraint reads

$$\begin{aligned} c_{it} + x_{kit} + x_{hit} &= \tilde{r}_{kt}k_{it} + \tilde{r}_{ht}h_{it} \\ k_{i,t+1} &= (1-\delta_k)k_{it} + x_{kit},\ k_{it} \geq 0 \\ h_{i,t+1} &= (1-\delta_h + \eta_{it})h_{it} + x_{hit},\ h_{it} \geq 0 \\ &(k_{i0},h_{i0}) \text{ given,} \end{aligned} \tag{2}$$

where $\delta_k$ and $\delta_h$ denote the average depreciation rate of human and physical capital, respectively. The term $\eta_{it}$ describes a household-specific shock to human capital. We assume that these idiosyncratic shocks are identically and independently distributed across households and across time (unpredictability of idiosyncratic shocks).

The budget constraint (2) makes three implicit assumptions about the accumulation of human capital. First, it lumps together general human capital (education, health) and specific human capital (on-the-job training). Second, it neglects the decision of households to allocate a fixed amount of time across different activities. Third, (2) does not impose a nonnegativity constraint on human capital investment ($x_{hit} \geq 0$).

The random variable $\eta_{it}$ represents uninsurable idiosyncratic labor income risk. A negative human capital shock, $\eta_{it} < 0$, can occur when a worker loses firm- or sector-specific human capital subsequent to job termination (worker displacement). In order to preserve the tractability of the model, the budget constraint (2) rules out extended periods of unemployment because it assumes that the wage payment is received in each period. Thus, the emphasis is on earnings uncertainty, not employment uncertainty. A decline in health (disability) provides a second example for a negative human capital shock. In this case, both general and specific human capital might be lost. Internal promotions and upward movement in the labor market provide two examples of positive human capital shocks ($\eta_{it} > 0$).

The budget constraint (2) permits households to save ($x_{kit} > 0$) and dissave ($x_{kit} < 0$) at the going interest rate, but rules out the possibility of negative financial wealth (no credit market). Thus, one might conjecture that the equilibrium will change once households are allowed to accumulate debt. However, this is not the case for the model analyzed here. More precisely, the introduction of a risk-free bond in zero net supply (default-free credit) does not change the equilibrium allocation (see equation (10)).

The budget constraint can be rewritten in a way that shows how the households's optimization problem is basically a standard intertemporal portfolio choice problem. To see this, define the following variables: $w_{it} \doteq k_{it} + h_{it}$ (total wealth) and $\tilde{k}_{it} \doteq k_{it}/h_{it}$ (the capital-to-labor ratio). With this new notation, the fraction of total wealth invested in physical capital is $\theta(\tilde{k}_{it}) = \tilde{k}_{it}/(1 + \tilde{k}_{it})$ and the fraction of total wealth invested in human capital is $1 - \theta(\tilde{k}_{it}) = 1/(1 + \tilde{k}_{it})$. Introduce further the following (average) rates of return on the two investment opportunities: $r_{kt} \doteq \tilde{r}_{kt} - \delta_k$ and $r_{ht} \doteq \tilde{r}_{ht} - \delta_h$. Using this notation, the budget constraint reads

$$
\begin{aligned}
w_{i,t+1} &= [1 + \theta(\tilde{k}_{it})r_{kt} + (1 - \theta(\tilde{k}_{it}))(r_{ht} + \eta_{it})]w_{it} - c_{it} \\
&w_{it} \geq 0,\ \tilde{k}_{it} \geq 0, \\
&(w_{i0}, \tilde{k}_{i0}) \text{ given.}
\end{aligned}
\tag{3}
$$

Households have identical preferences over consumption plans $\{c_{it}\}$. These preferences allow for a time-additive expected utility representation with one-period utility function of the logarithmic type:

$$U(\{c_{it}\}) = E\left[\sum_{t=0}^{\infty} \beta^t \log c_{it}\right]. \tag{4}$$

*II.B. Equilibrium*

An equilibrium is a list of sequences, $\{r_{kt},r_{ht}\}$, $\{k_t,h_t\}$, and $\{c_{it},w_{it},\tilde{k}_{it}\}$, so that $\{k_t,h_t\}$ maximizes profit (solves 1), $\{c_{it},w_{it},\tilde{k}_{it}\}$ maximize expected lifetime utility (4) subject to the sequential budget constraint (3), and markets clear. A stationary equilibrium is an equilibrium with $r_{kt} = r_k$ and $r_{ht} = r_h$. We now construct a stationary equilibrium.

Introduce the aggregate capital-to-labor ratio $\tilde{k}_t \doteq k_t/h_t$ and the production function $f = f(\tilde{k})$ with $f(\tilde{k}) \doteq F(\tilde{k}, 1)$. The first-order conditions associated with the firm's static maximization problem are

$$\begin{aligned} r_{kt} &= f'(\tilde{k}_t) - \delta_k \\ r_{ht} &= f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t) - \delta_h. \end{aligned} \tag{5}$$

Clearly, equation (5) shows that in a stationary equilibrium we must have $\tilde{k}_t = \tilde{k}$. Let $r_k = r_k(\tilde{k})$ and $r_h = r_h(\tilde{k})$ be the return functions defined by the first-order conditions (5), and let $r_{it} = r(\tilde{k}_{it},\tilde{k},\eta_{it})$ be the total investment return defined as $r_{it} = \theta(\tilde{k}_{it})r_k(\tilde{k}) + (1 - \theta(\tilde{k}_{it}))(r_h(\tilde{k}) + \eta_{it})$.

The Euler equations associated with the household's utility maximization problem read

$$\begin{aligned} c_{it}^{-1} &= \beta E[(1 + r(\tilde{k}_{it},\tilde{k},\eta_{i,t+1}))c_{i,t+1}^{-1}] \\ 0 &= E[(r_h(\tilde{k}) + \eta_{i,t+1} - r_k(\tilde{k}))c_{i,t+1}^{-1}]. \end{aligned} \tag{6}$$

The first equation in (6) says that the utility cost of investing (saving) one more unit of the good must be equal to the expected discounted utility gain of doing so, and the second equation states the equality of expected (marginal utility weighted) returns on the two investment opportunities. Notice that we take the unconditional expectations over $\eta_{i,t+1}$ in (6) because of the assumption that idiosyncratic shocks are independently distributed over time. It is straightforward to show that any plan satisfying the budget constraint (3) yields finite expected lifetime utility. Hence, any solution to (3) and (6) that satisfies a corresponding transversality condition is also a solution to the utility maximization problem. Direct calculation shows that the plan $c_{it} = (1 - \beta)r(\tilde{k}_i,\tilde{k},\eta_{it})w_{it}$ solves (3), (6), and a corresponding transversality condition if $\tilde{k}_i$ is the solution to

$$E\left[\frac{r_h(\tilde{k}) + \eta - r_k(\tilde{k})}{1 + r(\tilde{k}_i,\tilde{k},\eta)}\right] = 0. \tag{7}$$

In (7) we dropped the time- and household-index on $\eta_{i,t+1}$ because of the i.i.d. assumption, and we dropped the time index on $\tilde{k}_{it}$ to indicate the time-independence of the solution.

The market clearing condition can be written as

$$\frac{(1/I)\ \Sigma_i\ \theta(\tilde{k}_{it})w_{it}}{(1/I)\ \Sigma_i\ (1 - \theta(\tilde{k}_{it}))w_{it}} = \tilde{k}_t. \tag{8}$$

Notice that budget constraint (2) and market clearing condition (8) imply the market clearing conditions $(1/I)\ \Sigma_i\ k_{it} = k_t$ and $(1/I)\ \Sigma_i\ h_{it} = h_t$ as well as $y_t = c_t + x_{kt} + x_{ht}$. Suppose now that $\tilde{k}_i = \tilde{k}$; that is, the stationary equilibrium is symmetric. Clearly, in this case the market clearing condition (8) is automatically satisfied. Moreover, $\tilde{k}_i = \tilde{k}$ is a utility maximizing choice if it solves (7), which now becomes

$$E\left[\frac{r_h(\tilde{k}) + \eta - r_k(\tilde{k})}{1 + r(\tilde{k},\tilde{k},\eta)}\right] = 0. \tag{9}$$

The assumption that the production function is a neoclassical production function implies that the return functions $r_k = r_k(\tilde{k})$, $r_h = r_h(\tilde{k})$, and $r = r(\tilde{k},\tilde{k},\eta)$ exhibit properties which ensure the existence and uniqueness of a solution to (9). Thus, we have found a stationary equilibrium. We summarize the preceding discussion in the following proposition.

PROPOSITION 1. Let $\tilde{k}$ be the unique solution to (9), $r_k(\tilde{k})$ and $r_h(\tilde{k})$ be the values of the return functions (5) at $\tilde{k}$, and $r(\tilde{k},\tilde{k},\eta_{it}) = \theta(\tilde{k})r_k(\tilde{k}) + (1 - \theta(\tilde{k}))(r_h(\tilde{k}) + \eta_{it})$ with $\theta(\tilde{k}_{it}) = \tilde{k}_{it}/(1 + \tilde{k}_{it})$. Then the following is a stationary equilibrium:

i) Equilibrium allocation:

$$\begin{aligned} \tilde{k}_{it} &= \tilde{k} \\ w_{i,t+1} &= \beta[1 + r(\tilde{k},\tilde{k},\eta_{it})]w_{it} \\ c_{it} &= (1 - \beta)[1 + r(\tilde{k},\tilde{k},\eta_{it})]w_{it} \end{aligned}$$

$$k_{it} = \theta(\tilde{k})w_{it}$$
$$h_{it} = (1 - \theta(\tilde{k}))w_{it};$$

ii) Equilibrium asset returns:

$$r_{kt} = r_k(\tilde{k}), r_{ht} = r_h(\tilde{k}).$$

An interesting question is why in equilibrium ex post different households choose the same ratio $\tilde{k}_{it} = \tilde{k}$. To understand the intuition behind this strong result, notice first that for given returns, the decision problem of each individual household is a standard intertemporal portfolio choice problem with two assets (linear investment opportunities), one risk-free asset (physical capital) with return $r_k$ and one risky asset (human capital) with return $r_h + \eta_{i,t+1}$. Because households have CRRA-preferences, the optimal portfolio share $\theta_{it}$, and therefore the optimal ratio $\tilde{k}_{it}$, is independent of the wealth level $w_{it}$. Because the random variable $\eta_{i,t+1}$ is serially uncorrelated and identically distributed across households, the optimal portfolio share depends neither on $\eta_{it}$ nor on $i$. In short, the optimal portfolio share, and therefore the optimal capital-to-labor ratio, is the same for all households.

