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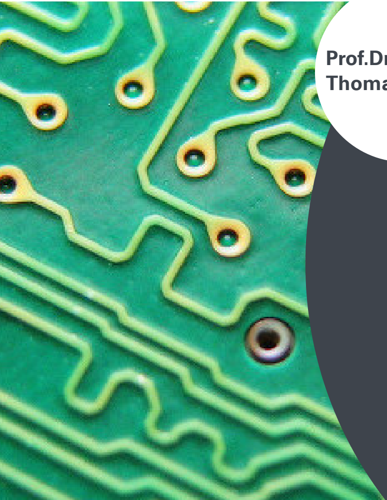

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Integrated Positioning and Navigation

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Rotational Motion

Rotational Motion

If two coordinate systems rotate with respect to each other, then the DCM has a non-zero time derivative. Rotational motion can be represented by an angular velocity vector. This module relates the angular velocity vector to the time derivative of the DCM.

The coordinates (in the s-system $\rightarrow$ superscript "s") of the angular velocity vector of the s-system with respect to the t-system (subscript "t" followed by subscript "s") reads:

The coordinate system in which the components are given

$$\boldsymbol{\omega}_{ts}^{s} = \left[\omega_{ts1}^{s} \;\; \omega_{ts2}^{s} \;\; \omega_{ts3}^{s}\right]^{T} \tag{3.1}$$

Direction of rotation: s rotates with respect to t

The angular velocity vector (in the s-system) of the t-system with respect to the s-system is then

$$\boldsymbol{\omega}_{st}^{s} = -\boldsymbol{\omega}_{ts}^{s} \tag{3.2}$$

Transformation of angular velocity vector coordinates (see Module 2):

$$\boldsymbol{\omega}_{ts}^{s} = \boldsymbol{C}_{t}^{s} \cdot \boldsymbol{\omega}_{ts}^{t} = -\boldsymbol{C}_{t}^{s} \cdot \boldsymbol{\omega}_{st}^{t} \tag{3.3}$$

Rotational Motion - cont't

For some algebraic operations it is more convenient to use the matrix representation of the angular velocity vector coordinates:

$$\boldsymbol{\omega}_{ts}^{s} = \begin{bmatrix} \omega_{ts1}^{s} \\ \omega_{ts2}^{s} \\ \omega_{ts3}^{s} \end{bmatrix} \quad \Rightarrow \quad \boldsymbol{\Omega}_{ts}^{s} = \begin{bmatrix} 0 & -\omega_{ts3}^{s} & \omega_{ts2}^{s} \\ \omega_{ts3}^{s} & 0 & -\omega_{ts1}^{s} \\ -\omega_{ts2}^{s} & \omega_{ts1}^{s} & 0 \end{bmatrix} \tag{3.4}$$

$$\boxed{\text{Background:} \quad \boldsymbol{\omega} \times \boldsymbol{b} = \boldsymbol{\Omega} \cdot \boldsymbol{b} = -\boldsymbol{B} \cdot \boldsymbol{\omega}}$$

$$\boldsymbol{\Omega}_{st}^{s} = -\boldsymbol{\Omega}_{ts}^{s} \tag{3.5}$$

This skew-symmetric matrix is transformed between coordinate systems according to (compare to equ. (3.3)):

$$\boldsymbol{\Omega}_{ts}^{s} = \boldsymbol{C}_{t}^{s} \cdot \boldsymbol{\Omega}_{ts}^{t} \cdot \boldsymbol{C}_{s}^{t} = -\boldsymbol{C}_{t}^{s} \cdot \boldsymbol{\Omega}_{st}^{t} \cdot \boldsymbol{C}_{s}^{t} \tag{3.6}$$

Rotational Motion - cont't

If two coordinate systems rotate with respect to each other, then the corresponding DCM has a non-zero time derivative.

The definition of the time derivative of a DCM:

$$\dot{\boldsymbol{C}}_t^s(t) = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{C}_t^s(t, t+\Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\boldsymbol{C}_t^s(t+\Delta t) - \boldsymbol{C}_t^s(t)}{\Delta t} \tag{3.7}$$

The DCM at time $t + \Delta t$ can be written as the product of the DCM at time t, and an additional DCM accounting for the change of the s-system with respect to the t-system in the time interval Δt.

$$\boldsymbol{C}_t^s(t+\Delta t) = \Delta \boldsymbol{C}_t^s(t, t+\Delta t) \cdot \boldsymbol{C}_t^s(t) \tag{3.8}$$

For sufficiently small Δt, the structure of $\Delta \boldsymbol{C}$ is (see. Equ. (2.8))

$$\Delta \boldsymbol{C} = \boldsymbol{I} - \Delta \boldsymbol{A} \tag{3.9}$$

Rotational Motion - cont't

ΔA is the skew-symmetric matrix containing the small rotation angles about the coordinate axes of the s-system during the time interval Δt.

$$\Delta A = \begin{bmatrix} 0 & -\Delta\alpha_3 & \Delta\alpha_2 \\ \Delta\alpha_3 & 0 & -\Delta\alpha_1 \\ -\Delta\alpha_2 & \Delta\alpha_1 & 0 \end{bmatrix} \tag{3.10}$$

Inserting equ. (3.8) and (3.9) into equ. (3.7):

$$\dot{\boldsymbol{C}}_t^s(t) = \lim_{\Delta t \to 0} \frac{-\Delta A \cdot \boldsymbol{C}_t^s}{\Delta t} = -\lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} \cdot \boldsymbol{C}_t^s = -\boldsymbol{\Omega}_{ts}^s \cdot \boldsymbol{C}_t^s$$

$$\boldsymbol{\Omega}_{ts}^s = \begin{bmatrix} 0 & -\omega_{ts3}^s & \omega_{ts2}^s \\ \omega_{ts3}^s & 0 & -\omega_{ts1}^s \\ -\omega_{ts2}^s & \omega_{ts1}^s & 0 \end{bmatrix} \tag{3.11}$$

Applying the transformation (equ. (3.6)) gives finally:

$$\dot{\boldsymbol{C}}_t^s = \boldsymbol{C}_t^s \cdot \boldsymbol{\Omega}_{st}^t \tag{3.12}$$