A Conceptual Framework for "Purpose at Work"

Wenzhi Wang

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1. The Principal-Agent Model from Akerlof and Kranton (2005)

1.1. Assumptions

The agent's actions affect profits but the principal cannot observe them. The agent can take either action A at cost of effort e_A or action B at cost of effort e_B, where $e_A > e_B$. Revenues are random, and conditional on the agent's action. When the agent takes action A, with probability $\frac{1}{2}$ revenues are high, π_H, and with probability $\frac{1}{2}$ revenues are low, π_L. When the agent takes action B, revenues are always low. The principal can observe realized revenues, but not the agent's action.

There are two social categories $\mathbf{C} = \{N, O\}$, where N denotes those people who think of themselves as part of the firm —Insiders —and O denotes those who think of themselves as not a part of the firm —Outsiders. Prescriptions **P** dictate the ideal effort level for each of the two categories: Insiders should act in the firm's best interest, and Outsiders should take the least cost action. Denote these prescribed ideal efforts as $e^*(c)$, where $e^*(N) = e_A$ and $e^*(O) = e_B$.

Then we can define an agent's utility as the following:

$$U(y, e; c) = \ln(y) - e + I_c - t_c \left| e^*(c) - e \right|, \tag{1}$$

for $c \in \{N, O\}$, where y is the agent's income, e is her choice of effort, I_c denotes the constant utility the agent derives from belonging to social category c, and the utility she loses from diverging from the ideal effort level is captured by $t_c |e^*(c) - e|$. Also, the agent's outside opportunities yield utility of amount $\bar{u}$.

1.2. Model Solution

Assuming that the principal finds it optimal to give the worker incentives to do action A. Then, in this principal-agent model, income y is the monetary incentive to induce this action. That is to

say, the principal specifies a high wage, w_H^c, to pay when the realization of revenues is π_H, and a low wage, w_L^c, to pay when the realization of revenues is π_L. Then the principal's problem is to maximize the following expected profits:

$$\Pi(c) = \frac{1}{2}(\pi_H + \pi_L) - \frac{1}{2}\left(w_H^c + w_L^c\right) \tag{2}$$

subject to the following participation constraint (PC) and incentive constraint (IC) for a worker with identity c:

$$\begin{aligned} &\frac{1}{2}\ln w_H^c + \frac{1}{2}\ln w_L^c - e_A + I_c - t_c\left|e^*(c) - e_A\right| \geq \bar{u}, \\ &\frac{1}{2}\ln w_H^c + \frac{1}{2}\ln w_L^c - e_A + I_c - t_c\left|e^*(c) - e_A\right| \geq \ln w_L^c - e_B + I_c - t_c\left|e^*(c) - e_B\right|. \end{aligned} \tag{3}$$

We can see in these constraints why the principal could prefer a worker with an insider identity. An insider feels that she should act in the interest of the firm, i.e., $e^*(N) = e_A$. Hence, for $t_N > 0$, an insider loses utility when she takes action B. This loss loosens the agent's incentive constraint, and the principal can pay wages with less variation to induce the agent to take action A. In addition, when $I_N > I_O$ the worker directly gains utility from being an insider, and this utility gain loosens the participation constraint.

When all constraints are binding, the optimal wages for an O agent are,

$$\begin{aligned} w_L^O &= \exp\left[\bar{u} - I_O + e_B\right] \\ w_H^O &= \exp\left[\bar{u} - I_O + e_B + 2\left(1 + t_O\right)\left(e_A - e_B\right)\right]. \end{aligned} \tag{4}$$

If $t_N < 1$, the low and the high wages for an N agent are respectively:

$$\begin{aligned} w_L^N &= \exp\left[\bar{u} - I_N + e_A - \left(1 - t_N\right)\left(e_A - e_B\right)\right] \\ w_H^N &= \exp\left[\bar{u} - I_N + e_A + \left(1 - t_N\right)\left(e_A - e_B\right)\right] \end{aligned} \tag{5}$$

If $t_N > 1$, then there is no difference between high and low wage: $w^N = \exp\left[\bar{u} - I_N + e_A\right]$.

Then, we can draw the first important conclusion in Akerlof and Kranton (2005) (the last paragraph in Page 14 of the original paper): if the worker has an identity as an insider, the presence of identity utility will *reduce* the wage differential needed to induce the worker to take the high-effort action A. Correspondingly, if the worker identifies as an outsider, the presence of identity utility will *increase* the necessary variation of the contingent rewards. *When identity incentive works, there is less usage of monetary incentive!*

Some comparative static results discussed in the paper: Changes in each parameter of the model, and any extension to it, will affect the firm's profits from investing in identity. If inculcating identity is cheap, if there is much uncertainty, if workers' effort is hard to observe, if revenues/output depend upon special exertion at peak times, if workers are especially risk averse, if high effort is critical to the organization's output, we would expect a firm to find it more profitable to use an identity-oriented

incentive scheme.

