# Path Loss in Wireless Channels

Radio propagation conditions play a critical role in the operation of mobile wireless systems. They determine the performance of these systems, whether they are being used to transmit real-time voice messages, data, or other types of communication traffic. Thus, channel characterization is important as it determines the quality of the signal as well as the level of interference.

Unlike wireline networks where there is a clear notion of a link, in wireless networks it is a bunch of stations (transmitters and receivers) sharing the radio frequency, from which we have to define "links."

### Signal Propagation Factors

We need to know how signals propagate, and how interference accumulates in order to:

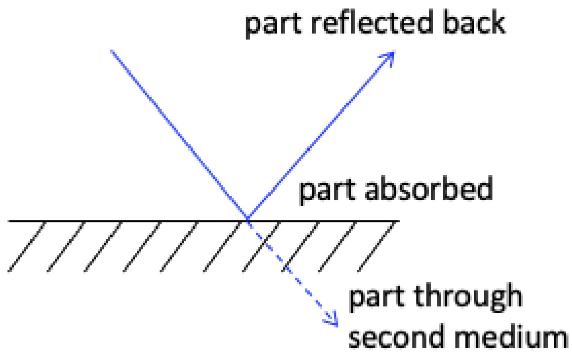
- ensure good coverage,
- plan the size and location of cells,
- determine the spectrum reuse pattern, and
- choose the right modulation/coding/multi-access schemes.

Radio transmissions over wireless channels are in general very difficult to characterize. There are a few complex factors that affect it.

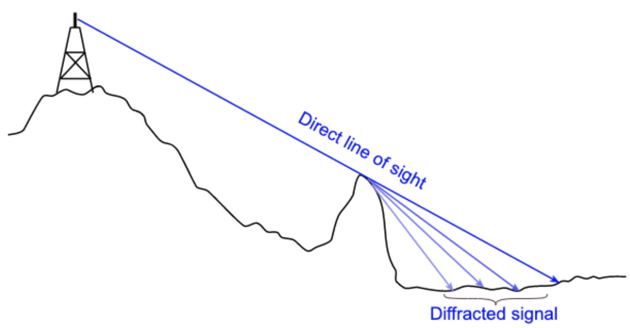
**(1) Obstacles:** An electromagnetic signal will encounter obstacles during the transmission, causing (i) reflection, (ii) diffraction, and (iii) scattering.

- **Reflection:** When a plane wave is incident on a dielectric, part of the wave is reflected back to the first medium, part of the wave is transmitted through the second medium, and part of the wave is absorbed, as illustrated below.

A *perfect dielectric* surface is one which has no absorption. A *perfect conductor* is one which will only have reflection (i.e., no electromagnetic energy can pass through).

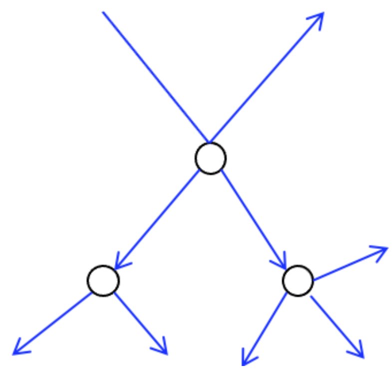



- **Diffraction:** Diffraction occurs when the radio path between the transmitter and receiver is obstructed by a surface that has sharp irregularities (edges), as illustrated below. The secondary waves resulting from the obstructing surface are present throughout the space and even behind the obstacle.




  Diffraction becomes less of an effect for small wavelengths (more like light), i.e., as we consider higher frequency signals (like mmWave).

- **Scattering:** Scattering occurs when the medium through which the EM wave travels consists of objects that are small compared to the wavelength, and where the number of obstacles per unit volume is large (e.g., a rough surface, leaves, street signs, light posts, winter snow). The result is that the EM wave travels in all directions, as depicted below.



**(2) Mobility:** Simply put, as transmitting and receiving terminals move, the conditions of reception at either end change.

**(3) Multipath:** Different resulting paths of the EM signals can enhance or cancel each other, depending on their amplitude/phase. **Fading** refers to the fact that the received signal strength can fluctuate significantly over time and space.

