Capital-Skill Complementarity, Income Distribution, and Output Accounting

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This article presents evidence for the view that physical capital is more complementary to educated labor than to less educated labor. For this reason previous estimates of elasticities of substitution between educated and less educated labor are too high. Using a two-level CES production function, we run international cross-sectional regressions at the level both of the economy and of individual sectors. The whole-economy model allows for the effects of wages on educational choices as well as vice versa. It predicts that, as total capital per head rises, the share of physical capital in national income falls and that of human capital rises. The production function also shows that intercountry differences in output per head are due more to differences in physical capital than in human capital.

Introduction and Summary

How quickly do the returns to education fall when the number of educated people rises? This has been a crucial question for the philosophy of educational planning, since the case for manpower forecasting and planning is stronger the less easy the process of substituting educated for less educated people.

In answer to the question, Blaug (1967) and others pointed out that U.S. rates of return to education had been remarkably constant over time despite a vast increase in the educated labor force. This suggested that substitution was relatively easy. Likewise, cross-sectional data on countries (Bowles 1970) and on U.S. states (Dougherty 1972) showed that the relative wages of the educated tend to vary with their relative numbers

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by only small amounts—evidence again, it was claimed, of easy substitution.

However, the inference, whether from time series or cross section, may not be valid if other things are varying at the same time. And the relative use of physical capital always varies, the ratio of physical capital to raw labor generally rising with the ratio of educated to raw labor. If physical capital is more complementary to educated than to raw labor, this could then explain why the relative wages of the educated are not much lower when their relative numbers are much higher.[1] Equally, if this is the explanation, planners should avoid excessive educational expansion not accompanied by physical investment, since this could produce a rapid fall in relative wages.

This "capital-skill complementarity hypothesis" has been advanced by Griliches (1969, 1970) and partially confirmed on a cross section of U.S. states. Because of lack of data, he assumed constancy across states in the absolute wage of educated manpower and in the rate of return on physical capital in any given industry. The present study relies instead on international data on 23 countries, including detailed information on national education-specific wages and data on physical capital stocks and rentals at the national, though not at the sectoral, level. The data have the advantages of wide variation but obvious problems of comparability. As regards the form of the relationships involved, Griliches restricted these by assuming constant own- and cross-elasticities of demand in each factor demand equation, while we restrict them by assuming an explicit form of production function—the two-level CES function.[2] This has the advantage, apart from yielding meaningful results, that we can use the explicit function to account for income differences between countries and for differences in the functional distribution of income.

The paper is constructed as follows. Section I examines capital-skill

[1] Other possible hypotheses explaining the constancy of relative wages over time include the following: (*a*) Luxuries are more education-intensive than necessities. Here Folger and Nam (1967) show that shifts in the sectoral composition of the labor force account for a negligible proportion of the overall rise in relative proportions of educated people in the United States. (*b*) Technical progress is biased (in the Hicksian sense) toward educated rather than raw labor. (*c*) Technical progress is fastest in education-intensive industries having demand elasticities greater than unity. Both (*b*) and (*c*) are extremely difficult to study. Other possible hypotheses explaining the cross-sectional stability of relative wages are: (*a*) as above; here Fallon (1974, chap. 8) shows the same as Folger and Nam's time series; (*b*) the factor-price equalization theorem; however, this implies identical factor proportions in all countries in any one industry, which is inconsistent with the data used in our Section II.

[2] We also tried the Griliches approach (modified so that prices are the dependent variables and quantities the independent variables), but found that the results for the richer and poorer countries taken separately were grossly inconsistent with each other and with the results for the pooled sample, whether judged by *F*-tests or by consistency of the signs of the coefficients.

complementarity and the ease of substitution between types of labor at the level of the whole economy. The problem here is that factor prices affect educational choices as well as vice versa, and a complete model is therefore specified. Section II examines the same question at the sectoral level, factor prices being taken here as exogenous. Both Sections I and II confirm the capital-skill complementarity hypothesis and suggest much lower elasticities of substitution between educated and raw labor than either Bowles or Dougherty. Sections III and IV revert to the whole-economy model. Section III illustrates how the production function, together with the relation determining the supply of educated people, determines the evolving pattern of income distribution as countries get richer (as measured by their total capital per head). If the supply of educated people is always such that the rates of return to education and to human capital are in fixed proportion, the parameters of the production function correctly predict that as economic progress occurs, human capital per head will grow faster than physical capital per head and the share of physical capital in national income will fall. In Section IV we examine the relative importance of physical and human capital in explaining income differences between countries and conclude (contrary to the claims of Krueger [1968]) that physical capital is generally the more important.

I. The Pattern of Substitution: Whole-Economy Level

The Problem

Suppose there are only three factors of production, physical capital (K), skill (S—to be defined), and other labor (N—to be defined), and output (Y) is determined by: $Y = f(K, S, N)$. Then the capital-skill complementarity hypothesis, in the form in which we are interested, states that:

$$\frac{\partial}{\partial_K}\left(\frac{f_S}{f_N}\right) > 0,$$

which implies

$$\frac{f_{SK}}{f_S} > \frac{f_{NK}}{f_N}.$$

By contrast, the production function used by Bowles and Dougherty assumes that physical capital has no effect on the relative marginal products of skill and other labor, the function having the form

$$Y = f(K, [bS^{\theta} + (1 - b)N^{\theta}]).$$

To examine the capital-skill complementarity hypothesis we need to use a form of function which permits it to be confirmed or rejected. The

Cobb-Douglas does not, and the simplest form which does is the two-level CES function,

$$Y = A[aQ^{\rho} + (1 - a)X_1^{\rho}]^{1/\rho} \qquad (\rho \leq 1),$$

where

$$Q = [bX_2^{\theta} + (1 - b)X_3^{\theta}]^{1/\theta} \qquad (\theta \leq 1),$$

and X_1, X_2, and X_3 are any permutation of K, S, and N.[3] The Bowles function would correspond to the version where K played the role of X_1 ("the odd man out") and was equally complementary to skill and other labor. Only if S or N played the role of X_1 could capital be more (or less) complementary to skill than to other labor. We have tested all three permutations of the two-level function using the two-stage least-square models developed later and found that the production function estimates where N played the role of X_1 were the most satisfactory on a number of criteria.[4]

Our preferred functional form is thus

$$Y = A[aQ^{\rho} + (1 - a)N^{\rho}]^{1/\rho} \qquad (\rho \leq 1),$$

where (1)

$$Q = [bK^{\theta} + (1 - b)S^{\theta}]^{1/\theta} \qquad (\theta \leq 1).$$

If the capital-complementarity hypothesis is true, $\rho > \theta$, and vice versa.[5] In other words, the direct elasticity of substitution between N and Q (i.e., $1/[1 - \rho]$) must exceed the direct elasticity of substitution, within the "nest," between K and S (i.e., $1/[1 - \theta]$).

