

# An iterated local search algorithm for the minimum differential dispersion problem



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## ABSTRACT

Given a set of  $n$  elements separated by a pairwise distance matrix, the minimum differential dispersion problem (*Min-Diff DP*) aims to identify a subset of  $m$  elements ( $m < n$ ) such that the difference between the maximum sum and the minimum sum of the inter-element distances between any two chosen elements is minimized. We propose an effective iterated local search (denoted by ILS\_MinDiff) for *Min-Diff DP*. To ensure an effective exploration and exploitation of the search space, ILS\_MinDiff iterates through three sequential search phases: a fast descent-based neighborhood search phase to find a local optimum from a given starting solution, a local optima exploring phase to visit nearby high-quality solutions around a given local optimum, and a local optima escaping phase to move away from the current search region. Experimental results on six data sets of 190 benchmark instances demonstrate that ILS\_MinDiff competes favorably with the state-of-the-art algorithms by finding 131 improved best results (new upper bounds).

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## 1. Introduction

Let  $N = \{e_1, e_2, \dots, e_n\}$  be a set of  $n$  elements and  $d_{ij}$  be the distance between  $e_i$  and  $e_j$  according to a given distance metric such that  $d_{ij} > 0$  if  $i \neq j$  and  $d_{ij} = 0$  otherwise. The minimum differential dispersion problem (*Min-Diff DP*) is to identify a subset  $S \subset N$  of a given cardinality  $m$  ( $m < n$ ), such that the difference between the maximum sum and the minimum sum of the inter-element distances between any two elements in  $S$  is minimized. Formally, *Min-Diff DP* can be described in the following way.

Let  $\Delta(e_v)$  be the sum of pairwise distances between an element  $e_v \in S$  and the remaining elements in  $S$ , that is:

$$\Delta(e_v) = \sum_{e_u \in S, u \neq v} d_{uv}. \quad (1)$$

The objective value  $f$  of the solution  $S$  is then defined by the following differential dispersion:

$$f(S) = \max_{e_u \in S} \{\Delta(e_u)\} - \min_{e_v \in S} \{\Delta(e_v)\}. \quad (2)$$

Then, *Min-Diff DP* is to find a subset  $S^* \subset N$  of size  $m$  with the minimum differential dispersion, i.e.,

$$S^* = \arg \min_{S \in \Omega} f(S). \quad (3)$$

where  $\Omega$  is the search space including all possible subsets of size  $m$  in  $N$ , i.e.,  $\Omega = \{S : S \subset N \text{ and } |S| = m\}$ . The size of  $\Omega$  is extremely large, up to a maximum number of  $\binom{n}{m} = \frac{n!}{m!(n-m)!}$ .

*Min-Diff DP* is one of many diversity or dispersion problems [1,30] which basically aim to find a subset  $S$  from a given set of elements, such that a distance-based objective function over the elements in  $S$  is maximized or minimized. These problems can be further classified according to two types of objective functions:

- *Efficiency-based* measures which consider some dispersion quantity for all elements in  $S$ . This category mainly includes the *maximum diversity problem* (MDP) and the *max-min diversity problem* (MMDP), which respectively maximizes the total sum of the inter-element distances of any two chosen elements and the minimum distance of any two chosen elements.
- *Equity-based* measures which guarantee equitable dispersion among the selected elements. This category includes three problems: (i) the *maximum mean dispersion problem* (Max-Mean DP) maximizes the average inter-element distance among the chosen elements; (ii) the *maximum min-sum dispersion problem* (Max-Min-sum DP) maximizes the minimum sum of the inter-element distances between any two chosen elements; (iii) the

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minimum differential dispersion problem considered in this work. It is worth noting that the cardinality of subset  $S$  is fixed except for Max-Mean DP.

In addition to their theoretical significance as NP-hard problems, diversity problems have a variety of real-world applications in facility location [21], pollution control [9], maximally diverse/similar group selection (e.g., biological diversity, admission policy formulation, committee formation, curriculum design, market planning) [17,27,28], densest subgraph identification [20], selection of homogeneous groups [8], web pages ranking [19,33], community mining [32], and network flow problems [7].

In this study, we focus on *Min-Diff DP*, which is known to be strongly NP-hard [30]. *Min-Diff DP* can be formulated as a 0–1 mixed integer programming problem (MIP) [30]. Thus it can be conveniently solved by MIP solvers like IBM ILOG CPLEX Optimizer (CPLEX). However, being an exact solver, CPLEX is only able to solve instances of small size (up to  $n = 50$  and  $m = 15$ ), while requiring high CPU times (more than 2500 s) [30]. To handle larger instances, heuristic and meta-heuristic algorithms are often preferred to find near-optimal solutions. In recent years, several heuristic approaches have been proposed in the literature [13,28,30]. In particular, in 2015, based on greedy randomized adaptive search procedure (GRASP), variable neighborhood search (VNS) and exterior path relinking (EPR), Duarte et al. proposed several effective hybrid heuristics [13]. Very recently (2016), Mladenović et al. proposed an improved VNS algorithm which uses the swap neighborhood both in its descent and shaking phases [28]. This new VNS algorithm significantly outperforms the previous best heuristics reported in [13] and is the current best performing algorithm available in the literature for *Min-Diff DP*. We will use it as our main reference for the computational studies.

Our literature review showed that, contrary to other diversity problems like MDP and Max-Min DP, for which many exact and heuristic methods have been investigated, there are only a very limited number of studies for *Min-Diff DP*. In this work, we partially fill the gap in terms of heuristic solving of *Min-Diff DP*, by proposing an effective heuristic algorithm for the problem (named ILS\_MinDiff). We identify the main contributions of this work as follows.

First, in terms of algorithmic design, the proposed ILS\_MinDiff algorithm is the first adaptation of the general three-phase search method to *Min-Diff DP* (Section 2). Compared to other meta-heuristic framework, this method has the main advantage of being conceptually simple and easy implementation. The proposed algorithm iterates through a descent-based local optimization phase, a local optima exploring phase, and a local optima escaping phase (Section 3). Like [13,28], the descent-based local optimization phase is based on the conventional swap neighborhood. Meanwhile, ILS\_MinDiff distinguishes itself from the existing heuristic algorithms by at least two specific features. Its local optima exploring phase applies a weak perturbation technique (based on deterministic tournament selection) to discover nearby local optima around a given local optimum (Section 3.4). On the other hand, the local optima escaping phase calls for a strong perturbation technique (based on parametric random selection) to move away from deep local optimum traps (Section 3.5).

Second, in terms of performance, we present 131 improved best-known results (i.e., new lower bounds) out of 190 popular *Min-Diff DP* benchmarks ( $\approx 69\%$ ). These improved lower bounds constitute a valuable contribution to *Min-Diff DP* studies since they can be used as new reference values to assess other *Min-Diff DP* heuristic algorithms. These tightened lower bounds could also be used within an exact algorithm for better bounding and possibly leading to an optimality proof of some benchmark instances. Together with the equal best-known results for other 42

instances, the computational outcomes confirm the relevance of the proposed algorithm.

Third, the availability of the source code of our ILS\_MinDiff algorithm (see Section 4.2) contributes favorably to future research on *Min-Diff DP* and related problems. Specifically, the code can be used to perform comparative studies or solve other problems that can be formulated as *Min-Diff DP*. It can also serve as a key component (local optimizer) of more sophisticated *Min-Diff DP* algorithms.

The remainder of the paper is organized as follows. In the next section, we present a brief literature review on the iterated local search framework and its two recent variants. In Section 3, we describe the general framework and the key components of the proposed algorithm. In Section 4, we show an extensive experimental comparison between our algorithm and state-of-the-art algorithms. A parameter analysis is provided in Section 5, followed by conclusions in Section 6.

## 2. Related work on applications of iterated local search

As one of the most widely-used meta-heuristic approaches, *Iterated local search* (ILS) [24] has been successfully applied to solve a variety of combinatorial optimization problems. In spite of its conceptual simplicity, it has led to a number of state-of-the-art results. Fig. 1 shows that over the last two decades, there has been an increasing interest in ILS, as witnessed by the number of publications related to ILS.

ILS is a two-phase approach which explores iteratively the search zones around the last local optimum discovered by a local search procedure. Typically, an ILS algorithm repeats a local search phase to find a local optimal solution from a starting point and a perturbation phase to modify the local optimum to generate a new starting point for the next iteration (Algorithm 1). Based on the

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### Algorithm 1 Iterated local search.

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```

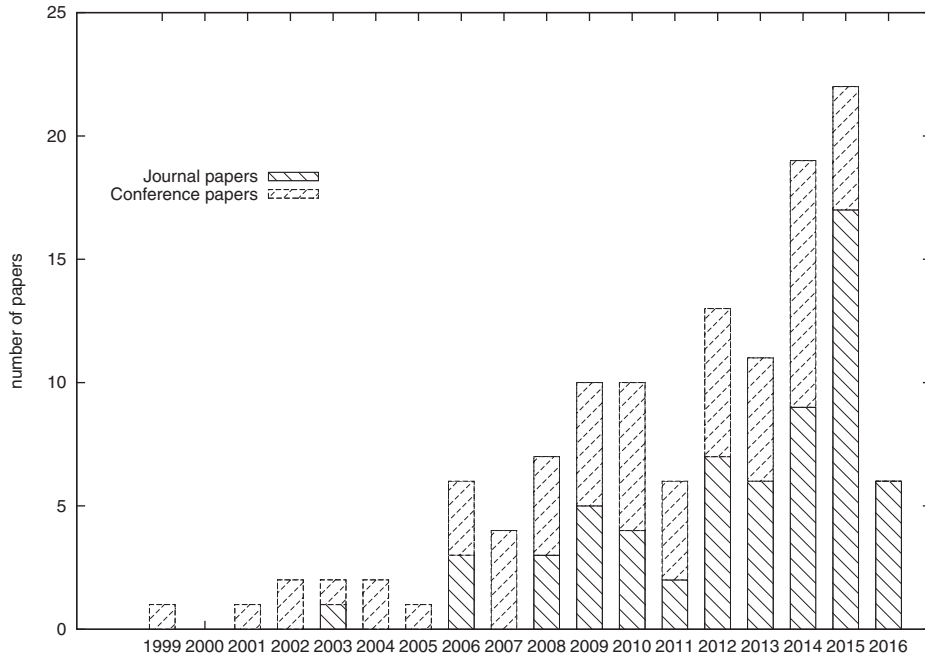
1:  $S_0 \leftarrow \text{GenerateInitialSolution}()$ 
2:  $S^* \leftarrow \text{LocalSearch}(S_0)$ 
3: while a stopping condition is not reached do
4:    $S' \leftarrow \text{Perturbation}(S^*, \text{history})$ 
5:    $S^{*'} \leftarrow \text{LocalSearch}(S')$ 
6:    $S^* \leftarrow \text{AcceptanceCriterion}(S^*, S^{*'}, \text{history})$ 
7: end while

```

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general ILS framework, several variants and extended approaches have recently been proposed, including two representative methods called breakout local search (BLS) [2,6] and three-phase search (TPS) [15]. The effectiveness of BLS and TPS have been verified on a variety of hard optimization problems and applications (see examples of Table 1). In the following, we present a brief review of these ILS variants.

*Breakout local search* introduced in [2,6] combines local search with a *dedicated and adaptive perturbation* mechanism. Its basic idea is to use a descent-based local search procedure to intensify the search in a given search region, and to perform dedicated perturbations to jump into a new promising search region once a local optimum is encountered. BLS is characterized by its adaptive perturbation. At the perturbation phase, BLS attempts to achieve the most suitable degree of diversification by dynamically determining the number of perturbation moves (i.e., the jump magnitude) and by adaptively selecting between several types of pre-defined perturbation operations of different intensities, which is achieved through the use of information from specific memory structures. As summarized in Table 1, BLS has reported excellent performances for several well-known combinatorial optimization problems. Algorithm 2 describes the general framework of BLS.



**Fig. 1.** Journal and conference publications by years on ILS. Data was extracted from the DBLP database <http://dblp.uni-trier.de/search/publ> under the keywords “iterated local search” on 29 June 2016.

**Table 1**

A summary of the applications of breakout local search and three-phase search.

Breakout local search	Three-phase search
minimum sum coloring problem [2]	quadratic minimum spanning tree problem [15]
quadratic assignment problem [3]	maximally diverse grouping problem [22]
maximum clique problem [4]	capacitated clustering problem [23]
max-cut problem [5]	max-k-cut problem [25]
vertex separator problem [6]	clique partitioning problem [34]
Steiner tree problem [14]	minimum differential dispersion problem
assembly sequence planning problem [16]	
single-machine total weighted tardiness problem [11]	

#### Algorithm 2 Breakout local search.