The expression for equilibrium consumption immediately implies that the introduction of a risk-free bond in zero net supply (borrowing and lending) will have no effect on the equilibrium allocation. More precisely, if households have the opportunity to trade a bond in zero net supply, then the equilibrium bond price,

$$Q = \beta E[(c_{i,t+1}/c_{it})^{-1}] = E[1 + r(\tilde{k},\tilde{k},\eta)], \tag{10}$$

ensures that the bond return is equal to the return on physical capital investment, $r_k = r_b$, and that households will optimally choose not to trade the bond. In other words, the equilibrium allocation in Proposition 1 is also the equilibrium allocation for an economy in which households can accumulate physical and human capital and trade a risk-free bond in zero net supply. This no-trade result is the production analog of the no-trade result of Constantinides and Duffie [1996]. The intuition for it is similar to the intuition underlying the result $\tilde{k}_{it} = \tilde{k}$. With CRRA-preferences and unpredictable shocks, the individual bond demand is the same across households. Thus, for bonds in zero net supply, the only way to achieve market clearing is through no trade.

### *II.C. Cobb-Douglas/Normal-Distribution Example*

Suppose that idiosyncratic shocks are normally distributed: $\eta_{it} \sim N(0,\sigma_\eta^2)$. Using the approximation $\log(1 + r_{i,t+1}) \approx r_{i,t+1}$ with $r_{i,t+1} = \theta_i r_k + (1 - \theta_i)(r_h + \eta_{i,t+1})$, we can write the second Euler equation in (6) as

$$E[e^{(1-\theta_i)\eta_i}(r_h + \eta_i - r_k)] = 0. \tag{11}$$

Integrating and solving for $\theta_i$, we find the following optimal portfolio choice (for given returns):

$$1 - \theta = (r_h - r_k)/\sigma_\eta^2, \tag{12}$$

where we dropped the index $i$ to indicate that portfolio choices are identical across households. Expression (12) is exactly the optimal portfolio choice for the corresponding continuous-time investment problem when returns follow a Brownian motion; that is, it is the solution to the Merton problem when the investor has log-utility preferences and there are two assets (one risky and one risk-free asset).

Suppose further that the production function is of the Cobb-Douglas type: $f(\tilde{k}) = A\tilde{k}^\alpha$. Using this assumption and $\theta = \tilde{k}/(1 + \tilde{k})$, we find

$$\frac{1}{1+\tilde{k}} = \frac{1}{\sigma_\eta^2}[((1-\alpha)A\tilde{k}^\alpha - \delta_h) - (\alpha A\tilde{k}^{\alpha-1} - \delta_k)]. \tag{13}$$

Equation (13) is the analog of equilibrium condition (9). It determines the equilibrium value of the capital-to-labor ratio $\tilde{k}$, from which the equilibrium values of all other variables follow as before (Proposition 1).

## III. Idiosyncratic Risk and Macroeconomic Outcome

This section studies the effect of changes in idiosyncratic risk on investment, growth, and welfare. The section begins with a general qualitative analysis, and then turns to the quantitative analysis based on a calibrated version of the model.

### *III.A. Qualitative Results*

Equation (9) determines a unique $\tilde{k}$ for each random variable $\eta$. A change in idiosyncratic risk amounts to a movement from an economy $\eta$ to an economy $\eta'$ inducing a corresponding movement from $\tilde{k}$ to $\tilde{k}'$. We assume that $\eta'$ is a mean-preserving spread of $\eta$;

that is, $\eta' = \eta + \epsilon$ with $E[\epsilon|\eta] = 0$. Thus, economy $\eta'$ has more idiosyncratic risk than economy $\eta$. Notice that for the special case $\eta \sim N(0, \sigma_\eta'^2)$ and $\eta' \sim N(0, \sigma_\eta^2)$, this means that $\sigma' > \sigma$. We assume that $\epsilon \neq 0$ to avoid the trivial case $\eta' \neq \eta$.

PROPOSITION 2. A mean-preserving spread in idiosyncratic human capital risk, $\eta \rightarrow \eta' = \eta + \epsilon$, increases the capital-to-labor ratio $\tilde{k}$ and the return to human capital (wage rate), $r_h(\tilde{k})$, and decreases the return to physical capital (interest rate), $r_k(\tilde{k})$, the growth rate, $E[c_{i,t+1}/c_{it}]$, and welfare $E[\Sigma_{t=0}^{\infty} \beta^t \log c_{it}]$.

*Proof.* See Appendix 1.

Proposition 2 holds for any random variables $\eta$ and $\eta'$ satisfying the stated condition, and its proof is based on manipulating equation (9). For the special case $\eta \sim N(0, \sigma_\eta^2)$ and $f(\tilde{k}) = A\tilde{k}^\alpha$, Proposition 2 also follows from implicitly differentiating equation (13) with respect to $\sigma_\eta^2$.

*III.B. Calibration*

The quantitative analysis is based on an economy with normally distributed human capital shocks, $\eta \sim N(0,\sigma_\eta^2)$, and a Cobb-Douglas production function, $f(\tilde{k}) = A\tilde{k}^\alpha$. We use $\alpha = .36$ to match capital's share in income and $\delta = .06$ (annually) as a compromise between the higher depreciation rate of physical capital used in the literature (but also see Cooley and Prescott [1995] for an argument that $\delta_k = .05$) and the probably lower depreciation rate of human capital. The values of the remaining parameters $A$, $\sigma_\eta^2$, and $\beta$ are chosen so that the model is roughly consistent with the U. S. evidence along three dimensions: saving, growth, and labor income risk. More specifically, we require that per capita consumption growth satisfies $\mu_g = E[c_{i,t+1}/c_{it}] - 1 = .02$ and that the implied saving rate is $s_k = x_{kt}/y_t = .25$. For the annual U. S. data on saving and growth, see Summers and Heston [1991]. Finally, we match observed labor income risk by requiring $\sigma_y = \sigma_\eta/(1 + \tilde{k}) = .15$.

The choice of $\sigma_y = \sigma_\eta/(1 + \tilde{k}) = .15$ is made to ensure consistency with the empirical results of a number of micro studies on labor income risk.[10] More specifically, in the model econ-

10. Notice that this implies a standard deviation of human capital returns of $\sigma_\eta = (1 + \tilde{k})\sigma_y = .2633$.

omy log-labor income of household $i$, $y_{hit}$, is given by $y_{hit} = (r_h + \delta)h_{it}$. Using the equilibrium condition (Proposition 1) $h_{i,t+1} = \beta[1 + \theta r_k + (1 - \theta)(r_h + \eta_{it}]h_{it}$, we find that

$$
\begin{aligned}
(14)\quad \log y_{hi,t+1} - \log y_{hit} &= \log h_{i,t+1} - \log h_{it} \\
&= \log \beta + \log (1 + \theta r_k + (1 - \theta)(r_h + \eta_{it})) \\
&\approx d + \tilde{\eta}_{it},
\end{aligned}
$$

where $d = \log \beta + \theta r_k + (1 - \theta)r_h$ and $\{\tilde{\eta}_{it}\}$ is a sequence of i.i.d. random variables with $\tilde{\eta}_{it} = (1 - \theta)\eta_{it}$. Hence, the logarithm of labor income follows (approximately) a random walk with drift $d$ and error term $\tilde{\eta}_{it} \sim N(0,\sigma_y^2)$, $\sigma_y = (1 - \theta)\sigma_\eta$.[11] The random walk specification is often used by the empirical literature to model the permanent component of labor income risk [Carroll and Samwick 1997; Hubbard, Skinner, and Zeldes 1994; Meghir and Pistaferri 2001; Storesletten, Telmer, and Yaron 2001]. Thus, their estimate of the standard deviation of the error term for the random walk component of annual labor income corresponds to the value of $\sigma_y = (1 - \theta)\sigma_\eta$. In our baseline model we use $\sigma_y = .15$, which lies on the lower end of the spectrum of estimates found by the empirical literature. For example, Carroll and Samwick and Hubbard, Skinner, and Zeldes find .15, Meghir and Pistaferri estimate .19, and Storesletten, Telmer, and Yaron have .25 (averaged over age-groups and, if applicable, over business cycle conditions).

