



Human capital risk in life-cycle economies

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ABSTRACT

The aggregate effects of market incompleteness are studied in a model where agents face idiosyncratic, uninsurable human capital investment risk. Using a life-cycle model with a version of a Ben-Porath (1967) human capital accumulation technology, stationary equilibria of calibrated cases are analyzed in which risk arises from specialization risk and career risk. With career risk only, stationary equilibria resemble those studied by Aiyagari (1994), and the impact of uninsurable idiosyncratic risk is relatively small. With a significant amount of specialization risk, however, stationary equilibria are severely distorted, with human capital about 57 percent as large as its complete markets counterpart.

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1. Introduction

Dispersion of labor earnings increases over the life-cycle, a well documented feature of the U.S. data. According to Deaton and Paxson (1994), within-cohort labor income inequality increases with age. In related work, Huggett et al. (2009) investigate the reasons behind this rise in earnings dispersion and find that about one-third of the variation in lifetime earnings is due to idiosyncratic human capital shocks. Other cross sectional studies also indicate that agents face a great deal of uncertainty when making their schooling decisions.¹ Taken together, it appears that investment in human capital is risky and part of the labor income uncertainty that agents face over their life-cycle is a manifestation of this idiosyncratic human capital risk. In addition, it is widely understood that human capital investment is uninsurable. One main consequence of this type of labor income uncertainty is that it could deter investment in human capital. If a mechanism like this is at work in actual economies, the impact of market incompleteness on the aggregate economy could be large,² possibly calling for policy intervention to mitigate the effect of this risk on household decisions to invest in training.

This paper studies the macroeconomic implications of labor income uncertainty arising from the risky nature of human capital investment. It uses a specification that allows a direct analysis of the impact of risk on the accumulation of human capital, helps isolate and quantify the effect of risk on individual decisions, and allows commentary on the divergent views in the literature on the role of market incompleteness in the aggregate economy.

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¹ Carneiro et al. (2003) find that the substantial heterogeneity in the returns to schooling is unpredictable at the time when schooling decisions are made. In related work, Cunha et al. (2005) conclude that 40 percent of the variability in the returns to schooling is unforecastable at the time students decide to go to college.

² Even more so if one takes the view that human capital is an engine of growth. With the exception of few, Eaton and Rosen (1980), Krebs (2003), Benabou (2002), for example, this strand of the literature typically abstracts from the risky nature of human capital investment.

1.1. Main ideas and findings

A version of a Ben-Porath (1967) production function for human capital that allows for risky human capital is used here to understand how uninsurable risk impacts an individual's decision to train in a general equilibrium life-cycle model. Two types of uninsurable idiosyncratic risks, namely, specialization risk and career risk are considered in this paper. The specialization risk is such that the endogenous decision to train increases both the expected returns from training and its variance, while in the case of career risk the training decision only affects the mean return from training. The career risk is additive in the human capital accumulation technology and is the most common formulation in the literature which studies the impact of uninsurable idiosyncratic risk on the aggregate economy. Risks that look like the multiplicative specialization risk of this paper were first studied by Angeletos and Calvet (2006), but not in a human capital setting.

Following Calvet (2001) and Angeletos and Calvet (2006), constant absolute risk aversion (CARA) utility function and normally distributed shocks is used in this paper. In addition, an individual's risk-taking decision is independent of wealth. These features ensure that the endogenous decisions have closed form solutions and are independent of wealth.

In this paper, calibrated versions of the model are studied to clearly assess how each of these risks, the specialization and the career risk, influence the aggregate economy by altering individual decisions. Stationary equilibrium with only career risk has properties similar to Aiyagari (1994). In particular, the precautionary savings induced by this risk has only a small quantitative impact on the macroeconomy.³ In the baseline calibration where both shocks play a role, the effects of the specialization risk dominate and there is a very large impact on macroeconomic variables in the stationary equilibrium. In particular, there is a 43 percent underaccumulation of human capital relative to the complete markets case. Accordingly, since labor quality is dramatically lower, output, physical capital, consumption and other variables are also drastically impacted. This suggests that specialization risk has a large impact on macroeconomic equilibrium, but that career risk does not.

1.2. Recent related literature

Using an incomplete markets framework and abstracting from aggregate uncertainty, several papers since Bewley (1977) have investigated the core implications of uninsurable idiosyncratic labor income risk on the aggregate economy. While Aiyagari (1994) finds that the quantitative implications of the labor income risk are not significant, a relatively recent view associated with Angeletos and Calvet (2006), Pijoan-Mas (2006) and Marcet et al. (2007), suggests that these effects could be large.⁴

A related paper, Krebs (2003), studies the impact of labor income risk in a model with risky human capital, but where the risky human capital is also the engine of growth. He finds that risk lowers investment in human capital which in turn lowers growth and welfare. Based on Krebs (2003) it is not clear whether the quantitatively significant macroeconomic implications of risky human capital are due to market incompleteness or because human capital is the engine of growth. To isolate the role of uninsurable risky human capital, this paper abstracts from growth in the stationary equilibrium. Even though this paper asks a question similar to Krebs (2003), the framework used here is quite different and is more consistent with the traditional analyses of human capital investment.