### Modeling Approaches

Given these complexities, how do we model the radio channel and propagation? In general, there are three approaches we could use:

**(1) EM modeling:** We could turn to Maxwell's equations for electromagnetism. While this is a deterministic approach, it is too complex to use in practice at scale, and suffers from a lack of insights.

**(2) Ray-tracing**, using the geometry of the paths. This can also become rather complex in practice.

**(3) Probabilistic models:** This class of approaches is much simpler, and gives good rules of thumb. As an analogy, consider flipping a coin. There are many physical factors that can affect exactly how the coin will land. Thus, instead of using a complex deterministic model based on mechanics, we use an elegant probabilistic model that the coin will land on its heads side with probability $p$ (with i.i.d. assumptions, etc.).

Using probabilistic models, we want to capture (at least) three components that affect the signal once it is transmitted:

- Path loss, i.e., attenuation of the signal.
- Shadow fading, i.e., large-scale fading, which is often modeled as log-normal fading.
- Multi-path fading, i.e., small-scale fading.

**Path Loss**

Path loss refers to attenuation of the EM signal due to the distance it has traveled. We will study two models: (1) the free-space model, where there are no obstacles, and (2) the two-ray model. In both cases, we will see the following underlying relationship for the loss:

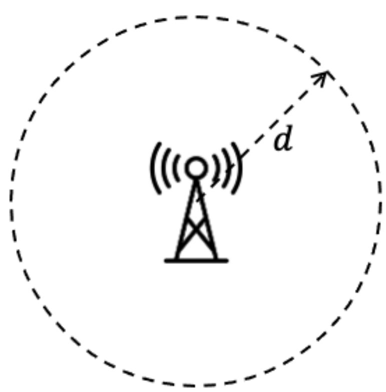
$$L(d) = K/d^n, \tag{1}$$

where $K$ is a constant, and $d$ is the distance from the transmitter.

With the free-space model, $n = 2$, and in the 2-ray model, $n = 4$. In general, $2 < n < 4$, i.e., the cases we study give the lower and upper bounds.

**Free Space Model**

First, assume that power is emitted from the transmitting antenna **isotropically** (i.e., it is an **omni-directional antenna**). Let $P_T$ be the transmission power in Watts (W). From the conservation of power, the receiver power density at distance $d$ meters (m) is $P_T/(4\pi d^2)$ W/m$^2$. We get this denominator by finding the area of the ball, drawn in 2D below:



The received power $P_R$ is then given by

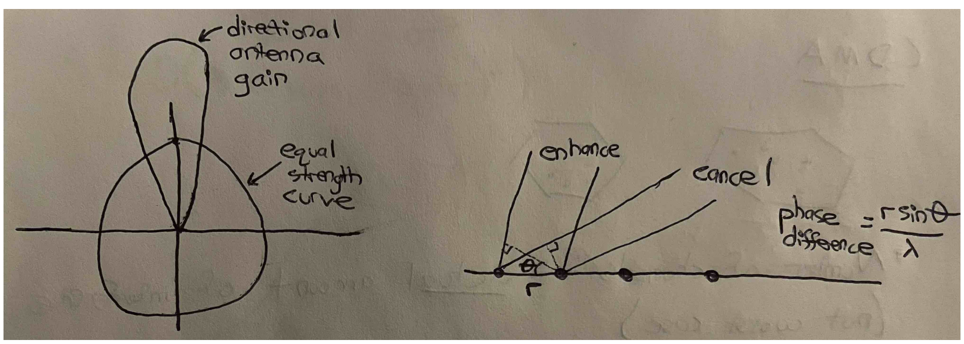
$$P_R(d) = \frac{P_T}{4\pi d^2} \cdot A_R \cdot \eta_R. \tag{2}$$

Here, $0 < \eta_R < 1$ is the efficiency parameter of the receiving antenna, which encapsulates several factors including the transmission line attenuation, filter loss, antenna loss, etc. $A_R$ is the **aperture** (or effective area) of the receiving antenna, in m$^2$. Based on the effective area of an isotropic antenna, we can further express the received power as follows:

$$A_R = \frac{\lambda^2}{4\pi}, \qquad P_R(d) = P_T \cdot \eta_R \cdot \left(\frac{\lambda}{4\pi d}\right)^2, \tag{3}$$

where $\lambda = c/f$ is the wavelength, $f$ is the frequency, and $c \approx 3 \times 10^8$ m/s is the speed of light. Note that $P_R$ decreases as $\lambda$ decreases (or as $f$ increases).

