Lecture 12
Feature Extraction

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Image analysis fundamental steps

image acquisition

preprocessing,
enhancement

segmentation

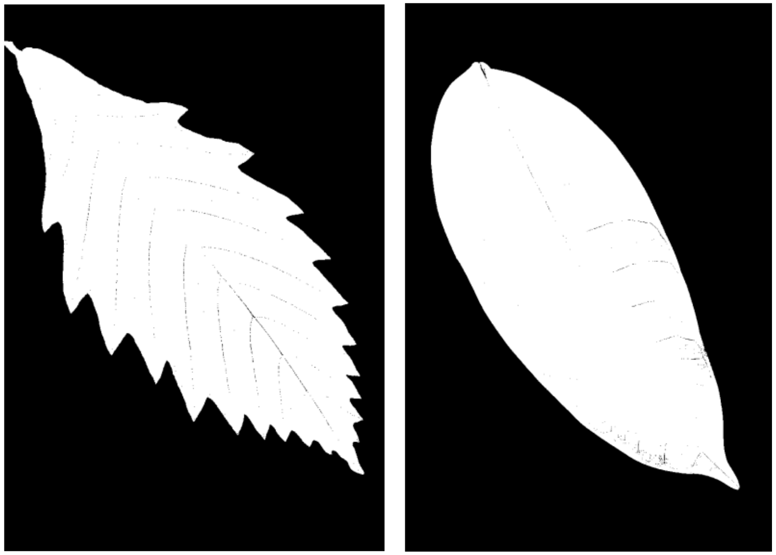
Representation, description, feature
extraction

classification,
interpretatio,
recognition

result

Boundary and region description

- Commonly after segmentation one needs to represent objects in order to describe them



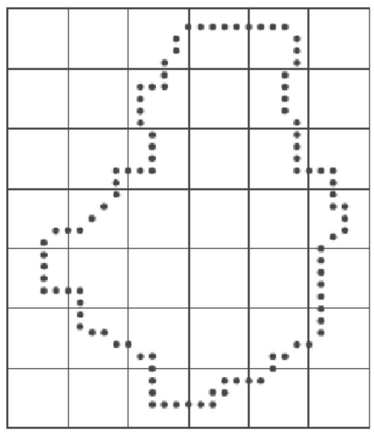
- **External (boundary):**
  - Representation: Polygon of the boundary
  - Description: The circumference
- **Internal (regional)**
  - Representation: Pixels inside the object
  - Description: The average color

Boundary and region description

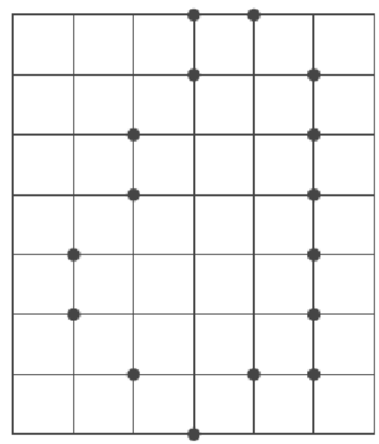
- Outline:
  - Boundary and region description
  - Topology (Euler number)
  - Skeleton
  - Statistic on histogram of intensity
  - Gray-level co-occurrence matrix (GLCM)

Boundary representation: Chain code

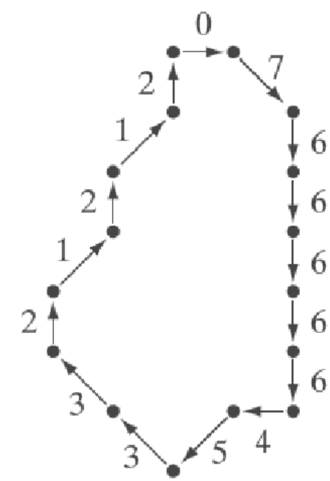
Boundary representation = 076666453321212