```

1:  $S \leftarrow \text{GenerateInitialSolution}()$ 
2:  $L \leftarrow L_0$ 
3: while a stopping condition is not reached do
4:    $S' \leftarrow \text{DescentBasedSearch}(S)$ 
5:    $L \leftarrow \text{DetermineJumpMagnitude}(L, S', \text{history})$ 
6:    $T \leftarrow \text{DeterminePerturbationType}(S', \text{history})$ 
7:    $S \leftarrow \text{Perturb}(L, T, S', \text{history})$ 
8: end while

```

BLS distinguishes itself from the conventional ILS approach by the following two aspects. First, multiple types of perturbations are used in BLS, which are triggered according to the search states, achieving variable levels of diversification. Second, the local optimal solution returned by the local search procedure is always accepted as the new starting solution in BLS regardless of its quality, which completely eliminates the acceptance criterion component of ILS (Algorithm 1, line 6).

Three-phase search proposed in [15] follows and generalizes the basic ILS scheme. TPS iterates through three distinctive and sequential search phases. The basic idea of TPS is described as

follows. Starting from an initial solution, a descent-based neighborhood search procedure is first employed to find a local optimal solution. Then, a local optima exploring phase is triggered with the purpose of discovering nearby local optima of better quality. When the search stagnates in the current search zone, TPS turns into a diversified perturbation phase, which strongly modifies the current solution to jump into a new search region. The process iteratively runs the above three phases until a given stopping condition is met. Compared to BLS, TPS further divides the perturbation phase into a local optima exploring phase (to discover more local optima within a given region) and a diversified perturbation phase (to displace the search to a new and distant search region). TPS has been successfully used to solve several optimization problems, as shown in the right column of Table 1. The general framework of TPS is outlined in Algorithm 3. Actually, the ILS\_MinDiff algorithm

#### Algorithm 3 Three-phase search.

```

1:  $S \leftarrow \text{GenerateInitialSolution}()$ 
2:  $S^* \leftarrow S$ 
3: while a stopping condition is not reached do
4:    $S \leftarrow \text{DescentBasedSearch}(S)$ 
5:    $S \leftarrow \text{LocalOptimaExploring}(S, \text{history})$ 
6:    $S^* \leftarrow \text{BestOne}(S, S^*)$ 
7:    $S \leftarrow \text{DiversifiedPerturb}(S, \text{history})$ 
8: end while

```

proposed in this work follows the TPS framework.

### 3. An iterated local search algorithm for Min-Diff DP

#### 3.1. General framework

Given a set of elements  $N = \{e_1, e_2, \dots, e_n\}$ , any subset set  $S \subset N$  of  $m$  elements is a legal or feasible solution of Min-Diff DP and can be represented by  $S = \{e_{S(1)}, e_{S(2)}, \dots, e_{S(m)}\}$  ( $1 \leq S(i) \neq S(j) \leq n$  for all  $i \neq j$ ) where  $S(l)$  ( $1 \leq l \leq m$ ) is the index of each selected element in  $S$ .

Following the three-phase search framework [15], the proposed ILS\_MinDiff algorithm is composed of three main components: a descent-based neighborhood search procedure, a local optima exploring procedure and a local optima escaping procedure. Starting from a (good) initial solution provided by an initial solution generation procedure (Section 3.2), ILS\_MinDiff first employs the descent neighborhood search procedure to quickly attain a local optimal solution (Section 3.3). Then it switches to the local optima exploring procedure which attempts to discover better solutions around the attained local optimum (Section 3.4). When no improved solution can be found (the search is stagnating in the current search zone), ILS\_MinDiff tries to jump to a new region with the help of a strong perturbation operation (Section 3.5). The best solution encountered during the search is recorded in  $S^b$  and updated whenever it is needed. ILS\_MinDiff repeats the above three phases until a stopping condition (in our case, a time limit  $t_{\max}$ ) is reached (Algorithm 4). The composing procedures of

**Algorithm 4** Framework of the ILS\_MinDiff algorithm for *Min-Diff DP*.

---

```

1: Input: a problem instance and the time limit  $t_{\max}$ 
2: Output: the best solution  $S^b$  found
3:  $S \leftarrow \text{generate\_initial\_solution}()$  /* generate a good initial so-
   lution, Section 3.2 */
4:  $S^b \leftarrow S$  /* record the best solution found so far in  $S^b$  */
5: while a stopping condition is not reached do
6:    $S \leftarrow \text{descent\_based\_neighborhood\_search}(S)$  /* local
     search, Section 3.3 */
7:    $S \leftarrow \text{local\_optima\_exploring}(S)$  /* explore nearby local op-
     tima, Section 3.4 */
   // update the best solution found so far
8:   if  $f(S) < f(S^b)$  then
9:      $S^b \leftarrow S$ 
10:  end if
11:   $S \leftarrow \text{local\_optima\_escaping}(S)$  /* escape from local
     optima, Section 3.5 */
12: end while

```

---

ILS\_MinDiff are presented in the next subsections.

### 3.2. Initialization

The ILS\_MinDiff algorithm requires an initial solution to start its search. In general, the initial solution can be generated by any means (e.g., a random procedure or a greedy heuristic). In this work, the search starts from an elite solution of good quality, which is obtained in the following way. From a random solution  $S \in \Omega$  (i.e., any subset of  $m$  elements), we apply the descent-based neighborhood search procedure (Section 3.3) to improve  $S$  until a local optimum is reached. We repeat the process ten times to obtain ten local optimal solutions among which we select the best one (i.e., having the smallest objective value) as the initial solution. This procedure allows us to obtain an initial solution of relatively high quality. However, for large instances with  $n \geq 3000$ , this initialization process becomes too time-consuming. In this case, the algorithm starts its search with a random solution, instead of an elite solution.

### 3.3. Descent-based neighborhood search phase

To obtain a local optimum from a given starting solution, a neighborhood search procedure is needed. In our case, we employ a simple and fast *descent\_based\_neighborhood\_search()* procedure. This search procedure iteratively makes transitions from the incumbent solution to a new neighbor solution according to a given

neighborhood relation such that each transition leads necessarily to a better solution. This improvement process runs until no improving neighbor solution is available in the neighborhood, in which case the current solution corresponds to a local optimum with respect to the neighborhood.

For the design of such a search procedure, two important issues must be considered: the neighborhood structure and the technique for a fast evaluation of neighboring solutions. The neighborhood *Neighbor* explored by the *descent\_based\_neighborhood\_search()* procedure is based on the *swap* operation, which has been used in previous studies [13,28]. Given a solution  $S$ , we define *swap*( $p, q$ ) as the move that exchanges an element  $e_p \in S$  with an element  $e_q \in N \setminus S$ . Each *swap*( $p, q$ ) brings about a variation  $\Delta f(S, p, q)$  in the objective function  $f$ . Let  $S'$  be the neighbor solution obtained by applying *swap*( $p, q$ ) to the solution  $S$ , then the objective variation  $\Delta f(S, p, q)$  (also called the move gain) is given by  $\Delta f(S, p, q) = f(S') - f(S)$ . Obviously, the size of this swap-based neighborhood is bound by  $\mathcal{O}(m(n-m))$ .

To evaluate the neighborhood as fast as possible, we adopt a popular incremental evaluation technique [13,22,28] to streamline the calculation of  $\Delta f(S, p, q)$ . Once a *swap*( $p, q$ ) move is performed, only the elements related to  $e_p$  and  $e_q$  are needed to be considered. Before calculating  $\Delta f(S, p, q)$  caused by *swap*( $p, q$ ), we first estimate the  $\Delta$  value of each element  $e_w$  in  $S$  as follows:

$$\Delta'(e_w) = \begin{cases} \Delta(e_w) - d_{wp} + d_{wq} & \forall e_w \in S \setminus \{e_p\} \\ \sum_{e_z \in S \setminus \{e_p\}} d_{qz} & e_w = e_q \end{cases} \quad (4)$$

Therefore, with the *swap*( $p, q$ ) operation, the objective value of the resulting neighbor solution  $S' = S \setminus \{e_p\} \cup \{e_q\}$  can be conveniently calculated according to the following formula:

$$f(S') = \max_{e_w \in S'} \{\Delta'(e_w)\} - \min_{e_w \in S'} \{\Delta'(e_w)\}. \quad (5)$$

Correspondingly, the move gain of *swap*( $p, q$ ) can be finally computed as:

$$\Delta f(S, p, q) = f(S') - f(S). \quad (6)$$

With the help of this fast updating strategy, we can calculate  $\Delta f(S, p, q)$  in  $\mathcal{O}(m)$  because one only needs to check the  $m-1$  elements adjacent to the removed element  $e_p$  in  $S$  and the  $m-1$  elements adjacent to the added element  $e_q$  in  $S$ . Since the total number of the neighboring solutions evaluated is  $m(n-m)$ , therefore the computational complexity to evaluate the whole neighborhood is  $\mathcal{O}(m^2n)$ .

To explore the neighborhood, the *descent\_based\_neighborhood\_search()* procedure uses the *best improvement strategy*. In other words, the best improving neighbor solution (with the smallest negative move gain) is selected at each iteration (ties are broken at random). After each solution transition, the search is resumed from the new solution. When no improving neighbor solution exists in the neighborhood, the current solution is a local optimum. In this case, the *descent\_based\_neighborhood\_search()* procedure terminates and returns the last solution as its output. Finally, after each swap operation, the  $\Delta$  value of each element  $e_i \in N$  is updated, which is achieved in  $\mathcal{O}(n)$ .

### 3.4. Local optima exploring phase

Obviously, the descent-based neighborhood search phase will quickly fall into a local optimum because it only accepts improving solutions during the search. To intensify the search around the attained local optimum and discover other solutions of higher quality, we introduce the *local\_optima\_exploring()* procedure, which iterates through a moderate perturbation operation and the descent-based neighborhood search procedure (Algorithm 5).



**Algorithm 5** `local_optima_exploring()` procedure.

---

