Explaining Rising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents, Review of Economic Dynamics, 1998

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1 General Equilibrium Approach to Studying Wage Inequality

1.1 A Dynamic General Equilibrium Model of Earnings, Schooling, and On-the-Job Training

We generalize the microeconomic model of earnings, schooling, and on-the-job training developed by Ben Porath. In his model, income maximizing agents combine time, goods, and the current stock of human capital to produce new human capital.

We extend his model in several ways. (1) In contrast to his model, we distinguish between schooling capital and job training capital at a given schooling level. In our model, school human capital is an input to the production of human capital acquired on the job and is also directly productive in the market. However, the tight link between schooling and on-the-job training investments which is characteristics of Ben Porath's model is broken. (2) Skills produced at different schooling levels command different prices, and wage inequality among persons is generated by differences in skill levels, differences in investment, and differences in the prices of different skills. In Ben Porath's model, wage inequality can only be generated by differences in skill levels and investment behavior, because all skill commands the same price. In our model, different levels of schooling enable individuals to invest in different skills through on-the-job training in the post-schooling period. In the aggregate, the skills associated with different schooling groups are not perfect substitutes. Within school groups, however, persons with different amounts of skill are perfect substitutes. (3) Persons choose among schooling levels with associated post-school investment functions. (4) Among persons of the same schooling level, there is heterogeneity both in initial stocks of human capital and in the ability to produce job-specific human capital. (5) We embed our model of individual human capital production into a general equilibrium setting so that the relationship between the capital market and the markets for human capital of different skill levels is explicitly developed.

[*]This note is written down during my M.phil. period at the University of Oxford.

1.2 The Microeconomic Model

We first derive the optimal consumption, on-the-job training, and schooling choices for a given individual of type θ who takes skill prices as given. We then aggregate the model to a general equilibrium setting. Throughout this paper, we simplify the tax code and assume that income taxes are proportional. Individuals live for $\bar{a}$ years and retire after $a_R \leq \bar{a}$ years. Retirement is mandatory. In the first portion of the life cycle, a prospective student decides whether or not to remain in school. Once he has left school, he cannot return. He chooses the schooling option that gives him the highest level of lifetime utility.

Define K_{at} as the stock of physical capital held at time t by a person age a; H_{at}^{s} is the stock of human capital at time t of type S at age a. The optimal life-cycle problem can be solved in two stages. First, condition on schooling and solve for the optimal path of consumption (C_{at}) and post-school investment (I_{at}^{s}) for each type of schooling level. Individuals then select among schooling levels to maximize lifetime welfare. Given S an individual age a at time t has the value function

$$V_{at}\left(H_{at}^{S}, K_{at}, S\right) = \max_{C_{at}, I_{at}^{S}} U\left(C_{at}\right) + \delta V_{a+1,t+1}\left(H_{a+1,t+1}^{S}, K_{a+1,t+1}, S\right) \tag{1}$$

This function is maximized subject to the budget constraint

$$K_{a+1,t+1} \leq K_{a,t}\left(1 + (1-\tau) r_t\right) + (1-\tau) R_t^{S} H_{at}^{S}\left(1 - I_{at}^{S}\right) - C_{at}, \tag{2}$$

where τ is the proportional tax rate on capital and labor earnings, R_t^{s} is the rental rate on human capital of type S, and r_t is the net return on physical capital at time t. In this paper, we abstract from labor supply.

In the empirical analysis in this paper, we use the conventional power utility specification of preferences

$$U\left(C_{at}\right) = \frac{C_{at}^{\gamma} - 1}{\gamma}. \tag{3}$$

On-the-job human capital for a person of schooling level S accumulates through the human capital production function

$$H_{a+1,t+1}^{S} = A^{S}(\theta)\left(I_{a,t}^{S}\right)^{\alpha_S}\left(H_{a,t}^{S}\right)^{\beta_S} + \left(1 - \sigma^{S}\right) H_{a,t}^{S}, \tag{4}$$

where the conditions $0 < \alpha_S < 1$ and $0 < \beta_S < 1$ guarantee that the problem is concave in the control variable, and σ^S is the rate of depreciation of job-S-specific human capital.

For simplicity, we ignore the input of goods into the production of human capital on the job. We explicitly allow for tuition costs of college which we denote by D_{at}^{S}. The same good that is used to produce capital and final output is used to produce schooling human capital. After completion of schooling, time is allocated to two activities, both of which must be nonnegative: on-the-job investment, I_{at}^{S}, and work, $(1 - I_{at}^{S})$. The agent solves a life-cycle optimization problem given initial stocks of human and physical capital, $H^S(\theta)$ and K_0, as well as his ability to produce human capital on the job, $A^S(\theta)$.

