BUBBLES AND FINANCIAL CRISIS[‡]

Bubbles and Total Factor Productivity[†]

By Jianjun Miao and Pengfei Wang*

Evidence shows that asset price bubbles typically precede financial crises and financial crises are accompanied by the collapse of bubbles. In addition, stock market booms and busts are typically associated with credit market booms and busts. The recent US housing and stock market bubbles and the subsequent Great Recession that began in late 2007 provide an example. The stock market booms in the early 1990s and the subsequent Asian financial crisis in 1997 provide another example. There are also many other related episodes of financial crises in emerging economies. Motivated by these observations, Miao and Wang (2011a) propose a theory of credit-driven stock price bubbles. We show that a positive feedback loop mechanism generates stock price bubbles when firms borrow against the stock market value of their collateralized assets to finance investment. We assume that firms face stochastic investment opportunities and bubbles improve investment efficiency. We also study stochastic bubbles and policy implications.

In the present paper, we apply the theory developed by Miao and Wang (2011a) to an environment in which firms face idiosyncratic productivity shocks and credit constraints. We show that both bubbleless and bubbly equilibria can exist. In the bubbly equilibrium, more productive firms have higher stock price bubbles. These bubbles allow firms to relax credit constraints and improve investment efficiency and capital allocation. Consequently, more capital is allocated to more productive firms, leading to a rise in total factor productivity (TFP).[1] On the other hand, the collapse of bubbles tightens the credit constraints and worsens investment efficiency. Thus, the collapse of bubbles leads to a recession and a fall of TFP. Our theory may help explain the empirical evidence documented by Meza and Quintin (2005); Pratap and Urrutia (forthcoming); and Queralto (2011), who find that TFP fell markedly during East Asian, Mexican, and Argentine financial crises in the 1990s.

I. The Model

Consider an infinite-horizon economy consisting of households and firms. There is no aggregate uncertainty, but firms face idiosyncratic productivity shocks. Time is discrete and denoted by $t = 0, 1, 2, \ldots$.

A. *Households*

There is a continuum of identical households of unit mass. Each household is risk neutral and derives utility from a consumption stream $\{C_t\}$ according to the utility function, $\sum_{t=0}^{\infty} \beta^t C_t$, where $\beta = 1/(1 + r)$ is the subjective discount factor. Households supply labor inelastically. The labor supply is normalized to one. Households trade firm stocks and risk-free bonds. The net supply of bonds is zero and the net supply of any stock is one. Because there is

[1] See Liu and Wang (2011) for an alternative model without bubbles based on a similar mechanism.

no aggregate uncertainty, r is equal to the risk-free rate (or interest rate) and also to the rate of return of each stock.

B. *Firms*

There is a continuum of firms of unit mass. Firms are indexed by $j \in [0, 1]$. Each firm j combines labor N_t^j and capital K_t^j to produce output according to the following Cobb-Douglas production function:

$$Y_t^j = (A_t^j K_t^j)^\alpha (N_t^j)^{1-\alpha}, \quad \alpha \in (0, 1),$$

where A_t^j represents idiosyncratic productivity shocks. These shocks follow a Markov process with the state space $\{A_1, A_2\}$ and with the transition probabilities given by

$$(1) \quad \Pr(A_{t+1}^j = A_1 | A_t^j = A_1) = 1 - \lambda\rho,$$

$$(2) \quad \Pr(A_{t+1}^j = A_2 | A_t^j = A_2) = 1 - \rho,$$

where $\rho, \lambda > 0$. Assume that A_t^j is independent across firms and thus idiosyncratic risks wash out in the aggregate. Let A_0^j be drawn from the stationary distribution $(1/(1 + \lambda), \lambda/(1 + \lambda))$. Assume that $A_1 > A_2$ and $\rho < 1 - \rho\lambda$, meaning that the chance of being productive is higher if the firm is relatively more productive in the previous period.

After solving the static labor choice problem, we obtain the operating profits

$$(3) \quad A_t^j R_t K_t^j = \max_{N_t^j} (A_t^j K_t^j)^\alpha (N_t^j)^{1-\alpha} - w_t N_t^j,$$

where w_t is the wage rate and

$$(4) \qquad R_t = \alpha \left(\frac{w_t}{1 - \alpha} \right)^{\frac{\alpha-1}{\alpha}}.$$