```

1: Input: a starting solution  $S$  and the given search depth  $nbr_{\max}$ 
2: Output: the best solution  $S^*$  found during the current local
   optima exploring phase
3:  $S^* \leftarrow S$  /* record the best
   solution found so far */
4:  $nbr \leftarrow 0$ 
5: while  $nbr < nbr_{\max}$  do
6:    $S \leftarrow \text{weak\_perturb\_operator}(S)$  /* perform a weak
     perturb operation */
7:    $S \leftarrow \text{descent\_based\_neighborhood\_search}(S)$  /* attain
     a local optimum */
     // update the best solution found
8:   if  $f(S) < f(S^*)$  then
9:      $S^* \leftarrow S$ 
10:     $nbr \leftarrow 0$ 
11:   else
12:      $nbr \leftarrow nbr + 1$ 
13:   end if
14: end while

```

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The `local_optima_exploring()` procedure uses the principle of deterministic tournament selection to perturb (slightly) the input local optimal solution  $S$  with the `weak_perturb_operator()`. At each perturbation step, we first generate at random  $n + 1$  candidate neighbor solutions of the incumbent solution and then select the best one among these solutions to replace the incumbent solution. The `weak_perturb_operator()` repeats  $p_w$  times ( $p_w$  is a parameter called *weak perturbation strength*), and returns the last perturbed solution which serves as the starting point of the descent-based neighborhood search. It is clear that a small (large)  $p_w$  leads to a perturbed solution which is close to (far away from) the input solution  $S$ . In this work, we set  $p_w = 2, 3$ .

Starting from the perturbed solution delivered by the `weak_perturb_operator()`, the `descent_based_neighborhood_search()` procedure is run to attain a new local optimum, which becomes the incumbent solution of next iteration of the current local optima exploring phase. The best local optimum  $S^*$  found during the local optima exploring phase is updated each time a new local optimum better than the recorded  $S^*$  is encountered. The `local_optima_exploring()` procedure terminates when the recorded best local optimum  $S^*$  cannot be updated for  $nbr_{\max}$  consecutive iterations ( $nbr_{\max}$  is a parameter called *search depth*), indicating that the region around the initial input solution  $S$  is exhausted and the search needs to move into a more distant region, which is the purpose of the local optima escaping procedure described in the next section.

### 3.5. Local optima escaping phase

To move away from the best local optimum  $S^*$  found by `local_optima_exploring()`, we call for the `local_optima_escaping()` procedure which applies a strong perturbation mechanism. Specifically, the `local_optima_escaping()` procedure takes  $S^*$  as its input, and then randomly performs  $p_s$  swap operations.  $p_s$ , called strong perturbation strength, is defined by  $p_s = \alpha \times n/m$ , where  $\alpha \in [1.0, 2.0]$  is a parameter called *strong perturbation coefficient*. Since the objective variations are not considered during the perturbation operations, the perturbed solution may be greatly different from the input local optimum. This is particularly true with large  $p_s$  value (e.g.,  $p_s > 10$ ), which definitively helps the search to jump into a distant search region.

## 4. Computational experiments

This section presents a performance assessment of the ILS\_MinDiff algorithm. For this purpose, we carry out an extensive experimental comparison between the proposed algorithm and two most recent and best performing algorithms (GRASP\_EPR [13] and VNS\_MinDiff [28]), based on six data sets of 190 benchmark instances.

### 4.1. Benchmark instances

MDPLIB<sup>1</sup> provides a comprehensive set of instances which are widely used for testing algorithms for diversity and dispersion problems. By excluding the small and easy instances, the remaining 190 benchmark instances tested in this work include the following three types and are classified into six data sets.

- **SOM** (SOM-b): This data set includes 20 test instances whose sizes range from  $(n, m) = (100, 10)$  to  $(n, m) = (500, 200)$ . The instances of this set were created with a generator developed by Silva et al. [31].
- **GKD** (GKD-b and GKD-c): These two data sets include 70 test instances whose sizes range from  $(n, m) = (25, 2)$  to  $(n, m) = (500, 50)$ . The distance matrices are built by calculating the Euclidean distance between each pair of randomly generated points from the square  $[0, 10] \times [0, 10]$ . These instances were introduced by Glover et al. [18] and generated by Duarte and Martí [12], and Martí et al. [26].
- **MDG** (MDG-a, MDG-b and MDG-c): The whole data set is composed of 100 test instances whose sizes range from  $(n, m) = (500, 50)$  to  $(n, m) = (3000, 600)$ . The distance matrices in these instances are generated by selecting real numbers between 0 and 10 from an uniform distribution. These instances have been widely used in, e.g., Duarte and Martí [12], Palubeckis [29], and Martí et al. [27].

### 4.2. Experimental settings

The ILS\_MinDiff algorithm<sup>2</sup> was implemented in C++ and compiled using g++ compiler with the '-O2' flag. All experiments were carried out on an Intel Xeon E5440 processor with 2.83 GHz and 2 GB RAM under Linux operating system. Without using any compilation flag, running the DIMACS machine benchmark procedure `dfmax.c`<sup>3</sup> on our machine requires 0.44, 2.63 and 9.85 s to solve the benchmark graphs r300.5, r400.5 and r500.5 respectively.

Given its stochastic nature, ILS\_MinDiff was independently executed, like [28], forty times with different random seeds on each test instance. Each run stops if the running time reaches the cut-off time limit ( $t_{\max}$ ). Following the literature [28], we set the time limit  $t_{\max}$  to  $n$ , where  $n$  is the number of elements in the considered test instance. To run the ILS\_MinDiff algorithm, there are three parameters to be determined, including search depth  $nbr_{\max}$ , weak perturbation strength  $p_w$  in the local optima exploring phase and strong perturbation coefficient  $\alpha$  in the local optima escaping phase. These parameters were fixed according to the experimental analysis of Section 5:  $nbr_{\max} = 5$  for instances of all six data sets;  $p_w = 3$  for instances with  $n < 500$  or  $n = 500, n/m < 10$ , and  $p_w = 2$  for the remaining instances;  $\alpha = 1.0$  for all instances. A detailed description of the parameter settings is provided in Table 2. It would be possible that fine tuning these parameters or using relaxed stopping conditions would lead to better results. As

<sup>1</sup> <http://www.opticom.es/mindiff/>.

<sup>2</sup> The best solution certificates and the code of our ILS\_MinDiff algorithm will be made available at <http://www.info.univ-angers.fr/pub/ha0/mindiffdp.html>.

<sup>3</sup> `dfmax`: <ftp://dimacs.rutgers.edu/pub/dsj/cliique>.

**Table 2**  
Parameter settings of the proposed ILS\_MinDiff algorithm.

Parameters	Section	Description	Value
$t_{\max}$	3.1	time limit	$n$
$nbr_{\max}$	3.4	search depth	5
$p_w$	3.4	weak perturbation strength	{2, 3}
$\alpha$	3.5	strong perturbation coefficient	1.0

we show below, with the adopted parameter settings, ILS\_MinDiff already performs very well relative to the state-of-the-art results.

#### 4.3. Comparison with the state-of-the-art algorithms

As indicated in the introduction, several main approaches have been recently proposed in the literature to solve *Min-Diff DP*, including mixed integer programming (MIP) and greedy randomized adaptive search procedure (GRASP) in 2009 [30], GRASP with exterior path relinking (GRASP\_EPR) in 2015 [13] and variable neighborhood search (VNS\_MinDiff) in 2016 [28]. The computational results reported in [13] indicate that GRASP\_EPR dominates MIP and GRASP of [30]. In [28], it was shown that the latest VNS\_MinDiff algorithm performs the best by updating the best-known solutions for 170 out of 190 benchmark instances which were previously established by GRASP\_EPR of [13]. Consequently, we adopted GRASP\_EPR and VNS\_MinDiff as our main reference algorithms to assess the performance of the proposed ILS\_MinDiff algorithm. Detailed computational results of GRASP\_EPR and VNS\_MinDiff were extracted from <http://www.opticom.es/mindiff/> and <http://www.mi.sanu.ac.rs/~nenad/mddp/> respectively. GRASP\_EPR was implemented in Java 7 and the experiments were conducted on an Intel Core i7 2600 (3.4 GHz) with 4GB of RAM. Each test instance was solved only one time with the time limit  $t_{\max} = n$ . VNS\_MinDiff was coded in C++ and run on a computer with an Intel Core i7 2600 CPU (3.4 GHz) and 16 GB of RAM. Each instance was solved forty times with the time limit  $t_{\max} = n$  per run.

Given that the compared algorithms were executed on different platforms with different configurations, it seems difficult to strictly compare the runtimes. Therefore, we use *solution quality* as the main criterion for our comparative studies. Nevertheless, we also report the computing times consumed by the compared