$H^S(\theta)$ and $A^S(\theta)$ represent ability to "earn" and ability to "learn," respectively, measured after completion of school. They embody the contribution of schooling to subsequent learning and earning in the schooling-level S-specific skills as well as any initial endowments. Notably absent from our model are the short-run credit constraints that are often featured in the literature on schooling and human capital accumulation. The α and β are also permitted to be S-specific, which emphasizes that schooling affects the process of learning on the job in the variety of different ways.

Assuming interior solutions conditional on the choice of schooling, we obtain the following first order conditions:

$$U_{C_{a,t}} = \delta \frac{\partial V_{a+1,t+1}}{\partial K_{a+1,t+1}}(1-\tau), \tag{5}$$

$$\frac{\partial V_{a+1,t+1}}{\partial K_{a+1,t+1}} = \delta \frac{\partial V_{a+1,t+1}}{\partial H_{a+1,t+1}} \left[\frac{A\alpha_S \left(I^S_{a,t}\right)^{\alpha_S - 1} \left(H^S_{a,t}\right)^{\beta_S}}{R^S_t H^S_{a,t}(1-\tau)} \right], \tag{6}$$

$$\frac{\partial V_{a,t}}{\partial K_{a,t}} = \delta \frac{\partial V_{a+1,t+1}}{\partial K_{a+1,t+1}} \left(1 + r_t(1-\tau)\right), \tag{7}$$

$$\frac{\partial V_{a,t}}{\partial H^S_{a,t}} = \delta \frac{\partial V_{a+1,t+1}}{\partial K_{a+1,t+1}} R^S_t \left(1 - I^S_{a,t}\right)(1-\tau) + \delta \frac{\partial V_{a+1,t+1}}{\partial H^S_{a+1,t+1}} \left(A\beta_S \left(I^S_{a,t}\right)^{\alpha_S} \left(H^S_{a,t}\right)^{\beta_S - 1} + \left(1 - \delta^S\right)\right) \tag{8}$$

At the end of working life, the final term, which is the contribution of human capital to earnings, has zero marginal value. We assume mandatory retirement at age a_R, leaving ages $\bar{a} - a_R$ as the retirement period during RR which there are no labor earnings.

At the beginning of life, agents choose the value of S that maximizes lifetime utility,

$$\hat{S} = \underset{S}{\mathrm{Arg\,max}} \left[V^S(\theta) - D^S - \varepsilon^S \right], \tag{9}$$

where $V^S(\theta)$ is the present value of earnings for schooling at level S, D^S is the discounted direct cost of schooling, and ε^S represents nonpecuniary benefits expressed in present value terms. Discounting of V^S and D^S is back to the beginning of life to account for different ages of completing school. Tuition costs are permitted to change over time so that different cohort face different environments for schooling costs. Given optimal investment in physical capital, schooling, investment in job-specific human capital, and consumption, we calculate the path of savings. For a given return on capital and rental rates on human capital, the solution to the S-specific optimization problem is unique given concavity of the production of in terms of $I^S_{at}(0 < \alpha_S < 1)$, the restriction that human capital be self-productive, but not too strongly $(0 \leq \beta_S \leq 1)$, the restriction that investment is in the unit interval $(0 \leq I^S_{at} \leq 1)$, and concavity of U in terms of $C(\gamma < 1)$.

The choice of S is unique almost surely if ε^S is a continuous random variable, as we assume in our empirical analysis. The dynamic problem is of split-endpoint form. We know the initial condition for human and physical capital and optimality implies that investment is zero at the end of life. In this paper, we numerically solve this problem using the method of shooting. For any terminal value of H^S and K, we solve backward to the initial period and obtain the implied initial conditions. We iterate until the simulated initial condition equals the prespecified value.

1.3 Aggregating the Model

The prices of skills and capital are determined as derivatives of an aggregate production function. To compute rental prices for capital and the different types of human capital, it is necessary to construct aggregates of each of the skills. Given the solution to the individual's problem for each value of θ and each path of prices, we use the distribution of θ, $G(\theta)$, to construct aggregates of human and physical capital. We embed our human capital model into an overlapping generations framework in which the population at any given time is composed of $\bar{a}$ overlapping generations, each with an identical ex-ante distribution of heterogeneity, $G(\theta)$.