After observing A_t^j, firm j may make investment I_t^j so that the law of motion for capital is given by

$$(5) \qquad K_{t+1}^j = (1 - \delta) K_t^j + I_t^j,$$

where $\delta > 0$ is the depreciation rate of capital. Assume that investment is subject to the following constraint:

$$(6) \qquad -\mu K_t^j \leq I_t^j \leq A_t^j R_t K_t^j + L_t^j,$$

where $\mu \in (0, 1 - \delta)$ and $L_t^j > 0$. The first inequality captures the assumption that investment is partially irreversible. The second inequality says that firms can finance investment by internal funds $A_t^j R_t K_t^j$ and external borrowing L_t^j. Assume that external equity is so costly that no firms will raise new equity to finance investment. Like Carlstrom and Fuerst (1997); Jermann and Quadrini (forthcoming); and Miao and Wang (2011a), we consider intratemporal loans for simplicity. These loans are taken at the beginning of the period and repaid at the end of the period. They do not incur interest.[2]

To capture financial frictions, we follow Miao and Wang (2011a) to introduce credit constraints. Let $V_t(K_t^j, A_t^j)$ denote the date-t cum-dividends stock market value of firm j with assets K_t^j and the realized productivity shock A_t^j. Then we write the credit constraint as

$$(7) \qquad L_t^j \leq \beta E_t V_{t+1}(\xi K_t^j, A_{t+1}^j),$$

where E_t is the conditional expectation operator with respect to the shock A_{t+1}^j. The motivation of this constraint is similar to that in Kiyotaki and Moore (1997): firm j pledges a fraction $\xi \in (0, 1]$ of its assets (capital stock) K_t^j at the beginning of period t as the collateral. The parameter ξ may represent the degree of pledgeability or the extent of financial market imperfections. It is the key parameter for our analysis below. At the end of period t, the stock market value of the collateral is equal to $\beta E_t V_{t+1}(\xi K_t^j, A_{t+1}^j)$. The lender never allows the loan repayment L_t^j to exceed this value. If this condition is violated, then firm j may take loans L_t^j and walk away, leaving the collateralized assets ξK_t^j behind. In this case, the lender runs the firm with the collateralized assets ξK_t^j at the beginning of period $t + 1$ and obtains the smaller firm value $\beta E_t V_{t+1}(\xi K_t^j, A_{t+1}^j)$ at the end of period t. Unlike Kiyotaki and Moore (1997), we have implicitly assumed that firm assets are not specific to a particular owner. Any owner can operate the assets using the same technology. Thus, the lender does not have to liquidate the collateralized assets in the event of default.

Following Miao and Wang (2011a), we may interpret the collateral constraint in (7)

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[2] Miao and Wang (2011a) study intertemporal bonds with interest payments and allow firms to save. This extension does not change our key insights.

as an incentive constraint in an optimal contract between firm j and the lender with limited commitment:[3] given a history of information at date t and after observing the idiosyncratic shock A_t^j, the contract specifies investments I_t^j and loans L_t^j at the beginning of period t, and repayments L_t^j at the end of period t. Firm j may default on debt at the end of period t. If this happens, then the firm and the lender renegotiate the loan repayment. In addition, the lender reorganizes the firm. Because of default costs, the lender can only seize a fraction ξ of capital K_t^j. The lender can run the firm with these assets at the beginning of period $t + 1$ after observing the productivity shock A_{t+1}^j. The date-t stock market value of the firm is given by $\beta E_t V_{t+1}(\xi K_t^j, A_{t+1}^j)$. This value is the threat value (or the collateral value) to the lender at the end of period t. Following Jermann and Quadrini (2010), we assume that the firm has all the bargaining power in the renegotiation and the lender gets only the threat value. The key difference between our modeling and that of Jermann and Quadrini (2010) is that the threat value to the lender is the going concern value in our model, while Jermann and Quadrini (2010) assume that the lender liquidates the firm's assets and obtains the liquidation value in the event of default.

Enforcement requires that the continuation value to the firm of not defaulting is not smaller than the continuation value of defaulting; that is,

$$\beta E_t V_{t+1}(K_{t+1}^j, A_{t+1}^j) - L_t^j$$

$$\geq \ \beta E_t V_{t+1}(K_{t+1}^j, A_{t+1}^j)$$

$$- \ \beta E_t V_{t+1}(\xi K_t^j, A_{t+1}^j),$$

where E_t is the conditional expectation operator with respect to A_{t+1}^j. This incentive constraint is equivalent to the collateral constraint in (7). This constraint ensures that there is no default in an optimal contract.

Firm value $V_t(K_t^j, A_t^j)$ satisfies the following Bellman equation:

$$(8) \quad V_t(K_t^j, A_t^j) = \max_{I_t^j} A_t^j R_t K_t^j - I_t^j$$

$$+ \ \beta E_t V_{t+1}(K_{t+1}^j, A_{t+1}^j),$$

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[3] See Albuquerque and Hopenhayn (2004) and Alvarez and Jermann (2000) for related contracting problems.

subject to (5), (6), and (7).

