Three Papers on Individual Contributions to Team Productivities

Wenzhi Wang

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Some Thoughts about Our Topic

In our context, with nursing data, to estimate individual contributions to team productivities, *the most important issue is to think of and justify an appropriate team production function*, that is, how individuals in a team determine overall team production.

If the production is of a linear additive form, then this topic is simply to *estimate fixed effects for each worker*, and the only concern here is the typical incidental parameter problem, and Bonhomme (2021) is a good start. It also provides a clean framework about how to decompose total variance into heterogeneity and sorting.

When we want to add interaction and complementarity into our production function, then Müller and Upmann (2022) is a good start, which *assumes that individual productivity is a linear combination of all coworkers' productivities in that given team*. This simple linearity form allows us to transform this identification and estimation problem into an eigenvalue-finding problem, that is why, the authors refer to this type of coworker productivity as eigenvalue productivities (EVP). However, this assumption about how coworkers in the same team affect one's own productivity may not make much sense in some cases, maybe especially in our context of medical care.

In fact, there are other potential production functions that can be justified in our case. Consider the production function proposed by Kremer (1993) [1] as an example, in our context, a patient's mortality outcome is determined by the lease able nurse, since if he/she makes a tiny mistake, no matter how competent other members in the team are, the patient may be dead. If we think of this production process as our underlining assumption, then we will have a totally different story about how individuals contribute to team output.

[1] The spirit of this paper is that many production processes consist of a series of tasks, mistakes in any of which can dramatically reduce the product's value. The name of this type of production function comes from the space shuttle Challenger. It had thousands of components: it exploded because it was launched at a temperature that caused one of those components, the O-rings, to malfunction.

With all kinds of possible production functions, one thing can be sure. The estimation procedure can be quite simple and intuitive once we impose a strong parametric assumption to our model. However, when we want more flexibility of the functional form, the estimation can be quite tough (e.g., the second part of Bonhomme (2021)).

Another thing deserves some attention is our choice of outcome variable. Right now, the outcome of interest is patient death. Since it is an adverse outcome and a binary variable, it may need some extra work to directly apply the methods in the above papers I have mentioned, specially if we want to do an "eigenvalue productivity" approach as the presentation slides said.

Lastly, the variation these papers utilize is that the same persons join different teams across time. I couldn't find a good way to use the variation in the original paper, which comes from the gap between actual and planned staffing rota.

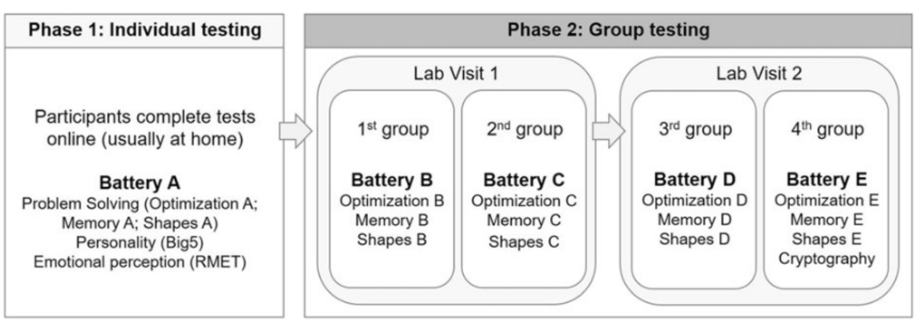
1. Weidmann and Deming (2021)

Paper: Team Players: How Social Skills Improve Team Performance, Econometrica

Summary: This paper develops a new experimental method for estimating individual contributions to team performance. The authors repeatedly randomly assign people to teams and find that some people consistently cause their teams to exceed predicted performance. These people are good "team players." The team player effect is not predicted by demographic characteristics such as age, gender, education, ethnicity, or IQ. Yet it is strongly related to individual scores on the Reading the Mind in the Eyes test (RMET), a widely used measure of social intelligence.

Experimental Design:

Figure 1-1: Overview of experiment



In the first phase, participants completed a series of online tests to measure their individual skill at three problem-solving tasks: Memory, Optimization, and Shapes. Participants' social intelligence / emotional perceptiveness [2] and personalities are also assessed.

[2] It is scored using a shortened version of the Reading the Mind in the Eyes Test (RMET), which measures participants'

The second phase of the experiment focused on testing participants in groups. Participants came to the lab and were randomly assigned to groups of 3 people. Each group completed a collective version of the individual problem-solving tasks.