A feature of this paper is that it can also produce some of the life-cycle features seen in the data in a general equilibrium framework. There is growing empirical-theoretic, primarily partial equilibrium literature studying the life-cycle features in the data. Deaton and Paxson (1994) and Huggett et al. (2009) among others study these issues.

The paper is organized as follows. Section 2 introduces the model and Section 3 presents the quantitative results. The final section concludes.

2. Model

The economy has an infinite sequence of overlapping generations of agents who live for T periods. Time is discrete and is indexed by $t=0,1,2,\dots$. In each period, a continuum of *ex ante* identical young agents with unit mass is born. Each agent is endowed with two units of time in each period until they retire. There is no population growth, physical capital is risk-free and there are no credit market imperfections.^{5,6} In terms of notation, subscripts indicate when the agent is born and the

³ This is despite the fact that unlike Aiyagari (1994), the present model with human capital accumulation has no borrowing constraints or wealth effects. Aiyagari (1994) studied the macroeconomic impact of uninsurable idiosyncratic labor income risk arising due to shocks to labor endowments in a model where households live forever and where there is no human capital.

⁴ None of these authors explicitly discussed human capital. Aiyagari (1994), Pijoan-Mas (2006), and Marcet et al. (2007) studied labor income risk whereas Angeletos and Calvet (2006) analyzed capital income risk. This paper is closer to Aiyagari (1994) and Marcet et al. (2007) but provides a different mechanism to explain variability in labor income. It also nests both the views in the literature and argues that the nature of risk, specialization versus career risk, determines which view prevails in the quantitative analysis.

⁵ Underlying the assumption that physical capital is risk-free is the notion that for an individual human capital investment is more risky than investment in physical capital. Typically investment in human capital cannot be easily diversified or directly traded in the market since it is non-separable from the owner.

⁶ Other papers on incomplete markets, for example Krebs (2003) and Angeletos and Calvet (2006) also assume that credit markets are perfect.

stage in an agent's life-cycle or the time in the model is in the parenthesis. In general, the aggregate variables do not have a subscript.

The preferences of agent i born at date t are given by

$$U = E_t \sum_{j=0}^{T-1} \beta^j u(c_t^i(t+j)) \quad (1)$$

where $c_t^i(t+j)$ is the date $t+j$ consumption of agent i who is born at date t , the discount factor is β , and E is the expectation operator. The period utility function is of the CARA form, $u(c) = -(1/a)\exp[-ac]$, with a as the coefficient of absolute risk aversion. The agent's budget constraint for $0 \leq j < J_R$, where J_R is the exogenous retirement date, is given by

$$c_t^i(t+j) = w(t+j)((1-\tau_t^i(t+j))\underline{x} + x_t^i(t+j)) + R(t+j-1)s_t^i(t+j-1) - s_t^i(t+j) \quad (2)$$

and for $J_R \leq j \leq T-1$, it is given by

$$c_t^i(t+j) = R(t+j-1)s_t^i(t+j-1) - s_t^i(t+j). \quad (3)$$

Time allocated to training by agent i is $0 \leq \tau_t^i(t+j) \leq 1$, $s_t^i(t+j)$ is the holdings of the risk-free asset (capital) and $x_t^i(t+j)$ is the labor quality, measured in efficiency units, at date $t+j$. To ensure that all agents within a generation make same training and asset holding decisions, decisions that are independent of the actual realization of the two shocks, it is assumed that the agent's labor quality $x_t^i(t+j)$ enters additively such that it does not interact with the training decision in period $t+j$. The parameter $\underline{x} > 0$ converts units of time into efficiency units.⁷ The wage rate per efficiency unit in period $t+j$ is $w(t+j)$ and $R(t+j)$ is the gross return on the risk-free asset holdings from period $t+j$ to $t+j+1$. Since there are no bequests, agents enter the first period without any assets and do not save in the last period of their life. Agents born at any date t , however, inherit the average aggregate labor quality $\bar{x}(t)$ prevalent at that date and as a result $x_t(t) = \bar{x}(t)$. Capital income is the only source of income after retirement.

This paper uses a modified Ben-Porath (1967) production function for human capital with time allocated to training as the input in the production technology.⁸ Human capital investments require agents to give up labor income early in the life-cycle in order to generate higher future efficiency units. When allocating $\tau_t^i(t+j)$ units of time to training at date $t+j$, an agent is, however, uncertain about the number of efficiency units received in the subsequent period, making these investments risky. The training technology is given by

$$h_t^i(t+j+1) = \gamma(t+j)\tau_t^i(t+j)^\phi e_t^i(t+j+1) + \eta_t^i(t+j+1) \quad (4)$$

for $0 \leq j < J_R-1$. Here $0 < \phi < 1$ is the return elasticity of training and $\gamma(t+j)$ is the productivity which is the same for all the agents in the same stage of the life-cycle but could vary over different stages of an agents's life-cycle. The two idiosyncratic shocks, the specialization shock $e_t^i(t+j+1)$, and the career shock $\eta_t^i(t+j+1)$, are independent and identical across agents within a generation and independent across generations. In particular, $e_t^i(t+j) \sim N(1, \sigma_e(t+j))$ and $\eta_t^i(t+j) \sim N(0, \sigma_\eta(t+j))$. In the case of complete markets, $e_t^i(t+j) = 1$ and $\eta_t^i(t+j) = 0$ for all the individuals and at all dates.

