Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis, ECMA, 2000

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1 Introduction

We conduct our analysis using a neoclassical aggregate production function in which the key feature of the technology is *capital-skill complementarity*. This means that the elasticity of substitution between capital equipment and unskilled labor is higher than that between capital equipment and skilled labor. A key implication of capital-skill complementarity is that growth in the stock of equipment increases the marginal product of skilled labor, but decreases the marginal produce of unskilled labor. In our framework, skilled-biased technological change reflects the rapid growth of the stock of equipment, combined with the different ways equipment interacts with different types of labor in the production technology.

For example, consider a three-factor production function. Output (y_t) is produced with capital equipment (k), unskilled labor (u), and skilled labor (s). Equipment and unskilled labor are perfect substitutes and have unit elasticity of substitution with skilled labor: $y_t = f(k_t, u_t, s_t) = (k_t + u_t)^\theta s_t^{1-\theta}$. The ratio of the marginal product of skilled labor to the marginal product of unskilled labor is

$$\frac{f_{s_t}}{f_{u_t}} = \left(\frac{1-\theta}{\theta}\right)\frac{k_t + u_t}{s_t}.$$

In this paper, we quantitatively evaluate how much capital-skill complementarity has affected the skill premium in the postwar period. To do this, we first modify the standard two-factor capital and labor aggregate production function by developing a four-factor aggregate production function that distinguishes among capital equipment, capital structures, skilled labor, and unskilled labor and that allows for different elasticities of substitution among the factors. Then, with time series observations, we read factor prices off the marginal product schedules and compare the skill premium in the model with the skill premium in the data.

2 The Model

In this model, there are three final goods: consumption c_t, structures investment x_{st}, and equipment investment x_{et}. Consumption and structures are produced with a constant returns to scale

*This note is written down during my M.phil. period at the University of Oxford.

technology, and equipment is produced with the same technology scaled by equipment-specific technological process q_t. Under these assumptions, the relative price of equipment is equal to $1/q_t$ and the aggregate production function is given by

$$y_t = c_t + x_{st} + \frac{x_{et}}{q_t} = A_t G(k_{st}, k_{et}, u_t, s_t). \tag{1}$$

The production function G has constant returns to scale in capital structures k_s, capital equipment k_e, unskilled labor input u_t, and skilled labor input s_t. In addition to equipment-specific technological change, there is neutral technological change, A.

For the production, we use:

$$G(k_{st}, k_{et}, u_t, s_t) = k_{st}^{\alpha}\left[\mu u_t^{\sigma} + (1-\mu)\left(\lambda k_{et}^{\rho} + (1-\lambda) s_t^{\rho}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}}. \tag{2}$$

In this specification, μ and λ are parameters that govern income shares, and σ and $\rho (\sigma, \rho < 1)$ govern the elasticity of substitution between unskilled labor, capital equipment, and skilled labor. The elasticity of substitution between equipment (or skilled labor) and unskilled labor is $\frac{1}{1-\sigma}$, and the elasticity of substitution between equipment and skilled labor is $\frac{1}{1-\rho}$. Capital-skill complementarity requires that $\sigma > \rho$. If either σ or ρ equals zero, the corresponding nesting is Cobb-Douglas.

The labor input of each type is measured in efficiency units: each input type is a product of the raw number of labor hours and an efficiency index: $s_t \equiv \psi_{st} h_{st}$ and $u_t \equiv \psi_{ut} h_{ut}$, where h_{it} is the number of hours worked and ψ_{it} is the unmeasured quality per hour of type i at date t.

2.1 The Skill Premium From the Model

We denote the skill premium by pi. Since factor prices are equal to marginal products per unit of work, the skill premium can be expressed as a function of input ratios:

$$\pi_t = \frac{(1-\mu)(1-\lambda)}{\mu}\left[\lambda\left(\frac{k_{et}}{s_t}\right)^{\rho} + (1-\lambda)\right]^{\frac{\sigma-\rho}{\rho}}\left(\frac{h_{ut}}{h_{st}}\right)^{1-\sigma}\left(\frac{\psi_{st}}{\psi_{ut}}\right)^{\sigma}. \tag{3}$$

To illustrate the implications of this expression for the skill premium, we log-linearize it and differentiate with respect to time. Log-linearizing yields

$$\ln \pi_t \simeq \lambda \frac{\sigma-\rho}{\rho}\left(\frac{k_{et}}{s_t}\right)^{\rho} + (1-\sigma)\ln\left(\frac{h_{ut}}{h_{st}}\right) + \sigma \ln\left(\frac{\psi_{st}}{\psi_{ut}}\right).$$

Differentiating with respect to time and denoting the growth rate of variable x by g_x, we obtain,

$$g_{\pi t} \simeq (1-\sigma)\left(g_{h_{ut}} - g_{h_{st}}\right) + \sigma\left(g_{\psi_{st}} - g_{\psi_{ut}}\right) + (\sigma-\rho)\lambda\left(\frac{k_{et}}{s_t}\right)^{\rho}\left(g_{k_{et}} - g_{h_{st}} - g_{\psi_{st}}\right). \tag{4}$$

Equation 4 decomposes the growth rate of the skill premium into three components:

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