algorithms, which can still provide some useful indications about the computational efficiency of each algorithm. We note the computers used to run GRASP\_EPR and VNS\_MinDiff are roughly 1.2 ( $3.4/2.83 \approx 1.2$ ) times faster than our computer according to their CPU frequencies. So the stopping condition  $t_{\max} = n$  used is slightly more favorable for the reference algorithms than for our ILS\_MinDiff algorithm.

The comparative results of the proposed ILS\_MinDiff algorithm and the reference algorithms (VNS\_MinDiff and GRASP\_EPR) are presented in Tables 3–8. In these tables, column 1 gives the name of each instance (Instance). Columns 2–3 show the results of GRASP\_EPR, i.e., the best objective value ( $f_{\text{best}}$ ) attained during a single run and the CPU time ( $t_{\text{best}}$ ) to attain the best objective value. Columns 4–7 report the results of VNS\_MinDiff, including the best objective value ( $f_{\text{best}}$ ), the average objective value ( $f_{\text{avg}}$ ), the worst objective value ( $f_{\text{worst}}$ ) and the average time ( $t_{\text{avg}}$ ) to attain the best objective value of the forty runs (i.e.,  $t_{\text{avg}} = \sum_{i=1}^{40} t_{\text{best}}^i / 40$  where  $t_{\text{best}}^i$  is the time of  $i$ th run to attain its best objective value. The same statistics of the proposed ILS\_MinDiff algorithm are shown in columns 8–11. For ILS\_MinDiff, we also report the standard deviation ( $\sigma$ ), while this information is not available for VNS\_MinDiff and not applicable for GRASP\_EPR (since it was run once). The last column indicates the difference  $\Delta f_{\text{best}}$  between the best solution values found by ILS\_MinDiff and the best known solution values reported in the literature (a negative/positive value indicates an improved/worsen result). The best values among the results of the compared algorithms are highlighted in bold. The last row of each table indicates the average value of each comparison indicator. We complete this information by including between the parentheses the number of instances for which an algorithm obtained the best performance compared to the other algorithms (if two algorithms report the same best performance on one instance, the instance was counted 0.5 for each algorithm).

To analyze the experimental results, we resort to the well-known *two-tailed sign test* [10] to check the significant difference on each comparison indicator between the compared algorithms. The two-tailed sign test is a popular technique to compare the overall performances of algorithms by counting the number of instances for which an algorithm is the overall winner. When two algorithms are compared, the corresponding null-hypothesis is

**Table 3**  
Comparative results of the proposed ILS\_MinDiff algorithm and reference algorithms on SOM-b.

Instance	GRASP_EPR [13] (2015)		VNS_MinDiff [28] (2016)				ILS_MinDiff					
	$f_{best}$	$t_{best}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$\sigma$	$\Delta f_{best}$
SOM-b_1_n100_m10	2.00	0.70	1.00	1.00	<b>1.00</b>	1.24	<b>0.00</b>	<b>0.83</b>	<b>1.00</b>	12.06	0.38	−1.00
SOM-b_2_n100_m20	6.00	3.04	<b>4.00</b>	4.50	<b>5.00</b>	23.80	<b>4.00</b>	<b>4.40</b>	<b>5.00</b>	29.62	0.49	0.00
SOM-b_3_n100_m30	10.00	5.80	8.00	8.60	<b>9.00</b>	18.06	<b>7.00</b>	<b>7.88</b>	<b>9.00</b>	23.82	0.40	−1.00
SOM-b_4_n100_m40	13.00	8.72	12.00	12.20	13.00	30.55	<b>10.00</b>	<b>10.98</b>	<b>12.00</b>	34.41	0.57	−2.00
SOM-b_5_n200_m20	5.00	5.93	<b>3.00</b>	<b>3.90</b>	<b>4.00</b>	68.52	<b>3.00</b>	4.00	5.00	64.52	0.39	0.00
SOM-b_6_n200_m40	13.00	24.92	10.00	10.50	<b>11.00</b>	87.63	<b>9.00</b>	<b>10.40</b>	<b>11.00</b>	47.47	0.54	−1.00
SOM-b_7_n200_m60	19.00	51.93	16.00	16.70	18.00	75.09	<b>15.00</b>	<b>15.88</b>	<b>17.00</b>	67.74	0.51	−1.00
SOM-b_8_n200_m80	27.00	74.15	22.00	24.00	26.00	58.43	<b>19.00</b>	<b>21.50</b>	<b>23.00</b>	76.80	0.78	−3.00
SOM-b_9_n300_m30	9.00	23.38	7.00	7.40	<b>8.00</b>	83.01	<b>6.00</b>	<b>7.03</b>	<b>8.00</b>	84.45	0.42	−1.00
SOM-b_10_n300_m60	17.00	88.86	15.00	16.20	17.00	97.65	<b>14.00</b>	<b>15.55</b>	<b>16.00</b>	103.76	0.59	−1.00
SOM-b_11_n300_m90	27.00	173.61	22.00	24.10	26.00	94.03	<b>21.00</b>	<b>22.98</b>	<b>24.00</b>	107.44	0.72	−1.00
SOM-b_12_n300_m120	36.00	300.00	29.00	31.90	34.00	135.42	<b>28.00</b>	<b>30.13</b>	<b>32.00</b>	110.24	0.95	−1.00
SOM-b_13_n400_m40	12.00	53.34	10.00	10.40	11.00	80.32	<b>9.00</b>	<b>9.85</b>	<b>10.00</b>	118.24	0.36	−1.00
SOM-b_14_n400_m80	24.00	239.43	<b>19.00</b>	21.30	23.00	165.73	<b>19.00</b>	<b>20.53</b>	<b>21.00</b>	123.43	0.55	0.00
SOM-b_15_n400_m120	38.00	400.00	30.00	31.70	34.00	204.59	<b>28.00</b>	<b>30.03</b>	<b>32.00</b>	160.33	0.76	−2.00
SOM-b_16_n400_m160	54.00	400.00	40.00	43.40	47.00	255.04	<b>36.00</b>	<b>38.73</b>	<b>41.00</b>	188.42	1.27	−4.00
SOM-b_17_n500_m50	13.00	114.40	12.00	12.80	<b>13.00</b>	195.87	<b>11.00</b>	<b>12.33</b>	<b>13.00</b>	158.23	0.52	−1.00
SOM-b_18_n500_m100	26.00	500.00	<b>23.00</b>	25.10	27.00	232.08	<b>23.00</b>	<b>24.70</b>	<b>26.00</b>	238.64	0.87	0.00
SOM-b_19_n500_m150	47.00	500.00	36.00	39.60	45.00	248.05	<b>34.00</b>	<b>37.00</b>	<b>38.00</b>	224.23	1.00	−2.00
SOM-b_20_n500_m200	69.00	500.00	49.00	56.40	63.00	268.57	<b>43.00</b>	<b>47.10</b>	<b>50.00</b>	264.56	1.50	−6.00
Average value (wins)	23.35	173.41	18.40(2)	20.09(1)	21.75(4)	121.47	<b>16.95(18)</b>	<b>18.59(19)</b>	<b>19.70(16)</b>	111.92	0.68	−1.45

**Table 4**

Comparative results of the proposed ILS\_MinDiff algorithm and reference algorithms on GKD-b.

Instance	GRASP_EPR [13] (2015)		VNS_MinDiff [13] (2015)				ILS_MinDiff					
	$f_{best}$	$t_{best}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$\sigma$	$\Delta f_{best}$
GKD-b_1_n25_m2	<b>0.00</b>	0.00	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	0.00	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	0.00	0.00	0.00
GKD-b_2_n25_m2	<b>0.00</b>	0.00	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	0.00	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	0.00	0.00	0.00
GKD-b_3_n25_m2	<b>0.00</b>	0.02	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	0.00	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	0.00	0.00	0.00
GKD-b_4_n25_m2	<b>0.00</b>	0.02	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	0.00	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	0.00	0.00	0.00
GKD-b_5_n25_m2	<b>0.00</b>	0.02	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	0.00	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	0.00	0.00	0.00
GKD-b_6_n25_m7	<b>12.72</b>	0.17	<b>12.72</b>	<b>12.72</b>	<b>12.72</b>	0.00	<b>12.72</b>	<b>12.72</b>	<b>12.72</b>	0.12	0.00	0.00
GKD-b_7_n25_m7	<b>14.10</b>	0.16	<b>14.10</b>	<b>14.10</b>	<b>14.10</b>	0.00	<b>14.10</b>	<b>14.10</b>	<b>14.10</b>	5.26	0.00	0.00
GKD-b_8_n25_m7	<b>16.76</b>	0.16	<b>16.76</b>	<b>16.76</b>	<b>16.76</b>	0.00	<b>16.76</b>	<b>16.76</b>	<b>16.76</b>	6.37	0.00	0.00
GKD-b_9_n25_m7	<b>17.07</b>	0.17	<b>17.07</b>	<b>17.07</b>	<b>17.07</b>	0.00	<b>17.07</b>	<b>17.07</b>	<b>17.07</b>	1.07	0.00	0.00
GKD-b_10_n25_m7	<b>23.27</b>	0.31	<b>23.27</b>	<b>23.27</b>	<b>23.27</b>	0.00	<b>23.27</b>	<b>23.27</b>	<b>23.27</b>	5.72	0.00	0.00
GKD-b_11_n50_m5	<b>1.93</b>	0.19	<b>1.93</b>	<b>1.93</b>	<b>1.93</b>	0.01	<b>1.93</b>	<b>1.93</b>	<b>1.93</b>	12.52	0.00	0.00
GKD-b_12_n50_m5	<b>2.12</b>	0.17	<b>2.05</b>	<b>2.05</b>	<b>2.05</b>	0.04	<b>2.05</b>	<b>2.05</b>	<b>2.05</b>	3.90	0.00	0.00
GKD-b_13_n50_m5	<b>2.36</b>	0.19	<b>2.36</b>	<b>2.36</b>	<b>2.36</b>	0.02	<b>2.36</b>	<b>2.36</b>	<b>2.36</b>	1.79	0.00	0.00
GKD-b_14_n50_m5	<b>1.66</b>	0.19	<b>1.66</b>	<b>1.66</b>	<b>1.66</b>	0.01	<b>1.66</b>	<b>1.66</b>	<b>1.66</b>	10.80	0.00	0.00
GKD-b_15_n50_m5	<b>2.85</b>	0.19	<b>2.85</b>	<b>2.85</b>	<b>2.85</b>	0.00	<b>2.85</b>	<b>2.85</b>	<b>2.85</b>	5.25	0.00	0.00
GKD-b_16_n50_m15	<b>42.75</b>	1.39	<b>42.75</b>	<b>42.75</b>	<b>42.75</b>	0.01	<b>42.75</b>	<b>42.75</b>	<b>42.75</b>	13.74	0.00	0.00
GKD-b_17_n50_m15	<b>48.11</b>	1.61	<b>48.11</b>	<b>48.11</b>	<b>48.11</b>	0.00	<b>48.11</b>	<b>48.11</b>	<b>48.11</b>	8.83	0.00	0.00
GKD-b_18_n50_m15	<b>43.20</b>	1.34	<b>43.20</b>	<b>43.20</b>	<b>43.20</b>	0.02	<b>43.20</b>	<b>43.20</b>	<b>43.20</b>	9.53	0.00	0.00
GKD-b_19_n50_m15	<b>46.41</b>	1.36	<b>46.41</b>	<b>46.41</b>	<b>46.41</b>	0.25	<b>46.41</b>	<b>46.41</b>	<b>46.41</b>	13.40	0.00	0.00
GKD-b_20_n50_m15	<b>47.72</b>	1.27	<b>47.72</b>	<b>47.72</b>	<b>47.72</b>	0.00	<b>47.72</b>	<b>47.72</b>	<b>47.72</b>	16.15	0.00	0.00
GKD-b_21_n100_m10	13.83	1.17	<b>9.33</b>	9.38	9.50	32.65	<b>9.33</b>	<b>9.34</b>	<b>9.43</b>	30.15	0.02	0.00
GKD-b_22_n100_m10	13.66	1.17	<b>8.04</b>	9.01	10.22	38.35	<b>8.04</b>	<b>8.46</b>	<b>9.58</b>	45.43	0.41	0.00
GKD-b_23_n100_m10	15.35	1.08	<b>6.91</b>	7.63	9.10	29.66	<b>6.91</b>	<b>7.16</b>	<b>8.64</b>	52.55	0.42	0.00
GKD-b_24_n100_m10	8.64	1.20	7.17	7.65	<b>7.94</b>	43.32	<b>5.79</b>	<b>7.24</b>	8.94	53.52	0.76	-1.38
GKD-b_25_n100_m10	17.20	1.33	<b>6.91</b>	8.65	10.43	46.92	6.92	<b>8.43</b>	<b>10.28</b>	58.28	0.91	+0.01
GKD-b_26_n100_m30	168.73	9.44	<b>159.19</b>	160.37	166.31	9.41	<b>159.19</b>	<b>159.19</b>	<b>159.19</b>	43.59	0.00	0.00
GKD-b_27_n100_m30	127.10	9.72	<b>124.17</b>	125.05	127.10	25.58	<b>124.17</b>	<b>124.17</b>	<b>124.17</b>	35.40	0.00	0.00
GKD-b_28_n100_m30	<b>106.38</b>	10.42	<b>106.38</b>	<b>106.38</b>	<b>106.38</b>	16.34	<b>106.38</b>	<b>106.38</b>	<b>106.38</b>	31.37	0.00	0.00
