Pysical Geodesy Lab4

YiWang 3371561 Yiming Chen 3371493 XiaoTan 3371707

Task 1. Equation to quantify the disturbing mass

1.1 Gradient field

$$a = \nabla \phi$$

1.2 Poisson equation

$$div\ \boldsymbol{a} = \nabla \cdot \boldsymbol{a} = \nabla \cdot \nabla \phi = div\ grad\phi = \Delta \phi = -4\pi G \rho$$

1.3 Gauss's divergence identity

$$\iiint_V \nabla \cdot \boldsymbol{a} dV = \iint_S \boldsymbol{a} \cdot d\boldsymbol{S} \Leftrightarrow \iiint_V \Delta \phi dV = \iint_S \nabla \phi \cdot \boldsymbol{n} dS$$

1.4 Total mass

$$-4\pi G \rho \iiint_V \rho dV = \iint_S \frac{\partial \phi}{\partial n} dS \Rightarrow -4\pi G M = \iint_S -g dS \Rightarrow M = \frac{1}{4\pi G} \iint_S g dS$$

1.5 Disturbance mass

$$\delta M = \frac{1}{4\pi G} \iint_{S_0} \delta g dS$$

Task 2. Compute the disturbing mass using Gauss's theorem

2.1 Principle

From Task 1 we get the equation for disturbing mass:

$$\delta \mathrm{M} = \frac{1}{4\pi \mathrm{G}} \iint_{S_0} \delta \mathrm{g}\, \mathrm{dS} = \frac{1}{4\pi \mathrm{G}} \sum_{i=1}^{imax} \sum_{j=1}^{jmax} \delta g_{ij}\, \Delta x \Delta y = \frac{\Delta x \Delta y}{4\pi G} \sum_{i=1}^{imax} \sum_{j=1}^{jmax} \delta g_{ij}$$

Where

$$\delta g_{ij} = g_{ij} - g_0$$

2.2 Result

$$\delta \mathrm{M} = 1.0924 \times 10^{12} kg$$

Task 3. Compute the disturbing mass using shape and density

The valley has V-shaped profile with the depth of at most 300 m and its surface has a shape of a parallelogram. The density difference between the sedimentary rock and rock is $\delta\rho = -700\text{kg}/m^3$. Therefore, we first calculate the volume:

$$V = \frac{2000 \cdot 300}{2} \cdot 5000 = 1.5 \times 10^9\ m^3$$

And then calculate the mass:

$$\delta\text{M} = \text{V} \cdot \rho = 1.5 \times 10^9 \cdot 700 = 1.05 \times 10^{12}\ kg$$

Compare this result to that we get from Task2, it can be seen that they are almost the same, while the second result is a little bit smaller. This might come from the underestimation of the shape and density of the area.