Human capital of type S is a perfect substitute for any other human capital of the same schooling type, whatever the age or experience level of the agent, but it is not perfectly substitutable with human capital from other schooling levels. In our model, cohorts differ from each other only because they face different price paths and policy environments within their lifetimes. We assume perfect foresight and not myopic expectations. Let c index cohorts, and denote the date at with cohort c is born by t_c. Their first period of life is t_c+1. Let P_{t_c} be the vector of paths of rental prices of physical and human capital confronting cohort c over its lifetime from time t_c+1 to $t_c+\bar{a}$. The rental rate on physical capital at time t is r_t. The rental rate on human capital is R_t^S. The choices made by individuals depend on the prices they face, P_{t_c}, their type θ, and hence their endowment and their nonpecuniary costs of schooling, ε^S. Let $H_{at}^S(\theta, P_{t_c})$ and $K_{at}^S(\theta, P_{t_c})$ be the amount of human and physical capital possessed, respectively, and let $I_{at}^S(\theta, P_{t_c})$ be the time devoted to investment by an individual with schooling level S, at age a, of type θ, in cohort c.

By definition, the age at time t of a person born at time t_c is $a=t-t_c$. Let $N^S(\theta, t_c)$ be the number of persons of type θ, in cohort c, of schooling level S. In this notation, the aggregate stock of employed human capital of type S at time t is accumulated over the non-retired cohorts in the economy at time t,

$$\bar{H}_t^S = \sum_{t_c=t-a_R}^{t-1} \int H_{t-t_c,t}^S\left(\theta, P_{t_c}\right)\left(1-I_{t-t_c,t}^S\left(\theta, P_{t_c}\right)\right) N^S(\theta, t_c)\, dG(\theta) \tag{10}$$

where $a=t-t_c$, $S=1,\ldots,\bar{S}$, where $\bar{S}$ is the maximum number of years of schooling. The aggregate potential stock of human capital of type S is obtained by setting $I_a^S(\theta, P_{t_c})$ in the preceding expression:

$$\bar{H}_t^S(\text{ potential }) = \sum_{t_c=t-a_R}^{t-1} \int H_{t-t_c,t}^S\left(\theta, P_{t_c}\right) N^S(\theta, t_c)\, dG(\theta). \tag{11}$$

The aggregate capital stock is the capital held by persons of all ages:

$$\bar{K}_t = \sum_{t_c=t-\bar{a}}^{t-1} \sum_{s=1}^{\bar{S}} \int K_{t-t_c,t}^S\left(\theta, P_{t_c}\right) N^S\left(\theta, P_{t_c}\right) dG(\theta). \tag{12}$$

1.4 Equilibrium Conditions under Perfect Foresight

To close the model, it is necessary to specify the aggregate production function $F(\bar{H}_t^1, \ldots, \bar{H}_t^{\bar{S}}, \bar{K}_t)$, which is assumed to exhibit constant returns to scale. The equilibrium conditions require that marginal products equal pre-tax prices $R_t^S = F_{\bar{H}^S}\left(\bar{H}_t^1, \dots, \bar{H}_t^{\bar{S}}, \bar{K}_t, t\right), S = 1, \dots, \bar{S}$ and $r_t^S = F_{\bar{K}_t}\left(\bar{H}_t^1, \dots, \bar{H}_t^{\bar{S}}, \bar{K}_t, t\right)$. In the two-skill economy estimated below, we specialize the production function to

$$F\left(\bar{H}_t^1, \bar{H}_t^2, \bar{K}_t\right) = a_3\left(a_2\left(a_1\left(\bar{H}_t^1\right)^{\rho_1} + (1-a_1)\left(\bar{H}_t^2\right)^{\rho_1}\right)^{\frac{\rho_2}{\rho_1}} + (1-a_2)\,\bar{K}_t^{\rho_2}\right)^{\frac{1}{\rho_2}}. \tag{13}$$

When $\rho_1 = \rho_2 = 0$, the technology is Cobb-Douglas. When $\rho_2 = 0$, we obtain a model consistent with the constancy of capital's share irrespective of the value of ρ_1.

Activities of the government, apart from its role in subsidizing human capital, are not central to our analysis. The government collects taxes at a fixed level and does not redistribute them.

1.5 Linking the Earnings Function to Prices and Market Aggregates

The earnings for a person of age a of cohort c of type θ with human capital $H_{at}^S(\theta, P_{t_c})$ at time t are

$$W\left(a, t, H_{a,t}^S\left(\theta, P_{t_c}\right)\right) = R_t^S H_{a,t}^S\left(\theta, P_{t_c}\right)\left(1 - I_{a,t}^S\left(\theta, P_{t_c}\right)\right). \tag{14}$$

They are determined by aggregate rental rates (R_t^S), individual endowments ($H_{a,t}^S\left(\theta, P_{t_c}\right)$), and individual investment decisions ($I_a^S(\theta, P_{t_c})$). The last two components depend on agent expectations of future prices. Different cohorts facing different price paths will invest differently and have different human capital stocks.

2 Determining the Parameters of the Model