Universität Stuttgart

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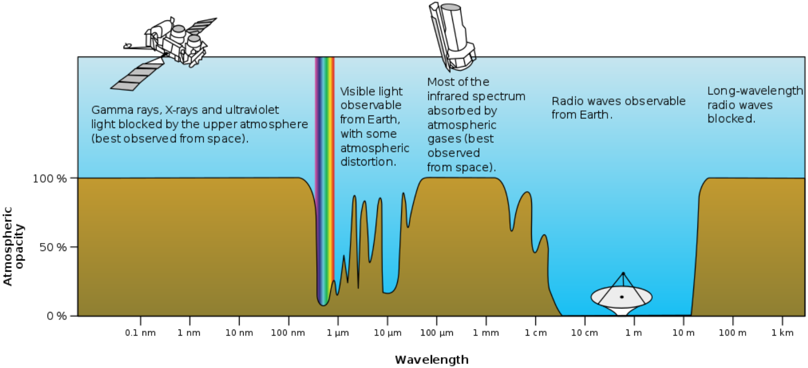
Satellite Navigation

Signals

4

Signals

Atmospheric electromagnetic opacity



(source: Wikipedia.org)

Signals

The spectrum of radio signals (ITU)

Band	Frequencies	Wave lengths
VLF (very low frequency)	3 kHz - 30 kHz	100 km - 10 km
LF (low frequency)	30 kHz - 300 kHz	10 km - 1 km
MF (medium frequency)	300 kHz - 3 MHz	1 km - 100 m
HF (high frequency)	3 MHz - 30 MHz	100 m - 10 m
VHF (very high frequency)	30 MHz - 300 MHz	10 m - 1 m
UHF (ultra high frequency)	300 MHz - 3 GHz	1 m - 10 cm
SHF (super high frequency)	3 GHz - 30 GHz	10 cm - 1 cm
EHF (extra high frequency)	30 GHz - 300 GHz	1 cm - 1 mm

IEEE L band

Frequency range	1 - 2 GHz
Wavelength range	30 - 15 cm
Related bands	D(NATO), UHF (ITU)

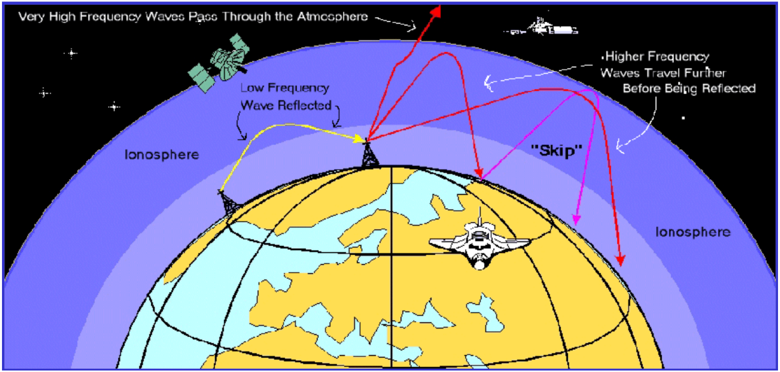
RNSS (Radio Navigation Satellite Service)

- Upper L-band (1559–1610MHz)
- Lower L-band (1164–1300MHz)

The GNSS signals are in the UHF band.

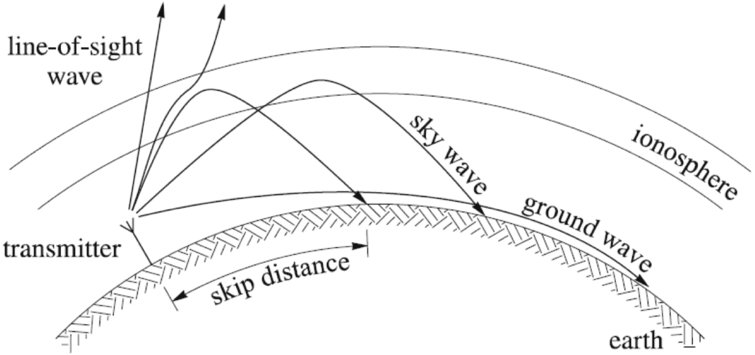
Signals

The spectrum of radio signals (propagation characteristics)



