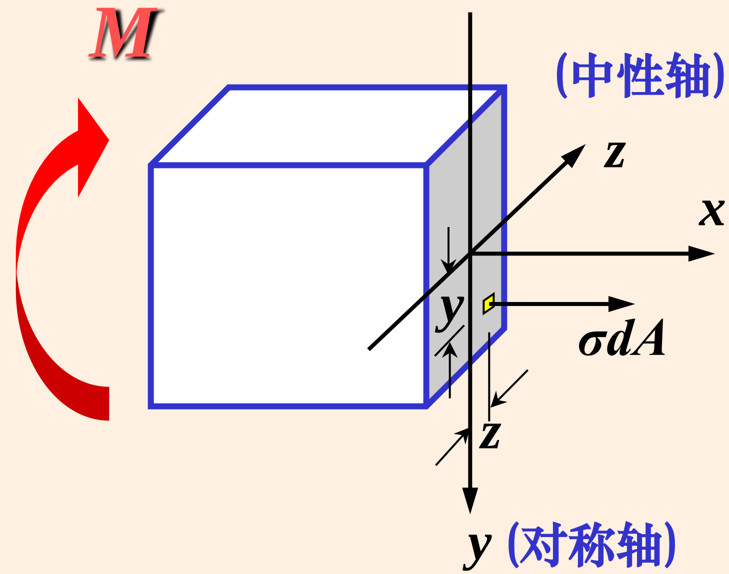
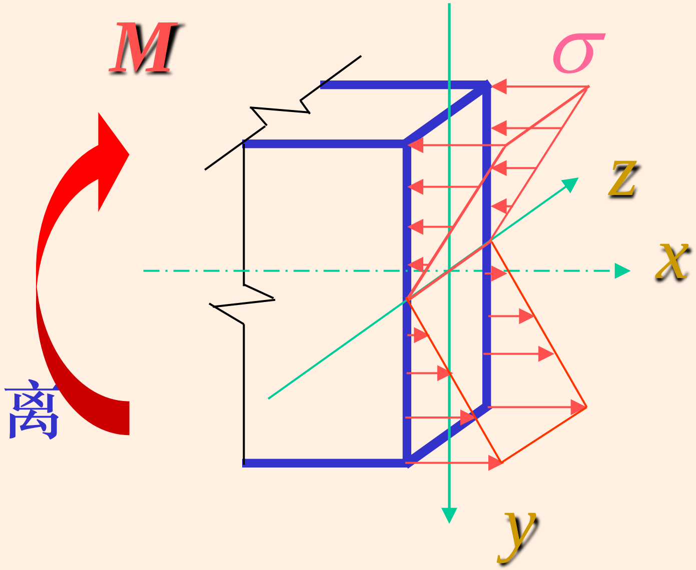
梁横截面上正应力分布：

$$\sigma = \frac{M}{I_z} y$$





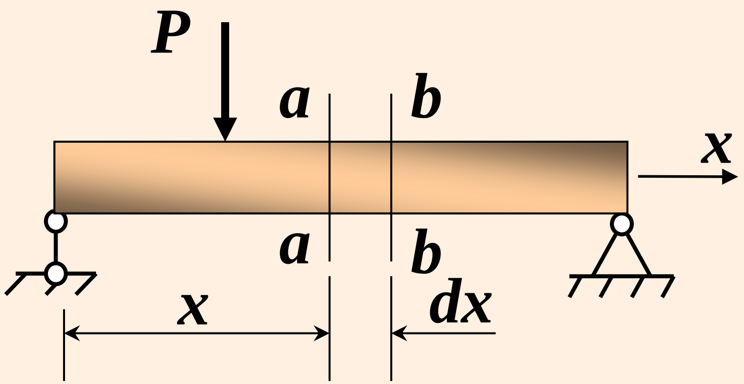
M — 截面上的弯矩;

