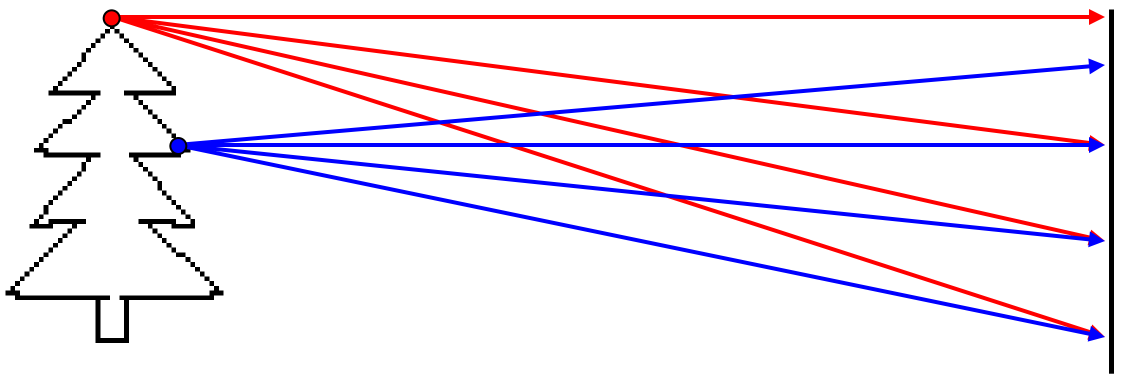
Overview

- The pinhole projection model
  - Qualitative properties
  - Perspective projection matrix
- Cameras with lenses
  - Depth of focus
  - Field of view
  - Lens aberrations
- Digital cameras
  - Sensors
  - Color
  - Artifacts

Let's design a camera

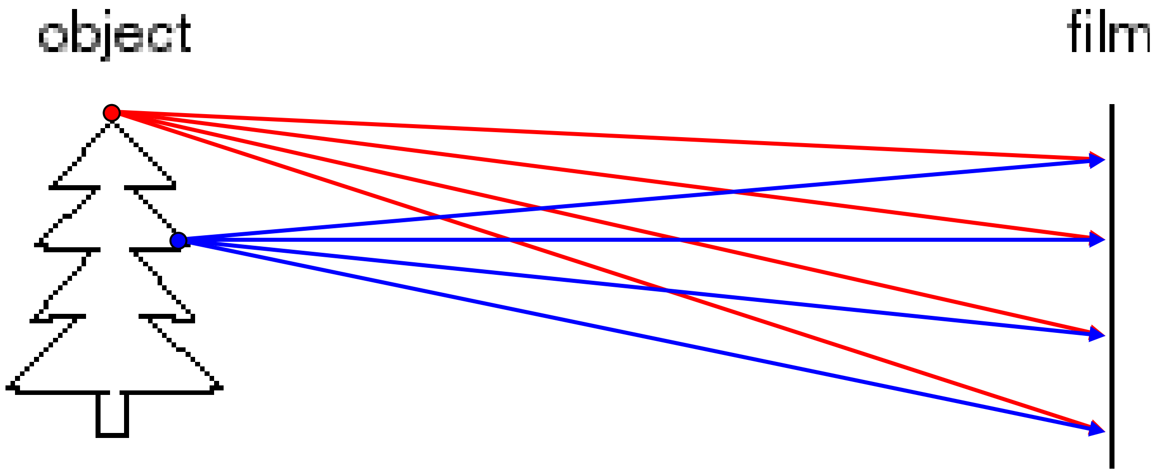


Idea 1: put a piece of film in front of an object

Do we get a reasonable image?

