CDMA Capacity Analysis

We have mentioned before that one of the reasons CDMA was pioneered in 2G cellular is due to its potential capacity improvements over FDMA/TDMA. In this lecture, we will build upon our analysis from last time to quantify the capacity of a CDMA system, and discuss which assumptions are necessary to make them true.

To begin, note that, given a particular modulation scheme, a certain SINR for the received signal $r(t)$ will be required for correct detection. We quantify this in terms of the received signal bit energy, E_b, and the power spectral density of the noise/interference, I_0. The SINR is E_b/I_0, for which 7 dB (≈ 5) is a typical requirement. Why this particular value? Without going into details, $E_b/I_0 = 5$ gives us a bit error probability at the receiver of 10^{-3} under common assumptions (see Schwartz Ch. 6.4).

Single Cell, No Fading

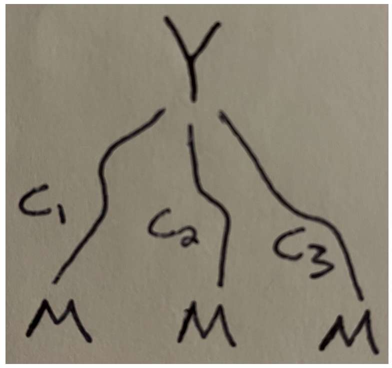
We will start with the simplest case of interference due to mobile users within a single cell only. We will make the following assumptions:

- There are K users in the cell, each with their own pseudorandom code.
- We will focus on the uplink of the cell, i.e., from mobile to BS, since this direction is the one which is more difficult to control.
- All users in the cell have power control, so that the same received power P_R is received at the BS.
- We assume the effects of noise and fading are negligible.

The received signal bit energy is given by

$$E_b = P_R \cdot T_s. \tag{1}$$

The interference power spectral density due to $K-1$ interfering users, ac-



counting for the spreading gain $W = T_s/T_c$, is given by

$$I_0 = \frac{P_R}{W} \cdot (K-1) \cdot T_s. \tag{2}$$

Dividing these two quantities, we have

$$\frac{E_b}{I_0} = \frac{P_R \cdot T_s}{\frac{P_R}{W} \cdot (K-1) \cdot T_s} = \frac{W}{K-1}. \tag{3}$$

The requirement on E_b/I_0 then limits the maximum number of users the cell can support:

$$K = \frac{W}{E_b/I_0} + 1. \tag{4}$$

Note that frequency reuse plays no role here, because all frequencies are used in cells: spread spectrum CDMA gives us universal reuse.

Example: In IS-95, the information bit rate is specified as $1/T_s = 9.6$ kbps, and the transmission bandwidth is $1/T_c = 1.2288$ Mbps over each 1.25 MHz band. This gives $W = T_s/T_c = 128$. With a requirement of $E_b/I_0 = 5$, from (4), we can support $K = 26$ users/cell for each 1.25 MHz. For a 25 MHz band, then, we have

$$K = \frac{25 \text{ MHz}}{1.25 \text{ MHz}} \cdot 26 = 520 \text{ users/cell}, \tag{5}$$

compared to 248 users/cell in the FDMA/TDMA GSM example for 4-reuse that we did previously.

These numbers look very promising in favor of CDMA. However, we have not taken into account the interference from other cells, or fading. As we will see, both of these factors can be extremely significant!

We should say a bit more about the importance of the assumption on perfect power control. Power control is critical to CDMA systems. To see why, consider the case without it, where all users transmit at the same power. Then, the received power P_i at the BS will be different, with close-in users have a larger received power than far-away users. Based on the resulting SINRs, i.e.,

$$\text{SINR for user } i = \frac{P_i}{\sum_{j \neq i} \frac{P_j}{W}}, \tag{6}$$

we see that close-in users will also have a higher SINR than far-away users, making the signal of far-away users difficult to decode. Thus, without power control, users close to the BS would have a built-in advantage in CDMA. We will discuss power-control techniques later on.