This is really all that needs to be said about the capital-skill complementarity hypothesis given the two-level CES function However, it may be useful to relate the condition $\rho > \theta$ to the other elasticities used in general discussions of the issue. Of these, the most relevant is the elasticity of complementarity (Hicks 1970). This is defined for a constant-returns production function $f(X_1, \ldots, X_n)$ as

$$c_{ij} = \frac{1}{v_j}\frac{\partial \log f_i}{\partial \log X_j}\left(= \frac{1}{v_i}\frac{\partial \log f_j}{\partial \log X_i} = c_{ji}\right),$$

where v_j is the share of the jth factor in output, and $\partial \log f_i/\partial \log X_j$ indicates the proportional effect on the marginal product of the ith factor of a change in the quantity of the jth factor, holding all other input quantities constant. Clearly

$$c_{SK} = \frac{1}{v_K} \cdot \frac{f_{SK}}{f_S} \cdot K$$

[3] Another function which does is the log-quadratic, but this does not enable one to take advantage of data on relative prices.

[4] The evidence is presented in Appendix A.

[5] Taking derivatives in (1), if $(f_{SK}/f_S) > (f_{NK}/f_N)$,

$$a(1 - a)b(1 - b)A^{2\rho}Y^{2(1-\rho)}K^{\theta-1}S^{\theta-1}N^{\rho-1}Q^{\rho-2\theta}(\rho - \theta) > 0.$$

and

$$c_{NK} = \frac{1}{v_K} \cdot \frac{f_{NK}}{f_N} \cdot K,$$

so that our form of the capital-skill complementarity hypothesis is fulfilled if $c_{SK} > c_{NK}$. In the case of our two-level CES function,[6]

$$c_{SK} = 1 - \rho + \frac{1}{v_Q} (\rho - \theta)$$

and

$$c_{NK} = c_{SN} = 1 - \rho.$$

This confirms that the capital-skill complementarity hypothesis requires that $\rho > \theta$. We may also note that, in Hicks's language, the pairs N, K and N, S are q-complements whatever the value of ρ (< 1), while capital and skill (K and S) may or may not be q-complements but must be if $\rho > \theta$.

From our point of view, the elasticity of complementarity is a more useful concept than the better-known Allen elasticity of substitution, used in the Griliches (1969) analysis. The reason is that the former deals with the effect of a factor quantity change on factor prices (other factor quantities and output price constant), while the latter deals with the effect of a factor price change on factor quantities (other factor prices and output quantity constant). The Allen elasticity of substitution is defined as

$$\sigma_{ij} = \frac{1}{v_j} \cdot \frac{\partial \log X_i}{\partial \log p_j} \left(= \frac{1}{v_i} \cdot \frac{\partial \log X_j}{\partial \log p_i} = \sigma_{ji} \right).$$

However, under constant returns to scale it can be shown that, even in the n-factor case,[7] $c_{SK} > c_{NK}$ implies and is implied by $\sigma_{SK} < \sigma_{NK}$. In our two-level CES function

$$\sigma_{SK} = \frac{1}{1 - \rho} + \frac{1}{v_Q} \left(\frac{1}{1 - \theta} - \frac{1}{1 - \rho} \right)$$

and

$$\sigma_{NK} = \sigma_{SN} = \frac{1}{1 - \rho}.$$

This confirms again that the capital-skill complementarity hypothesis requires $\rho > \theta$. In addition we may note that, in Hicks's language, the

[6] $$c_{SS} = 1 - \rho + \frac{1}{v_Q} (\rho - \theta) + \frac{1}{v_S} (\theta - 1) < 0 \qquad \text{(and analogously for } c_{KK}\text{);}$$
$$c_{NN} = 1 - \rho + \frac{1}{v_N} (\rho - 1) < 0.$$

[7] On the relationship between elasticities of complementarity and substitution, see Sato and Koizumi (1973).

pairs N, K and N, S must be p-substitutes, while capital and skill (K and S) may or may not be p-substitutes but must be if $\rho < \theta$.

So much for capital-skill complementarity—that is, the effect of changes in *capital* on rates of return to education. Turning to the effect of *education* on rates of return to education, the measure normally used is the direct elasticity of substitution, defined as

$$d_{ij} = -\frac{\partial \log (X_i/X_j)}{\partial \log (f_i/f_j)} \left(= -\frac{\partial \log (X_j/X_i)}{\partial \log (f_j/f_i)} = d_{ji} \right),$$

that is, the proportional change in the relative quantities of the ith and jth factors for a given change in their relative marginal products, all other factors and output being held constant. To measure the rate at which the returns to education will fall as the relative number of educated people rises, we take the inverse

$$\frac{1}{d_{SN}} = -\frac{\partial \log (f_S/f_N)}{\partial \log (S/N)}.$$

In the case of our function,[8]

$$\frac{1}{d_{SN}} = \frac{v_S v_N}{v_S + v_N} (2c_{SN} - c_{SS} - c_{NN}).$$

Like all direct elasticities, this is positive and lies between $1/(1 - \theta)$ and $1/(1 - \rho)$. It may not be strictly what is needed from a planning point of view, since, if the number of skilled people is increased in a country, the number of unskilled is constrained to decline by the same number. So output rises and substitution does not occur along an isoquant. Thus, unless the production function is homothetic in skilled and unskilled labor, the proportional change in f_S/f_N for a change in S/N such that $dS + dN = 0$ slightly exceeds $1/d_{SN}$ (provided $f_S > f_N$). However, in our estimated function the difference is small for most countries,[9] and we shall simply quote the direct elasticities of substitution as the occasion arises.

The Model with More Educated and Less Educated Labor

In order to estimate the parameters of the production function (1), we must specify the model generating our observations. It is convenient to begin with some definitions:

[8] The expression for $1/d_{KN}$ is analogous, while $1/d_{SK} = 1 - \theta$.

[9] If S' is defined as human capital (see below) and N' as the total labor force, there is no constraint of the form $dS' + dN' = 0$, and the rate at which the returns to education fall is given by

$$-\frac{d \log (f_{S'}/f_{N'})}{d \log S'},$$

which exceeds $1/d_{S'N'}$.

Quantity	Price (1963 \$)	Definition of Quantity
K	r	Physical capital (\$000, 1963 prices)
S	z	Workers with 8 or more years of education (000s)
N	w	Workers with 7 or less years of education (000s)
Q	q	$(bK^{\theta} + (1 - b)S^{\theta})^{1/\theta}$
Y	...	Output (\$000, 1963 prices)
P	...	Population (000s)

The following two basic estimating equations follow from the condition that price equals marginal product.

$$\log \frac{r}{z} = \log \frac{b}{1 - b} + (\theta - 1) \log \frac{K}{S}. \tag{2}$$

$$\log \frac{q}{w} = \log \frac{a}{1 - a} + (\rho - 1) \log \frac{Q}{N}. \tag{3}$$

Clearly (2) has to be estimated before (3), since it provides the parameters needed to construct the variables Q and q.