**Directional antennas** can provide a gain factor over omnidirectional antennas by focusing the energy in one direction. A directional antenna can be built from **antenna arrays**, where the phase difference among the received signals of each antenna element is used to enhance the signal in one direction and cancel it in others. This is shown at a high level in the figure below.




Both the transmitter and the receiver can have such types of gains. At the transmitter side, we define $G_T$ as the gain factor of the transmitting antenna. $G_T$ is proportional to the effective radiating area $A_T$ of the transmitting antenna (i.e., the antenna size in wavelengths). Specifically,

$$G_T = \frac{4\pi \cdot \eta_T \cdot A_T}{\lambda^2}, \tag{4}$$

where $\eta_T$ is the efficiency factor for the transmitting antenna. In the isotropic case, we simply have $\frac{4\pi \cdot A_T}{\lambda^2} = 1$. The received power is then

$$P_R(d) = \frac{P_T}{4\pi d^2} \cdot G_T \cdot A_R \cdot \eta_R. \tag{5}$$

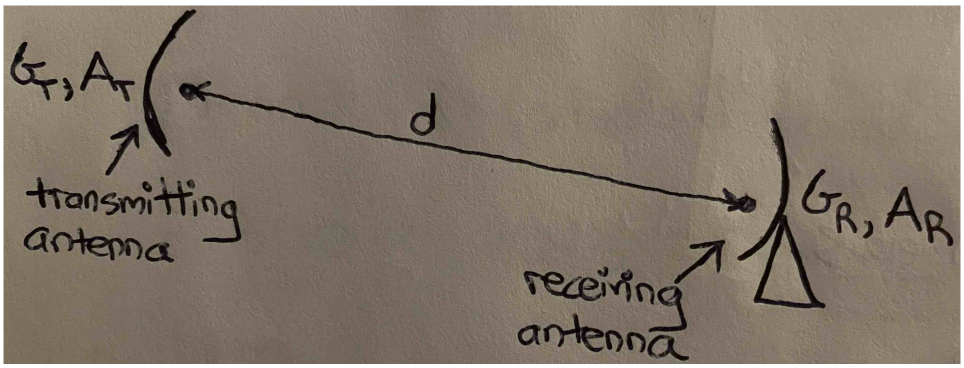
The effective area $A_R$ of the receiver obeys a similar relationship to $A_T$. If we define the antenna gain at the receiver end as

$$G_R = \frac{4\pi \cdot \eta_R \cdot A_R}{\lambda^2}, \tag{6}$$

then we get the well-known **Friis free-space equation**:

$$P_R(d) = P_T \cdot G_T \cdot G_R \cdot \left(\frac{\lambda}{4\pi d}\right)^2. \tag{7}$$

This equation captures the free-space loss in the diagram below.




Note that the Friis equation is only valid for the **far field** (or the **Fraunhofer region**), where electromagnetic radiation dominates. The far field distance is

$$d_F = \frac{2D^2}{\lambda} \tag{8}$$

where $D$ is the longest physical linear dimension of the antenna.

Sometimes, it is convenient to write $P_R(d)$ as

$$P_R(d) = P_R(d_0) \cdot \left(\frac{d_0}{d}\right)^2, \tag{9}$$

where $d_0$ is a reference distance that we have measured the power at. In decibels (dB), taking $10\log_{10}(P_R(d))$, we see that the power decreases by 20 dB as the distance is increased by 10 m.