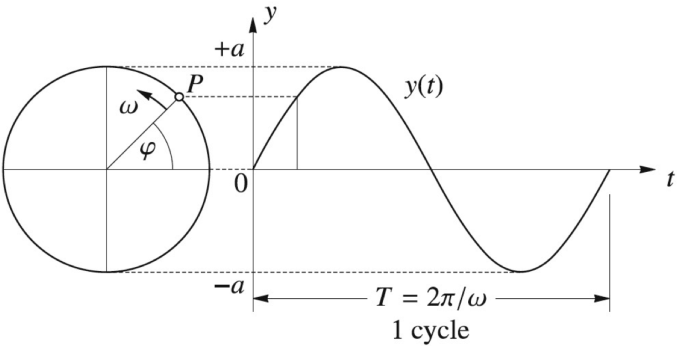
Signals

The spectrum of radio signals (propagation characteristics)



Signals

Structure of radio signals



$$y(t) = a(t)\sin(2\pi f(t) + \varphi(t))$$

Signals

Structure of radio signals

A basic radio signal:

$$\begin{aligned} y(t) &= A \cdot \cos(2\pi f_c t) \\ y(t) &= A \cdot \cos(\omega_c t) \end{aligned} \tag{4.1}$$

A:	amplitude
f_c:	frequency
ω_c:	angular frequency
$2\pi f_c t$:	phase

Equ. (4.1) describes the temporal variation of the signal at a fixed location. The signal can also be represented in its spatial variation (a snap shot in time).

$$y(x) = A \cdot \cos\left(\frac{2\pi}{\lambda} x\right) \tag{4.2}$$

λ:	wave length
x:	distance from signal source

In the frequency domain the signal according to equ. (4.1) or (4.2) is characterized by a single spectral line at the frequency f_c (or ω_c).

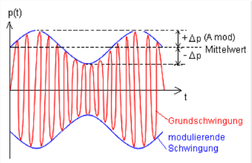
Signals

Modulation of radio signals

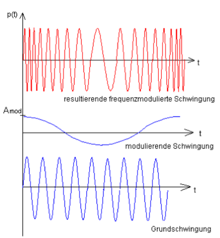
In order to transmit information with a radio signal, the signal must be modulated. For this purpose:

- amplitude,
- frequency or
- phase

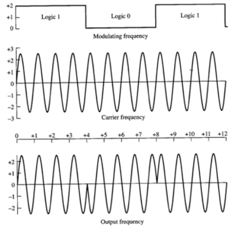
of the signal can be modulated.



amplitude modulation (AM)



frequency modulation (FM)



phase modulation (PM)

Signals

Modulation of radio signals – Amplitude modulation

Example for AM:
periodic variation of signal amplitude (used in EDM for the modulation of IR signals)

$$
\begin{aligned}
A(t) &= A_0 + \delta A \cos(2\pi f_M t) \\
y(t) &= (A_0 + \delta A \cos(2\pi f_M t)) \cos(2\pi f_c t)
\end{aligned}
\tag{4.3}
$$

Evaluation of the products of trigonometric functions:

$$
y(t) = A_0 \cos(2\pi f_c t) + \frac{\delta A}{2} \cos(2\pi (f_c - f_M) t) + \frac{\delta A}{2} \cos(2\pi (f_c + f_M) t) \tag{4.4}
$$

Question: What is the representation of the modulated signal in the space domain, the time domain and in the frequency domain?

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Modulation of radio signals – Frequency modulation

Example for FM:
periodic variation of signal amplitude (used in EDM for the modulation of IR signals)

$$f(t) = f_0 + \delta f \cos(2\pi f_M t)$$
$$y(t) = A\cos(2\pi(f_0 + \delta f \cos(2\pi f_M t))t) \tag{4.5}$$

Evaluation of the momentary frequency = time derivative of signal phase $\varphi(t)$

$$f(t) = \frac{1}{2\pi}\frac{d\varphi}{dt} \;\Rightarrow\; \varphi(t) = 2\pi\int\limits_0^t f(\tau)d\tau \tag{4.6}$$

With (4.6) in (4.5):

$$\varphi(t) = 2\pi f_0 t + \frac{\delta f}{f_M}\sin(2\pi f_M t)$$

$$y(t) = A\cos\left(2\pi f_0 t + \frac{\delta f}{f_M}\sin(2\pi f_M t)\right)$$

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Modulation of radio signals – Phase modulation

For the transmission of digital data (like in the GPS signals), phase modulation is the preferred method:

$$y(t) = A\cos(2\pi(f_c t + \varphi(t)) \tag{4.7}$$

$\varphi(t)$ is the modulation signal. For the transmission of binary data, the modulation signal must have two distinct values (representing the binary 0 and the binary 1). Such modulation is known as Binary Phase Shift Keyed (BPSK).
$\Rightarrow$The two states of the modulation signal are often chosen to be 0 and π.

$\varphi(t) = 0$: the signal to be modulated remains unchanged
$\varphi(t) = \pi$: the signal to be modulated is phase shifted by π (180°)

Equivalent is the statement, that for $\varphi(t) = \pi$ the sign of the signal is reversed.