The specialization risk $e_t^i(t+j+1)$ is such that for the same level of risk, if individuals devote more time to training, they face a positive risk-return trade-off. Allocating more time to training increases the expected future returns but at the same time it also increases the variance of the returns from training.⁹ In Palacios-Huerta (2003), average real human capital return for white males in the U.S. with less than 5 years of experience with no high school, high school and college education are: 5.9 (6.2), 13.6 (9.7), 14.2 (11.3), respectively, with the standard deviation of returns reported in the parenthesis.¹⁰ The career risk $\eta_t^i(t+j+1)$ can be interpreted as the general uncertainty associated with working in a job, whether the match is good, whether the agent is compatible with other workers, and other aspects of the labor market that are not incorporated in the model but are independent of the level of training. If there is only career risk, $\sigma_e(t) = 0$, then for the same level of risk, acquiring more training only increases the mean return from training but leaves the variance unaltered.

The uncertainty in the returns from training makes an agent's human capital, represented by the labor quality, random. Random labor quality makes labor income uncertain in this paper. The labor quality of agent i at date $t+j$ depends on the undepreciated level of inherited average aggregate labor quality $\bar{x}(t)$ and the human capital accumulated in the previous

⁷ One aspect of this specification is that in a stationary equilibrium, the marginal cost of acquiring training is the same across all agents and is independent of their own individual labor quality.

⁸ Human capital is homogenous here and there is no heterogeneity due to occupation choice. Therefore, one could potentially interpret the model as a representative occupation model. See Willis (1986) for more details. The type of the human capital investment considered here is also distinct from the general skills and specific skills that typically refer to aspects of training that are transferable across different jobs.

⁹ For instance, a college graduate who invests more years in training faces a higher return from training but also faces a greater risk relative to a high school graduate who devotes fewer years in training and hence faces lower risk-return trade-off.

¹⁰ Other studies, for example a cross country study by Pereira and Martins (2002) and a study by Christiansen et al. (2007) using Danish data, also show that there exists a positive risk-return trade-off in human capital investment. However, typically most studies estimate the returns to schooling by ignoring the riskiness of these investments. See Card (2001) for a survey of this literature.

periods via training, the first and second terms, respectively, in the following equation

$$x_t^i(t+j) = (1-\delta_h)^j x(t) + \sum_{m=0}^{j-1} h_t^i(t+m+1) \quad (5)$$

where $x(t)$ depreciates at rate δ_h .¹¹

The production technology of the final good is standard, given by $Y(t) = AK(t)^\alpha L(t)^{1-\alpha}$, where $K(t)$ is the aggregate capital stock and $L(t)$ is total labor supply measured in efficiency units at date t . The intensive form representation of the production technology is standard, $y(t) = Ak(t)^\alpha$, where $y(t)$ is the output per efficiency unit and $k(t)$ is the capital per efficiency unit. Total labor supply at date t is given by $L(t) = \sum_{j=0}^{J_R-1} \{(1-\tau_{t-j}(t))\underline{x} + x_{t-j}(t)\}$.¹² Inputs are hired in competitive markets. The wage rate per efficiency unit is $w(t) = (1-\alpha)Ak(t)^\alpha$ and the rental rate of capital is $r(t) = \alpha Ak(t)^{\alpha-1}$. Let δ_k be the rate at which capital depreciates, then $R(t) = r(t+1) + 1 - \delta_k$.

At the aggregate level there is no uncertainty. The aggregate market clearing condition for physical capital can be written as $L(t)k(t) = \sum_{j=1}^{T-1} s_{t-j}(t-1)$. The law of motion of average aggregate labor quality is described by $x(t) = (1/J_R)E_t \sum_{j=0}^{J_R-1} x_{t-j}(t)$. At date t , the average labor quality of any generation born in period $t-j$ is $E_t x_{t-j}(t)$, for $j=0,1,\dots,J_R-1$. Averaging over all the generations gives the average aggregate labor quality at date t which is inherited by the agents born at date t .

3. Quantitative analysis

The calibration strategy, similar to Aiyagari (1994), is to first choose parameters such that they match some targets in the U.S. data for the complete markets benchmark. To calibrate risk, the life-cycle features of the model are used. Two calibrated versions of the model are considered here, the *career risk only* economy and the *baseline calibration* economy, that primarily differ along one dimension, the calibration of risk, which is chosen to endogenously generate the variance of logarithm of labor income that matches data. In spite of the fact that both these economies endogenously generate variance in labor income that matches the relevant data, the aggregate implications are strikingly different across these two economies.

3.1. Calibration of complete markets benchmark

In the calibration, the model period is eight years and an agent lives for 8 periods from age 16–80. In the last two periods, agents retire from the labor market. An agent trains for 6 years between the ages 16 and 32. Standard values of α, β and δ_k are used: α equals 0.4, the annual discount factor β is 0.975 and the annual depreciation rate of physical capital is 8 percent. In addition, the annual depreciation rate of human capital is 4 percent and the return elasticity of training ϕ is $1/2$.¹³ The other five parameters $A, a, \underline{x}, \gamma(t)$ and $\gamma(t+1)$ are chosen to match the targets of physical capital and human capital in the U.S. data. The capital-output ratio is targeted at 4.02, consumption-output ratio at 0.748, and the gross annual risk-free rate at 1.03 percent.¹⁴ According to the Current Population Survey on school enrollment, 95.7 percent of the Americans between the age 15 and 17, 72.5 percent from the age 18 to 19, 39.4 percent from 20 to 24 and 12.1 percent between the age 25 and 34 were enrolled in school in 2006.¹⁵ The enrollment rate data allows calibration of the time devoted to training within each period. Using that, a target of 5.5 years and about half year of training in the first and the second period, respectively, is set. As a consequence, an individual spends 6 years, two years in secondary and four years in post-secondary education, in school between the age 16 and 32.¹⁶ The relative risk aversion implied by the model is in the range (1,3).