Tasks are chosen to satisfy three criteria. First, tasks must be feasible to administer to both individuals and groups, with only minor modifications between the individual and group versions. Second, tasks needed to be objective in the sense that we could easily rank performance across individuals and groups. Third, to finish the tasks, cooperation among group members would plausibly improve performance.

Identification Strategy:

Notations: Let individuals be indexed by $i = 1, \ldots, n$. Let groups be indexed by g, with n_g groups. Let I_g^i be an indicator of whether participant i is in group g. Let X_{ik} denote the performance of individual i on task type $k \in$ Optimization; Memory; Shapes. et G_{gk} denote the performance of group g on task type k. Rescale G_{gk} using sample mean and standard deviation and obtain $\widetilde{G}_{gk}$, which is the main measure of group performance.

Statistical Model:

1. First, consider the following model:

$$\begin{aligned} \widetilde{G}_{gk} &= \alpha_k \sum_i I_g^i X_{ik} + \epsilon_{gk}, \\ \epsilon_{gk} &\sim N\left(0, \sigma_G^2\right). \end{aligned} \tag{1-1}$$

   The residuals $\widehat{\epsilon}_{gk}$ from equation (1-1) provide an estimate of whether each group under- or over-performed on task k relative to the prediction based on task-specific skills.

2. Then, the authors average this residual performance across tasks.

$$\widehat{T}_g = \frac{1}{3} \sum_k \widehat{\epsilon}_{gk} \tag{1-2}$$

3. Next, the authors estimate the *team player index* $\widehat{\beta}_i$ as the average $\widehat{T}_g$ across all groups that i participated in:

$$\widehat{\beta}_i = \frac{1}{4} \sum_g I_g^i \widehat{T}_g. \tag{1-3}$$

   $\widehat{\beta}_i$ is an estimate of the causal contribution of individual i to team performance.

4. With enough randomizations, β_i could be precisely estimate for each participant. However, with only four team assignments, $\widehat{\beta}_i$ is relatively noisy at the individual level. Thus, the authors

ability to recognize emotions in others and, more broadly, their "theory of mind" (i.e., their ability to reason about the mental state of others).

focus on σ_β, the standard deviation of the β estimates.

$$\begin{aligned}\widehat{T}_{gi} &= \beta_i + e_{gi}, \\ \beta_i &\sim N\left(0, \sigma_\beta^2\right), \\ e_{gi} &\sim N\left(0, \sigma^2\right).\end{aligned} \tag{1-4}$$

Main Results: The main results can be divided into two parts. The first part investigates whether some people are good team players by fitting model (1-4) to estimate the team player effect (σ_β) and evaluating their results against the null that this effect – conditional on individual skill, as in equation (1-1) – is equal to zero. The authors then draw a conclusion that some people consistently cause their team to exceed its predicted performance. The second part investigates what factors can predict one person being a good team player by regressing $\widehat{\beta}_i$ on individual characteristics. They find out that team players score significantly higher on a well-established measure of social intelligence, but do not differ across a variety of other dimensions, including IQ, personality, education, and gender.

How is it related to our topic?

Not so much. The statistical model in this paper is quite simple. But it builds on the assumption that the composition of groups is fully randomized so that there won't be any sorting or self-selection effects in the estimation of the residuals.

This paper is useless since we can never get a measure of individual productivities in our setting, which is a requirement in this framework.

2. Bonhomme (2021)

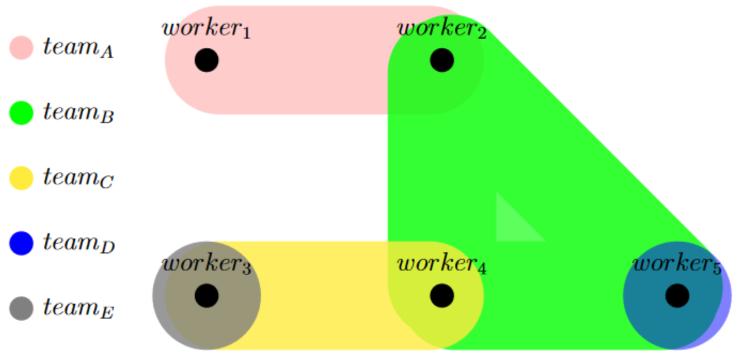
Paper: Teams: Heterogeneity, Sorting, and Complementarity, Working Paper

2.1. Framework

Consider a set of workers $1, \ldots, N$ who collaborate and produce output in teams. Workers are linked to each other by "hyperedges," which are referred to as "teams". Denote the workers in a team j of size $n_j = n$ as $\left(i_1\left(j, \ldots, i_n\left(j\right)\right)\right)$. Denote the number of teams as J. For example, the following figure illustrates five teams and five workers.