Original boundary

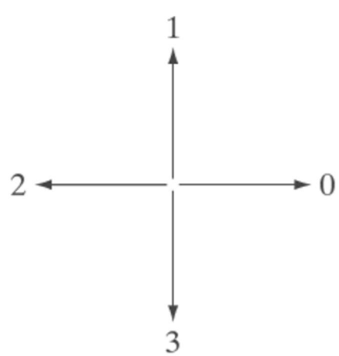


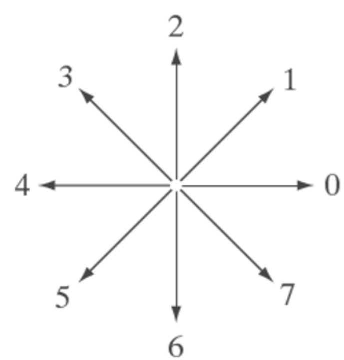
Sub-sampled boundary



Chain code of boundary

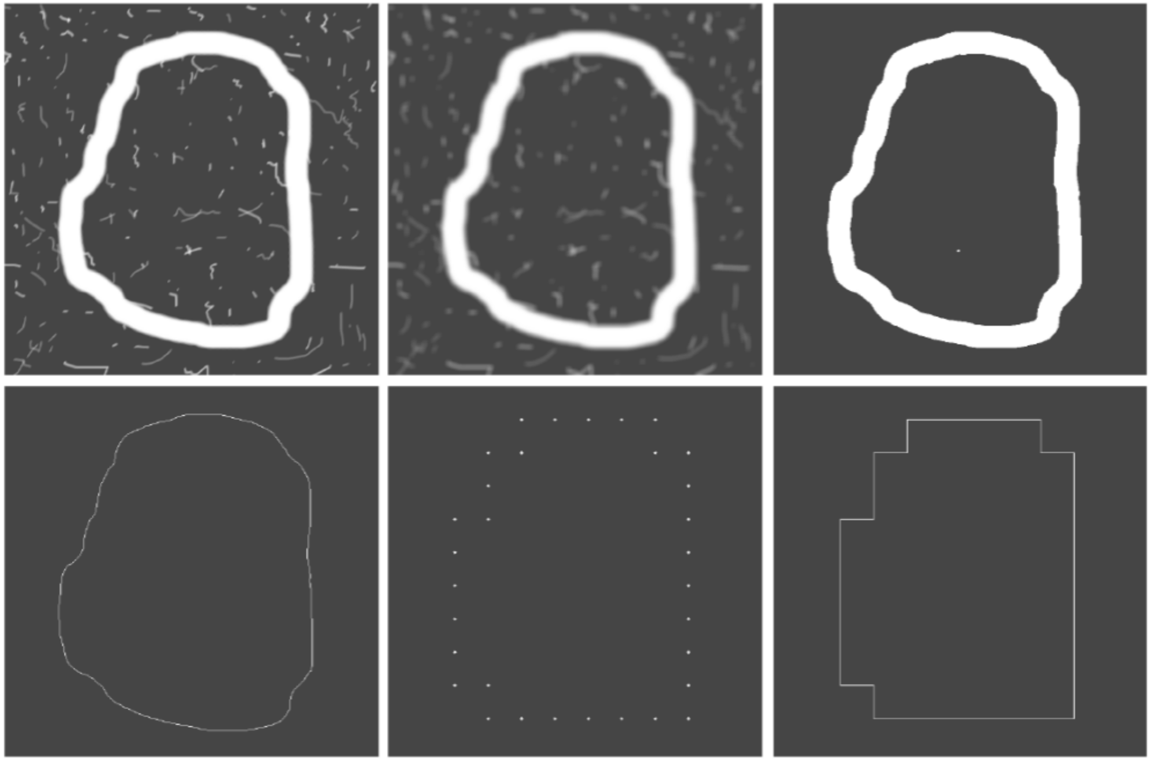
Chain code for
4-neighborhood





Chain code for
8-neighborhood

Chain code: example

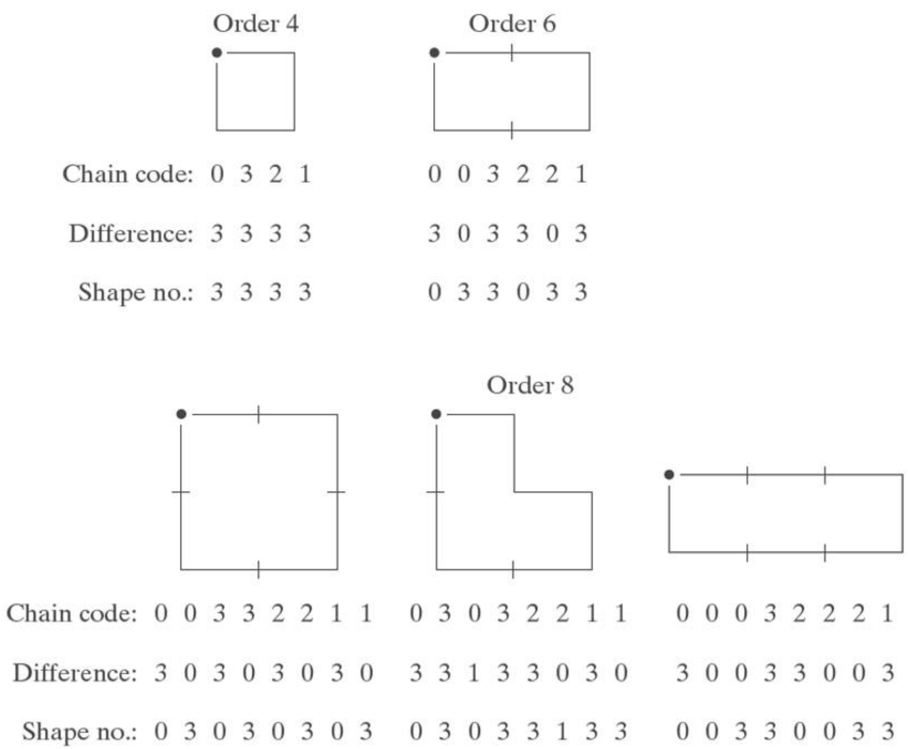


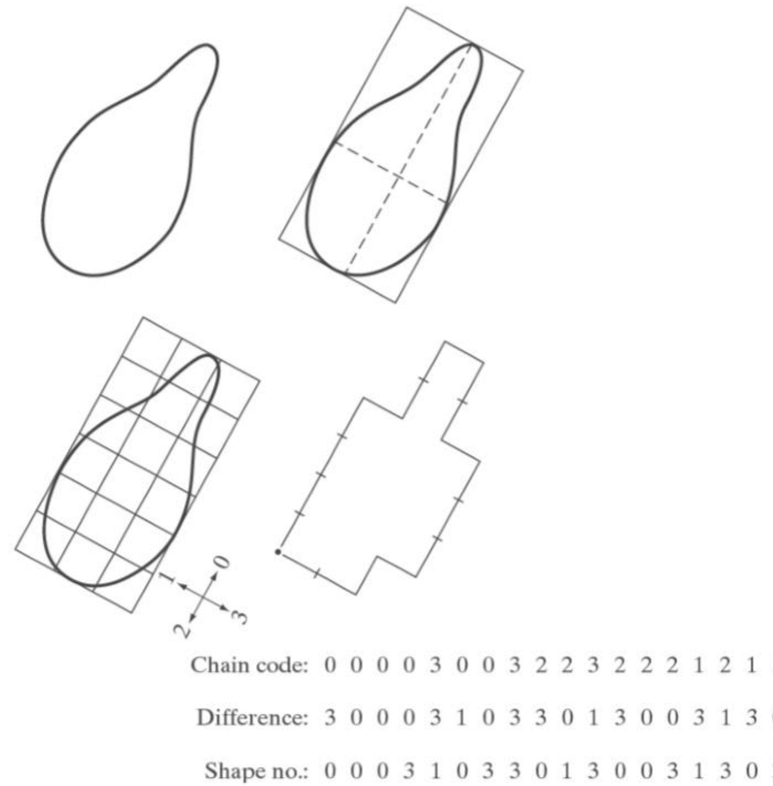
8-directional chain code → 00006066666666444444242222202202

Starting point normalized chain code → 00006066666666444444242222202202

Rotation normalized chain code → 0006200000006000006260000620626

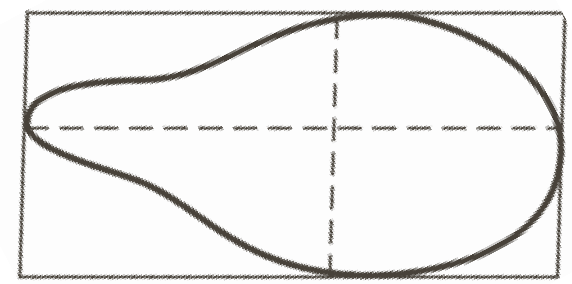
Shape number: A boundary descriptor

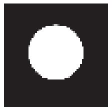
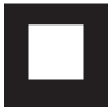
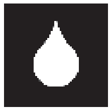




Simple Boundary Descriptors

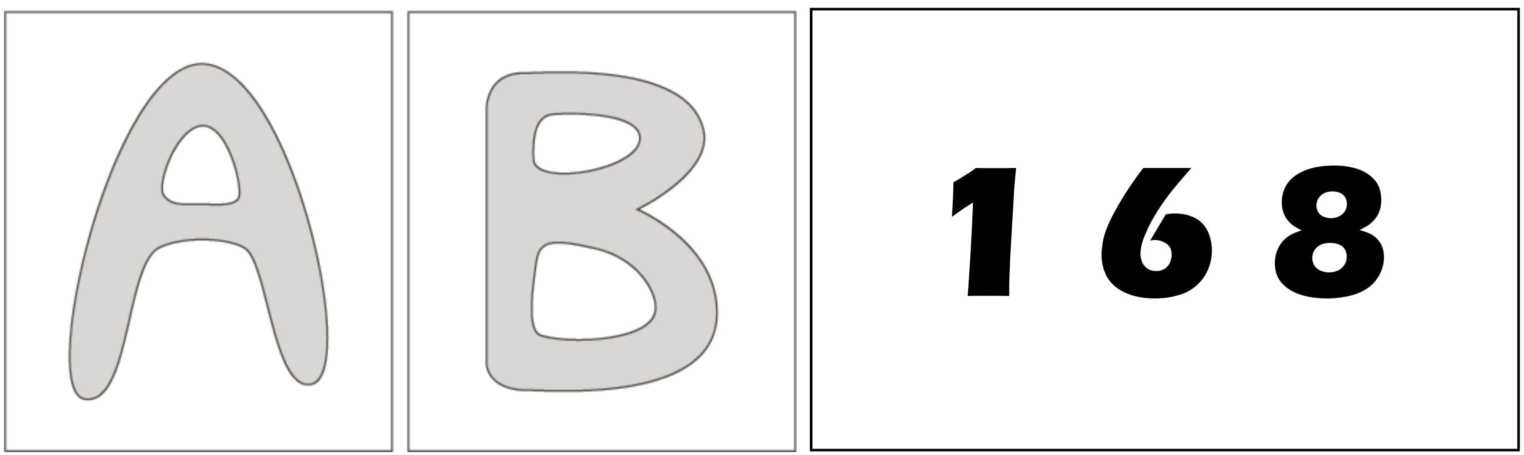
- Perimeter (周长)
- Area
- Bounding Box
- Diameter (直径): longest path between two edge pixels.
- Compactness : $\frac{(Perimeter)^2}{(area)}$
- Circularity: $\frac{4\pi(area)}{(Perimeter)^2}$
- Centroid (形心): $c(x,y) = \frac{1}{k}\sum_{p\in Object} p(x,y)$
- Major Axis (长轴) & Minor Axis（短轴）
- Eccentricity (偏心率)



Descriptor				
Compactness	10.1701	42.2442	15.9836	13.2308
Circularity	1.2356	0.2975	0.7862	0.9478
Eccentricity	0.0411	0.0636	0	0.8117

Topological Descriptors（拓扑描绘子）

- Euler Number (欧拉数): $E = C - H$
- C stands for # of components and H stands for # of Holes.



Fourier Descriptors（傅里叶描绘子）

- Represent the boundary by a sequence of points (assume clockwise order)

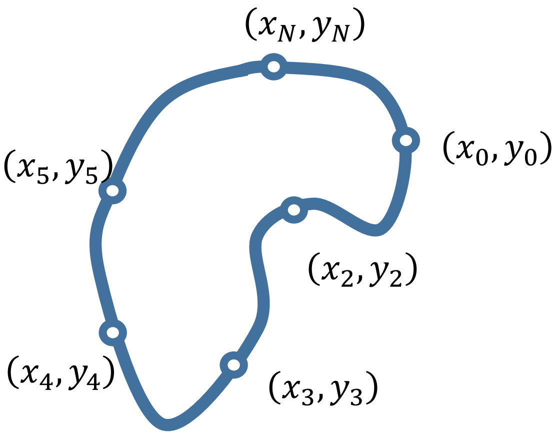
$$\{(x_0, y_0), (x_1, y_1), \cdots, (x_N, y_N)\}$$

- Write each point (x_n, y_n) as a complex number

$$s(n) = x(n) + jy(n)$$

- Take 1D Fourier series of $s(n)$ to get coefficient $a(u)$

$$a(u) = \sum_{n=1}^{N} s(n) e^{-j2\pi un/N}$$



- Fourier descriptors are a concise description of (object) contours
- Can be used for
  - Contour processing (filtering, interpolation, morphing)
  - Image analysis (characterizing and recognizing shapes)