GKD-b_29_n100_m30	137.45	10.05	<b>135.85</b>	<b>135.85</b>	<b>135.85</b>	33.27	<b>135.85</b>	<b>135.85</b>	<b>135.85</b>	39.90	0.00	0.00
GKD-b_30_n100_m30	127.48	9.28	<b>127.27</b>	<b>127.27</b>	<b>127.27</b>	10.03	<b>127.27</b>	<b>127.27</b>	<b>127.27</b>	38.36	0.00	0.00
GKD-b_31_n125_m12	11.75	3.14	<b>11.05</b>	<b>11.05</b>	<b>11.05</b>	0.85	<b>11.05</b>	<b>11.05</b>	<b>11.05</b>	38.21	0.00	0.00
GKD-b_32_n125_m12	18.79	2.22	11.60	12.62	<b>13.78</b>	55.97	<b>10.36</b>	<b>12.41</b>	14.80	65.07	1.07	-1.24
GKD-b_33_n125_m12	18.53	2.50	<b>9.18</b>	11.47	12.95	48.14	<b>9.18</b>	<b>10.16</b>	<b>12.58</b>	63.48	0.88	0.00
GKD-b_34_n125_m12	19.49	2.26	11.81	<b>12.78</b>	<b>13.49</b>	62.27	<b>10.79</b>	12.81	14.37	59.28	0.79	-1.02
GKD-b_35_n125_m12	18.11	2.31	<b>7.53</b>	9.39	11.69	47.38	<b>7.53</b>	<b>8.72</b>	<b>10.41</b>	52.72	0.91	0.00
GKD-b_36_n125_m37	155.43	17.74	<b>125.55</b>	148.48	181.35	32.20	<b>125.55</b>	<b>129.58</b>	<b>133.17</b>	59.51	1.79	0.00
GKD-b_37_n125_m37	198.89	19.44	<b>194.22</b>	205.60	232.00	32.58	<b>194.22</b>	<b>194.30</b>	<b>195.80</b>	48.05	0.35	0.00
GKD-b_38_n125_m37	187.97	18.71	<b>184.27</b>	190.19	213.88	16.69	<b>184.27</b>	<b>184.27</b>	<b>184.27</b>	48.78	0.00	0.00
GKD-b_39_n125_m37	168.59	18.43	159.48	168.84	181.13	62.17	<b>155.39</b>	<b>158.74</b>	<b>160.83</b>	60.80	1.60	-4.09
GKD-b_40_n125_m37	178.19	18.18	174.34	186.65	205.13	56.29	<b>172.80</b>	<b>178.58</b>	<b>181.78</b>	60.16	1.63	-1.54
GKD-b_41_n150_m15	23.35	4.39	17.40	19.60	<b>20.98</b>	64.52	<b>15.85</b>	<b>19.56</b>	22.44	69.33	1.54	-1.55
GKD-b_42_n150_m15	26.79	4.59	18.20	19.42	<b>20.94</b>	73.41	<b>13.96</b>	<b>17.96</b>	21.18	76.20	1.63	-4.24
GKD-b_43_n150_m15	26.75	4.15	15.57	17.66	19.93	58.30	<b>11.83</b>	<b>16.56</b>	<b>19.56</b>	62.96	1.76	-3.74
GKD-b_44_n150_m15	25.94	4.32	15.16	<b>17.58</b>	<b>19.28</b>	87.71	<b>11.74</b>	17.73	21.33	71.02	1.87	-3.42
GKD-b_45_n150_m15	27.77	4.36	15.23	18.38	21.77	65.09	<b>12.84</b>	<b>17.83</b>	<b>20.50</b>	75.93	1.87	-2.39
GKD-b_46_n150_m45	227.75	34.37	<b>207.81</b>	225.89	258.53	41.74	<b>207.81</b>	<b>209.38</b>	<b>214.56</b>	72.08	2.79	0.00
GKD-b_47_n150_m45	228.60	34.57	214.42	228.62	259.86	56.65	<b>211.77</b>	<b>214.14</b>	<b>218.14</b>	74.34	1.61	-2.65
GKD-b_48_n150_m45	226.75	30.27	180.00	221.04	239.52	48.94	<b>177.29</b>	<b>184.01</b>	<b>192.36</b>	69.80	3.71	-2.71
GKD-b_49_n150_m45	226.41	36.04	205.39	227.51	288.07	37.33	<b>197.88</b>	<b>201.12</b>	<b>206.75</b>	80.44	3.07	-7.51
GKD-b_50_n150_m45	248.86	33.04	<b>220.76</b>	245.17	279.55	80.03	<b>220.76</b>	<b>225.78</b>	<b>234.81</b>	78.84	4.54	0.00
Average value (wins)	66.47	7.18	60.26(19)	64.36(14)	70.76(18)	26.28	<b>59.51(31)</b>	<b>60.82(36)</b>	<b>62.27(32)</b>	36.60	0.72	-0.75

that two algorithms are equivalent. The null-hypothesis is accepted if and only if each algorithm wins on approximately  $X/2$  out of  $X$  instances. Since tied matches support the null-hypothesis [10], we split them evenly between the two compared algorithms, i.e., each one wins 0.5.

We first compare GRASP\_EPR with VNS\_MinDiff and ILS\_MinDiff. Since GRASP\_EPR was run only *once* to solve each instance while VNS\_MinDiff and ILS\_MinDiff were run *forty times* per instance, it is unfair to compare the best results of GRASP\_EPR with the best results of VNS\_MinDiff and ILS\_MinDiff. Instead, we can compare the *best* objective values of GRASP\_EPR with the *average* objective values of VNS\_MinDiff and ILS\_MinDiff since the average result can be reasonably considered as a representative single run. As we observe from Tables 3–8, both VNS\_MinDiff and ILS\_MinDiff dominate GRASP\_EPR for all six data sets according to this indicator. The results also indicate that even the

worst results of VNS\_MinDiff and ILS\_MinDiff (in particular, those of ILS\_MinDiff) are better than the best results of GRASP\_EPR apart from a few exceptions. Concerning the computing efficiency, given that the compared algorithms obtain solutions of quite different quality, it is not meaningful to compare their computing times. Based on this comparison, we decided to exclude GRASP\_EPR for further comparisons and focus our study on a performance assessment of ILS\_MinDiff with respect to VNS\_MinDiff.

Based on the results of ILS\_MinDiff and VNS\_MinDiff reported in Tables 3–8, we observe first that the two-tailed sign test rejected the null hypothesis in all six data sets, suggesting these two algorithms do not have an equal performance. At a significance level of 0.05, the *Critical Values* (CV) of the two-tailed sign test were respectively  $CV_{0.05}^{20} = 15$ ,  $CV_{0.05}^{40} = 27$ , and  $CV_{0.05}^{50} = 32$  when the number of instances in each data set is  $X = 20$ ,  $X = 40$ , and

**Table 5**

Comparative results of the proposed ILS\_MinDiff algorithm and reference algorithms on GKD-c.

Instance	GRASP_EPR [13] (2015)		VNS_MinDiff [13] (2015)				ILS_MinDiff					
	$f_{best}$	$t_{best}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$\sigma$	$\Delta f_{best}$
GKD-c_1_n500_m50	16.85	186.20	10.09	12.09	13.63	344.96	<b>9.85</b>	<b>10.85</b>	<b>11.93</b>	221.70	0.54	−0.24
GKD-c_2_n500_m50	16.53	189.93	10.79	12.98	15.41	351.47	<b>10.06</b>	<b>11.64</b>	<b>12.79</b>	202.54	0.71	−0.73
GKD-c_3_n500_m50	18.50	181.71	<b>8.84</b>	11.81	16.29	366.57	9.87	<b>11.71</b>	<b>13.26</b>	262.49	0.68	+1.03
GKD-c_4_n500_m50	18.87	173.69	<b>9.42</b>	12.82	15.99	436.28	9.55	<b>11.51</b>	<b>13.60</b>	262.46	0.70	+0.13
GKD-c_5_n500_m50	18.45	182.91	11.26	13.53	16.00	235.73	<b>10.36</b>	<b>12.03</b>	<b>13.11</b>	267.60	0.68	−0.90
GKD-c_6_n500_m50	17.92	183.83	10.63	12.14	14.49	367.35	<b>10.21</b>	<b>11.36</b>	<b>12.37</b>	254.12	0.56	−0.42
GKD-c_7_n500_m50	17.54	173.60	11.58	13.71	16.65	158.51	<b>10.47</b>	<b>12.27</b>	<b>13.45</b>	253.01	0.66	−1.11
GKD-c_8_n500_m50	19.86	186.61	11.31	13.79	20.50	341.60	<b>9.89</b>	<b>11.55</b>	<b>12.88</b>	287.74	0.70	−1.42
GKD-c_9_n500_m50	17.96	169.37	10.45	13.71	17.64	320.03	<b>9.69</b>	<b>11.29</b>	<b>12.61</b>	262.99	0.68	−0.76
GKD-c_10_n500_m50	17.10	180.95	<b>9.21</b>	13.94	18.45	358.62	10.99	<b>12.14</b>	<b>13.24</b>	288.20	0.56	+1.78
GKD-c_11_n500_m50	15.77	184.50	11.03	12.39	14.59	370.01	<b>9.03</b>	<b>10.99</b>	<b>12.10</b>	206.79	0.76	−2.00
GKD-c_12_n500_m50	17.71	179.79	<b>9.48</b>	12.98	15.06	303.82	9.82	<b>11.42</b>	<b>12.60</b>	203.12	0.63	+0.34
GKD-c_13_n500_m50	17.04	184.04	10.04	12.51	15.38	351.28	<b>9.94</b>	<b>12.10</b>	<b>13.86</b>	261.22	0.78	−0.10
GKD-c_14_n500_m50	19.27	181.15	11.28	12.88	15.60	274.72	<b>9.24</b>	<b>11.54</b>	<b>12.48</b>	300.43	0.62	−2.04
GKD-c_15_n500_m50	17.65	177.48	10.85	13.78	17.59	278.53	<b>9.53</b>	<b>12.11</b>	<b>13.51</b>	219.81	0.88	−1.32
GKD-c_16_n500_m50	16.32	179.78	<b>8.39</b>	12.30	15.11	349.12	10.04	<b>11.62</b>	<b>12.51</b>	268.00	0.58	+1.65
GKD-c_17_n500_m50	17.56	180.31	10.14	11.52	13.29	350.12	<b>9.90</b>	<b>11.16</b>	<b>12.31</b>	261.73	0.60	−0.24
GKD-c_18_n500_m50	19.03	180.01	<b>9.77</b>	13.47	17.06	289.34	10.56	<b>12.04</b>	<b>13.03</b>	217.42	0.56	+0.79
GKD-c_19_n500_m50	18.15	192.12	11.11	13.56	16.36	276.61	<b>10.25</b>	<b>11.73</b>	<b>12.67</b>	219.49	0.62	−0.86
GKD-c_20_n500_m50	18.53	182.48	10.44	12.82	15.12	354.73	<b>10.10</b>	<b>11.52</b>	<b>12.74</b>	273.15	0.62	−0.34
Average value (wins)	17.83	181.52	10.31(6)	12.94(0)	16.01(0)	323.97	<b>9.97(14)</b>	<b>11.63(20)</b>	<b>12.85(20)</b>	249.70	0.65	−0.34

$X = 50$ . This means that algorithm A is significantly better than algorithm B if A wins at least  $CV_{0.05}^X$  instances for a data set of  $X$  instances. Below, we analyze the performances of ILS\_MinDiff and VNS\_MinDiff for each of the six data sets.

From Table 3 which concerns the 20 instances of data set *SOM-b*, it can be observed that the proposed ILS\_MinDiff algorithm competes very favorably with the reference VNS\_MinDiff algorithm in all listed indicators. For this data set, ILS\_MinDiff easily dominates VNS\_MinDiff by improving the best-known results for 16 out of 20 instances and matching the best-known upper bounds for the remaining 4 instances. In addition, ILS\_MinDiff outperforms the reference algorithm in terms of the average solution value as well as the worst solution value. To achieve these best results, ILS\_MinDiff also spends much less time than that of the VNS\_MinDiff algorithm. A small standard deviation over forty runs demonstrates that ILS\_MinDiff has a stable performance. In this case,  $CV_{0.05}^{20} = 15$ , the two-tailed sign test confirms the statistical significance of the differences between these two algorithms in all comparison indicators on solution values.

Table 4 displays the comparative results of GRASP\_EPR, VNS\_MinDiff and ILS\_MinDiff on the 50 instances of data set *GKD-b*. As we can see from the table, the proposed ILS\_MinDiff algorithm achieves new best solutions (improved upper bounds) for 13 out of 50 instances and matches the best-known solutions for the remaining 37 instances. Moreover, compared to VNS\_MinDiff, ILS\_MinDiff respectively obtains a better average solution value and worst solution value for 36 and 32 out of 50 instances. It is worth mentioning that ILS\_MinDiff usually achieves a small standard deviation  $\sigma$ , and even  $\sigma = 0.0$  for 27 out of 50 instances, which shows that ILS\_MinDiff performs stably on this data set. Furthermore, the two-tailed sign test confirms that ILS\_MinDiff is significantly better than VNS\_MinDiff in terms of the average and worst values, while for the best solution value, the number of instances won by ILS\_MinDiff is only slightly smaller than the critical value, i.e.,  $31.5 < CV_{0.05}^{50} = 32$ .