There are at least two reasons why the above approach might underestimate human capital risk. First, a constant $\sigma_y = .15$ represents less uncertainty than a $\sigma_y$ that fluctuates with business cycle conditions and has a mean of .15. Second, the assumption of normally distributed innovations understates the amount of idiosyncratic risk households face if the actual distribution has a fat lower tail. For strong evidence for such a deviation from the normal-distribution framework, see Brav, Constantinides, and Geczy [2002], and Geweke and Keane [2000]. There is, however, also an argument that the current approach might overestimate human capital risk because it assumes that all labor income is

11. We have $\tilde{\eta}_{it}$ instead of $\tilde{\eta}_{i,t+1}$ in equation (14), and the latter is the common specification for a random walk. However, this is not a problem if the econometrician observes the idiosyncratic depreciation shocks with a one-period lag. In this case, (14) is the correct equation from the household's point of view, but a modified version of (14) with $\tilde{\eta}_{i,t+1}$ replacing $\tilde{\eta}_{it}$ is the specification estimated by the econometrician.

return to human capital investment. If some component of labor income is independent of human capital investment, as argued in Mankiw, Romer, and Weil [1992], and if this component is random (random endowment of genetic skills), then some part of the variance of labor income is not human capital risk.

Using the capital accumulation equation, we find that

$$s_k = x_{kt}/y_t = ((g + \delta)/A)\tilde{k}^{1-\alpha}. \tag{15}$$

Given the already assigned values for $\alpha$, $\delta$, $\mu_g$, $s_k$, and $\sigma_y$, equation (15) in conjunction with the equilibrium condition (13) determine the values of $A$ and $\tilde{k}$. We find that $A = .2674$ and $\tilde{k} = .7554$. Finally, the value for $\beta$ is determined by the expression for the equilibrium per capita growth rate,

$$\mu_g = \beta\left[1 + \frac{\tilde{k}}{1+\tilde{k}}A\alpha\tilde{k}^{\alpha-1} + \frac{1}{1+\tilde{k}}A(1-\alpha)\tilde{k}^{\alpha} - \delta\right] - 1. \tag{16}$$

We find that $\beta = .9465$.

The above calibration procedure ensures that the model economy matches as many features of the U. S. economy as there are free parameters. It is also interesting to investigate how the calibrated model performs in matching additional features of the U. S. economy. For example, the implied values for the average return on physical and human capital are $r_k = 5.52$ percent and $r_h = 9.47$ percent, respectively. The return $r_k = 5.52$ percent is higher than the observed real interest rate on short-term U. S. government bonds (1 percent), but lower than the observed real return on U. S. equity (8 percent). Given that there is no aggregate risk, and therefore no equity premium, in the model, it is not clear which one of the many financial return variables should be used as a basis for calibration, and we therefore conclude that the implied value is within the range of reasonable values.[12] The implied average return on investment in human capital, $r_h = 9.47$ percent, is in line with the estimates of rate of returns to schooling.[13] Notice that the implied excess return on human capital investment is $r_h - r_k = 3.95$ percent. Thus, the model generates a substantial "human capital premium." Finally, according to the model, individual consumption growth is normally

12. The RBC literature usually strikes a compromise and chooses the parameter values so that the implied return on capital is 4 percent, which is somewhat lower than the value used here.

13. The estimates vary considerably across households and studies, with an average of about 10 percent [Krueger and Lindhal 2001].

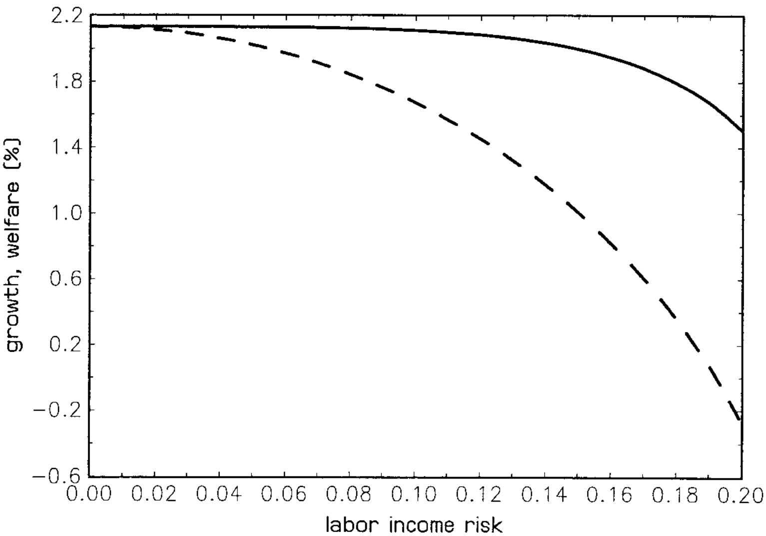


FIGURE I
Growth (Solid Line) and Welfare (Dashed Line) as a Function of Labor Income Risk

distributed, $g = c_{i,t+1}/c_{it} - 1 \sim N(\mu_g, \sigma_g^2)$, and has a standard deviation of $\sigma_g = \beta\sigma_y = .1420$. This amount of consumption volatility is somewhat lower than what is found in the data. For example, using CEX data on consumption of nondurables and services, Brav, Constantinides, and Geczy [2002] find that the standard deviation of quarterly consumption growth ranges from .06 to .12 for different household groups with an average of about .09. If quarterly consumption growth is i.i.d., then this corresponds to a standard deviation of annual consumption growth of .18.

### *III.C. Quantitative Results*

Figure I shows the average growth rate $\mu_g$ as a function of idiosyncratic labor income risk, $\sigma_y$.[14] Although we calibrated the model under the conservative assumption $\sigma_y = .15$, we present the results for values up to $\sigma_y = .2$ since this is still within the range of empirical estimates. Figure I also depicts the risk-ad-

14. In Figures I–III we change $\sigma_\eta$, respectively $\sigma_y$, keeping the other parameter values constant.

justed average growth rate, $\tilde{\mu}_g$, which is defined as the certain growth rate for which expected lifetime utility is equal to the expected lifetime utility associated with the uncertain growth rate $g \sim N(\mu_g,\sigma_g)$ (keeping initial consumption $c_{i0}$ constant). Direct calculation using the welfare formula (17) below yields $\tilde{\mu}_g = \exp(E[\log(1+g)]) - 1$. The change in this risk-adjusted growth rate, $\Delta\tilde{\mu}_g = \tilde{\mu}'_g - \tilde{\mu}_g$, measures the total welfare cost of an increase in idiosyncratic risk from $\sigma_y$ to $\sigma'_y$ expressed in growth-rate units.[15] This total welfare cost can be decomposed into two parts. The first part measures the welfare cost due to the decrease in average consumption growth, $\mu_g$, and the second part reflects the welfare cost due to the increase in consumption volatility, $\sigma_g$. Notice that the difference, $\Delta\tilde{\mu}_g - \Delta\mu_g$, measures the second part, namely the welfare cost of an increase in idiosyncratic risk for unchanged $\mu_g$ (exchange economy).

Expected lifetime utility (welfare) is calculated as follows. Let $\{c_{it}\}$ stand for a consumption plan defined by an initial consumption level $c_{i0}$ and a sequence of i.i.d. consumption growth rates $\{g_{it}\}$. Expected lifetime utility associated with this consumption plan is

$$E_0\left[\sum_{t=0}^{\infty} \beta^t \log c_{it}\right] = a \log c_{i0} + bE[\log(1+g)], \tag{17}$$

where $a = 1/(1-\beta)$ and $b = \beta/(1-\beta)^2$ and we dropped the time- and household index on the growth rate $g$.[16] Notice that (17) is additively separable in $g$ and $c_{i0}$. Thus, a change in government policy that leads to a change in $g$ will change the welfare of all households by the same amount regardless of their initial consumption level $c_{i0}$. Moreover, the risk-adjusted growth rate, $\tilde{\mu}_g = \exp(E[\log(1+g)]) - 1$, is the same for all households regardless of initial condition $c_{i0}$.

Figure I and Table I reveal that the growth and welfare effects of uninsurable idiosyncratic labor income risk are substantial. For example, starting from the baseline economy ($\sigma_y = .15$), the total elimination of idiosyncratic risk increases growth by $\Delta\mu_g = .13$ percent. Starting from the incomplete-markets economy with $\sigma_y = .20$, this growth rate effect becomes $\Delta\mu_g = .50$

15. When calculating $\tilde{\mu}_g$, the expression $E[\log(1+g)]$ is evaluated using a simple two-state approximation for the normally distributed random variable $g$.

16. Using the equilibrium conditions $1 + g = \beta[1 + r(\tilde{k},\tilde{k},\eta)]$ and $c_{i0} = (1-\beta)[1 + r(\tilde{k},\tilde{k},\eta_{i0})]w_{i0}$, equation (17) defines the value function $v = v(w_{i0},\eta_{i0},\tilde{k})$.

TABLE I
ONE-SECTOR MODEL

| | $\sigma_y = 0$ | $\sigma_y = .15$ | $\sigma_y = .20$ |
|---|---|---|---|
| $\tilde{k}$ | .5625 | .7554 | 1.065 |
| $\mu_g$ | 2.13% | 2.00% | 1.50% |
| $\tilde{\mu}_g$ | 2.13% | 1.01% | −.28% |
| $s_k$ | 21.05% | 25.00% | 29.22% |
| $s_h$ | 37.42% | 33.10% | 27.43% |
| $r_k$ | 7.91% | 5.52% | 3.25% |
| $r_h$ | 7.91% | 9.47% | 11.51% |

percent. The corresponding welfare changes, expressed as changes in the risk-adjusted growth rate, are $\Delta\tilde{\mu}_g = 1.12$ percent and $\Delta\tilde{\mu}_g = 2.41$ percent, respectively. It is instructive to express these welfare changes in terms of changes in consumption levels. Using (17), we find the following relationship $\Delta c_{i0}/c_{i0} \approx \beta/(1 - \beta)\Delta\tilde{\mu}_g$. Thus, in order to express the welfare effects in terms of equivalent changes in consumption levels, we simply multiply the change in the risk-adjusted growth rate by the factor $\beta/(1 - \beta) = 17.68$. For example, for the baseline incomplete-markets economy ($\sigma_y = .15$), the welfare gain from eliminating idiosyncratic risk is equivalent to an increase of consumption by almost 20 percent. In comparison, Lucas [1987] finds that the welfare gain of eliminating aggregate risk in a representative-agent exchange economy is only .07 percent of consumption (assuming log-utility preferences). In other words, the welfare effects discussed here are at least two orders of magnitude larger than the welfare effects discussed in Lucas [1987]. Finally, notice that the relationship between $\mu_g$, respectively, $\tilde{\mu}_g$, and $\sigma_y$ is highly nonlinear; that is, the main part of the growth and welfare effect of idiosyncratic risk occurs for high levels of idiosyncratic risk.

