Exercise on 31.10.2019

Task 1 (3 points)

Show, that for two consecutive rotations with angles ϕ_1 and ϕ_2 (around the same vector) the following equation holds:

$$\boldsymbol{R}_{\phi_1+\phi_2} = \boldsymbol{R}_{\phi_1} \cdot \boldsymbol{R}_{\phi_2}$$

Annotation: $\boldsymbol{R}_i \in SO(3)$

Task 2 (4 points)

For the given matrix

$$\boldsymbol{A} = \begin{bmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} & A_{13} \\ \frac{\sqrt{6}}{8} & \frac{5\sqrt{2}}{8} & -\frac{\sqrt{2}}{4} \\ \frac{3\sqrt{6}}{8} & -\frac{\sqrt{2}}{8} & \frac{\sqrt{2}}{4} \end{bmatrix}$$

i) Calculate A_{13}, such that $\boldsymbol{A}$ is a rotation matrix.

ii) Calculate the euler angles using the parametrisation given in formula (2.6) from the lecture. Are the rotations unique?

iii) Explain the effect of $\boldsymbol{A}$ on a vector $\boldsymbol{v}$ if A_{13} differs from the solution above.

Task 3 (3 points)

Show, that the quaternion rotation defined by

$$\boldsymbol{p}' = \boldsymbol{q}\boldsymbol{p}\bar{\boldsymbol{q}}$$

corresponds to the DCM (2.14) from the lectures.

Annotation: $\boldsymbol{p} = \begin{pmatrix} 0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}$ contains the coordinates of the vector $\vec{p} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$