1. Pseudorange observation value

$$P_r^s = \rho_r^s + c(dt_r - dt^s) + I_{r,f}^s + T_r^s + c \cdot dR^s + dE_r^s + \varepsilon_r^s.$$

Ionospheric delay can be corrected by Klobuchar model, and tropospheric delay can be corrected by Saastamoinen model. In addition, when using broadcast ephemeris, the relativistic effect correction formula can be expressed as follows. In the process of program implementation, the relativistic effect correction is included in the calculation of clock error:

$$dR^s = -\frac{2e\sqrt{au}}{c^2} sinE,$$

The earth rotation correction can be expressed as follows. In the process of program implementation, it has been corrected to calculate the distance between the earth and the satellite:

$$dE_r^s = \frac{w}{c}(x^s \cdot y_r - y^s \cdot x_r).$$

Function model

$$P_r^s = \rho_{r,0}^s - cdt^s + I_{r,f}^s + T_r^s + c \cdot dR^s + dE_r^s + \frac{\partial P_r^s}{\partial dt_r}\delta dt_r + \frac{\partial P_r^s}{\partial x_r}\delta x_r + \frac{\partial P_r^s}{\partial y_r}\delta y_r + \frac{\partial P_r^s}{\partial z_r}\delta z_r,$$

where,

$$\frac{\partial P_r^s}{\partial x_r} = \frac{\Delta x}{\rho_0},$$
$$\frac{\partial P_r^s}{\partial y_r} = \frac{\Delta y}{\rho_0},$$
$$\frac{\partial P_r^s}{\partial z_r} = \frac{\Delta z}{\rho_0},$$
$$\frac{\partial P_r^s}{\partial dt_r} = c,$$

where,

$$\rho_0 = \sqrt{(x_r - x^s)^2 + (y_r - y^s)^2 + (z_r - z^s)^2},$$
$$\Delta x = x_r - x^s,$$
$$\Delta y = y_r - y^s,$$
$$\Delta z = z_r - z^s.$$

Stochastic model

There are two common weighting methods in GNSS single-point positioning, weighting based on altitude angle and weighting based on signal-to-noise ratio.

2. Doppler observation

According to the Doppler effect, there is the following relationship between the receiver received signal and the satellite transmitted signal:

$$\frac{f_r}{f_s} = \frac{1 - \frac{\left(\overrightarrow{v_r} - \overrightarrow{v^s}\right) \cdot \left(\overrightarrow{p_r} - \overrightarrow{p^s}\right)}{c\left\|\overrightarrow{p_r} - \overrightarrow{p^s}\right\|}}{\sqrt{1 - \frac{\left\|\overrightarrow{v_r} - \overrightarrow{v^s}\right\|}{c^2}}},$$

since the receiver-satellite velocity is much smaller than the speed of light, it can be expressed as:

$$\frac{f_r}{f_s} = 1 - \frac{\left(\overrightarrow{v_r} - \overrightarrow{v^s}\right) \cdot \left(\overrightarrow{p_r} - \overrightarrow{p^s}\right)}{c\left\|\overrightarrow{p_r} - \overrightarrow{p^s}\right\|},$$

therefore, Doppler frequency shift can be expressed as:

$$D = f_r - f_s = -\frac{\left(\overrightarrow{v_r} - \overrightarrow{v^s}\right) \cdot \left(\overrightarrow{p_r} - \overrightarrow{p^s}\right)}{c\left\|\overrightarrow{p_r} - \overrightarrow{p^s}\right\|} f_s = -\frac{\left(\overrightarrow{v_r} - \overrightarrow{v^s}\right) \cdot \left(\overrightarrow{p_r} - \overrightarrow{p^s}\right)}{\lambda_s\left\|\overrightarrow{p_r} - \overrightarrow{p^s}\right\|} = -\frac{\dot{P}_r^s}{\lambda_s}.$$

In addition, Doppler frequency shift can also be expressed as the derivative of distance, then:

$$-\lambda_s D = \dot{P}_r^s = \frac{\left(\overrightarrow{v_r} - \overrightarrow{v^s}\right) \cdot \left(\overrightarrow{p_r} - \overrightarrow{p^s}\right)}{\left\|\overrightarrow{p_r} - \overrightarrow{p^s}\right\|} + c\left(d\dot{t}_r - d\dot{t}^s\right) + \dot{I}_{r,f}^s + \dot{T}_r^s + c \cdot d\dot{R}^s + d\dot{E}_r^s$$
$$+ \dot{\varepsilon}_r^s,$$

where,

$$d\dot{E}_r^s = \frac{w}{c}\left(v^{s,x} \cdot y_r + v_{r,y} \cdot x^s - v^{s,y} \cdot x_r - v_{r,x} \cdot y^s\right).$$