Multicell Case with Shadow Fading

We now generalize our model to the multi-cell case, with propagation effects. We will make the following assumptions:

- We will focus again on the uplink.
- We assume that power control is executed in each cell, such that the same average receiving power P_R is received from the mobiles in the cell.
- We assume that shadow fading takes place, and that the channel gain changes slowly due to shadow fading.

  Due to power control, the transmitted power of the mobile must be adjusted according to the fading gain, in order to maintain the same received power.

  We assume that power control is fast enough to compensate the changes in channel gain due to shadow fading.

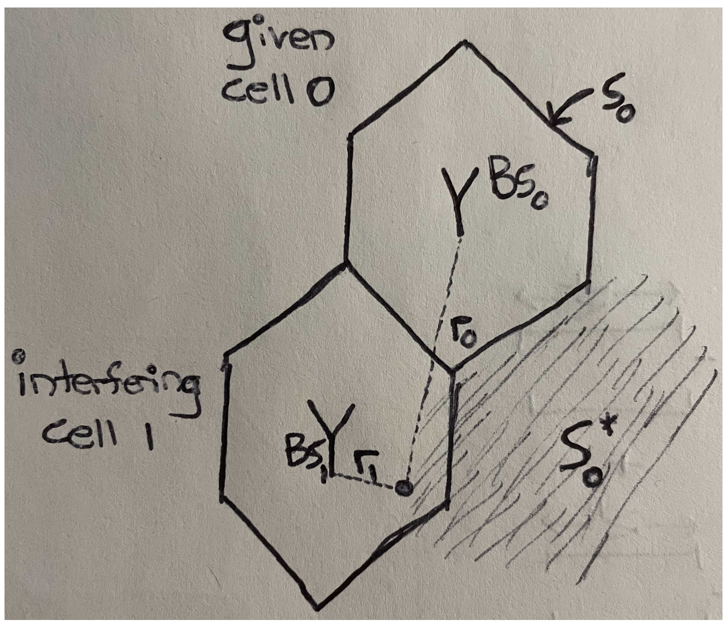
  We assume that the shadow fading and path-loss parameters are the same across cells:

  $$P_R = P_T \cdot r^{-n} \cdot 10^{z/10}, \quad z \sim \mathcal{N}(0, \sigma^2). \tag{7}$$

- To model the distance r, we assume mobiles are uniformly distributed throughout each cell. The average number of users in each cell is K.

- We focus initially on **hard-handoffs**, where a mobile communicates with the BS in its geometric cell only. This is in contrast to **soft-handoffs**, where a mobile may be in communication with two or more BSs, and then assigned to the one which the propagation loss is the least. CDMA allows soft-handoffs.
- We ignore multipath fading: We assume it is taken into account in the SINR threshold.

We need to derive the interference from other cells. Referring to cell 0 as our cell of interest, consider a given mobile in interfering cell 1 from the following diagram:



Let r_1 be the distance from the mobile to its own BS (in meters), and r_0 be its distance to the BS in our given cell 0. Let us calculate the interference caused by this mobile at the BS of cell 0.

At the interfering cell, since we assume perfect power control,

$$P_R = P_{T_1} \cdot r_1^{-n} \cdot 10^{z_1/10}. \tag{8}$$

Therefore, the transmission power of the mobile is

$$P_{T_1} = P_R \cdot r_1^{n} \cdot 10^{-z_1/10}. \tag{9}$$

At the BS of cell 0, the interference created by this mobile is then given by

$$P_{T_1} \cdot r_0^{-n} \cdot 10^{z_0/10} = P_R \cdot \left(\frac{r_1}{r_0}\right)^n 10^{(z_0 - z_1)/10}. \tag{10}$$

Recalling that the area of a cell with radius R is $(3\sqrt{3}/2)R^2$, then with K users/cell, the traffic density of the cell is

$$\rho = \frac{2K}{3\sqrt{3}R^2} \quad \frac{\text{users}}{\text{m}^2}, \tag{11}$$

which by assumption is uniform throughout the cell. Hence, the total average instantaneous power at the BS of cell 0 from mobiles *outside* S_0 (the region of cell 0) is given by integrating the interference density over $S_0^\star$ (the region outside cell 0):

$$I_{S_0^\star} = \frac{2K}{3\sqrt{3}R^2} P_R \cdot \mathbb{E}\left[\iint_{S_0^\star} \left(\left(\frac{r_1}{r_0}\right)^n \cdot 10^{(z_1 - z_0)/10}\right) d\mathrm{A}\right]. \tag{12}$$