However, ordinary least-square estimates of these equations could be seriously biased if relatively more people get educated in countries where the private rate of return to education is high relative to the rate of return on physical capital. We therefore need an equation for the supply of educated labor, relating this to the relative rate of return and (owing to the consumption aspects of education) to income per head. The private rate of return to education is approximately proportional to the ratio of more educated to less educated wages (z/w), so we have

$$\log \frac{S}{N} = \log c_0 + c_1 \log \frac{z/w}{r} + c_2 \log \frac{Y}{P}. \tag{4}$$

Clearly S, N and P are not unrelated, since $S + N$ is the total labor force. There is, therefore a total labor supply equation,[10]

$$\log (S + N) = \log d_0 + d_1 \log P; \tag{5}$$

and the model is completed by the product exhaustion condition

$$Y = rK + zS + wN. \tag{6}$$

Treating Q and q not as separate variables but as weighted aggregates of K, S, r, and w, we have six endogenous variables (Y, S, N, r, z, w) and two exogenous variables (K and P). If the system were linear, we

[10] In many ways it would have been simpler to treat the total labor supply as exogenous, but we should then have had to explain S/N in eq. (4) by income per worker rather than by income per head; the latter seems more plausible. Fortunately, the estimated value of d_1 for the whole sample was 1.03, making the solution values of real prices and quantity ratios approximately independent of the size of country and dependent only on the ratio of the two exogenous variables K and P.

should conclude that equation (2), which includes six linear restrictions, was overidentified. The same holds for (3), as can be seen by explicitly adding two equations defining log Q and log q. Equation (4) would be just identified. However, the system is not linear; but Fisher (1966, p. 149) has shown that, in general, nonlinear systems are not less identified than the corresponding linear systems. We therefore estimate equations (2), (3), and (4) using two-stage least squares, log K and log P being the exogenous variables.

The Model with Human Capital and Raw Labor

The preceding model is rather crude in the way it adds together all workers with higher education (L_H) and with secondary education (L_S) to form a single category for more educated workers (S), and all workers with primary education (L_P) and with no education (L_0) to form a single category of less educated workers (N).[11] If we want to take into account differences between countries in L_H/L_S and L_P/L_0 while continuing to work within a three-factor model, the natural approach is to differentiate between human capital (S') and raw labor (N'). The latter (N') consists simply of the number of workers in the labor force, irrespective of how much human capital, if any, each worker has; and its price (w') is the wage of workers with no education ($= w_0$). Human capital (S') is a weighted sum of the human capital formed by primary education (L_P), by secondary education (L_S), and by higher education (L_H). Since we are treating each type of human capital as a perfect substitute for each other, we need a constant set of weights, the natural weights being the average world price for the services of each type of capital. So

$$S' = (\bar{w}_H - \bar{w}_0)L_H + (\bar{w}_S - \bar{w}_0)L_S + (\bar{w}_P - \bar{w}_0)L_P,$$

where $\bar{w}_H$ is the world average wage of higher-educated workers (the unweighted country average), and likewise for the other categories. The dimension of S' is units of skill. Its price (z') equals its income divided by its quantity and is a money flow per unit of time. Thus

$$z' = \frac{(w_H - w_0)L_H + (w_S - w_0)L_S + (w_P - w_0)L_P}{S'},$$

where w_H are the wages of higher-educated people in the country in question and likewise for the other categories. This price (z') is not the same as the private rate of return to education, which depends on the return to education (z') relative to its cost, which in turn is roughly proportional to the earnings of the uneducated (w'). Thus, if all educated workers had primary education only, the rate of return would be pro-

[11] $L_H = 13+$ years of education, $L_S = 8$–12, $L_P = 1$–7, $L_0 =$ less than 1.

portional to $(w_P - w_0)/w'$, which would be proportional to z'/w'. It is this latter expression which we take as our proxy for the rate of return to education.[12]

We can now set up our model. The key feature is that now total capital (C), both human and physical, is taken as exogenous rather than physical capital (K) only, as in our first model. Physical capital is measured in the same way as before under the assumption that the price of a capital good is equalized across the sample by international trade. On the other hand, human capital must be measured in units of domestic value rather than, as S', in physical units. Since the monetary rate of return on education is influenced by its nonmonetary returns, it seems reasonable to take the rate of return on physical capital (r) as the rate of time preference, so that the value of human capital is $z'S'/r$. The total capital identity is then

$$C = K + \frac{z'S'}{r}. \tag{7}$$

The balance between human and nonhuman capital is determined as before by relative rates of return and income per head:

$$\log \frac{z'S'/r}{K} = \log c_0 + c_1 \log \frac{z'/w'}{r} + c_2 \log \frac{Y}{P}. \tag{4'}$$

The total labor force (N) is related to the total population (P) by

$$\log N' = \log d_0 + d_1 \log P, \tag{5'}$$

and the model is completed by equations (1′), (2′), (3′), and (6′), which are identical with (1), (2), (3), and (6) except that N', w', S', z', Q', and q' replace the corresponding variables. This model has one equation more than the first and one additional variable (C). The identification

[12] In the case of more than one type of education, if the relative prices of the different types of human capital are constant, as assumed in constructing S', then the relative rates of return on the different types of human capital vary. Thus suppose there are two levels of education, secondary and primary, each lasting 1 year. By assumption,

$$\frac{w_S - w_0}{w_P - w_0} = k = \frac{\bar{w}_S - \bar{w}_0}{\bar{w}_P - \bar{w}_0}.$$

By definition

$$r_P = \frac{w_P - w_0}{w_0} = \frac{w_P}{w_0} - 1$$

$$r_S = \frac{w_S - w_0}{w_P} = \frac{k(w_P - w_0)}{w_P} = k - \frac{kw_0}{w_P} = k - \frac{k}{r_P + 1}.$$

Thus r_S and r_P vary positively, r_P/r_S rising as r_P rises (an observation confirmed by Psacharopoulos 1973). Therefore

$$\frac{z'}{w'} = \frac{(w_S - w_0)L_S + (w_P - w_0)L_P}{(\bar{w}_S - \bar{w}_0)L_S + (\bar{w}_P - \bar{w}_0)L_P} \cdot \frac{1}{w_0} = \left(\frac{w_P - w_0}{\bar{w}_P - \bar{w}_0}\right)\frac{1}{w_0} = \frac{r_P}{\bar{w}_P - \bar{w}_0}.$$

So z'/w' is proportional to r_P and varies positively with r_S.

TABLE 1

SUBSTITUTION PARAMETERS: WHOLE ECONOMY

	Eqq. (2), (2′) $\theta - 1$	Eqq. (3), (3′) $\rho - 1$	$\rho - \theta$	No. of Observations
All countries:				
Using S, N	−3.45	−0.67	2.78	22
	(−1.8)	(−11.0)	(1.5)	
Using S', N'	−1.72	−0.96	0.76	22
	(−4.3)	(−8.5)	(1.8)	
Richer countries:				
Using S, N	−1.81	−0.54	1.27	9
	(−2.1)	(−3.8)	(1.4)	
Using S', N'	−0.94	−1.27	−0.33	9
	(−4.6)	(−5.7)	(−1.1)	
Poorer countries:				
Using S, N	−1.50	−0.47	1.03	13
	(−1.2)	(−4.1)	(0.8)	
Using S', N'	−2.03	−0.41	1.62	13
	(−1.6)	(−2.0)	(1.3)	

NOTE.—Figures in parentheses are t-statistics (see text re: third column).

of equations (2′), (3′), and (4′) is thus the same as for (2), (3), and (4). Estimation is by two-stage least squares, log C and log P being the exogenous variables.