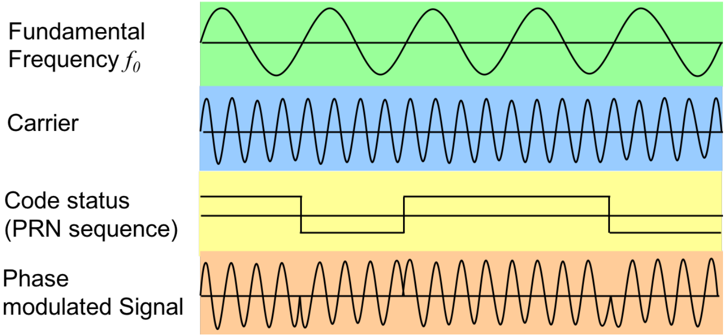
$$\begin{aligned} y(t) &= A\cos(2\pi f_c t + c(t)\pi) \\ y(t) &= [1 - 2c(t)]A\cos(2\pi f_c t) \end{aligned} \tag{4.8}$$

The modulation sequence c(t) consists of the two states 0 and 1.

Signals

Modulation of radio signals – Phase modulation

Binary Phase Shift Keyed (BPSK) for GPS signal



Signals

GPS Carrier Signals

The GPS signal structure is defined in the Interface Control Document IS-GPS-200 (`https://www.gps.gov/technical/icwg/IS-GPS-200J.pdf`)

All GPS signal components are derived from the fundamental frequency f_0, which is generated by a cesium or rubidium frequency standard. The fundamental frequency for GPS is:

$$f_0 = 10.23 \text{ MHz} \tag{4.9}$$

The frequency standards are adjusted before the launch of the satellite to account for the effects of Special and General Relativity.

Two coherent carrier signals are derived from the fundamental frequency, the carrier signals are usually denoted L_1 and L_2.

$$L_1(t) = A_1 \cos(2\pi f_1 t), \text{ [Quadrature comp. } L_1(t) = A_1 \sin(2\pi f_1 t)]$$

$$L_2(t) = A_2 \cos(2\pi f_2 t), \text{ [Quadrature comp. } L_2(t) = A_2 \sin(2\pi f_2 t)]$$

$$f_1 = 154 \cdot f_0 = 1575.42 \text{ MHz} \Rightarrow \lambda_1 \approx 19 \text{ cm} \tag{4.10}$$

$$f_2 = 120 \cdot f_0 = 1227.60 \text{ MHz} \Rightarrow \lambda_2 \approx 24 \text{ cm}$$

$$f_5 = 115 \cdot f_0 = 1176.45 \text{ MHz} \Rightarrow \lambda_5 \quad \text{IIF only!}$$

Signals

GPS Pseudo Random Noise Codes

The GPS carrier signals are phase modulated with several Pseudo Random Noise (PRN) codes, e.g. the C/A-code (coarse/acquisition), the P-code (precise), the L2C(M)-code (civil-moderate) or the L2C(L)-code (civil-long).

All codes are pseudo-random sequences of 0 and 1; they cannot be truly random, since they are generated with a mathematical algorithm.

The code chipping rate of the C/A-code is 1/10 of the fundamental frequency f0, the code chipping rate for the P-code is equal to the fundamental frequency, the code chipping rate for the L2C(M)- and the L2C(L)-code is 1/20 of the fundamental frequency.

$$\text{code chipping rates} \qquad \begin{aligned} f_{\mathrm{C/A}} &= 1.023 \text{ MHz} \\ f_{\mathrm{P}} &= 10.23 \text{ MHz} \\ f_{\mathrm{CM}} = f_{\mathrm{CL}} &= 0.5115 \text{ MHz} \end{aligned} \tag{4.11}$$

Signals

GPS Pseudo Random Noise Codes

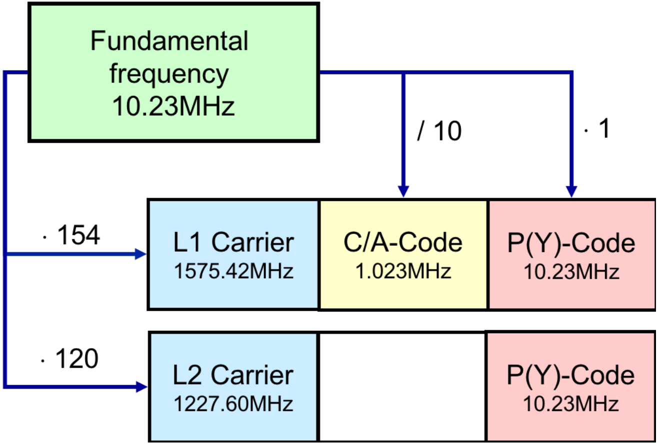
- The C/A-code is transmitted by all GPS satellites on the L_1 carrier signal (on the quadrature component Q)
- The P-code is transmitted by all GPS satellites on the L_1 and the L_2 carrier signal (on the in-phase component I)
- The L2C(M)-code and the L2C(L)-code are transmitted, starting by the generation of Block IIR-M satellites, on the L2 carrier signal (on the quadrature component Q).