Fourier Descriptors（傅里叶描绘子）

- We have Fourier transform coefficients $a(u)$

$$a(u) = \sum_{n=1}^{N} s(n) e^{-j2\pi un/N}$$

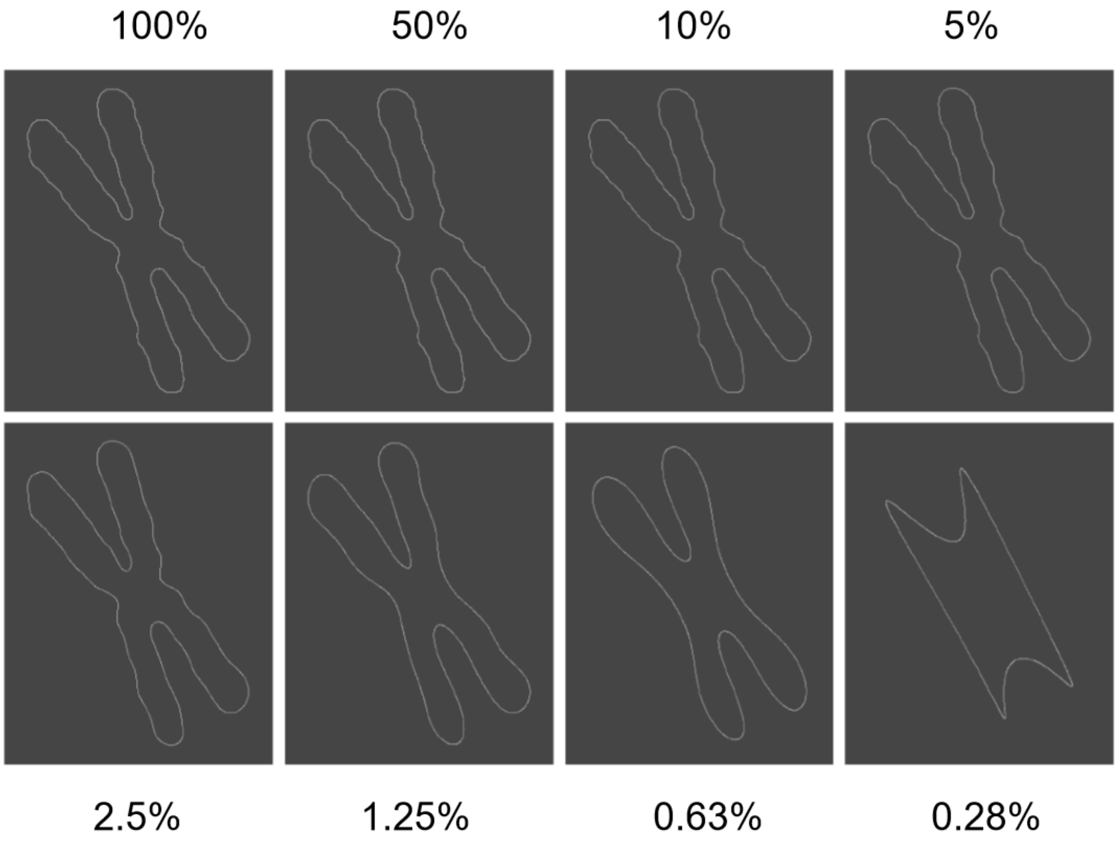
- Given coefficients, we can reconstruct boundary

$$s(n) = \frac{1}{N} \sum_{u=1}^{N} a(u) e^{j2\pi un/N}$$

- Higher order coefficients can be truncated for a more concise representation (e.g. low pass filter)
- *Other filters: Sharpening, edge extraction……*

Boundary Reconstruction using Fourier Descriptors

2868 descriptors



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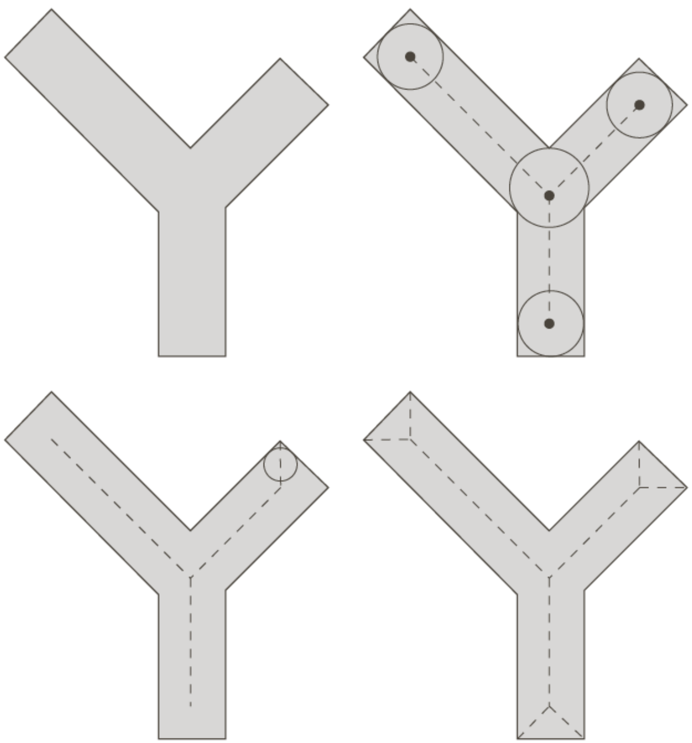
Fourier Descriptors

Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy}\delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

Skeletons（骨架）



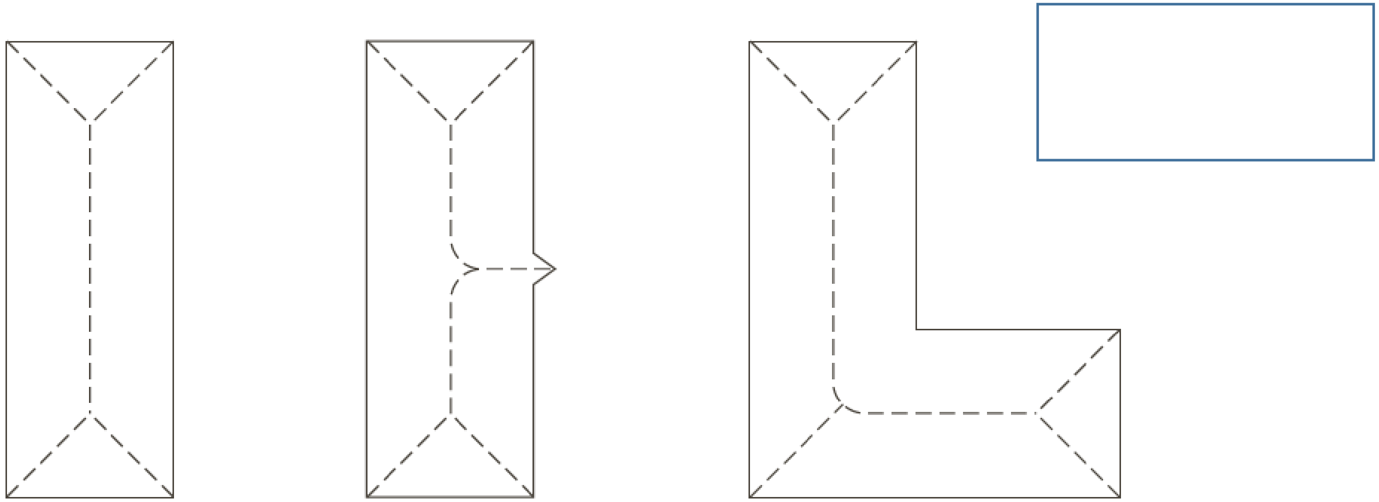
- Estimation:
  - Successive erosions
  - Distance transform
  - Points that have more than one nearest neighbor.

Matlab code:

```
Bw = bwmorph(im,'skel',Inf);
```

Skeletons（骨架）

- Get topology information:
  - The number of connection nodes contained in a structure.
  - Number of branches from a node.
  - Distance between branches and nodes.



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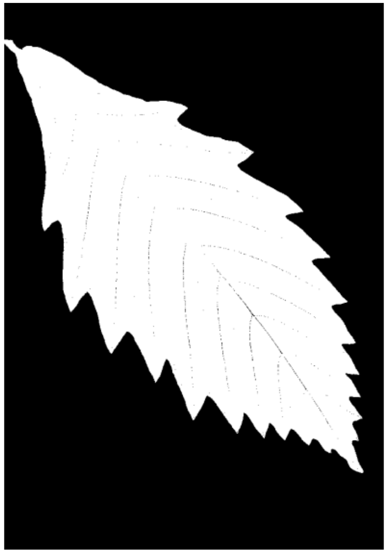
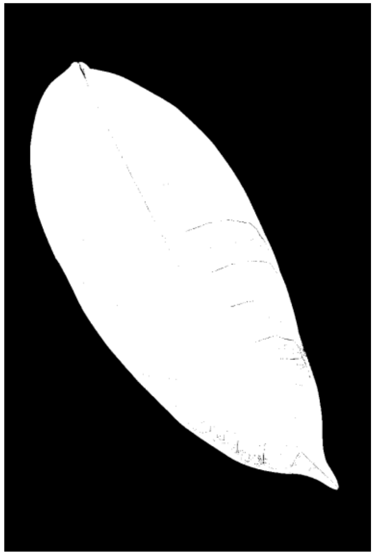
Boundary and region description

- Commonly after segmentation one needs to represent objects in order to describe them





- **External (boundary):**
  - Representation: Polygon of the boundary
  - Description: The circumference
- **Internal (regional)**
  - Representation: Pixels inside the object
  - Description: The average color

