EECS487 Lab0

Xiaofei Wang 56209720

September 12, 2015

Angle Between Vectors

Denote the angle by $\alpha(\vec{u}, \vec{v})$. We have

$$\begin{aligned}\cos\alpha(\vec{u},\vec{v}) &= \frac{\langle \vec{u}, \vec{v}\rangle}{||\vec{u}|| \cdot ||\vec{v}||} \\ &= \frac{1\times(-5)+2\times 8+3\times(-2)}{\sqrt{(1^2+2^2+3^2)((-5)^2+8^2+(-2)^2)}} \\ &= \frac{5}{\sqrt{1302}}\end{aligned}$$

Therefore $\alpha(\vec{u}, \vec{v}) = \arccos\frac{5}{\sqrt{1302}} \approx 1.43\text{rad} = 82.035\text{degree}$

Linear Equations With Matrices

The equations can be denoted by

$$\begin{pmatrix} 2 & 6 & 2 \\ 1 & 9 & 3 \\ -3 & -3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$$

We have

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 6 & 2 \\ 1 & 9 & 3 \\ -3 & -3 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$$

denote

$$A = \begin{pmatrix} 2 & 6 & 2 \\ 1 & 9 & 3 \\ -3 & -3 & 1 \end{pmatrix}$$

and Cof$A$ the cofactor of A. We have

$$A^{-1} = \frac{1}{\det A}(\text{Cof}A)^T$$

$$\det A = 2\times(9+9) - 1\times(6+6) - 3\times(18-18) = 24$$

$$\begin{aligned}&\text{Cof}A \\ &= \begin{pmatrix} 9+9 & -(1+9) & -3+27 \\ -(6+6) & 2+6 & -(-6+18) \\ 18-18 & -(6-2) & 18-6 \end{pmatrix} \\ &= \begin{pmatrix} 18 & -10 & 24 \\ -12 & 8 & -12 \\ 0 & -4 & 12 \end{pmatrix}\end{aligned}$$

$$\text{Cof}A^T = \begin{pmatrix} 18 & -12 & 0 \\ -10 & 8 & -4 \\ 24 & -12 & 12 \end{pmatrix}$$

$$\begin{aligned}A^{-1} &= \frac{1}{\det A}(\text{Cof}A)^T \\ &= \begin{pmatrix} \frac{3}{4} & -\frac{1}{2} & 0 \\ -\frac{5}{12} & \frac{1}{3} & -\frac{1}{6} \\ 1 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}\end{aligned}$$

Therefore

$$\begin{aligned}\begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} \frac{3}{4} & -\frac{1}{2} & 0 \\ -\frac{5}{12} & \frac{1}{3} & -\frac{1}{6} \\ 1 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} -\frac{3}{2} - \frac{1}{2} + 0 \\ \frac{5}{6} + \frac{1}{3} + \frac{1}{6} \\ -2 - \frac{1}{2} - \frac{1}{2} \end{pmatrix} \\ &= \begin{pmatrix} -2 \\ \frac{4}{3} \\ -3 \end{pmatrix}\end{aligned}$$

Projection

The component of $\vec{u}$ orthogonal to $\vec{v}$ is $\vec{u} - \frac{\vec{v}}{||\vec{v}||^2}(\vec{u}\cdot\vec{v})$.

$$\begin{aligned}\vec{u} - \frac{\vec{v}}{||\vec{v}||^2}(\vec{u}\cdot\vec{v}) &= \langle -3, 2, 4\rangle + \frac{10}{5}\langle 2, 0, -1\rangle \\ &= \langle 1, 2, 2\rangle\end{aligned}$$

Triangle Area

$$S = \frac{1}{2}||\vec{u}\times\vec{v}|| = \frac{3\sqrt{5}}{2}$$

Matrix Multiplication

$$\begin{aligned}\hat{M}\hat{N} &= \begin{pmatrix} -5+21-18 & 6+35-12 & 10-49-6 \\ -10+6-24 & 12+10-16 & 20-14-8 \\ -5+0+42 & 6+0+28 & 10+0+14 \end{pmatrix} \\ &= \begin{pmatrix} -2 & 29 & -45 \\ -28 & 6 & -2 \\ 37 & 34 & 24 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\hat{N}\hat{M} &= \begin{pmatrix} -5+12+10 & -35+12+0 & 15-24+70 \\ 3+10-7 & 21+10+0 & -9-20-49 \\ 6+8+2 & 42+8+0 & -18-16+14 \end{pmatrix} \\ &= \begin{pmatrix} 17 & -23 & 61 \\ 6 & 31 & -78 \\ 16 & 50 & -20 \end{pmatrix}\end{aligned}$$

Vector Multiplication

$$\vec{u}^T\vec{v} = -6 + 0 - 4 = -10$$

$$\vec{u}\vec{v}^T = \begin{pmatrix} -6 & 0 & 3 \\ 4 & 0 & -2 \\ 8 & 0 & -4 \end{pmatrix}$$

Plane

Assume the equation of this plane is $f(x, y, z) = Ax + By + Cz + D = 0$. We have

$$\begin{cases} A + 2B + 3C + D = 0 \\ 3A - 2B + 7C + D = 0 \\ -2A - 2B + 4C + D = 0 \end{cases}$$

Solving these equations we can get

$$A = -\frac{3}{5}C, B = \frac{7}{10}C, D = -\frac{19}{5}C$$

So the equation of this plane is $f(x, y, z) = 6x - 7y - 10z + 38 = 0$