Estimates

The data used relate to 1963 and are described in Appendix B, while the countries included are shown in table 4. Broadly the wage data are based on the sources used in the rate-of-return studies summarized in Psacharopoulos (1973),[13] while the employment data are based on the 1961 Censuses. The capital measure is the sum of gross fixed capital formation from 1949 to 1963 inclusive, at 1963 prices, and capital rental is the ratio of estimated capital income to capital so measured.

Table 1 shows the estimates of θ and ρ provided by equations (2) and (3) (and [2′] and [3′]). The regressions are done for the whole sample and for the richer and poorer countries separately. We thus have six estimates of $\rho - \theta$, all but one positive, as predicted by the capital-skill complementarity hypothesis. To measure the significance of the differences between $\hat{\rho}$ and $\hat{\theta}$, table 1 assumes that they are independent. The t-statistics shown in the third column are computed on this assumption, but a sensitivity analysis has suggested that a weak positive relation exists between $\hat{\rho}$ and $\hat{\theta}$, in which case these t-statistics are (absolutely) too low. At the crude aggregate level there is thus some mild confirmation of the capital-skill complementarity hypothesis.

As for the substitutability of more and less educated people, for the whole sample the direct elasticity of substitution is 0.61, much less than

[13] We are very grateful to the author for making the data available to us.

the values of 6–8 found by Bowles. This is partly due to differences in data and partly due to differences in specification. If on our data we use the two-level function with K taking the role of X_1, which is analogous to Bowles's procedure, we find an elasticity of 3.54.[14]

II. The Pattern of Substitution: Sectoral Level

Estimating Approach

Economy-wide functions are, however, necessarily crude and we turn now to the sectoral level. Apart from the advantage of disaggregation, we can also drop the simultaneous model, treating each sector as a price taker. However, we lack data on physical capital stocks and rentals at the sectoral level and have therefore to estimate our marginal productivity equations in their absolute rather than relative price form. This time we begin by estimating ρ from the condition that

$$\frac{\partial Y}{\partial N} = w = (1 - a)A^{\rho}\left(\frac{Y}{N}\right)^{1-\rho}$$

or, in its estimating form,

$$\log \frac{Y}{N} = -\frac{1}{1-\rho}\log[(1 - a)A^{\rho}] + \frac{1}{1-\rho}\log w. \tag{8}$$

We then need to estimate θ. This requires some tedious manipulations. Our remaining marginal productivity condition is for S, since we have no data on K or r. This gives

$$\frac{\partial Y}{\partial S} = z = aA^{\rho}\left(\frac{Y}{Q}\right)^{1-\rho}(1 - b)\left(\frac{Q}{S}\right)^{1-\theta}.$$

This cannot be estimated, since Q depends on θ. To eliminate Q we use Euler's theorem:[15]

$$Y = wN + \frac{\partial Y}{\partial Q}\cdot Q.$$

Now

$$\frac{\partial Y}{\partial Q} = aA^{\rho}\left(\frac{Y}{Q}\right)^{1-\rho}$$

$$\begin{aligned}\therefore Q &= a^{-(1/\rho)}A^{-1}(Y - wN)^{1/\rho}Y^{(\rho-1)/\rho}\\ &= a^{-(1/\rho)}A^{-1}X,\end{aligned}$$

[14] The comparable estimates using human capital and raw labor are 1.09 in our formulation and 1.67 where K takes the role of X_1.

[15] An alternative approach is to obtain Q from the production function relationship.

$$aQ^{\rho} = \left(\frac{Y}{A}\right)^{\rho} - (1 - a)N^{\rho}.$$

This leads to a different estimating equation from (9) which, unlike (9), uses the constant-term estimate from (8) and does not use wage data. However, the estimated values of $1/(1 - \theta)$ are similar to those given by (9).

where X is defined by

$$X = (Y - wN)^{1/\rho} Y^{(\rho - 1)/\rho}.$$

We now substitute for Q in the marginal productivity condition above and rearrange to obtain

$$\log \frac{X}{S} = - \frac{1}{1 - \theta} \log [(1 - b) a^{\theta/\rho} A^{\theta}] + \frac{1}{1 - \theta} \log \left(\frac{z}{X^{\rho - 1} Y^{1 - \rho}} \right). \tag{9}$$

As long as A (the "efficiency parameter") is regarded as a constant across countries, equations (8) and (9) provide our basic estimating equations, the first term on the right-hand side being in each case a constant. If, however, A is considered variable, then the estimates of ρ and θ will in general be biased. Two ways were used for dealing with this. First, a *dummy variable* was included distinguishing the poorer from the richer countries. Second, an *efficiency variable* (A_i) was calculated from the whole-economy regression of Section I as follows:[16]

$$A_i = \frac{Y_i}{[a Q_i^{\rho} + (1 - a) N_i^{\rho}]^{1/\rho}}.$$

We then assumed that in each sector, the ratio of the efficiency parameters of the different countries was the same as in the whole economy (as in Arrow et al. 1961). The estimating equations are got by rearranging (8) so that $1/(1 - \rho)$ is the regression coefficient of log Y/NA on log w/A, and by rearranging (9) so that $1/(1 - \theta)$ is the regression coefficient of log X/SA on log $z/X^{\rho - 1} Y^{1 - \rho} A$.

Estimates

Owing to lack of data, we do not at the sectoral level distinguish between richer and poorer countries. So for each sector we have four estimates

[16] The relative values are shown in table 4 (below). Since A varies between countries but is not directly observable, the model in Section I should strictly be completed by a function explaining A. In Section III we assume in a model with S' and N' that

$$\log A = \log e_0 + e_1 \log \frac{C}{N'}. \tag{10}$$

This alters the model by adding one variable and one equation and does not change the identifiability of eqq. (2′), (3′), and (4′).

of the parameters, one pair using the dummy variables and the efficiency variable (A) on equations (8) and (9) and another pair using them on equations (8′) and (9′), in which S' (human capital) and N' (raw labor) appear. Since we are studying four sectors, this gives 16 estimates altogether of $[1/(1 - \rho)] - [1/(1 - \theta)]$, which must be positive if $\rho > \theta$. Of these 16 estimates, 15 are positive and seven significantly so at at least the 90 percent level (see table 2). The negative estimate is not significant. A sensitivity analysis again suggested that our t-statistics for $[1/(1 - \hat{\rho})] - [1/(1 - \hat{\theta})]$ are downward biased. Thus the capital-skill complementarity hypothesis receives strong support at the sectoral level.

As regards the direct elasticities of substitution between skill and other labor, these lie between $1(1 - \rho)$ and $1/(1 - \theta)$ and are generally not above unity.

