# Chapter 8

# Externalities and Public Goods

## 8.1 What is an Externality?

We just showed that competitive markets result in Pareto optimal allocations – that is the market acts to make sure that those who value goods the most receive them, and those that can produce goods at the least cost produce them, and there is no way that everybody in society could be made better off. This gave us the first and second welfare theorems – the market allocates commodities efficiently, and any efficient allocation can be derived by a market with suitable ex ante transfers of wealth. Now we will take a look at one important circumstance where the welfare theorems do not hold.

When we talked about commodities in the past, they were always what are called "private goods." That is, they were such that they were consumed by only one person, and that person's consumption of the good had no effect on other people's utility. But, this is not true of all goods. Think, for example, of a local bakery that produces bread. Earlier, we said that each person purchases the quantity of bread where the marginal benefit of consuming an additional loaf is just equal to the price of a loaf, and each firm produces bread up to the point where the marginal cost of producing the loaf is just equal to its price. In equilibrium, then, the marginal benefit of eating an additional loaf of bread is just equal to the marginal cost of producing an additional loaf. But, think about the following. People who walk by the bakery get the benefit from the pleasant smell of baking bread, and this is not incorporated into the price of bread. Thus at the equilibrium, the marginal social benefit of another loaf of bread is equal to the benefit people get from eating the bread as well as the benefit people get from the pleasant smell of baking bread. However, since bread purchasers do not take into account the benefit provided to people who do not purchase

bread, at the equilibrium price the total marginal benefit of additional bread will be greater than the marginal cost. From a social perspective, too little bread is produced.

We can also consider the case of a negative externality. One of the standard examples in this situation is the case of pollution. Suppose that a factory produces and sells tires. In the course of the production, smoke is produced, and everybody that lives in the neighborhood of the factory suffers because of it. The price consumers are willing to pay for tires is given by the benefit derived from using the tires. Hence at the market equilibrium, the marginal cost of producing a tire is equal to the marginal benefit of using the tire, but the market does not incorporate the additional cost of pollution imposed on those who live near the factory. Thus from the social point of view, too many tires will be produced by the market.

Another way to think about (some types of) goods with external costs or benefits is as public goods. A public good is a good that can be consumed by more than one consumer. Public goods can be classified based on whether people can be excluded from using them, and whether their consumption is rivalrous or not. For example, a non-excludable, non-rivalrous public good is national defense.[1] Having an army provides benefits to all residents of a country. It is non-excludable, since you cannot exclude a person from being protected by the army, and it is non-rivalrous, since one person consuming national defense does not diminish the effectiveness of national defense for other people.[2] Pollution is a non-rivalrous public good (or public bad), since consumption of polluted air by one person does not diminish the "ability" of other people to consume it. A bridge is also a non-rivalrous public good (up to certain capacity concerns), but it may be excludable if you only allow certain people to use it. Another example is premium cable television. One person having HBO does not diminish the ability of others to have it, but people can be excluded from having it by scrambling their signal.

Examples of externalities and public goods tend to overlap. It is hard to say what is an externality and what is a public good. This is as you would expect, since the two categories are really just different ways of talking about goods with non-private aspects. It turns out that a useful way to think about different examples is in terms of whether they are rivalrous or non-rivalrous, and whether they are excludable or not. Based on this, we can create a 2-by-2 matrix describing

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[1] Goods of this type are often called "pure public goods."

[2] This is true in the case of national missile defense, which protects all people equally. However, in a nation where the military must either protect the northern region or the southern region, the army may be a rivalrous public good.

goods.[3]

|  | Non-rivalrous | Rivalrous |
| --- | --- | --- |
| Non-Excludable | (Pure) Public Goods | Common-Pool Resources |
| Excludable | Club Goods | Private Goods |

Private goods are goods where consumption by one person prevents consumption by another (an extreme form of rivalrous consumption), and one person has the right to prevent the other from consuming the object. When consumption is non-rivalrous but excludable, as in the case of a bridge, such goods are sometimes called **club goods**. Because club goods are excludable, inefficiencies due to external effects can often be addressed by charging people for access to the club goods, such as charging a toll for a bridge or a membership fee for a club. Pure public goods are goods such as national defense, where consumption is non-rivalrous and non-excludable. **Common-pool resources** are goods such as national fisheries or forests, where consumption is rivalrous but it is difficult to exclude people from consuming them. Both pure public goods and common-pool resources are situations where the market will fail to allocate resources efficiently. After considering a simple, bilateral externality, we will go on to study pure public goods and common pool resources in greater detail.

## 8.2   Bilateral Externalities

We begin with the following definition. An **externality** is present whenever the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy (and this interaction is not mediated by the price mechanism). An important feature of this definition is the word "directly." This is because we want to differentiate between a true externality, and what is called a **pecuniary externality**. For example, return to the example of the bakery we considered earlier. We can think of three kinds of external effects. First, there is the fact, as we discussed earlier, that consumers walking down the street may get utility from the smell of baking bread. This is true regardless of whether the people participate in any market. Second, if the smells of the bread are pleasant enough, the bakery may be able to charge more for the bread it sells, and, the fact that the price of bread increases may have harmful effects on people who buy the bread because they must pay more for the bread. We call this type of effect a pecuniary externality, since it works through the price mechanism. Effects such as this are not

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[3]Based on Ostrom, *Rules, Games, and Common Pool Resources*, University of Michigan Press, 1994.

really externalities, and will not have the distortionary effects we will find with true externalities.[4] Third, there is the fact that being next to a bakery may increase rents in the area around it. While this is a situation where the bakery has effects outside of the bread market, this effect is captured by the rent paid by other stores in the area. Whether this is an externality or not depends on the particular situation. For example, if you own an apartment building next to the bakery before it opens and are able to increase rents after it begins to produce bread, they you have realized an external benefit from the bakery (since the bakery has increased the value of your property). On the other hand, if you purchase the building next to the bakery once it is already opened, then you will pay a higher price for the building, but this is the fair price for a building next to a bakery. Thus this situation is really more of a pecuniary externality than a true externality.

