

# STATEMENT OF PURPOSE

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Under the supervision of Professor Vasily Dolgushev, I conducted the research that introduced a more streamlined version of the groupoid GTSh of GT-shadows and contributed to the SageMath package for working with GT-shadows. The groupoid GTSh is a 'computable' approximation to the gentle version of the Grothendieck-Teichmüller group  $\widehat{\mathrm{GT}}_{\mathrm{gen}}$ . The set of objects of GTSh is the poset  $\mathrm{NFI}^{B_3}(F_2)$  of  $B_3$ -invariant finite index normal subgroups of the free group  $F_2$  on two generators. This object is worth studying because it is very 'likely' that in most cases, the morphism of GTSh lifts to the absolute Galois group  $G_{\mathbb{Q}}$ . And it is already known that for a specific subposet  $\mathrm{Dih}_2$  of dihedral posets, if  $K$  is an element of  $\mathrm{Dih}_2$ , every morphism of the connected component of  $K$  in GTSh lifts to an element of  $G_{\mathbb{Q}}$ .

Mentored by Professor Maxence Mayrand, I participated in a research internship that constructed new GKP code from the Abelian varieties on the cyclotomic field  $K = \mathbb{Q}(\zeta_n)$ . The construction  $(K \otimes_{\mathbb{Q}} \mathbb{R})/\bar{O}_K$  is an abelian variety of CM-type. We defined a skew-symmetric form  $E$  and extended it to a hermitian inner product  $H$  on  $(K \otimes_{\mathbb{Q}} \mathbb{R})/\bar{O}_K$ . Under certain restrictions, this defines a GKP code. There is a homomorphism from  $\mathrm{Aut}(X, L)$  to  $\{\text{unitary operators on } H^0(X, L \otimes L)\}$ , the gates. It is already known that this construction produces Clifford gates. I computed the decomposition of these Clifford gates as the Hadamard gate, S-gate and CNOT-gate. An interesting phenomenon is that the expression of the gate produced by the case  $n = 5$  contains CNOT-gate.

Under the supervision of Professor Lisa Berger, Ajmain Yamin and Connor Stewart, I participated in an online collaborative research to obtain equations for  $K_9$  dessin, that is the bipartification dessin of the complete regular map from  $K_9$  graph. We proved that the Riemann surface of the  $K_9$  dessin is a degree 9 cover of the Bolza surface  $y^2 = x(x^4 - 1)$  with covering group  $(\mathbb{Z}/3\mathbb{Z})^2$ . We used the fact that any unramified abelian cover of a Riemann surface  $S$  is obtained from pulling back  $S$  along an unramified abelian cover of

the Jacobian  $\text{Jac}(S)$  of  $S$ . So, we computed the equation for the Abel-Jacobi map of the Bolza surface  $S_B$  and constructed a degree 9 cover of the Jacobian  $\text{Jac}(S_B)$  and obtained the equation.

These research experiences let me explore a wide range of mathematics and significantly improve my self-learning, writing, programming and speaking skills. It is these research experiences that boost my determination to seek admission to a graduate program and do mathematical research.

I think University of Milan is a great place to study Number theory. My undergraduate advisor recommended University of Milan to me.