Worker i contributes to the team a quantity α_i, which is unobserved to the econometrician. Assume α_i is constant across collaborations, which is referred to as the type of worker i. The output of a team

Figure 2-1: Overview of experiment



j with n workers is denoted as Y_{nj}, and is given by

$$Y_{nj} = \phi_n \left(\alpha_{i_1(j)}, \ldots, \alpha_{i_n(j)}, \varepsilon_{nj} \right) \tag{2-1}$$

where ϕ_n is the production function of an n-worker team, and ε_{nj} represent other factors, or shocks, unobserved to the econometrician, that affect team output beyond workers' inputs α_i.

Assumption 1. (Network exogeneity) $\{\varepsilon_{nj} : (n, j)\}$ are independent of $\{(i_1(j), \ldots, i_n(j)) : (n, j)\}$ conditional on $\{\alpha_i : i\}$.

Assumption 1 restricts the team formation process. It states that team formation is independent of the team-specific shocks ε_{nj}, conditional on the worker types $\alpha_1, \ldots, \alpha_N$. Hence, while the probability of joining a team may depend unrestrictedly on worker types and other factors that are independent of the ε_{nj}'s, it cannot depend on the ε_{nj}'s themselves. Assumption 1 will thus fail if, before forming a team or joining one, workers have advance information about team-specific shocks.

Assumption 2. (Independent shocks) $\{\varepsilon_{nj} : (n, j)\}$ are mutually independent, and independent of $\{(i_1(j), \ldots, i_n(j)) : (n, j)\}$ and $\{\alpha_i : i\}$.

Assumption 2 rules out dependence among team-specific shocks. As an implication, shocks to a team of workers who collaborate repeatedly over time are assumed serially independent. *Another implication of Assumption 2 is the absence of "team effects" in the model.* In the framework, a team j is only a collection of workers, and there is no effect of j per se except through the independent shocks ε_{nj}.

Lastly, another assumption implicit in the production function (2-1) is the absence of covariates, except for team size n. Covariates can be incorporated as additional inputs to production.

2.2. Additive Production

Suppose that the production function in (2-1) takes the following additive form

$$Y_{nj} = \lambda_n \left(\alpha_{i_1(j)} + \ldots + \alpha_{i_n(j)}\right) + \varepsilon_{nj}, \tag{2-2}$$

where λ_n is a team-size scaling factor. Adopt the normalization $\lambda_1 = 1$. In (2-2), output depends on the sum of worker types, or equivalently on the mean worker type in the team (given the presence of λ_n). It is useful to write it in matrix form,

$$Y = D_\lambda A\alpha + \varepsilon, \tag{2-3}$$

where D_λ is a diagonal matrix, and A is a matrix of zeros and ones where a one indicates that a worker (i.e., a column) participates in a team (i.e., a row). Network exogeneity then takes the form

$$\mathbb{E}\left(\varepsilon_{nj} \mid A, \alpha\right) = \mu_n, \tag{2-4}$$

where μ_n is a team-size-specific constant.

Identification:

Treat α, λ, μ, and A as non-stochastic quantities. Suppose, to start with, that the λ_n's are known, and that $\mu_n = 0$ in (2-4). Since model (2-3)-(2-4) is a linear regression, the identification of the worker effects $\alpha_1, \ldots, \alpha_N$ is determined by the rank properties of A.

Estimation:

I will focus on the following decomposition of output variance, for every team size n:

$$\underbrace{\mathrm{Var}_n\left(Y_{nj}\right)}_{\text{Total variance}} = \underbrace{\lambda_n^2 \sum_{m=1}^{n} \mathrm{Var}_n\left(\alpha_{i_m(j)}\right)}_{\text{Heterogeneity}} + \underbrace{2\lambda_n^2 \sum_{m=1}^{n} \sum_{m'=m+1}^{n} \mathrm{Cov}_n\left(\alpha_{i_m(j)}, \alpha_{i_{m'}(j)}\right)}_{\text{Sorting}} + \underbrace{\mathrm{Var}_n\left(\varepsilon_{nj}\right)}_{\text{Other factors}}. \tag{2-5}$$

In this decomposition, the component labelled "heterogeneity" reflects the variation in worker effects on output, keeping team composition constant. Equivalently, it represents the effect of worker heterogeneity if the allocation of workers to teams were random, in which case covariances would be zero. In turn, the component labelled "sorting" reflects the variance contribution due to team composition not being random.