We will use the following example for our externality model. There are two consumers, $i = 1, 2$. There are $L$ traded goods in the economy with price vector $p$, and the actions taken by these two consumers do not affect the prices of these goods. That is, the consumers are price takers. Further, consumer $i$ has initial wealth $w_i$.

Each consumer has preferences over both the commodities he consumes and over some action $h$ that is taken by consumer 1. That is,

$$u_i \left( x_1^i, ..., x_L^i, h \right).$$

Activity $h$ is something that has no direct monetary cost for person 1. For example, it could be playing loud music. Loud music itself has no cost. In order to play it, the consumer must purchase electricity, but electricity can be captured as one of the components of $x^i$.

From the point of view of consumer 2, $h$ represents an external effect of consumer 1's action. In the model, we assume that

$$\frac{\partial u_2}{\partial h} \neq 0.$$

Thus the externality in this model lies in the fact that $h$ affects consumer 2's utility, but it is not priced by the market. For example, $h$ is the quantity of loud music played by person 1.

Let $v_i (p, w_i, h)$ be consumer $i$'s indirect utility function:

$$
\begin{aligned}
v_i (w_i, h) &= \max_{x_i} u_i (x_i, h) \\
\text{s.t. } p \cdot x^i &\leq w_i.
\end{aligned}
$$

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[4] The key to being a true externality is that the external effect will usually be on parties that are not participants in the market we are studying, in this case the market for bread.

We will also make the additional assumption that preferences are quasilinear with respect to some numeraire commodity. If this were not so, then the optimal level of the externality would depend on the consumer's level of wealth, significantly complicating the analysis.

When preferences are quasilinear, the consumer's indirect utility function takes the form:

$$v_i\left(w_i, h\right) = \bar{\phi}_i\left(h\right) + w_i.^5$$

Since we are going to be concerned with the behavior of utility with respect to $h$ but not $p$, we will suppress the price argument in the utility function. That is, let $\phi_i\left(h\right) = \bar{\phi}_i\left(p, h\right)$, when we hold the price $p$ constant. We will assume that utility is concave in $h : \phi_i^{''}\left(h\right) < 0$.

Now, we want to derive the competitive equilibrium outcome, and show that it is not Pareto optimal. How will consumer 1 choose $h$? The function $v_1$ gives the highest utility the consumer can achieve for any level of $h$. Thus in order to maximize utility, the consumer should choose $h$ in order to maximize $v_1$. Thus the consumer will choose $h$ in order to satisfy the following necessary and sufficient condition (assuming an interior solution):

$$\phi_1'\left(h^*\right) = 0.$$

Even though consumer 2's utility depends on $h$, it cannot affect the choice of $h$. Herein lies the problem.

What is the socially optimal level of $h$? The socially optimal level of $h$ will maximize the sum of the consumers' utilities (we can add utilities because of the quasilinear form) :
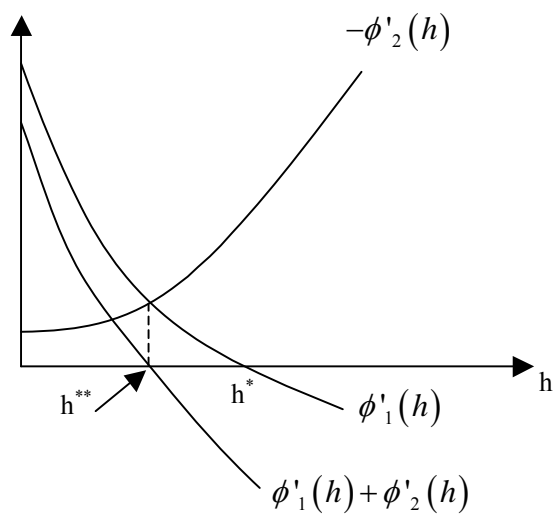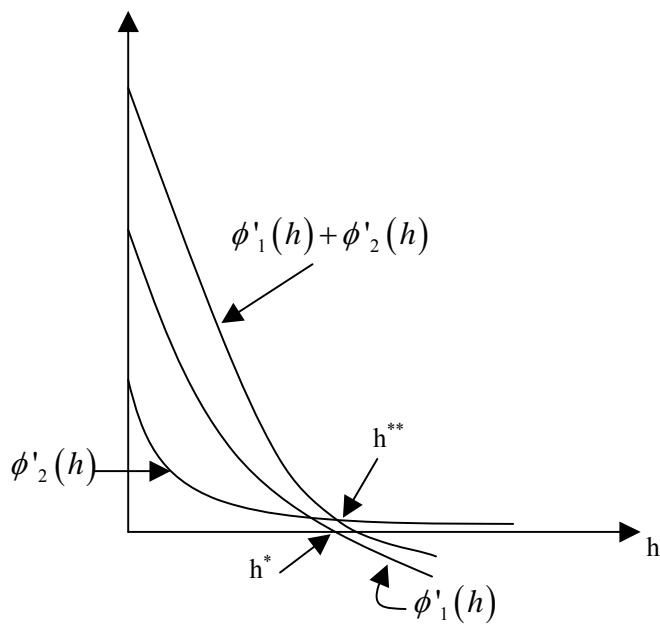
$$\max_h \phi_1\left(h\right) + \phi_2\left(h\right).$$

The first-order condition for an interior maximum is:

$$\phi_1'\left(h^{**}\right) + \phi_2'\left(h^{**}\right) = 0,$$

where $h^{**}$ is the Pareto optimal amount of $h$.