Table 5 presents the comparative results of GRASP\_EPR, VNS\_MinDiff and ILS\_MinDiff on the 20 instances of data set *GKD-c*. From Table 5, we observe that ILS\_MinDiff achieves a new improved solution for 14 out of 20 instances. ILS\_MinDiff fully dominates the reference algorithm by obtaining a better average solution value and worst solution value for all 20 instances. In

addition, ILS\_MinDiff needs much less CPU times to achieve these results for almost all 20 instances. Although no significant difference is observed on the best solution values between ILS\_MinDiff and VNS\_MinDiff ( $14 < CV_{0.05}^{20} = 15$ ), ILS\_MinDiff significantly performs better than VNS\_MinDiff in terms of the average solution value and the worst solution value.

Table 6 shows the comparative results of ILS\_MinDiff with the reference algorithms on the 40 instances of data set *MDG-a*. These results show that ILS\_MinDiff again outperforms the reference algorithms. Specifically, in terms of the best solution value, ILS\_MinDiff finds a new best solution for 32 out of 40 instances and matches the best-known solutions for 2 instances. Concerning the average solution value, ILS\_MinDiff remains competitive, achieving a better average solution value for 30 out of 40 instances. In addition, ILS\_MinDiff also wins much more instances than VNS\_MinDiff, i.e., 21 instances for the worst solution value. With the two-tailed sign test, at a significance level of 0.05, we find that ILS\_MinDiff is significantly better than VNS\_MinDiff algorithm in terms of the best solution value ( $33 > CV_{0.05}^{40} = 27$ ) and average solution value ( $30 > CV_{0.05}^{40} = 27$ ).

Table 7 on the 40 instances of data set *MDG-b* demonstrates that ILS\_MinDiff achieves a highly competitive performance by obtaining a new best solution for 37 out of 40 instances. Moreover, ILS\_MinDiff achieves a better average solution value and worst solution value, and respectively wins 38 and 30 out of 40 instances with respect to VNS\_MinDiff. The two-tailed sign test confirms that the differences of these two algorithms are statistically significant. ILS\_MinDiff uses less time than VNS\_MinDiff for 26 out of 40 instances. It is only slightly smaller than the critical value of the statistical test at the significance level of 0.05, i.e.,  $26 < CV_{0.05}^{40} = 27$ . It is worth noting that the ILS\_MinDiff algorithm achieves these results with a relatively large average standard deviation 98.97, suggesting that ILS\_MinDiff is less stable on this data set than on the previous data sets.

Table 8 summaries the comparative results of ILS\_MinDiff, GRASP\_EPR and VNS\_MinDiff on the 20 instances of data set *MDG-c*.<sup>4</sup> From this table, we observe that our ILS\_MinDiff algorithm

<sup>4</sup> We observe that VNS\_MinDiff (and GRASP\_EPR) report abnormal computing times for several instances, exceeding the allowed time limit ( $t_{max} = n$ ). No information is available to explain these anomalies.



**Table 6**

Comparative results of the proposed ILS\_MinDiff algorithm and reference algorithms on MDG-a.

Instance	GRASP_EPR [13] (2015)		VNS_MinDiff [28] (2016)				ILS_MinDiff					
	$f_{best}$	$t_{best}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$\sigma$	$\Delta f_{best}$
MDG-a_1_n500_m50	13.53	179.47	11.34	12.26	<b>12.67</b>	151.84	<b>11.11</b>	<b>12.09</b>	12.99	259.36	0.43	−0.23
MDG-a_2_n500_m50	12.99	176.56	11.67	12.45	12.94	189.10	<b>11.00</b>	<b>12.10</b>	<b>12.86</b>	227.06	0.40	−0.67
MDG-a_3_n500_m50	13.34	172.91	11.71	12.22	12.82	311.65	<b>11.31</b>	<b>12.08</b>	<b>12.68</b>	253.43	0.32	−0.40
MDG-a_4_n500_m50	13.41	178.02	11.56	12.34	12.94	258.35	<b>11.35</b>	<b>12.05</b>	<b>12.65</b>	253.37	0.34	−0.21
MDG-a_5_n500_m50	13.50	164.69	12.05	12.50	12.77	233.59	<b>10.75</b>	<b>12.08</b>	<b>12.67</b>	275.97	0.41	−1.30
MDG-a_6_n500_m50	12.95	180.56	<b>10.87</b>	12.15	12.74	328.59	11.11	<b>12.07</b>	<b>12.72</b>	255.47	0.40	+0.24
MDG-a_7_n500_m50	13.09	173.27	10.95	12.14	13.17	292.69	<b>10.64</b>	<b>12.08</b>	<b>12.76</b>	234.95	0.46	−0.31
MDG-a_8_n500_m50	13.89	170.31	11.80	12.41	<b>13.00</b>	204.47	<b>10.16</b>	<b>11.99</b>	<b>13.00</b>	251.16	0.52	−1.64
MDG-a_9_n500_m50	13.61	176.66	11.54	12.37	12.80	207.14	<b>10.98</b>	<b>11.98</b>	<b>12.61</b>	250.43	0.40	−0.56
MDG-a_10_n500_m50	12.56	175.50	11.60	12.33	13.00	248.57	<b>11.04</b>	<b>12.01</b>	<b>12.57</b>	252.52	0.36	−0.56
MDG-a_11_n500_m50	13.21	174.29	11.25	12.12	<b>12.68</b>	117.75	<b>10.83</b>	<b>11.96</b>	12.71	237.47	0.47	−0.42
MDG-a_12_n500_m50	13.01	182.68	12.17	12.53	12.87	251.91	<b>11.12</b>	<b>12.03</b>	<b>12.69</b>	244.03	0.37	−1.05
MDG-a_13_n500_m50	12.70	170.06	12.05	12.41	12.99	298.51	<b>10.63</b>	<b>12.00</b>	<b>12.74</b>	216.85	0.46	−1.42
MDG-a_14_n500_m50	12.89	181.77	11.60	12.42	13.06	164.87	<b>10.77</b>	<b>11.94</b>	<b>12.79</b>	289.92	0.41	−0.83
MDG-a_15_n500_m50	13.51	178.36	11.55	12.39	12.91	221.15	<b>10.97</b>	<b>12.01</b>	<b>12.77</b>	274.32	0.42	−0.58
MDG-a_16_n500_m50	13.19	176.83	12.15	12.64	13.12	240.76	<b>10.65</b>	<b>12.04</b>	<b>12.76</b>	242.01	0.43	−1.50
MDG-a_17_n500_m50	12.48	180.14	11.76	12.32	<b>12.73</b>	276.39	<b>10.88</b>	<b>12.12</b>	12.94	293.07	0.45	−0.88
MDG-a_18_n500_m50	11.49	169.06	11.95	12.42	12.90	317.89	<b>10.88</b>	<b>12.12</b>	<b>12.60</b>	286.69	0.35	−1.07
MDG-a_19_n500_m50	13.50	177.66	<b>11.50</b>	<b>12.34</b>	12.93	241.46	11.57	<b>12.21</b>	12.71	249.01	0.28	+0.07
MDG-a_20_n500_m50	13.20	175.63	11.66	12.18	<b>12.60</b>	253.48	<b>11.10</b>	<b>12.17</b>	13.20	284.12	0.42	−0.56
MDG-a_21_n2000_m200	68.00	2000.00	<b>50.00</b>	<b>53.10</b>	<b>57.00</b>	1359.44	<b>50.00</b>	53.43	<b>57.00</b>	1342.38	1.79	0.00
MDG-a_22_n2000_m200	70.00	2000.01	51.00	53.60	<b>56.00</b>	1490.53	<b>50.00</b>	<b>53.55</b>	58.00	1446.83	2.20	−1.00
MDG-a_23_n2000_m200	63.00	2000.00	52.00	54.30	<b>57.00</b>	959.98	<b>49.00</b>	<b>53.60</b>	58.00	1380.25	2.49	−3.00
MDG-a_24_n2000_m200	63.00	2000.00	<b>48.00</b>	<b>53.00</b>	<b>58.00</b>	1348.31	50.00	53.63	<b>58.00</b>	1447.23	2.03	+2.00
MDG-a_25_n2000_m200	57.00	2000.00	51.00	54.30	<b>58.00</b>	1255.08	<b>50.00</b>	<b>53.60</b>	<b>58.00</b>	1420.26	1.95	−1.00
MDG-a_26_n2000_m200	68.00	2000.00	<b>49.00</b>	<b>53.00</b>	<b>57.00</b>	1136.47	50.00	53.58	<b>57.00</b>	1410.15	1.69	+1.00
MDG-a_27_n2000_m200	62.00	2000.00	50.00	54.70	<b>58.00</b>	1196.13	<b>49.00</b>	<b>53.73</b>	<b>58.00</b>	1406.19	2.18	−1.00
MDG-a_28_n2000_m200	64.00	2000.00	<b>48.00</b>	53.10	<b>57.00</b>	1280.44	50.00	<b>52.98</b>	59.00	1464.31	2.03	+2.00
MDG-a_29_n2000_m200	63.00	2000.01	51.00	<b>53.00</b>	<b>56.00</b>	1097.72	<b>47.00</b>	53.48	58.00	1542.65	2.61	−4.00
MDG-a_30_n2000_m200	65.00	2000.00	50.00	<b>54.00</b>	<b>57.00</b>	804.95	<b>49.00</b>	54.28	59.00	1469.70	2.93	−1.00
MDG-a_31_n2000_m200	67.00	2000.00	50.00	54.50	60.00	1084.71	<b>49.00</b>	<b>53.88</b>	<b>58.00</b>	1517.10	2.32	−1.00
MDG-a_32_n2000_m200	57.00	2000.00	51.00	54.60	61.00	1049.13	<b>48.00</b>	<b>53.25</b>	<b>57.00</b>	1383.72	1.84	−3.00
MDG-a_33_n2000_m200	67.00	2000.01	49.00	<b>53.70</b>	60.00	1315.45	<b>48.00</b>	53.80	<b>59.00</b>	1634.04	2.66	−1.00
MDG-a_34_n2000_m200	59.00	2000.00	<b>49.00</b>	<b>53.30</b>	<b>57.00</b>	1058.42	<b>49.00</b>	53.48	59.00	1388.50	2.23	0.00
MDG-a_35_n2000_m200	67.00	2000.00	53.00	54.70	<b>56.00</b>	1020.57	<b>48.00</b>	<b>54.08</b>	59.00	1431.79	2.18	−5.00
MDG-a_36_n2000_m200	57.00	2000.00	51.00	54.10	<b>57.00</b>	1300.08	<b>48.00</b>	<b>53.73</b>	59.00	1439.09	2.41	−3.00
MDG-a_37_n2000_m200	57.00	2000.00	49.00	<b>52.90</b>	<b>56.00</b>	1294.62	<b>48.00</b>	53.85	58.00	1398.92	2.37	−1.00
MDG-a_38_n2000_m200	65.00	2000.00	<b>48.00</b>	<b>53.60</b>	<b>57.00</b>	1263.25	49.00	53.83	58.00	1505.69	2.24	+1.00
MDG-a_39_n2000_m200	60.00	2000.00	51.00	54.00	<b>58.00</b>	1234.13	<b>48.00</b>	<b>53.48</b>	<b>58.00</b>	1451.99	2.36	−3.00
MDG-a_40_n2000_m200	62.00	2000.00	50.00	<b>53.20</b>	<b>56.00</b>	1056.70	<b>49.00</b>	54.03	59.00	1361.78	2.30	−1.00
Average value (wins)	38.08	1087.86	30.84(7)	33.04(10)	<b>35.17(19)</b>	709.86	<b>29.92(33)</b>	<b>32.86(30)</b>	35.49(21)	849.34	1.32	−0.92

overall performs competitively. In terms of the best solution value, ILS\_MinDiff significantly outperforms the reference algorithms by finding 19 improved best solutions. In terms of the average solution value and the worst solution value (only available for ILS\_MinDiff and VNS\_MinDiff), ILS\_MinDiff achieves a competitive performance compared with VNS\_MinDiff, by winning 11 and 9 out of 20 instances respectively. For the last 9 out of 10 large instances, VNS\_MinDiff has a better performance in terms of the average solution value and the worst solution value, but consumes much more time than ILS\_MinDiff. However, according to the explanation given at the beginning of this section, the computer used to run VNS\_MinDiff is about 1.2 times faster than our computer. To verify if ILS\_MinDiff can improve its results on these 9 instances with a running time comparable to that used by VNS\_MinDiff (i.e.,  $3000 \times 1.2 = 3600$  s), we re-ran ILS\_MinDiff with this relaxed time limit of 3600 s. ILS\_MinDiff actually achieved better results than VNS\_MinDiff in terms of the best, average and worst solution values. For example, the best, average, and worst values of ILS\_MinDiff for MDG-c\_11\_n3000\_m500 are respectively improved to 10295.00, 11123.25 and 12214.00 with an average time  $t_{avg} = 3237.36$  s. Similarly, for MDG-c\_12\_n3000\_m500, we obtained  $f_{best} = 9909.00$ ,  $f_{avg} = 10894.98$ ,  $f_{worst} = 12328.00$  and  $t_{avg} = 3132.11$ .