The comparative static related to our empirical study: Those workers for whom t_N is high should receive relatively little variation in their wages.

1.3. Relationship to Our Study

This paper, and the subsequent paper Akerlof and Kranton (2008) both consider a full principal-agent model, where the income is contracted by the firm as one kind of incentives. The authors' contribution is to introduce "identity" into the agent's utility function, and then investigate how to use identity to motivate workers to pay more effort. Their main finding is that, identity can be a substitute to the monetary incentives in the sense that insiders need less variation in wages $\left(w_H^N - w_L^N < w_H^O - w_L^O\right)$ to induce higher level of effort.

However, we don't want to incorporate elements of asymmetric information into our story; thus, we cannot link our framework to the principal-agent model. What we want is an explicit and contractable relationship between one's effort and his wage.

Even though, from the modelling perspective, these papers are somehow not so relevant to our study, it still has some implications for our empirical study.

- We can investigate *the heterogeneous effects based on whether the worker's effort can be easily observed*. If effort is perfectly observable, there may be little need to invest in identity. If so, we may also guess that the treatment effects of the workshop is less salient.
- Furthermore, generating identification should be relatively cheap for successful firms with a homogeneous workforce, but perhaps too costly for firms with a heterogeneous workforce. Therefore, we can *investigate whether the treatment effect of the workshop is heterogeneous among workplaces with different levels of homogeneity (race, gender, etc)*.

2. The Simplest Possible Model

As Akerlof and Kranton (2005, 2008); Shayo (2020), we model identities as preferences: In an organization, a worker's utility not only depend on income and effort, but also on how she thinks of herself as a member of the firm and how she treats herself in the work-life balance.

To keep our illustration the simplest, assume that there are only two "social categories" – *workaholic* (W) and *non-workaholic* (N). Like the names suggested, the workers who identify themselves as workaholic can gain more utility from working; thus their ideal choice of effort level is higher than non-workaholic, anything else holding constant. Suppose that *there are only two choices of effort level, e_H and e_L, where $e_H > e_L$*. The norms for workaholics are to put more weights into their work and to choose e_H, while a non-workaholic's ideal effort level is e_L since she cares more about her

personal life.

Workers also differ from each other in their ability, α, which dictates the level of disutility from exerting effort into their work. In this firm, each worker's ability is an independent draw from distribution $F(\alpha)$. And, like in all standard economic theories, workers care about their income y, which is a deterministic function of her effort choice. In other words, we assume that the firm can observe workers' effort level and wage can be contracted based on workers' effort, so that we can avoid the problem of asymmetric information.

Broadly, in our conceptual framework, there are three components of a worker's utility: the utility from her income, the disutility from taking effort, and the identity (dis-)utility from deviation from her ideal choice of effort level: $U\big(y,e;c\big) = u\big(y\big) - t(e) - f(e - e^*(c))$, where $c \in \{N, O\}$ is the worker's social category, y is her income, e is her choice of effort, and $e^*(c)$ is her ideal choice of effort level. Recall that $e^*(W) = e_H$ and $e^*(N) = e_L$. More specifically, we specify the following functional form in terms of one's effort level:

$$U(e;c) = \ln(e) - \alpha e^2 - \beta\left(e - e^*(c)\right)^2, \tag{1}$$

where $\alpha > 0$ is the ability parameter drawn from distribution $F(\alpha)$ and $\beta > 0$ determines the loss of utility for a worker of category c by deviating from $e^*(c)$. Also assume the worker's outside opportunities yield utility of amount $\bar{u}$. Our experiment seeks to shock β by prompting workers to think about what is the purpose of work actively.

To illustrate how one's identity c, ability α, and β affect her choice of effort level, let's first consider a workaholic. To minimize the utility loss from deviating from her ideal effort level, she must choose the higher effort, e_H and gain more wage. However, even though her workaholic identity induces her to exert more effort, if her ability parameter α is too large, that is, she is not competent enough, then she will still choose the lower effort. Our purpose workshop increases β and thus increases the weight of losing identity utility in the overall utility and thus those workaholics with marginal ability parameters will turn to exert more effort. Besides, increasing β will prompt those workers who are at the margin of leaving current work to pursue outside opportunities, and thus there is a significant treatment effect on workers' exit and move.