The social optimum requires that the sum of the two consumers' marginal utilities for $h$ is zero (for an interior solution). On the other hand, the level of the externality that is actually chosen depends only on person 1's utility. Thus the level of the externality will not generally be the socially optimal one. In the case where the externality is bad for consumer 2 (loud music), the level of $h^* > h^{**}$. That is, too much $h$ is produced. In the case where the externality is good for consumer 2 (baking bread smell or yard beautification), too little will be provided, $h^* < h^{**}$. These situations are illustrated in Figures 8.1 and 8.2.

$-\phi'_2(h)$

$h^{**}$          $h^*$          $\phi'_1(h)$

$\phi'_1(h)+\phi'_2(h)$

Figure 8.1: Negative Externality: $h^{**} < h^*$

$\phi'_1(h)+\phi'_2(h)$

$h^{**}$

$\phi'_2(h)$

$h^*$

$\phi'_1(h)$

Figure 8.2: Positive Externality: $h^{**} > h^*$

Note that the social optimum is not for the externality to be eliminated entirely. Rather, the social optimum is where the sum of the marginal benefit of the two consumers equals zero. In the case where there is a negative externality, this is where the marginal benefit to person 1 equals the marginal cost to person 2. In the case of a positive externality, this is where the sum of the marginal benefit to the two people is equal to zero.

The fact that the optimal level of a negative externality is greater than zero is true even in the case where the externality is pollution, endangered species preservation, etc. Of course, this still leaves open for discussion the question of how to value the harm of pollution or the benefit of saving wildlife. Generally, those who produce the externality (i.e., polluters) think that the optimal level of the externality is larger than those who are victims of it.

### 8.2.1   Traditional Solutions to the Externality Problem

There are two traditional approaches to solving the externality problem: quotas and taxes. Quotas impose a maximum (or minimum) amount of the externality good that can be produced. Taxes impose a cost of producing the externality good on the producer. Positive taxes will tend to decrease production of the externality, while negative taxes (subsidies) will tend to increase production of the externality.
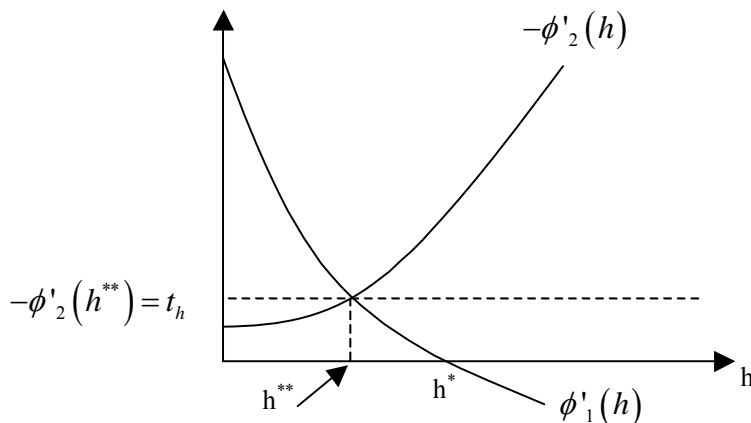
Let's begin by considering a quota. Suppose that activity $h$ generates a negative external effect, so that the privately chosen quantity $h^*$ is greater than the socially optimal quantity $h^{**}$. In this case, the government can simply pass a quota, prohibiting production in excess of $h^{**}$. In the case of a positive externality, the government can require consumer 1 to produce at least $h^{**}$ units of the externality (although this is less often seen in practice).

While the quota solution is simple to state, it is less simple to implement since it requires the government to enforce the quota. This involves monitoring the producer, which can be difficult and costly. One thing that would be nice would be if there were some adjustment we could make to the market so that it worked properly. One way to do this, known as Pigouvian Taxation, is to impose a tax on the production of the externality good, $h$.

Suppose consumer 1 were charged a tax of $t_h$ per unit of $h$ produced. His optimization problem would then be

$$\max \phi_1\left(h\right) - t_h h$$

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[5]The argument for why is contained in footnote 3 on p. 353 in MWG.

Figure 8.3: Implementing $h^{**}$ Using a Tax on $h$

with first-order condition

$$\phi_1' \left( h^t \right) = t_h$$

Thus setting $t_h = -\phi_2' \left( h^{**} \right)$ (which is positive) will lead consumer 1 to choose $h^t = h^{**}$, implementing the social optimum. See Figure 8.3.

Note that the proper tax is equal to the marginal externality at the optimal level of $h$. By forcing consumer 1 to pay this, he is required to internalize the externality. That is, he must pay the marginal cost imposed on consumer 2 when the externality is set at its optimal level, $h^{**}$. When the tax rate is set in this way, consumer 1 chooses the Pareto optimal level of the externality.

In the case of a positive externality, the tax needed to implement the Pareto optimal level of the externality is negative. Consumer 1 is subsidized in the amount of the marginal external effect at the optimal level of the externality activity. And, when he internalizes the benefit imposed on the other consumer, he chooses the (larger) optimal level of $h$.

Another equivalent approach would be for the government to pay consumer 1 to reduce production of the externality. In this case, the consumer's objective function is:

$$\phi_1 \left( h \right) + s_h \left( h^* - h \right) = \phi_1 \left( h \right) - s_h h + s_h h^*$$

By setting $s_h = -\phi_2' \left( h^{**} \right)$, the socially optimal level of $h$ is implemented.