Finally, Table 9 summarizes the performances of the proposed ILS\_MinDiff algorithm against the best performing reference

VNS\_MinDiff algorithm on all 190 benchmark instances. The significant differences are marked in bold. From this table, we can make the following observations:

- ILS\_MinDiff performs highly competitively compared to the state-of-the-art results, by winning 152.5, 154 and 128 out of 190 instances in terms of the best solution value, the average solution value and the worst solution value respectively.
- ILS\_MinDiff performs significantly better than the current best algorithm (VNS\_MinDiff) in terms of the best solution value for 4 out of 6 data sets (SOM-b, MDG-a, MDG-b and MDG-c). For GKD-b and GKD-c, ILS\_MinDiff respectively wins 31.5 and 14 instances, which are just slightly smaller than the corresponding critical values (i.e.,  $Cv_{0.05}^{50} = 32$  and  $Cv_{0.05}^{20} = 15$ ).
- Compared to the reference algorithm, ILS\_MinDiff significantly performs better in terms of the average solution value for all data sets except for MDG-c. Although the difference between ILS\_MinDiff and VNS\_MinDiff is not significant on MDG-c, ILS\_MinDiff still achieves a better average solution value for 11 out of 20 instances.
- ILS\_MinDiff performs significantly better than VNS\_MinDiff in terms of the worst solution value on SOM-b, GKD-b, GKD-c and MDG-b. For MDG-a and MDG-c, ILS\_MinDiff achieves a competitive performance, but the difference is not statistically significant.

**Table 7**

Comparative results of the proposed ILS\_MinDiff algorithm and reference algorithms on MDG-b.

Instance	GRASP_EPR [13] (2015)		VNS_MinDiff [28] (2016)				ILS_MinDiff					
	$f_{best}$	$t_{best}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$\sigma$	$\Delta f_{best}$
MDG-b_1_n500_m50	1350.08	178.54	1185.11	1246.78	1296.49	266.17	<b>1118.48</b>	<b>1209.70</b>	<b>1265.10</b>	198.74	35.72	−66.63
MDG-b_2_n500_m50	1368.54	189.36	1182.48	1256.77	1322.03	245.14	<b>1082.03</b>	<b>1199.86</b>	<b>1249.69</b>	279.30	35.52	−100.45
MDG-b_3_n500_m50	1286.01	186.81	<b>1070.87</b>	1243.84	1310.09	331.21	1104.07	<b>1203.60</b>	<b>1265.66</b>	237.08	39.72	+33.20
MDG-b_4_n500_m50	1300.24	185.34	1153.93	1240.57	1287.46	239.95	<b>1052.27</b>	<b>1193.79</b>	<b>1258.01</b>	226.04	43.67	−101.66
MDG-b_5_n500_m50	1258.79	185.03	1209.80	1262.90	1317.82	186.06	<b>1051.91</b>	<b>1204.83</b>	<b>1275.55</b>	225.86	48.06	−157.89
MDG-b_6_n500_m50	1272.73	182.13	1071.61	1227.71	1319.86	298.82	<b>1061.50</b>	<b>1202.49</b>	<b>1292.94</b>	242.20	41.70	−10.11
MDG-b_7_n500_m50	1279.10	193.63	1099.68	1215.38	1311.55	256.32	<b>1076.57</b>	<b>1205.87</b>	<b>1260.31</b>	229.67	38.66	−23.11
MDG-b_8_n500_m50	1315.79	185.12	1185.59	1245.45	1316.97	247.01	<b>1005.45</b>	<b>1207.23</b>	<b>1278.51</b>	250.86	47.25	−180.14
MDG-b_9_n500_m50	1346.91	175.09	1154.33	1232.61	1261.83	243.90	<b>1116.63</b>	<b>1210.19</b>	<b>1260.14</b>	269.01	35.30	−37.70
MDG-b_10_n500_m50	1339.82	179.28	1198.08	1242.15	1289.55	272.05	<b>1092.25</b>	<b>1195.55</b>	<b>1264.05</b>	237.71	44.78	−105.83
MDG-b_11_n500_m50	1305.28	182.65	1145.73	1221.54	1275.68	249.64	<b>1090.59</b>	<b>1202.24</b>	<b>1256.12</b>	236.51	36.85	−55.14
MDG-b_12_n500_m50	1274.36	169.72	1165.43	1238.15	1294.60	252.95	<b>1093.89</b>	<b>1206.29</b>	<b>1287.75</b>	232.29	39.64	−71.54
MDG-b_13_n500_m50	1337.33	185.02	1180.43	1238.00	<b>188.52</b>	130.92	<b>1123.26</b>	<b>1205.87</b>	1296.97	268.81	37.48	−57.17
MDG-b_14_n500_m50	1291.06	191.77	1166.81	1247.25	1315.79	150.65	<b>1089.38</b>	<b>1200.78</b>	<b>1258.22</b>	264.85	36.19	−77.43
MDG-b_15_n500_m50	1278.86	186.00	1220.83	1273.74	1314.10	281.51	<b>1108.10</b>	<b>1207.73</b>	<b>1287.25</b>	231.45	41.54	−112.73
MDG-b_16_n500_m50	1328.66	180.79	1176.16	1248.31	1317.30	295.13	<b>1132.34</b>	<b>1217.24</b>	<b>1266.44</b>	313.70	31.32	−43.82
MDG-b_17_n500_m50	1299.00	179.15	1174.66	1252.52	1297.05	319.81	<b>1062.69</b>	<b>1200.76</b>	<b>1275.49</b>	228.43	44.04	−111.97
MDG-b_18_n500_m50	1321.87	174.22	1187.82	1267.94	1338.04	152.27	<b>1032.54</b>	<b>1198.20</b>	<b>1271.48</b>	253.42	45.10	−155.28
MDG-b_19_n500_m50	1333.22	172.76	1175.26	1257.81	1291.88	369.35	<b>1076.27</b>	<b>1206.94</b>	<b>1272.92</b>	236.22	39.22	−98.99
MDG-b_20_n500_m50	1328.53	172.66	1151.34	1233.04	1285.53	271.70	<b>1050.33</b>	<b>1199.99</b>	<b>1260.42</b>	242.06	42.23	−101.01
MDG-b_21_n2000_m200	5073.98	2000.00	4083.16	4468.62	4737.59	1025.80	<b>3978.52</b>	<b>4299.38</b>	<b>4552.68</b>	1366.04	138.11	−104.64
MDG-b_22_n2000_m200	5062.07	2000.00	4187.77	4540.42	4952.63	1039.80	<b>3911.34</b>	<b>4377.97</b>	<b>4678.26</b>	1466.30	178.38	−276.43
MDG-b_23_n2000_m200	4899.35	2000.00	4237.38	4489.07	5171.39	1327.39	<b>4127.34</b>	<b>4422.12</b>	<b>4858.86</b>	1591.88	183.29	−110.04
MDG-b_24_n2000_m200	4780.51	2000.00	4212.28	4452.33	4708.87	1002.08	<b>4088.26</b>	<b>4421.77</b>	<b>4705.92</b>	1513.38	147.62	−124.02
MDG-b_25_n2000_m200	5021.93	2000.00	4152.88	4435.36	4713.25	1375.81	<b>3892.67</b>	<b>4340.78</b>	<b>4577.78</b>	1463.02	132.66	−260.21
MDG-b_26_n2000_m200	4959.65	2000.00	<b>4039.92</b>	4497.39	4798.83	1317.73	4116.90	<b>4423.07</b>	<b>4775.00</b>	1535.76	147.65	+76.98
MDG-b_27_n2000_m200	4874.36	2000.00	<b>4010.77</b>	4486.90	4855.86	1079.71	4126.90	<b>4424.59</b>	<b>4711.53</b>	1599.67	150.02	+116.13
MDG-b_28_n2000_m200	5245.69	2000.00	4206.07	4498.25	<b>4798.32</b>	844.44	<b>4112.43</b>	<b>4446.16</b>	4975.79	1579.61	173.71	−93.64
MDG-b_29_n2000_m200	4955.58	2000.00	4214.79	4505.51	<b>4809.00</b>	1037.28	<b>4057.62</b>	<b>4377.08</b>	4846.05	1450.96	164.65	−157.17
MDG-b_30_n2000_m200	5045.63	2000.00	4272.07	4564.38	<b>4786.12</b>	1022.86	<b>4110.61</b>	<b>4470.64</b>	4885.52	1519.61	178.31	−161.47
MDG-b_31_n2000_m200	4962.72	2000.00	4328.97	4474.43	4710.96	1248.66	<b>4074.80</b>	<b>4323.11</b>	<b>4539.19</b>	1155.72	120.41	−254.17
MDG-b_32_n2000_m200	4833.29	2000.00	4226.55	4484.07	4664.58	1069.63	<b>3929.49</b>	<b>4301.35</b>	<b>4628.75</b>	1278.11	159.57	−297.06
MDG-b_33_n2000_m200	4973.32	2000.39	4037.50	4387.64	4786.52	1281.73	<b>3985.32</b>	<b>4351.01</b>	<b>4624.97</b>	1396.08	148.77	−52.18
MDG-b_34_n2000_m200	4880.74	2000.00	4279.58	4480.58	<b>4850.85</b>	1038.31	<b>4084.46</b>	<b>4402.11</b>	4965.12	1611.59	183.27	−195.12
MDG-b_35_n2000_m200	5061.54	2000.00	4018.60	<b>4367.05</b>	<b>4679.23</b>	1582.57	<b>4000.31</b>	4396.43	4824.11	1432.83	163.53	−18.30
MDG-b_36_n2000_m200	4963.93	2000.00	4231.38	<b>4433.05</b>	<b>4674.14</b>	1067.68	<b>4095.13</b>	4435.33	4955.35	1678.21	172.50	−136.25
MDG-b_37_n2000_m200	4801.03	2000.00	4100.54	4472.45	<b>4834.64</b>	1479.05	<b>4035.74</b>	<b>4409.06</b>	4873.27	1524.78	164.91	−64.80
MDG-b_38_n2000_m200	4946.67	2000.00	4136.67	4506.89	<b>4802.26</b>	1262.67	<b>4126.69</b>	<b>4418.53</b>	4817.94	1443.40	171.30	−9.98
MDG-b_39_n2000_m200	5095.33	2000.44	4242.30	4450.95	<b>4635.06</b>	1219.16	<b>4131.87</b>	<b>4403.46</b>	4807.67	1486.56	137.81	−110.43
MDG-b_40_n2000_m200	5001.89	2000.00	4249.76	4556.52	4804.78	1422.06	<b>4306.02</b>	<b>4306.02</b>	<b>4549.61</b>	1232.07	138.38	−226.72
Average value (wins)	3141.39	1090.90	2668.12(3)	2861.11(2)	3045.47(10)	720.12	<b>2565.75(37)</b>	<b>2795.73(38)</b>	<b>3013.91(30)</b>	856.57	98.97	−102.37

**Table 8**

Comparative results of the proposed ILS\_MinDiff algorithm and reference algorithms on MDG-c.

Instance	GRASP_EPR [13] (2015)		VNS_MinDiff [28] (2016)				ILS_MinDiff					
	$f_{best}$	$t_{best}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$f_{best}$	$f_{avg}$	$f_{worst}$	$t_{avg}$	$\sigma$	$\Delta f_{best}$
MDG-c_1_n3000_m300	7429.00	3001.50	6344.00	6595.40	7135.00	1455.45	<b>5772.00</b>	<b>6265.60</b>	<b>6794.00</b>	2322.74	290.56	−572.00
MDG-c_2_n3000_m300	7781.00	3001.59	6109.00	6651.50	7183.00	1320.28	<b>5936.00</b>	<b>6539.33</b>	<b>7114.00</b>	2772.71	300.02	−173.00
MDG-c_3_n3000_m300	7438.00	3001.63	6365.00	6828.70	7221.00	1639.72	<b>5585.00</b>	<b>6243.03</b>	<b>7006.00</b>	2501.82	277.13	−780.00
MDG-c_4_n3000_m300	7212.00	3001.71	6304.00	6787.10	7215.00	1294.78	<b>5969.00</b>	<b>6636.75</b>	<b>7168.00</b>	2841.41	299.52	−335.00
MDG-c_5_n3000_m300	7346.00	3001.48	5954.00	6729.30	<b>7282.00</b>	1648.19	<b>5750.00</b>	<b>6663.25</b>	7405.00	2750.35	343.32	−204.00
MDG-c_6_n3000_m400	10559.00	3002.86	8403.00	9422.10	10592.00	1861.19	<b>7648.00</b>	<b>8412.98</b>	<b>8999.00</b>	2781.03	334.32	−755.00
MDG-c_7_n3000_m400	9738.00	3003.16	8606.00	9308.60	9770.00	1847.39	<b>7829.00</b>	<b>8457.15</b>	<b>9522.00</b>	2837.35	373.83	−777.00