Slide by Steve Seitz

Image formation

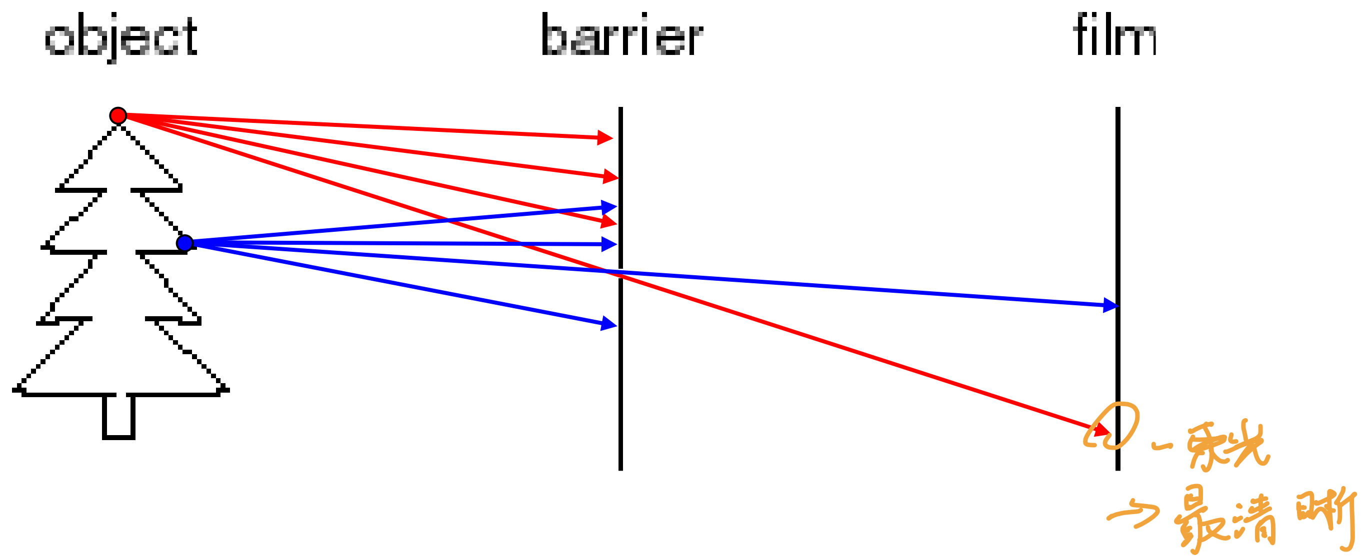


Let's design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

Slide source: Seitz

Pinhole camera

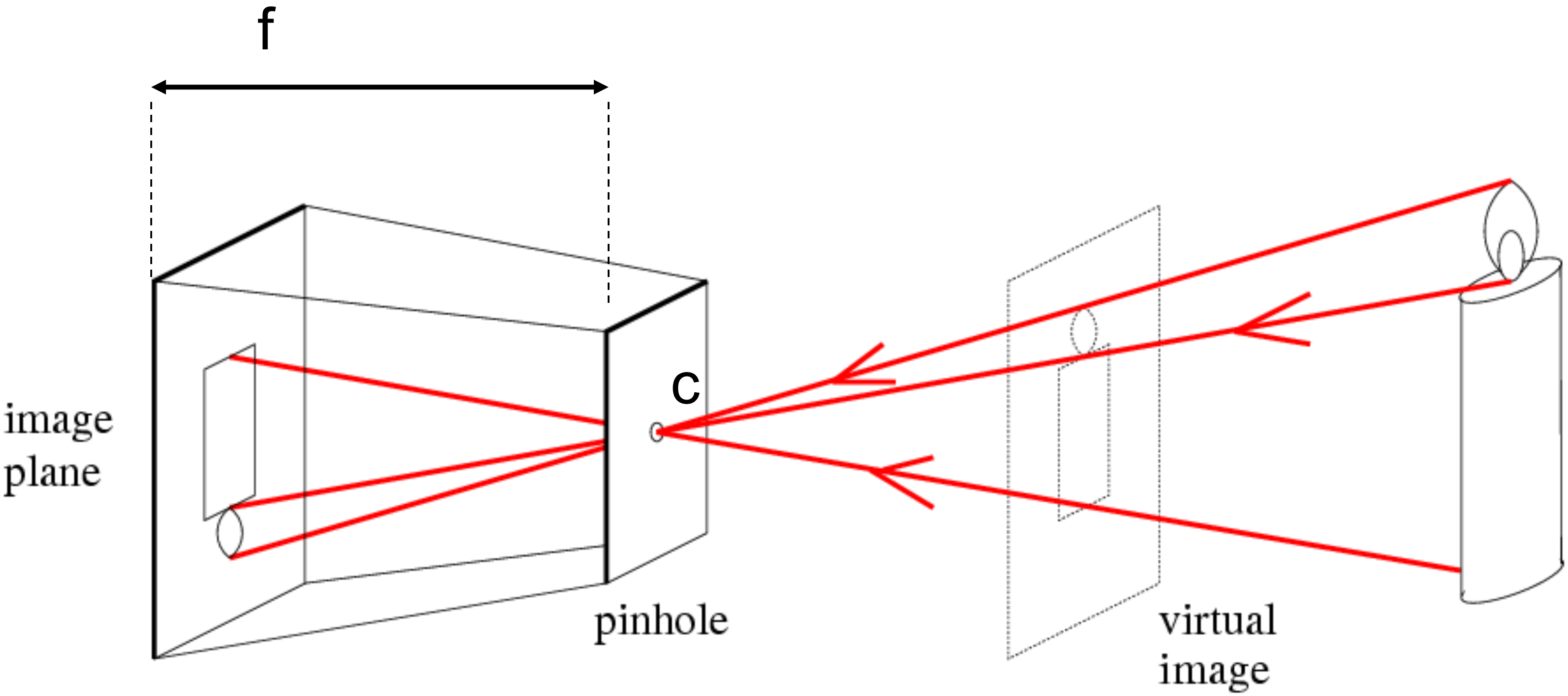


Idea 2: add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**

Slide source: Seitz

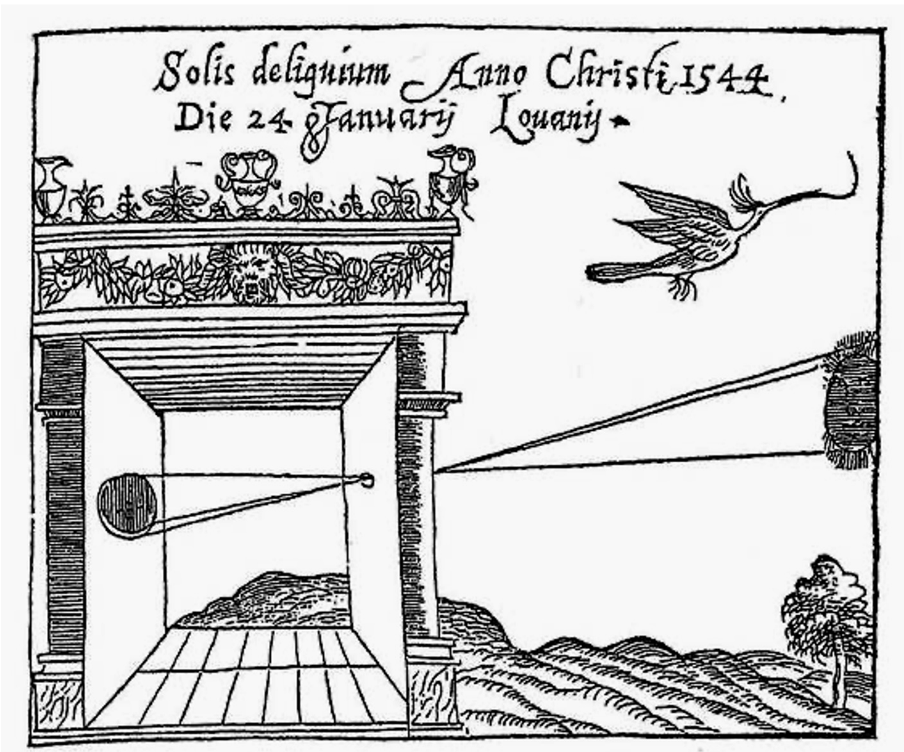
Pinhole camera



f = focal length
c = center of the camera

Figure from Forsyth

Camera obscura



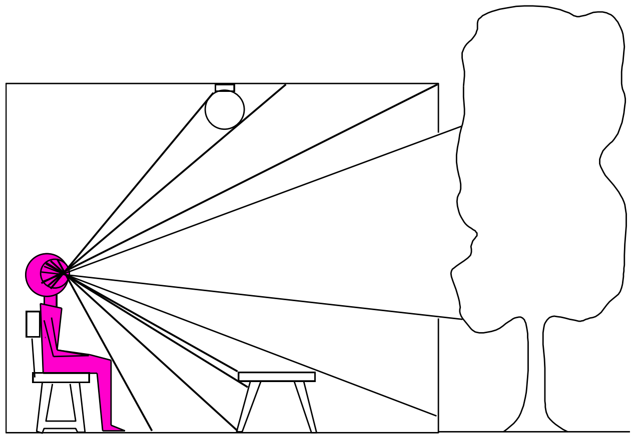
Gemma Frisius, 1558

- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

Source: A. Efros

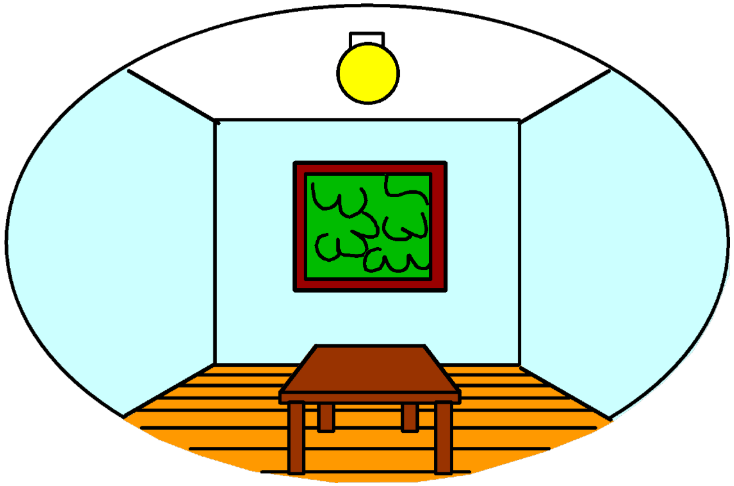
Dimensionality reduction: from 3D to 2D

3D world



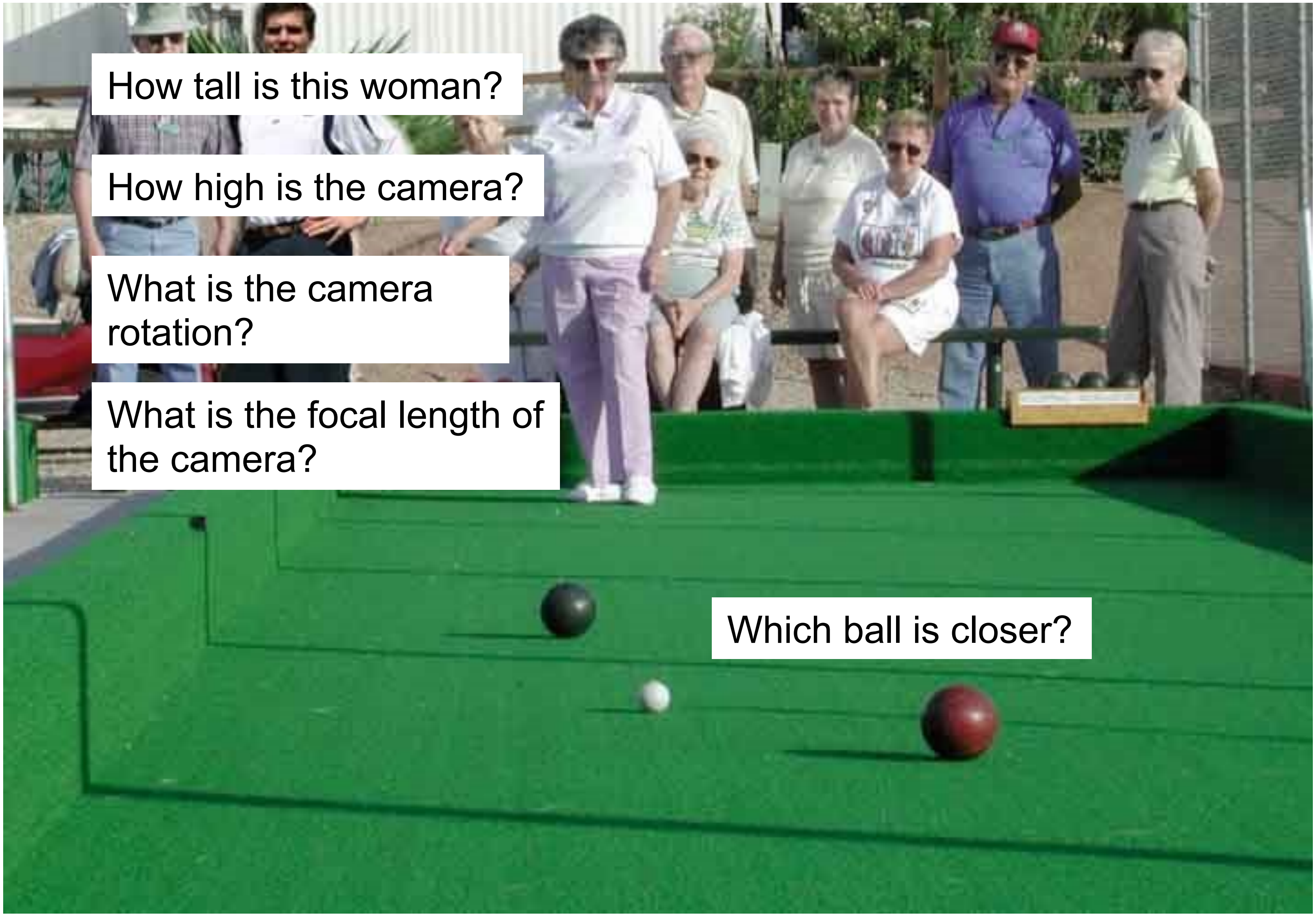
Point of observation

2D image



Slide by A. Efros

Single-view Geometry



Slide source: Seitz

Projection can be tricky…

Slide source: Seitz

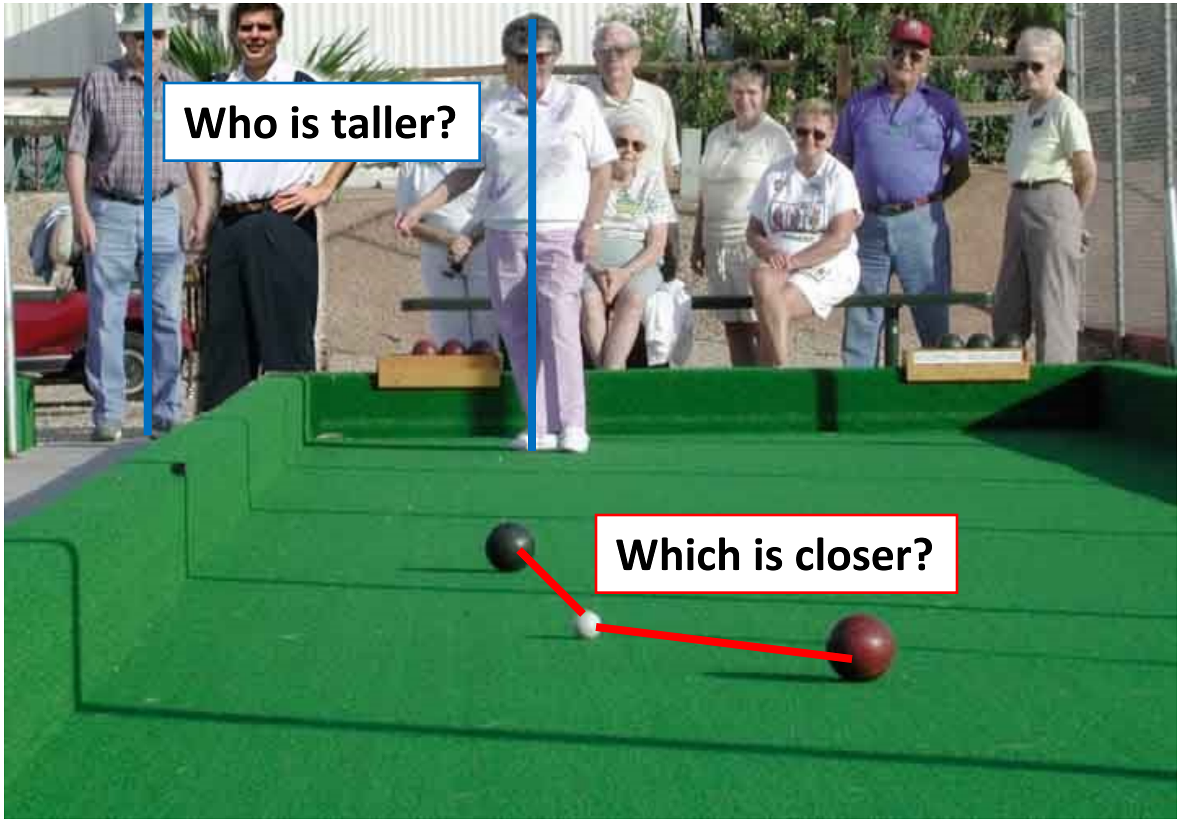
Projection can be tricky…

Making of 3D sidewalk art: http://www.youtube.com/watch?v=3SNYtd0Ayt0

Projective Geometry

What is lost?

- Length



Length is not preserved

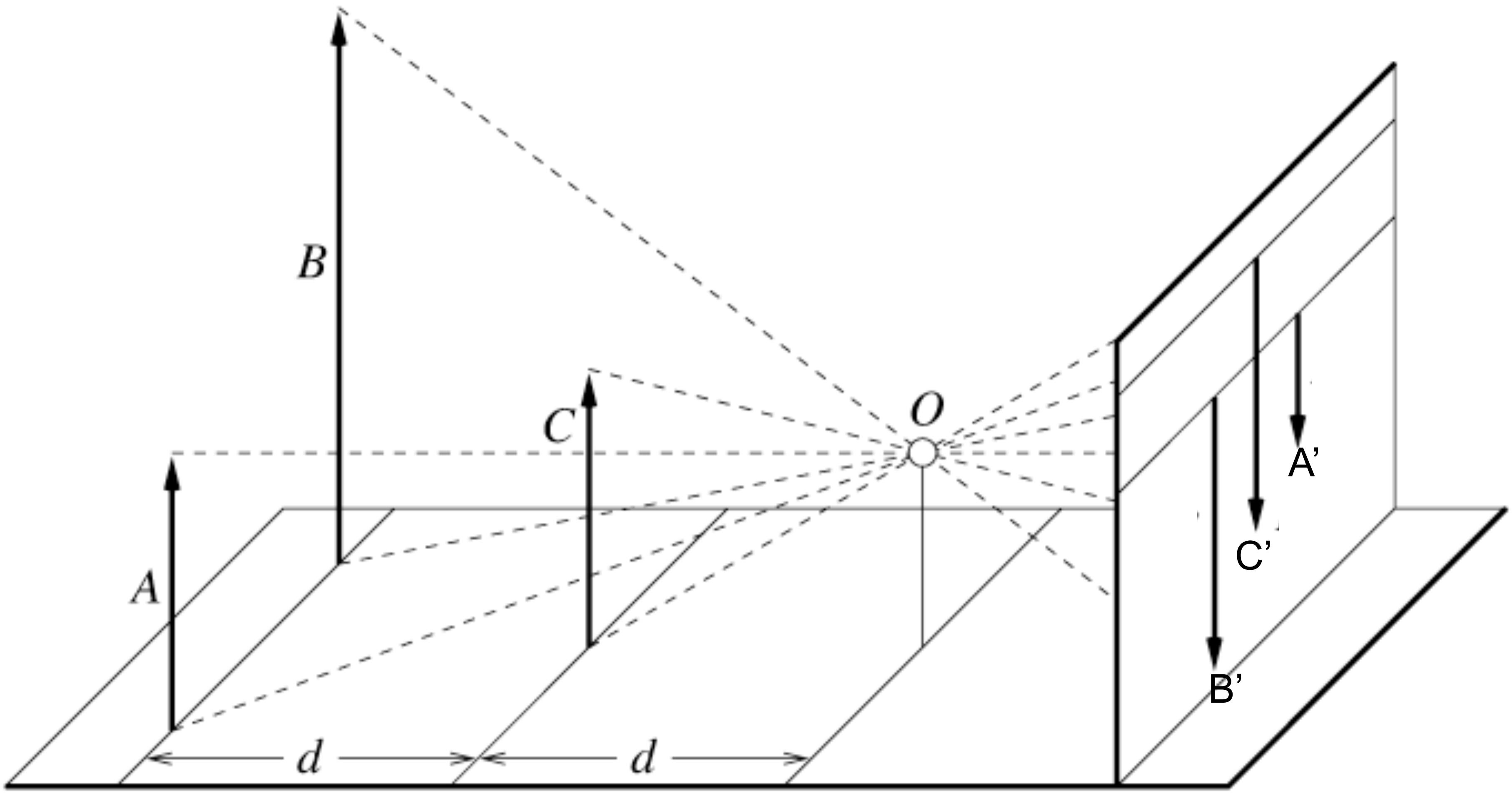
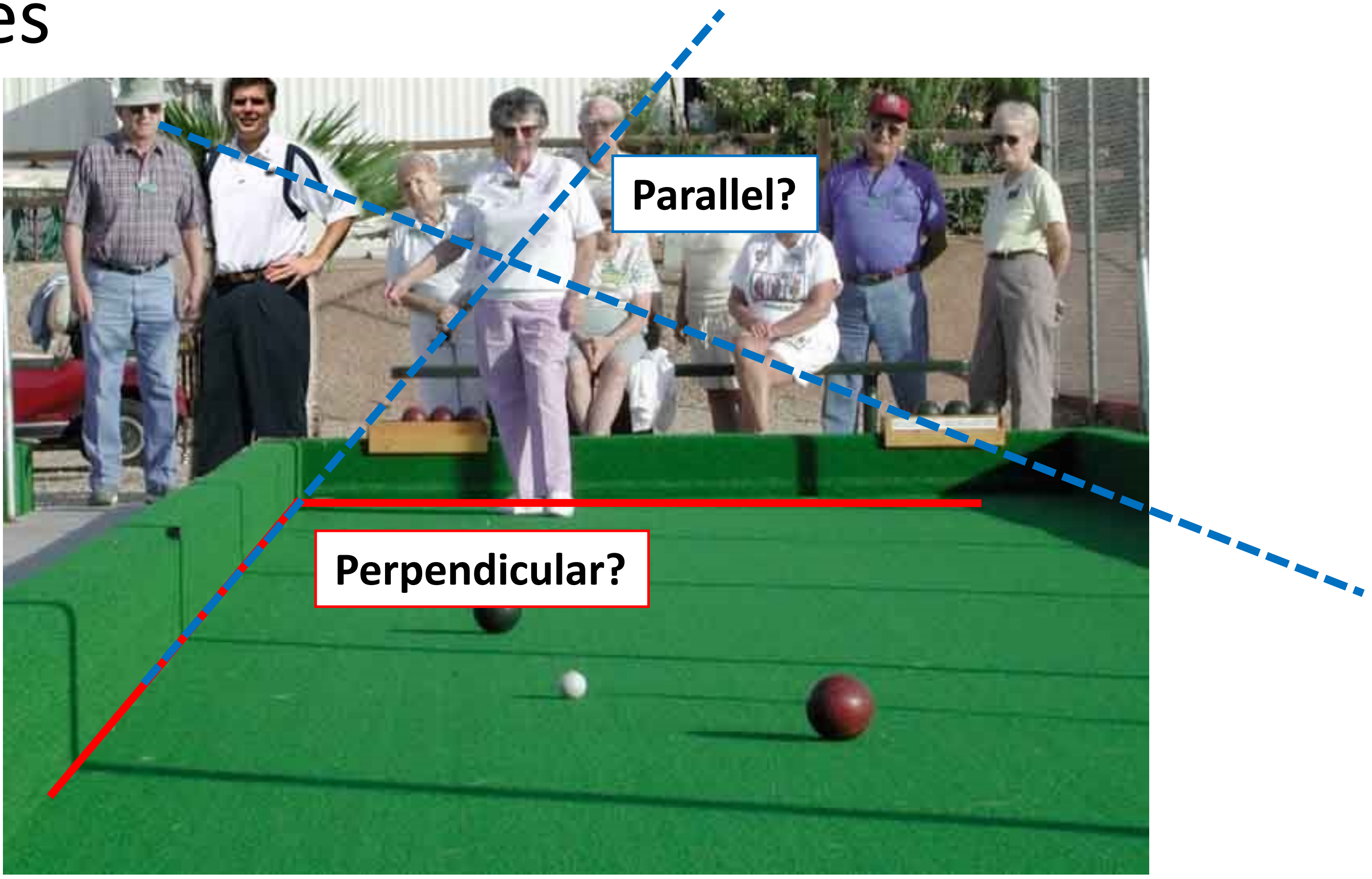


Figure by David Forsyth

Projective Geometry

What is lost?

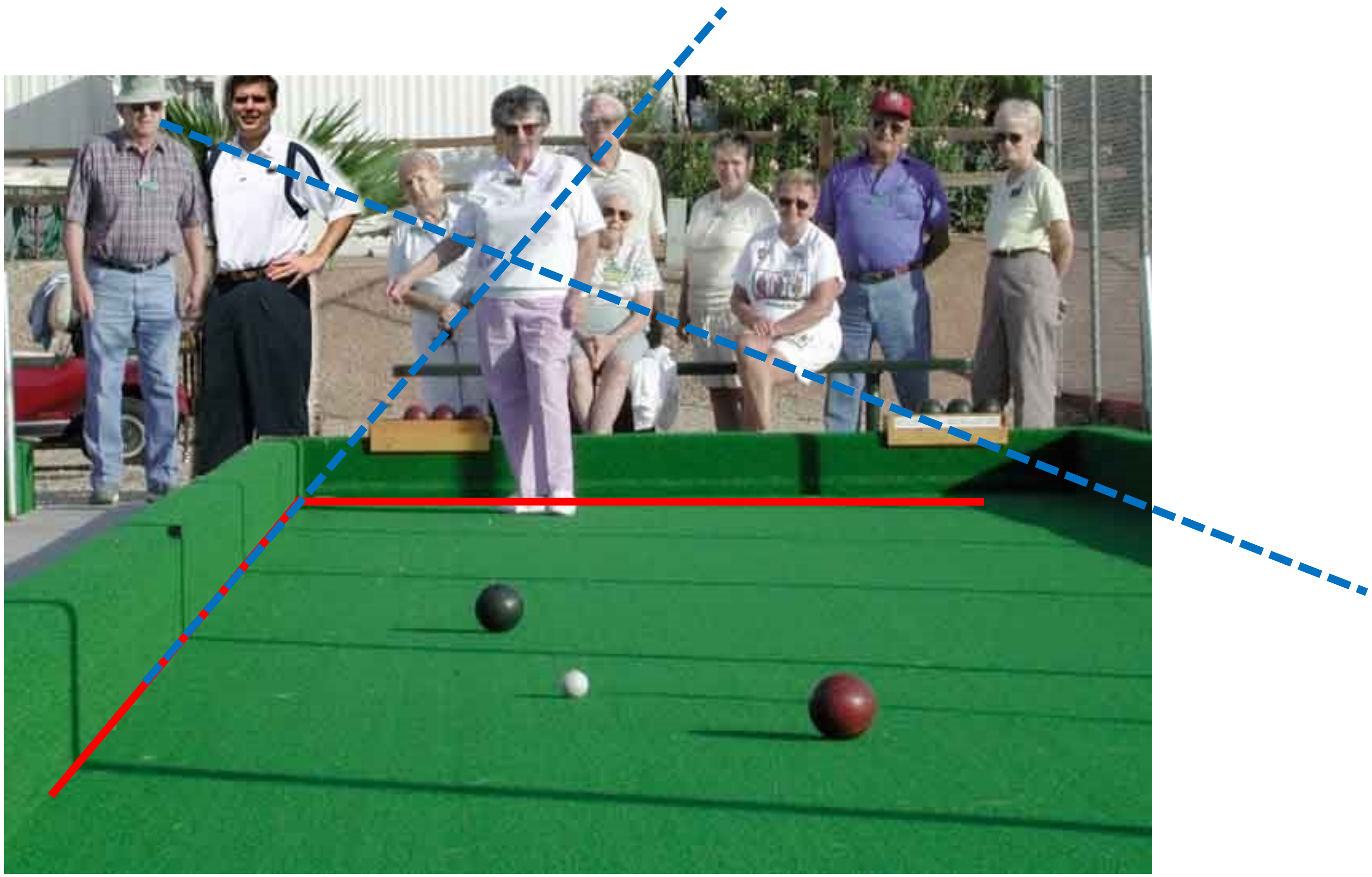
- Length
- Angles



Projective Geometry

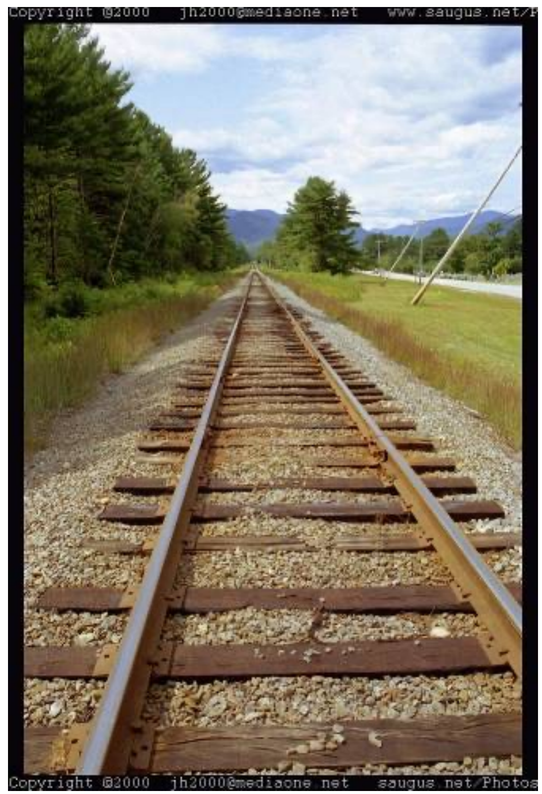
What is preserved?