To estimate the variance components above, the authors first estimate the team-size effects λ_n, then estimate α using OLS.

The calculation details are quite simple. In this framework, the authors can construct the fixed-effects (FE) estimator for any variance component $V_Q = \alpha' Q\alpha$, where Q is a known $N \times N$ matrix.

2.3. Nonlinear Production

To keep tractability, this paper models types α_i as discrete, with K points of support. Suppose that $\alpha_1, \ldots, \alpha_N$ are drawn from the joint distribution $\prod_{i=1}^{N} \pi(\alpha_i)$, where $\pi(\alpha_i)$ are type probabilities. The authors interpret π as a prior on individual types.

Estimation:

Estimating the random-effects model of team production is challenging. To see this, consider a setup where the density $f^n_{\alpha_{i_1(j)},\ldots,\alpha_{i_n(j)}}(y)$ of Y_{nj} conditional on $\alpha_{i_1(j)}, \ldots, \alpha_{i_n(j)}$ is parametric, indexed by a finite-dimensional vector θ. The RE likelihood is

$$\mathscr{L}(\theta, \pi) = \sum_{\alpha_1} \ldots \sum_{\alpha_N} \prod_i \pi(\alpha_i) \prod_n \prod_j f^n_{\alpha_{i_1(j)},\ldots,\alpha_{i_n(j)}}\left(Y_{nj}; \theta\right) \tag{2-6}$$

involves an intractable N-dimensional sum over all possible worker type realizations. The likelihood does not factor in simple ways, except in the special case where all teams consist of one worker.

To reduce computational complexity, the authors follow a mean-field variational approach that is increasingly popular in related settings in machine learning and statistics.

To illustrate the methods, the authors then estimate the impact of economists on their research output, and the contributions of inventors to the quality of their patents.

2.4. How is it related to our topic?

If we want an additive production function, this paper has a very good conceptual framework. And the main goal is to calculate the degree of sorting, and heterogeneity in the authors' applications. However, if we want some complementarity in the production function, that is, if we want a nonlinear model, this paper may not be so constructive. In their Bayesian approach, we need first specify the number of types (that is, the number of all possible individual productivities) of the nurses, and due to computational complexity, the authors use only 2-6 types as an example.

3. Müller and Upmann (2022)

Paper: Eigenvalue Productivity: Measurement of Individual Contributions in Teams, PLOS ONE

3.1. Framework

Consider a team consisting of a fixed set of n workers $\mathscr{N} = \{1, 2, \ldots, n\}$. Assume that the individual productivity of each worker $i \in \mathscr{N}$ is nonnegative, and depends linearly on the productivity of i's teammates. The coworker productivity of worker i in team $\mathcal{N}$, $p^i(\mathcal{N})$, is defined as

$$p^i(\mathcal{N}) = \frac{1}{\lambda} \sum_{j \in \mathcal{N}} g_{ij}(\mathcal{N}) p^j(\mathcal{N}) \quad \forall i \in \mathcal{N}, \tag{3-1}$$

where $g_{ij}(\mathcal{N}) \geq 0$ denotes the extent to which worker i benefits from the coworker productivity of worker j, while $\lambda > 0$ is a strictly positive normalization factor.

In matrix notation, we have

$$\mathbf{p}(\mathcal{N}) \equiv \frac{1}{\lambda} \mathbf{G}(\mathcal{N}) \mathbf{p}(\mathcal{N}), \tag{3-2}$$

where the vector of the coworker productivities is denoted by $\mathbf{p}(\mathcal{N}) \equiv \left(p^1, \ldots, p^n\right)(\mathcal{N})$, and the matrix of the coefficients measuring the extent to which the individual productivities affect each other. Then, we can rewrite (3-2) as

$$\lambda \mathbf{p} = \mathbf{G}\mathbf{p} \quad \Leftrightarrow \quad (\mathbf{G} - \lambda \mathbf{I})\mathbf{p} = \mathbf{0}, \tag{3-3}$$

where $\mathbf{I}$ denotes the identity matrix.