III. Income Distribution in the Whole Economy Model

Some Stylized Facts

We now revert to the whole-economy model in its human capital version, and look at some of its implications. First we set out some stylized facts from the data and then show how the model helps to explain them. The stylized facts can be usefully summarized by stating how our key variables change when a country gets richer in terms of total capital per man (see table 3). (1) The prices of skill (z') and of raw labor (w') rise, but the price of raw labor rises faster. This reduced wage differential (z'/w') does not of itself, of course, guarantee that inequality of earnings as measured by the Gini coefficient is reduced,[17] but taken with the actual changes in relative numbers it does in fact have the effect of reducing inequality so measured.[18] This is consistent with the findings of others (Lydall 1968). It also, of course, implies a reduction in the rate of return to investing in skill. (2) The rate of return to physical capital (r) also falls absolutely, but remains roughly proportional to the rate of return on skill. (3) The ratio of physical capital relative to skill measured in physical units (K/S) rises. But the ratio of physical capital to the value of human capital ($rK/z'S'$) falls. (4) The share of raw labor in national income does not change in any systematic way, while the share of physical capital falls and that of human capital rises.

[17] For example, if there are two categories of labor and the poorer category grows in relative number, then the Gini coefficient can easily rise even if wage differentials are reduced.

[18] The Gini coefficient used takes no account of variations in wages within educational categories. It is simply measured as

$$\frac{1}{2}\frac{1}{(L_H + L_S + L_P + L_O)^2}\frac{1}{\bar{w}}\sum_i\sum_j L_iL_j|w_i - w_j| \qquad (i, j = H, S, P, O).$$

TABLE 2

Substitution Parameters: Sectoral Level

	Using Dummy			Using Variable A			
	Eqq. (8), (8′) $\frac{1}{1-\rho}$	Eqq. (9), (9′) $\frac{1}{1-\theta}$	$\frac{1}{1-\rho}-\frac{1}{1-\theta}$	Eqq. (8), (8′) $\frac{1}{1-\rho}$	Eqq. (9), (9′) $\frac{1}{1-\theta}$	$\frac{1}{1-\rho}-\frac{1}{1-\theta}$	No. of Observations
Mining:							
Using S, N	1.45 (7.1)	0.76 (3.9)	0.69 (2.9)	0.84 (4.0)	0.54 (3.1)	0.30 (1.1)	15
Using S', N'	0.93 (5.2)	0.84 (9.6)	0.09 (0.8)	0.48 (2.8)	−0.09 (−0.6)	0.57 (2.5)	15
Manufacturing:							
Using S, N	1.66 (7.9)	0.74 (3.2)	0.92 (2.4)	1.11 (6.7)	0.85 (8.3)	0.26 (1.3)	16
Using S', N'	1.16 (4.2)	0.91 (8.6)	0.25 (0.4)	0.63 (3.8)	0.40 (2.6)	0.23 (1.03)	16
Construction:							
Using S, N	0.90 (3.5)	0.25 (1.3)	0.65 (2.0)	0.89 (3.6)	0.66 (5.3)	0.23 (0.8)	14
Using S', N'	0.70 (5.4)	0.08 (0.6)	0.62 (3.4)	0.44 (2.7)	−0.44 (−2.09)	0.88 (3.29)	14
Electricity, gas, water:							
Using S, N	1.03 (3.6)	1.06 (39.6)	−0.03 (−0.10)	1.09 (6.1)	1.01 (8.3)	0.08 (0.4)	16
Using S', N'	0.63 (3.2)	0.23 (1.66)	0.40 (1.66)	0.69 (4.1)	0.22 (1.4)	0.47 (2.0)	16

Note.—Figures in parentheses are t-statistics (see text). The sample of countries for the *mining* sector consists of United States, Sweden, Canada, New Zealand, France, Norway, United Kingdom, Japan, Greece, Israel, Puerto Rico, Chile, Philippines, Ghana, and Thailand. The samples for the other sectors are identical except that *manufacturing* excludes Greece and includes India and Korea; *construction* excludes Greece and Ghana and includes India and Korea; *electricity and gas* includes Korea. The last column shows the number of observations for regressions using the dummy variable. In the regressions using variable A, Israel was excluded.

TABLE 3

CORRELATION COEFFICIENTS BETWEEN C/N' AND THE VARIABLES SHOWN

Variable	Coefficient	Variable	Coefficient
w'	.86	S'/N'	.83
r	$-.79$	K/N'	.96
z'	.70	K/S'	.76
z'/w'	$-.45$	$rK/z'S'$	$-.39$
r/w'	$-.54$	rK/Y	$-.36$
r/z'	$-.52$	$z'S'/Y$	.26
$z'/w'r$	.00	$w'N'/Y$	.06

An Explanation

Let us now see how our model relates to these facts. The estimated production function taking all countries together is

$$Y = A(0.87K^{-0.72} + 0.13S'^{-0.72})^{0.62/-0.72}N'^{0.38}. \tag{1'}$$

As can be seen, this is a Cobb-Douglas function in Q' and N', the reason being that the estimate of ρ is 0.04 (see table 1), which is not significantly different from zero ($t = 0.4$). This function therefore implies that w' rises proportionately to Y/N' and that the share of raw labor is constant.

To go further than this we need to know how growth in total capital per man is allocated between growth in physical and human capital. In our model this is determined in equation (4′), but before turning to this it is instructive to look at the implication of a simpler model in which the rates of return on human and nonhuman capital are constrained to a fixed ratio (m).[19] So

$$\frac{z'/w'}{r} = m \tag{4''}$$

or $(z'/r) = mw'$. Given this, increased wealth per head must raise not only w' but also z'/r. And since Q is homothetic in K and S', this must raise K/S'. However, we know from table 1 that the direct elasticity of substitution between K and S' is less than unity (1/1.72), so that when K/S' rises, the ratio $rK/z'S'$ falls. In this way our production function, together with one simple behavioral constraint, predicts that the relative share of physical capital will fall with economic progress and that the shares of human capital and earnings will rise (fact 4). It also predicts that the value of human capital will rise faster than physical capital (fact 3).

As for the absolute rate of return on physical capital (r), this is bound to fall, provided A is constant, since both K/S' and K/N' are rising

[19] In our sample the rate of return (z'/w') is positively correlated with r, the correlation coefficient being .41. The ratio of z'/w' to r is completely uncorrelated with C/N'.

(fact 2). And by the same token the rate of return on human capital also falls (fact 1). However, as we have seen, the efficiency parameter A is not constant, and r falls with total capital per head only because the effects of changing factor proportions outweigh those of changes in efficiency.

The preceding discussion has used an exceedingly crude behavioral equation (4″) in order mainly to highlight some features of the production function. We now simulate the behavior of our key variables as C/N' changes, applying equations (1), (2′), (3′), (4′), (6′), and (7) to a country having a participation rate (N'/P) of 0.34. The supply equation estimated by two-stage least squares is

$$\log \frac{z'S'}{rK} = -0.21 + \underset{(t = 0.89)}{0.51} \log \frac{z'}{w'r} + \underset{(t = 3.4)}{0.36} \log \frac{Y}{P}. \tag{4'}$$

As can be seen, more of total capital is allocated to human capital the higher the rate of return to human capital relative to physical capital, though the estimated effect is not highly significant. Likewise, for given relative profitabilities of human and physical capital, relatively more human capital is chosen the richer the country. This reflects the psychic value attaching to human capital as such.