3.2. Two types of risk

In the second step of the calibration, the life-cycle features of the model are used to match the variance of labor earnings over the life-cycle which is primarily due to age effects in the U.S. data. In the model since there is no growth and at age 16 all agents are *ex ante* identical, variance in labor earnings over the life-cycle is entirely due to age effects. But in the data there may be numerous other reasons why dispersion in labor earnings increases over the life-cycle, for example, time effects, cohort effects, differences in initial conditions, human capital risk, and other employment related risks. For this

¹¹ To ensure agents within a generation make decisions that are independent of the actual shock realizations, it is assumed that only inherited labor quality depreciates.

¹² Superscript i is suppressed here since all the agents within a generation are identical with respect to their endogenous decisions.

¹³ In the human capital literature the estimated elasticity lies in the range (0.5,0.9). See Table 2.3 and 2.4 in Browning et al. (1999). The quantitative findings reported here are robust to alternative calibrations of ϕ and δ_h (see supplementary appendix).

¹⁴ In Table 1 the per-period capital-output ratio and gross risk-free rate are reported as annual variables to facilitate comparison with these targets.

¹⁵ See Table S1401.

¹⁶ The total number of years spent in school by an average American (see the estimates of the United Nation Educational, Scientific and Cultural Organization's (UNESCO) Global Education Digest 2004, Fig. 1) over this time period is consistent with these targets.

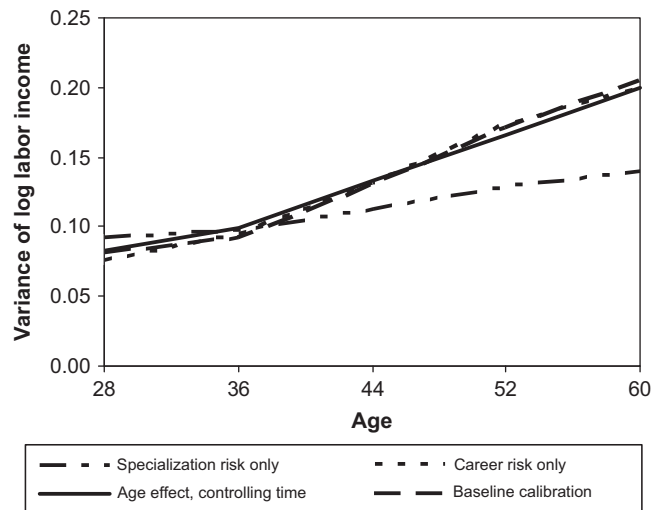


Fig. 1. A comparison of the variance of logarithm of labor income in different calibrated cases of the incomplete markets version of the model with the relevant data from Huggett et al. (2009).

analysis, the Huggett et al. (2009) paper provides a close approximation of the age effects in the variance of labor earnings.¹⁷ Moreover, in this model there are no other sources of within generation heterogeneity except for the heterogeneity due to the idiosyncratic human capital shocks. Therefore the variance of labor earnings over the life-cycle which is due to age effects must be entirely due to these shocks. Huggett et al. (2009) investigate different sources of labor income dispersion and conclude that approximately 1/3 of the variance in lifetime earnings due to age effects is actually explained by idiosyncratic human capital risk. As a result, the variance of the two idiosyncratic human capital shock processes is chosen in a way such that the life-cycle pattern of labor earning variability generated by the model under market incompleteness is in fact 1/3 of the variance in labor earnings in the data.

The quantitative analysis considers two calibrated economies, the *career risk only* economy and the *baseline calibration* economy.¹⁸ There is no risk in the first period of life and in the last two periods. Fig. 1 illustrates how this calibration strategy helps in matching the model generated earnings dispersion with Huggett et al. (2009) measure of the labor earning variability which is due to idiosyncratic human capital shocks.¹⁹

3.3. Macroeconomic implications of human capital risk

Following the calibration procedure described above, stationary equilibria of calibrated economies are studied to determine the quantitative macroeconomic consequences of market incompleteness due to human capital risk in a general equilibrium life-cycle model.²⁰

3.3.1. Career risk only economy

Can market incompleteness produce small and quantitatively insignificant results in a model with risky human capital investment? First consider the *career risk only* economy. A comparison of columns 2 and 3 in Table 1, clearly illustrates that relative to the complete markets benchmark, incomplete markets have small and quantitatively insignificant effects on the aggregate economy—a result similar to Aiyagari (1994). Due to market incompleteness, the savings rate and output rise relative to the complete markets benchmark. Somewhat surprising is that market incompleteness leads to *overaccumulation* of human capital, though quantitatively insignificant but in compliance with Angeletos and Calvet (2006).²¹

These results demonstrate that allowing agents to optimally decide how much time to allocate to training when returns from training are uncertain and uninsurable is not always likely to generate large quantitative effects. Moreover, when individuals face career risk, the risk where a change in training only affects the mean return, the effects of market incompleteness are marginal.