The effects of idiosyncratic risk on saving and investment are summarized in Figure II. Again, the effects are substantial. For example, in the baseline incomplete-market economy, investment in human capital is 3.5 percent (of GDP) lower and investment in physical capital is 5 percent higher than its first-best level. That is, at least 5 percent of observed saving (as measured in the NIPA) is due to uninsurable idiosyncratic labor income risk. The corresponding values for the capital-to-labor ratio, $\tilde{k}$, are .755 (incomplete-market economy) and $\alpha/(1 - \alpha) = .562$ (first best allocation). Notice that the investment and saving effects are

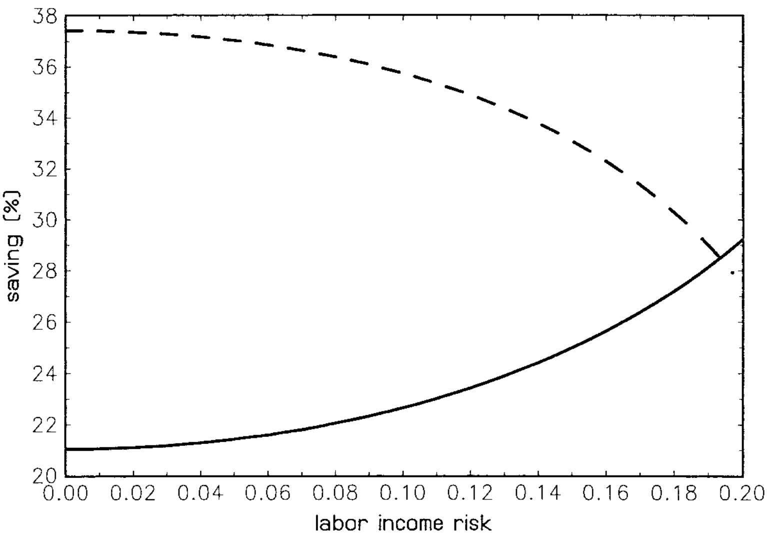


FIGURE II
Saving in Physical Capital (Solid Line) and Human Capital (Dashed Line) as a Function of Labor Income Risk

again larger for larger values of $\sigma_y$, but this nonlinearity is somewhat less pronounced than the nonlinearity for growth rates and welfare.

Figure III depicts the effect of idiosyncratic risk on the interest rate and wages. The effects are substantial. For example, in the baseline incomplete-market economy the excess return of human capital investment over physical capital investment, $r_h - r_k$, is 3.95 percent and for $\sigma_y = .20$ this excess return increases to 8.26 percent, whereas in the complete-market economy these two returns are equalized. Notice also that the growth-rate effect of idiosyncratic risk becomes much larger if the endogenous response of asset returns is neglected. For example, if asset returns are held constant, then the total elimination of idiosyncratic risk (starting from $\sigma_y = .15$) increases the growth rate by 1.61 percent, compared with .13 percent when asset returns are fully flexible. This result demonstrates the existence of very strong general equilibrium effects in closed economies with perfectly competitive labor (and financial) markets. Put differently, a model that allows for international capital flows or real-wage rigidities is likely to imply substantially larger growth effects of market incompleteness than the ones reported here.

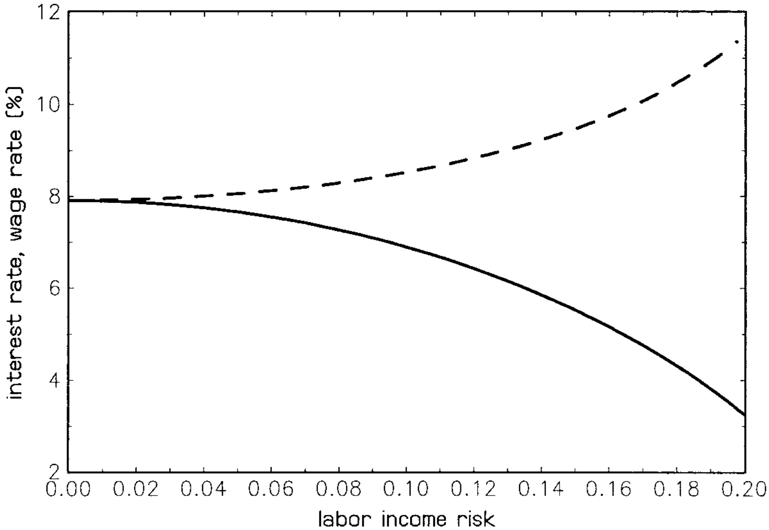


FIGURE III
Interest Rate (Solid Line) and Wage Rate (Dashed Line) as a Function of Labor Income Risk

## IV. GOVERNMENT POLICY AND MACROECONOMIC OUTCOME

This section introduces a government that uses proportional income taxation to finance a transfer program that consists of making payments to workers who have experienced above-average depreciation of human capital. These government payments can be interpreted as either severance payments made to displaced workers, the interpretation favored in this paper, or public training assistance for displaced workers.[17]

### IV.A. Model

Suppose that the model is as described in Section II with the only difference that now there is a government that taxes capital

17. Rogerson and Schindler [2002] conduct a quantitative study of government-sponsored severance payments using a calibrated macromodel with exogenous stochastic earnings process (exogenous human capital). Alvarez and Veracierto [2001], Lazear [1990], and Hopenhayn and Rogerson [1993] analyze government-regulated severance payments made by firms, in which case the severance payments have an impact on the hiring and firing decision of firms. Notice also that severance payments amount to ex post redistribution of income, which differs from the case in which ex ante redistribution of income enhances growth [Alesina and Rodrik 1994; Galor and Zeira 1993].

and labor income at the uniform rate $\tau$ and makes transfer payments $T(\eta_{it})h_{it}$ with $T(\eta_{it}) \geq 0$. The modified budget constraint of household $i$ then reads

$$
\begin{aligned}
c_{it} + x_{kit} + x_{hit} &= (1-\tau)\tilde{r}_{kt}k_{it} + (1-\tau)\tilde{r}_{ht}h_{it} + T(\eta_{it})h_{it} \\
k_{i,t+1} &= (1-\delta)k_{it} + x_{kit},\ k_{it} \geq 0 \\
h_{i,t+1} &= (1-\delta+\eta_{it})h_{it} + x_{hit},\ h_{it} \geq 0 \\
&(k_{i0},h_{i0}) \text{ given.}
\end{aligned} \tag{18}
$$

The budget constraint (18) assumes that gross income is taxed, which neglects possible tax deduction for capital depreciation. To simplify the notation, (18) also assumes an equal average depreciation rate of physical and human capital $\delta_k = \delta_h = \delta$.

As in Section II, the budget constraint can be rewritten so that the utility maximization problem of each household becomes an intertemporal portfolio choice problem with two assets. Repeating the steps that led to equation (9), we find the following modified equation:

$$
E\left[\frac{(1-\tau(\tilde{k}))(r_h(\tilde{k}) - r_k(\tilde{k})) + \eta + T(\eta)}{1 + (1-\tau(\tilde{k}))r(\tilde{k},\tilde{k}) - \tau(\tilde{k})\delta + (1/(1+\tilde{k}))(\eta + T(\eta))}\right] = 0. \tag{19}
$$

In (19) the equilibrium tax rate is determined by the requirement that the government runs a balanced budget in every period. This leads to $\tau(\tilde{k}) = E[T(\eta)]/((r_k(\tilde{k}) + \delta)\tilde{k} + r_h(\tilde{k}) + \delta)$, which in conjunction with (19) determines a unique $\tilde{k}$ for any government insurance program $T(\cdot)$.

### *IV.B. Qualitative Results*

For any $\tilde{k}$ determined by (19) the total investment return of household $i$ is $r_{it} = r(\tilde{k},\tilde{k}) + (\eta_{it} + T(\eta_{it}) - E[T(\eta_{it})])/(1+k)$, where $r(\tilde{k},\tilde{k}) = \theta(\tilde{k})r_k(\tilde{k}) + (1-\theta(\tilde{k}))r_h(\tilde{k})$. As in Section II, individual consumption growth is given by $c_{i,t+1}/c_{it} = \beta(1 + r_{i,t+1})$. Taking the expectations over idiosyncratic shocks, we derive the following expression for per capita growth:

$$
E\left[\frac{c_{i,t+1}}{c_{it}}\right] = \beta(1 + r(\tilde{k},\tilde{k})). \tag{20}
$$

Thus, for given $\tilde{k}$, growth is unaffected by the government program. The reason for this neutrality result is that both income taxation and transfer payments are proportional to capital levels, which implies that the net effect on total investment (in physical and human capital) is nil.

Although the government program has no effect on growth for fixed $\tilde{k}$, it does affect $\tilde{k}$ and therefore growth. To understand the nature of this indirect growth effect most clearly, consider a government policy of proportional income taxation that uses the proceeds to make state-independent payments proportional to human capital: $T(\eta_{it}) = T > 0$. This policy amounts to a subsidy to human capital investment and therefore reduces $\tilde{k}$. The proof of this statement immediately follows from equation (19). As already noticed in Proposition 2, a decrease in $\tilde{k}$ is growth enhancing. Thus, in contrast to the complete-markets case, in which the growth maximum is achieved without government intervention, in the incomplete-markets case any government policy that subsidizes human capital investment increases growth.