- Straight lines are still straight

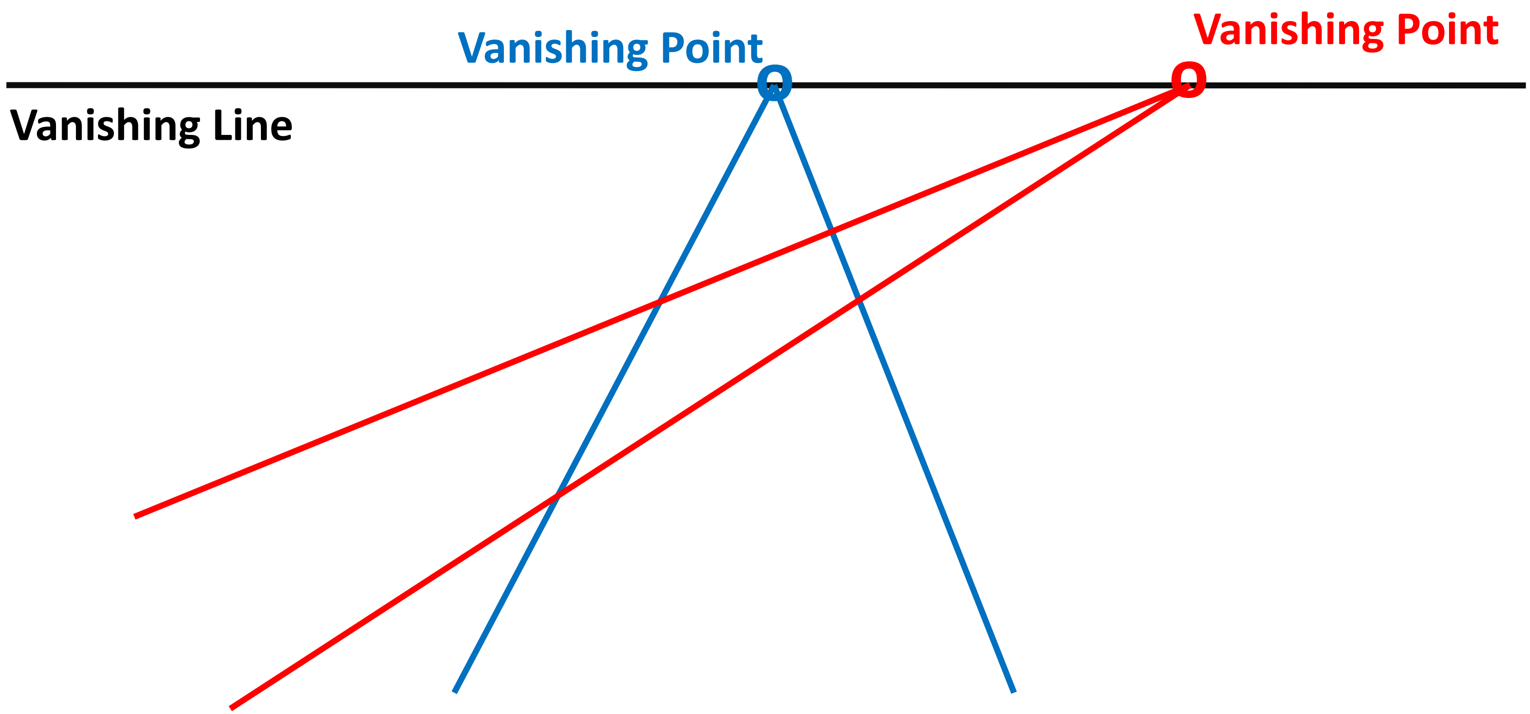


Vanishing points

- All parallel lines converge to a *vanishing point*
  - Each direction in space is associated with its own vanishing point
  - Exception: directions parallel to the image plane

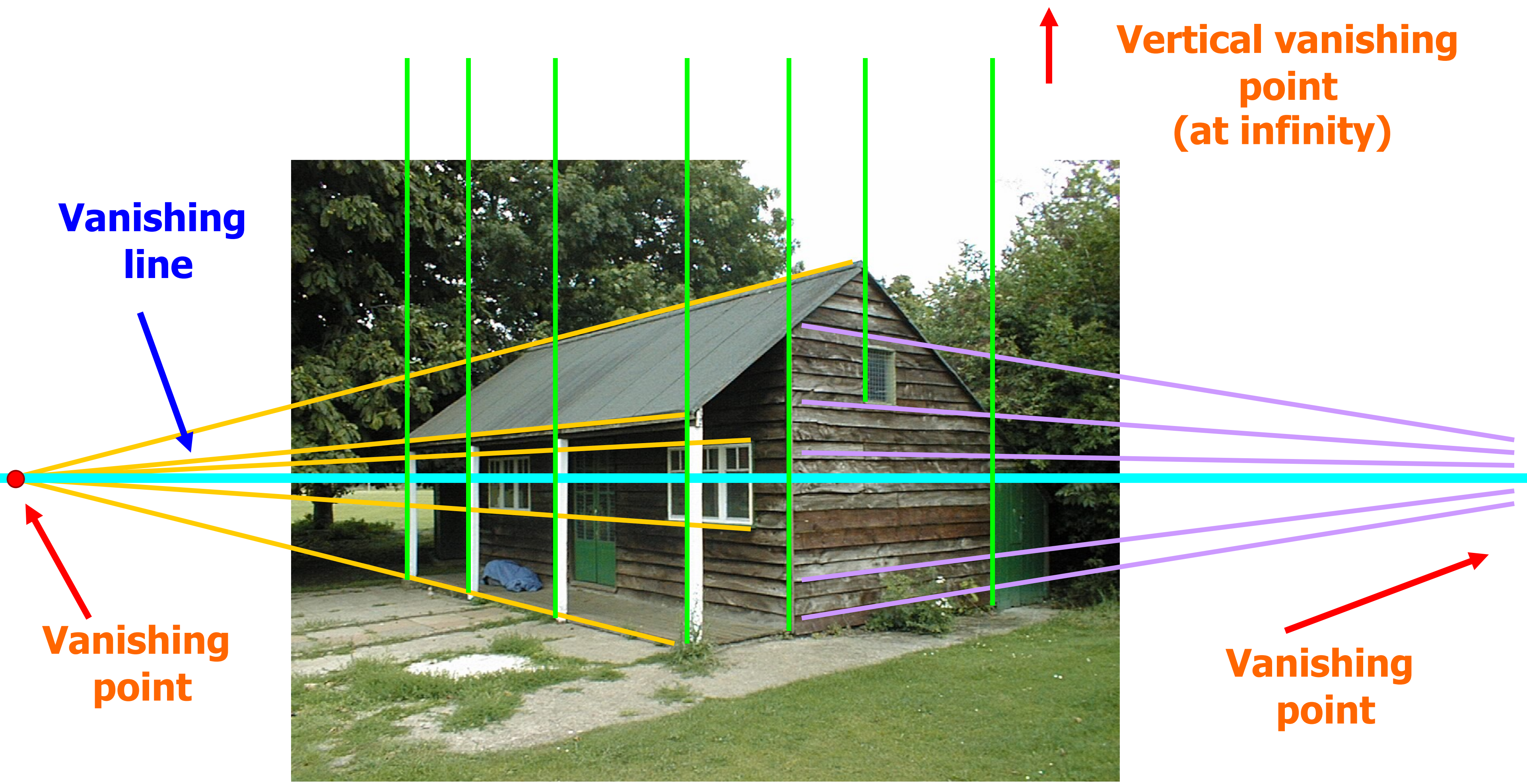


Vanishing points and lines



- The projections of parallel 3D lines intersect at a **vanishing point**
- The projection of parallel 3D planes intersect at a **vanishing line**
- Not all lines that intersect are parallel

Vanishing points and lines

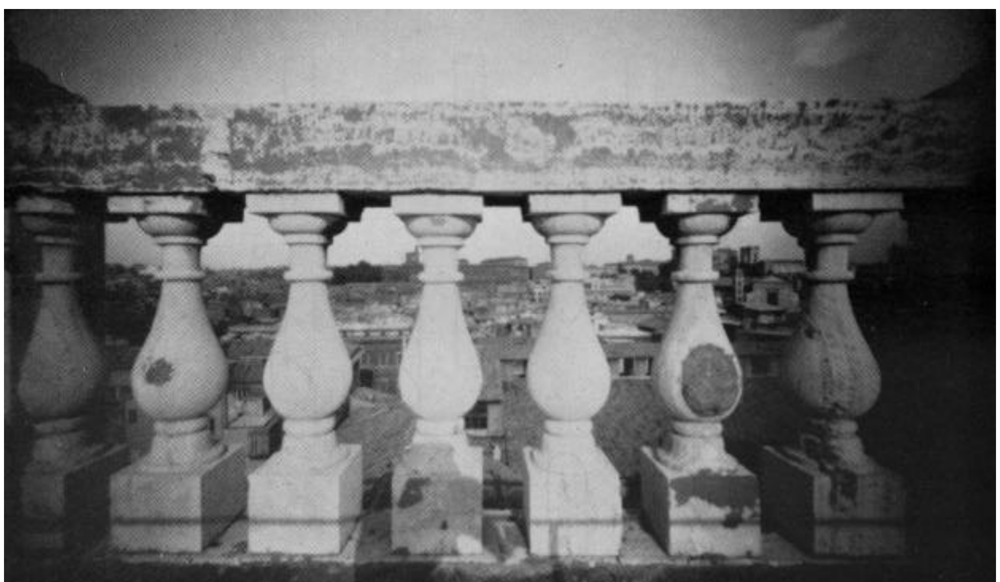


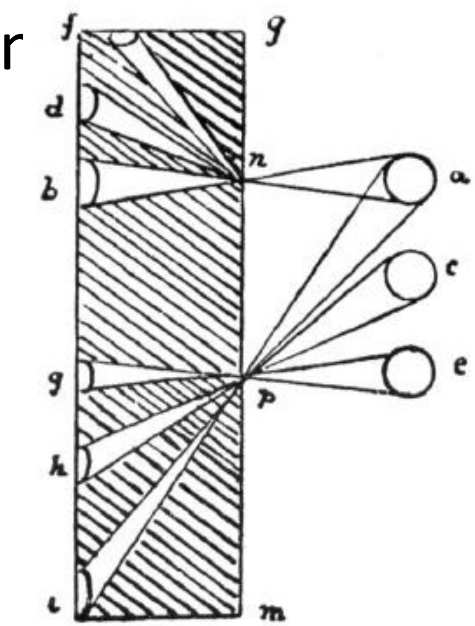
Slide from Efros, Photo from Criminisi

Vanishing objects

Perspective distortion

- Are the widths of the projected columns equal?
  - The exterior columns are wider
  - This is not an optical illusion, and is not due to lens flaws
  - Phenomenon pointed out by Da Vir





Source: F. Durand

Perspective distortion

- What is the shape of the projection of a sphere?

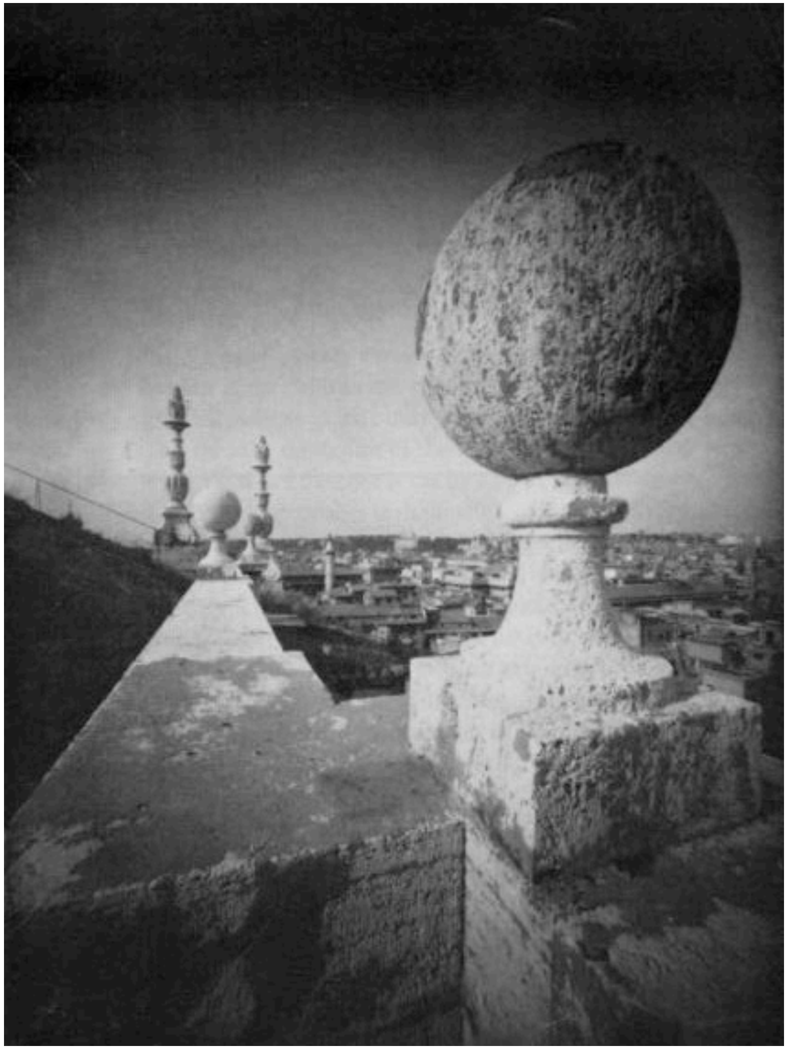
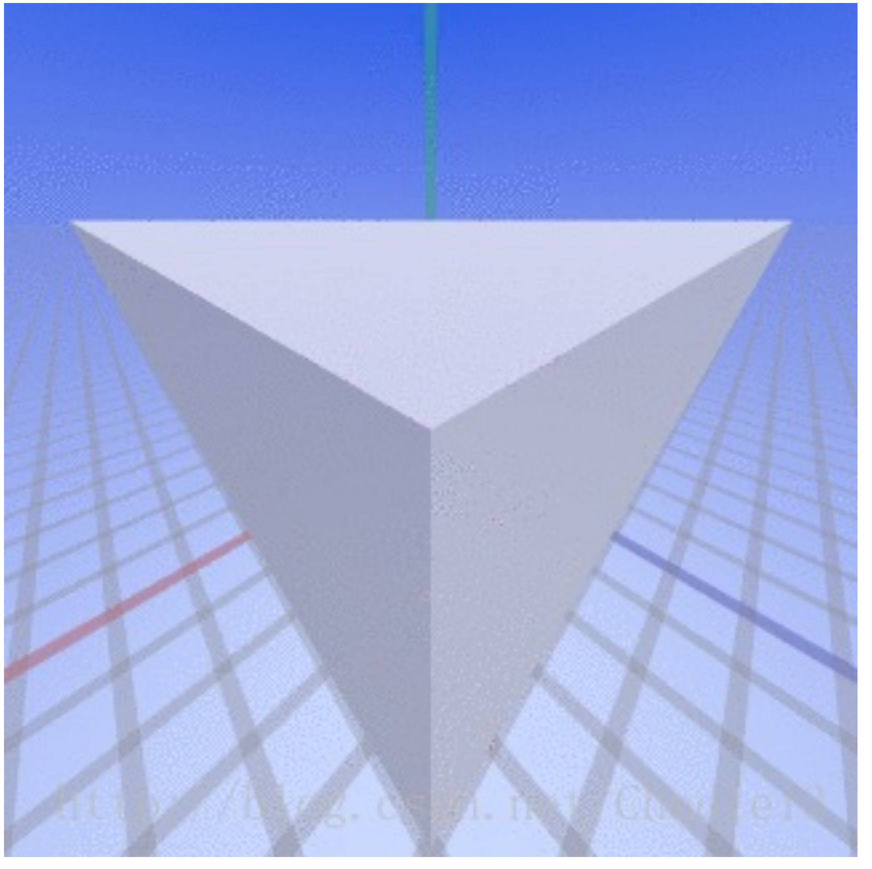
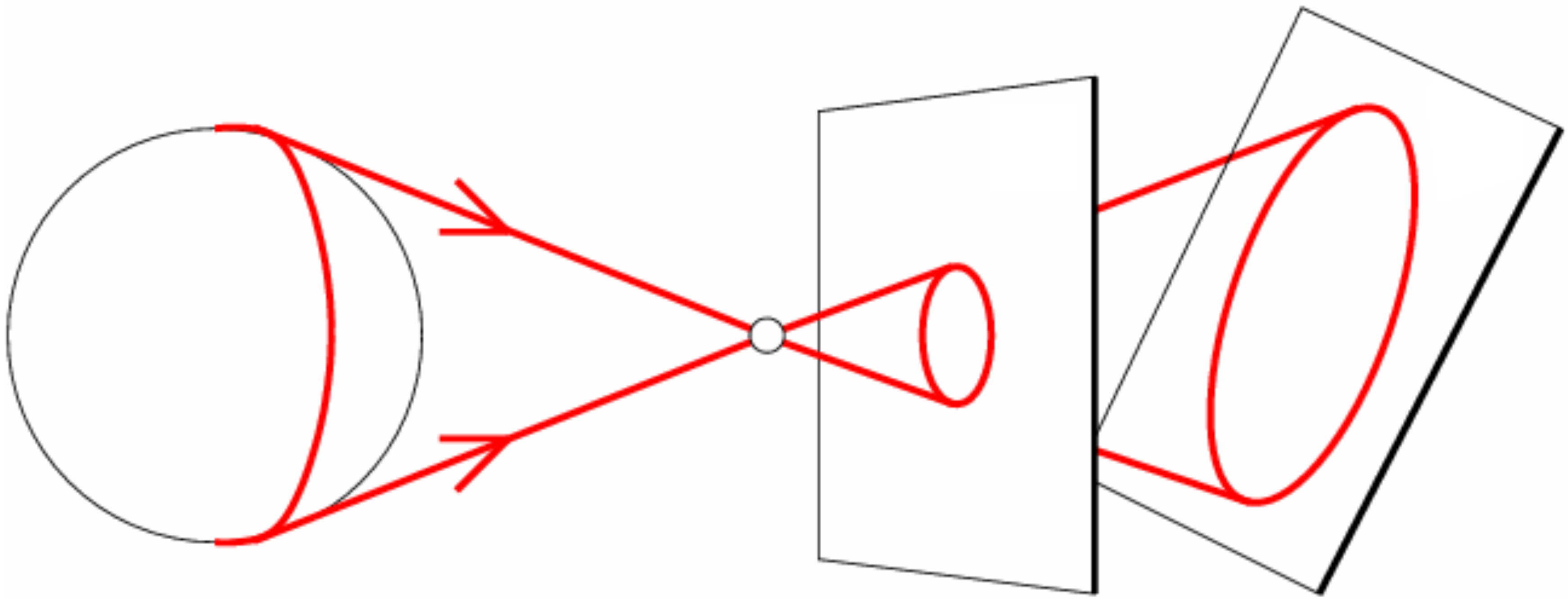