The length of the code chips can be expressed in the time domain and in the space domain:

$$
\begin{aligned}
&\text{code chip lengths}\\
&\Delta t_{\text{C/A}} = (1.023\ \text{MHz})^{-1} \approx 1\ \mu\text{s},\ \Delta s_{\text{C/A}} \approx 300\ \text{m}\\
&\Delta t_{\text{P}} = (10.23\ \text{MHz})^{-1} \approx 0.1\ \mu\text{s},\ \Delta s_{\text{P}} \approx 30\ \text{m}\\
&\Delta t_{\text{CM}} = \Delta t_{\text{CL}} = (0.5115\ \text{MHz})^{-1} \approx 2\ \mu\text{s},\ \Delta s_{\text{CM}} = \Delta s_{\text{CL}} \approx 600\ \text{m}
\end{aligned}
\tag{4.12}
$$

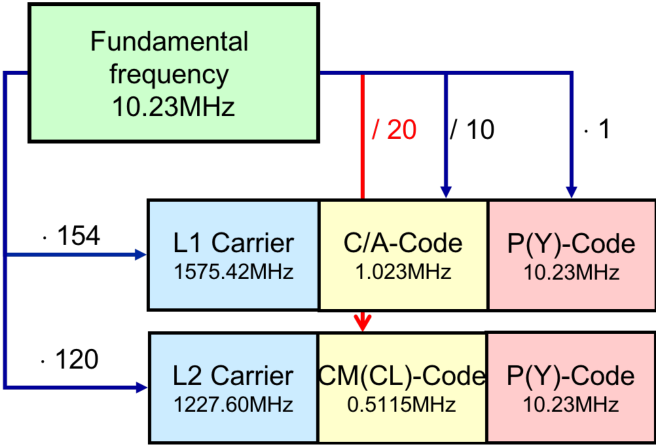
Signals

GPS Pseudo Random Noise Codes – Block IIA and IIR satellites



Signals

GPS Pseudo Random Noise Codes – Block IIR-M satellites



Signals

GPS Pseudo Random Noise Codes - C/A Code

The C/A-code, the P-code, the CM-code and the CL-code are generated from the output of shift registers. For the C/A-code 10-stage shift registers are used.

The output of a 10-stage shift register is a PRN sequence with $2^{10} - 1 = 1023$ chips in length; then the PRN sequence repeats.

- The C/A-code length is 1 ms (300,000 m)
- The C/A-code is generated by adding (modulo 2) the output of two 10-stage registers (two so-called Gold-codes G1 and G2).
- Each GPS satellite is assigned a different C/A$_i$-code by delaying the G2 sequence by a well defined number of chips.
- For more details on C/A$_i$-code generation see the corresponding pages of the IS-GPS-200.

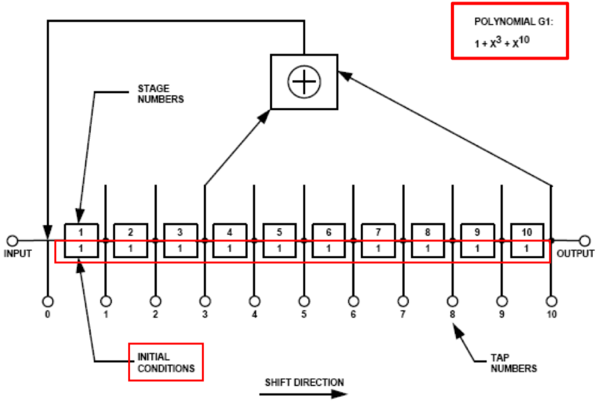
Signals

GPS Pseudo Random Noise Codes - C/A Code

The C/A-code is the modulo-2 addition of the output G1 generated by a 10-stage shift register with the output of two stages of the G2 shift register, also a 10-stage register.