Statistic on histogram of intensity in a region

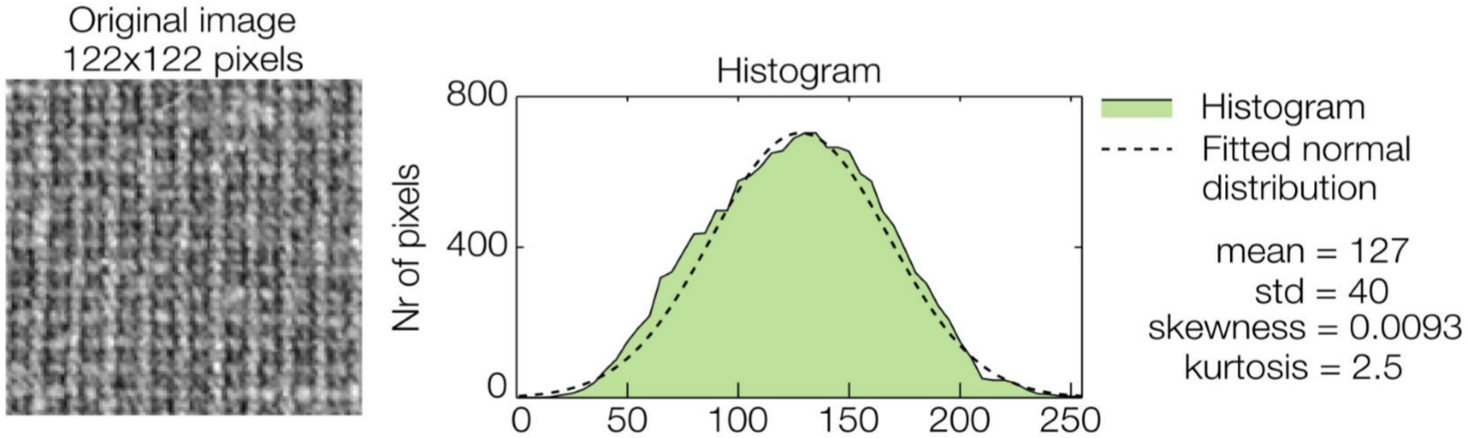
- **There is also underlying intensities/ colors inside each region we found**
- **Texture can also be filtered**
  - Flat
  - Noisy
  - Stripy

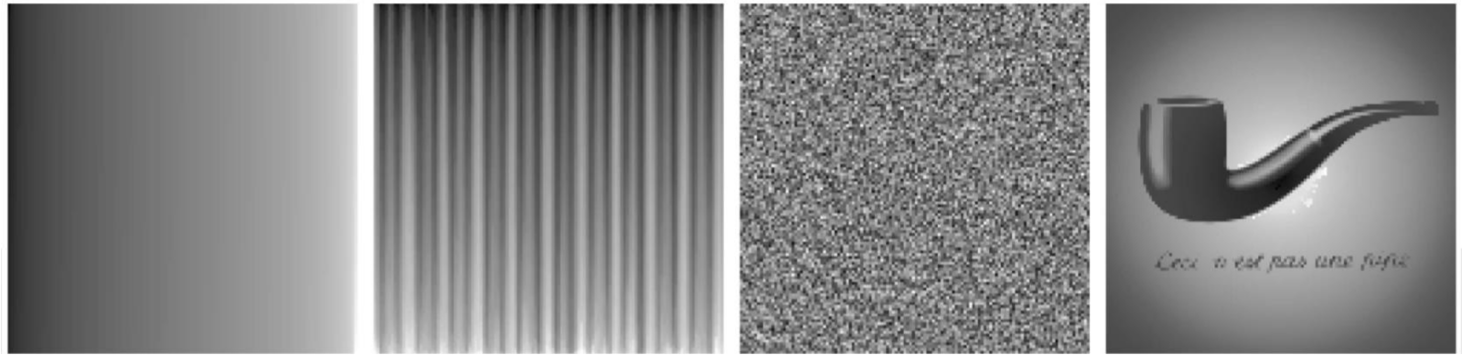
Statistic on histogram of intensity in a region

- Statistics on histogram of intensity in the region:
  - Mean & variance (contrast)
  - Flat -- var=0; Noisy -- var = high;
  - Skewness (locally bright or dark)
  - Entropy (how random)

Intensity histogram: a problem

- Intensity histogram says nothing about the spatial distribution of the pixel intensities





Gray-level co-occurence matrix (GLCM)

- How pixels' intensity correlate to each other.
- 1) Specify an operation *Q* (spatial relationship between 2 pixels).
  - e.g. *Q* = "1 pixel to the right".
  - If N gray levels, this makes NxN matrix.
- 2) $P\big((x_0, y_0), (x_1, y_1)\big) = [intensity1, intsnsity2]$, the pair of $(x_0, y_0), (x_1, y_1)$ depends on the operation *Q*.
  - Where P stands for possibility. e.g. How often do I see (1,1) in the given pixel pairs.
  - Matlab commend: graycomatrix ()
- 3) In practice, # of gray levels is quantized, the quantization depends on the area of the region of interest. (e.g. 8 or 16)

Gray-level co-occurence matrix (GLCM)

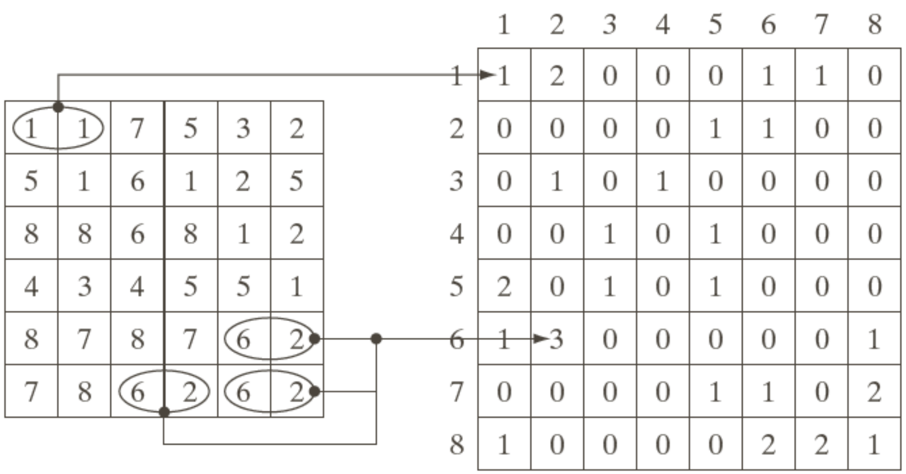
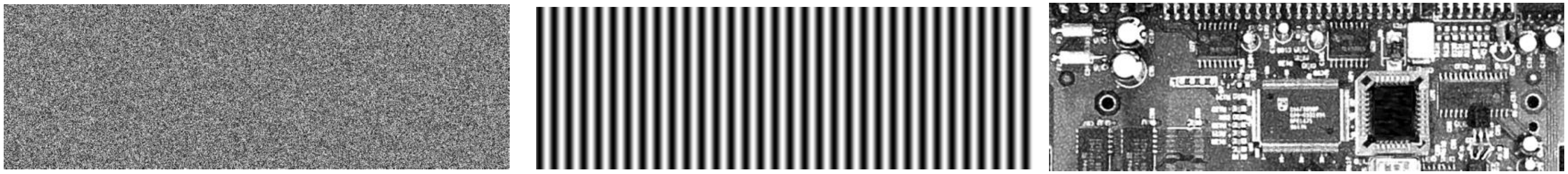
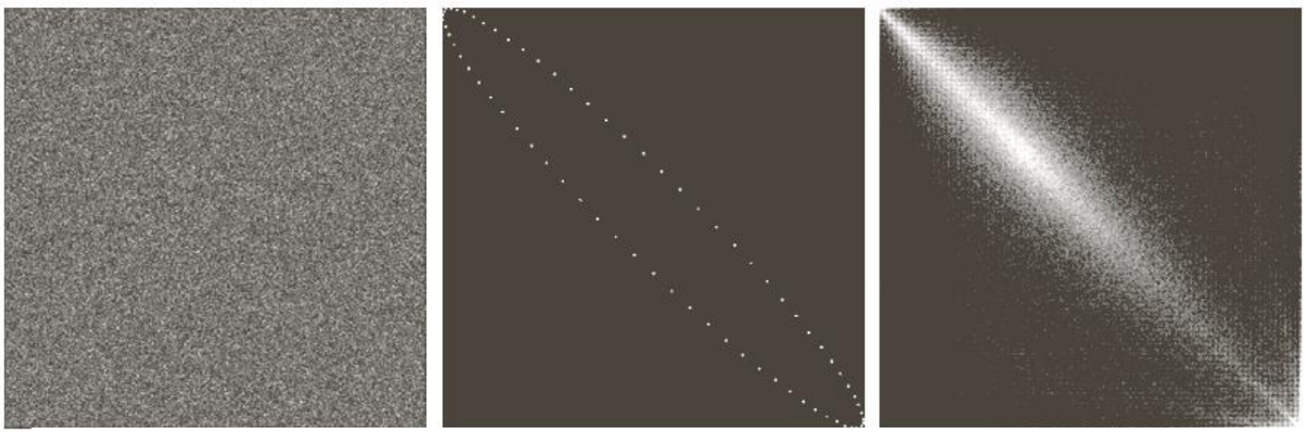


Image f Co-occurrence matrix **G**

- 1) Specify an operation Q (spatial relationship between 2 pixels).
  - e.g. Q = "1 pixel to the right".
  - If N gray levels, this makes NxN matrix.
- 2) $P\big((x_0,y_0),(x_1,y_1)\big) = [intensity1, intsnsity2]$, the pair of $(x_0,y_0),(x_1,y_1)$ depends on the operation Q.