¹⁷ The regression results of Huggett et al. (2009) where they control for time effects (see Fig. 1b) are used here.

¹⁸ The standard deviation of the two shocks in the career risk only economy are $\{\sigma_\epsilon(t+j)\}_{j=1}^5 = \{0\}_{j=1}^5$ and $\{\sigma_\eta(t+j)\}_{j=1}^5 = \{0.08, 0.03, 0.03, 0.06, 0.03\}$. Similarly, the sequence of shocks in the baseline calibration economy are $\{\sigma_\epsilon(t+j)\}_{j=1}^5 = \{1.41, 1.84, 0, 0, 0\}$ and $\{\sigma_\eta(t+j)\}_{j=1}^5 = \{0, 0.01, 0.02, 0.02, 0.02\}$.

¹⁹ The specialization risk only economy cannot be calibrated to match the data because specialization shocks interacts with time allocated to training and agents in this model train only in the first two periods of their life-cycle.

²⁰ These results are based on an exact solution of the model.

²¹ Section 3.4 explores why there is overaccumulation of human capital in the career risk only economy.

Table 1
Economies with human capital risk.

	Complete markets	Incomplete markets	
		Career risk only	Baseline calibration
\hat{x}	0.1540	0.1542	0.088
\hat{k}	1.275	1.276	1.381
\hat{K}/\hat{Y}	4.023	4.039	4.221
\hat{C}/\hat{Y}	0.755	0.755	0.742
\hat{L}/\hat{Y}	0.395	0.394	0.382
\hat{R}	1.034	1.034	1.030
\hat{w}	1.521	1.521	1.575
\hat{Y}	4.272	4.276	3.366

Comparing the quantitative macroeconomic effects of incomplete markets with the complete markets in the stationary equilibria of the career risk only economy and the baseline calibration economy.

3.3.2. Baseline calibration economy

In the incomplete markets case of the *baseline calibration* economy, agents face both risks. Since the empirical literature provides no guidance on how to assign weights to these shocks, these weights are assigned in a way that attributes almost all the variability in labor earnings to specialization risk early in the life-cycle and later, the additional risk that agents face is entirely due to the career shocks. The reason behind such an extreme calibration is to help understand the role of specialization risk while still matching the data on life-cycle labor income variability. From Fig. 1 it is clear that specialization risk alone cannot match the data and hence it is combined with career risk.

Since these two economies differ only in the calibration of risk, the complete markets case is identical for the two economies. However, note that the stationary equilibria with uninsurable idiosyncratic human capital risk are strikingly different, see column 4, Table 1. The stationary equilibrium of the baseline calibration economy clearly illustrates that the specialization risk plays a crucial role in determining the aggregate implications of market incompleteness. Market incompleteness has a significant impact on human capital accumulation causing the average labor quality to decline by 43 percent. This is in stark contrast with the findings of Aiyagari (1994) where the quantitative effect of incomplete markets was marginal. Agents self-insure in this economy by allocating less time to training. Due to such a sharp decrease in average labor quality, capital and the labor supply reduce dramatically by 12 and 19 percent, respectively. Unlike the other economy studied here, output *falls* and the decrease is substantial—17 percent relative to the complete markets benchmark. Like Aiyagari (1994) and other studies in this literature, agents in this economy also insure themselves against the uninsurable risk by holding more risk-free assets. As a result, capital-output ratio increases by 4.9 percent, even though in the aggregate both capital and output fall relative to the complete markets case.

The quantitative analysis therefore compares two different economies, the career risk only economy and the baseline calibration economy. Such a comparison is convincing in that both economies endogenously generate the same life-cycle pattern of labor income variance even when they have extremely different aggregate macroeconomic implications. In the career risk only economy, incomplete markets have very small quantitative effects. On the contrary, in the baseline calibration economy with higher weight on specialization risk, market incompleteness decreases labor quality dramatically and all other aggregate variables are also impacted considerably. Thus depending on the weights of the two shocks, the quantitative implications of incomplete markets could be as large as our baseline case or as small as the case with only career shocks. Unless empirical studies isolate the relative weights of these shocks in the data, it is hard to take a stand on the exact role of market incompleteness.

3.4. Life-cycle features of the model

This subsection takes a closer look at the underlying training and asset holding decisions which are identical for all the agents within a generation. In addition, the cross sectional distribution of earnings and consumption implied by the model is also explored here.

3.4.1. Individual decisions

Time allocated to training is reported in Table 2. Panel A reports the overall effects, whereas panel B isolates the partial equilibrium effects of idiosyncratic uninsurable human capital risk by holding the wage and the interest rate fixed at the complete markets level.

When there is no uncertainty, agents spend 6 years in training, 5.5 years between the age 16 and 24 and 0.5 year between 24 and 32.²² In the case when labor income uncertainty is due to career risk, time allocated to training does not

²² Time devoted to training in the first and second period is $0.683 \times 8 = 5.5$ and $0.066 \times 8 = 0.5$ years, respectively.

Table 2
Time in training.