The G1 code generation:

Initial condition of the shift register: a "1" everywhere

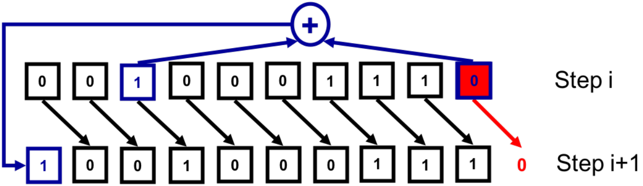


Signals

GPS Pseudo Random Noise Codes - C/A Code

The G1 code generation (*Tapped Feedback Shift Register*)

- G1 Gold Code (polynomial) $G1 = 1 + x^3 + x^{10}$



- XOR (Modulo 2 Addition):

$$\boxed{0} \oplus \boxed{0} = 0 \qquad \boxed{0} \oplus \boxed{1} = 1$$

$$\boxed{1} \oplus \boxed{0} = 1 \qquad \boxed{1} \oplus \boxed{1} = 0$$

- Code Chip Status (becomes pseudo randomly)

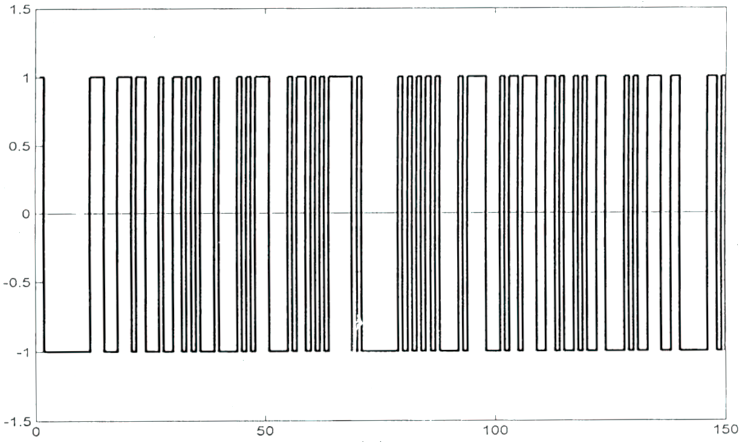
$$0 \rightarrow +1$$

$$1 \rightarrow -1$$

Signals

GPS Pseudo Random Noise Codes - C/A Code

PRN-Code Chip Status (0, 2, . . ., 150) for G1-Code)



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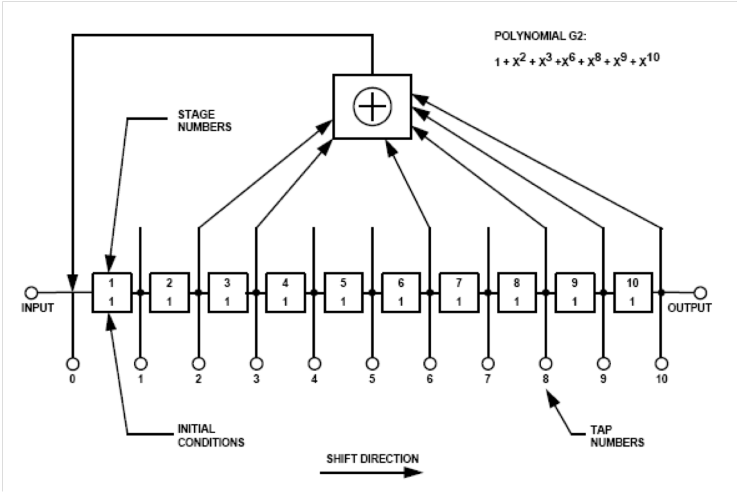
GPS Pseudo Random Noise Codes - C/A Code

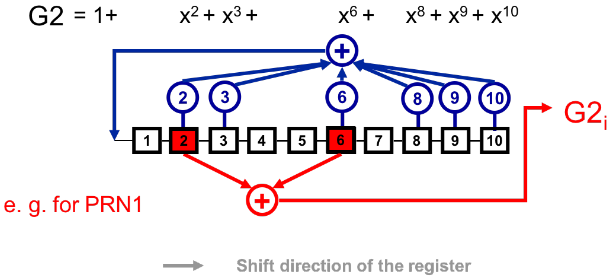
The G2 shift register:



Signals

GPS Pseudo Random Noise Codes - C/A Code

The G2 code generation (*Tapped Feedback Shift Register*) :



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GPS Pseudo Random Noise Codes - C/A Code

Table 3-I IS-GPS-200:

The C/A-code #1 is the modulo-2 addition of the output of stages 2 and 6 with the output of G1

The C/A-code #2 is the modulo-2 addition of the output of stages 3 and 7 with the output of G1

The C/A-code #3 is the modulo-2 addition of the output of stages 4 and 8 with the output of G1

Each satellite has a different C/A-code!