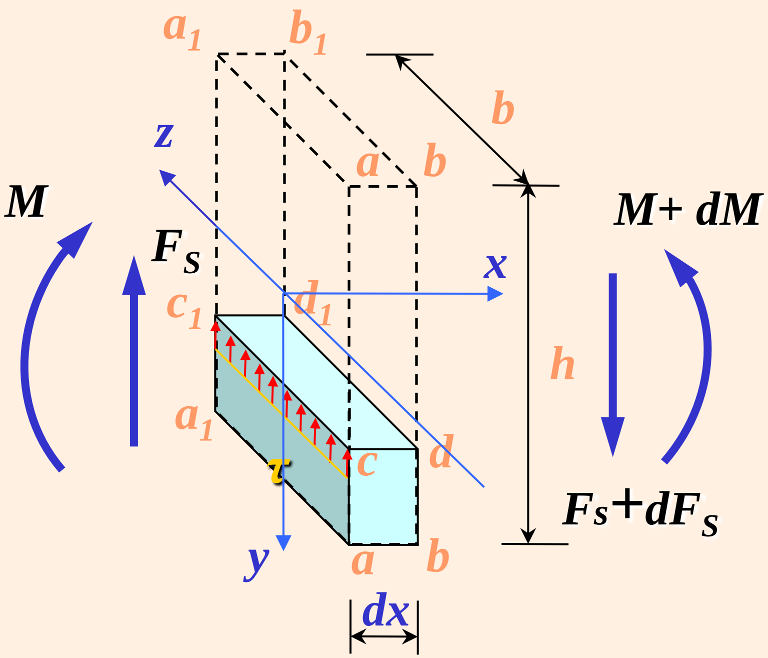
I_z — 截面对中性轴的惯性矩

y — 所求应力点到中性轴的距离

§7-3 梁的剪应力及其强度校核

1. 矩形截面梁的剪应力



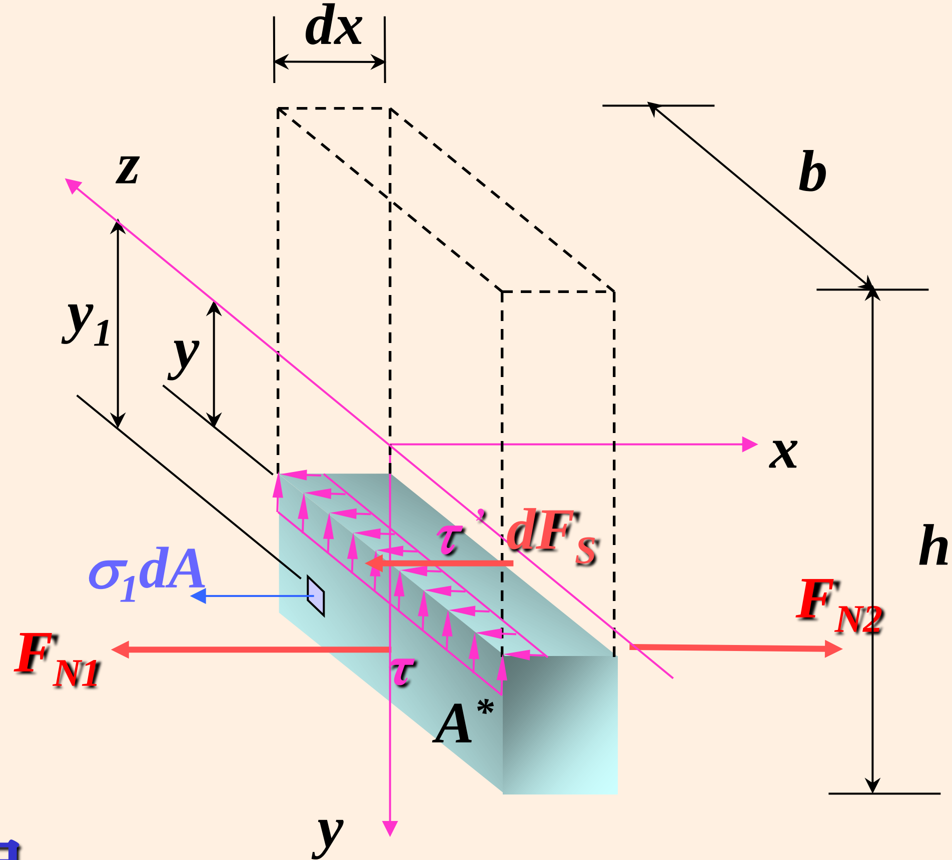


假设:

①**方向**：剪应力与剪力F_S一致；

②**分布**：沿截面宽度均布。

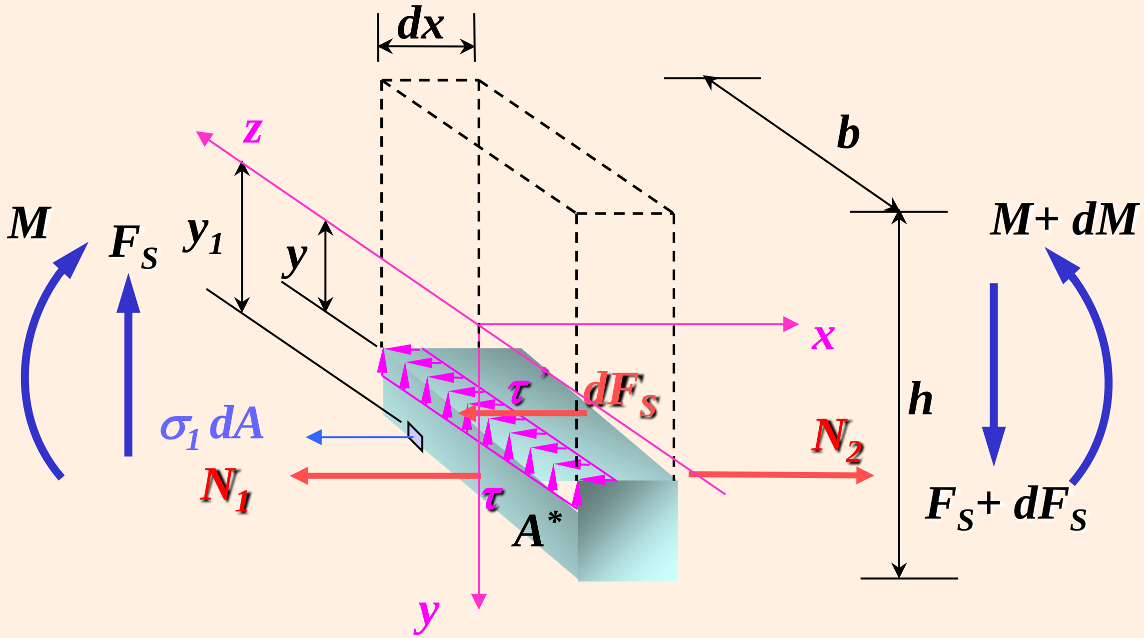
•取隔离体：



•列平衡方程：

$$\sum x = 0: \quad F_{N2} - F_{N1} - dF_S = 0 \qquad \text{--- (1)}$$

$$dF_S = \tau' b dx \qquad \text{--- (2)}$$



$$F_{N1} = \int_{A^*} \sigma_1 dA = \int_{A^*} \frac{My_1}{I_z} dA = \frac{M}{I_z} \cdot S_z^* \quad \text{---} \ (3)$$

其中：$$S_z^* = \int_{A^*} y_1 dA \quad \text{---}$$ A^*对z轴的静矩

同理：$$F_{N2} = \frac{M + dM}{I_z} \cdot S_z^* \quad \text{---} \ (4)$$

(2)、(3)、(4)代入(1)得:

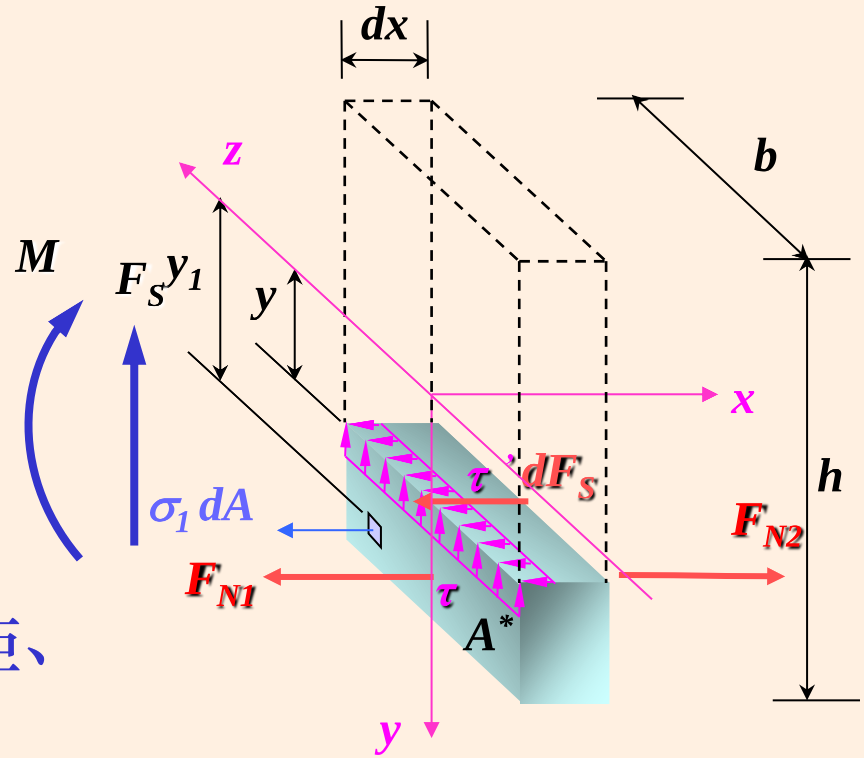
$$\frac{M+dM}{I_z}S_z^* - \frac{M}{I_z}S_z^* - \tau' bdx = 0$$

整理得: $$\tau' = \frac{dM}{dx}\cdot\frac{S_z^*}{I_z b}$$

$$\because \frac{dM}{dx} = F_S \;, \quad \tau' = \tau$$

$$\therefore \tau = \frac{F_S S_z^*}{I_z b} \qquad \text{--- (5)}$$

$$\therefore \tau = \frac{F_S S_z^*}{I_z b} \qquad \text{---} \ (5)$$



F_S — 截面上的剪力;

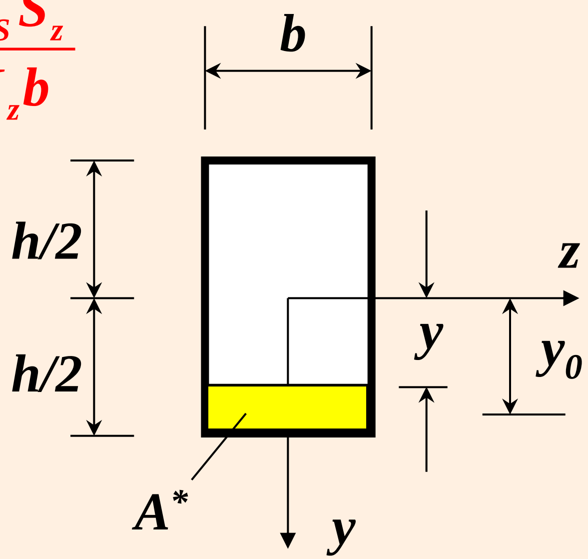
I_z, b — 截面对中性轴的惯性矩、截面宽度;