Note that it is key to tax the externality producing activity directly. If you want to reduce pollution from cars, you have to tax pollution, not cars. Taxes on cars will not restore optimality of pollution (since it does not affect the marginal propensity to pollute) and will distort people's car purchasing decisions. Similarly, if you want a tractor factory to reduce its pollution, you need

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to tax pollution, not tractors. Taxing tractors will generally lead the firm to reduce output, but it won't necessarily lead it to reduce pollution (what if the increased costs lead it to adopt a more polluting technology?).

Note that taxes and quotas will restore optimality, but this result depends on the government knowing exactly what the correct level of the externality-producing activity is. In addition, it will require detailed knowledge of the preferences of the consumers.

### 8.2.2   Bargaining and Enforceable Property Rights: Coase's Theorem

A different approach to the externality problem relies on the parties to negotiate a solution to the problem themselves. As we shall see, the success of such a system depends on making sure that property rights are clearly assigned. Does consumer 1 have the right to produce $h$? If so, how much? Can consumer 2 prevent consumer 1 from producing $h$? If so, how much? The surprising result (known as Coase's Theorem) is that as long as property rights are clearly assigned, the two parties will negotiate in such a way that the optimal level of the externality-producing activity is implemented.

Suppose, for example, that we give consumer 2 the right to an externality-free environment. That is, consumer 2 has the right to prohibit consumer 1 from undertaking activity $h$. But, this right is contractible. Consumer 2 can sell consumer 1 the right to undertake $h_2$ units of activity $h$ in exchange for some transfer, $T_2$. The two consumers will bargain both over the size of the transfer $T_2$ and over the number of units of the externality good produced, $h_2$.[6]

In order to determine the outcome of the bargaining, we first need to specify the bargaining mechanism. That is, who does what when, what are the other consumer's possible responses, and what happens following each response.[7]

Suppose bargaining mechanism is as follows:

1. Consumer 2 offers consumer 1 a take-it-or-leave-it contract specifying a payment $T_2$ and an activity level $h_2$.

2. If consumer 1 accepts the offer, that outcome is implemented. If consumer 1 does not accept the offer, consumer 1 cannot produce any of the externality good, i.e., $h = 0$.

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[6] The subscript 2 is used here because we will compare this with the case where 1 has the right to produce as much of the externality as it wants, and we'll denote the outcome with the subscript 1 in that case.

[7] Those of you familiar with game theory will recognize that what we are really doing here is setting up a game. If you don't know any game theory, revisit this after you see some, and it will be much clearer.

To analyze this, begin by considering which offers $(h, T)$ will be accepted by consumer 1. Since in the absence of agreement, consumer 1 must produce $h = 0$, consumer 1 will accept $(h_2, T_2)$ if and only if it offers higher utility than $h = 0$. That is, 1 accepts if and only if:[8]

$$\phi_1(h) - T \geq \phi_1(0).$$

Given this constraint on the set of acceptable offers, consumer 2 will choose $(h_2, T_2)$ in order to solve the following problem.

$$\max_{h,T} \phi_2(h) + T$$
$$\text{subject to} \quad : \quad \phi_1(h) - T \geq \phi_1(0).$$

Since consumer 2 prefers higher $T$, the constraint will bind at the optimum. Thus the problem becomes:

$$\max_h \phi_1(h) + \phi_2(h) - \phi_1(0).$$

The first-order condition for this problem is given by:

$$\phi_1'(h_2) + \phi_2'(h_2) = 0.$$

But, this is the same condition that defines the socially optimal level of $h$. Thus consumer 2 chooses $h_2 = h^{**}$, and, using the constraint, $T_2 = \phi_1(h^{**}) - \phi_1(0)$. And, the offer $(h_2, T_2)$ is accepted by consumer 1. Thus this bargaining process implements the social optimum.

Now, we can ask the same question in the case where consumer 1 has the right to produce as much of the externality as she wants. We maintain the same bargaining mechanism. Consumer 2 makes consumer 1 a take-it-or-leave-it offer $(h_1, T_1)$, where the subscript indicates that consumer 1 has the property right in this situation. However, now, in the event that 1 rejects the offer, consumer 1 can choose to produce as much of the externality as she wants, which means that she will choose to produce $h^*$. Thus the only change between this situation and the previous example is what happens in the event that no agreement is reached.

In this case, consumer 2's problem is:

$$\max_{h,T} \phi_2(h) + T$$
$$\text{subject to} \quad : \quad \phi_1(h) - T \geq \phi_1(h^*)$$

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[8]In the language of game theory, this is called an incentive compatibility constraint.

Again, we know that the constraint will bind, and so consumer 2 chooses $h_1$ and $T_1$ in order to maximize

$$\max \phi_1(h) + \phi_2(h) - \phi_1(h^*)$$

which is also maximized at $h_1 = h^{**}$, since the first-order condition is the same. The only difference is in the transfer. Here $T_1 = \phi_1(h^{**}) - \phi_1(h^*)$.

While both property-rights allocations implement $h^*$, they have different distributional consequences. The transfer is larger in the case where consumer 2 has the property rights than when consumer 1 has the property rights. The reason for this is that consumer 2 is in a better bargaining position when the non-bargaining outcome is that consumer 1 is forced to produce 0 units of the externality good. However, note that in the quasilinear framework, redistribution of the numeraire commodity has no effect on social welfare.

The fact that regardless of how the property rights are allocated, bargaining leads to a Pareto optimal allocation is an example of the Coase Theorem: If trade of the externality can occur, then bargaining will lead to an efficient outcome no matter how property rights are allocated (as long as they are clearly allocated). Note that well-defined, enforceable property rights are essential for bargaining to work. If there is a dispute over who has the right to pollute (or not pollute), then bargaining may not lead to efficiency. An additional requirement for efficiency is that the bargaining process itself is costless.