Image source: F. Durand



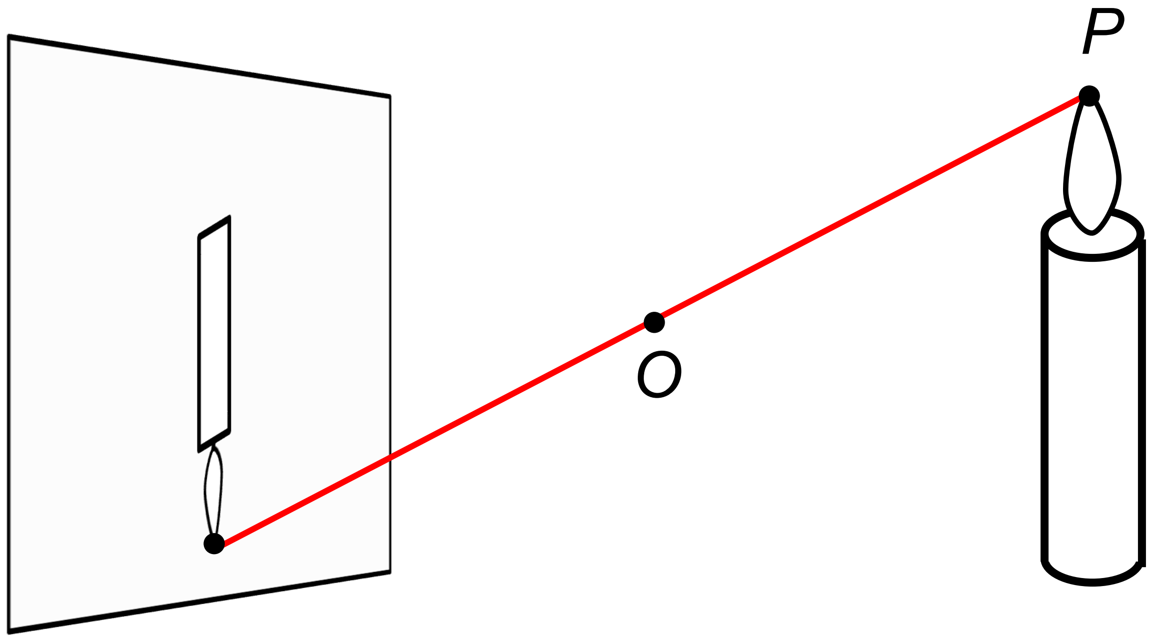
Perspective distortion

- What is the shape of the projection of a sphere?



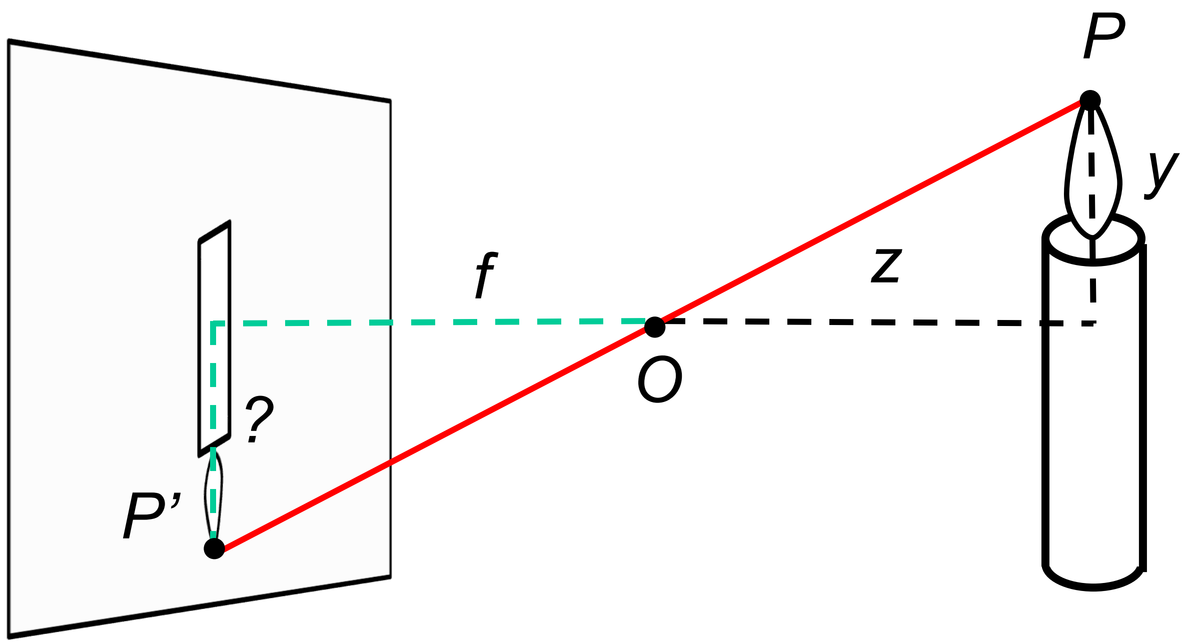
Perspective distortion: People

Modeling projection



- To compute the projection *P'* of a scene point *P*, form the **visual ray** connecting *P* to the camera center *O* and find where it intersects the image plane
  - All scene points that lie on this visual ray have the same projection in the image
  - Are there scene points for which this projection is undefined?

Modeling projection



The coordinate system

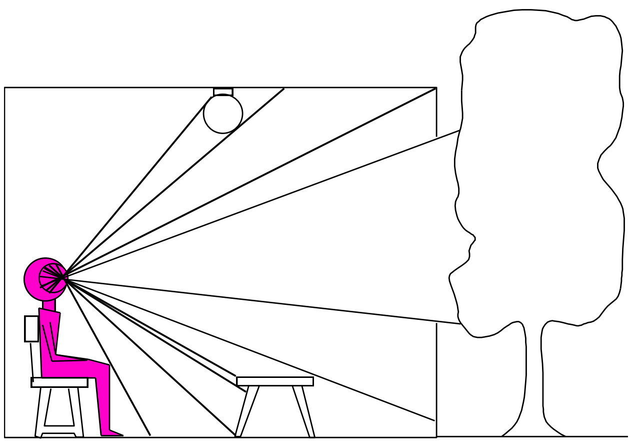
- The optical center (***O***) is at the origin
- The image plane is parallel to xy-plane or perpendicular to the z-axis, which is the *optical axis*

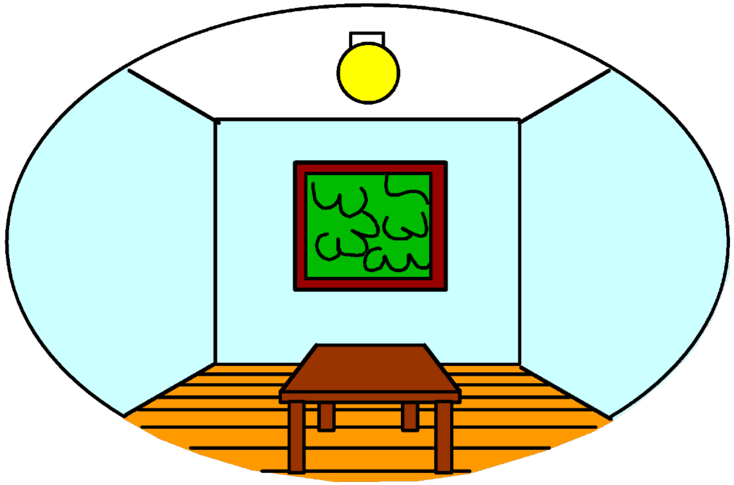
Projection equations

- Derived using similar triangles $$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Fronto-parallel planes

- What happens to the projection of a pattern on a plane parallel to the image plane?
  - All points on that plane are at a fixed *depth z*
  - The pattern gets scaled by a factor of *f* / *z*, but angles and ratios of lengths/areas are preserved

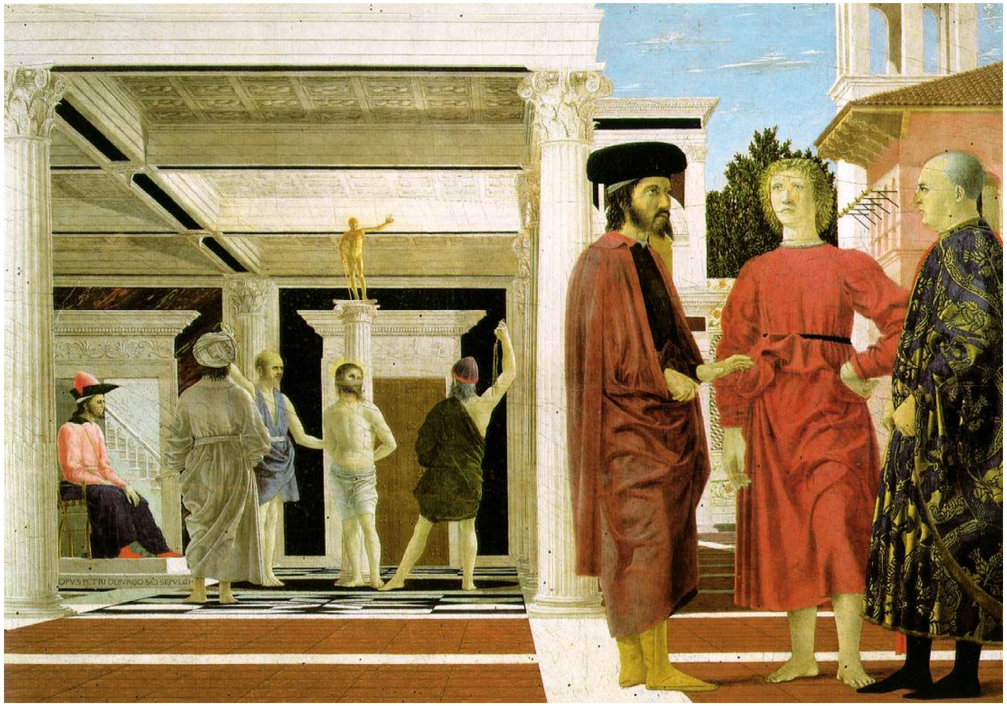




$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Fronto-parallel planes

- What happens to the projection of a pattern on a plane parallel to the image plane?
  - All points on that plane are at a fixed *depth z*
  - The pattern gets scaled by a factor of *f* / *z*, but angles and ratios of lengths/areas are preserved



Piero della Francesca, *Flagellation of Christ*, 1455-1460

Jan Vermeer, *The Music Lesson*, 1662-1665

Perspective Projection (pinhole projection)

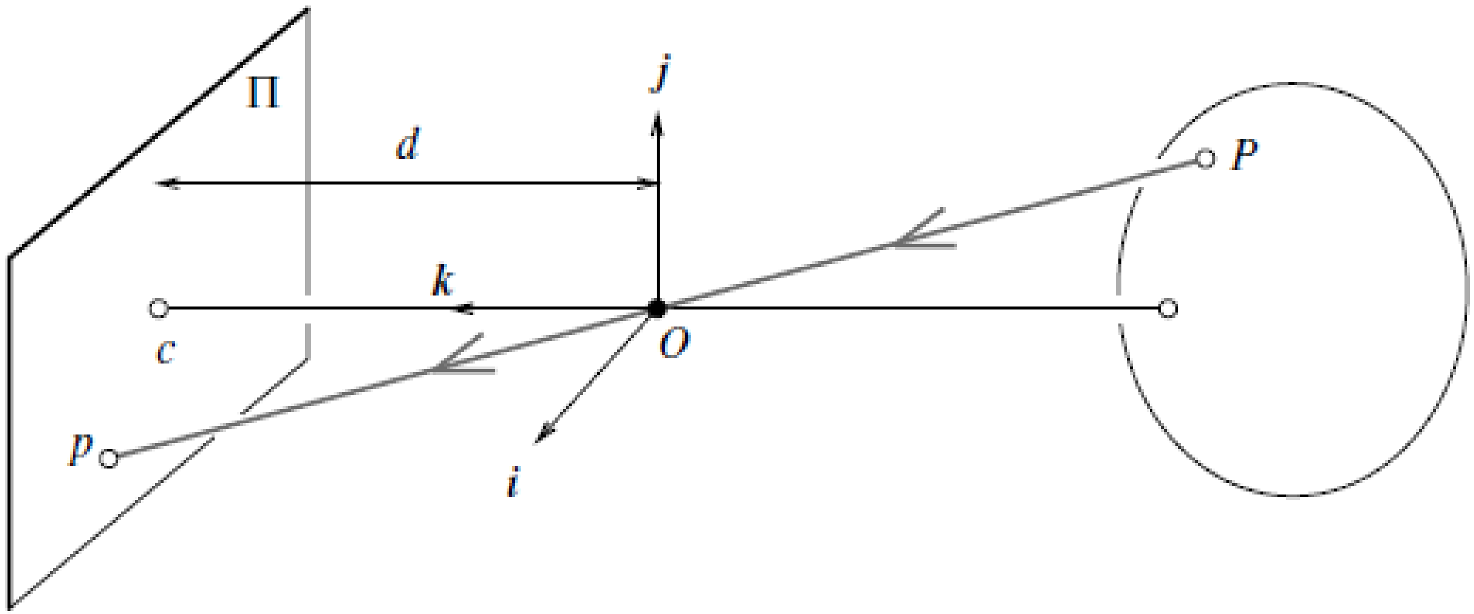


FIGURE 1.4: The perspective projection equations are derived in this section from the collinearity of the point P, its image p, and the pinhole O.

$$\begin{cases} x = \lambda X \\ y = \lambda Y \\ d = \lambda Z \end{cases} \iff \lambda = \frac{x}{X} = \frac{y}{Y} = \frac{d}{Z},$$

$$\begin{cases} x = d\dfrac{X}{Z}, \\ y = d\dfrac{Y}{Z}. \end{cases}$$

Non-homogenous Coordinates

A point P in some coordinate frame (F) = (O, ***i, j, k***) is represented as:

$$\overrightarrow{OP} = X\boldsymbol{i} + Y\boldsymbol{j} + Z\boldsymbol{k}.$$

The same point P in different coordinate systems (A) and (B):

$$^{A}\boldsymbol{P} = \mathcal{R}\,^{B}\boldsymbol{P} + \boldsymbol{t},$$

旋转平移

Here ***R*** is a rotation matrix, ***t*** is a translation vector.