S_z^* —过所求应力点水平线到截面边缘所包围面积对中性轴的静矩。

•剪应力分布：

对某一截面：F_S、I_z、b是常量，τ只随S_z^*改变。

$$\tau = \frac{F_S S_z^*}{I_z b}$$



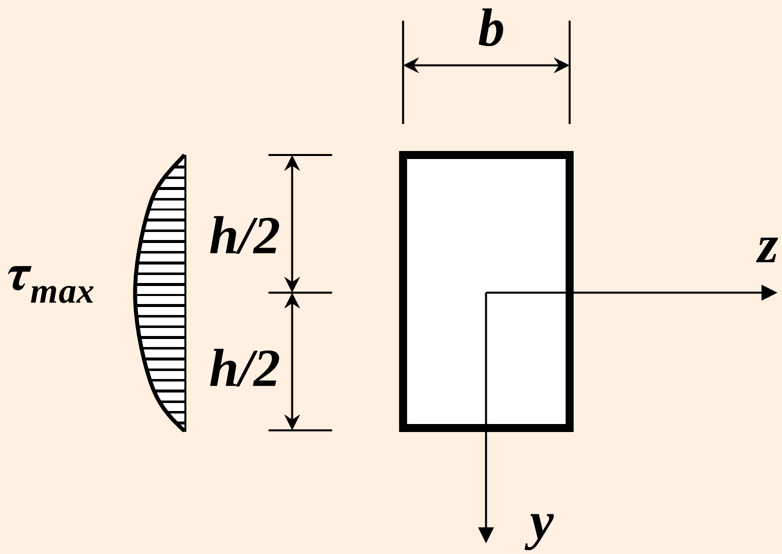
$$S_z^* = A^* y_o$$

$$= b(\frac{h}{2} - y)[y + (\frac{h}{2} - y)/2]$$

$$= \frac{b}{2}(\frac{h^2}{4} - y^2)$$

代入τ得：

$$\tau = \frac{6F_s}{bh^3}(\frac{h^2}{4} - y^2)$$



$$\tau = \frac{6F_s}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

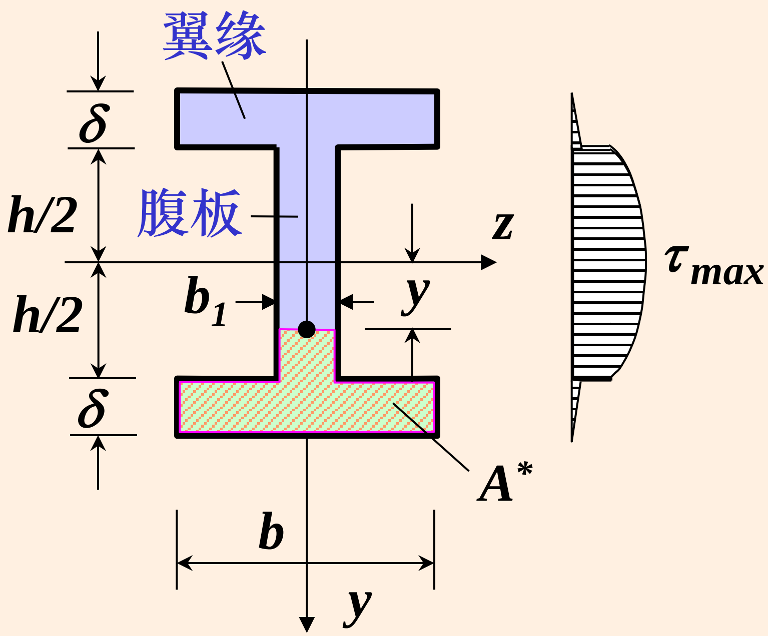
讨论：

$$(\tau)_{y=\frac{h}{2}} = 0$$ 边缘应力为零

$$(\tau)_{y=0} = \tau_{\max} = \frac{3}{2}\frac{F_s}{A}$$

中性轴剪应力最大，为平均剪应力的1.5倍。

2.工字形截面梁的剪应力



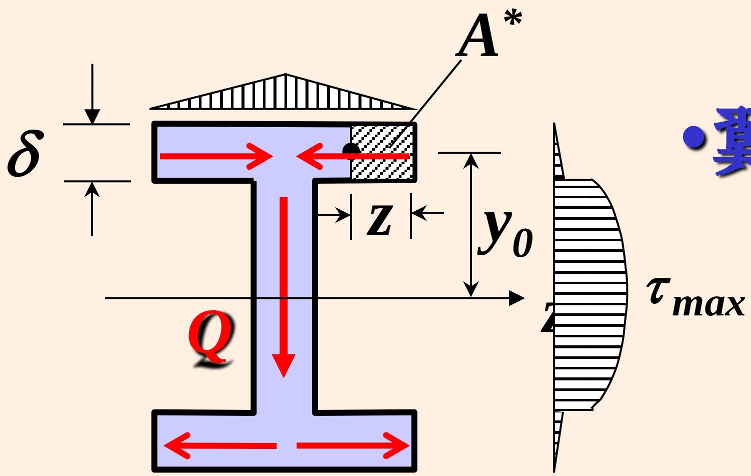
• **腹板剪应力:**

$$\tau = \frac{F_S S_z^*}{I_z b_1}$$

b_1 — 腹板宽度

I_z — 全截面对中性轴的惯性矩

S_z^* — 图示阴影面积对z轴的静矩



•翼缘水平剪应力:

$$\tau_H = \frac{F_S S_z^*}{I_z \delta}$$

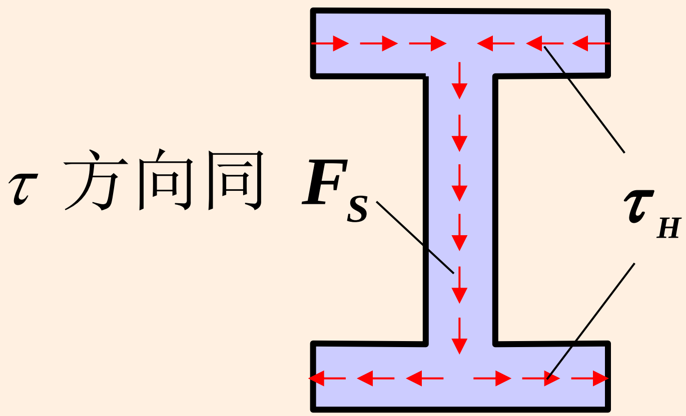
翼缘竖向剪应力忽略

δ — 翼缘宽度

I_z — 全截面对中性轴的惯性矩

$S_z^* = \delta \cdot z \cdot y_o$ — 图示阴影面积对 z 轴的静矩

•**剪应力方向:**



剪力流:

方向由腹板τ确定。

3.剪应力强度条件:

•**最大剪应力**

$$\tau_{\max} = \frac{F_S S^*_{z\max}}{I_z b} \qquad \text{截面}$$

$$\tau_{\max} = \frac{F_{S\max} S^*_{z\max}}{I_z b} \qquad \text{全梁}$$

•强度条件

$$\frac{F_{S\max}S^{*}_{z\max}}{I_z b}\leq[\tau]$$

梁的强度计算:

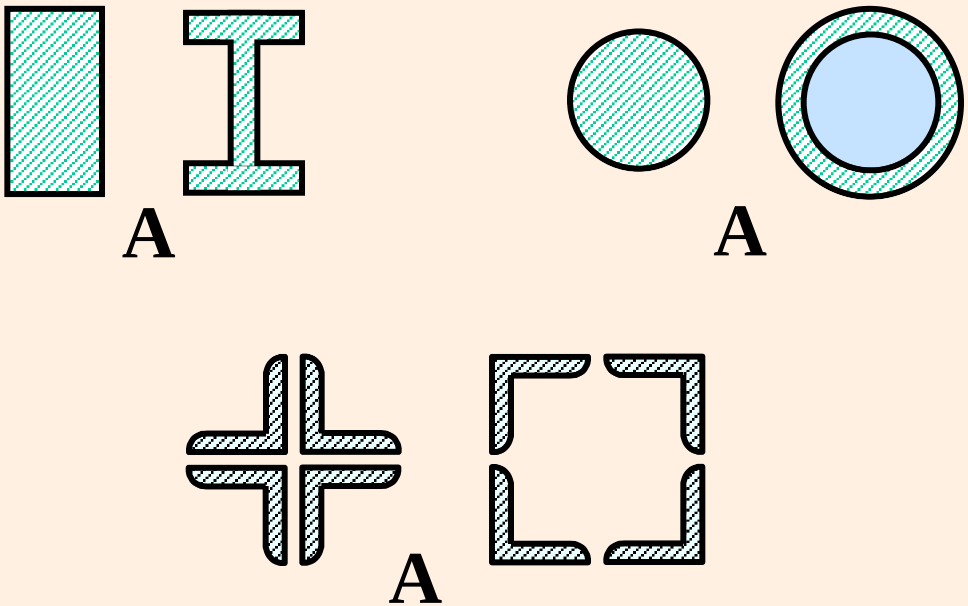
同时满足正应力、剪应力强度条件。

步骤:

•按正应力条件选择截面;

•再按剪应力强度条件校核。

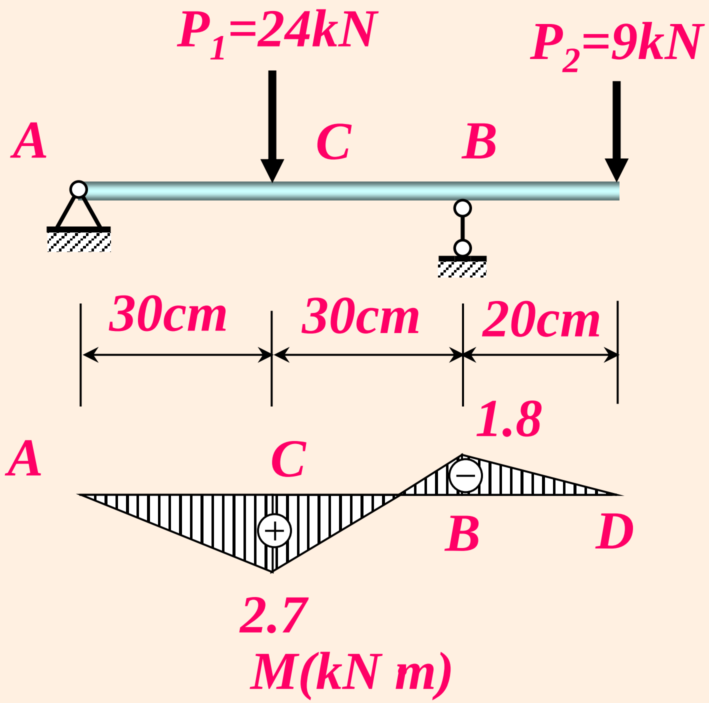
4. 梁的合理截面形式:

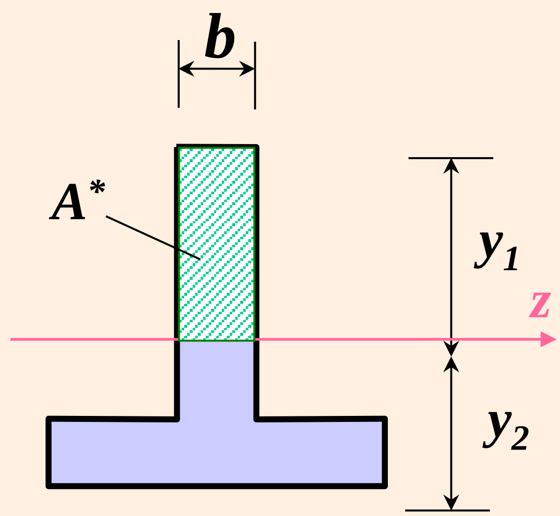


尽量使材料分布在远离中性轴的位置。

例： P115 例6-7

$y_1 = 0.072m,\ \ y_2 = 0.038m, b = 0.03m$



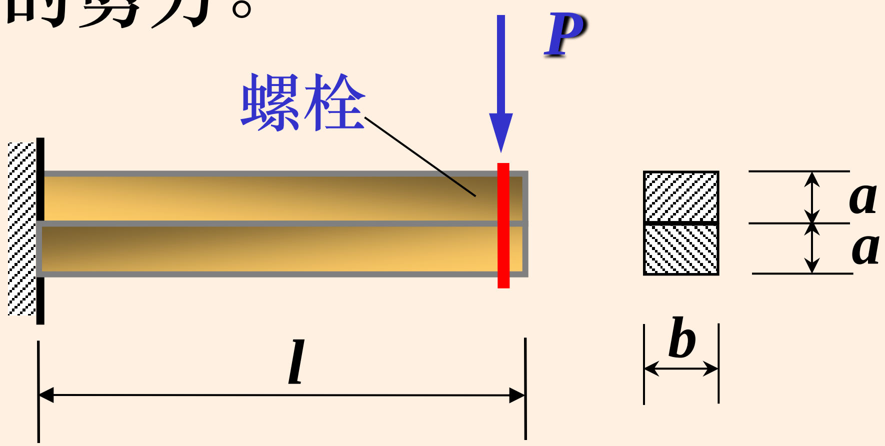


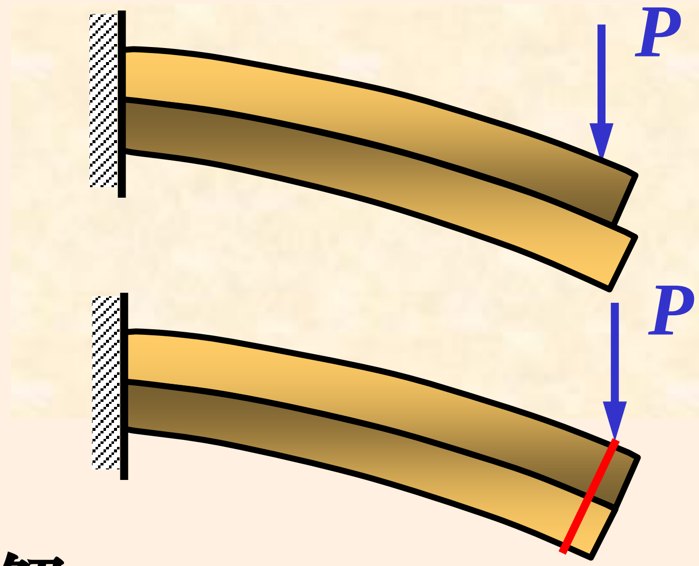
求τ_{max}：

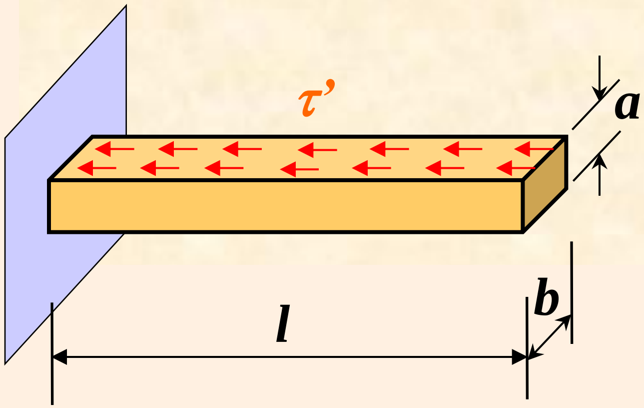
找全梁的F_{Smax}；

求中性轴处的τ。

例： 求螺栓受的剪力。







解:

横截面剪力:　$F_S = P$

横截面剪应力:　$\tau = \dfrac{F_S S_z^*}{I_z b} = \dfrac{P \cdot ab \cdot a/2}{\dfrac{b \cdot (2a)^3}{12} \cdot b} = \dfrac{3}{4} \cdot \dfrac{P}{ab}$

中性层剪应力:　$\tau' = \tau$

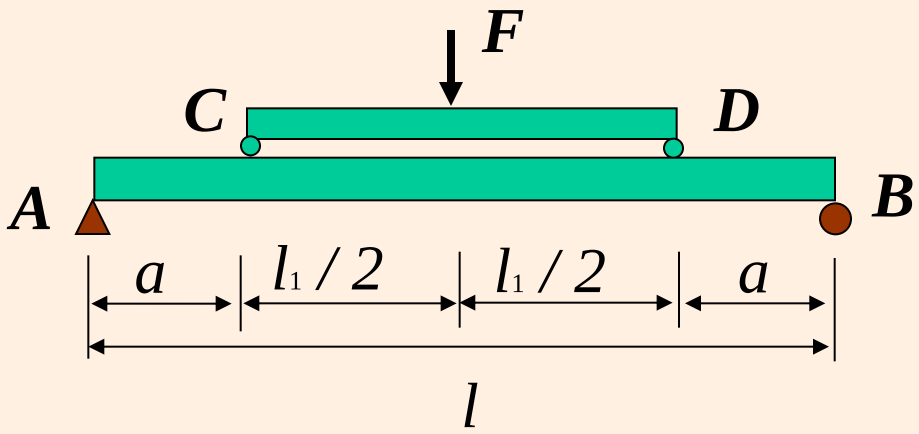
螺栓受力:　$F_S' = \tau' \cdot bl = \dfrac{3}{4}\dfrac{Pl}{a}$

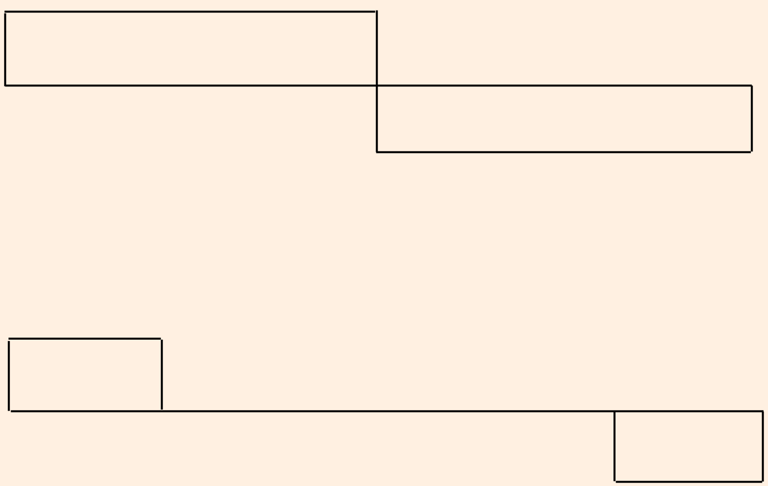
$F = 13\text{kN}$, $l = 4\text{m}$, AB梁高 $h = 180\text{mm}$, 宽 $b = 120\text{mm}$,

a取何值, 使AB梁满足强度要求.