Gray-level co-occurence matrix (GLCM)





a b c

FIGURE 11.31 256 × 256 co-occurrence matrices, $\mathbf{G}_1$, $\mathbf{G}_2$, and $\mathbf{G}_3$, corresponding from left to right to the images in Fig. 11.30.

TABLE 11.4
Descriptors evaluated using the co-occurrence matrices displayed as images in Fig. 11.32.

Normalized Co-occurrence Matrix	Maximum Probability	Correlation	Contrast	Uniformity	Homogeneity	Entropy
$\mathbf{G}_1/n_1$	0.00006	−0.0005	10838	0.00002	0.0366	15.75
$\mathbf{G}_2/n_2$	0.01500	0.9650	00570	0.01230	0.0824	06.43
$\mathbf{G}_3/n_3$	0.06860	0.8798	01356	0.00480	0.2048	13.58

GLCM quantification metrics

TABLE 11.3
Descriptors used for characterizing co-occurrence matrices of size $K \times K$. The term p_{ij} is the ij-th term of **G** divided by the sum of the elements of **G**.

Descriptor	Explanation	Formula
Maximum probability	Measures the strongest response of **G**. The range of values is [0, 1].	$\max_{i,j}(p_{ij})$
Correlation	A measure of how correlated a pixel is to its neighbor over the entire image. The range of values is 1 to −1 corresponding to perfect positive and perfect negative correlations. This measure is not defined if either standard deviation is zero.	$\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{(i-m_r)(j-m_c)p_{ij}}{\sigma_r \sigma_c}$ $\sigma_r \neq 0;\ \sigma_c \neq 0$
Contrast	A measure of intensity contrast between a pixel and its neighbor over the entire image. The range of values is 0 (when **G** is constant) to $(K-1)^2$.	$\sum_{i=1}^{K}\sum_{j=1}^{K}(i-j)^2 p_{ij}$
Uniformity (also called Energy)	A measure of uniformity in the range [0, 1]. Uniformity is 1 for a constant image.	$\sum_{i=1}^{K}\sum_{j=1}^{K}p_{ij}^2$
Homogeneity	Measures the spatial closeness to the diagonal of the distribution of elements in **G**. The range of values is [0, 1], with the maximum being achieved when **G** is a diagonal matrix.	$\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{p_{ij}}{1+\lvert i-j\rvert}$
Entropy	Measures the randomness of the elements of **G**. The entropy is 0 when all p_{ij}'s are 0, and is maximum when the p_{ij}'s are uniformly distributed. The maximum value is thus $2\log_2 K$.	$-\sum_{i=1}^{K}\sum_{j=1}^{K}p_{ij}\log_2 p_{ij}$

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Take home message

- Representation of the Object:
  - An encoding of the object
  - Truthful but possible approximation
- Descriptor of the Object:
  - Only an aspect of the object
  - Suitable for classification
  - Consider invariance to e.g. noise, translation

Object & feature detection

- Template matching
- Harris corner and Shi-tomasi corner detection
- What is an interesting point?
- Blob detection
- Scale Invariant Feature Transform (SIFT)

Template matching

- Correlation between template and image
- Template: $\omega(x, y)$; Image: $I(x, y)$.
- Correlation coefficient:

$$\boldsymbol{\gamma(x,y)} = \frac{\boldsymbol{cov(\omega, I)}}{\boldsymbol{\sigma_\omega \sigma_I}} = \frac{\boldsymbol{E(\omega - \overline{\omega})(I - \overline{I}))}}{\boldsymbol{\sigma_\omega \sigma_I}}$$

$\overline{\omega}$: average value of template; $\overline{I}_{xy}$: average value of image inside window; $\gamma \in [-1, 1]$; $\gamma = 1$: template perfectly match the window; $\gamma = 0$: no correlation/no match.

Image

Template

Template matching

Image

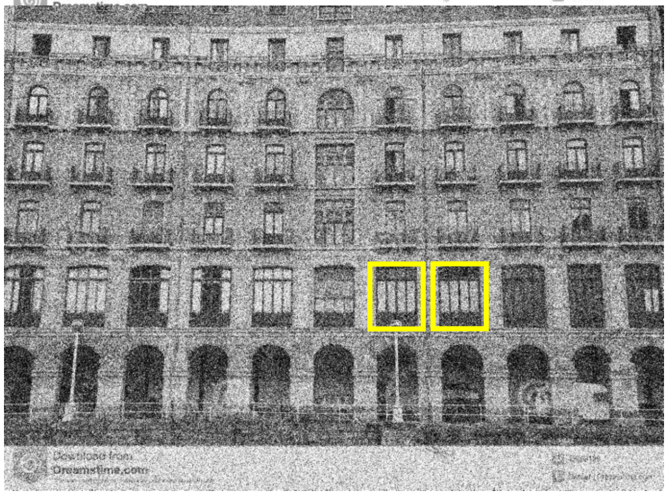
Template

Thresh=0.9

Thresh=0.6

Template matching

Template selected



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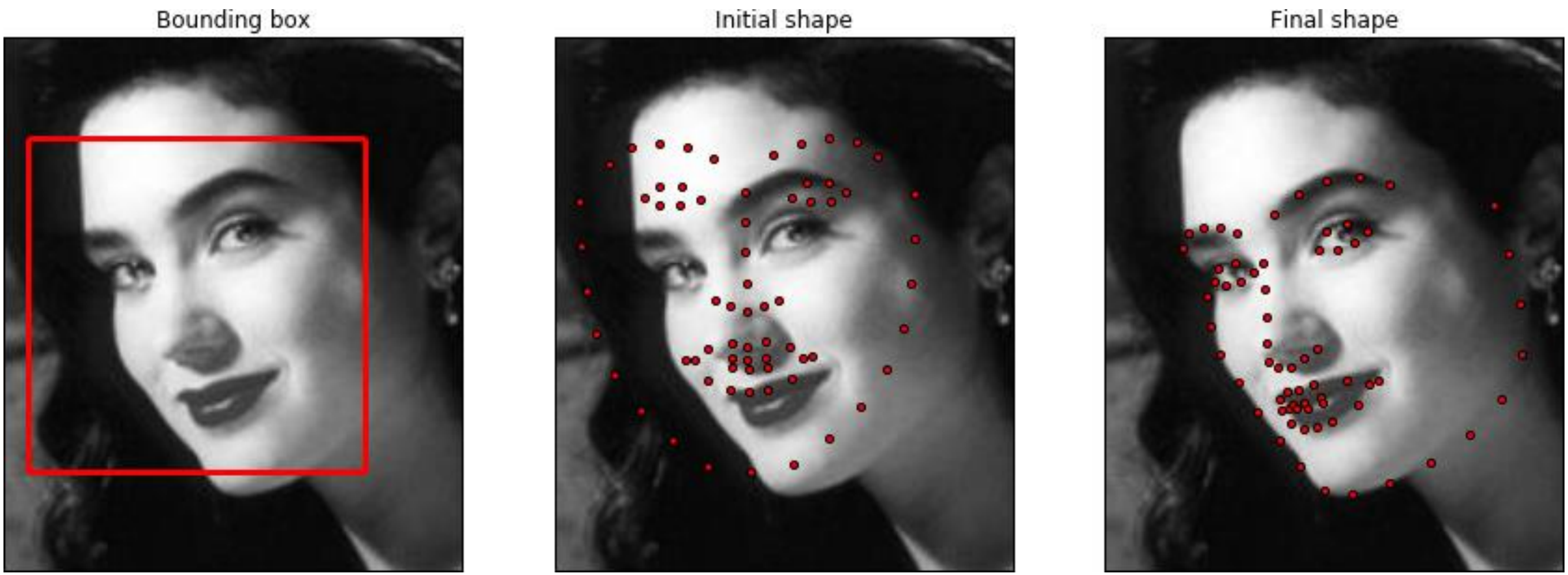
Template matching: applications