Homogenous Coordinates

Add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \; w$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

By introducing homogenous coordinates, we have

$$ {}^{A}\boldsymbol{P} = \mathcal{T}\,{}^{B}\boldsymbol{P}, \quad \text{where} \quad \mathcal{T} = \begin{pmatrix} \mathcal{R} & \boldsymbol{t} \\ \boldsymbol{0}^T & 1 \end{pmatrix},$$

$$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}\begin{pmatrix} P \\ 1 \end{pmatrix} = RP + t$$

Projection Equation in Homogenous Coordinates

投影

For a point **P** in some fixed world coordinate

P=(X, Y, Z, 1)^T, and its image **p** in the camera's reference frame (normalized image plane) \hat{*p*}= (x,y,1)^T, the projection equation is represented as:

投影矩阵M=相机内参（本身硬件）+外参（空间上方向位置）3*4

$$p = \frac{1}{Z}\mathcal{M}\underbrace{P}_{三维}.$$

三维点映射到二维平面

Intrinsic Parameters

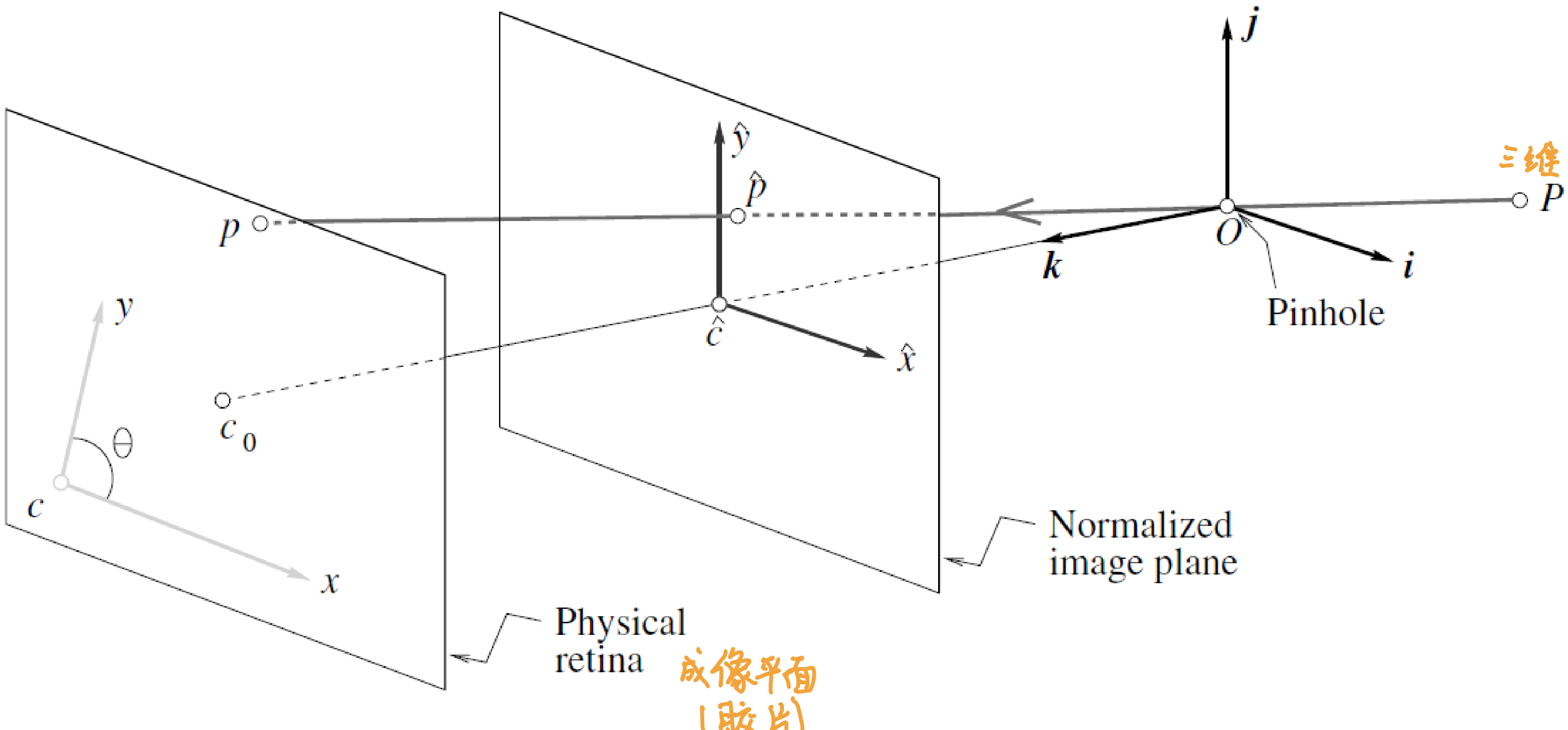


FIGURE 1.14: Physical and normalized image coordinate systems.

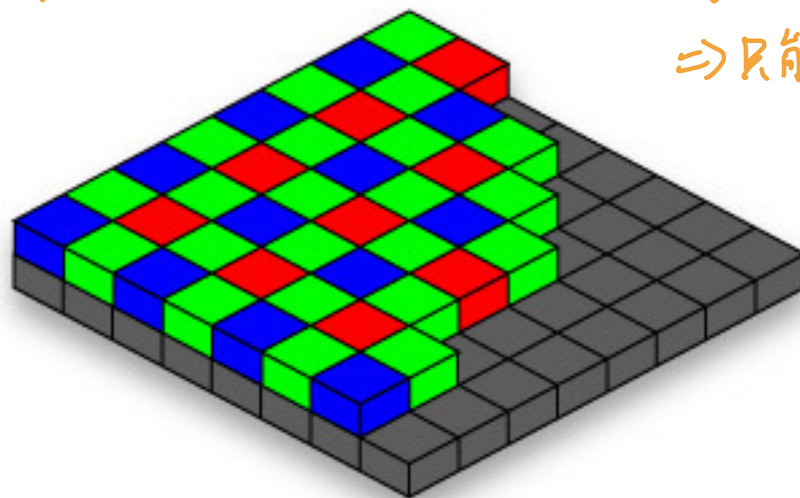
Point at normalized image plan

$$\begin{cases} \hat{x} = \dfrac{X}{Z} \\ \hat{y} = \dfrac{Y}{Z} \end{cases} \iff \hat{\boldsymbol{p}} = \frac{1}{Z}\begin{pmatrix} \mathrm{Id} & \mathbf{0} \end{pmatrix} \boldsymbol{P}$$

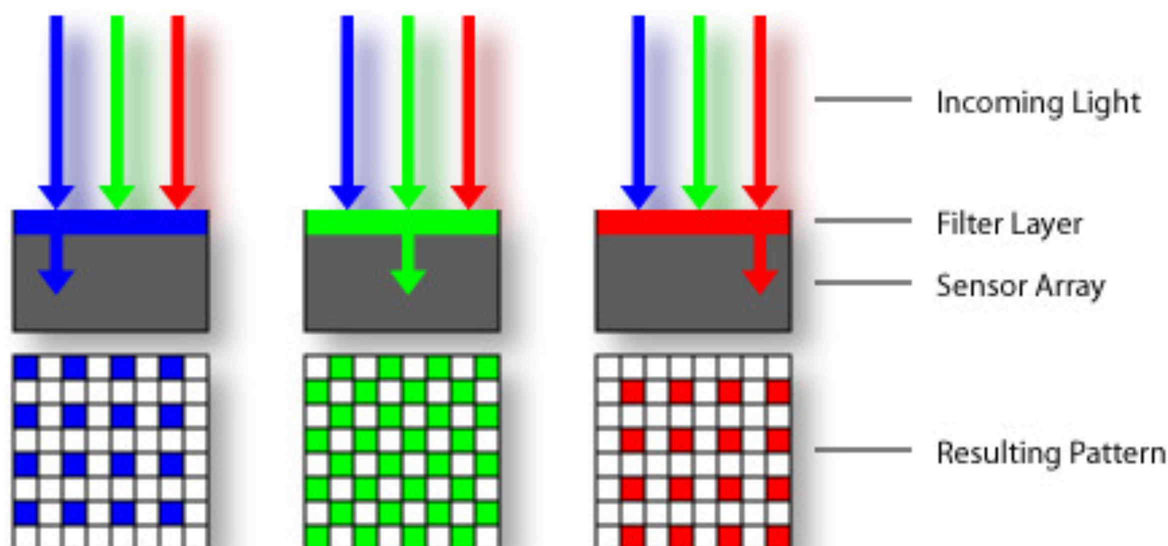
Color Sensing: Bayer Grid

交错排列，不是每个pixel都有green感光

⇒只能捕捉一种信号



Estimate RGB at each cell from neighboring values



http://en.wikipedia.org/wiki/Bayer_filter

Slide by Steve Seitz

Intrinsic Parameters 内参

- The coordinates (x, y) of the image point p are expressed in pixel units (not meters).
- Pixels may be rectangular instead of square(skewed).

$$\begin{cases} x = kf\dfrac{X}{Z} = kf\hat{x}, \\ y = lf\dfrac{Y}{Z} = lf\hat{y}. \end{cases}$$

$$\alpha = kf \text{ and } \beta = lf$$

乘个比例)

- The center of the CCD matrix usually does not coincide with the image center c_0 (不在同一位置)

同时发生

$$\begin{cases} x = \alpha\hat{x} + x_0, \\ y = \beta\hat{y} + y_0. \end{cases}$$

对齐

- Due to manufacturing error, the angle between two image axes is not 90 degrees.

$$\begin{cases} x = \alpha\hat{x} - \alpha\cot\theta\hat{y} + x_0, \\ y = \dfrac{\beta}{\sin\theta}\hat{y} + y_0. \end{cases}$$

Intrinsic Parameters

Putting all equations together, we get

$$\boldsymbol{p} = \mathcal{K}\hat{\boldsymbol{p}}, \quad \text{where} \quad \boldsymbol{p} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \quad \text{and} \quad \mathcal{K} \stackrel{\text{def}}{=} \begin{array}{c} \begin{array}{ccc} \text{比例} & \text{角度} & \text{位置} \end{array} \\ \begin{pmatrix} \alpha & -\alpha \cot\theta & x_0 \\ 0 & \dfrac{\beta}{\sin\theta} & y_0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}.$$

Here $\mathcal{K}$ is called (Internal) calibration matrix of the camera.

标定

$$\boldsymbol{p} = \frac{1}{Z}\mathcal{K}(\text{Id} \quad \mathbf{0})\boldsymbol{P} = \frac{1}{Z}\mathcal{M}\boldsymbol{P}, \quad \text{where} \quad \mathcal{M} \stackrel{\text{def}}{=} (\mathcal{K} \quad \mathbf{0})$$

Intrinsic parameters: $\alpha,\ \beta,\ \theta,\ x_0,\ \text{and}\ y_0$

Extrinsic Parameters:

Camera coordinate frame:C

$$p = \frac{1}{Z}\mathcal{M}\,{}^{C}P$$

World coordinate frame:W rotation & translation

$$\underbrace{{}^{C}P}_{\text{P在 camera coordinate下}} = \underbrace{\begin{pmatrix} \mathcal{R} & t \\ 0^{T} & 1 \end{pmatrix}}\,{}^{W}P,$$

Taking P = ^{W}P

$$p = \frac{1}{Z}\overset{3\times4}{\mathcal{M}}\overset{4\times1}{P}, \quad \text{where} \quad \mathcal{M} = \overset{3\times3}{\mathcal{K}}\overset{3\times4}{(\mathcal{R} \quad t)}. = 3\times4$$

Extrinsic Parameters

Extrinsic Parameters: 3 independent parameters in rotation matrix ***R*** and 3 parameters in translation vector ***t.***

Denote the columns in M as $\boldsymbol{m}_1^T$, $\boldsymbol{m}_2^T$ and $\boldsymbol{m}_3^T$

Then we have

$$\begin{cases} x = \dfrac{\boldsymbol{m}_1 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}}, \\ y = \dfrac{\boldsymbol{m}_2 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}}. \end{cases}$$

