STATEMENT OF PURPOSE

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My primary research interest lies in the area of number theory and arithmetic geometry, especially about the absolute Galois group. I am actively involved in a research project that introduces a more streamlined version of the groupoid GTSh which already shows its power in constructing the first non-abelian quotients of Grothendieck–Teichmüller group that receive surjective homomorphisms from $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$. But I am open to exploring new ideas as well.

Under the supervision of Professor Vasily Dolgushev, I conducted the research that introduced a more streamlined version of the groupoid GTSh of GT-shadows and contributed to the SageMath package for working with GT-shadows. The groupoid GTSh is a ‘computable’ approximation to the gentle version of the Grothendieck–Teichmüller group $\widehat{\mathsf{GT}}_{gen}$. The set of objects of GTSh is the poset $\mathrm{NFI}^{\mathrm{B}_3}(\mathrm{F}_2)$ of B_3-invariant finite index normal subgroups of the free group F_2 on two generators. This object is worth studying because it is very ‘likely’ that in most cases, the element of GTSh lifts to the absolute Galois group. And it is already known that for dihedral posets, every element of GTSh lifts to the absolute Galois group.

Mentored by Professor Maxence Mayrand, I participated in a research internship that constructed new GKP code from the Abelian varieties on the cyclotomic field $K = \mathbb{Q}(\zeta_n)$. The construction $(K \otimes_{\mathbb{Q}} \mathbb{R})/\mathcal{O}_K$ is an abelian variety of CM-type. We defined a skew-symmetric form E and extended it to a hermitian inner product H on $(K \otimes_{\mathbb{Q}} \mathbb{R})/\mathcal{O}_K$. Under certain restrictions, this defines a GKP code. There is a homomorphism from $\mathrm{Aut}(X, L)$ to $\{\text{unitary operators on } H^0(X, L \otimes L)\}$, the gates. It is already known that this construction produces Clifford gates. I computed the decomposition of these Clifford gates as the Hadamard gate, S-gate and CNOT-gate. An interesting phenomenon is that the expression of the gate produced by the case $n = 5$ contains CNOT-gate.

Under the supervision of Professor Lisa Berger, Ajmain Yamin and Connor Stewart, I participated in an online collaborative research to obtain equations for K_9 dessin, that is the bipartification dessin of the complete regular map from K_9 graph. We proved that the

Riemann surface of the K_9 dessin is a degree 9 cover of the Bolza surface $y^2 = x(x^4 - 1)$ with covering group $(\mathbb{Z}/3\mathbb{Z})^2$. We used the fact that any unramified abelian cover of a Riemann surface S is obtained from pulling back S along an unramified abelian cover of the Jacobian $\mathrm{Jac}(S)$ of S. So, we computed the equation for the Abel-Jacobi map of the Bolza surface S_B and constructed a degree 9 cover of the Jacobian $\mathrm{Jac}(S_B)$ and obtained the equation.

These research experiences let me explore a wide range of mathematics and significantly improve my self-learning, writing, programming and speaking skills. It is these research experiences that boost my determination to seek admission to a graduate program and do mathematical research.

My short-term plan after admission is to try to pass the qualification exam in the first year, finish the required courses and teaching duties, attend colloquia, find an advisor, and begin my dissertation work as soon as possible. My long-term plan is to to become a Mathematics professor at a research institution and make original contributions to the mathematics community. I hope to learn from and do research with Professor Frauke Bleher. I am interested in Professor Frauke Bleher's paper "On representations of Gal(Q/Q), {GT}^ and Aut({F2}^),". This paper constructed non-abelian representations for finite index subgroup of $\mathrm{Aut}(\hat{F}_2)$ that contains the image of the Belyi's embedding. I like the University of Iowa's picturesque and historic architecture and large areas of green spaces and trees. I believe the University of Iowa is the best place for me to build a successful mathematical career.