C. *Competitive Equilibrium*

Let $N_t = \int_0^1 N_t^j \, dj$, and $Y_t = \int_0^1 Y_t^j \, dj$ denote the aggregate labor demand, and aggregate output. Let $K_{it} = \int_{A_t^j = A_i} K_t^j \, dj$ and $I_{it} = \int_{A_t^j = A_i} I_t^j \, dj$ denote the aggregate capital stock and the aggregate investment for firms with productivity A_i, $i = 1, 2$. Then a competitive equilibrium is defined as sequences of $\{Y_t\}$, $\{C_t\}$, $\{K_{it}\}$, $\{I_{it}\}$, $\{N_t\}$, $\{w_t\}$, $\{R_t\}$, $\{V_t(K_t^j, A_t^j)\}$, $\{I_t^j\}$, $\{K_t^j\}$, $\{N_t^j\}$, and $\{L_t^j\}$ such that households and firms optimize and markets clear; i.e., $N_t = 1$, $C_t + I_{1t} + I_{2t} = Y_t$, and

$$(9) \quad K_{1t+1} = [(1 - \delta) K_{1t} + I_{1t}] (1 - \rho\lambda)$$

$$+ \ [(1 - \delta) K_{2t} + I_{2t}] \rho,$$

$$(10) \quad K_{2t+1} = [(1 - \delta) K_{2t} + I_{2t}] (1 - \rho)$$

$$+ \ [(1 - \delta) K_{1t} + I_{1t}] \lambda\rho.$$

II. Model Analysis

We show that there are two types of equilibrium. In the first type of equilibrium, there is no stock price bubble. In the second type, all firms have bubbles. We then show that bubbles help raise total factor productivity. We shall present results directly and relegate the detailed derivation to an online Appendix.

A. *Bubbleless Equilibrium*

In a bubbleless equilibrium, firm value takes the following form as in Hayashi (1982) given a constant-returns-to-scale technology:

$$V_t(K, A_i) = v_{it} K, \quad i = 1, 2,$$

where v_{it} is to be determined below. Define Tobin's marginal Qs as

$$(11) \quad Q_{1t} = \beta[v_{1t+1}(1 - \rho\lambda) + v_{2t+1}\rho\lambda],$$

$$(12) \quad Q_{2t} = \beta[v_{1t+1} \rho + v_{2t+1}(1 - \rho)].$$

They are also equal to Tobin's average Qs given the constant-returns-to-scale technology. We shall focus on the equilibrium in which

$Q_{1t} > 1$ and $Q_{2t} \leq 1$ in the neighborhood of a steady state.

In this case, any firm with productivity A_1 chooses the maximal investment level and the credit constraint binds. Thus, aggregate investment for these firms is given by

$$I_{1t} = A_1 R_t K_{1t} + \xi Q_{1t} K_{1t}.$$

If $Q_{2t} < 1$, then the investment level of any firm with productivity A_2 reaches the lower bound $-\mu K_t^j$. If $Q_{2t} = 1$, then its investment level is indeterminate. Only the aggregate investment level I_{2t} of these firms is determined in equilibrium. Using the Bellman equation (8), we can show that Q_{1t} and Q_{2t} satisfy the following asset pricing equations:

$$(13) \quad Q_{1t} = \beta(1 - \rho\lambda)[A_1 R_{t+1} + (1 - \delta)Q_{1t+1}$$
$$+ (Q_{1t+1} - 1)(A_1 R_{t+1} + \xi Q_{1t+1})]$$
$$+ \beta[A_2 R_{t+1} + (1 - \delta)Q_{2t+1}$$
$$+ (Q_{2t+1} - 1) i_{2t}]\rho\lambda,$$

$$(14) \quad Q_{2t} = \beta\rho[A_1 R_{t+1} + (1 - \delta)Q_{1t+1}$$
$$+ (Q_{1t+1} - 1)(A_1 R_{t+1} + \xi Q_{1t+1})]$$
$$+ \beta[A_2 R_{t+1} + (1 - \delta)Q_{2t+1}$$
$$+ (Q_{2t+1} - 1) i_{2t}](1 - \rho),$$

where in equilibrium R_t satisfies

$$R_t = \alpha(A_1 K_{1t} + A_2 K_{2t})^{\alpha-1}.$$

In addition, if $Q_{2t} < 1$, then $i_{2t} \equiv I_t^j/K_t^j = -\mu$, and if $Q_{2t} = 1$, then i_{2t} is indeterminate. The bubbleless equilibrium is characterized by four equations, (9), (10), (13), and (14) for four variables Q_{1t}, I_{2t}, K_{1t}, and K_{2t} if $Q_{2t} = 1$, and for Q_{1t}, Q_{2t}, K_{1t} and K_{2t} if $Q_{2t} < 1$. Moreover, the usual transversality condition must be satisfied.