Note that the government doesn't need to know about individual consumers here – it only needs to define property rights. However, it is critical that it do so clearly. Thus the Coase Theorem provides an argument in favor of having clear laws and well-developed courts.

### 8.2.3   Externalities and Missing Markets

The externality problem is frequently called a "missing market" problem. To see why, suppose now that there were a market for activity $h$. That is, suppose consumer 2 had the right to prevent all activity $h$, but could sell the right to undertake 1 unit of $h$ for a price of $p_h$. In this case, in deciding how many rights to sell, player 2 will maximize

$$\phi_2(h) + p_h h$$

This has the first-order condition for an interior solution

$$\phi_2'(h) = -p_h,$$

which implicitly defines a supply function: $h_2(p_h)$.

In deciding how many rights to purchase, consumer 1 maximizes

$$\phi_1(h) - p_h h$$

This has the first-order condition for an interior solution:

$$\phi_1'(h) = p_h,$$

which implicitly defines a demand function, $h_1(p_h)$.

The market-clearing condition says that $h_1(p_h) = h_2(p_h)$, or that:

$$\phi_1'(h^m) = -\phi_2'(h^m)$$

at the equilibrium, $h^m$. But, note that this is the defining equation for the optimal level of the externality, $h^{**}$. Thus if we can create the missing market, that market will implement the Pareto optimal level of the externality.

This result depends on the assumption of price taking, which is unreasonable in this case. But, in most real markets with externalities, this is not an unreasonable assumption, since (as in the case of air pollution), there are many producers and consumers. This is the basic approach that is used in the case of tradeable pollution permits. The government creates a market for the right to pollute, and, once the missing market has been created, the market will work in such a way that it implements the socially optimal level of the externality good.

## 8.3   Public Goods and Pure Public Goods

Previously we looked at a simple model of an externality where there were only two consumers. We can also think of externalities in situations where there are many consumers. In situations such as these, it is useful to think of the externality-producing activity as a public good. Public goods are goods that are consumed by more than one consumer. As we described earlier, public goods can take a number of forms. Basically, the most useful way to classify them (I have found) is based on whether the consumption of the good is rivalrous (i.e., whether consumption by one person affects consumption by another person) and whether consumption is excludable (i.e., whether a person can be prevented from consuming the public good). We begin our study with pure public goods. A **pure public good** is a non-rivalrous, non-excludable public good. Consumption of the good

by one person does not affect its consumption by others, and it is difficult (impossible) to exclude a person from consuming it.  The prototypical example is national defense.

Many goods are public goods, but are not pure public goods because their consumption is either rivalrous or excludable.  Consider, for example, public grazing land or an open-access fishery.  The more people who use this resource, the less benefit people get from using it.  Resources like this are **common-pool resources**.  We'll look at an example of a common-pool resource in Section 8.4.

A public good can also differ from a pure public good if its consumption is excludable.  For example, you can exclude people from using a bridge or a park.  Excludable, non-rivalrous public goods are called **club goods**.  Excludability will play an important role in whether you can get people to pay for a public good or not.  For example, how do you expect to get people to voluntarily pay for a pure public good like national defense when they cannot be excluded from consuming it?  If there is no threat of being excluded, people will be tempted to free ride off of the contributions of others.  On the other hand, in the case of a club good such as a park, the fact that people who do not contribute will be excluded from consuming the public good can be used to induce everybody, not just those who value the club good the most, to contribute.

Finally, not all public goods need to be "good."  You can also have a public bad:  pollution, poor quality roads, overgrazing on public land, etc.  However, it will frequently be possible to redefine a public bad, such as pollution, in terms of a public good, pollution abatement or clean air.  Thus the models we use will work equally well for public goods and public bads.

### 8.3.1   Pure Public Goods

Consider the following simple model of a pure public good.  As usual, there are $I$ consumers, and $L$ commodities.  Preferences are quasilinear with respect to some numeraire commodity, $w$.  Let $x$ denote the quantity of the public good.  In this case, indirect utility takes the form

$$v_i\left(p, x, w\right) = \bar{\phi}_i\left(p, x\right) + w.$$

As in the case of the bilateral externality, we will not be interested in prices, and so we will let $\phi_i\left(x\right) = \bar{\phi}_i\left(p, x\right)$.  Assume that $\phi_i$ is twice differentiable and concave at all $x \geq 0$.  In the case of a public good, $\phi_i' > 0$, in the case of a public bad, $\phi_i' < 0$.

Assume that the cost of supplying $q$ units of the public good is $c\left(q\right)$, where $c\left(q\right)$ is strictly increasing, convex, and twice differentiable.  In the case of a public good whose production is

costly, $\phi_i' > 0$ and $c' > 0$. In the case of a public bad whose prevention is costly (such as garbage on the front lawn or pollution), $\phi_i' < 0$ and $c' < 0$.

In this model, a Pareto optimal allocation must maximize the aggregate surplus and therefore must solve

$$\max_{q \geq 0} \sum_{i=1}^{I} \phi_i(q) - c(q).$$

This yields the necessary and sufficient first-order condition, where $q^0$ is the Pareto optimal quantity,

$$\sum_{i=1}^{I} \phi_i'\left(q^0\right) - c'\left(q^0\right) = 0$$

for an interior solution.[9] Thus the total marginal utility due to increasing the public good is equal to the marginal cost of increasing it.