### Two-Ray Model

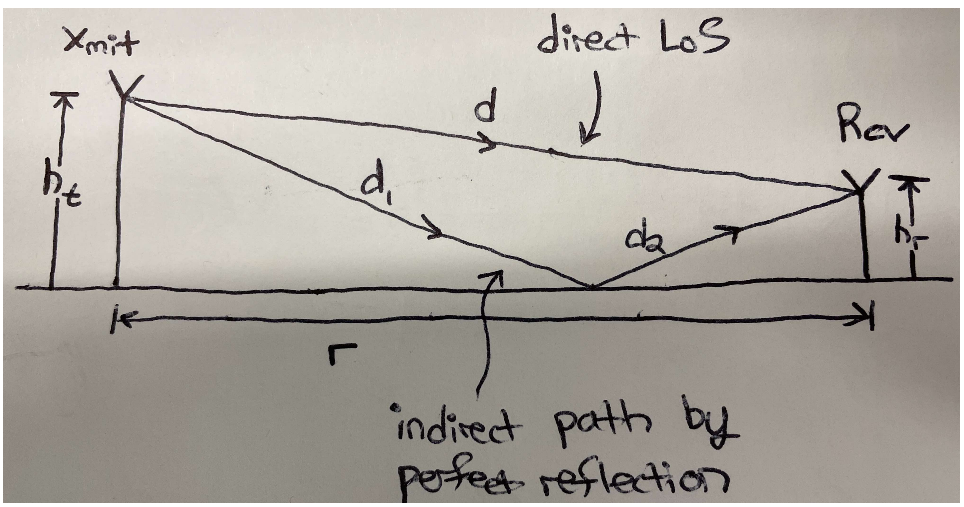
In the free-space model, there is only one path between the sender and receiver. This is often not the case for terrestrial communication, where as discussed there can be several obstacles.

The two-ray propagation model is the simplest one that can be adopted in demonstrating the effect of the average received power of multiple rays due to reflection, diffraction, and scattering. It treats the case of a single reflected ray. We can think of this as the ground, which becomes a natural reflector in terrestrial communications. Despite its relative simplicity, it has been found to provide reasonably accurate results in macrocellular systems.

In the 2-ray model, the transmitted EM wave reaches the receiver through two paths:

- the direct line of sight path, and
- indirectly by perfect reflection from a flat ground surface.

The geometry of the setup is depicted below:




The transmitting antenna (e.g., located at the base station), is radiating from a height $h_t$ m above a perfectly reflecting, flat ground surface. The receiving antenna, a free-space distance $d$ m away, is situated at a height $h_r$ m above the ground. The indirect ray travels a distance $d_1$ before being reflected at the ground surface. The reflected ray then travels a distance $d_2$ further before reaching the receiving antenna. The parameter $r$ represents the ground distance between transmitting and receiving antennas.

We will show that $P_R(d) \propto 1/d^4$.

Say the electric field is a sinusoidal wave at frequency $f_c$ with amplitude $E_T$. In complex notation, we write it as $E_T e^{j\omega_c t}$. The transmitted power $P_T$ is proportional to the magnitude squared of the electric field and is thus just $KE_T^2$. Then, the direct-path received signal is

$$\frac{E_T}{d} e^{j\omega_c (t - \frac{d}{c})}. \tag{10}$$

Since perfect reflection leads to a phase shift of $\pi$, and $e^{j\pi} = -1$, The indirect-path received signal is

$$-\frac{E_T}{d_1 + d_2} e^{j\omega_c (t - \frac{d_1+d_2}{c})}. \tag{11}$$

The sum of these two gives the total received signal:

$$\frac{E_T}{d} e^{j\omega_c (t - \frac{d}{c})} \left[ 1 - \frac{d}{d_1 + d_2} e^{-j\omega_c \left( \frac{d_1 + d_2 - d}{c} \right)} \right]. \tag{12}$$

Thus, the total received power is

$$\frac{E_T^2}{d^2} \cdot \left| 1 - \frac{d}{d_1 + d_2} e^{-j\omega_c \left( \frac{d_1 + d_2 - d}{c} \right)} \right|^2. \tag{13}$$

Here, the first term comes from the free-space loss. For the directional case, we can use (7) to write

$$\frac{E_T^2}{d^2} = P_T G_T G_R \left( \frac{\lambda}{4\pi d} \right)^2. \tag{14}$$

The second term in (13) comes from the fact that we have two rays. We will show that when $h_t, h_r \ll d$,