Matters are different for welfare. More specifically, the state-independent transfer payment $T(\eta_{it}) = T > 0$ reduces expected lifetime utility of all households. This follows from the observation that without government intervention the equilibrium consumption plan of each household is also the solution to a one-agent decision problem in which the household directly invests human and physical capital in the production technology $F$. Hence, from a welfare point of view, any government program that does not provide additional insurance opportunities is distortionary. We summarize the preceding discussion in the following proposition.

PROPOSITION 3. A government program that uses uniform proportional income taxation to finance a state-independent subsidy to human capital investment increases growth, $E[c_{i,t+1}/c_{it}]$, but decreases welfare, $E[\Sigma_{t=0}^{\infty} \beta^t \log c_{it}]$.

Once the government uses the tax revenues to make state-dependent transfer payments, the welfare effect of the government program becomes ambiguous. On the one hand, there is still the distortionary effect discussed above. On the other hand, there is now a positive effect due to the reduction in human capital risk. Clearly, the net effect depends on the relative strength of these two effects. The next subsection analyzes the sign and size of this net welfare effect for the calibrated model economy.

*IV.C. Quantitative Results*

The calibrated model economy without government program is the baseline economy ($\sigma_y = .15$) discussed in Section III. However, we use the exact equilibrium formula (19) and a four-state approximation for normally distributed random variables based on Gauss-Hermite quadrature [Judd 1998].[18] We choose this procedure over the approach taken in Section III because the introduction of the transfer scheme means that even with the approximation $\log(1 + r) \approx r$ no simple formula analogous to (13) for the equilibrium variables is available.

Calibrating the model according to the principles laid out in Section III and using the four-state approximation yields the parameter values $A = .26587$, $\beta = .94708$, and $\sigma_\eta = .26229$. Notice that these values are very close to the values obtained in Section III. Furthermore, the initial distortion in the capital-to-labor ratio obtained here is also very close to the one obtained previously: $\tilde{k} = .74852$ compared with $\tilde{k} = .75535$. This shows that for the degree of idiosyncratic risk considered here, the approximation $\log(1 + r) \approx r$ used in Section III only introduces small approximation errors.

We first present in Figure IV the quantitative effects of a government program that uses uniform proportional income taxation to make state-independent subsidies to human capital investment: $T(\eta_{it}) = T > 0$. As discussed before, this type of policy increases growth (as long as $\tilde{k}$ is smaller than its first-best level), but decreases welfare. This decrease in welfare represents the distortionary effect of the government policy. A comparison with Figure I shows that these distortionary effects are of the same order of magnitude as the beneficial growth effects of labor income insurance discussed in Section III. Figure IV (dashed-dotted line) also depicts the growth and welfare effects of the same government program for the corresponding complete-markets (representative-agent) economy. The calculations are based on a complete-markets economy with values for preference and technology parameters chosen so that the model matches observed U. S. growth and saving. Clearly, in this case the growth and welfare effects are identical. Moreover, they are always

18. More precisely, we replace the random variable $\eta \sim N(0,\sigma_\eta^2)$ by a random variable that has four possible realizations, $-\eta_1$, $-\eta_2$, $+\eta_1$, $+\eta_2$, where $\eta_1 = .7420\sigma_\eta$ and $\eta_2 = 2.3344\sigma_\eta$. These four realizations occur with probabilities prob $(-\eta_1)$ = prob $(+\eta_1) = .4541$ and prob $(-\eta_2)$ = prob $(+\eta_2) = .0459$.

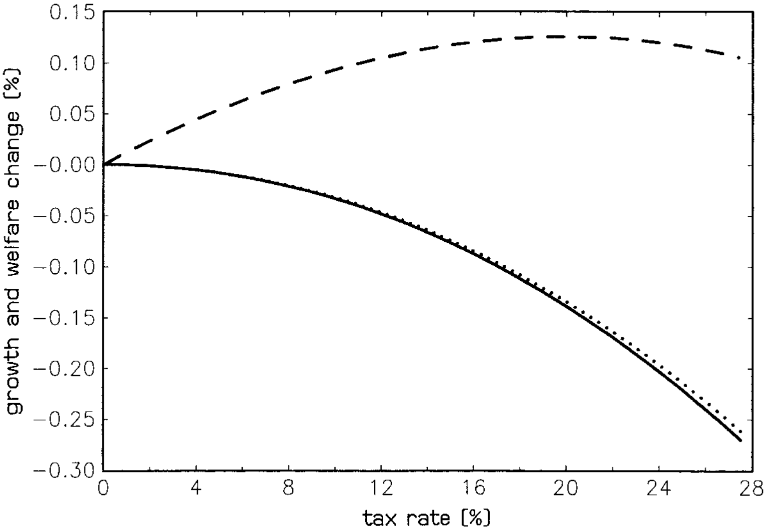


FIGURE IV
State-Independent Transfers: Change in Growth and Welfare for Incomplete Markets (Dashed Line and Solid Line) and Complete Markets (Dashed-Dotted Line)

negative since the equilibrium allocation without government intervention is first-best. In a certain sense, the dashed-dotted line in Figure IV depicts the distortionary effects of (unequal) proportional income taxation discussed by the extensive literature on tax reform in representative-agent models of endogenous growth [Jones, Manuelli, and Rossi 1993; Lucas 1990; King and Rebelo 1990; Rebelo 1991; Stokey and Rebelo 1995]. Notice that the decrease in welfare in the incomplete-markets economy is roughly equal to the welfare reduction in the complete-markets economy.

We now turn to an analysis of a government program that is potentially welfare enhancing. More specifically, consider a program of public assistance to workers who have experienced a large human capital loss. That is, consider the case $T(\eta) = 0$ if $\eta = +\eta_1, +\eta_2, -\eta_1$ and $T(\eta) = \nu\eta$ if $\eta = -\eta_2$. The parameter $\nu$ varies from 0 to 1 and measures the size of the government program. If we want to interpret this program as a policy of making severance payments to displaced workers, then according to this interpretation a fraction prob $(-\eta_2) = .0459$ of the workers are displaced in each year, and each of the displaced workers

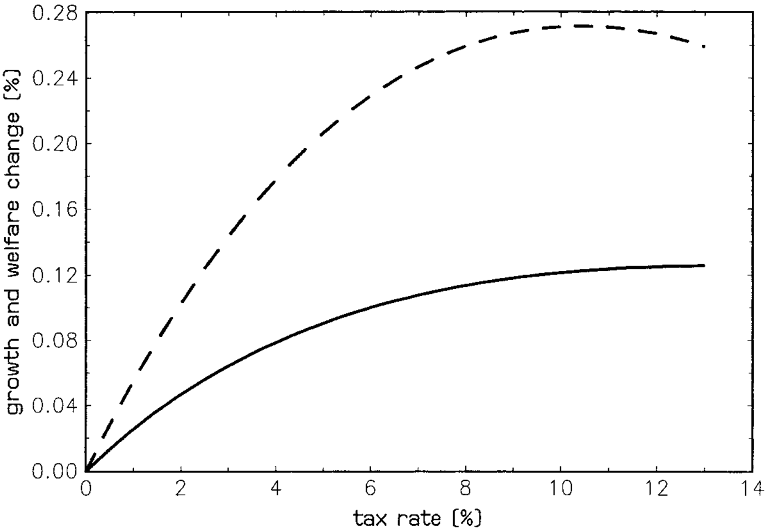


FIGURE V
State-Dependent Transfers: Change in Growth (Solid Line) and Welfare (Dashed Line) as a Function of Tax Rate for Incomplete Markets (sigma_*h* = .15)

loses a fraction $2.3344\sigma_h = .3502$ of his labor income. In comparison, Jacobson, LaLonde, and Sullivan [1993] study high-tenure Pennsylvania workers and find that job displacement results in a permanent income loss of 25 percent. Using PSID data, Topel [1991] estimates that ten years of job seniority raise the wage of the typical male worker by at least 25 percent, and perhaps up to 40 percent. The displacement rate of high-tenure workers is about 4 percent according to the Displaced-Workers' Survey [Farber 1993]. Thus, our values for displacement rate and income loss are somewhat on the high end of the spectrum of empirical estimates.[19]

Figure V shows the growth and welfare effects of the government program as a function of the tax rate, $\tau$, necessary to finance the program. Notice that there is a unique welfare maximum at $\tau = 10.49$ percent. In other words, for small values of $\tau$, the

19. A lower replacement rate of 2 percent or a smaller income loss of 30 percent does not change in a significant way the relationship between tax rate and growth (welfare) depicted in Figure V (results not reported here). However, in this case $\nu = 1$ corresponds to a significantly lower tax rate, which implies that the welfare maximum in Figure V might not be achieved.