MDG-c_8_n3000_m400	10262.00	3002.85	8217.00	9206.80	10219.00	2009.81	<b>7984.00</b>	<b>8497.28</b>	<b>9033.00</b>	2699.22	240.93	−233.00
MDG-c_9_n3000_m400	10202.00	3003.00	8478.00	9140.50	10337.00	2082.95	<b>7657.00</b>	<b>8259.35</b>	<b>9128.00</b>	2626.88	371.85	−821.00
MDG-c_10_n3000_m400	9266.00	3003.02	8244.00	9372.30	<b>10129.00</b>	1821.70	<b>7672.00</b>	<b>8646.00</b>	10432.00	2992.54	589.01	−572.00
MDG-c_11_n3000_m500	13203.00	3005.46	11145.00	<b>11998.90</b>	<b>13151.00</b>	3034.47	<b>11031.00</b>	12223.38	14281.00	2840.16	679.63	−114.00
MDG-c_12_n3000_m500	13458.00	3005.06	11366.00	<b>12001.40</b>	<b>12709.00</b>	3138.85	<b>10604.00</b>	12103.03	13987.00	2832.65	782.51	−762.00
MDG-c_13_n3000_m500	11930.00	3004.86	10942.00	<b>11832.40</b>	<b>12427.00</b>	3322.91	<b>10743.00</b>	12228.58	14838.00	2867.29	949.06	−199.00
MDG-c_14_n3000_m500	13734.00	3005.04	10903.00	<b>11455.20</b>	<b>12095.00</b>	2736.52	<b>9941.00</b>	11643.90	15043.00	2833.35	1207.82	−962.00
MDG-c_15_n3000_m500	12091.00	3004.80	11051.00	<b>12311.90</b>	<b>13282.00</b>	3255.82	<b>10870.00</b>	12365.85	14298.00	2883.88	757.94	−181.00
MDG-c_16_n3000_m600	16682.00	3007.55	13934.00	<b>14732.10</b>	<b>15278.00</b>	3446.15	<b>13910.00</b>	15801.65	18285.00	2585.17	1065.75	−24.00
MDG-c_17_n3000_m600	16673.00	3007.45	14086.00	<b>14882.70</b>	<b>16184.00</b>	3549.47	<b>13676.00</b>	15284.10	17002.00	2899.82	712.02	−410.00
MDG-c_18_n3000_m600	15307.00	3007.09	<b>13415.00</b>	<b>14515.20</b>	<b>15385.00</b>	3536.68	14011.00	15547.08	16893.00	2925.46	589.07	+596.00
MDG-c_19_n3000_m600	14812.00	3007.68	13850.00	<b>14821.90</b>	<b>15976.00</b>	3905.60	<b>13338.00</b>	15526.85	16729.00	2890.80	753.32	−312.00
MDG-c_20_n3000_m600	14462.00	3007.16	13532.00	14651.80	15396.00	3653.64	<b>12415.00</b>	<b>13545.33</b>	<b>14697.00</b>	2879.54	709.70	−1117.00
Average value (wins)	11479.25	3004.25	9862.40(1)	10662.29(9)	<b>11448.30(11)</b>	2528.08	<b>9427.05(19)</b>	<b>10544.52(11)</b>	11932.70(9)	2791.86	581.37	−435.35

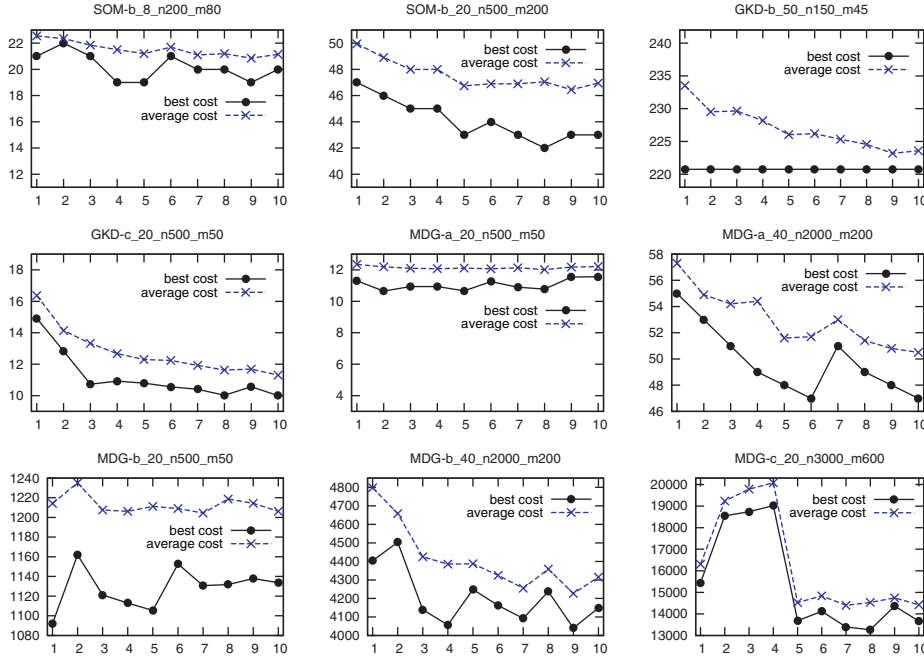


Fig. 2. Effect of the search depth  $nbr_{\max}$  on the performance of the ILS\_MinDiff algorithm.

Table 9

A summary of win statistics between the proposed ILS\_MinDiff algorithm (left part of each column) and the reference VNS\_MinDiff algorithm (right part of each column) on all six data sets.

Indicator	SOM-b	GKD-b	GKD-c	MDG-a	MDG-b	MDG-c
$f_{\text{best}}$	18 2	31.5 18.5	14 6	33 7	37 3	19 1
$f_{\text{avg}}$	19 1	36 14	20 0	30 10	38 2	11 9
$f_{\text{worst}}$	16 4	32 18	20 0	21 19	30 10	9 11

## 5. Experimental analysis

Now, we study the role of the parameters of ILS\_MinDiff: the search depth  $nbr_{\max}$ , the weak perturbation strength  $p_w$  and the strong perturbation coefficient  $\alpha$ . The analysis was based on a set of 9 instances selected from all six data sets, i.e., *SOM-b\_8\_n200\_m80*, *SOM-b\_20\_n500\_m200*, *GKD-b\_50\_n150\_m45*, *GKD-c\_20\_n500\_m50*, *MDG-a\_20\_n500\_m50*, *MDG-a\_40\_n2000\_m200*, *MDG-b\_20\_n500\_m50*, *MDG-b\_40\_n2000\_m200* and *MDG-c\_20\_n3000\_m600*. For each instance with  $n < 2000$ , we solved it 20 times, otherwise we solved it 10 times.

### 5.1. Effect of the search depth $nbr_{\max}$

To investigate the effect of the search depth  $nbr_{\max}$  on the performance of ILS\_MinDiff, we first fixed the weak perturbation strength  $p_w$  to 3 and the strong perturbation coefficient  $\alpha$  to 1.0, i.e.,  $p_s = 1.0 \times n/m$ , and then varied  $nbr_{\max}$  from 1.0 to 10.0 with a step size of 1. Fig. 2 shows the behavior of ILS\_MinDiff relative to  $nbr_{\max}$ , where the X-axis indicates the values of  $nbr_{\max}$  while the Y-axis shows the best/average objective values.

Fig. 2 reveals that the average solution values of ILS\_MinDiff continuously decrease when the values of  $nbr_{\max}$  increase from 1.0 to 10.0 for all tested instances except for *MDG-c\_20\_n3000\_m600*. For *MDG-c\_20\_n3000\_m600*, the curve shows a large variation and it also decreases when  $nbr_{\max}$  increases to 5. We also observe that ILS\_MinDiff obtains a superior best solution value when  $nbr_{\max}$  is 4 or 5 on all tested instances. This justifies the adopted setting ( $nbr_{\max} = 5$ ) shown in Table 2.

### 5.2. Effect of the weak perturbation strength $p_w$

To study the effect of the weak perturbation strength  $p_w$  on the performance of the proposed algorithm, we first fixed the search depth  $nbr_{\max}$  to 5 according to the outcomes of Section 5.1, and the strong perturbation coefficient  $\alpha$  to 1.0, i.e.,  $p_s = 1.0 \times n/m$ . Then, we varied the value of  $p_w$  from 1.0 to 10.0 with a step size of 1. Fig. 3 shows the performances of the ILS\_MinDiff with different values of  $p_w$  varying from 1.0 to 10.0, where the X-axis and Y-axis respectively shows the weak perturbation strength  $p_w$  and the best/average objective values.

Fig. 3 indicates that almost for all tested instances, the best performance was attained when  $p_w = 2$  or  $p_w = 3$ . A too large or small  $p_w$  value gave a poor performance of the ILS\_MinDiff algorithm. This can be understood given that a small  $p_w$  value cannot enable the search to jump out of the current local optimum trap, while a too large value of  $p_w$  will have an effect similar to a random restart. This justifies the adopted setting for  $p_w$  shown in Table 2.

### 5.3. Effect of the strong perturbation coefficient $\alpha$

The ability of the ILS\_MinDiff algorithm to escape deep local optimum traps depends on the strong perturbation strength  $p_s = \alpha \times n/m$ . To analyze the influence of the strong perturbation coefficient  $\alpha$  on the performance of the algorithm, we fixed the search depth  $nbr_{\max}$  and the weak perturbation strength  $p_w$  respectively according to the values determined from Figs. 2 and 3, and varied the strong perturbation coefficient  $\alpha$  from 1.0 to 1.9 with a step size of 0.1. Fig. 4 displays the outcomes of this experiment, where the X-axis and the Y-axis show the values of  $\alpha$  and the best/average objective values respectively.

As we observe from Fig. 4, the average solution values are not really sensitive to  $\alpha$  on almost all tested instances except for *MDG-c\_20\_n3000\_m600*. To achieve better average solution values, we roughly set  $\alpha = 1.0$  for all instances.

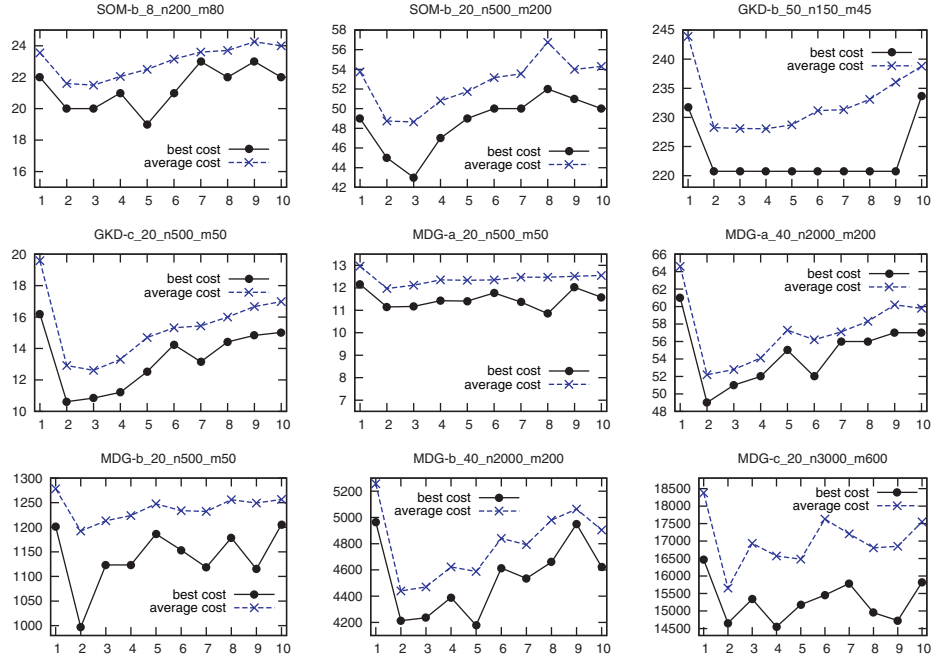


Fig. 3. Effect of the weak perturbation strength  $p_w$  on the performance of the ILS\_MinDiff algorithm.

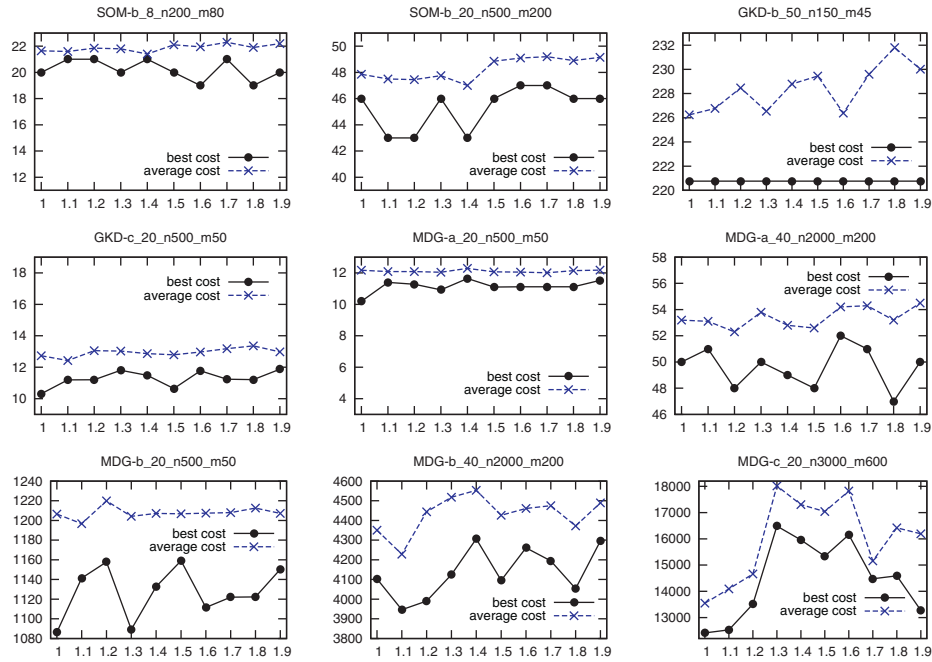


Fig. 4. Effect of the strong perturbation coefficient  $\alpha$  on the performance of the ILS\_MinDiff algorithm.

## 6. Conclusions

The minimum differential dispersion problem (*Min-Diff DP*) is a useful model for a variety of practical applications. However, finding high-quality solutions to large *Min-Diff DP* instances represents an imposing computational challenge. In this study, we have proposed ILS\_MinDiff, a highly effective iterated local search algorithm for *Min-Diff DP*, which adopts for the first time the general three-phase search framework to *Min-Diff DP*. The proposed algorithm uses the conventional swap neighborhood and integrates two specific features which distinguish itself from existing algorithms. It uses a local optima exploring procedure to locate nearby local optima within a limited search region and a local optima escaping

procedure to displace the search to distant region. In addition, being conceptually simple, ILS\_MinDiff was implemented with ease.

Extensive computational experiments on six data sets of 190 benchmark instances have demonstrated that despite its simplicity, the proposed algorithm competes very favorably with state-of-the-art methods in the literature. In particular, ILS\_MinDiff is able to find new best results for 131 out of 190 instances (improved upper bounds) and match the best-known results for 42 out of the remaining 59 instances. These improved results constitute useful new references for evaluating other *Min-Diff DP* heuristic algorithms. The computational results indicate that the three-phase search framework is a very suitable for the studied problem when



it is combined with the basic swap-based descent procedure and the local optima exploring and escaping procedures.

From this study, several perspectives can be contemplated for future studies. First, given its simplicity and effectiveness, the proposed algorithm can be advantageously employed as the local optimization component of a more complex hybrid algorithm (e.g., memetic search, evolutionary path-relinking). Second, this study demonstrates the effectiveness of the general iterated three-phase search framework for the minimum differential dispersion problem. It would be interesting to investigate the usefulness of this framework to solve other diversity problems like those presented in the introduction. Third, a more fundamental perspective concerns studies on a deep understanding of the proposed algorithm, which could provide useful information for additional improvements of the algorithm and the general search framework.

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