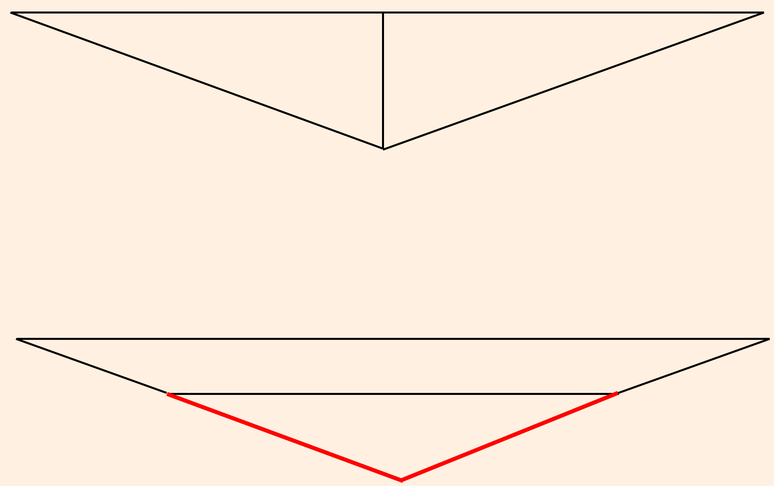
材料容许应力 :

$[\sigma] = 10\text{MPa}$, $[\tau] = 2.2\text{MPa}$



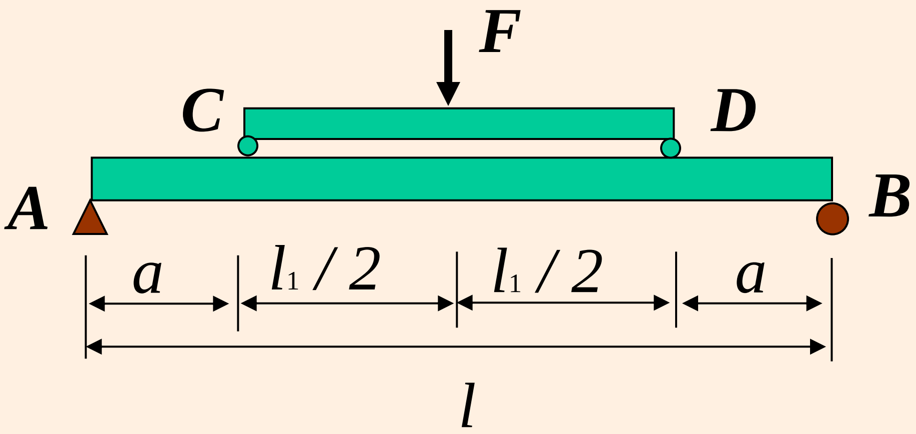


F_s 图



M图

$$\sigma_{\max} = \frac{M_C}{W_z} = \frac{M_C}{bh^2/6} \le [\sigma]$$



$$M_C \le \frac{1}{6} bh^2 [\sigma]$$

$$M_C = \frac{1}{2} Fa \qquad a \le \frac{bh^2[\sigma]/6}{F/2} \approx 1m$$

再校核AB梁的切应力满足要求。

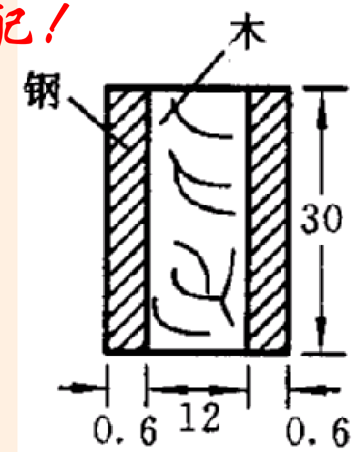
CD梁的存在对AB梁满足抗剪能力没有帮助。

CD梁自身的抗弯抗剪强度也需要校核

$$\frac{M_{钢}}{E_{钢} I_{钢}} = \frac{M_{木}}{E_{木} I_{木}}$$

各自承担的内力按抗弯刚度分配！

剪力如何分配？



又 $I_{钢} = \left(\frac{1}{12} \times 0.6 \times 30^3\right) \times 2\ \text{cm}^4 = 2700\ \text{cm}^4$

$$I_{木} = \frac{12 \times 30^3}{12}\ \text{cm}^4 = 27000\ \text{cm}^4$$

将已知条件代入，得

$$M_{钢} = \frac{40}{3}\ \text{kN} \cdot \text{m}, \quad M_{木} = \frac{20}{3}\ \text{kN} \cdot \text{m}$$

$$(\sigma_{钢})_{max} = \frac{M_{钢}\, y_{max}}{I_{钢}} = \frac{\frac{40}{3} \times 10^3 \times 15 \times 10^{-2}}{2700 \times 10^{-8}}\ \text{Pa} = 74\ \text{MPa}$$

$$(\sigma_{木})_{max} = \frac{M_{木}\, y_{max}}{I_{木}} = \frac{\frac{20}{3} \times 10^3 \times 15 \times 10^{-2}}{27000 \times 10^{-8}}\ \text{Pa} = 3.7\ \text{MPa}$$

