Model

Variables

$tasks$	The number of tasks to schedule
$machines$	The number of machines available
$T = \{1 \dots tasks\}$	The tasks
$M = \{1 \dots machines\}$	The machines
$d(task_i) \in \{0 \dots 2^{63} - 1\}, \quad task_i \in T$	The duration of task i
$s(task_i) \in \{0 \dots 2^{63} - 1\}, \quad task_i \in T$	The start time of task i
$m(task_i) \in M, \quad task_i \in T$	The machine task i uses
$dep(task_i, task_j) \in \{0, 1\}, \quad task_i, task_j \in T$	The dependency between task i and task j. If 1, task i needs to be completed before task j
$End = \{0..D\}, \quad D = sum(\{d(task_i) \vert \forall task_i \in T\})$	Marks the end of the schedule, i.e. makespan. The variable that is minimized.
$same\_machine_k \subset T$	Set of tasks needing the same machine
$concurrent\_tasks_k \subset T$	Set of tasks needing concurrent execution
$occupied_k \subset T$	Set of tasks occupying a machine in the order specified by *dep*
$overlap(task_i, task_j), \quad task_i, task_j \in T$	$task_i$ overlaps $task_j$

Constraints

$End = max(\{s(task_i) + d(task_i) \mid \forall task_i \in T\})$	The end is where the last task ends
$task_i \prec task_j, \quad task_i, task_j \in T \wedge dep(task_i, task_j) = 1$	$task_i$ has to preceed $task_j$ if specified in *dep*.
$overlap(task_i, task_j) \implies m(task_i) \neq m(task_j),$ $task_i, task_j \in T$	If $task_i$ and $task_j$ overlap, they cannot use the same machine.
$m(task_i) = m(task_j), \quad \forall task_i, task_j \in same\_machine_k$	Every task in a *same_machine* set should use the same machine.
$s(task_i) = s(task_j) \wedge m(task_i) \neq m(task_j),$ $\forall task_i, task_j \in concurrent\_tasks_k$	Every task in a *concurrent_tasks* set should execute at the same time, but they cannot use the same machine.
$m(task_k) = m(task_i) \implies task_k \prec task_i \vee task_k \succ task_j,$ $\forall task_k \in T, \quad task_i, task_j \in occupied_k, \quad dep(task_i, task_j) = 1,$ $task_i \neq task_k, \quad task_j \neq task_k$	If two tasks are set to occupy a machine following each other, there cannot be another task between them using the same machine.