Since $\mathbf{p}$ is the vector of individual productivities of interest, the authors refer to this concept of coworker productivities as *eigenvalue productivity*. Hence, assuming that all information relevant for the productivity measure is contained in the matrix $\mathbf{G}$, they define:

Definition 1. (Eigenvalue productivity (EVP)) Let the set of team members be $\mathcal{N} = \{1, \ldots, n\}$ with $n \in \mathbb{N}$. Let $\mathbf{G} \equiv \mathbf{G}(\mathcal{N}) \equiv \left[g_{ij}(\mathcal{N})\right]_{i,j \in \mathcal{N}} \geq \mathbf{0}$ denote a nonnegative, nonzero matrix - the matrix of pairwise, directional production coefficients – and let $\rho(\mathbf{G})$ denote the spectral radius of $\mathbf{G}$. Let $\lambda = \rho(\mathbf{G})$ be a maximal eigenvalue of $\mathbf{G}$. We call an associated eigenvector $\mathbf{p}(\lambda)$ a vector of eigenvalue productivities (EVP-vector).

The concept of EVP thus uses the information on the productivity relations among the team members contained in $\mathbf{G}$, and assigns each member a nonnegative real number, λ, assigns the team a nonnegative vector, which is unique upon multiplication by a constant (or upon scaling).

3.2. Existence and properties of eigenvalue productivity

Under some regularity conditions, we can guarantee existence and uniqueness of the eigenvalue productivity vector.

And it satisfies some good properties. Specifically, EVP nicely respects the following ideas: a player's productivity is increasing in the productivity of their teammates; the productivity of a player who fails to contribute anything to a team's success is zero; symmetric players are treated identically; and the productivity of a player decreases as there are other players who provide exactly the same productivity profile, and thus are perfect substitutes for the player.

3.3. Calculation

In real world applications, because we cannot directly observe the (marginal) effect of worker i on worker j's productivity, so that the productivity matrix $\mathbf{G}$ is not readily available, but has to be calculated from available data.

First, delete from the team, i.e., from the set $\mathcal{N}$,those workers who have never been in action during the given period. Then, calculate the entries of $\mathbf{G}$ as follows: For each given unordered pair of workers $\{i, j\}$, $i, j \in \mathcal{N}$, consider those projects in which i and j have worked together, that is, where both were included in the team composition. Calculate the ratio of the points (which is a generic term for any success measure) achieved in these projects to the maximal number of points the teams including the pair $\{i, j\}$ could have potentially achieved. Let s_{ij} denote this point ratio, which measures the success (performance) of the pair $\{i, j\}$ over all team compositions.

Next, consider those projects where worker i was a member of the team (worker j may or may not have been a member of the team), and calculate the points the respective teams have achieved in these projects divided by the maximal number of points those teams could have potentially achieved; denote this point ratio by $s_i \equiv s_{ii}$,which measures the success (performance) of worker i. As a convention, for pairs of workers $\{i, j\}$ that have never been jointly included in some team composition during the period, we set $s_{ij} = \sqrt{s_i s_j}$.

Finally, define the ratio $g_{ij} \equiv \frac{s_{ij}}{s_j}, i, j \in \mathcal{N}$, which represents the relative performance of the pair $\{i, j\}$ compared to the overall performance of worker j, an effect which can be attributed to the cooperation with worker i. In this way, the quadratic, non symmetric matrix $\mathbf{G}$ represents all relative normalized pairwise performance measures.

3.4. How is it related to our topic?

We can simply apply this framework to our data without using the expected rota data. However, to calculate $\mathbf{G}$, we need a properly defined outcome variables. The variable in the original paper, i.e., mortality is a binary variable, thus may not be suitable.

References

Bonhomme, Stéphane (2021) "Teams: Heterogeneity, Sorting, and Complementarity," *Working Paper*, 10.2139/ssrn.3778749.

Kremer, Michael (1993) "The O-Ring Theory of Economic Development," *The Quarterly Journal of Economics*, 108 (3), 551–575, 10.2307/2118400.

Müller, Julia and Thorsten Upmann (2022) "Eigenvalue Productivity: Measurement of Individual Contributions in Teams," *PLOS ONE*, 17 (9), e0273623, 10.1371/journal.pone.0273623.

Weidmann, Ben and David J. Deming (2021) "Team Players: How Social Skills Improve Team Performance," *Econometrica*, 89 (6), 2637–2657, 10.3982/ECTA18461.