SV ID No.	GPS PRN Signal No.	Code Phase Selection		Code Delay Chips		First 10 Chips Octal*	First 12 Chips Octal
		C/A($G2_i$)***	($X2_i$)	C/A	P	C/A	P
1	1	2 ⊕ 6	1	5	1	1440	4444
2	2	3 ⊕ 7	2	6	2	1620	4000
3	3	4 ⊕ 8	3	7	3	1710	4222
4	4	5 ⊕ 9	4	8	4	1744	4333
5	5	1 ⊕ 9	5	17	5	1133	4377
6	6	2 ⊕ 10	6	18	6	1455	4355
7	7	1 ⊕ 8	7	139	7	1131	4344
8	8	2 ⊕ 9	8	140	8	1454	4340
9	9	3 ⊕ 10	9	141	9	1626	4342
10	10	2 ⊕ 3	10	251	10	1504	4343
11	11	3 ⊕ 4	11	252	11	1642	
12	12	5 ⊕ 6	12	254	12	1750	
13	13	6 ⊕ 7	13	255	13	1764	
14	14	7 ⊕ 8	14	256	14	1772	
15	15	8 ⊕ 9	15	257	15	1775	
16	16	9 ⊕ 10	16	258	16	1776	
17	17	1 ⊕ 4	17	469	17	1156	
18	18	2 ⊕ 5	18	470	18	1467	
19	19	3 ⊕ 6	19	471	19	1633	4343

* In the octal notation for the first 10 chips of the C/A-code as shown in this column, the first digit (1) represents a "1" for the first chip and the last three digits are the conventional octal representation of the remaining 9 chips. (For example, the first 10 chips of the C/A-code for PRN Signal Assembly No. 1 are: 1100100000).

** C/A-codes for 34 and 37 are identical.

*** The two-tap coder utilized here is only an example implementation that generates a limited set of valid C/A-codes.

⊕ = "exclusive or"

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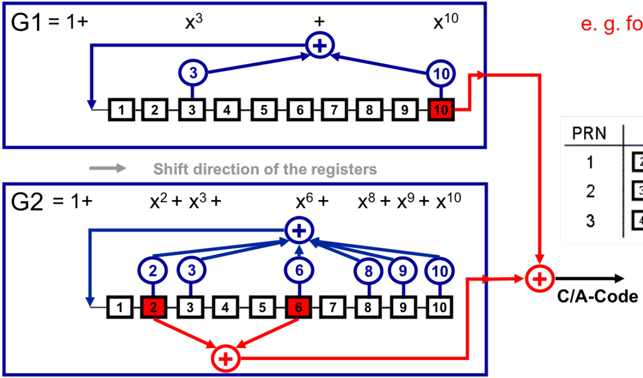
GPS Pseudo Random Noise Codes - C/A Code

The C/A-code generation:



e. g. for PRN1

PRN	TAPS
1	2 ⊕ 6
2	3 ⊕ 7
3	4 ⊕ 8

Signals

GPS Pseudo Random Noise Codes - P Code

- For the P-code generation four 12-stage registers are used in a more complicated manner.
- The output of these calculations is a PRN sequence with $\sim 2.35 \cdot 10^{14}$ chips in length ($\sim$266 days!).
- Each GPS satellite is assigned a one week long segment of the complete code.
- The P-code segment P_i of each satellite is $6,187,104,000,000$ chips long.
- The P_i-code is further encoded with an (unknown) W-code to form the Y_i-code.

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GPS Pseudo Random Noise Codes - CM- and CL-Code (IIR-M satellites)

- All the CL- and the CM-codes are generated with the same 27-stage shift register
- The polynomial for the shift register operation (see C/A-code descr.)

$$1 + x^3 + x^4 + x^5 + x^6 + x^9 + x^{11} + x^{13} + x^{16} + x^{19} + x^{21} + x^{24} + x^{27}$$

- The individual CM_i and CL_i for the different satellites are obtained by using different initial states for the 27-stage shift register (see IS-GPS-200)
- The CM_i-code is reset after $10,230$ chips (= 20 ms)
- The CL_i-code is reset after $767,250$ chips (= 1.5 s)

Signals

GPS Pseudo Random Noise Codes - CM- and CL-Code (IIR-M satellites)

Initial states (octal) of the 27 stage shift register (binary)

Example:
CM$_7$: 124510070

The end states (after 10230 cycles or after 767250 cycles) are also shown in octal numbers.