Assuming that shadow fading is independent of the location, this expression breaks up into the product of two terms:

$$I_{S_0^\star} = \frac{2K}{3\sqrt{3}R^2} P_R \cdot \underbrace{\mathbb{E}\left[10^{(z_1 - z_0)/10}\right]}_{\text{shadow fading}} \cdot \underbrace{\iint_{S_0^\star} \left(\frac{r_1}{r_0}\right)^n d\mathrm{A}}_{\text{mobile location}}. \tag{13}$$

For the shadow fading term, when there is no fading, we see that $\mathbb{E}[10^{(z_1 - z_0)/10}] = 1$, as expected. Otherwise, when there is fading, $\mathbb{E}[10^{(z_1 - z_0)/10}] > 1$.

Effect of Shadow Fading

Referring to (13), we will first derive an expression for the shadow fading term. The two random variables $z_0, z_1 \sim \mathcal{N}(0, \sigma^2)$ represent power variations measured at the two base stations of cells 0 and 1, respectively. In general, z_1 and z_0 may be correlated because the base stations are close to each other. A common approach for modeling this is to express the random variables as:

$$z_i = ah + bh_i, \quad i = 0, 1, \quad a^2 + b^2 = 1, \tag{14}$$

where h, h_0, h_1 are Gaussian random variables also distributed as $\mathcal{N}(0, \sigma^2)$:

- h represents the fading term common to z_1 and z_0, in the vicinity of the transmitting mobile.
- The h_i's represent the independent fading terms, due to the (independent) propagation conditions encountered along the two paths to the base stations.

A common assumption is that $a^2 = b^2 = 1/2$, i.e., that half of the effect of shadow fading is due to the common region around the transmitting mobile, and the other half due to the independent terms received at the two base stations. Then, the exponent in (13) can be expressed as

$$y = z_1 - z_0 = b(h_1 - h_0), \tag{15}$$

which is a Gaussian random variable with variance $\sigma_y^2 = 2b^2\sigma^2$. Knowing σ_y^2, we can then calculate

$$\begin{aligned}
\mathbb{E}\left[10^{(z_1-z_0)/10}\right] &= \mathbb{E}\left[10^{y/10}\right] = \int_{-\infty}^{\infty} e^{y\cdot\frac{\ln 10}{10}} \cdot \frac{1}{\sqrt{2\pi\sigma_y^2}} \cdot e^{\frac{-y^2}{2\sigma_y^2}}\,dy & (16)\\
&= \frac{1}{\sqrt{2\pi}\sigma_y} \cdot \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma_y^2}(y-\sigma_y^2\cdot\frac{\ln 10}{10})^2} \cdot e^{\frac{1}{2}\sigma_y^2\cdot(\frac{\ln 10}{10})^2}\,dy & (17)\\
&= e^{\frac{1}{2}\sigma_y^2\cdot(\frac{\ln 10}{10})^2}, & (18)
\end{aligned}$$

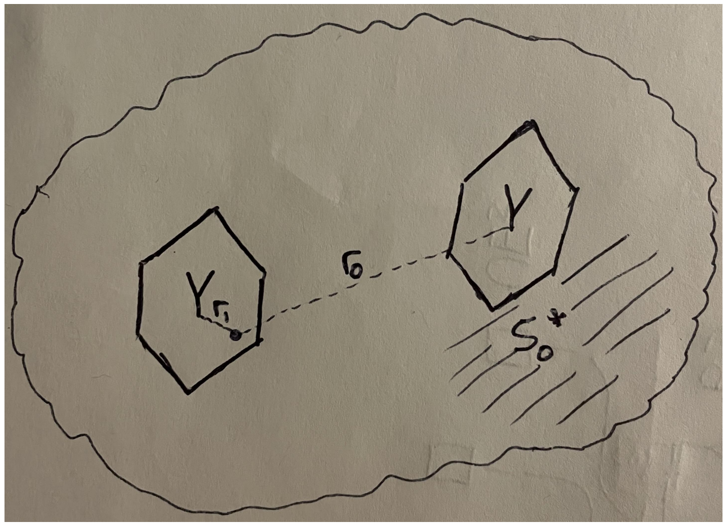
where the last step follows from $\int_{-\infty}^{\infty} f_X(x)dx = 1$ with $f_X(x)$ being the probability density function of random variable X. This is consistent with our previous interpretation from (13): $\mathbb{E}[10^{(z_1-z_0)/10}] = 1$ if $\sigma_y = 0$ (i.e., no shadow fading), and $\mathbb{E}[10^{(z_1-z_0)/10}] > 1$ in general.