Building a Real Camera

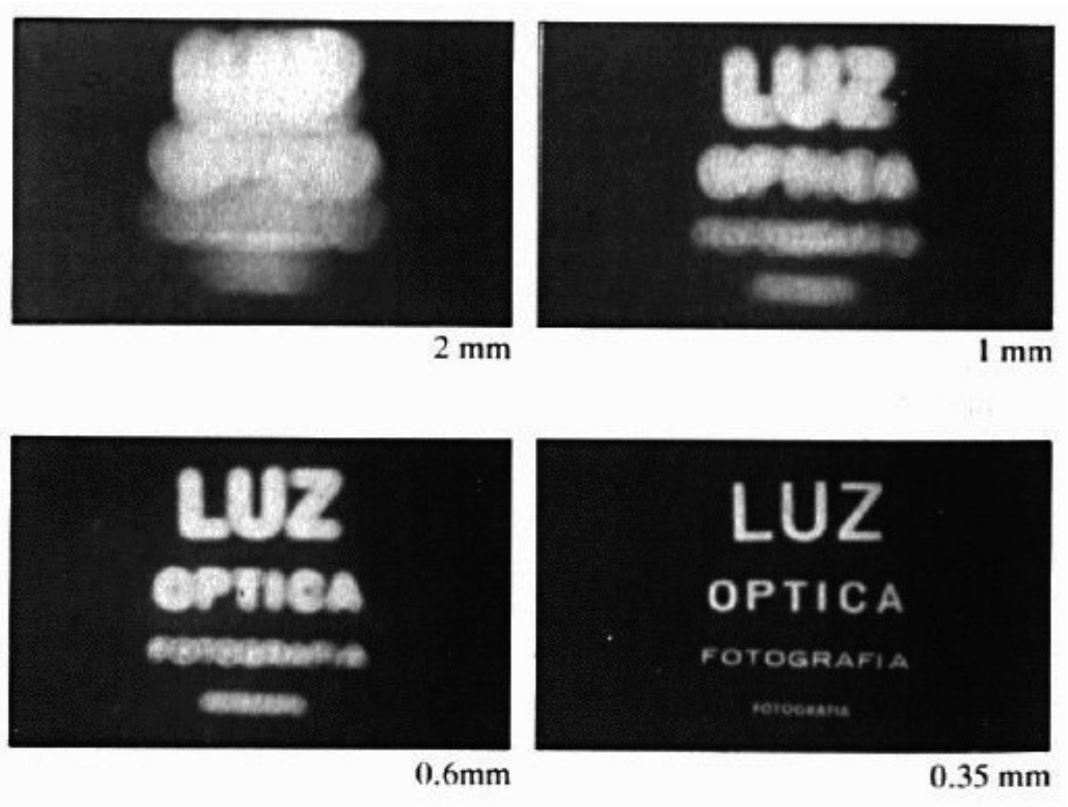
Home-made pinhole camera

Slide by A. Efros

http://www.debevec.org/Pinhole/

Shrinking the aperture

光圈越小，成像越清晰

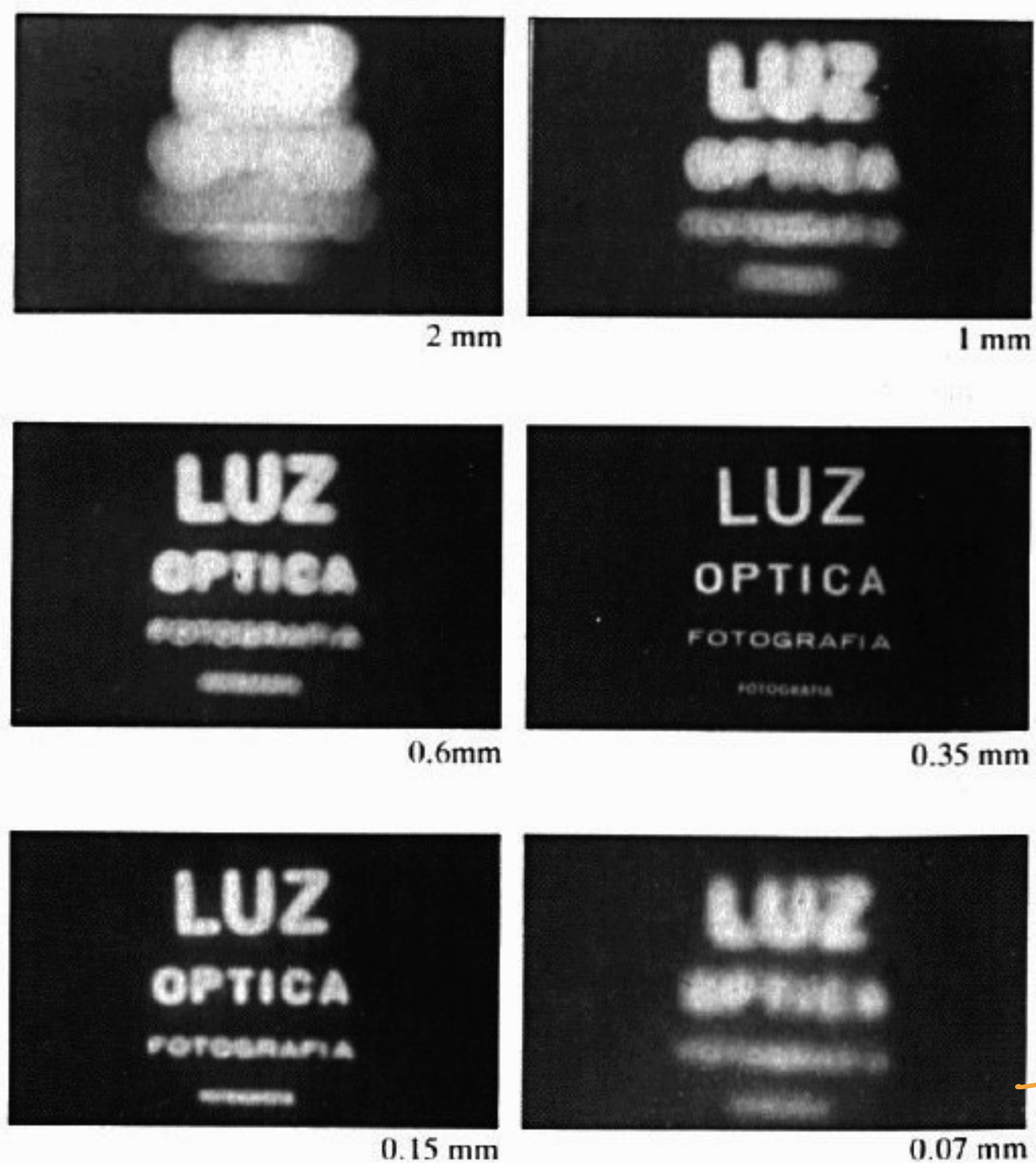


2 mm
1 mm
0.6mm
0.35 mm

Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects…

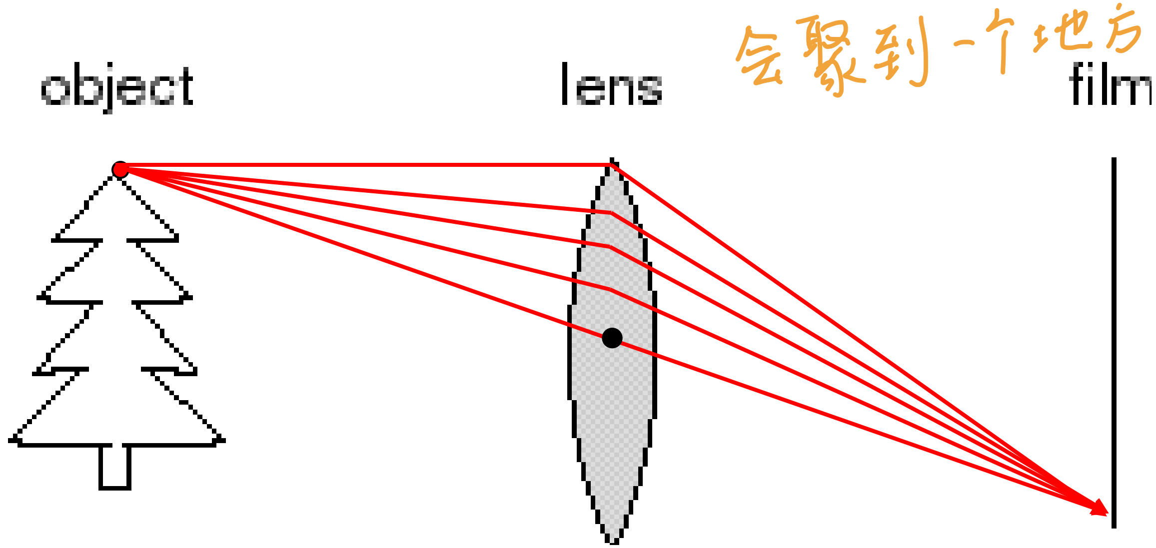
Shrinking the aperture



但越来越暗

再减 → 过小 → 衍射

Adding a lens 透镜

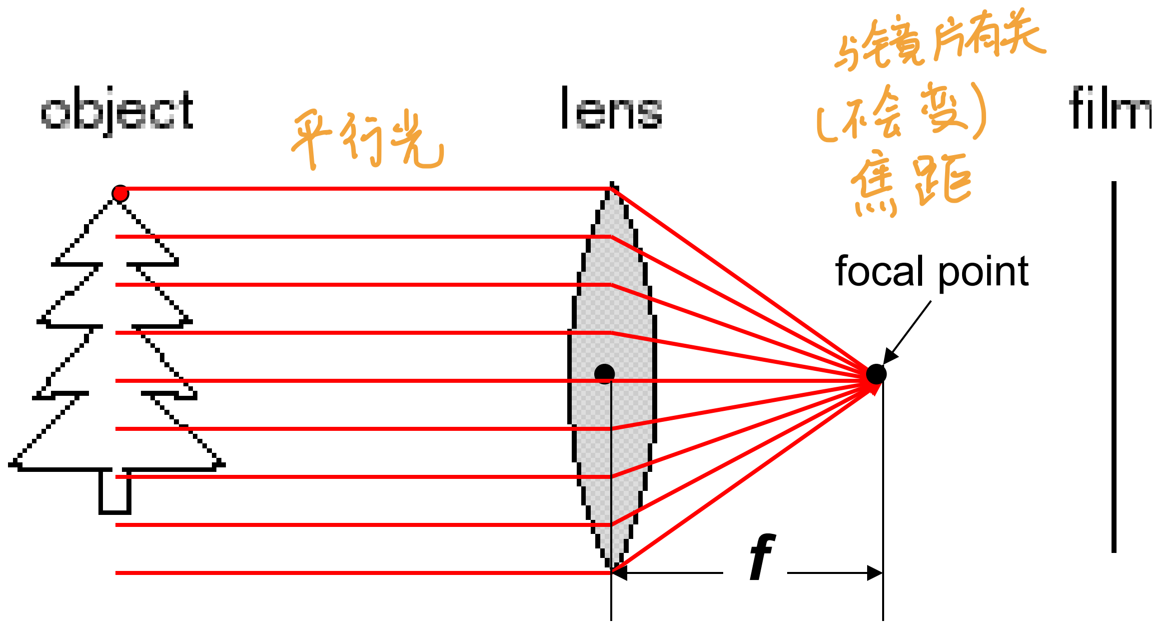


A lens focuses light onto the film

- Thin lens model:
  - Rays passing through the center are not deviated (pinhole projection model still holds)

Slide by Steve Seitz

Adding a lens

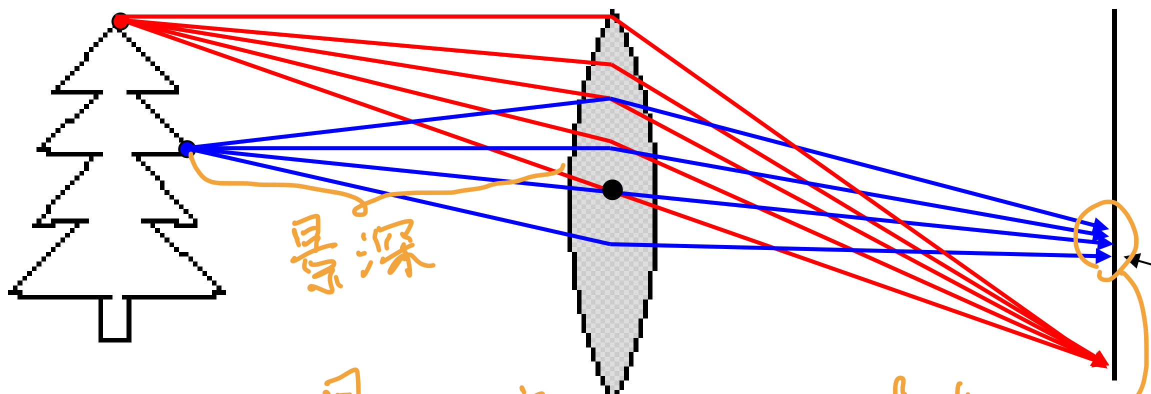


A lens focuses light onto the film

- Thin lens model:
  - Rays passing through the center are not deviated (pinhole projection model still holds)
  - All parallel rays converge to one point on a plane located at the *focal length f*

Slide by Steve Seitz

Adding a lens



景深变化 → 不 perfect

A lens focuses light onto the film

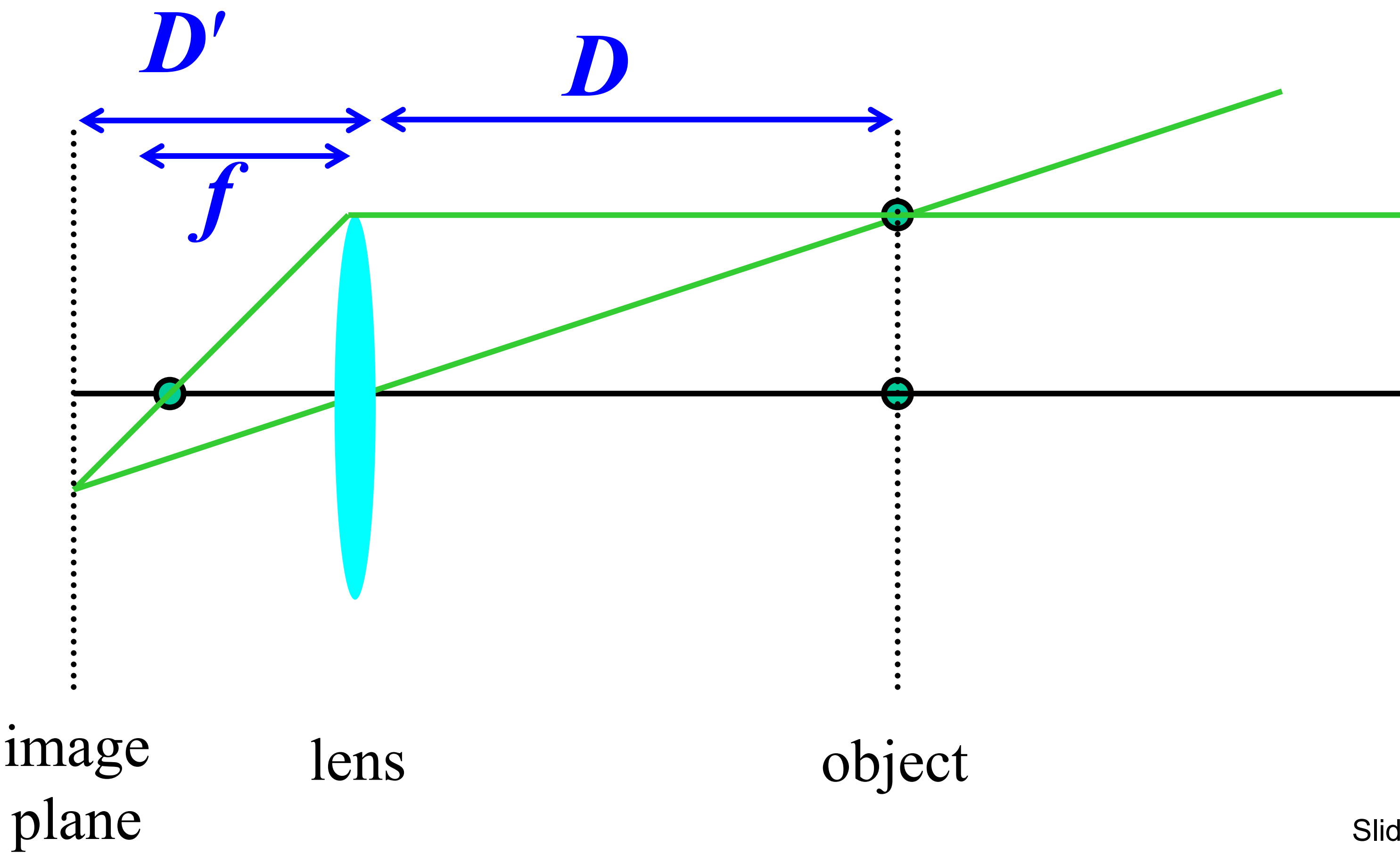
焦距在特定范围内

- There is a specific distance at which objects are "in focus"
  - other points project to a 'circle of confusion' in the image

Slide by Steve Seitz

Thin lens formula

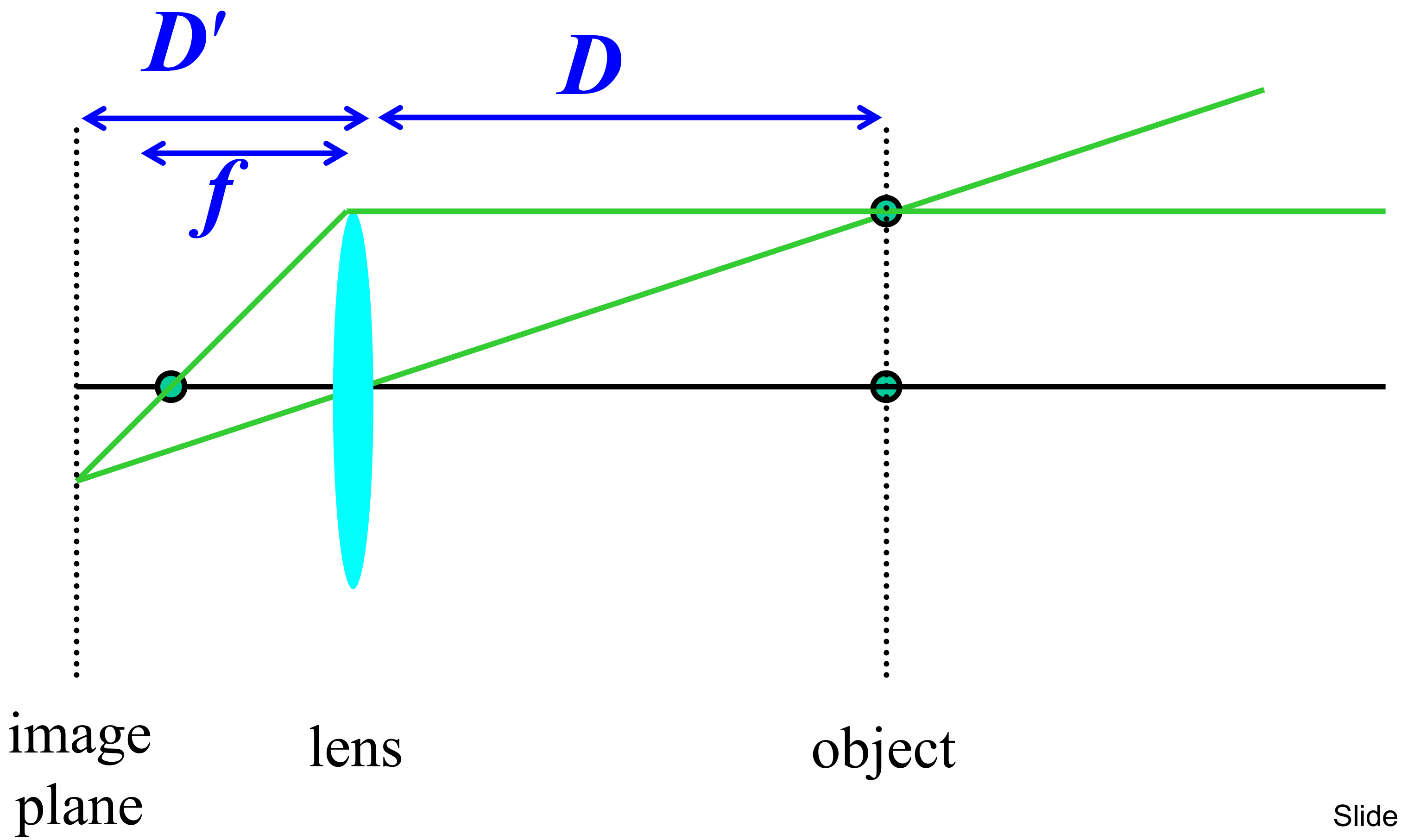
- What is the relation between the focal length (f), the distance of the object from the optical center (D), and the distance at which the object will be in focus (D')?



Slide by Frédo Durand

Thin lens formula

Similar triangles everywhere!