B. *Bubbly Equilibrium*

We now turn to the bubbly equilibrium in which

$$V_t(K, A_i) = v_{it}K + b_{it},$$

where v_{it} and b_{it} are to be determined. Due to limited liability, stock prices cannot be negative. Thus, we require $b_{it} > 0$ and interpret it as a bubble. Define Q_{1t} and Q_{2t} as in (11) and (12) and define

$$(15) \quad B_{1t} = \beta[b_{1t+1}(1 - \rho\lambda) + b_{2t+1}\rho\lambda],$$

$$(16) \quad B_{2t} = \beta[b_{2t+1}(1 - \rho) + b_{1t+1}\rho].$$

Because of the presence of bubbles, Tobin's marginal Q is not equal to the average Q even though the technology has constant returns to scale.

We will construct a bubbly equilibrium in which $Q_{1t} > 1$, and $Q_{2t} < 1$ around a steady state. In such an equilibrium, $i_{2t} = I_{2t}/K_{2t} = -\mu$ and

$$(17) \quad I_{1t} = A_1 R_t K_{1t} + \xi Q_{1t} K_{1t} + B_{1t}/(1 + \lambda),$$

$$(18) \quad B_{1t} = \beta[B_{1t+1}Q_{1t+1}(1 - \rho\lambda) + \rho\lambda B_{2t+1}],$$

$$(19) \quad B_{2t} = \beta[B_{2t+1}(1 - \rho) + \rho B_{1t+1}Q_{1t+1}].$$

The bubbly equilibrium is characterized by 6 equations (9), (10), (13), (14), (18), and (19) for 6 variables, $Q_{1t}, Q_{2t}, K_{1t}, K_{2t}, B_{1t}$, and B_{2t}. In addition, the usual transversality condition must be satisfied.

Equation (17) shows that bubbles relax the credit constraint and allow firms to make more investment when they are more productive. The increased investment allows firms to accumulate more capital, make more profits, and distribute more dividends. This in turn makes firms' assets indeed more valuable, justifying the initial beliefs that assets contain bubbles. As discussed by Miao and Wang (2011a), this positive feedback loop mechanism supports stock price bubbles. Of course, there is another equilibrium as studied in Section A in which no one believes in bubbles.

To interpret (18), we rewrite it as

$$(20) \quad r = (1 - \rho\lambda)\frac{B_{1t+1}(Q_{1t+1} - 1)}{B_{1t}}$$
$$+ \frac{(1 - \rho\lambda)B_{1t+1} + \rho\lambda B_{2t+1} - B_{1t}}{B_{1t}}.$$

This equation says that the rate of return on the more productive firm's bubble is equal to the

interest rate r. This return consists of two components: the first component is the capital gains represented by the second term on the right-hand side of equation (20). The second component is the dividend yield represented by the first term on the right-hand side of equation (20). When the current more-productive firm continues to be more productive in the next period, it can use one dollar of the bubble to relax the credit constraint by one dollar and hence it can make one more dollar of investment, generating a net benefit $(Q_{1t+1} - 1)$. When the current more-productive firm becomes less productive in the next period, however, it sells capital and the bubble does not help generate dividends because the firm does not borrow. The interpretation of equation (19) is similar.

Note that the equilibrium restriction on the credit-driven stock price bubbles studied here is different from that on bubbles on intrinsically useless assets or on assets with exogenous payoffs often studies in the literature (e.g., Tirole 1985). Because the latter type of bubbles do not help generate dividends, there is no dividend yield component and hence the growth rate of these bubbles is equal to the interest rate. See Miao and Wang (2011a) for a more detailed discussion on this point.

