Likelihood for Seed Production

Growth data is measured on the individual tree-level, while seed data is collected via traps that capture multiple individual trees across a stand of trees. Thus I plan to model individual tree radial growth and stand-level seed production by introducing $\mu_{j,sp,y}$, representing the mean radial growth for stand j, species sp, and year y.

For each individual tree i in stand j, species sp, and year y, the observed radial growth is modeled as, with σ being the variance across individuals:

$$\text{Growth}_{j,sp,y[i]} \sim \text{Normal}(\mu_{j,sp,y}, \sigma^2)$$

The seed count for a particular stand j, species sp, and year y follows a negative binomial distribution with mean $\lambda_{j,sp,y}$ and dispersion term ϕ:

$$\text{Seed}_{j,sp,y} \sim \text{NegBinomial}(\lambda_{j,sp,y}, \phi)$$

The modeled seed count ($\lambda_{j,sp,y}$) for stand j, species sp, and year y is:

$$\log(\lambda_{j,sp,y}) = \alpha_0 + \alpha_{sp} + \alpha_j + (\beta_{1sp} + \beta_{1j}) \cdot \mu_{j,sp,y} + (\beta_{2sp} + \beta_{2j}) \cdot \mu_{j,sp,y-1} + \beta_{3sp} \cdot \text{Elevation}_j$$

This approach allows variation in the seed count due to species and year—independent of tree growth—via

α_0: a baseline seed count (i.e., the grand mean across all species, stands and years not explained by any other components of the model);

α_{sp}: the effect of each species on this baseline (that is a species-specific intercept for each species sp, which is modeled as an offset from the baseline);

α_j: the effect of each stand on this baseline (that is a stand-specific intercept for each stand j, which is modeled as an offset from the baseline);

It also allows variation in seed count due to tree growth in both the current previous years:

β_{1sp}: the effect of current-year growth on seed production for species sp (that is a species-specific slope for the relationship between current-year growth and seed production);

β_{1j}: the effect of current-year growth on seed production for stand j (that is a stand-specific slope for the relationship between current-year growth and seed production);

$\mu_{j,sp,y}$: the mean growth of species sp in stand$j$ in year y;

β_{2sp}: the effect of previous-year growth (lag effect) on seed production for species sp (that is a species-specific slope for the relationship between previous growth and seed production);

β_{2j}: the effect of previous-year growth (lag effect) on seed production for stand j (that is a stand-specific slope for the relationship between previous growth and seed production);

β_{3sp}: the effect of elevation on seed production for species sp (that is a species-specific slope for the relationship between elevation and seed production).

Priors for varying intercepts and slopes

We specify normal priors for both species- and stand-level deviations in intercepts and slopes. These priors allow species and stands to vary around the overall mean relationships in a structured way:

Species and stand-level intercepts

$$\alpha_{sp} \sim \text{Normal}(0, \sigma_{\alpha_{sp}})$$

$$\alpha_j \sim \text{Normal}(0, \sigma_{\alpha_j})$$

Species and stand-level slopes for growth effects

$$\beta_{1sp} \sim \text{Normal}(0, \sigma_{\beta_1 sp})$$

$$\beta_{2sp} \sim \text{Normal}(0, \sigma_{\beta_2 sp})$$

$$\beta_{1j} \sim \text{Normal}(0, \sigma_{\beta_1 j})$$

$$\beta_{2j} \sim \text{Normal}(0, \sigma_{\beta_2 j})$$

Where:

$\sigma_{\alpha_{sp}}$: how much variation is expected from the baseline seed count across species (species level standard deviation);

σ_{α_j}: how much variation is expected from the baseline seed count across stand (stand level standard deviation);

$\sigma_{\beta_1 sp}$ and $\sigma_{\beta_2 sp}$: the extent of variation in these growth–seed production relationships across species (species level variation around the slope);

$\sigma_{\beta_1 j}$ and $\sigma_{\beta_2 j}$: the extent of variation in these growth–seed production relationships across stand (stand level variation around the slope).