SV ID No.	GPS PRN Signal No.	Initial Shift Register State (Octal)		End Shift Register State (Octal)	
		L2 CM	L2 CL	L2 CM *	L2 CL **
1	1	742417664	624145772	552566002	267724236
2	2	756014035	506610362	034445034	167516066
3	3	002747144	220360016	723443711	771756405
4	4	066265724	710406104	511222013	047202624
5	5	601403471	001143345	463055213	052770433
6	6	703232733	053023326	667044524	761743665
7	7	124510070	652521276	652322653	133015726
8	8	617316361	206124777	505703344	610611511
9	9	047541621	015563374	520302775	352150323
10	10	733031046	561522076	244205506	051266046
11	11	713512145	023163525	236174002	305611373
12	12	024437606	117776450	654305531	504676773
13	13	021264003	606516355	435070571	272572634
14	14	230655351	003037343	630431251	731320771
15	15	001314400	046515565	234043417	631326563
16	16	222021506	671511621	535540745	231516360
17	17	540264026	605402220	043056734	030367366
18	18	205521705	002576207	731304103	713543613
19	19	064022144	525163451	412120105	232674654

* Short cycled period = 10230
** Short cycled period = 767250

NOTE: There are many other available initial register states which can be used for other signal transmitters including any additional SVs in the future.

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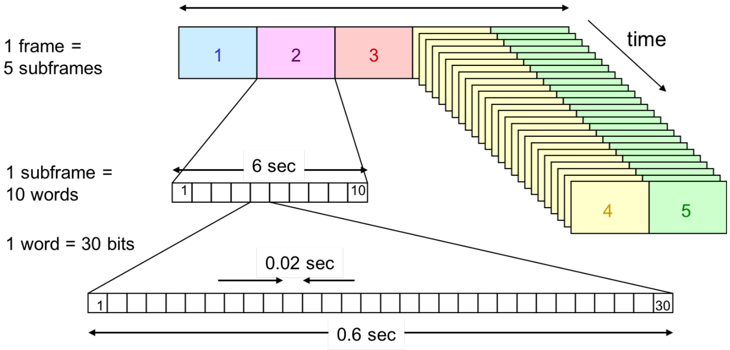
The Message - (NAV-data D(t) encoded on GPS signals)

Message Format:



The first three subframes repeat every 30 sec (identical content every time).
Subframes 4 and 5 have 25 different pages; one complete message set takes 12.5 min to transmit.

Signals

The Message - (NAV-data D(t) encoded on GPS signals)

Message content:

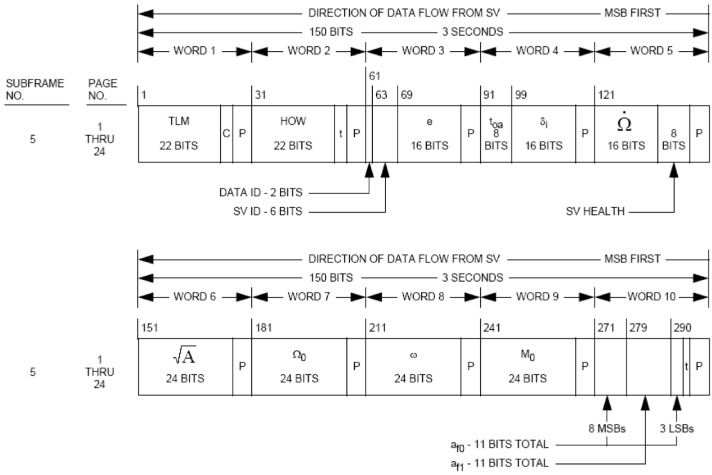
Subframe 1:	Flags (week number, satellite accuracy and health status,...) Age of data, satellite clock correction coefficients
Subframe 2 & 3 :	Satellite orbit parameters (broadcast ephemeris)
Subframe 4 & 5 :	Almanac for complete GPS constellation Health status for complete constellation

To use the satellite for positioning, only the data from subframes 1 , 2 , 3 are required; these are repeated every 30 sec.

For the prediction of satellite visibility, satellite geometrical distribution etc. subframe 4 & 5 data is required

Signals

The Message - (NAV-data D(t) encoded on GPS signals)



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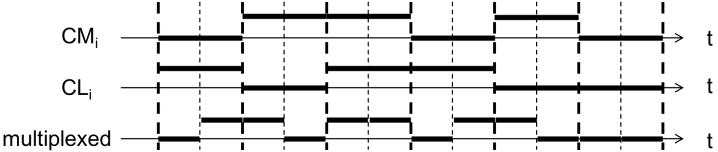
Message and Code additions for the IIR-M satellites

NAV-data D(t):
The message described on the previous pages is modulo-2 added to the C/A-code on the quadrature component of the L1 carrier signal and to the in-phase components of the L1 and the L2 carrier signal for the IIA and IIR satellites.

CNAV-data D_C(t):
IIR-M satellites have additionally the L2 quadrature signal component.