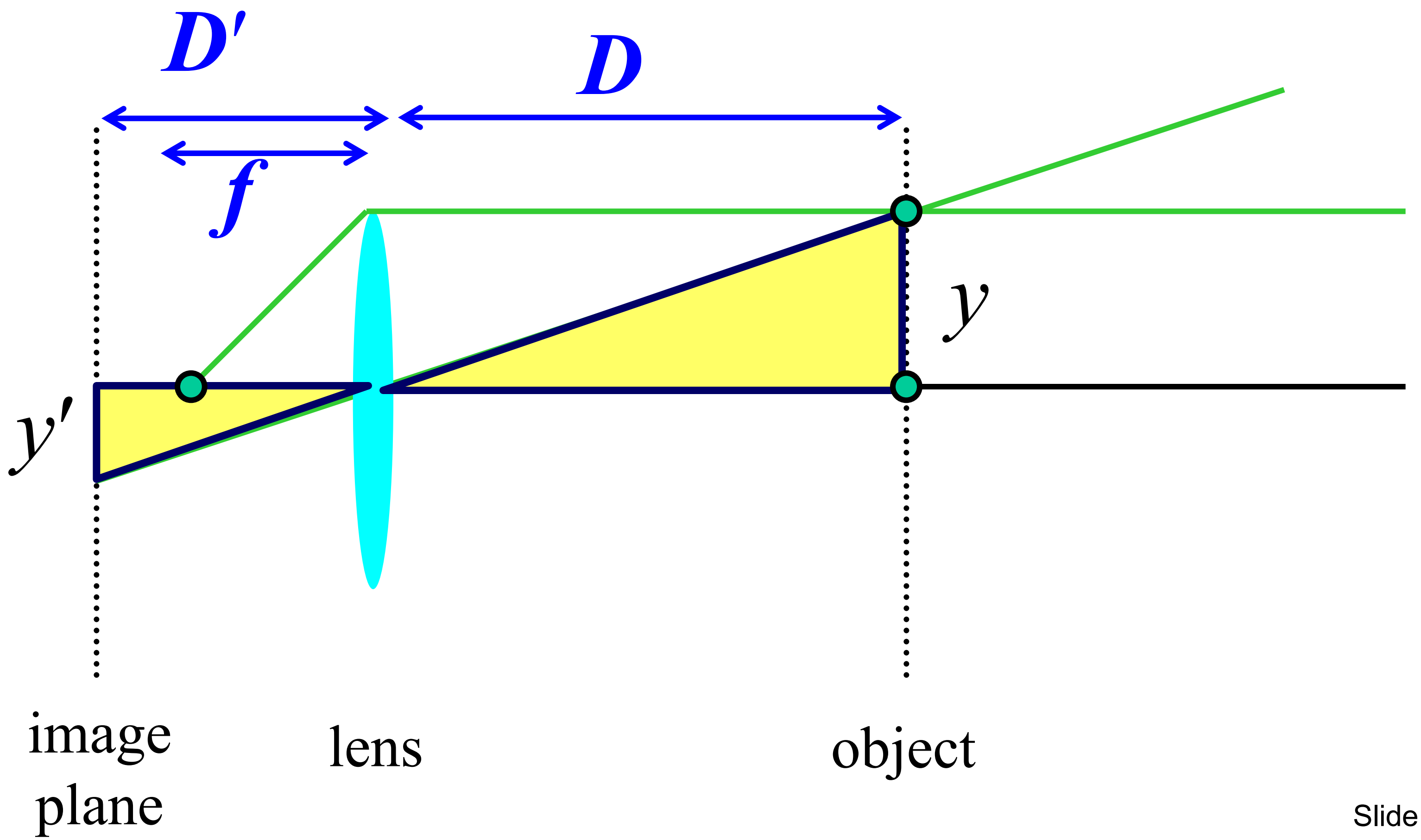


Slide by Frédo Durand

Thin lens formula

Similar triangles everywhere!

$$y'/y = D'/D$$



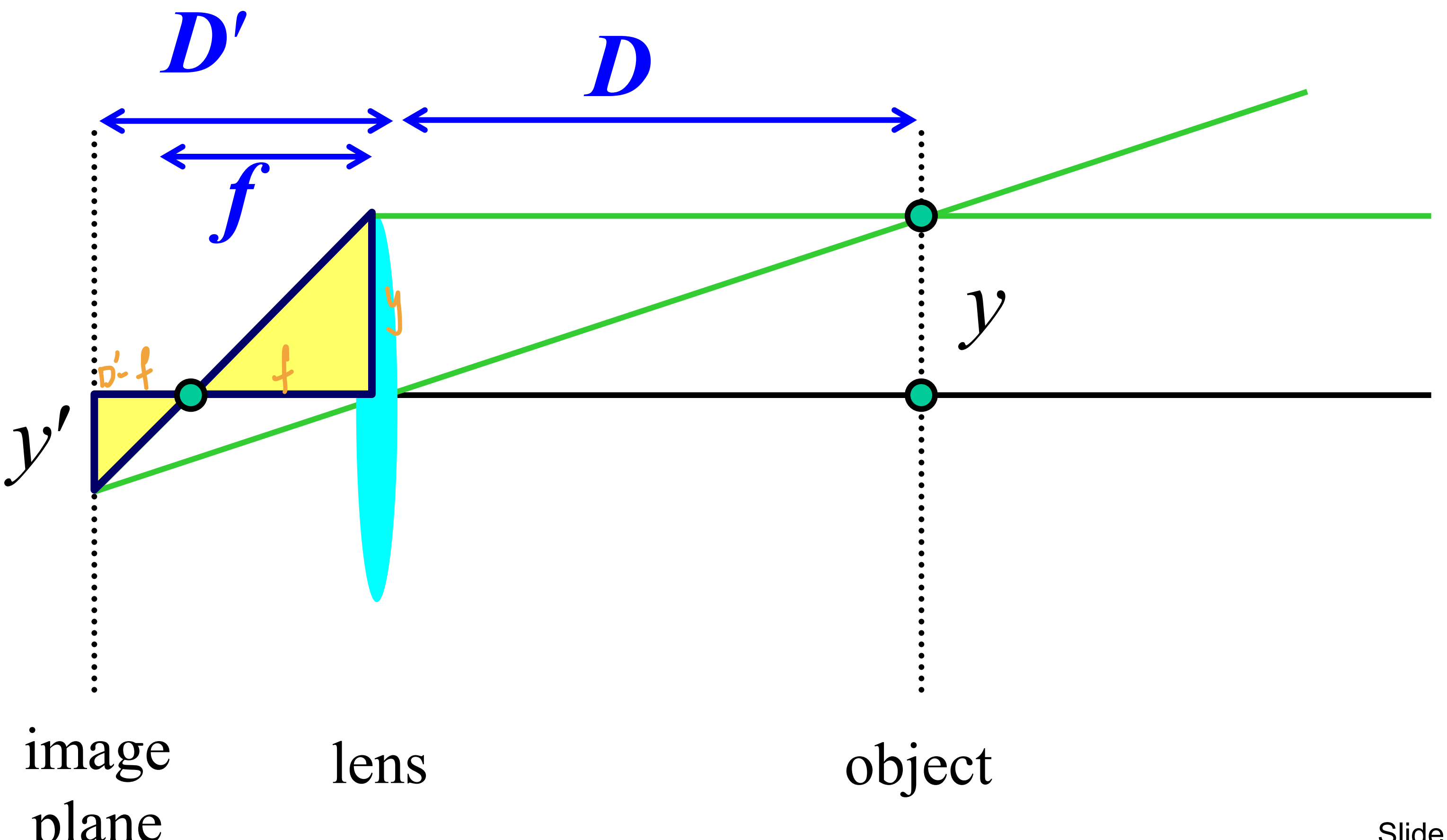
Slide by Frédo Durand

Thin lens formula

Similar triangles everywhere!

$$y'/y = D'/D$$

$$y'/y = (D'-f)/f$$

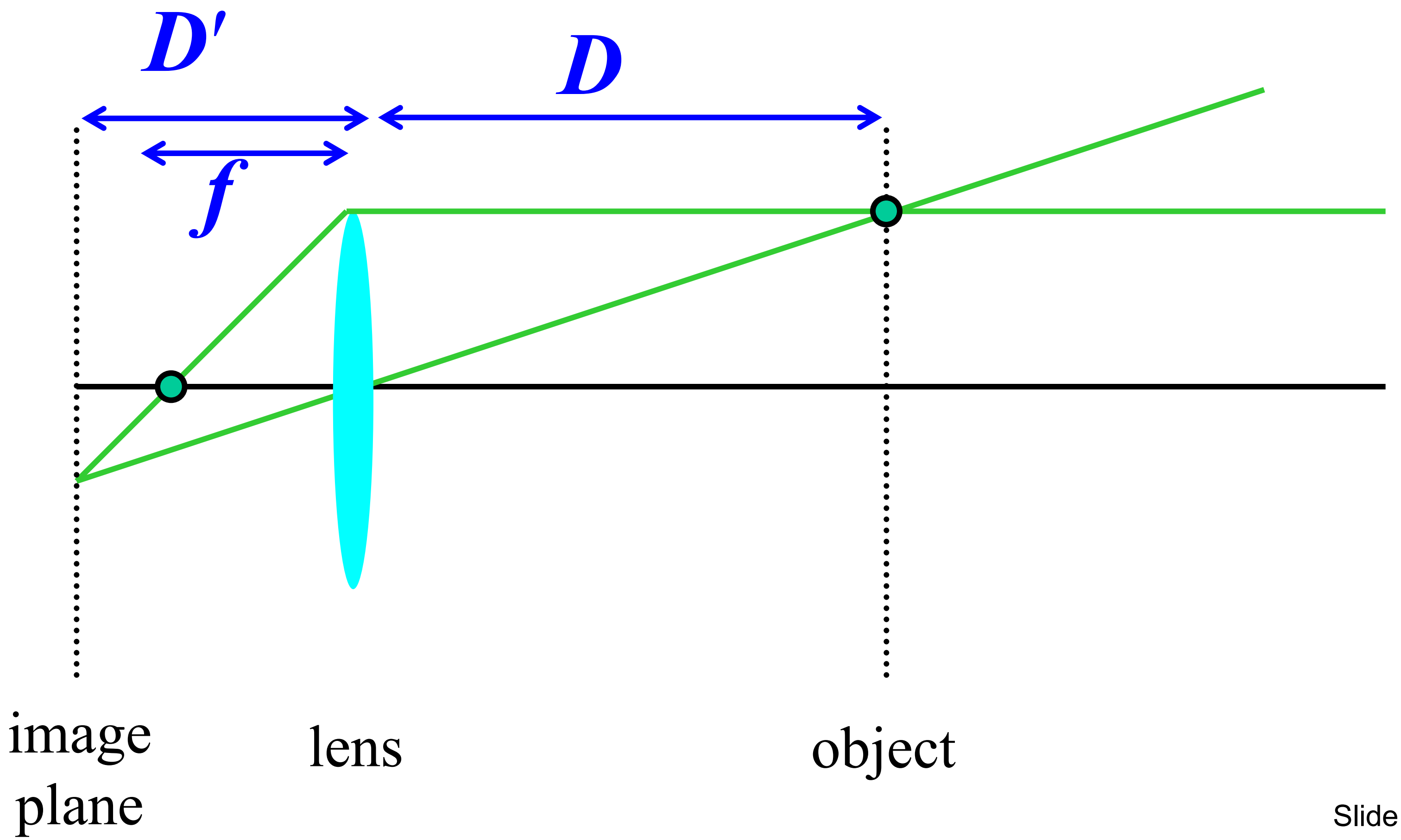


Slide by Frédo Durand

Thin lens formula

$$\boxed{\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}}$$

Any point satisfying the thin lens equation is in focus.



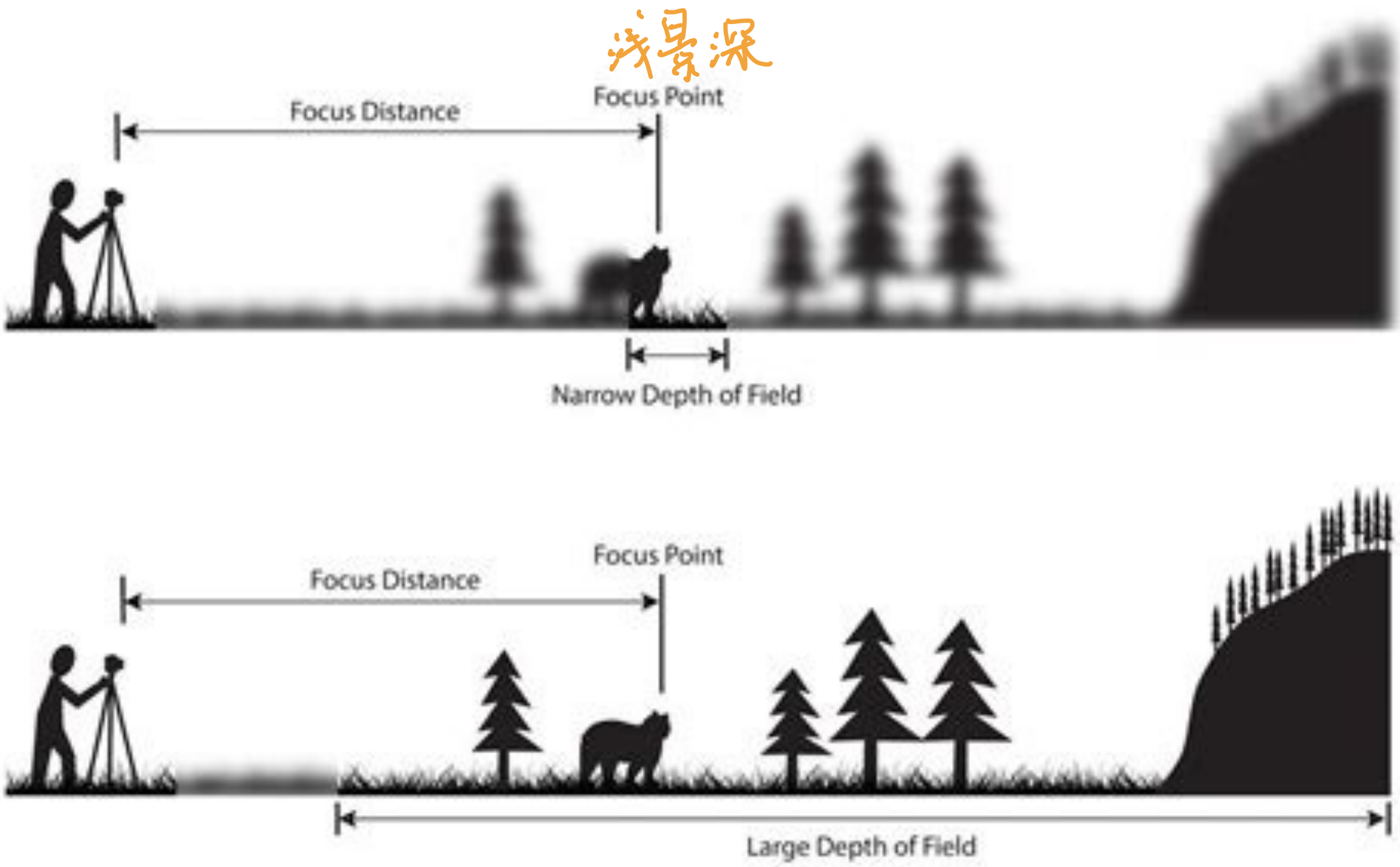
Slide by Frédo Durand

Depth of Field

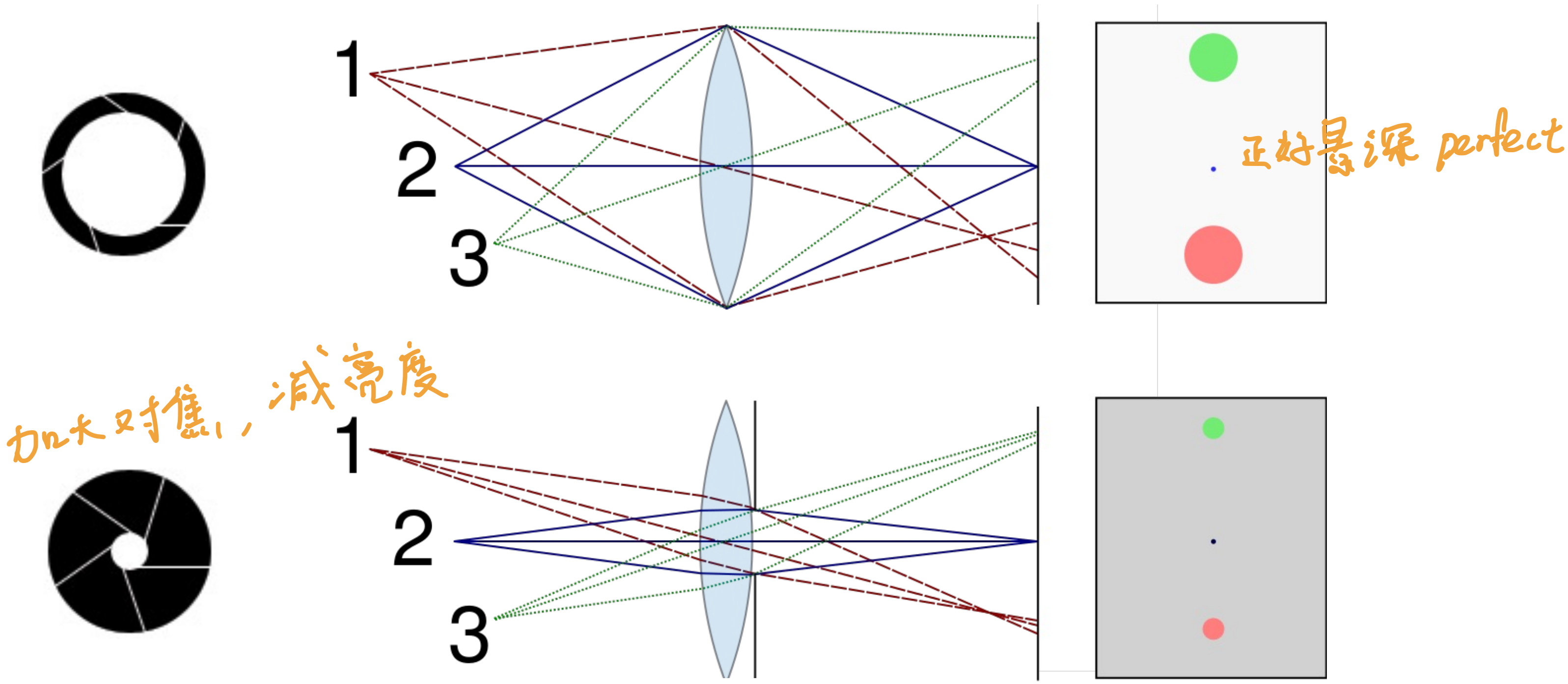
http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Slide by A. Efros