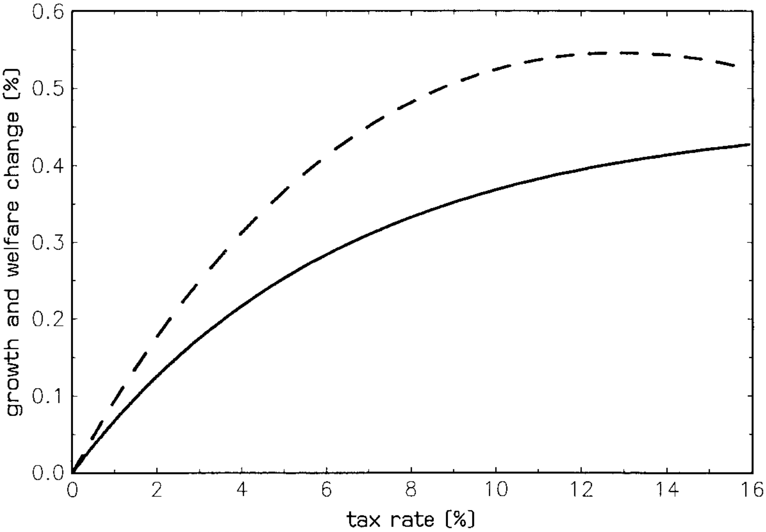


FIGURE VI
State-Dependent Transfers: Change in Growth (Solid Line) and Welfare (Dashed Line) as a Function of Tax Rate for Incomplete Markets (sigma_*h* = .20)

insurance effect dominates the distortionary effect, but for high values of $\tau$ the distortionary effect dominates the insurance effect. At this maximum a displaced worker receives public assistance of 82 percent of the incurred income loss. Note that the growth rate $\mu_g$ is monotonically increasing in $\tau$ as long as $\tilde{k}$ is smaller than its first-best value, which is the case for all tax rates depicted in Figure V. Thus, the welfare-maximizing government policy increases both growth and welfare, and there is no trade-off between efficiency (growth) and ex post inequality (welfare). For the welfare-maximizing tax rate the increase in welfare is $\Delta\tilde{\mu}_g =$ .2715 percent and the increase in growth is $\Delta\mu_g = .1227$ percent. Recall that a growth-rate increase of $\Delta\tilde{\mu}_g = .2715$ percent is equivalent to an increase in consumption levels of $17.68 \times .2715 = 4.8$ percent. Clearly, the welfare-maximizing government program is quite large, but even for a more modest tax rate of $\tau =$ 5 percent the growth and welfare gains are substantial: $\Delta\mu_g =$ .0896 percent and $\Delta\tilde{\mu}_g = .2044$ percent.

Finally, Figure VI presents the growth and welfare effects of the same type of government policy using an economy with a higher degree of human capital risk ($\sigma_y = .20$). In this case, the welfare-maximizing tax rate is $\tau = 12.83$ percent, and the corresponding gains in growth and welfare are $\Delta\mu_g = .4033$ percent and $\Delta\tilde{\mu}_g = .5461$ percent.

The above results have to be interpreted with caution since the current analysis assumes that severance payments have no effect on search and work effort.[20] We conclude this section with comments regarding possible moral hazard problems of the current insurance scheme.

First, there is the possibility that severance payments have an adverse effect on work effort. However, if severance payments are only made to workers who have lost their job due to plant closure (mass layoffs) or other "exogenous" events (displaced workers), then the effect on work effort might be small. Second, severance payments may reduce search effort. Clearly, unemployment insurance affects search effort because payments depend on unemployment duration, but in the case of severance payments, which are independent of unemployment duration, the issue is less clear-cut. However, if severance payments are made in proportion to observed income losses, they induce workers to accept lower wage offers and may also reduce search intensity.[21] Analyzing these effects in an extended version of the current model with search seems a promising avenue for future research. Alternatively, one can think of severance payments that are proportional to current salaries (before job loss). In this case, severance payments have no effect on search, but with heterogeneity in income losses among displaced workers they are less effective in providing consumption insurance. The current analysis disregarded such heterogeneity, but incorporating it into the analysis is an important topic for future research.

## V. Conclusion

This paper developed a tractable incomplete-markets model of economic growth and used the model to study the qualitative

20. However, the effect of severance payments on search and work effort might be quite small if the payment scheme is designed appropriately (see the discussion below). In the case of unemployment insurance, Hansen and Imrohoroglu [1992] have shown that the search effect is large and might lead to a net loss in output and welfare. Acemoglu and Shimer [1999, 2000] demonstrate that accounting for heterogeneity in worker-firm matches is likely to increase the output and welfare gains from unemployment insurance.

21. Severance payments can also be made in proportion to human capital losses, but these losses are in general not easily observable.

and quantitative effects of idiosyncratic labor income risk on investment in physical and human capital, economic growth, and welfare. The analysis showed that uninsurable idiosyncratic labor income risk is likely to be detrimental to growth and that its effect on investment, growth, and welfare are quantitatively important. Moreover, the gains from additional labor income insurance are of the same order of magnitude as the costs of distortionary income taxation and strong enough to justify the introduction of government-sponsored severance payments.

There are several applications of the basic framework that have not been analyzed because of space limitations. First, the model can be used to study optimal tax policy in incomplete-markets economies. Section IV provides a first step in this direction, but a more general analysis should allow for different tax rates on labor and capital income. Second, in this paper the model was calibrated to match observed labor income risk and then used to study the effect of labor income risk on economic growth. However, the model can also be calibrated using death probabilities and then be used to study the effect of mortality risk on economic growth when insurance markets are incomplete.[22] Third, the two-sector model outlined in Appendix 2 provides a useful framework for a more detailed quantitative analysis of entrepreneurial risk and economic growth. Introducing a fixed cost of becoming an entrepreneur is likely to add an important element of realism to such an analysis.

There are also several extensions of the current model that promise to yield additional insights into the relationship between human capital risk and economic growth. First, the assumption of log-utility preferences should be replaced by the more general assumption of CRRA-preferences. Second, the current framework disregards human capital externalities and difference in the production of human and physical capital (human capital production is more human capital intensive). Incorporating these features into the analysis is likely to increase the growth effect of a reduction in labor income risk. Third, this paper endogenized the human capital investment decision, but treated the choice of the number of hours worked as exogenous. Moreover, the model abstracted from extended periods of unemployment and job search.

22. Previous work on mortality risk and human capital investment has usually assumed full insurance [Ehrlich and Lui, 1991; Kalemei-Ozcan, Ryder, and Weil 2000].

Incorporating these additional dimensions into the model would allow for a more complete welfare analysis of severance payments, and also opens the door for a discussion of unemployment benefits. Fourth, as noticed in Section III, introducing international capital flows (open economy model) or real wage rigidities tends to increase the growth effect of uninsurable human capital risk, and an explicit model could be used to provide a quantitative assessment of this effect. Finally, there is an extensive literature on endogenous technological progress [Aghion and Howitt 1992] that implicitly assumes that the risk of developing and implementing new technologies is fully diversifiable. If the joint assumption of constant-returns-to-scale and competitive markets is maintained, then a variant of the current model with a third input factor (technology or general knowledge) can be used to study the interaction between financial market development and technological progress.

## Appendix 1: Proof of Proposition 2

Let $\tilde{k}$ stand for the solution of (9) if $\eta$ and $\tilde{k}'$ if $\eta'$, that is, $\tilde{k}$ and $\tilde{k}'$ solve

$$E[\varphi(\tilde{k},\eta)] = 0 \qquad \text{(A1)}$$
$$E[\varphi(\tilde{k}',\eta')] = 0,$$

where

$$\varphi(\tilde{k},\eta) \equiv \frac{r_h(\tilde{k}) + \eta - r_k(\tilde{k})}{1 + r(\tilde{k},\tilde{k},\eta)}.$$

Because agents are risk-averse, we must have $r_h(\tilde{k}) > r_k(\tilde{k})$. Using this and the fact that $r_h$ is increasing in $\tilde{k}$ and $r_k$ is decreasing in $\tilde{k}$, it is straightforward to show that $\varphi$ is decreasing in $\tilde{k}$ and strictly convex in $\eta$. Since $\eta'$ is a mean-preserving spread of $\eta$, we can write $\eta' = \eta + \epsilon$ with $E[\epsilon|\eta] = 0$. Thus, we find

$$\begin{aligned} \text{(A2)} \qquad E[\varphi(\tilde{k}',\eta')] &= E[\varphi(\tilde{k}',\eta + \epsilon)] \\ &= E[E[\varphi(\tilde{k}',\eta + \epsilon)|\eta]] \\ &> E[\varphi(\tilde{k}',E[\eta + \epsilon|\eta])] \\ &= E[\varphi(\tilde{k}',\eta)], \end{aligned}$$

where we used the strict convexity of $\varphi$ in $\eta'$. Combining (A1) and (A2), we find that

$$E[\varphi(\tilde{k}',\eta)] < E[\varphi(\tilde{k},\eta)]. \tag{A3}$$

Because $\varphi$ is decreasing in $\tilde{k}$, the inequality (A3) implies that $\tilde{k}' > \tilde{k}$. This proves the first part of the proposition, and straightforward differentiation proves the stated effect on all other variables. Expected lifetime utility (welfare) of all agents must increase because the equilibrium allocation is the solution to a one-agent decision problem, and Blackwell's theorem therefore ensures that more information cannot be worse. □

## Appendix 2: Risky Physical Capital

This section extends the model developed so far and allows for a second sector in which production is subject to nondiversifiable idiosyncratic risk (entrepreneurial risk). To streamline the analysis, the model is immediately laid out under the assumption of normally distributed depreciation shocks and Cobb-Douglas production functions. The extension to the more general case is straightforward.