### Private provision of a public good

Now suppose that the public good is provided by private purchases by consumers. That is, the public good is something like national defense, and we ask people to pay for it by saying to them, "Give us some money, and we'll use it to purchase national defense."[10] So, each consumer chooses how much of the public good $x_i$ to purchase. We treat the supply side as consisting of profit-maximizing firms with aggregate cost function $c()$.

At a competitive equilibrium:

1. Consumers maximize utility:[11]

$$\max \phi_i\left(x_i + \sum_{j \neq i} x_j^*\right) - px_i.$$

For an interior solution, the first-order condition is:

$$\phi_i'\left(x_i^* + x_{-i}^*\right) = p.$$

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[9] As usual, we assume an interior solution here, but generally you would want to look at the Kuhn-Tucker conditions and determine endogenously whether the solution is interior or not.

[10] To see how well this works, think about public television.

[11] In our treatment of the consumer's problem, we model the consumer as assuming all other consumers choose $x_j^*$. Consumer $i$ then chooses the level of $x_i$ that maximizes his utility, given the choices of the other consumers. While this seems somewhat strange at first, note that in equilibrium, consumer i's beliefs will be confirmed. The other consumers will really choose to purchase $x_j^*$ units of the public good. For those of you who know some game theory, what we're doing here is finding a Nash equilibrium.

2. Firms maximize profit:

$$\max pq - c(q).$$

For an interior solution, the first-order condition is

$$p = c'(q^*).$$

3. Market clearing: at a competitive equilibrium the price adjusts so that,

$$x^* = \sum_i x_i^* = q^*.$$

Putting conditions 1 and 3 together, we know that

$$\phi_i'(x^*) = c'(x^*)$$

for any $i$ that purchases a positive amount of the external good. Further, for all $i$ that do not purchase the good, $\phi_i(x^*) > 0$. Without loss of generality, suppose consumers 1 through $K$ do not contribute and consumers $K+1$ through $I$ do contribute. This implies that

$$\sum_i \phi_i'(x^*) = \sum_{i=1}^{K} \phi_i'(x^*) + \sum_{i=K+1}^{I} \phi_i'(x^*) \tag{8.1}$$

$$= \sum_{i=1}^{K} \phi_i'(x^*) + (I - (K+1))\, c'(x^*) > c'(x^*). \tag{8.2}$$

whenever a positive amount of the public good is provided.

Now, compare this with the condition defining the Pareto optimal quantity of the public good:

$$\sum_{i=1}^{I} \phi_i'(q^0) = c'(q^0).$$

Thus, when people make voluntary contributions, the market will provide too little of the public good: $q^0 > q^*$.

The fact that the market provides too little of the public good can be understood in terms of externalities. Purchase of one unit of the public good by one consumer provides an external benefit on all other consumers. More formally, provision of one unit of a public good by consumer $i$ involves a private cost of $p^*$, a private benefit, $\phi_i'(x^*)$, and a public benefit, $\sum_{j\neq i} \phi_j'(x^*)$. When purchasing units of the public good, individuals weigh the private benefit against the private cost. However, society as a whole is interested in weighing the total benefit against the cost (since $p^* = c'(q)$, the private cost is also the public cost). The fact that individuals do not consider the public benefit
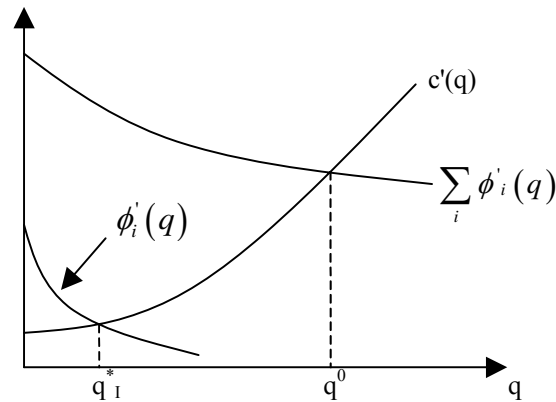
Figure 8.4: The Free-Rider Problem

results in underprovision of the public good. This is frequently called the **free-rider problem**, depicted in Figure 8.4.

For a striking example of the free-rider problem, consider the case where consumers' marginal utilities are increasing in their index:

$$\phi'_1(x) < \phi'_2(x) < ... < \phi'_I(x) \text{ for all } x.$$

In this case, condition

$$\phi'_i\left(x^*_i + x^*_{-i}\right) = p^*.$$

can hold for at most one consumer. Call the consumer who purchases the public good consumer $j^*$. All of the other consumers must choose $x^*_i = 0$. In the case where $x^*_i = 0$, the first-order condition is: $\phi'_i(x^*_{i*}) \leq p = \phi'_j\left(x^*_{j*}\right)$. This implies that $j^*$ must be consumer $I$, since $\phi'_I(x) > \phi'_i(x)$ for all $i \neq I$.

The previous example is a particularly stark example of the free-rider problem. The only person who pays for the public good is the person who values it the most (on the margin). All others contribute nothing toward the public good. Real-world examples of something like this include contributions to public television. For another example, think about whether you've ever shared an apartment with a person who either is much neater or much sloppier than you are. Who does all of the cleaning in this case?

### 8.3.2    Remedies for the Free-Rider Problem

As in the case of bilateral externalities, there are also a number of remedies for the free rider problem in public goods environments.   Some remedies include government intervention in the market for the public good.  For example, the government may mandate the amount of the public good that consumers must purchase.   The government may pass a law requiring inoculations or imposing limits on pollution.  For other public goods, such as roads and bridges or national defense, the government may simply take over provision of the public good, taking the decision out of the hands of individual consumers entirely.

