Cellular Capacity and SINR

Recall that the cellular concept significantly increases the capacity of wireless networks, since channels are *reused* across multiple cells. These channels can be frequency bands, timeslots, or codes. We will now turn to a more formal treatment of the cellular concept.

In particular, we would like to understand several questions:

1. How far apart do cells with the same channels need to be? This is determined by inter-cell interference, i.e., co-channel interference.
2. How big is the area of the cell? This is related to the traffic density.
3. How should we allocate channels to cells? Both fixed and dynamic channel allocations are possible.

The approaches we will discuss now are more geared towards voice systems. We will also point out their limitations for data services.

1D System Model

To understand the implications of channel reuse, it is informative to start with a one-dimensional cellular system (e.g., a deployment across a highway). With **C-reuse**, cells with the same channel allocations are separated by $C-1$ other cells. Thus, 4-reuse and 3-reuse have the following patterns:

4-reuse

1	2	3	4	1	2	3	4	1	2	•••

3-reuse

1	2	3	1	2	3	1	2	3	1	•••

Recall that AMPS has a total of 832 channels (30 kHz each across 25 MHz). Assuming we have N cells, then at 4-reuse, the total capacity is

$$\frac{832}{4} \cdot N = 208N.$$

On the other hand, at 3-reuse, the total capacity is

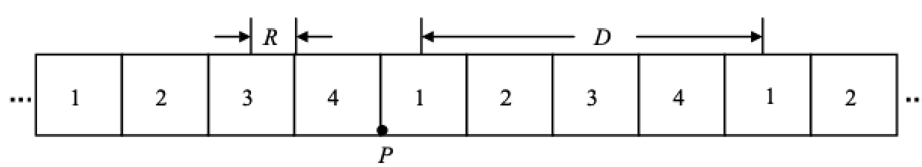
$$\frac{832}{3} \cdot N = 277N.$$

Which one should we choose? Based on capacity alone, a smaller reuse seems more attractive. However, we must also consider the required **signal to interference and noise ratio** (**SINR**). The SINR is the ratio of the desired signal power at the receiver to the total interference and noise power.

Typical modulation schemes need an SINR above a certain threshold to achieve an acceptable bit error probability. In AMPS, the threshold was 18 dB, while in GSM, 7-12 dB became feasible, due to improvements in the underlying communication scheme.

In modeling the SINR, we will make a number of simplifications:

1. We will ignore background noise and fading, i.e., focusing on average power. In this case, the SINR can be determined by the **reuse distance**. The reuse distance is denoted D in the diagram below, and refers to the spacing between interfering cells. The **radius** of each cell is denoted R.
2. We will focus on the downlink, and assume the base stations are at the centers of each cell. We assume that all base stations transmit at the same power P_T.
3. We will consider the worst case receiver at the edge of any given cell, i.e., mobile P in the diagram below.



Based on these assumptions, the received signal strength at mobile P is:

$$\frac{P_T}{R^n} \tag{1}$$

The interference from 1st-tier interferers, i.e., from the closest cells with channel assignments 1, is:

$$\frac{P_T}{(D-R)^n} + \frac{P_T}{(D+R)^n} \tag{2}$$

The interference from 2nd-tier interferers would then be:

$$\frac{P_T}{(2D-R)^n} + \frac{P_T}{(2D+R)^n} \tag{3}$$

In practice, only the 1st-tier interferers need to be considered (especially when n is large). Dividing the received signal strength by the interference, we thus get

$$\text{SINR} = \frac{R^{-n}}{(D-R)^{-n} + (D+R)^{-n}} = \frac{1}{(\frac{D}{R}-1)^{-n} + (\frac{D}{R}+1)^{-n}} \tag{4}$$

Note that for 3-cell reuse, $D = 6R$, and for 4-cell reuse, $D = 8R$. The results of the SINR calculation (in dB) appear in the table below.