	Complete markets	Incomplete markets	
		Career risk only	Baseline calibration
Panel A			
$\hat{\tau}_0$	0.683	0.684	0.202
$\hat{\tau}_1$	0.066	0.067	0.054
Panel B			
$\hat{\tau}_0$		0.683	0.193
$\hat{\tau}_1$		0.066	0.049

Fraction of one unit of time devoted to training in the first two periods of the life-cycle. No time is allocated to training thereafter. Panels A and B report general equilibrium and partial equilibrium results, respectively.

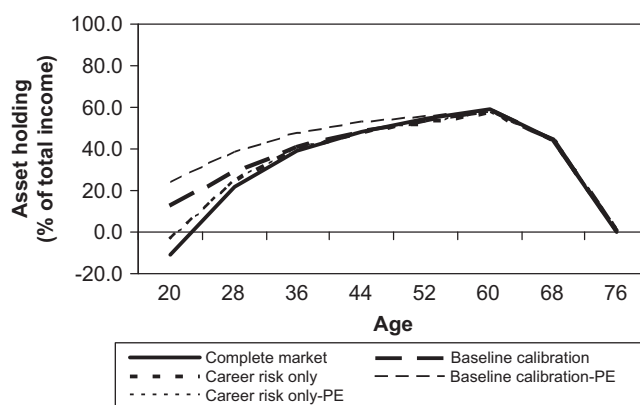


Fig. 2. Life-cycle pattern of asset holding as a fraction of total income in the different economies for the general equilibrium and the partial equilibrium (PE) case.

change much relative to the complete markets benchmark (column 3, panel A). However, in the baseline calibration agents train for 2 years, 1.6 and 0.4 years in the first and the second period, respectively, causing labor quality to fall dramatically.

Panel B evaluates the direct impact of risk on individual training decisions by holding the wage and the interest rate fixed at the complete markets level. Due to the additive nature of the career risk, the training decision is not impacted directly (column 3, panel B).²³ However, in the baseline calibration economy, time devoted to training is even lower relative to the comparable case in panel A. Reducing training time lowers efficiency units and at the same time it increases labor supplied to the market. Thus the overall impact on the equilibrium wage rate is ambiguous in a general equilibrium setting. However, from Table 1 it is clear that the wage rate increases in the baseline calibration economy relative to the complete markets case. Not allowing the wage rate to increase in panel B partly explains why training is lower in this panel in relation to the corresponding column in panel A.

The decision to hold risk-free assets for precautionary purposes when markets are incomplete has been studied extensively. Fig. 2 shows that the asset holding as a fraction of total income increases over the working life of an agent, irrespective of whether markets are complete or not. One reason is that agents accumulate human capital when young and therefore their asset holdings are relatively low in the earlier part of their life-cycle. Fig. 2 also shows that the asset holding decision gets distorted by risk, a common result in this literature. Agents tend to save more when markets are incomplete for precautionary purposes. In a partial equilibrium setting, in both economies asset holding as a fraction of total income is higher relative to their corresponding general equilibrium economies with incomplete markets, albeit marginally in the career risk only economy.²⁴

Therefore, *both* decisions, the decision to train less and the decision to save more allow agents to self-insure against uninsurable idiosyncratic human capital risk. In addition, the large quantitative effects for the baseline calibration economy are primarily due to the partial equilibrium effects of idiosyncratic human capital risk on these endogenous decisions.

²³ The overaccumulation of training in the career risk only economy is therefore due to the general equilibrium effects. See the two-period case in the supplementary appendix for more details.

²⁴ In the partial equilibrium case, total income is computed by using prices, wage and interest rate, from the stationary equilibrium of the complete markets economy.

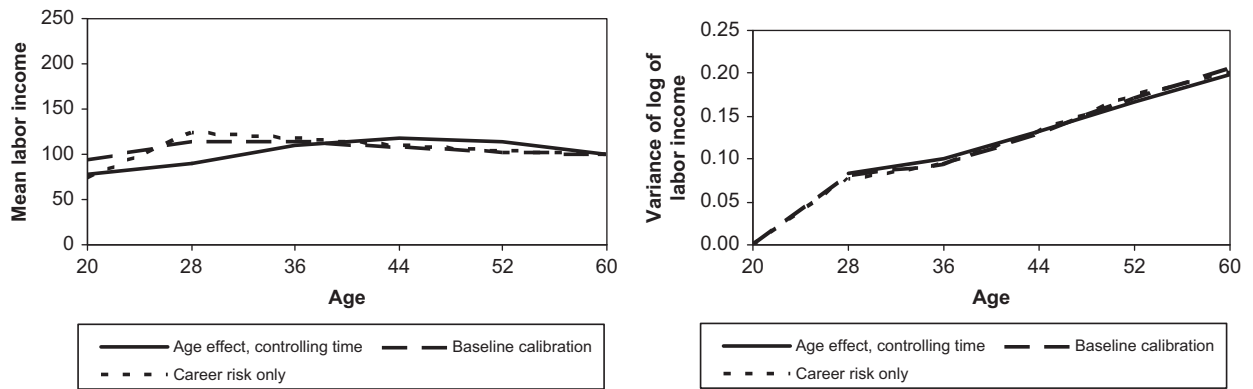


Fig. 3. The left graph plots mean labor income and the right plots the variance of logarithm of the labor income over the life-cycle in the different economies.