### *Model*

There are two sectors producing the one good, a corporate sector with one aggregate production function and a noncorporate sector (entrepreneurial sector) with one production function for each household. Output produced in the first sector is given by $y_1 = F_1(k_1,h_1)$, and physical and human capital invested in this sector by household $i$ depreciates at a rate $\delta_1$, respectively, $\delta_1 + \eta_{1it}$. The assumption that the average depreciation rate, $\delta_1$, is the same for physical and human capital is not essential, but simplifies the notation. We assume $F_1(k_1,h_1) = A_1 k_1^{\alpha_1} h_1^{1-\alpha_1}$ and $\eta_{1it} \sim N(0,\sigma^2_{\eta_1})$. In addition to investing in the corporate sector, each household also has the opportunity to invest physical and human capital in a second sector. If household $i$ has allocated $k_{2it}$ physical capital and $h_{2it}$ human capital to this entrepreneurial sector, then he receives $y_{2it} = A_2 k_{2it}^{\alpha_2} h_{2it}^{1-\alpha_2}$ additional income in period $t$. Physical and human capital allocated to the entrepreneurial sector depreciate at the common rate $\delta_2 + \eta_{2it}$. One example of a large negative shock to capital is the event of bankruptcy, in which case both physical and human capital allocated to the business "depreciate" at an above average rate. We assume that the idiosyncratic depreciation shocks, $\eta_{2it}$, are i.i.d. across households and time periods and that $\eta_{1it}$ and $\eta_{2it}$ are uncorrelated. Moreover, we assume that $\eta_{2it} \sim N(0,\sigma^2_{\eta_2})$. For notational simplicity, we do not introduce idiosyncratic productivity shocks, but the analysis is easily extended to this case as long as the random variables $A_{2it}$ are i.i.d. across households and time periods.

Given the assumption of equal depreciation rates of physical and human capital in the entrepreneurial sector, the optimal capital-to-labor ratio in this sector is $\tilde{k}_{2it} = \alpha_2/(1 - \alpha_2)$. Introducing new variables along the lines discussed in Section II, each household's utility maximization problem becomes a three-asset intertemporal portfolio choice problem. Using the Euler equations and the approximation $\log(1 + r) \approx r$, we find the following optimal portfolio choices:

$$\begin{aligned} z(1-\theta) &= (r_{h1} - r_{k1})/\sigma^2_{\eta_1} \\ 1 - z &= (r_2 - r_{k1})/\sigma^2_{\eta_2}, \end{aligned} \tag{A4}$$

where $z$ is the fraction of total wealth (human and physical) invested in the corporate sector and $\theta$ is the fraction of corporate-sector wealth that is physical capital. Notice again the close resemblance of (A4) to the solution to the corresponding continuous-time Merton problem.

Using $\theta = \tilde{k}_1/(1 + \tilde{k}_1)$ and the profit maximization condition of the corporate-sector firm, (A4) determines the equilibrium values of the two endogenous variables $\tilde{k}_1$ and $z$ for given $\sigma_{\eta_1}$. Moreover, the resulting nonlinear equation system defines functions $\tilde{k}_1 = \tilde{k}_1(\sigma_{\eta_1})$ and $z = z(\sigma_{\eta_1})$. Applying the implicit function theorem to the resulting equation system, we find

$$\begin{aligned} &\frac{d\tilde{k}}{d\sigma_{\eta_1}} > 0 \\ &\frac{dz}{d\sigma_{\eta_1}} < 0. \end{aligned} \tag{A5}$$

These results are intuitive. An increase in the risk of investing human capital in the corporate sector makes human capital investment less attractive than physical capital investment in this sector, and therefore reduces the relative share of human capital investment in the corporate sector (increases the capital-to-labor ratio in the corporate sector). It also makes investing in the corporate sector less attractive relative to investing in the noncorporate sector, and therefore reduces the share of total capital that is allocated to the corporate sector. If $r_2 \leq \theta r_{k1} + (1-\theta)r_{h1}$, then (A5) implies that economic growth is reduced. However, if $r_2 > \theta r_{k1} + (1-\theta)r_{h1}$, then it may be that economic growth is enhanced because more capital is shifted toward the high-return entrepreneurial sector.

*Calibration*

In the quantitative part, we assume that at the aggregate level the technologies employed in both sectors are identical: $\alpha_1 = \alpha_2 = \alpha$, $A_1 = A_2 = A$, and $\delta_2 = \delta_1 = \delta$. We continue to assume that $\alpha = .36$ and $\delta = .06$. The values of the remaining model parameters $A$, $\sigma_{\eta_1}$, $\sigma_{\eta_2}$, and $\beta$ are determined in conjunction with $\tilde{k}_1$ and $z$ by the two equilibrium conditions and the requirement that the model economy matches the U. S. data along the following four dimensions: aggregate saving, per capita growth, idiosyncratic risk, and relative size of the noncorporate sector. More precisely, we require that $s_k = .25$, $\mu_g = .02$,

$$\sigma_y = \sqrt{\frac{z^2}{(1+\tilde{k}_1)^2}\,\sigma_{\eta 1}^2 + (1-z)^2\sigma_{\eta 2}^2} = .15,$$

and $k_2/(k_1 + k_2) = .30$. The implied parameter values are $A = .25474$, $\beta = .95192$, $\sigma_{\eta_1} = .3131$, and $\sigma_{\eta_2} = .27991$. The implied equilibrium capital-to-labor ratios are $\tilde{k}_1 = .76324$ and $\tilde{k}_2 = .5625$ (first best), and we have $z = .6999$.

The restriction

$$\sigma_y = \sqrt{\frac{z^2}{(1+\tilde{k}_1)^2}\,\sigma_{\eta 1}^2 + (1-z)^2\sigma_{\eta 2}^2} = .15$$

ensures that the amount of total idiosyncratic risk is equal to the predetermined value of .15, which is the value also used in Section III. The condition $k_2/(k_1 + k_2) = .30$ is imposed to ensure that the size of the noncorporate sector is 30 percent. This number is taken from Quadrini [2000] who uses data on privately held tangible assets from the FED's *Flow of Funds Account* to estimate this capital ratio. The flow of fund account distinguishes five sectors, *nonfinancial corporations, financial institutions, households and nonprofit institutions, farm businesses,* and *nonfarm noncorporate businesses.* The estimate of 30 percent assumes that the capital stock of the corporate sector consists of the asset holdings of *nonfi-*

*nancial corporations, financial institutions* and *households* and *non-profit institutions,* and that the noncorporate sector consists of the two remaining classes: *farm business* and *nonfarm noncorporate business.* Given that the model assumes that there is no nondiversifiable idiosyncratic risk in the corporate sector, the estimate of 30 percent might be an underestimate of the size of the noncorporate sector. However, given that the model also assumes that there are no distortions in the noncorporate sector because there is no formal employer-employee relationship, 30 percent might still be an overestimate. Absent any further information on the relative importance of these two opposing effects, a point estimate of 30 percent appears to be a reasonable compromise.

A final comment about the calibration of the two-sector model is in order. The model implies that the average return on human capital investment is higher than the average return on entrepreneurial activity, $r_{h1} = 8.79$ percent and $r_2 = 7.25$ percent, and these two returns are in turn higher than the return on physical capital investment in the corporate sector $r_{k1} = 4.90$ percent. Recent empirical evidence [Moskowitz and Vissing-Jorgensen 2002] indicates that average entrepreneurial returns are not higher than stock returns, which suggests that the model overestimates $r_2$ relative to $r_{k1}$. Allowing for different technologies in the two sectors ($A_1 > A_2$ or $\delta_1 < \delta_2$) would allow for a lower $r_2$ relative to $r_{k1}$ and $r_{h1}$. However, since entrepreneurial activity has a large idiosyncratic risk component [Heaton and Lucas 2000], in this case households can only be induced to invest in the entrepreneurial sector if they either derive nonpecuniary benefits from being an entrepreneur or overestimate the probability of success. Both possibilities find some support by survey evidence [Moskowitz and Vissing-Jorgensen].

*Quantitative Results*

The effect of $\sigma_y$ on growth, welfare, investment, and asset returns in the two-sector economy is similar to the effect shown in Figures I–III for the one-sector economy (not reported here). In particular, all relationships are monotone and exhibit nonlinearity increasing in $\sigma_y$. Moreover, these effects are substantial, although somewhat smaller than in the one-sector case. For example, the growth-rate effect of a complete elimination of idiosyncratic human

TABLE II
TWO-SECTOR MODEL

| | $\sigma_y = 0, \nu = .3$ | $\sigma_y = .15, \nu = .3$ | $\sigma_y = 0, \nu = .5$ | $\sigma_y = .15, \nu = .5$ |
|---|---|---|---|---|
| $\tilde{k}$ | .5625 | .7632 | .5625 | .7668 |
| $\mu_g$ | 2.10% | 2.00% | 2.08% | 2.00% |
| $\tilde{\mu}_g$ | 2.10% | .99% | 2.08% | .98% |
| $s_k$ | 21.99% | 25.00% | 22.42% | 25.00% |
| $s_h$ | 39.10% | 35.56% | 39.86% | 36.68% |
| $r_k$ | 7.25% | 4.90% | 6.99% | 4.65% |
| $r_h$ | 7.25% | 8.79% | 6.99% | 8.52% |

capital risk is reduced from .13 percent to .10 percent (Table II).[23] The reason for this reduction in the growth-rate effect is that in the two-sector model a decrease in idiosyncratic labor income risk diverts capital away from the high-return entrepreneurial activity. To the extent that in the data entrepreneurial returns are not necessarily higher than corporate-sector returns (see discussion above), Table II might understate the growth-rate effect of human capital risk. Table II also includes results for an economy with half of capital invested in the entrepreneurial sector ($\nu = .5$). Clearly, these results establish a lower bound on the growth-rate effects of human capital risk.