Template matching: applications

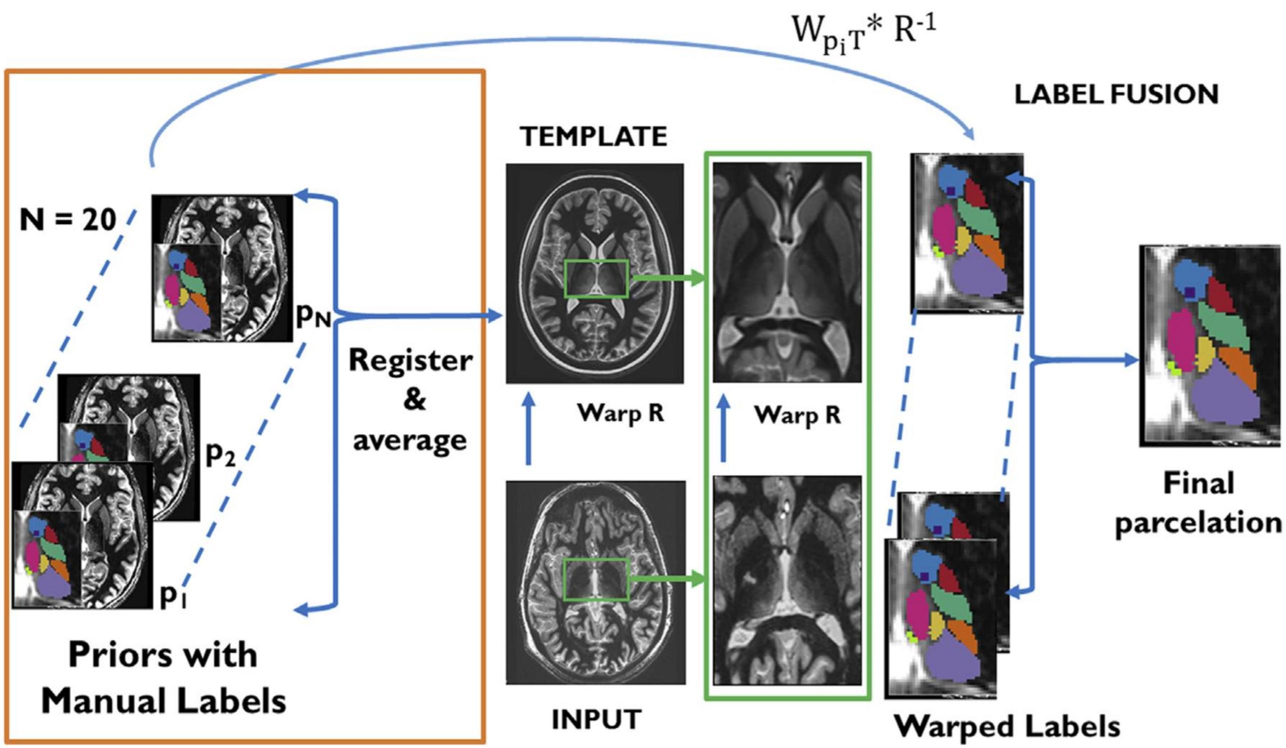


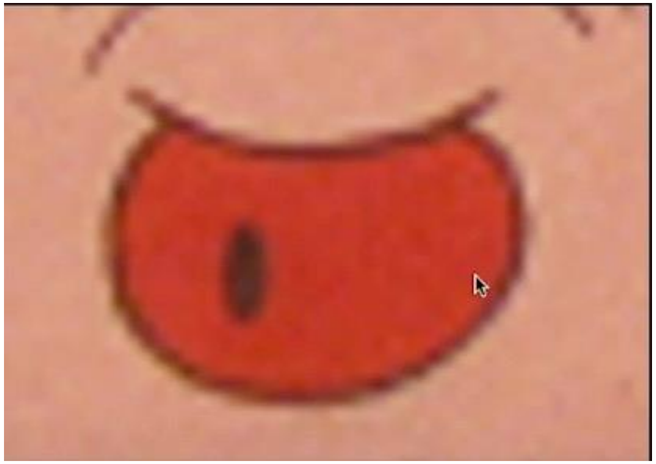
Fig. 1. THOMAS workflow- The input image is nonlinearly registered to the average brain template. Precomputed warps from each of the 20 priors to the template is then combined with the template-input warp (R^{-1}) to generate 20 candidate labels, which are then fused to get the final parcellation.

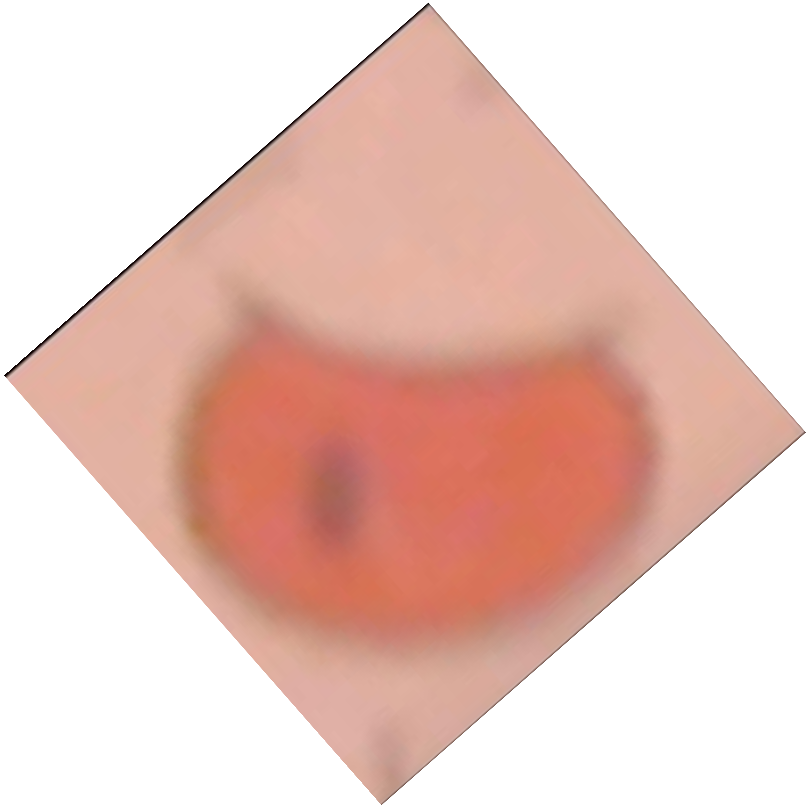
What is an interesting point?

Different size, orientation, lighting, brightness, etc

What is an interesting point? **Lines? Edges? Corners?**

What is an interesting point?





Scale and Rotation!

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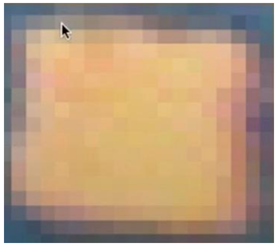
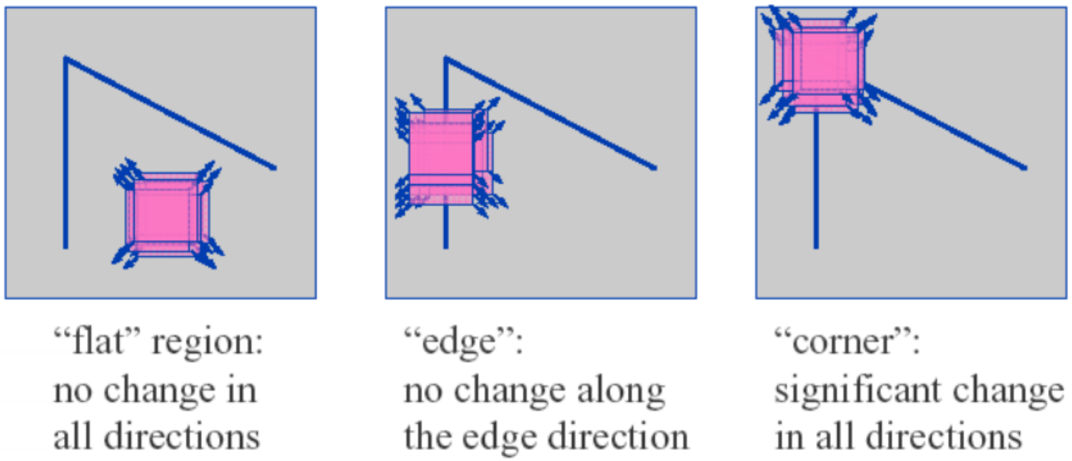
An interesting point & feature

- Rich image content (brightness variation, color variation, etc.) within the local window
- Well-defined representation (signature)
- Well-defined position in the image
- Invariant to image rotation and scaling
- Insensitive to lighting changes



Harris Corner



- A shifted corner produces some difference in the image.
- A shifted uniform region produces no difference.
- So, look for large difference in shifted image

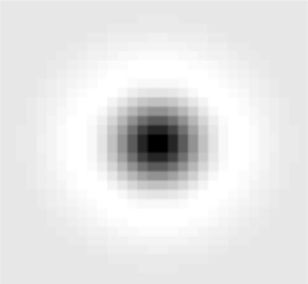
SIFT: Key point extraction

- Stands for scale invariant feature transform
- Patented by University of British Columbia
- Similar to the one used in primate visual system (human, ape, monkey, etc.)