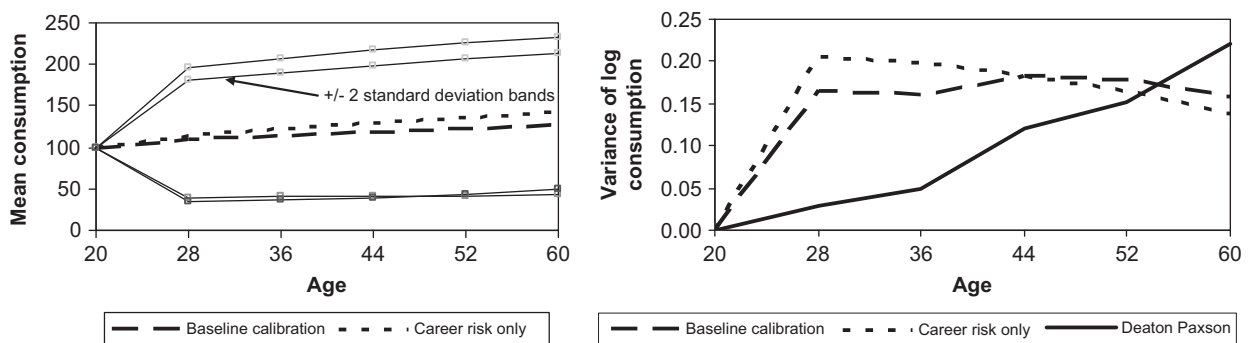


Fig. 4. The left graph plots mean consumption in the two economies along with ± 2 standard deviation bands which suggests that negative consumption is a rare event in these economies. The right graph plots the variance of the logarithm of consumption.

3.4.2. Life-cycle pattern of income and consumption

Fig. 3, left graph compares the distribution of life-cycle earnings generated by this model with the mean earnings implied by the estimates of Huggett et al. (2009).²⁵ The average labor income generated by this model has the familiar hump shape. However, the model generates the hump in the earlier periods of the life-cycle, close to age 30, but in the data labor earnings peak when agents are in their mid 40s. One reason for this mismatch could be that in this model labor quality is not allowed to increase due to reasons other than schooling, like on-the-job training, learning by doing and other mechanisms that improve labor quality over the entire working life of an agent. The variance of the logarithm of labor earnings, the right graph in Fig. 3, matches data in all the different economies considered here since the risk is calibrated to match this aspect of the data.

The model does not match the moments of life-cycle consumption data very well, see Fig. 4.²⁶ Mean age-consumption in our model has an upward slope, a common implication of life-cycle models with additively separable utility function defined over consumption alone. The model also generates higher consumption volatility relative to Deaton and Paxson (1994). In this model there is no form of social insurance and that could partly explain high consumption variance.²⁷ Another reason for high consumption variance in this model is that an agent's endogenous decisions are unaffected by the actual realization of the two shocks. Therefore consumption is residual and hence more volatile.²⁸

Even though the aggregate implications of these two economies with incomplete markets, the career risk only economy and the baseline calibration economy, are undoubtedly very different, the life-cycle features are somewhat similar.

²⁵ Following Huggett et al. (2009), mean labor income is scaled such that it is 100 at the end of an agent's work life.

²⁶ The two shocks have a normal distribution and consumption is residual here. Therefore, the quantitative analysis can have some instances in which consumption is negative. The bands in the left graph of Fig. 4 for mean consumption contain 95 percent of the values suggesting that the shocks in the model are sufficiently small to preclude high incidences of negative consumption.

²⁷ Storesletten et al. (2004) suggest that without a social security system, variance in consumption is roughly 20 percent higher relative to data.

²⁸ The variance of consumption generated by the model is also concave, whereas in the data it is somewhat linear. In the model when agents are young, they accumulate human capital which is risky and their holdings of risk-free assets are relatively low. As a result shocks to training have a larger impact on consumption in the early periods of the life-cycle. Over time as holdings of risk-free asset increases, human capital shocks tend to have a lower impact.

The two economies endogenously generate the same life-cycle pattern of labor income variability. Other life-cycle features, such as mean labor income, mean consumption and the variance of consumption over the life-cycle are also not very different across these two economies. One reason could be that these divergent decisions occur only in the early part of the life-cycle and their influence gets smoothed over many periods of the life-cycle.

3.5. A suggested empirical strategy to identify human capital risk

These quantitative results indicate that to understand the impact of uninsurable idiosyncratic human capital risk on the aggregate economy, it is useful to identify these risks in the data. The following subsection outlines an empirical strategy by briefly discussing (i) the key steps involved in such an estimation, (ii) issues that would have to be addressed when estimating the relevant equations, (iii) data availability and (iv) key econometric issues that would typically arise in such an analysis.

To quantify the aggregate implications of uninsurable human capital risk, consider a structural approach that directly estimates the standard deviation of the two shocks by using the generalized method of moments (GMM) approach. To use this method, the labor earnings implied by the model, Eqs. (2), (4) and (5), have to be appropriately adjusted such that the model can be taken to the data while still capturing the key differences between the two types of risk. As a first step, estimate σ_η by computing the difference in logarithm of earnings over subsequent periods of an agent's working life.²⁹ The difference in earnings will primarily depend on two factors (i) experience and other non-stochastic factors that impact wage changes but are not incorporated in the model and (ii) idiosyncratic career shock. The implied moment conditions (mean and variance) will allow estimation of σ_η . Given career risk, σ_ϵ can be estimated in the following manner. For each age–education group, compute the conditional variance of earnings.³⁰ Then take the difference in conditional variances of different education but same age group and sum it across all age groups. Using this procedure, the additional risk associated with higher levels of education can be identified.