- This signal is modulated by the CM_i-code, the CL_i-code and the message.
- The message is modulo-2 added to the CM_i-code, but not to the CL_i-code.
- The CM_i-code (plus message) and the CL_i-code are chip-by-chip time multiplexed (see below).
- The multiplexed binary sequence has a chipping rate of 1.023 MHz; it is phase modulated onto the quadrature component of the L2 carrier.



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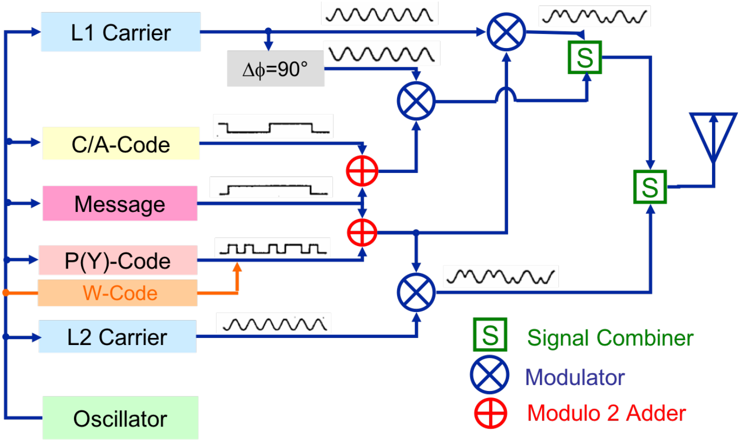
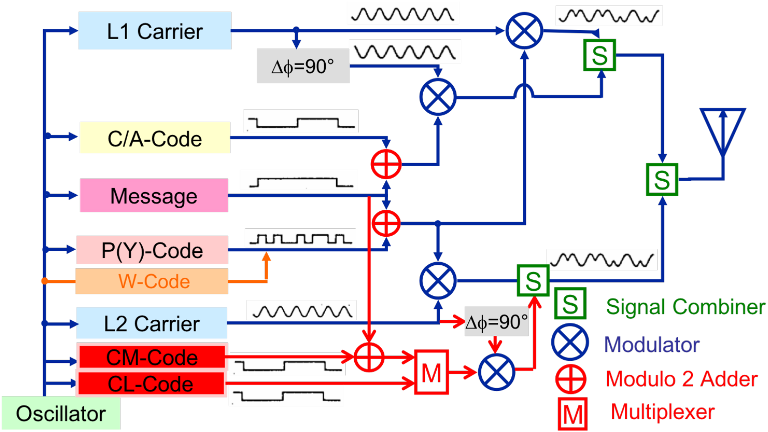
The Complete Signal for IIA and IIR satellites



Signals

The Complete Signal for IIR-M satellites



Signals

The Complete Signal

The combination of Carrier signals, P- and C/A-codes, and the message

$$\begin{aligned} S_{1i}(t) = \quad & \mathsf{P}_i(t) \oplus \mathsf{W}_i(t) \oplus \mathsf{D}_i(t) \otimes A_1 \cos(2\pi f_1 t) \\ & + \mathsf{C/A}_i(t) \oplus \mathsf{D}_i(t) \otimes A_1 \sin(2\pi f_1 t) \end{aligned} \tag{4.13}$$

$$\begin{aligned} S_{2i}(t) = \quad & \mathsf{P}_i(t) \oplus \mathsf{W}_i(t) \oplus \mathsf{D}_i(t) \otimes A_2 \cos(2\pi f_2 t) \\ & + \langle \mathsf{CM}_i(t) \oplus \mathsf{D}_i(t) \rangle \langle \mathsf{CL}_i(t) \rangle \otimes A_2 \sin(2\pi f_2 t) \end{aligned} \tag{4.14}$$

(Note: $\oplus$ binary addition, $\otimes$ BPSK modulation, $\langle\rangle\langle\rangle$ multiplexing)

$S_{1i}(t)$:	combined signal based on L1 carrier signal
$S_{2i}(t)$:	combined signal based on L2 carrier signal
$\mathsf{P}_i(t)$:	P-code of satellite number "i"
$\mathsf{C/A}_i(t)$:	C/A-code of satellite number "i'
$\mathsf{W}_i(t)$:	W-code of satellite number "i'
$\mathsf{CM}_i(t)$:	CM-code of satellite number "i'
$\mathsf{CL}_i(t)$:	CL-code of satellite number "i'
$\mathsf{D}_i(t)$:	message data of satellite number "i'
A_1, A_2:	amplitudes of the L1 and L2 carrier signals
f_1, f_2:	frequencies of the L1 and L2 carrier signals

The L2C quadrature signal is on IIR-M satellites only!