D. Lowe. Distinctive image features from scale invariant key points., International Journal of Computer Vision 2014

Effect of LoG Filter

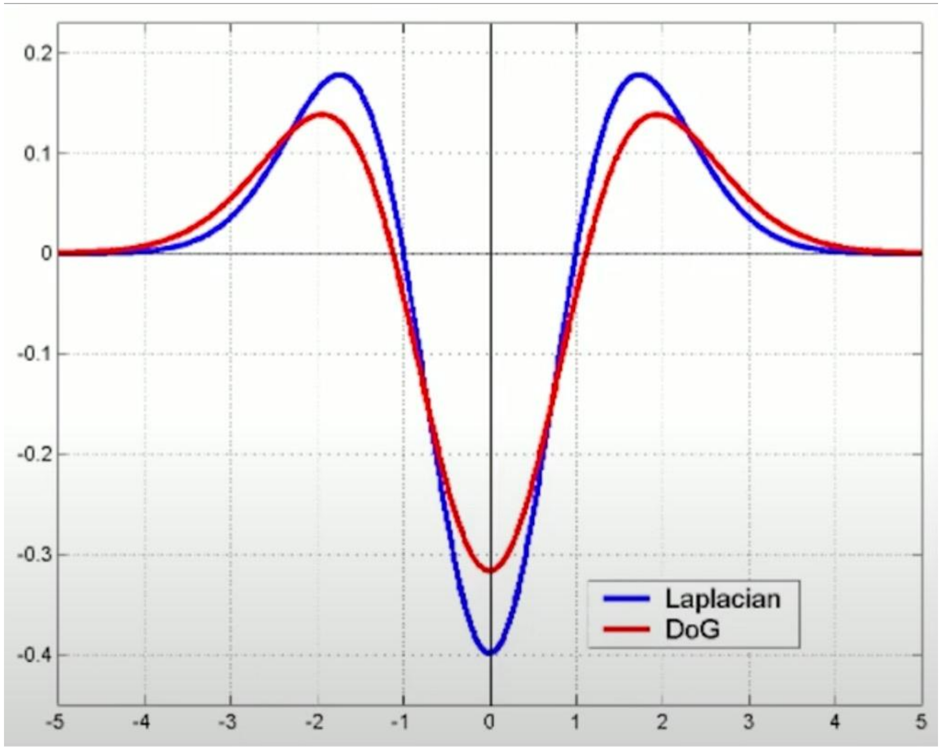
$$G''(x,y) = -\frac{1}{\pi\sigma^4}\left(1-\frac{x^2+y^2}{2\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$



Sigma = 1 Sigma = 4 Sigma = 10

Band-Pass Filter (suppresses both high and low frequencies)

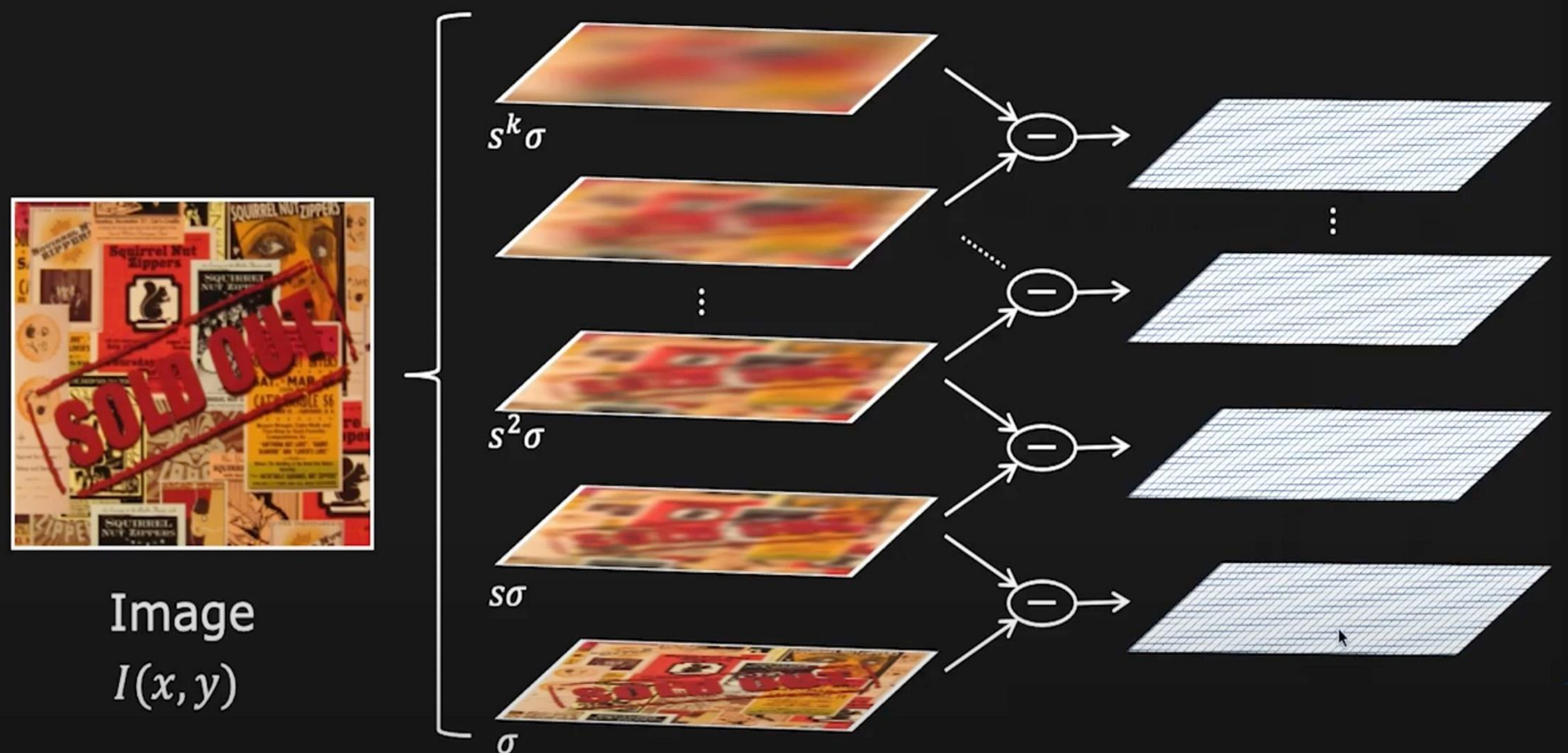
Fast NLoG Approximation: DoG



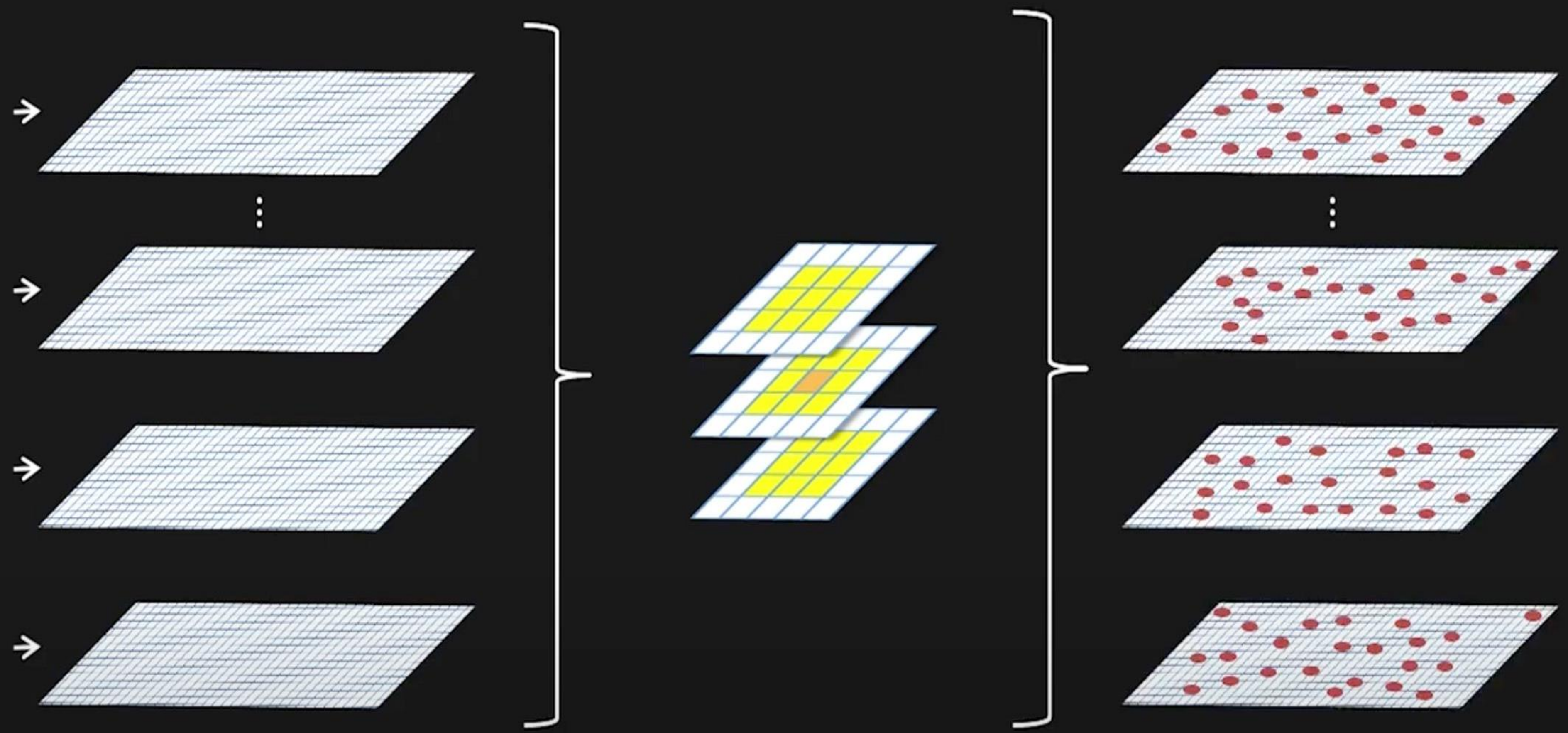
NLoG

$$\text{Difference of Gaussian (DoG)} = (n_{s\sigma} - n_{\sigma}) \approx (s-1)\boxed{\sigma^2 \nabla^2 n_{\sigma}}$$