Depth of Field


浅景深
Focus Point
Focus Distance
Narrow Depth of Field
Focus Point
Focus Distance
Large Depth of Field

Controlling depth of field → 靠光圈



Changing the aperture size affects depth of field

- A smaller aperture increases the range in which the object is approximately in focus
- But small aperture reduces amount of light – need to increase exposure

http://en.wikipedia.org/wiki/File:Depth_of_field_illustration.svg

Varying the aperture

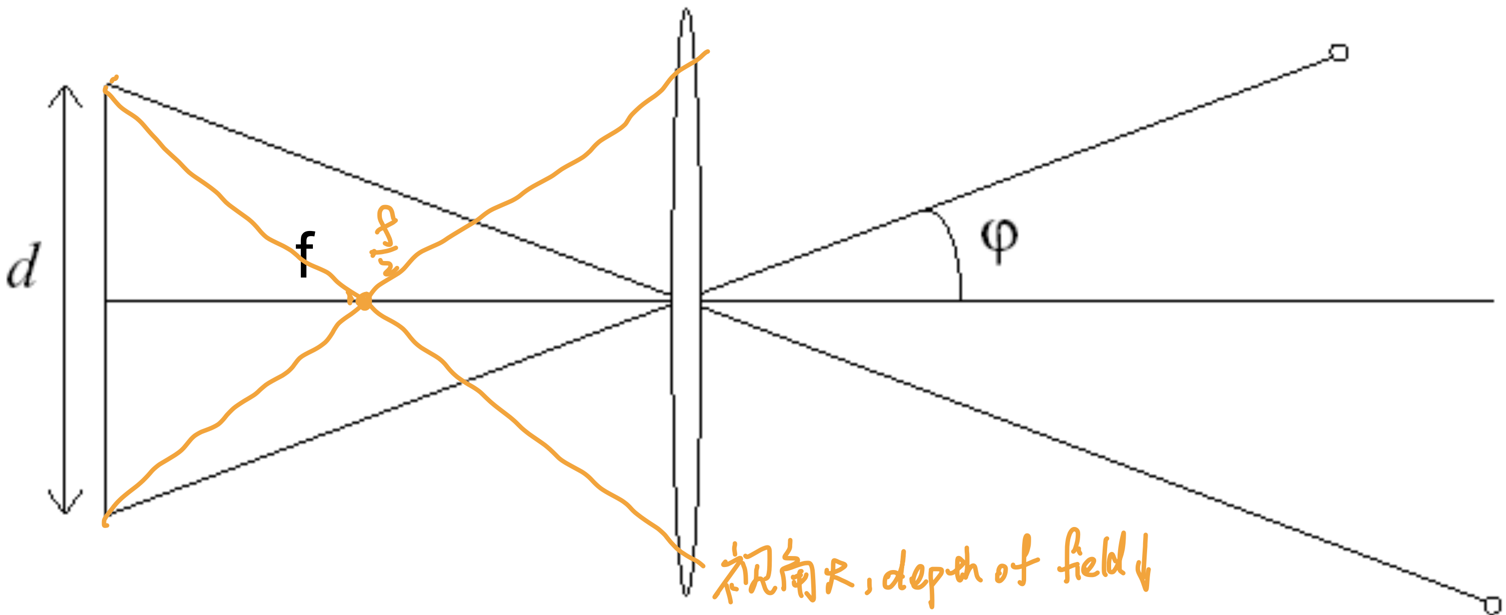
Large aperture = small DOF

DOF： depth of focus

Small aperture = large DOF

Slide by A. Efros

Field of View 视野



FOV depends on focal length and size of the camera retina

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

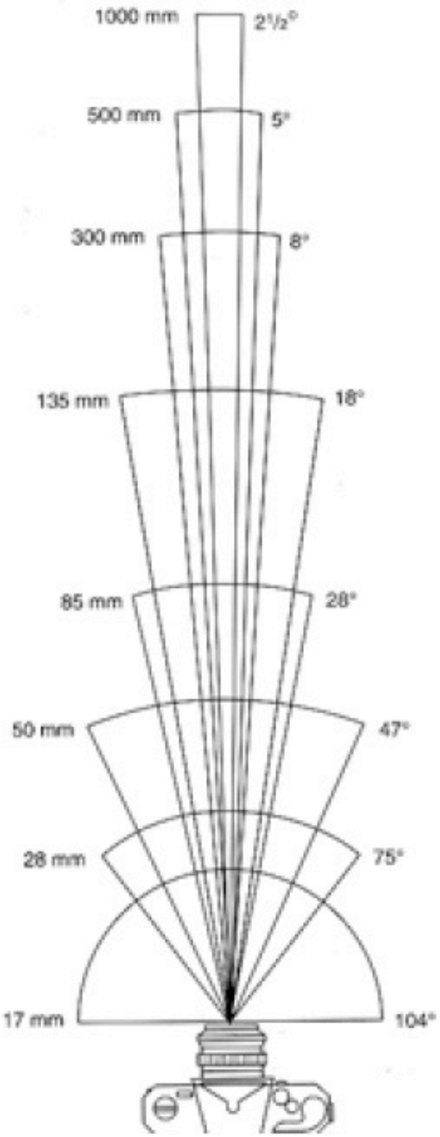
Larger focal length = smaller FOV

Slide by A. Efros

正常人視野

青光眼
(逐漸縮小的視野)

Field of View



17mm

28mm

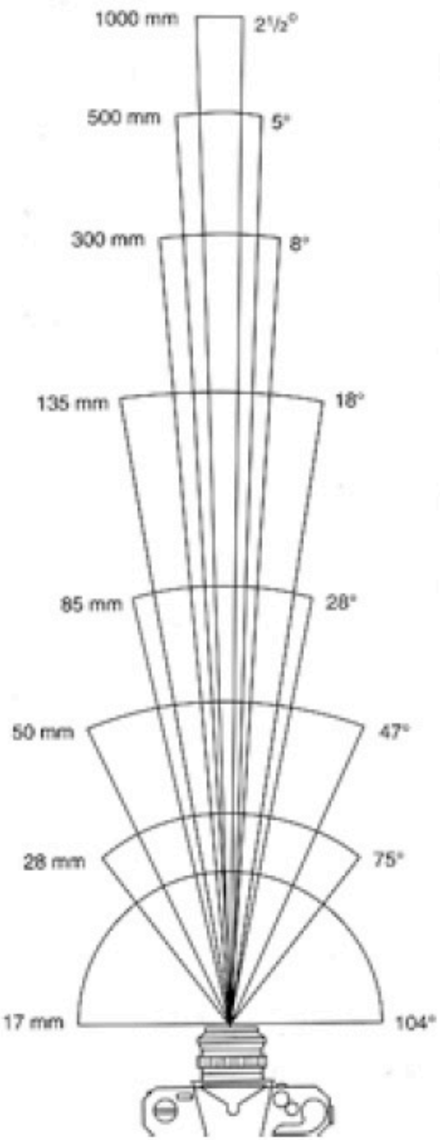
50mm

85mm

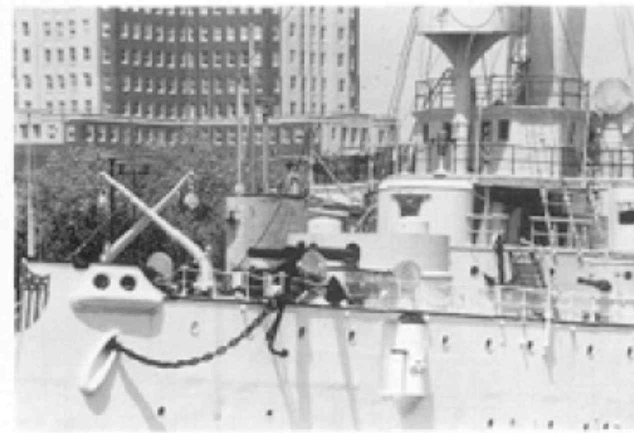
加焦距 → FOV↓

Slide by A. Efros

Field of View



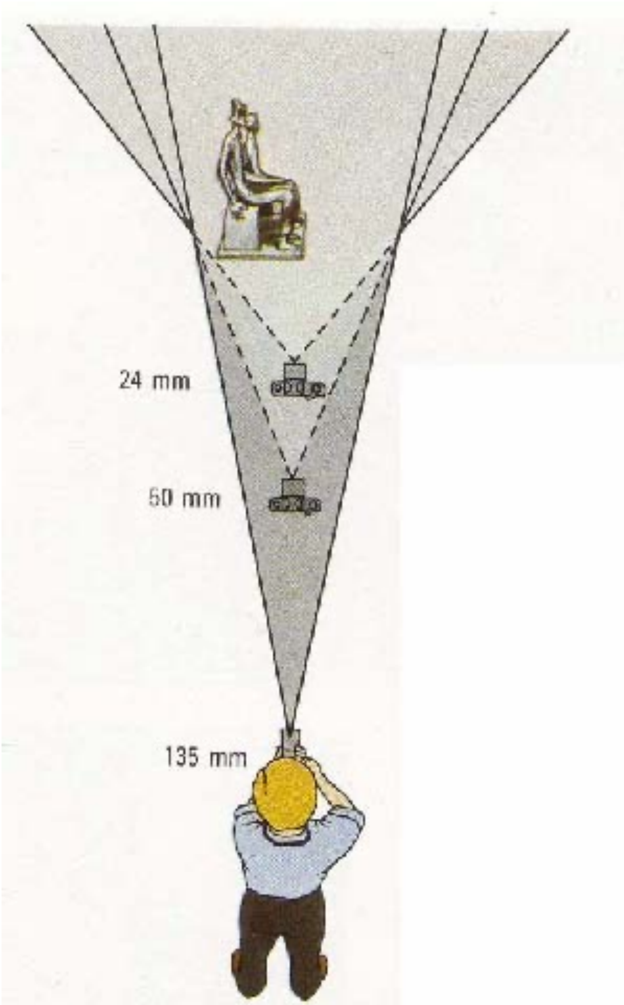
135mm



300mm

Slide by A. Efros

Field of View / Focal Length



Large FOV, small *f*
Camera close to car

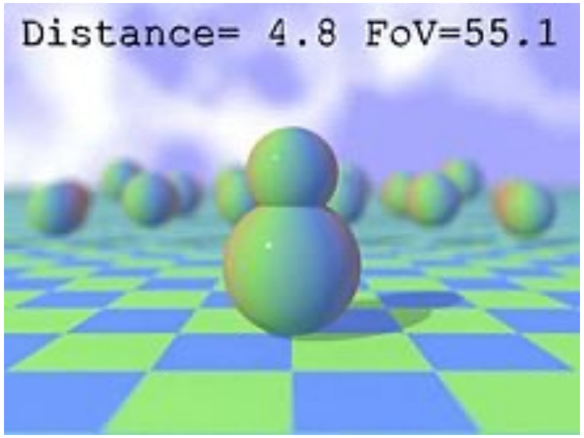
Small FOV, large *f*
Camera far from the car

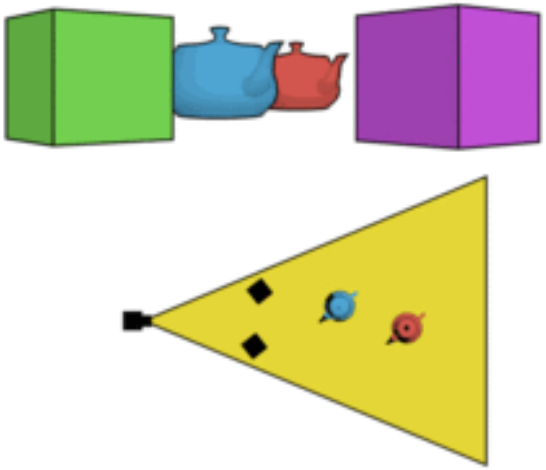
Sources: A. Efros, F. Durand

The dolly zoom(滑动变焦)

- Continuously adjusting the focal length while the camera moves away from (or towards) the subject

焦点





http://en.wikipedia.org/wiki/Dolly_zoom

The dolly zoom

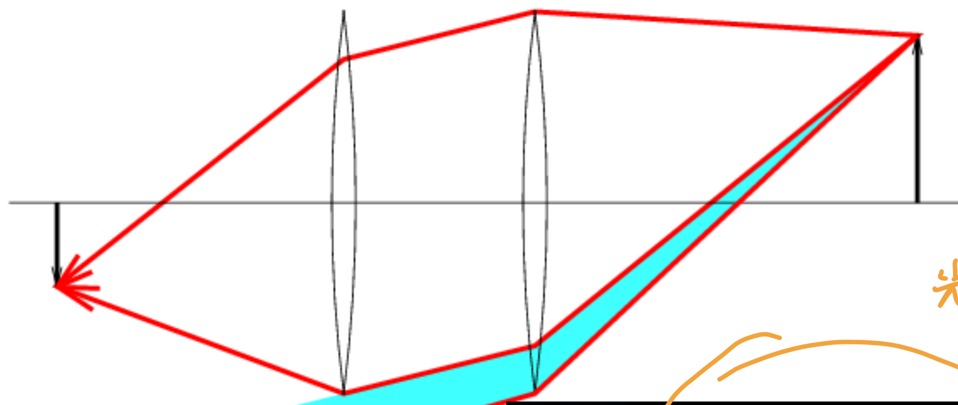
- Continuously adjusting the focal length while the camera moves away from (or towards) the subject
- 眩晕
  “The Vertigo shot”

[Example of dolly zoom from *Goodfellas*](#) (YouTube)

[Example of dolly zoom from *La Haine*](#) (YouTube)

Lens flaws: Vignetting（光晕）

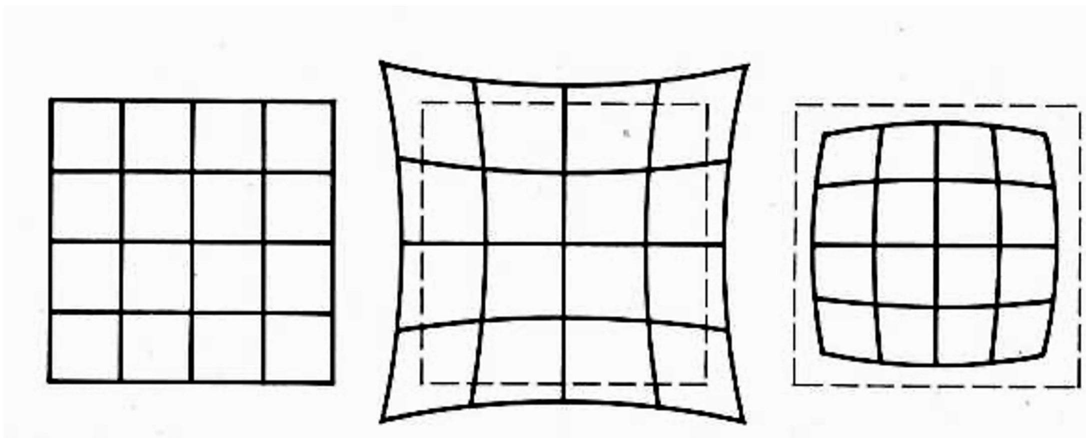
A photograph whose edges shade off gradually



光晕（直射光）
→ 遮光罩

Radial Distortion 镜像畸变

- Caused by imperfect lenses.
- Deviations are most noticeable near the edge of the lens



No distortion Pin cushion Barrel

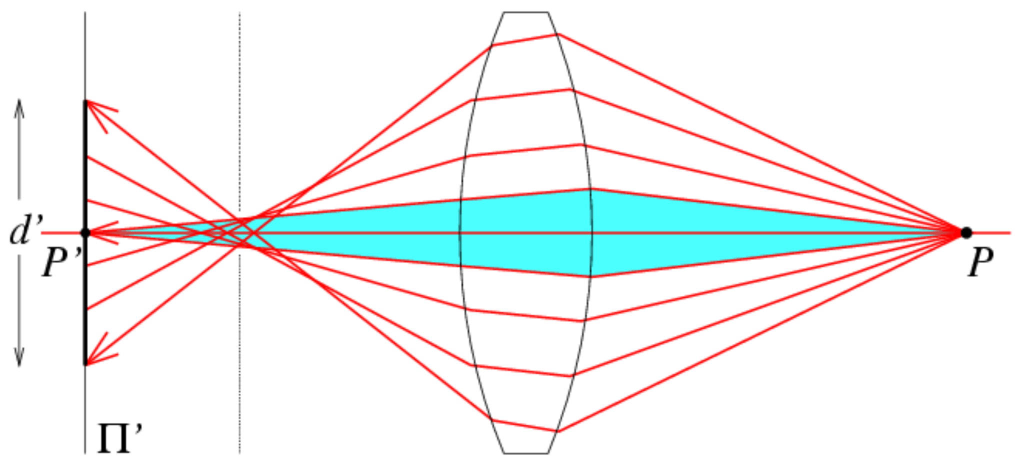
Lens flaws: Spherical aberration 球差

Spherical lenses don't focus light perfectly

Rays farther from the optical axis focus closer

焦不到一个位置

(散射光圈)



Lens Flaws: Chromatic Aberration 色差

Lens has different refractive indices for different wavelengths: causes color fringing

颜色不能会聚一点