To numerically illustrate how our purpose workshop increase workers' performance. Let's first focus on the workaholics. We have the following equations:

$$\begin{aligned} U(e_H;W) &= \ln(e_H) - \alpha e_H^2, \\ U(e_L;W) &= \ln(e_L) - \alpha e_L^2 - \beta(e_H - e_L)^2. \end{aligned} \tag{2}$$

Then obviously, there is a cutoff value of α, denoted by $\widetilde{\alpha}$, such that those workers with $\alpha < \widetilde{\alpha}$ will choose e_H while other workers with $\alpha > \widetilde{\alpha}$ will choose e_L (if the maximal utility still exceeds the outside utility). The expression for $\widetilde{\alpha}$ is as follows:

$$\widetilde{\alpha} = \frac{\ln(e_H/e_L) + \beta(e_H - e_L)^2}{e_H^2 - e_L^2}. \tag{3}$$

It is an increasing function with respect to β, thus increasing β will prompt those marginal workaholics who originally exert lower effort to choose higher effort level.

Now, I'll give one numerical example to see how changing β affect workers with different ability parameters. Consider the parameter values in Figure 1:

Figure 1: Effects of Changing β on Utility Values

```
clear all
e_H = 2; e_L = 1;               % two choices of effort level
beta = 1; beta_treated = 2;     % original beta and beta after the workshop
alpha = linspace(0, 1, 1e+3);   % ability parameter: uniform distribution in [0,1]
u_H_W = log(e_H) - alpha * e_H .^ 2;  % workaholic choosing e_H before and after the workshop
u_L_W = log(e_L) - alpha * e_L .^ 2 - beta .* (e_L - e_H) .^ 2;  % workaholic choosing e_L before the workshop
u_L_W_treated = log(e_L) - alpha * e_L .^ 2 - beta_treated .* (e_L - e_H) .^ 2; % workaholic with e_L after the workshop
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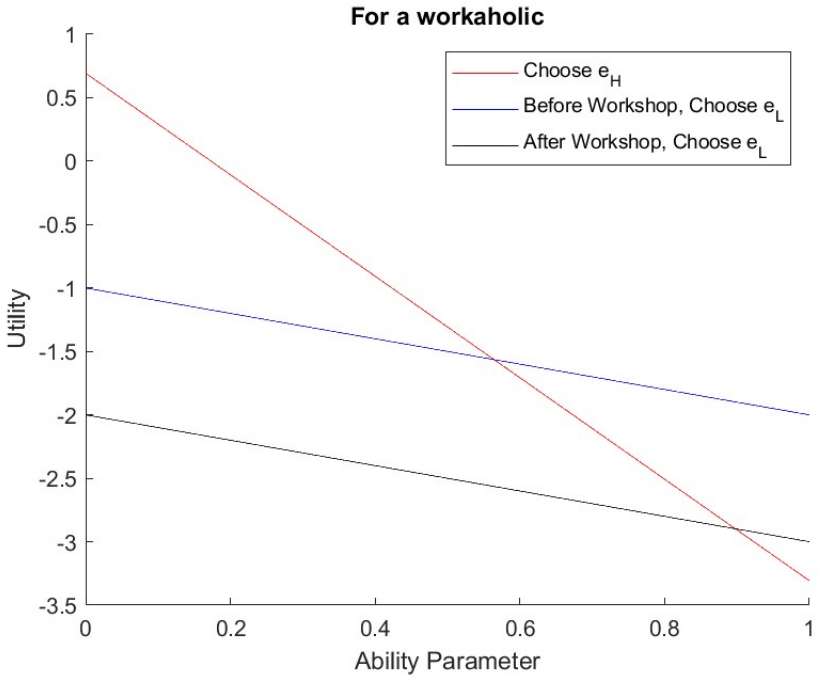
Then, the effects of changing β on the utility values from choosing e_H and e_L are in Figure 2. Note that the workshop treatment does not change the utility of choosing e_H.

Figure 2: Parameter Values



And the linear relationship between cutoff value $\tilde{\alpha}$ and β is in Figure 3. Since we assume that α has a uniform distribution, this cutoff value equals to the fraction of workers choosing higher-level effort, e_H.

One thing deserves to mention: changing β won't affect non-workaholics' decisions since they will always find lower-level effort suits their identity and induces less disutility directly from effort. See Figure 4 for an illustration.

Figure 3: Effects of Changing β on $\widetilde{\alpha}$ (Fraction of Workers Choosing e_H)

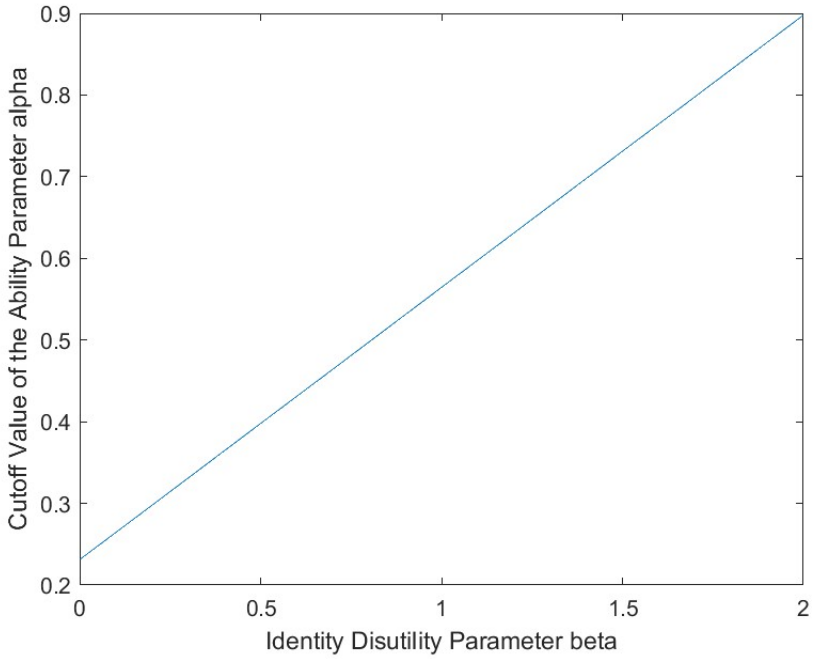
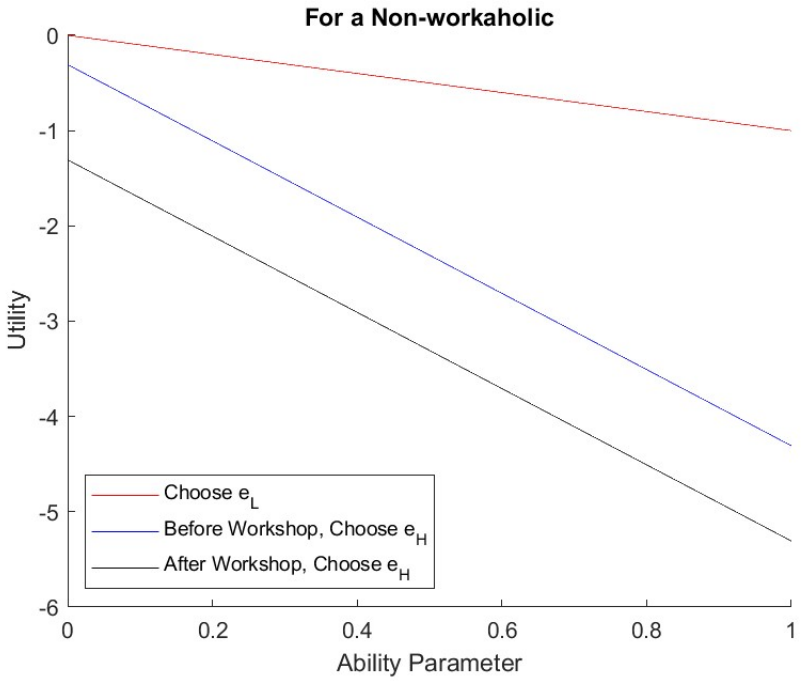