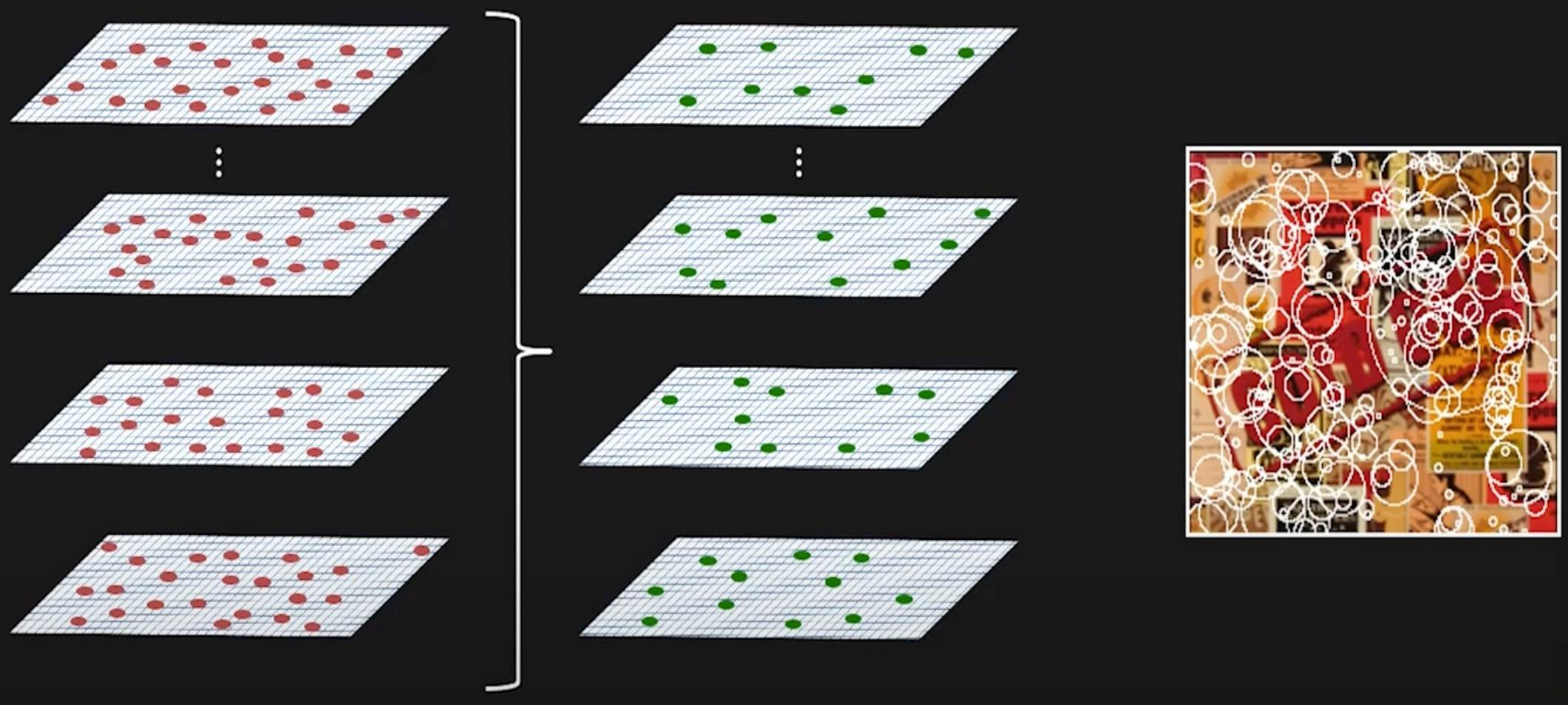
Extracting SIFT interesting points



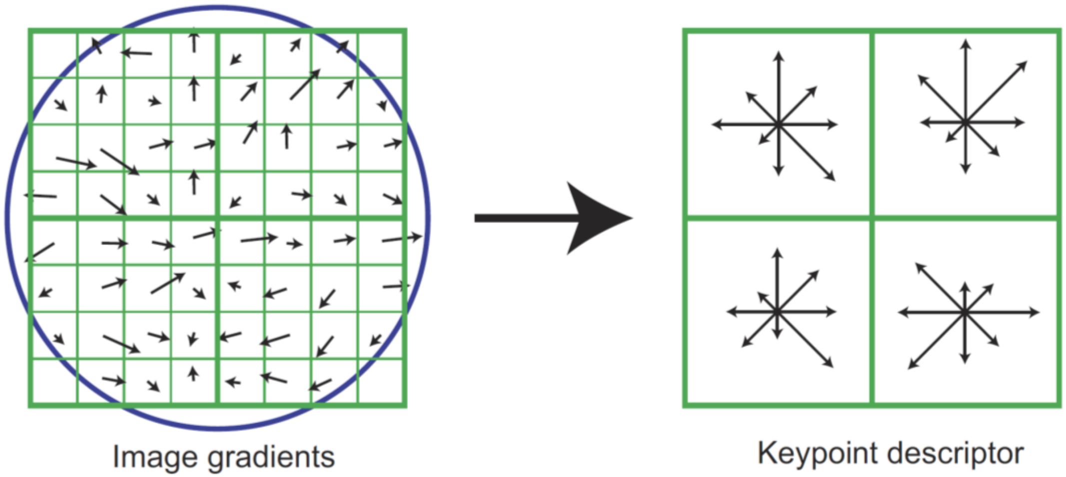
Extracting SIFT interesting points



Extracting SIFT interesting points

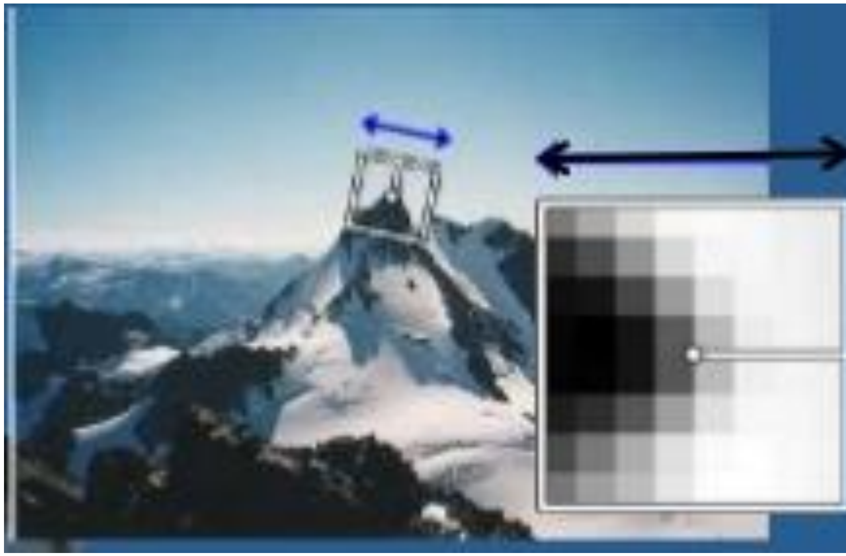


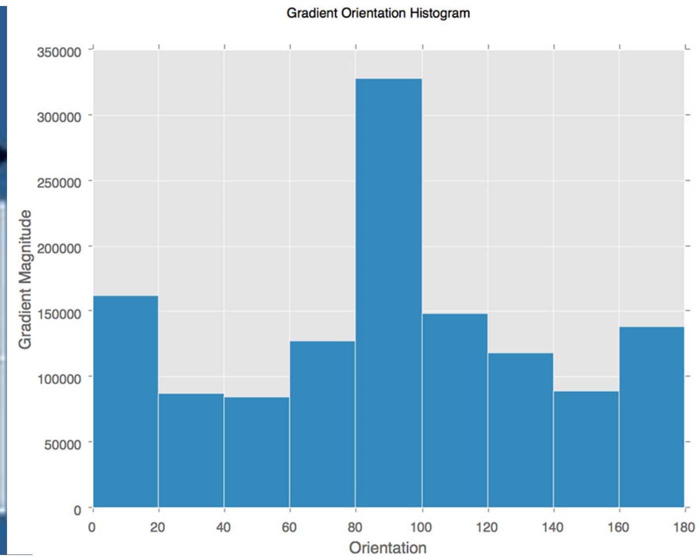
SIFT descriptors



- Crop key point local feature to 4x4 sub-regions.
- Compute the quantized orientation histogram in each subregion.
- This operation allows for significant shift and rotation for the key point.

Orientation assignment





- Compute orientation histogram, based on orientation quantization.
- Find the dominant orientation for key point.
- Extract local region around key point and orient the region to its right direction.

https://aishack.in/tutorials/sift-scale-invariant-feature-transform-keypoint-orientation/

SIFT detection