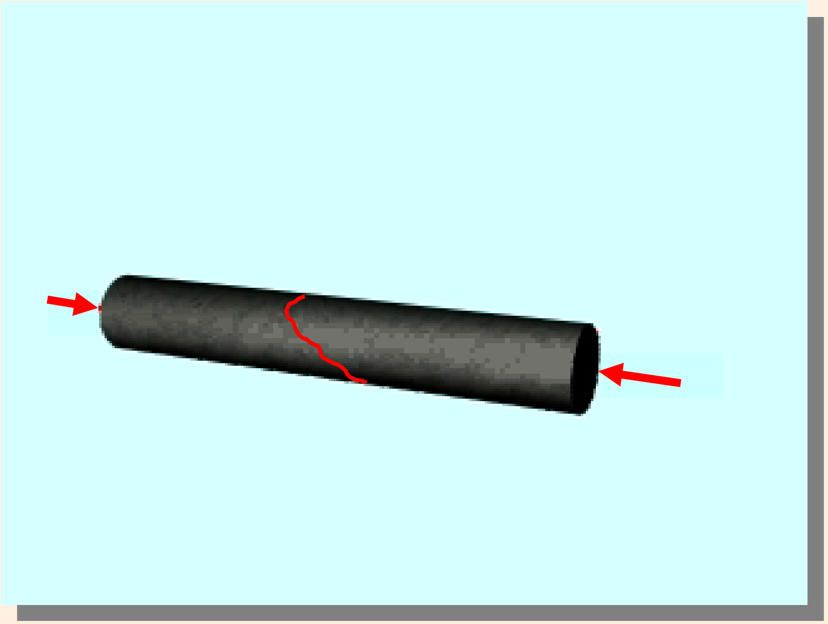
第八章

应力状态

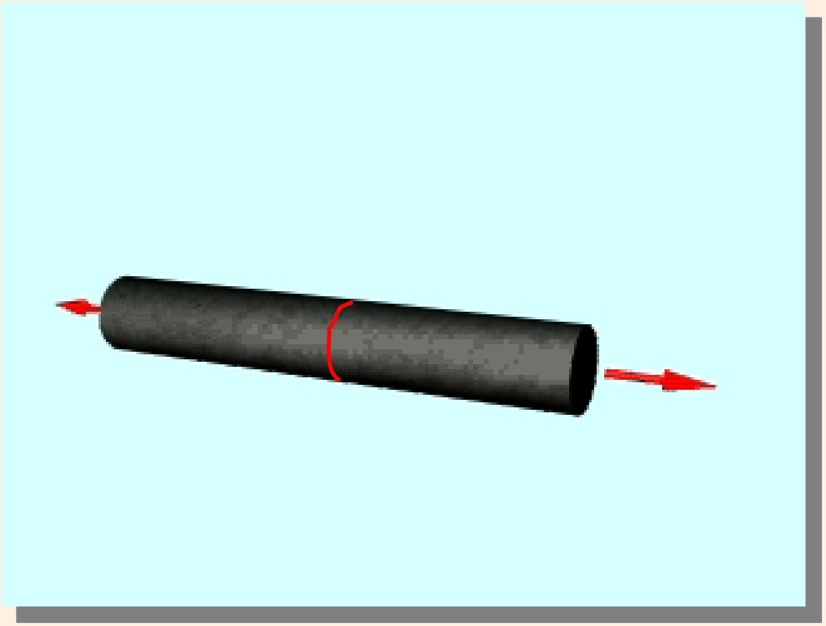
§8-1 应力状态的概念

• 铸铁的拉压实验

压缩

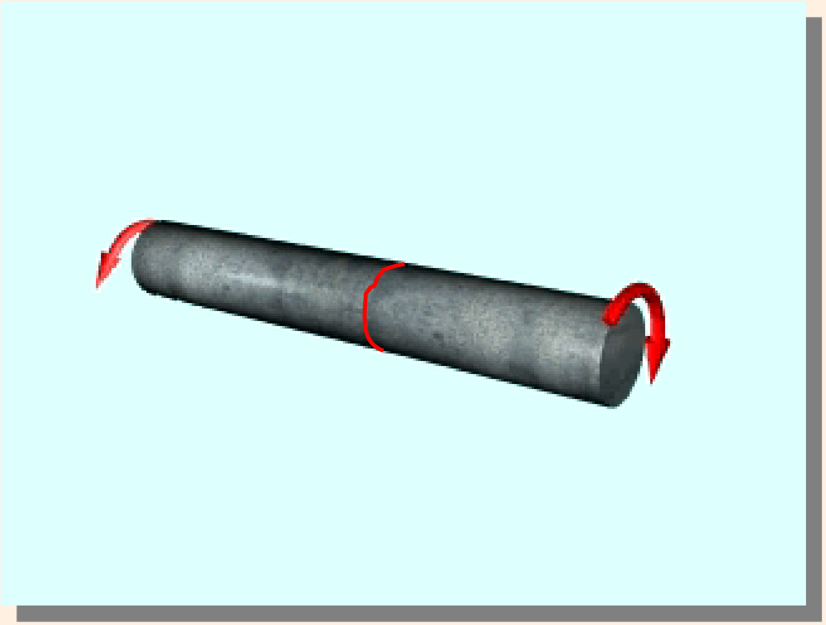


拉伸

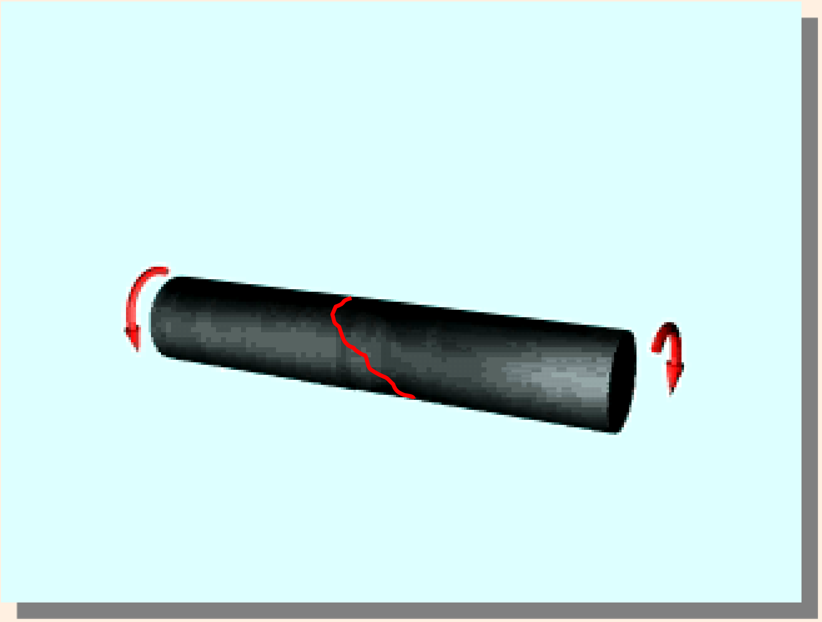


- 低碳钢和铸铁的扭转实验

低碳钢



铸铁



•地震荷载作用下的墙体破坏

说明:

破坏面不一定垂直于轴线 。

推论:

对同一点:一个方向上满足强度要求,并不能说明已经安全。