To implement the empirical strategy outlined above, consider a sample of full-time workers and abstract from inherited human capital.³¹ The logarithm of labor earnings implied by the model can be re-written as $\bar{e}_t^i(t+j) = \bar{w}(t+j) + \log[\sum_{m=0}^{j-1} h_t^i(t+m+1)]$, where the logarithm of a variable is denoted by bar. Therefore, to identify human capital risk described by the above procedure one would have to incorporate the following changes: (i) $w(t+j)\exp[\sum_{m=0}^{j-1} h_t^i(t+m+1)]$, and (ii) $h_t^i(t+j+1) = \gamma(t+j) + \tau_t^i(t+j)^\phi e_t^i(t+j+1) + \eta_t^i(t+j+1)$. Due to the first change, shocks enter linearly and the second change allows specialization shocks to be identified uniquely.

To conduct this analysis, using data from the family files of Panel Study on Income Dynamics (PSID) will help in keeping the analysis consistent with the related empirical life-cycle literature. As human capital heterogeneity due to occupation choice is not incorporated in this model, conducting a within-occupation analysis will give flexibility in the identification and interpretation of these human capital shocks in each occupation.³² PSID data on occupation is available at 1, 2 or 3 digit classification.³³

Separating heterogeneity from uncertainty would be important in this analysis in order to get a correct estimate of the human capital risk faced by agents when making the schooling decisions.³⁴ Controlling for observable factors (e.g., race, sex, marital status, birth year, parental education) is straightforward but unobservable ability poses a harder problem. Since PSID does not have measures of ability, to proxy for unobservable ability the following alternatives can be used: (i) use of individual fixed effects and (ii) combine NLSY, which has measures of ability, with PSID.³⁵ In this model the decision to invest in human capital is also affected by risk and uncertainty. The resulting endogeneity issue could be overcome by following Heckman et al. (1998) and considering a panel where agents no longer make this decision (e.g., working males above the age of 40).

²⁹ In describing this approach, for simplicity it is assumed that both shocks are homoskedastic.

³⁰ In this computation, control for factors (observable and unobservable) that make earnings heterogeneous in each age–education group but are not incorporated in the model (e.g., race, sex, marital status, birth year, parental education—a proxy for inherited human capital, ability).

³¹ The following component of labor earnings $w(t+j)[(1-\tau_t^i(t+j))\bar{x} + (1-\delta_h)\bar{x}(t)]$ is ignored in order to focus on earnings impacted by accumulation of human capital.

³² However, most empirical studies that find a positive risk–return trade-off in human capital investment, for example Palacios-Huerta (2003), do not conduct a within-occupation analysis. Some preliminary evidence on whether such a trade-off exists within occupations is provided in Section 3 in the supplementary appendix. Conducting a quantile regression analysis using PSID 1993 data, predicted wage attributable to education is computed for the last and first decile (0.9 and 0.1 quantile) across different occupations. The results indicate that within some occupations, most notably professionals and managers that are directly applicable to the analysis in this paper, dispersion of predicted wage is increasing in the level of education. This suggests that there is some preliminary evidence supporting the idea that specialization risk is greater when individuals spend more time in training. Underlying this rough measure of risk is the notion that when agents make human capital decisions they are uncertain about their position in the relevant wage distribution. However, this evidence is merely suggestive. It does not control for heterogeneity, discussed below, and therefore the dispersion in wages could be due to both heterogeneity and uncertainty, where the latter is the focus of this paper.

³³ In conducting a within-occupation analysis, occupations may need to be classified more broadly into 1-digit classification in order to overcome the issue of sparse coverage per occupation when using a 3-digit occupation classification.

³⁴ For more details see Carneiro et al. (2003) and Cunha et al. (2005)

³⁵ Cunha et al. (2005) adopt this methodology.

4. Conclusion

The impact of uninsurable idiosyncratic human capital risk in a general equilibrium life-cycle model is studied in this paper. The results indicate that the nature of risk, career versus specialization, is crucial in determining the quantitative implications of market incompleteness for the aggregate economy. Calibrated stationary equilibrium with only career risk has properties similar to Aiyagari (1994) reconfirming the long held belief that the quantitative implications of uninsurable idiosyncratic risk are inconsequential. However, the baseline calibration with high weight on the specialization risk illustrates that market incompleteness can have large, quantitatively significant, macroeconomic implications. Stationary equilibrium in this case is severely distorted relative to the complete markets benchmark.

The quantitative analysis also suggests that in order to comment on the aggregate effect of uninsurable idiosyncratic human capital risk, empirical studies based on micro data would need to determine the nature of human capital risk. Such studies could thereby alert us about the likely extent of distortion due to uninsurable idiosyncratic human capital risk.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at [doi:10.1016/j.jmoneco.2010.05.012](https://doi.org/10.1016/j.jmoneco.2010.05.012).

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