Function model

$$-\lambda_s D = \dot{P}_r^s = \dot{P}_0 + \frac{\partial \dot{P}_r^s}{\partial x_r}\delta x_r + \frac{\partial \dot{P}_r^s}{\partial y_r}\delta y_r + \frac{\partial \dot{P}_r^s}{\partial z_r}\delta z_r + \frac{\partial \dot{P}_r^s}{\partial v_{x,r}}\delta v_{r,x} + \frac{\partial \dot{P}_r^s}{\partial v_{y,r}}\delta v_{r,y}$$
$$+ \frac{\partial \dot{P}_r^s}{\partial v_{z,r}}\delta v_{r,z} + \frac{\partial \dot{P}_r^s}{\partial d\dot{t}_r}\delta d\dot{t}_r,$$

where,

$$\frac{\partial \dot{P}_r^s}{\partial x_r} = \frac{\Delta v_x}{\rho_0} - \frac{\Delta v_x \Delta x + \Delta v_y \Delta y + \Delta v_z \Delta z}{\rho_0^3}\Delta x,$$

$$\frac{\partial \dot{P}_r^s}{\partial y_r} = \frac{\Delta v_y}{\rho_0} - \frac{\Delta v_x \Delta x + \Delta v_y \Delta y + \Delta v_z \Delta z}{\rho_0^3}\Delta y,$$

$$\frac{\partial \dot{P}_r^s}{\partial y_r} = \frac{\Delta v_z}{\rho_0} - \frac{\Delta v_x \Delta x + \Delta v_y \Delta y + \Delta v_z \Delta z}{\rho_0^3} \Delta z,$$

$$\frac{\partial \dot{P}_r^s}{\partial v_{r,x}} = \frac{\Delta x}{\rho_0},$$

$$\frac{\partial \dot{P}_r^s}{\partial v_{r,y}} = \frac{\Delta y}{\rho_0},$$

$$\frac{\partial \dot{P}_r^s}{\partial v_{r,z}} = \frac{\Delta z}{\rho_0},$$

$$\frac{\partial \dot{P}_r^s}{\partial d\dot{t}_r} = c,$$

where,

$$\Delta v_x = v_{r,x} - v^{s,x},$$
$$\Delta v_y = v_{r,y} - v^{s,y},$$
$$\Delta v_z = v_{r,z} - v^{s,z}.$$

Using the EKF method for pseudorange and Doppler joint positioning, the parameters that need to be estimated include:

$$x = \begin{pmatrix} x_r & y_r & z_r & v_{r,x} & v_{r,y} & v_{r,z} & dt_r & d\dot{t}_r \end{pmatrix}^T$$

The extended Kalman filter includes state equations and observation equations. The expressions of the state equations and observation equations are as follows:

$$x(k) = \Phi_{k,k-1} x(k-1) + w(k-1)$$

$$L(k) = H_k x(k) + \varepsilon(k)$$

where,

$$\Phi_{k,k-1} = \left| \begin{matrix} 1 & & & t_k - t_{k-1} & & & & \\ & 1 & & & t_k - t_{k-1} & & & \\ & & 1 & & & t_k - t_{k-1} & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & t_k - t_{k-1} \\ & & & & & & & 1 \end{matrix} \right|$$

Since neither the pseudorange nor the Doppler observation equations are linear, the observation equations need to be linearized. The expressions of H_k and $L(k)$ are:

$$
H_k = \begin{vmatrix}
\frac{\partial P_r^s}{\partial x} & \frac{\partial P_r^s}{\partial y} & \frac{\partial P_r^s}{\partial z} & 0 & 0 & 0 & 1 & 0 \\
\frac{\partial \dot{P}_r^s}{\partial x} & \frac{\partial \dot{P}_r^s}{\partial y} & \frac{\partial \dot{P}_r^s}{\partial z} & \frac{\partial \dot{P}_r^s}{\partial v_{r,x}} & \frac{\partial \dot{P}_r^s}{\partial v_{r,y}} & \frac{\partial \dot{P}_r^s}{\partial v_{r,z}} & 0 & 1 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\frac{\partial P_r^s}{\partial x} & \frac{\partial P_r^s}{\partial y} & \frac{\partial P_r^s}{\partial z} & 0 & 0 & 0 & 1 & 0 \\
\frac{\partial \dot{P}_r^s}{\partial x} & \frac{\partial \dot{P}_r^s}{\partial y} & \frac{\partial \dot{P}_r^s}{\partial z} & \frac{\partial \dot{P}_r^s}{\partial v_{r,x}} & \frac{\partial \dot{P}_r^s}{\partial v_{r,y}} & \frac{\partial \dot{P}_r^s}{\partial v_{r,z}} & 0 & 1
\end{vmatrix}
L(k) = \begin{vmatrix}
P_r^s - P_{r,0}^s \\
-\lambda_s D - \dot{P}_{r,0}^s \\
\cdots \\
\cdots \\
P_r^s - P_{r,0}^s \\
-\lambda_s D - \dot{P}_{r,0}^s
\end{vmatrix}
$$