To see how equation (4′) brings about the rise of K/S' as C/N' rises (fact 3), we need simply combine it with (2′) to obtain

$$\log \frac{z'S'}{rK} = -3.17 + 2.22 \log w' - 1.57 \log \frac{Y}{N'}.$$

Since when Y/N' rises, w' rises at the same rate, the dependent variable rises also ($2.22 - 1.57 > 0$).

As for the fall in the absolute magnitude of r, this follows from the estimation of one further efficiency-determining equation which can be added to the model.[20]

$$\log A = 0.83 + \underset{(t = 5.84)}{0.20} \log \frac{C}{N'}. \tag{10}$$

IV. Accounting for Variations in Output per Head

Finally, we can use our production function to analyze the sources of differences between countries in their income per head. Unfortunately a substantial part of this remains unexplained, except insofar as we choose to attribute it to efficiency differences. However, the unexplained part is a great deal less than if the role of human capital had been ignored, as columns (1–3) of table 4 show. Column 1 shows the actual income per head of each country relative to that of the United States. For column 2

[20] The function was actually estimated in the form $\log A = \log e_0 + e_1 \log (C/P)$, and equation (10) is obtained by substituting $N' = 0.34\,P$.

TABLE 4

SOURCES OF DIFFERENCES IN INCOME PER HEAD

Country	$\frac{(Y/P)_i}{(Y/P)_{US}}$ (1)	A_i/A_{US} 2-Factor Model (2)	A_i/A_{US} 3-Factor Model (3)	D^K (4)	D^S (5)	D^N (6)	D^A (7)	$\sum D$ (8)
Richer:								
United States . . .	1.00	1.00	1.00	. . .	. . .	. . .	. . .	. . .
Sweden	0.70	0.73	0.63	0.28	−0.47	−0.28	1.27	0.80
Canada.	0.66	0.77	0.75	0.16	0.21	0.04	0.72	1.13
N. Zealand.	0.57	0.52	0.50	−0.39	0.05	−0.04	1.19	0.81
France	0.52	0.59	0.60	0.35	0.03	−0.13	0.85	1.10
Norway	0.50	0.58	0.60	0.16	0.26	−0.06	0.81	1.17
United Kingdom	0.49	0.59	0.61	0.52	0.03	−0.18	0.76	1.13
Japan	0.23	0.32	0.38	0.58	0.06	−0.18	0.81	1.27
Greece	0.17	0.33	0.37	0.64	0.24	−0.08	0.75	1.55
Poorer:								
Puerto Rico	0.32	0.66	0.61	0.53	0.17	0.07	0.61	1.38
Mexico	0.13	0.46	0.61	0.75	0.64	0.20	0.44	2.03
Chile	0.10	0.40	0.46	0.72	0.50	0.21	0.61	2.04
Columbia	0.09	0.29	0.33	0.72	0.38	0.12	0.73	1.95
Brazil	0.08	0.31	0.45	0.75	0.66	0.18	0.60	2.19
Turkey	0.08	0.26	0.38	0.84	0.52	−0.07	0.67	1.96
Philippines	0.08	0.37	0.47	0.86	0.38	0.26	0.57	2.07
Ghana	0.07	0.20	0.37	0.70	0.76	0.01	0.68	2.15
S. Korea.	0.05	0.27	0.37	0.88	0.51	0.04	0.66	2.09
Thailand	0.04	0.16	0.23	0.84	0.43	−0.06	0.80	2.01
Kenya.	0.03	0.18	0.28	0.85	0.74	0.13	0.74	2.46
India.	0.03	0.11	0.22	0.85	0.74	−0.05	0.81	2.35
Uganda	0.02	0.12	0.19	0.88	0.60	−0.02	0.83	2.29

we have estimated a simple CES function using capital and raw labor only,[21] and then shown the efficiency of each ith country relative to that of the United States (A_i/A_{US}), where

$$A_i = \frac{Y_i}{[aK_i^\rho + (1 - a)N_i'^\rho]^{1/\rho}}.$$

The third column shows the relative efficiencies we have already used in Sections II and III. As can be seen, the differences in income per head in column 1 are greater than the efficiency differences in column 2, which in turn are greater than the efficiency differences in column 3. This pattern can be summarized by the coefficients of variation for each column which are 1.03, 0.56, and 0.40, respectively.

But although differences in factor endowments leave much unexplained, it is still useful to summarize their effects. Following Krueger (1968),

[21] The regression equation was

$$\log \frac{r}{w''} = 0.06 - \underset{(t\,=\,-13.75)}{1.07} \log \left(\frac{K}{N'}\right),$$

where w'' is the average wage of all workers irrespective of skill. From these parameter values, A was computed for each country.

we focus on the differences in income per head in each ith country from that in the United States. For each factor in turn we ask how much the U.S. income per head would fall if the endowment per head of that factor were reduced to its level in the ith country, all other U.S. factor endowments held constant. This fall is then expressed as a proportion of the total gap in income per head between the two countries, so as to yield a measure D_i^j indicating the relative fall due to reducing the jth factor to its level in the ith country. If the production function is written as

$$\frac{Y}{P} = Af\left[\frac{K}{P}, \frac{S'}{P}, \frac{N'}{P}\right],$$

we have, for example, that

$$D_i^K = \frac{(Y/P)_{US} - A_{US}f[(K/P)_i, (S'/P)_{US}, (N'/P)_{US}]}{(Y/P)_{US} - (Y/P)_i};$$

D_i^S and D_i^N are calculated analogously. In addition, we have

$$D_i^A = \frac{(Y/P)_{US} - A_i f[(K/P)_{US}, (S'/P)_{US}, (N'/P)_{US}]}{(Y/P)_{US} - (Y/P)_i}.$$

Clearly the statistics D do not account for income differences in the sense that they generally sum to unity. In fact, for any country less well endowed in all factors than the United States and less efficient,[22] $D_i^K + D_i^{S'} + D_i^{N'} + D_i^A \geq 1$. Nevertheless, the relative magnitudes of the D terms do provide as good an indication as any of the "importance" of different factors in explaining income difference. Physical capital is more "important" than human capital in five out of eight richer countries, and in 12 out of the 13 poorer ones.[23] This finding was already implicit

[22] First there is the problem of the multiplicative relation of A and f.

$$\begin{aligned} A_{US}(f_{US} - f_i) + (A_{US} - A_i)f_{US} &= A_{US}f_{US} - A_{US}f_i + A_{US}f_{US} - A_i f_{US} - A_i f_i + A_i f_i \\ &= A_{US}f_{US} - A_i f_i + (A_{US} - A_i)(f_{US} - f_i) \\ &> A_{US}f_{US} - A_i f_i. \end{aligned}$$

Second, consider the interaction effects within f. To simplify, suppose f a function of two variables only, $f(K, N)$. Then, since N raises the marginal product of K,

$$f(K_{US}, N_{US}) - f(K_i, N_{US}) > f(K_{US}, N_i) - f(K_i, N_i).$$

Therefore, adding $f(K_{US}, N_{US})$ to both sides and rearranging,

$$[f(K_{US}, N_{US}) - f(K_i, N_{US})] + [f(K_{US}, N_{US}) - f(K_{US}, N_i)] > f(K_{US}, N_{US}) - f(K_i, N_i).$$

So the sum of the differences got by varying one factor at a time exceeds the overall difference $f_{US} - f_i$.