Effect of Location Term

We next focus on the location term in (13). To get this, we must compute the integral over the region (visualized on the next page).

This integration can be done either numerically, or analytically through the circular-cell interferer model (see the Schwartz textbook p. 152 and 155). For a pathloss parameter $n = 4$, we get the following result from numerical integration:

$$\frac{2}{3\sqrt{3}R^2} \iint_{S_0^\star} \left(\frac{r_1}{r_0}\right)^n d\mathrm{A} = 0.44. \tag{19}$$



CDMA Capacity

We now combine our results to come up with an expression for CDMA capacity.

The shadow fading term is actually typically quite large. To see this, consider the common shadow-fading modeling case of $\sigma = 8$ dB. Then, $\sigma^2 = 64$. With $b^2 = 1/2$,

$$\mathbb{E}\left[10^{(z_1 - z_0)/10}\right] = e^{\frac{1}{2}\cdot 2\cdot \frac{1}{2}\cdot 64\cdot (\frac{\ln 10}{10})^2} = 5.42. \tag{20}$$

Shadow fading is thus expected to add significantly to the interference power, and has a large effect on the resulting system capacity. Using this value, and combining it with the value of 0.44 found in (19), we get an average outside cell interference power in (13) of

$$I_{S_0^\star} = P_R \cdot K \cdot 0.44 \cdot 5.42 = 2.38 P_R \cdot K. \tag{21}$$

This term represents the interference power from outside a cell, which must be added to the in-cell interference power $P_R(K-1)$ that we had used in (2). The SINR is thus

$$\frac{E_b}{I_0} = \frac{P_R}{\left(P_R(K-1) + 2.38 P_R \cdot K\right)/W} = \frac{W}{3.38K - 1}. \tag{22}$$

Clearly, the effect of the outside cell interference and shadow fading is quite strong! Compared with (3) for the single cell case, there is more than a 3x reduction in E_b/I_0. Based on (22), we can calculate the capacity as

$$K = \frac{\frac{W}{E_b/I_0} + 1}{3.38}. \tag{23}$$

Example: Returning to the IS-95 example from earlier, with a required $E_b/I_0 = 5$, we could support $K = 7.8$ users/cell for each 1.25 MHz. For the 25 MHz band, we can support

$$K = 7.8 \cdot \frac{25 \text{ MHz}}{1.25 \text{ MHz}} = 156 \text{ users/cell}, \tag{24}$$

compared with 248 users/cell in GSM.

How, then, can we claim that CDMA has an advantage over TDMA/FDMA? Several factors that have not been taken into account work into our favor here. For one, the GSM capacity calculation has not taken into account fading, either, which would lower its performance. Also, CDMA has several other capacity improving features that we will discuss shortly.

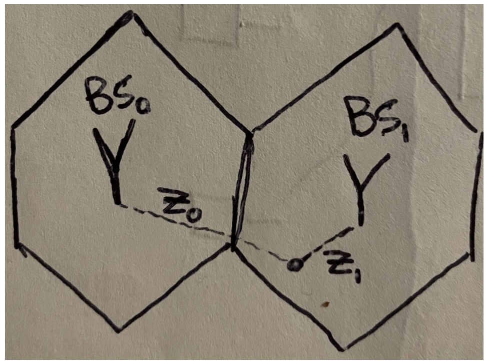
Note that $I_{S_0^\star}$ is the expected amount of interference from neighboring cells. A few points should be made here relating to CDMA vs. TDMA/FDMA:

- This interference is averaged over all node locations and shadow fading. The actual amount of interference can vary up or down.
- However, when K is large, a statistical multiplexing effect occurs, such that the instantaneous amount of interference will not be substantially different from the mean in CDMA.
- In GSM systems, interference comes from a small number of co-channel cells (i.e., we effectively have $K = 4$), so the variations from the mean tend to be larger.