C. *Total Factor Productivity*

To analyze the effect of stock price bubbles on TFP, we focus on the steady state. We use a variable without a subscript t to denote its steady state value. We can compute the steady state TFP as

$$TFP = \left(\frac{A_1 K_1 + A_2 K_2}{K_1 + K_2} \right)^\alpha.$$

Thus, to show that TFP rises in a bubbly equilibrium, we only need to show that K_1/K_2 rises too. Using equations (9) and (10), we can show that

$$\frac{K_1}{K_2} = \frac{1 - \lambda\rho - (1 - \delta + I_2/K_2)(1 - \rho - \rho\lambda)}{\lambda\rho}.$$

If $Q_2 = 1$ in a bubbleless equilibrium, then it is possible that aggregate investment for the less productive firms does not reach the lower bound

so that $I_{2t} > -\mu K_{2t}$. If $Q_2 < 1$ and $I_{2t} = -\mu K_{2t}$ in a bubbly equilibrium, then the above equation reveals that K_1/K_2 is higher in a bubbly equilibrium than in a bubbleless equilibrium given the assumption $1 - \rho - \rho\lambda > 0$, and hence TFP rises in a bubbly equilibrium. The intuition is that, relative to the bubbleless equilibrium, stock price bubbles induce less productive firms to sell more capital so that more productive firms can attract more capital and make more investment. This reallocation of capital raises TFP.

Given the above analysis, our goal is to impose assumptions on parameters such that the bubbly and bubbleless equilibria characterized in Sections IIA and IIB can coexist. General conditions are fairly complex. We thus consider a numerical example. For simplicity, assume that $A_1 = 1$ and $A_2 = 0$. Set $\beta = 0.99$, $\delta = 0.025$, $\alpha = 0.3$, $\rho = 0.2$, $\lambda = 2$, $\mu = 0.9$ and $\xi = 0.3$. We can then show that, in a bubbly equilibrium, $Q_1 = 1.0299$, $Q_2 = 0.98611$, $B_1 = 7.2382$, $B_2 = 7.0962$, $C = 1.5297$, $Y = 1.8764$, and $TFP = 0.85257$. There also exists a bubbleless equilibrium in which $Q_1 = 1.0695$, $Q_2 = 1$, $I_2/K_2 = -0.27595$, $C = 1.3909$, $Y = 1.7631$, and $TFP = 0.78421$. Clearly, the bubbly equilibrium generates a boom and Pareto-dominates the bubbleless equilibrium.

The crucial parameter for our analysis is ξ, the degree of pledgeability or the fraction of assets recovered by the lender in the event of default. It is possible to show that there are three cutoff values ξ_1, ξ_2, and ξ_3 such that four cases are possible: (i) If $\xi > \xi_1$, then the economy achieves the first best equilibrium in which $Q_1 = 1$ and $Q_2 < 1$; (ii) If $\xi_2 < \xi < \xi_1$, then there is a unique bubbleless equilibrium in which $Q_1 > 1$ and $Q_2 < 1$; (iii) If $\xi_3 < \xi < \xi_2$, then there is a bubbly equilibrium in which $Q_1 > 1$ and $Q_2 < 1$, and also a bubbleless equilibrium in which $Q_1 > 1$ and $Q_2 < 1$; (iv) If $\xi < \xi_3$, then there is a bubbleless equilibrium in which $Q_1 > 1$ and $Q_2 = 1$, and also a bubbly equilibrium in which $Q_1 > 1$ and $Q_2 < 1$. Note that TFP rises in a bubbly equilibrium only in the last case.

So far, we have focused on deterministic bubbles. Following Weil (1987) and Miao and Wang (2011a), we can construct an equilibrium with stochastic bubbles. Suppose that the economy has bubbles initially. All agents believe that there is a constant probability that bubbles will persist next period. Once bubbles burst, no bubble can reemerge. If this probability is sufficiently large,

then an equilibrium with stochastic bubbles can exist. After the bursting of bubbles, the economy will enter a recession and TFP will fall.

III. Concluding Remarks

The extant literature on rational bubbles typically studies bubbles on intrinsically useless assets or on assets with exogenous payoffs. Miao and Wang (2011a) develop a theory of credit-driven stock-price bubbles when dividends are endogenous in an infinite-horizon production economy. The present paper applies this theory to an environment in which firms face idiosyncratic productivity shocks and credit constraints. We show that stock price bubbles can make capital allocation more efficient among heterogeneous firms and help raise TFP. The collapse of bubbles leads to a recession and a fall in TFP.

In a related study, Miao and Wang (2011b) extend Miao and Wang (2011a) to a two-sector economy. They show that sectoral bubbles may misallocate capital between the two production sectors and hence are detrimental to economic growth. In a work in progress, Miao and Wang (2011c) build a dynamic stochastic general equilibrium model to study the quantitative implications of stock market bubbles for business cycles. Based on a similar positive feedback loop mechanism, Miao and Wang (2011d) show, in another work in progress, that banking bubbles can exist when banks have limited commitment and may default on deposit liabilities. We also show that the collapse of banking bubbles can lead to a recession even though there is no real or financial friction in a sound nonfinancial sector.

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