Figure 4: Effects of Changing β on on Utility Values (N-type)



2.1. What Can This Model Explain?

In this very simple model, there are only two categories and two choices of effort level. But still it can explain two main results in the paper: (1) Increasing β will increase workers' effort level. But note that *the only mechanism suggested by the model is workaholics with the marginal value of ability parameters α changing from e_L to e_H*. (2) Increasing β will increase the probability of people pursuing outside options. This is natural since the outside utility $\bar{u}$ is fixed while some workers will find paying less effort is less desirable than pursuing outside opportunities, no matter what their types are.

Under current functional form and assumptions, it seems impossible to extend the discrete effort set and social category set into a continuum set while still get a clean result, as the original PPT suggested. The point is that when effort level can be any value chosen from an interval, we need to take the functional form very seriously, and get the optimal effort level as $e^{\text{optimal}} = \frac{\beta E + \sqrt{\beta^2 E^2 + 2(\beta+\alpha)}}{2(\beta+\alpha)}$. If we further let E, the ideal effort level (or in other terminology, self-identity, social categories) and α, the ability parameter to be drawn from a distribution, the calculation is a mess!

It can also partly explain why the results are more pronounced in workers with worse baseline performance since when increasing β, the workers who respond to this change are those with higher α, that is, the utility cost of exerting effort is high, or in other words, they are less competent. Then it is natural to think of the scenario if we extend the effort set into a continuous set, and these workers are those who pay less effort before the workshop treatment.

2.2. What Cannot This Model Explain?

The shrinking gender heterogeneity. If we consider all men as workaholics while all women as non-workaholics, then actually, in current model, the gender wage gap expands since some men change from e_L to e_H, but all women remain to choose e_L.

Some Thoughts:

It is so hard to construct a model by myself even in such an intuitive context... The identity literature is not so helpful since these papers mainly consider a principal-agent model, where effort is not observable or contractable. The only lesson I learned from there is to choose only two categories and two possible effort levels. But, still, the model implications are not fully aligned with our empirical results.

References

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