The government may also engage in price-based interventions.   For example, the government could tax or subsidize the provision of public goods in such a way that private incentives are brought into line with public incentives.  Suppose there are $I$ consumers, each with benefit function $\phi_i(x)$. Using the Pigouvian taxation example from the bilateral externality case, we can implement the optimal consumption $x^0$ by setting the per unit subsidy to each consumer equal to

$$s_i = \sum_{j \neq i} \phi'_j\left(x^0\right).$$

This is because (assuming that the other consumers choose $x_j^0$) the consumer maximizes

$$\phi_i\left(x_i + x_{-i}^0\right) + s_i x_i - p x_i.$$

The necessary and sufficient first-order condition for this problem is:

$$\phi'_i\left(x_i + x_{-i}^0\right) + s_i = p.$$

Substituting in the above subsidy and combining with the market-clearing condition,

$$\phi'_i\left(x_i + x_{-i}^0\right) + \sum_{j \neq i} \phi'_j\left(x^0\right) = p^* = c'(x)$$

which is satisfied when $x = x^0$.   Thus the optimum is implemented.[12]

While the subsidies described above will implement the Pareto optimal level of the public good, it might be very expensive for the government to do so, since the subsidies can be quite large, and each person is paid the marginal value to all other consumers.

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[12] This is an example of the Groves-Clark mechanism.  The Groves-Clark mechanism is basically a class of mechanisms in which the external effect of each consumer's decision is added to the private effect, in order to bring individual preferences in line with social preferences.

**Lindahl Equilibrium**

As in the bilateral externality case, both the quantity- and price-based government interventions require the government to have detailed knowledge of the preferences of consumers and firms. We now present a market-based solution to the problem, known as the Lindahl equilibrium. The idea behind the Lindahl equilibrium is that the public good is unbundled into $I$ private goods, where each good is "person $i$'s enjoyment of the public good," each with its own price (known as the Lindahl price). The equilibrium in this market is known as the Lindahl equilibrium, and it turns out that the Lindahl equilibrium implements the Pareto optimal allocation of the public good.

Suppose that for each $i$, there is a market for "person $i$'s enjoyment of the public good." Denote the price of this personalized good as $p_i$. Given an equilibrium price, the consumer chooses the total amount of the good to maximize

$$\phi_i(x) - p_i x_i,$$

which has necessary and sufficient first-order condition

$$\phi_i'(x) = p_i \text{ for each } i. \tag{8.3}$$

Now, consider the producer side of the market. When the firm produces a single unit of the public good, it produces one unit of the personalized public good for every person. That is, one unit of national defense is a bundle of one unit of defense for person 1, one unit for person 2, etc. Hence for each unit of the public good that the firm produces and sells, it earns $\sum_i p_i$ dollars. Hence the firm's problem is written as

$$\max_q \left( \sum_i p_i q \right) - c(q),$$

which has first-order condition

$$\sum_i p_i = c'(q).$$

Combining this with the consumer's optimality condition, Equation 8.3, yields

$$\sum_i \phi_i'(q^{**}) = c'(q^{**}),$$

which is the defining equation for the efficient level of the public good. The corresponding prices are $p_i^{**} = \phi_i'(q^{**})$. Thus the Lindahl equilibrium results in the efficient level of the public good being provided.

The Lindahl equilibrium illustrates that the right kind of market can implement the Pareto optimal allocation, even in the public good case. However, Lindahl equilibrium may not be realistic. In particular, the Lindahl equilibrium depends on consumers behaving as price takers, even when they are the only buyers of a particular good. Still, it may be reasonable to think that the consumer has no power to force the producer of the public good to lower its price, especially if the producer is the government. Second, and more troubling, is the idea that in order for the Lindahl equilibrium to work, the consumer has to believe that if they do not purchase any of the public good, they will not be able to consume any of it. Of course, since one of the defining features of a public good is that it is non-excludable, it is unlikely that consumers will believe this.

### 8.3.3   Club Goods

However, while Lindahl equilibria may not be reasonable for pure (non-excludable) public goods, they are reasonable if the good is excludable, which we earlier called **club goods**. The Lindahl price $p_i^{**}$ can then be thought of as the price of a membership in the "club," i.e. the right for access to the club good. In this case, the market will result in efficient provision of the club good (although you still have to worry about the price-taker assumption).

## 8.4   Common-Pool Resources

A common-pool resource (CPR) is a good where consumption is rivalrous and non-excludable. Some of the prototypical examples of CPRs are local fishing grounds, common grazing land, or irrigation systems. In such situations, individuals will tend to overuse the CPR since they will choose the level of usage at which the individual marginal utility is zero, but the Pareto optimum is the level at which the total marginal benefit is zero, which is generally a lower level of consumption than the market equilibrium.

Consider the following example of a common-pool resource. The total number of fish caught in a non-excludable local fishery is given by $f(k)$, where $k$ is the total number of fishing boats that work the fishing ground. Assume that $f' > 0$ and $f'' < 0$, and that $f(0) = 0$. That is, we assume total fish production is an increasing concave function of the number of boats working the fishing ground. Also, note that as a consequence of the concavity of $f()$, $\frac{f(k)}{k} > f'(k)$. That is, the number of fish caught per boat is always larger than the marginal product of adding another boat. This follows from the observation that average product is decreasing for a concave production

function.  Let $AP(k) = \frac{f(k)}{k}$.  Then  $AP'(k) = \frac{1}{k}(f'(k) - AP(k)) < 0$.  Hence $AP(k) > f'(k)$.

Fishing boats are produced at a cost $c(k)$, where $k$ is the total number of boats, and $c()$ is a strictly increasing, strictly convex function.  The price of fish is normalized to 1.