[23] Exactly the same result is obtained when we start with the lower country's endowment and calculate, for example,

$$D_i^K = \frac{A_i f[(K/P)_{US}, (S/P)_i, (N/P)_i] - (Y/P)_i}{(Y/P)_{US} - (Y/P)_i}.$$

in Section III, which showed that the proportional differences between rich and poor countries are much greater for physical capital per head than for the skill input per head measured in physical units. Since the distribution parameter in the production function is much higher for physical capital than for skill, it follows that differences in physical capital per head account for a greater proportion of differences in output per head.

This finding conflicts with that of Krueger, who offered "minimum estimates" of $D^{S'}$ higher than our own for more than half the countries covered in both our studies. In most countries her estimate exceeded one-half, on the strength of which she claimed that human capital was more important than all other factors put together (including efficiency) in explaining differences in income per head. However, she appears to have calculated the statistic

$$\frac{\sum_j w_{US}^j (L_j/N')_i}{\sum_j w_{US}^j (L_j/N')_{US}},$$

the labor force being divided into a large number of age-education-sector categories. This statistic is then presented as a minimum estimate of the income per head that would obtain in country i relative to that of the United States if country i had the U.S. endowment of physical capital per head but its own labor-force composition. If this is a correct interpretation of the statistic (in Krueger 1968, table 2, col. 2), it overestimates the amount by which output per head would in fact be reduced.[24] For assuming with Krueger that all types of labor are perfect substitutes and can be combined into a measure (E) of efficiency units per head of labor, it follows that if the elasticity of output per head with respect to E is a constant e (< 1) over the relevant range, the relevant ratios of income will be [25]

$$\left(\frac{E_i}{E_{US}}\right)^e > \frac{E_i}{E_{US}}.$$

Our conclusion about the greater relative importance of physical than human capital therefore stands.

[24] We are grateful to C. R. S. Dougherty for discussions on this subject; see also Dougherty (1974).

[25] The only type of measure which would definitely underestimate the share of human capital in accounting for income differences in a constant-returns production function $Y/N' = f(K/N', E)$ would be

$$\frac{(E_{US} - E_i) \cdot f_E[(K/N)_{US}, E_{US}]}{(Y/N')_{US} - (Y/N')_i}.$$

This would be an underestimate in the sense that, if added to a similar measure for the share of K/N', the sum would be less than unity. For a proof, see Krueger (1968, p. 644, note).

Appendix A

Alternative Forms of Nested CES Function

The three alternative forms of nested CES production functions using K, S, and N are:

$$Y = A[aQ^{\rho} + (1 - a)N^{\rho}]^{1/\rho}, \quad \text{where} \quad Q = [bK^{\theta} + (1 - b)S^{\theta}]^{1/\theta}; \tag{A1}$$

$$Y = A[aQ^{\rho} + (1 - a)K^{\rho}]^{1/\rho}, \quad \text{where} \quad Q = [bN^{\theta} + (1 - b)S^{\theta}]^{1/\theta}; \tag{A2}$$

$$Y = A[aQ^{\rho} + (1 - a)S^{\rho}]^{1/\rho}, \quad \text{where} \quad Q = [bK^{\theta} + (1 - b)N^{\theta}]^{1/\theta}. \tag{A3}$$

Another three functions (1′)–(3′) correspond to the permutations on K, S', and N'. We use the following criteria to distinguish between functions (1)–(3) and between functions (1′)–(3′).

a) How well does the function, when estimated using the two-stage least-square model in Section I, explain output? To answer this we use equation (A1) to compute for each country i

$$A_i = \frac{Y_i}{[aQ_i^{\rho} + (1 - a)N_i^{\rho}]^{1/\rho}},$$

and then compute the coefficient of variation $\sqrt{\sum(A_i - \bar{A})^2}/\bar{A}$. An analogous computation is also made using equations (A2) and (A3). The results are as follows.

	Eqq. (A1), (A1′)	Eqq. (A2), (A2′)	Eqq. (A3), (A3′)
Using S, N	.41	.49	.80
Using S', N'	.41	.44	.43

Functions (A1) and (A1′) perform best.

b) How stable is the estimate of the effect of X_2/X_3 on p_2/p_3 when we also let X_1 influence p_2/p_3? To answer this, we first assume X_1 separable and estimate

$$\log \frac{p_2}{p_3} = e + f \log \frac{X_2}{X_3}. \tag{A4}$$

We then assume that this is a misspecification and estimate each of the two alternative specifications, the first with X_2 separable and the second with X_3 separable:

$$\log \frac{p_2}{p_3} = e + f \log \frac{X_2}{X_3} + g \log \frac{Q^{13}}{X_3}, \quad \text{where } Q^{13} = [bX_1^{\theta} + (1 - b)X_3]^{1/\theta}; \tag{A5}$$

$$\log \frac{p_2}{p_3} = e + f \log \frac{X_2}{X_3} + g \log \frac{Q^{12}}{X_3}, \quad \text{where } Q^{12} = [bX_1^{\theta} + (1 - b)X_2]^{1/\theta}. \tag{A6}$$

To estimate functions (A5) and (A6), we used prior estimates of b and θ obtained from the corresponding fully estimated model. The resulting estimates of f are shown in table A1. For each pair (p_2, p_3) the results of the three equations (A4)–(A6) are shown on a given row. There are three rows corresponding to the three pairs (p_2, p_3) among r, z, and w, and another three rows corresponding to the three pairs among r, z', w'. As the table shows, the estimates of f are far more stable (along a row) for the pair r/z than for its two rivals, and for r/z' than for its two rivals. This provides strong support for the corresponding functions (A1) and (A1′).

TABLE A1
ESTIMATES OF f FROM EQUATIONS (A4), (A5), AND (A6)

	ADDITIONAL VARIABLE OTHER THAN X_2/X_3			
p_2/p_3	None	Q^{KS}, $Q^{KS'}$	Q^{SN}, $Q^{S'N'}$	Q^{KN}, $Q^{KN'}$
Using S, N:				
r/z	-3.45	N.A.	-1.56	-2.48
z/w	-0.28	-1.50	N.A.	-1.38
r/w	-0.81	$-\infty$	-37.07	N.A.
Using S', N':				
r/z'	-1.72	N.A.	-4.41	-1.02
z'/w'	-0.60	693.44	N.A.	-11.82
r/w'	-1.07	1.93	0.45	N.A.

NOTE.—N.A. = not applicable.

c) How sensible are the estimates of $(\theta - 1)$ and $(\rho - 1)$. and how consistent are they between rich and poor countries? This is a trickier criterion. Clearly there is no reason why $(\theta - 1)$ should be the same in both groups of countries, though one might be surprised if it differed by a multiple of 10 or if it were significantly different from zero in one group and not in the other. The same applies to $(\rho - 1)$. The estimates for equations (A1) and (A1′) are in table 1. For equations (A2) and (A2′), $(\theta - 1)$ was significant when all countries were pooled, but both very different and insignificant in rich countries and poor countries taken separately. The same applies to $(\rho - 1)$. Equations (A3) and (A3′) performed even worse; $(\theta - 1)$ was significant for all countries and for rich and poor on their own, but $(\rho - 1)$ varied wildly and was often positive—implying isoquants in Q, S space that are concave to the origin.