Other Capacity Increasing Features

We will consider three capacity-increasing features of CDMA which help provide its advantage over TDMA/FDMA:

(1) Soft handoff. In our analysis based on hard handoff, shadow fading contributes to a significant increase in the amount of interference. Specifically, when $\sigma = 0$ dB, $\mathbb{E}[10^{(z_1 - z_0)/10}] = 1$, while when $\sigma = 8$ dB, we saw that $\mathbb{E}[10^{(z_1 - z_0)/10}] = 5.42$.



Why does shadow fading tend to increase interference?

Interference will increase when the fading coefficient z_0 to BS_0 is strong, and the fading coefficient z_1 to BS_1 is small (smaller meaning more fading). The mobile thus uses a large power to communicate with BS_1, and creates large interference at BS_0.

Instead, the mobile may very well communicate with BS_0 rather than BS_1. Soft handoff allows a mobile to measure signal strength from two or more BSs, and pick the one with the strongest signal to communicate with.

Two-cell handoff reduces the outside-cell interference from $2.38P_R \cdot K$ to $0.77P_R \cdot K$ in theory:

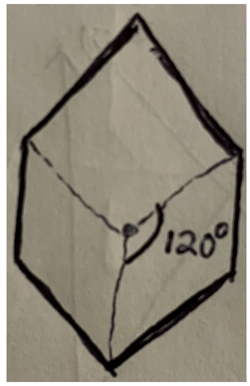
- This increases the capacity from 1/3.38 to 1/1.77, which is a factor of 1.90.
- In practice, due to measurement errors, a factor of 1.25 is typically achieved.

(2) Silence detection. In normal speech, users tend to alternate between silence and talk spurs. On average, speech is active about 40% of the time:

- This reduces both in-cell and outside-cell interference to 40% of their intensity, without the use of any additional equipment.
- As a result, the capacity will be increased by a factor of 2.5.

(3) 120° sectional antennas. Finally, assume that 120° sector antennas are used at the base station:

- This reduces both in-cell and outside-cell interference to 1/3, since interference is only experienced from mobiles in each sector.



- The system capacity is increased by a factor of 3 as a result.

In reality, power control, silence detection, and sectional antennas are not perfect solutions. After accounting for potential losses, the number of users per cell for IS-95 in each 1.25 MHz band is found to be 84 if two-cell handoff is used, or 96 if three-cell handoff is used.

(Again, these numbers are obtained with $\sigma = 8$ dB, $b^2 = 1/2, n = 4, E_b/I_0 =$ 7 dB.)

For the 25 MHz band, the number of users per cell is then

$$84 \times 20 = 1680 \text{ users/cell}, \tag{25}$$

compared with 248 users/cell in GSM.

So, the main capacity improvement is not due to CDMA itself, but rather due to silence detection, sectional antennas, and soft handoffs. CDMA allows us to deal with these features easily:

- Its capacity is limited by interference.
- It exploits statistical multiplexing.

This helps us understand why CDMA became the preferred multiple access technique for voice systems.

Can we use similar capacity-improving features in GSM systems? This is not always easy to do:

- Sectional antennas: Yes
- Power control: Yes

- Silence detection: More challenging. Requires e.g., putting 3 calls into 2 channels for oversubscription.
- Soft handoff: This is challenging as it would require a mobile device holding multiple frequency channels at once.

Summary

CDMA is a perfect example of physical-layer advances integrating with network-layer advances to attain significant throughput. When studying this subject, it is hard to stop wondering how the researchers realized the potential of CDMA, especially when the physical layer attribute itself does not directly contribute to the capacity increase.