

Stochastic Galerkin Method

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1 General Procedure

1.1 Problem

$$-\nabla \cdot (a(\omega, x) \nabla u(\omega, x)) = f(\omega, x) \quad (\omega, x) \in \Omega \times D$$

$$u(\omega, x) = 0 \quad x \in \partial D$$

1.2 Tensor Product

$$(f \otimes g)(x, y) = f(x)g(y)$$

$$((A \otimes B)_{i_1 j_1})_{i_2 j_2} = A_{i_1 j_1} B_{i_2 j_2}$$

1.3 Solution

$$\mathbf{E}[\int_D a \nabla u \nabla v dx] = E[\int_D f v dx]$$

$$v(x) = \sum_{i=1}^N V_i \phi_i(x)$$

$$\int_D a \nabla \left(\sum_{i=1}^N U_i \phi_i(x) \right) \cdot \nabla \phi_j dx = \int_D f \phi_j dx$$

$$\sum_{i=1}^N U_i \int_D a \nabla \phi_i \cdot \nabla \phi_j dx = \int_D f \phi_j dx$$

$$KU = F$$

$$a(\omega, x) = \bar{a} + \sum_{l=1}^{\infty} \varphi_l(x) y_l(\omega)$$

$$u(\omega, x) = \sum_{n=1}^Q \sum_{i=1}^N (U_n)_i \psi_n(\omega) \phi_i(x)$$

$$\mathbf{E}[\int_D a \nabla \sum_{i=1}^N \sum_{n=1}^Q (U_n)_i \psi_n \phi_i \cdot \nabla \psi_m \phi_j dx] = \mathbf{E}[\int_D f \psi_m \phi_j dx]$$

$$\sum_{i=1}^N \sum_{n=1}^Q (U_n)_i \mathbf{E}[\int_D a \nabla \psi_n \phi_i \cdot \nabla \psi_m \phi_j dx] = \mathbf{E}[\int_D f \psi_m \phi_j dx]$$

$$\sum_{i=1}^N \sum_{n=1}^Q (U_n)_i \mathbf{E}[\psi_n \psi_m \int_D a \nabla \phi_i \cdot \nabla \phi_j dx] = \mathbf{E}[\psi_m \int_D f \phi_j dx]$$

$$a(\omega, x) = \sum_{l=0}^S a_l(x) y_l(\omega)$$

$$\sum_{l=0}^S \sum_{i=1}^N \sum_{n=1}^Q (U_n)_i \mathbf{E}[y_l \psi_n \psi_m \int_D a_l \nabla \phi_i \cdot \nabla \phi_j dx] = \mathbf{E}[\psi_m \int_D f \phi_j dx]$$

$$\sum_{l=0}^S \sum_{i=1}^N \sum_{n=1}^Q (U_n)_i \mathbf{E}[y_l \psi_n \psi_m] \int_D a_l \nabla \phi_i \cdot \nabla \phi_j dx = \mathbf{E}[\psi_m \int_D f \phi_j dx]$$

$$\sum_{l=0}^S \sum_{i=1}^N \sum_{n=1}^Q (U_n)_i (G_l)_{nm} (K_l)_{ij} = (F_m)_j$$

$$\sum_{l=0}^S \sum_{i=1}^N \sum_{n=1}^Q (U_n)_i ((G_l \otimes K_l)_{nm})_{ij} = (F_m)_j$$

$$\sum_{l=0}^S (G_l \otimes K_l) U = F$$