Appendix B

Data Sources

a) *Wages and Labor Force*

The wage data come from Psacharopoulos (1973) except for New Zealand, where they are taken from the 1966 Census. For Japan, France, and Puerto Rico only relative wages are available; absolute wages are generated from estimates of total labor income (see below). Wage data are converted into U.S. dollars using free market exchange rates where these are available, or, otherwise, official exchange rates. They are adjusted to 1963 levels by multiplying them by the ratio of monetary GDP in 1963 to monetary GDP in the year to which the data relate (these dates vary from 1959 to 1968).

The labor force data come from Psacharopoulos (1973) or Organization for Economic Cooperation and Development (1969). For most countries the sectoral breakdown is taken from the "Education by Branch of Economic Activity" tables in OECD (1969). However, this breakdown has to be generated indirectly for Canada, France, Greece, the United Kingdom, Chile, Ghana, Korea, and Puerto Rico. The method is as follows. Let A be the occupation-by-sector matrix with n rows for occupations and m columns for sectors. Let B be the occupation-by-education matrix with each element expressing the proportion of those in a given occupation having a given level of educational attainment; B consists of k rows for educational levels and n columns for occupations. Then $BA = C$ gives us a $k \times m$ matrix C, which is our estimate of the "Education by Branch of Economic Activity" table. To examine the validity of this method, we have also applied it to some of those countries for which an "Education by Branch of Economic Activity" table is given in OECD (1969). A comparison of the elements in the actual and estimated tables tends to confirm the validity of the method.

A final problem is the treatment of age. As workers get older their earnings rise, and presumably they become more productive. We therefore subdivide each educational category, such as the higher-educated, into five age classes, $L_{H1}, \ldots, L_{H5}$. We then assume that the age classes are perfect substitutes for each other, and estimate the labor input, for example, from higher-educated people as

$$L_H = \frac{\bar{W}_{H1}L_{H1} + \cdots + \bar{W}_{H5}L_{H5}}{\frac{1}{5}(W_{H1} + \cdots + \bar{W}_{H5})},$$

where $\bar{W}_{Hi}$ refers to the world average wage of the higher-educated in the ith age class (strictly the unweighted intercountry average of the country average wages). The choice of denominator is of course immaterial, but the one chosen has the effect that L_H is of the same order of magnitude as the crude number of higher-educated people in a country. The corresponding wage per unit of higher-educated labor is

$$W_H = \frac{W_{H1}\ L_{H1}\ + \cdots + W_H L_H}{L_H},$$

where W_{Hi} is the wage of higher-educated people in the ith age class in the country in question. The age distributions are obtained either from the United Nations (1964, 1965) or from the original samples on which the various rate-of-return studies in Psacharopoulos (1973) were based. In many cases, data are lacking on the labor force as such, and total population statistics have to be used. We assume that the proportion of workers in each age group is the same in each sector. For illiterates however, the paucity of age data for either wages or labor force made the above procedures impossible; no age adjustment was therefore made—a justifiable procedure if age-earnings profiles are roughly flat. No data on L_0 and W_0 exist for several of the richer countries. In these cases L_0 can be safely set at zero; W_0 is approximated by an average of unskilled wage rates in various sectors of the economy, as quoted in the International Labour Office *Bulletin of Labour Statistics*.

b) Output, Capital Stock, and Capital Rental

On output (value added), United Nations (1966) provides a convenient source for mining, manufacturing, and electricity, gas, and water, while the United Nations *Yearbook of National Accounts Statistics* provides the remaining data. Physical capital is measured as the simple sum of gross fixed capital formation from 1949 to 1963 inclusive, measured always at 1963 prices. Most of the data on capital formation come from various editions of the United Nations *Yearbook of National Accounts Statistics*, although for some countries national sources are also used to obtain figures for the earlier years (especially before 1953). Data given at current year prices are converted to 1963 prices using wholesale price indices from either the United Nations *Monthly Bulletin of Statistics* or from local country sources. To compute the total income of capital, GDP at factor cost is divided into employee income, income from unincorporated enterprises, and "other." Income from unincorporated enterprises is then divided between labor and capital in the ratio of employee to "other" income, and total capital income is the capital share of unincorporated income plus "other" income. Labor income is the labor share of unincorporated income plus employee income. Capital rental (per unit of capital) is total capital income divided by total capital.

References

Arrow, K. J.; Chenery, H. B.; Minhas, B. S.; and Solow, R. M. "Capital-Labor Substitution and Economic Efficiency." *Rev. Econ. and Statis.* (August 1961).

Blaug, M. "Approaches to Educational Planning." *Econ. J.* (June 1967).

Bowles, S. "Aggregation of Labour Inputs in the Economics of Growth and Planning: Experiments with a Two-Level CES Function." *J.P.E.* 78, no. 1 (January/February 1970): 68–81.

Denison, E. *Why Growth Rates Differ*. Washington: Brookings Inst., 1967.

Dougherty, C. R. S. "Estimates of Labour Aggregation Functions." *J.P.E.* 80, no. 6 (November/December 1972): 1101–1119.

———. "Factor Endowments and Per Capita Income Differences among Countries: A Note." Mimeographed. London School of Econ., 1974.

Fallon, P. R. "The Productive Roles of Workers with Different Levels of Education: An International Cross-Section Study." Ph.D. thesis, Univ. London, 1974.

Fisher, F. M. *The Identification Problem in Econometrics*. New York: McGraw-Hill, 1966.

Folger, J. K., and Nam, C. B. *Education of the American Population: A 1960 Census Monograph*. Washington: Government Printing Office, 1967.

Griliches, Z. "Capital-Skill Complementarity." *Rev. Econ. and Statis.* (November 1969).

———. "Notes on the Role of Education in Production Functions and Growth Accounting." In *Education, Income, and Human Capital*, edited by W. L. Hansen. New York: Nat. Bur. Econ. Res., 1970.

Hicks, J. R. "Elasticity of Substitution Again: Substitutes and Complements." *Oxford Econ. Papers* (November 1970).

Krueger, A. O. "Factor Endowments and Per Capita Income Differences among Countries." *Econ. J.* (September 1968).

Lydall, H. *The Structure of Earnings*. Oxford: Clarendon, 1968.

Organization for Economic Cooperation and Development. *Occupational and Educational Characteristics of the Labour Force*. Washington: OECD, 1969.

Psacharopoulos, G. *Returns to Education: An International Comparison*. Amsterdam: Elsevier, 1973.

Sato, R., and Koizumi, T. "On the Elasticities of Substitution and Complementarity." *Oxford Econ. Papers* (March 1973).

United Nations. *Demographic Yearbook*. New York, 1964, 1965.

———. *Growth in World Industry*. New York, 1966.