BROWN UNIVERSITY

## REFERENCES

Acemoglu, Daron, and Robert Shimer, "Efficient Unemployment Insurance," *Journal of Political Economy,* CVII (1999), 893–928.
Acemoglu, Daron, and Robert Shimer, "Productivity Gains from Unemployment Insurance," *European Economic Review,* XLIV (2000), 1195–1224.
Acemoglu, Daron, and Jaume Ventura, "The World Income Distribution," *Quarterly Journal of Economics,* CXVII (2002), 654–694.
Acemoglu, Daron, and Fabrizio Zilibotti, "Was Prometheus Unbound by Chance? Risk, Diversification, and Growth," *Journal of Political Economy,* CV (1997), 709–751.
Aghion, Philippe, and Peter Howitt, "A Model of Growth through Creative Destruction," *Econometrica,* LX (1992), 323–351.
Aghion, Philippe, and Peter Howitt, *Endogenous Growth Theory* (Cambridge, MA: MIT Press, 1998).
Aiyagari, Rao, "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics,* CIX (1994), 659–684.
——, "Optimal Income Taxation with Incomplete Markets, Borrowing Con-

23. In the economy $\sigma_{\eta 1} = 0$, all capital is invested in the first (corporate sector). Notice also that the results reported in Table II do not change if we eliminate idiosyncratic risk in both sectors.

straints, and Constant Discounting," *Journal of Political Economy,* CIII (1995), 1158–1175.
Alesina, Alberto, and Dani Rodrik, "Distributive Politics and Economic Growth," *Quarterly Journal of Economics,* CIX (1994), 465–490.
Alvarez, Fernando, and Marcelo Veracierto, "Severance Payments in an Economy with Frictions," *Journal of Monetary Economics,* XLVII (2001), 477–498.
Angeletos, George-Marios, and Laurent Calvet, "Incomplete Markets, Growth, and the Business Cycle," Working Paper No. 1910, Harvard University, 2001.
Barro, Robert, and Xavier Sala-i-Martin, *Economic Growth* (New York: McGraw-Hill, 1995).
Benabou, Roland, "Tax and Education Policy in a Heterogenous-Agent Economy: What Levels of Redistribution Maximize Growth and Welfare," *Econometrica,* LXX (2002), 481–519.
Bencivenga, Valerie R., and Bruce D. Smith, "Financial Intermediation and Endogenous Growth," *Review of Economic Studies,* LVIII (1991), 195–209.
Brav, Alon, George M. Constantinides, and Christopher C. Geczy, "Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence," *Journal of Political Economy*, CX (2002), 793–824.
Caballero, Ricardo J., "Earnings Uncertainty and Aggregate Wealth Accumulation," *American Economic Review,* LXXXI (1991), 859–871.
Carroll, Christopher D., "Buffer-Stock Saving and the Life-Cycle/Permanent Income Hypothesis," *Quarterly Journal of Economics,* CXII (1997), 1–55.
Carroll, Christopher D., and Andrew A. Samwick, "The Nature of Precautionary Saving," *Journal of Monetary Economics,* XL (1997), 41–72.
Constantinides, George M., and Darrell Duffie, "Asset Pricing with Heterogeneous Consumers," *Journal of Political Economy,* CIV (1996), 219–240.
Cooley, Thomas F., and Edward C. Prescott, "Economic Growth and Business Cycles," in T. Cooley, ed., *Frontiers in Business Cycle Research* (Princeton, NJ: Princeton University Press, 1995).
Deaton, Angus, "Saving and Liquidity Constraints," *Econometrica,* LIX (1991), 1221–1248.
Deaton, Angus, and Christina Paxson, "Intertemporal Choices and Inequality," *Journal of Political Economy,* CII (1994), 437–467.
Devereux, Michael B., and Gregory W. Smith, "International Risk Sharing and Economic Growth," *International Economic Review,* XXXV (1994), 535–550.
Eaton, Jonathan, and Harvey S. Rosen, "Taxation, Human Capital, and Uncertainty," *American Economic Review,* LXX (1980), 705–715.
Ehrlich, Isaak, and Francis T. Lui, "Intergenerational Trade, Longevity, Intrafamily Transfers, and Economic Growth," *Journal of Political Economy,* XCIX (1991), 1029–1059.
Farber, Henry, "The Incidence and Costs of Job Loss: 1982–91," *Brookings Papers of Economic Activity: Microeconomics,* 1 (1993), 73–119.
Galor, Oded, and Joseph Zeira, "Income Distribution and Macroeconomics," *Review of Economic Studies,* LX (1993), 35–52.
Geweke, John, and Michael Keane, "An Empirical Analysis of Male Income Dynamics in the PSID: 1968–1989," *Journal of Econometrics,* XCVI (2000), 293–356.
Greenwood, Jeremy, and Boyan Jovanovic, "Financial Development, Growth, and the Distribution of Income," *Journal of Political Economy,* XCVIII (1990), 1076–1107.
Hansen, Gary D., and Ayse Imrohoroglu, "The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard," *Journal of Political Economy,* C (1992), 118–142.
Heaton, John, and Deborah J. Lucas, "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing," *Journal of Political Economy,* CIV (1996), 443–487.
Heaton, John, and Deborah J. Lucas, "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk," *Journal of Finance,* LV (2000), 1163–1198.
Hopenhayn, Hugo, and Richard Rogerson, "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy,* CI (1993), 915–938.

Hubbard, R. Glenn, Jonathan Skinner, and Stephen P. Zeldes, "Precautionary Saving and Social Insurance," *Journal of Political Economy,* CIII (1995), 360–399.
Huggett, Mark, "Wealth Distribution in Life-Cycle Economies," *Journal of Monetary Economics,* XXXIX (1996), 385–403.
Huggett, Mark, and Sandra Ospina, "Aggregate Precautionary Savings: When Is the Third Derivative Irrelevant?" *Journal of Monetary Economics,* XLVIII (2001), 373–396.
Jacobson, Louis S., Robert J. LaLonde, and Danile G. Sullivan, "Earnings Losses of Displaced Workers," *American Economic Review,* LXXXIII (1993), 685–709.
Jones, Larry E., and Rodolfo E. Manuelli, "A Convex Model of Equilibrium Growth: Theory and Policy Implications," *Journal of Political Economy,* XCVIII (1990), 1008–1038.
Jones, Larry E., Rodolfo E. Manuelli, and Peter E. Rossi, "Optimal Taxation in Models of Economic Growth," *Journal of Political Economy,* CI (1993), 485–517.
Judd, Kenneth L., *Numerical Methods in Economics* (Cambridge, MA: MIT Press, 1998).
Kalemli-Ozcam, Sebnem, Harl E. Ryder, and David Weil, "Mortality Decline, Human Capital, and Economic Growth," *Journal of Development Economics,* LXII (2000), 1–23.
King, Robert G., and Ross Levine, "Finance and Growth: Schumpeter Might Be Right," *Quarterly Journal of Economics,* CVIII (1993), 717–737.
Krueger, Alan B., and Mikael Lindhal, "Education for Growth: Why and for Whom," *Journal of Economic Literature,* XXXIX (2001), 1101–1136.
Krusell, Per, and Anthony A. Smith, "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy,* CVI (1998), 867–896.
Lazear, Edward P., "Job Security Provisions and Employment," *Quarterly Journal of Economics,* CV (1990), 699–726.
Levhari, David, and Yoram Weiss, "The Effect of Risk on the Investment of Human Capital," *American Economic Review,* LXIV (1974), 950–963.
Ljungqvist, Lars, and Thomas J. Sargent, *Recursive Macroeconomic Theory* (Cambridge, MA: MIT Press, 2000).
Lucas, Robert E. Jr., *Models of Business Cycles* (New York: Blackwell, 1987).
——, "Supply-Side Economics: An Analytical Review," *Oxford Economic Papers,* XLII (1990), 293–316.
Magill, Michael, and Martine Quinzii, *Theory of Incomplete Markets* (Cambridge, MA: MIT Press, 1996).
Mankiw, N. Gregory, David Romer, and David Weil, "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics,* CVII (1992), 407–437.
Meghir, Costas, and Luigi Pistaferri, "Income Risk Dynamics and Heterogeneity," Working Paper, Stanford University, 2001.
Moskovitz, Tobias J., and Annette Vissing-Jorgensen, "The Private Equity Premium Puzzle," *American Economic Review*, XCII (2002), 745–779.
Obstfeld, Maurice, "Risk-Taking, Global Diversification, and Growth," *American Economic Review,* LXXXIV (1994), 1310–1329.
Pries, Michael J., "Uninsurable Idiosyncratic Risk and Human Capital Accumulation," Working Paper, University of Maryland, 2001.
Quadrini, Vincenzo, "Entrepreneurship, Saving, and Social Mobility," *Review of Economic Dynamics,* III (2000), 1–40.
Rebelo, Sergio, "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy,* XCIX (1991), 500–521.
Rebelo, Sergio, and Robert King, "Public Policy and Economic Growth: Developing Neoclassical Implications," *Journal of Political Economy,* XCVIII (1990), 126–150.
Rogerson, Richard, and Martin Schindler, "The Welfare Costs of Worker Displacement," *Journal of Monetary Economics*, XLIX (2002), 1213–1235.
Stokey, Nancy L., and Sergio Rebelo, "Growth Effects of Flat-Rate Taxes," *Journal of Political Economy,* CIII (1995), 519–550.
Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron, "Asset Pricing with Idio-

syncratic Risk and Overlapping Generations," Working Paper, Carnegie Mellon University, 2001.
Summers, Robert, and Alan Heston, "The Penn World Table: An Expanded Set of International Comparisons, 1950–1988," *Quarterly Journal of Economics,* CVI (1991), 327–368.
Topel, Robert, "Specific Capital, Mobility, and Wages: Wages Rise with Job Seniority," *Journal of Political Economy,* XCIX (1991), 145–176.
Woodford, Michael, "Stationary Sunspot Equilibria in a Financed Constrained Economy," *Journal of Economic Theory,* XL (1986), 128–137.
Zeldes, Stephen P., "Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence," *Quarterly Journal of Economics,* CIV (1989), 275–298.