STATEMENT OF PURPOSE

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Under the supervision of Professor Vasily Dolgushev, I conducted the research that introduced a more streamlined version of the groupoid GTSh of GT-shadows and contributed to the SageMath package for working with GT-shadows. The groupoid GTSh is a 'computable' approximation to the gentle version of the Grothendieck–Teichmüller group $\widehat{\mathsf{GT}}_{gen} $. The set of objects of GTSh is the poset $\mathrm{NFI}^{B_3}(F_2)$ of B_3-invariant finite index normal subgroups of the free group F_2 on two generators. This object is worth studying because it is very 'likely' that in most cases, the morphism of GTSh lifts to the absolute Galois group $G_{\mathbb{Q}}$. And it is already known that for a specific subposet Dih_2 of dihedral posets, if K is an element of Dih_2, every morphism of the connected componenet of K in GTSh lifts to an element of $G_{\mathbb{Q}}$.

Mentored by Professor Maxence Mayrand, I participated in a research internship that constructed new GKP code from the Abelian varieties on the cyclotomic field $K = \mathbb{Q}(\zeta_n)$. The construction $(K \otimes_{\mathbb{Q}} \mathbb{R})/\mathcal{O}_K$ is an abelian variety of CM-type. We defined a skew-symmetric form E and extended it to a hermitian inner product H on $(K \otimes_{\mathbb{Q}} \mathbb{R})/\mathcal{O}_K$. Under certain restrictions, this defines a GKP code. There is a homomorphism from $\mathrm{Aut}(X, L)$ to $\{\text{unitary operators on } H^0(X, L \otimes L)\}$, the gates. It is already known that this construction produces Clifford gates. I computed the decomposition of these Clifford gates as the Hadamard gate, S-gate and CNOT-gate. An interesting phenomenon is that the expression of the gate produced by the case $n = 5$ contains CNOT-gate.

Under the supervision of Professor Lisa Berger, Ajmain Yamin and Connor Stewart, I participated in an online collaborative research to obtain equations for K_9 dessin, that is the bipartification dessin of the complete regular map from K_9 graph. We proved that the Riemann surface of the K_9 dessin is a degree 9 cover of the Bolza surface $y^2 = x(x^4 - 1)$ with covering group $(\mathbb{Z}/3\mathbb{Z})^2$. We used the fact that any unramified abelian cover of a Riemann surface S is obtained from pulling back S along an unramified abelian cover of

the Jacobian Jac(S) of S. So, we computed the equation for the Abel-Jacobi map of the Bolza surface S_B and constructed a degree 9 cover of the Jacobian Jac(S_B) and obtained the equation.

These research experiences let me explore a wide range of mathematics and significantly improve my self-learning, writing, programming and speaking skills. It is these research experiences that boost my determination to seek admission to a graduate program and do mathematical research.