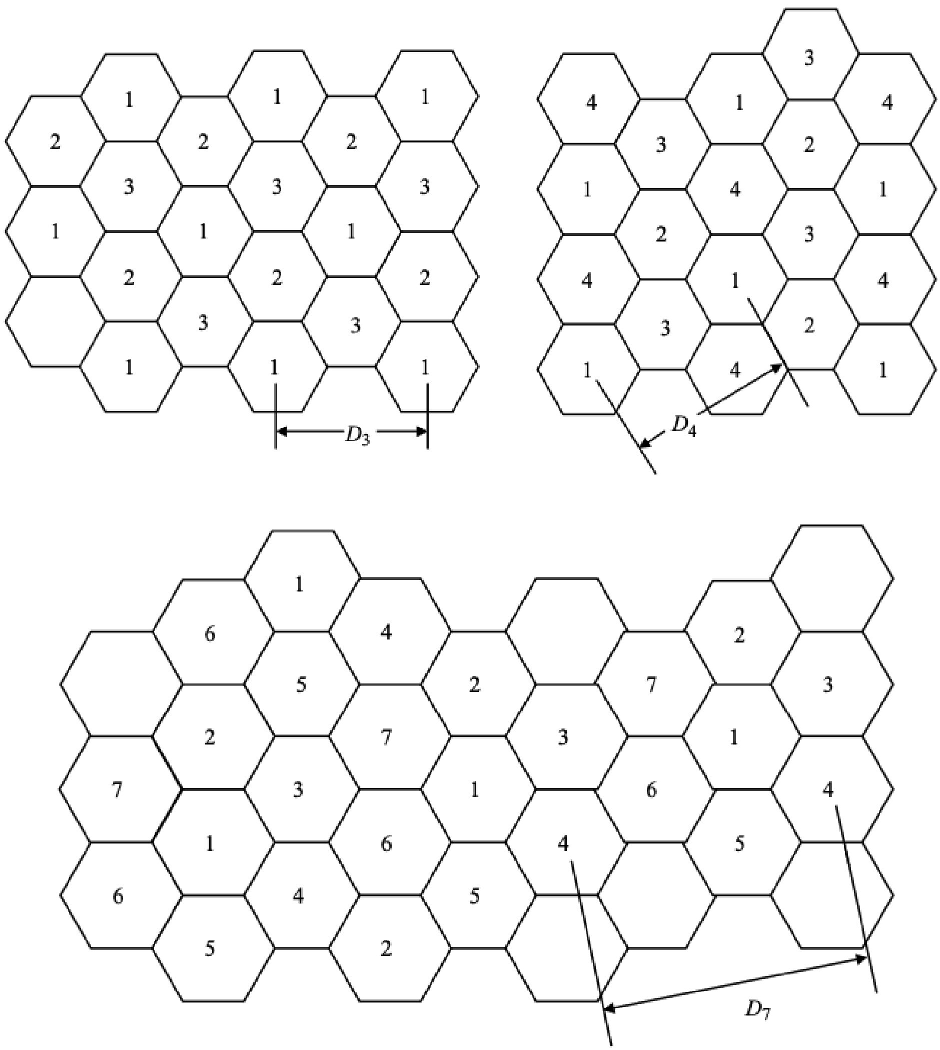
Reuse factor	SIR(dB)	
	$n = 3$	$n = 4$
3 cells	19.6	27
4 cells	23	32

As expected, as the distance variation parameter n increases, the interference decreases and the SIR increases. As the reuse factor increases, the interference decreases as well, which is the inverse of what happens to the capacity metric. Therefore, it is important to consider both capacity and SINR in cellular design.

2D System Model

More generally, we must consider two-dimensional cellular systems spanning a given geographic region. To come up with our model "building block," we need to ask: what equilateral geometric shapes can tesselate an infinite 2-dimensional space? The square, triangle, and hexagon can each do this. The hexagon is the closest to a circle, and has fairly simple geometric properties that enable SINR calculations to be readily carried out.

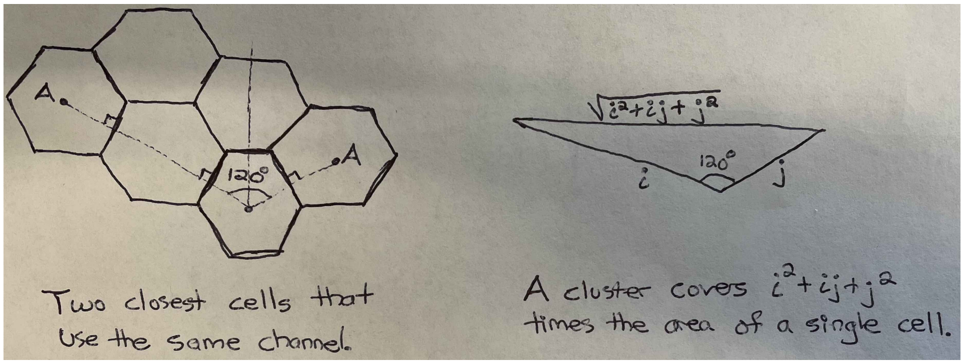
We focus therefore on a 2-dimensional structure with hexagonally shaped cells. The generalization to 3, 4, and 7-reuse patterns in 2D are given below:



Note that in 2D, we now must consider adjacent cells in each direction for making channel assignments. With a reuse factor of C, we see that every cluster of C cells repeats itself. We have labeled the reuse distance in each diagram, D_C, as the spacing between closest interfering cells. More generally, for uniform hexagon cells, it can be shown that a reuse factor C is feasible if and only if it satisfies

$$C = i^2 + ij + j^2, \qquad \text{where } i, j \text{ are nonnegative integers.}$$

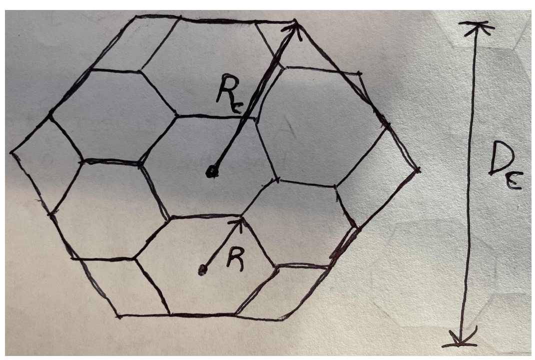
You will demonstrate this in a homework problem. The key idea is to relate the distance between the centers of two cells with the same channel allocations using the geometry of hexagons, as shown in the diagrams below:



Further, the reuse distance D is related to the reuse factor C by

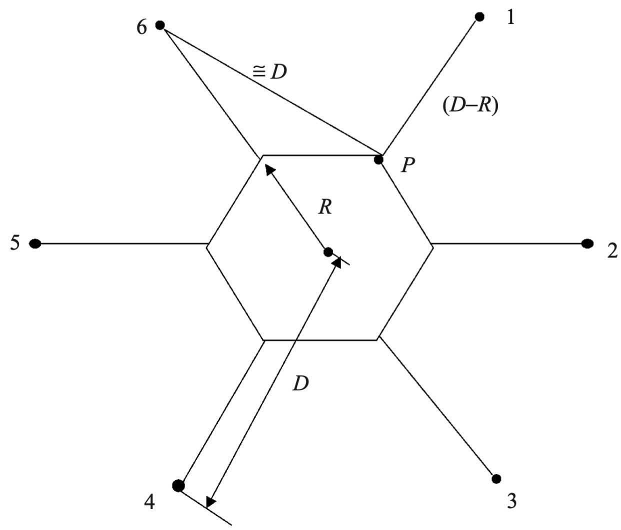
$$D = \sqrt{3C} \cdot R. \tag{5}$$

To see this, consider a cluster of seven cells, and a larger hexagon that circumscribes this cluster, as shown in the diagram below:



The height of the larger hexagon is also the reuse distance, D_C. Denoting the radius of the larger hexagon as R_C, we have $R_C = D_C/\sqrt{3}$, and its area is $A_c = (3\sqrt{3}/2)R_C^2 = (\sqrt{3}/2)D_C^2$, by hexagon geometry. Similarly, any of the smaller hexagons with radius R have area $a = (3\sqrt{3}/2)R^2$. Since the number of cells within a cluster is the ratio of the cluster area to the cell area, we have $C = A_C/a = D_C^2/3R^2$, from which we arrive at (5).

To calculate the two-dimensional SINR, note that any given cell has six first-tier interferers located about it. These are drawn in the figure below:



The distance from P to its own base station is R. Its distance to the closest interferer, the base station at 1, is then approximately $D - R$. The furthest away interferer, at 4, has a distance of $D + R$. The other four are varying distances away (by symmetry, the distances from P to 2 and 6 are the same, as are from P to 3 and 5). With $C \geq 3$, a good approximation for these distances is D. Thus, the first-tier interference power is

$$\approx \frac{P_T}{(D-R)^n} + \frac{P_T}{(D+R)^n} + 4 \cdot \frac{P_T}{D^n}, \tag{6}$$

which leads to an SIR of

$$\text{SIR} \approx \frac{1}{\left(\frac{D}{R} - 1\right)^{-n} + \left(\frac{D}{R} + 1\right)^{-n} + 4\left(\frac{D}{R}\right)^{-n}}. \tag{7}$$

The SINR calculations from (6) for a few values of C are given below:

C	D/R	SIR		SIR_{dB}	
		$n=3$	$n=4$	$n=3$	$n=4$
3	3	3.4	8.5	5.3	9.3
7	4.58	14.3	62.5	11.6	18
12	6	30.3	167	14.8	22.2

Various other approximations have been proposed in the literature as well. The relative difference between the approximations decreases as the reuse factor C becomes larger.

Implications and Assumptions

What can we learn from these results? Consider 1G AMPS, which required an SINR threshold of 18 dB. From the two-dimensional SINR table, we see that this would require 7-reuse at $n = 4$. In practice, we use $C = 7$ plus three antennas per cell, each covering a 120 degree sector, in order to attain the 18 dB performance level. This sectorization effectively reduces the number of first-tier interferers by three, as each sector only will have two. Next consider 2G GSM, which lowered the required SINR threshold to 7 dB. This would only need 3-reuse at $n = 4$.

Additionally, let's re-examine the assumptions we made at the start:

1. We assumed the noise is 0. If there is noise, then the SINR would naturally decrease.
2. We assumed there is no fading. When there is fading, only part of the cell (and/or for part of the time) will receive acceptable service. The probability or fraction of time with acceptable service can be calculated from the fading distribution, e.g., by finding the probability of the measured SIR being above a threshold. More details are given in the Schwartz book, Ch. 3.5.

   Alternatively, we may consider designing the reuse pattern according to the worst-case fading. However, this may be too pessimistic of an assumption, especially since we aim to exploit diversity gains.
3. We assumed the base stations transmit with equal power. Is it a good idea to keep all power the same? Alternatively, we can use power control to equalize the received signal powers.
4. We focused on the downlink. What about the uplink? The calculations are actually rather similar, as we will see when we study CDMA.
5. We are focusing on voice traffic, vs. data traffic. Data services will be able to take advantage of varying SINRs and diversity.