The Pareto efficient number of boats is found by solving

$$\max_{k} f(k) - c(k),$$

which implies the first-order condition for the optimal number of boats $k^0$:

$$f'(k^0) = c'(k^0).$$

Let $k_i$ be the number of boats that fisher $i$ employs, and assume that there are $I$ total fishers. Hence $k = \sum_i k_i$.  If $p$ is the market-clearing price of a fishing boat, the boat producers solve

$$\max_{k} pk - c(k),$$

which has optimality condition

$$c'(k) = p.$$

Under the assumption that each fishing boat catches the same number of fish, each fisher solves the problem

$$\max_{k_i} \frac{k_i}{k_i + k_{-i}} f(k) - pk_i,$$

where $k_{-i} = \sum_{j \neq i} k_j$. The optimality condition for this problem is

$$f'(k^*) \frac{k_i^*}{k^*} + \frac{f(k^*)}{k^*}\left(\frac{k_{-i}^*}{k^*}\right) = p.$$

Market clearing then implies that:

$$f'(k^*) \frac{k_i^*}{k^*} + \frac{f(k^*)}{k^*}\left(\frac{k_{-i}^*}{k^*}\right) = c'(k^*).$$

Since all of our producers are identical, the optimum will involve $k_i^* = k_j^*$ for all $i, j$. That is, all fishers will choose the same number of boats.[13]   If there are $n$ total fishers, we can rewrite this condition as:

$$f'(k^*) \frac{1}{n} + \frac{f(k^*)}{k^*}\left(\frac{n-1}{n}\right) = c'(k^*).$$

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[13]In fact, since $f()$ is strictly concave, we could show this if we wanted to.

Thus the left-hand side is a convex combination of the marginal product, $f'(k)$, and the average product, $\frac{f(k)}{k}$. And, since we know that $\frac{f(k)}{k} > f'(k)$, $f'(k^*)\frac{1}{n} + \frac{f(k^*)}{k}\left(\frac{n-1}{n}\right) > f'(k^*)$. Finally, since $c'(k)$ is increasing in $k$, this implies that $k^* > k^0$. **The market overuses the fishery**.

What causes people to overuse the CPR? Consider once again the fisher's optimality condition:

$$f'(k^*)\frac{k_i^*}{k^*} + \frac{f(k^*)}{k^*}\left(\frac{k_{-i}^*}{k^*}\right) = p.$$

The terms on the left-hand side correspond to two different phenomena. First, if fisher $i$ buys another boat, that increases the total catch, and fisher $i$ gets $\frac{k_i^*}{k^*}$ of that increase. This is the term $f'(k^*)\frac{k_i^*}{k^*}$. Second, because fisher $i$ now has one more boat, he has a greater proportion of the total boats, and so he gains from the fact that a greater proportion of the total catch is given to him. This second effect, which corresponds to the term $\frac{f(k^*)}{k^*}\left(\frac{k_{-i}^*}{k^*}\right)$, can be thought of as a "market stealing" effect.

Note that both of these effects can lead the market to act inefficiently. Since the fisher only gets $\frac{k_i^*}{k^*}$ of the increase in the number of fish due to adding another boat, this will tend to make the market choose too few boats. However, since adding another boat also results in stealing part of the catch from the other fishers, and this is profitable, this tends to make fishers buy too many boats. When $f()$ is concave, we know that the latter effect dominates the former, and there will be too many boats.

The previous phenomenon, that the market will tend to overuse common-pool resources, is known as the **tragedy of the commons**. We won't have time to formally go through all of the solutions to this problem, but they include the same sorts of tools that we have already seen. For example, placing a quota on the number of boats each fisher can own or the number of fish that each boat can catch would help to solve the problem. In addition, putting a tax on boats or fish would also help to solve the problem. The appropriate boat tax would be

$$t^* = \frac{k_{-i}^*}{k^*}\left(\frac{f(k^*)}{k^*} - f'(k^*)\right).$$

In this case, the fisher's problem becomes:

$$\max_{k_i} \frac{k_i}{k_i + k_{-i}}f(k) - pk_i - t^*k_i.$$

The first-order condition is:

$$f'(k)\frac{k_i}{k} + \frac{f(k)}{k}\left(\frac{k_{-i}}{k}\right) - t^* = p.$$

Substituting in the value for $t^*$

$$f'(k)\frac{k_i}{k} + \frac{f(k)}{k}\left(\frac{k_{-i}}{k}\right) - \frac{k^*_{-i}}{k^*}\left(\frac{f(k^*)}{k^*} - f'(k^*)\right) = p.$$

When combined with the market-clearing condition, this becomes:

$$f'(k)\frac{k_i}{k} + \frac{f(k)}{k}\left(\frac{k_{-i}}{k}\right) - \frac{k^*_{-i}}{k^*}\left(\frac{f(k^*)}{k^*} - f'(k^*)\right) = c'(k),$$

which is solved at $k_i = \frac{k^*}{n}$ for all $i$.

   Privatization (or nationalization) is another solution. If the whole CPR is held by one person, then they will choose $f'(k) = p$, and (assuming price-taking) $f'(k^*) = c(k^*)$. The owner of the CPR and the consumers can then bargain over usage. However, privatization can have problems of its own, including managerial difficulties and adverse distributional consequences.

   In addition to the tools I've already mentioned, it is important to note that while the tragedy of the commons is a real problem, people have been solving it for hundreds of years, at least in part. Frequently, when the people who have access to a CPR (such as common grazing land, an irrigation system, or an open-access fishery) are part of a close community, such as the residents of a village, they find informal ways to cooperate with each other. Since the villagers know each other, they can find informal ways to punish people who overuse the resources.