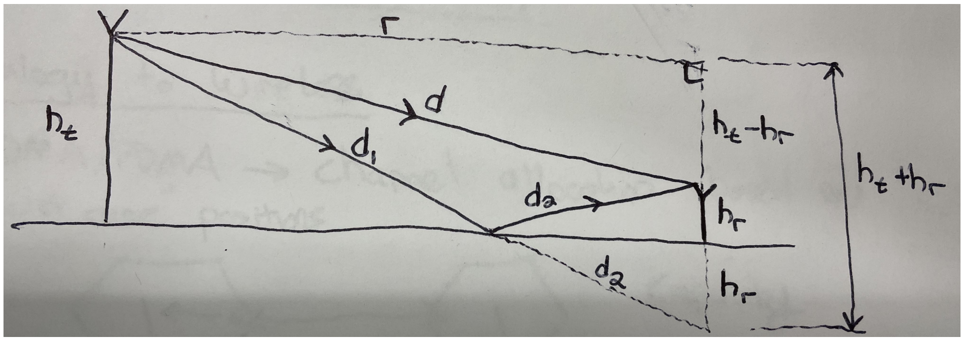
$$\left| 1 - \frac{d}{d_1 + d_2} e^{-j\omega_c \left( \frac{d_1 + d_2 - d}{c} \right)} \right|^2 \approx \left( \frac{4\pi h_t h_r}{\lambda d} \right)^2. \tag{15}$$

We can extend the geometry of the figure, as shown on the top of the next page. We see that:

$$d = \sqrt{r^2 + (h_t - h_r)^2}, \qquad d_1 + d_2 = \sqrt{r^2 + (h_t + h_r)^2} \tag{16}$$

Substituting for $r$, we have

$$d_1 + d_2 = \sqrt{d^2 + 4h_t h_r} = d \cdot \sqrt{1 + \frac{4h_t h_r}{d^2}} \tag{17}$$




Based on the Taylor series expansion $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \cdots$, if we assume that $4h_t h_r \ll d^2$, then $x \ll 1$ and we can approximate the expansion with the first term:

$$d_1 + d_2 \approx d + \frac{2h_t h_r}{d} \tag{18}$$

We will return to the validity of this assumption later.

Next, we consider the phase difference $\Delta\phi = \omega_c(\frac{d_1+d_2-d}{d})$ in (13). Based on (18), we can write $\Delta d = d_1 + d_2 - d \approx \frac{2h_r h_t}{d}$. Thus, since $c = \lambda(\omega_c/2\pi)$,

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot \frac{2h_t h_r}{d} = \frac{4\pi h_t h_r}{\lambda d}. \tag{19}$$

We will now make a second assumption, that $\Delta\phi \ll 1$, i.e., the phase difference is small. This implies that $d \gg h_t h_r/\lambda$, i.e., we are considering a long LoS distance. Again, we will return to its validity shortly. For a small $\Delta\phi$, we know based on trigonometric properties that $e^{-j\Delta\phi} = \cos(\Delta\phi) - j\sin(\Delta\phi) \approx 1 - j\Delta\phi$. Further, based on (17), we have $d/(d_1 + d_2) \approx 1$ for large $d$. It follows that

$$\left|1 - \frac{d}{d_1 + d_2} e^{-j\Delta\phi}\right|^2 \approx |\Delta\phi|^2. \tag{20}$$

This proves (15). Combining with (13), we thus have

$$P_R(d) = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 \left(\frac{4\pi h_t h_r}{\lambda d}\right)^2 = P_T G_T G_R \cdot \frac{(h_t h_r)^2}{d^4}. \tag{21}$$

The key point here is that the phase difference determines the combined effect of the two rays. We will see this again in our analysis of multipath fading.

**Validity of Assumptions**

In the two-ray model analysis, we made two assumptions:

A. $4h_t h_r \ll d^2$. For example, if the transmitting antenna height is $h_t =$ 50 m and the receiving height is $h_r = 2$ m, this implies that $d \gg 20$ m. If $h_t$ increases to 100 m, we must have $d \gg 30$ m. This appears quite reasonable when we consider typical distances between BS and UE.

B. $\frac{4\pi h_t h_r}{d\lambda} \ll 1$. For example, consider a frequency of operation of $f_c =$ 800 MHz ($\lambda = 3/8$ m), which around where most first- and second-generation cellular systems hover. For $h_t = 50$ m and $h_r = 2$ m as above, we get $d \gg 3.3$ km. If we increase the frequency to $f_c = 30$ GHz ($\lambda = 0.01$ m), this would require $d \gg 126$ km.

Assumption B thus is usually more stringent than Assumption A. It becomes more problematic for high frequencies: instead of seeing smooth $1/d^n$ behavior, we are more likely to see small-scale multi-path fading.