Exam: 14.2.2014, 10:00–13:00, M2.370

Exam questions

i) The gravitational attraction of a point mass $\boldsymbol{a} = -\frac{GM}{r^3}\boldsymbol{r}$ is a *conservative* field. What does this property physically mean? Demonstrate that, mathematically, it follows that $\boldsymbol{a}$ can be written as a gradient field.

ii) The gravitational attraction of a Bouguer plate of thickness h and density ρ reads $\boldsymbol{a}(z) = -2\pi G\rho h\boldsymbol{e}_z$, with $\boldsymbol{e}_z$ the vertical unit vector. Is this a conservative field? And is it a Laplace field?

iii) Suppose you are doing seaborne gravimetry on open ocean and you want to correct for the effect of bathymetry using the Bouguer plate. Given a depth of 4 km, calculate the attraction of the water layer.

iv) The Eötvös correction corrects for the vertical component of the Coriolis acceleration. Starting from the equation of rotational kinematics below, explain the steps that are needed to arrive at the correction term $2\omega\sin\theta v_{\mathrm{E}}$. (Explain the steps; don't do the derivations.) Calculate its value for an Eastbound ship survey around the equator with a velocity of 20 knots.

v) Explain the concept of geopotential number. Is this an observable quantity? If yes, explain how it is observed. If no, explain why it is not observable.

vi) During the gravity survey, the ship will obviously follow the ocean surface, which can be equated with the geoid. As such, her ellipsoidal height will be variable, because the geoid height varies roughly ± 100 m. Explain ellipsoidal, geoidal and orthometric heights graphically. Does the orthometric height change during the survey? What is the order of magnitude of the variation in geopotential number along the survey?

vii) The Earth's flattening is expressed by the spherical harmonic coefficient $C_{2,0}$. Using Rodrigues' formula, derive the Legendre polynomial $P_{2,0}(t) = P_2(t)$ and sketch its behaviour from pole to pole. Now explain, why the term with degree 2 and order 0 represents the Earth's flattening. Should the numerical value of $C_{2,0}$ be positive or negative?

Formulas and numbers

gravitational constant:	G	$=$	$6.672 \cdot 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$
	GM	$=$	$3.986\,005 \cdot 10^{14}\,\mathrm{m^3s^{-2}}$
Earth radius:	R	$=$	$6378\,137\,\mathrm{m}$
potential of sphere:	$V(r)$	$=$	$\frac{4}{3}\pi G\rho R^3\frac{1}{r}$
Bouguer plate:	$a(z)$	$=$	$-2\pi G\rho h$
Rodrigues:	$P_l(t)$	$=$	$\frac{1}{2^l\,l!}\frac{\mathrm{d}^l(t^2-1)^l}{\mathrm{d}t^l}$
kinematics:	$\ddot{\boldsymbol{r}}_e$	$=$	$R\ddot{\boldsymbol{r}}_i - 2\boldsymbol{\omega}\times\dot{\boldsymbol{r}}_e - \dot{\boldsymbol{\omega}}\times\boldsymbol{r}_e - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}_e)$
orthometric height:	H_P	$=$	$\frac{C_P}{g_P + 0.0424H_P}$

closed book
pocket calculator only

GIS

Exam: 9.8.2013, 10:00–12:00, M24.01

Exam questions

A standard tool of geophysical exploration is airborne gravimetry, which utilizes a gravimeter, mounted on an airplane.

i) The gravitational attraction of a point mass $\boldsymbol{a} = -\frac{GM}{r^3}\boldsymbol{r}$ can be written as the gradient of the potential $V = GM/r$. Why is that the case?

ii) Using the potential of the point mass, which equals the outer potential of a solid sphere, derive a formula for the vertical gradient of $\boldsymbol{a}$ at the surface of the spherical Earth.

iii) In case the airplane flies at 400 m altitude, calculate the correction, needed to reduce the gravity measurement at altitude down to the Earth surface.

The equation of rotational kinematics (see below) describes the several inertial accelerations in a moving reference frame.

iv) Identify and name the three inertial accelerations.

v) How is the Eötvös correction ($2\omega \sin\theta v_{\mathrm{E}}$) connected to this equation?

vi) Calculate the Eötvös correction for a gravity survey in case the airplane flies an East-West profile at the latitude of Stuttgart ($\phi_{\mathrm{St}} = 48°45'$) with a velocity of 100 km/h.

vii) **Bonus 10%:** If the accuracy of airborne gravimetry would be 1 mGal, calculate the required velocity accuracy for the Eötvös correction.

The Laplace equation below describes the external gravitational potential.

viii) Explain, which steps are necessary to go from the Laplace equation to the spherical harmonic solution, also given below. (Note: only explain steps, don't do derivations.)

ix) Using the spherical harmonic series expression, discuss the question, whether the airplane should fly as high or rather as low as possible.

x) Formulate the concept of airborne gravimetry as a boundary value problem. Is it a Neumann, a Robin, or a Dirichlet problem? Or none of them?

Formulas and numbers

gravitational constant: $G = 6.672 \cdot 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$

$GM = 3.986\,005 \cdot 10^{14}\,\mathrm{m^3 s^{-2}}$

Earth radius: $R = 6378\,137\,\mathrm{m}$

kinematics: $\ddot{\boldsymbol{r}}_e = R\ddot{\boldsymbol{r}}_i - 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_e - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_e) - \dot{\boldsymbol{\omega}} \times \boldsymbol{r}_e$

Laplace: $\Delta V = 0$

solid sphere: $V(r) = \dfrac{GM}{r}$

real Earth:
$$V(r,\theta,\lambda) = \frac{GM}{R}\sum_{l=0}^{L_{\max}}\sum_{m=0}^{l}\left(\frac{R}{r}\right)^{l+1}\bar{P}_{lm}(\cos\theta)\left[\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\right]$$

closed book
pocket calculator only

GIS

Exam: 15.03.2013, 11:00–13:00, M24.01

Exam questions

The gravitational attraction of a Bouguer plate of thickness h and density ρ reads $\boldsymbol{a}(z) = -2\pi G\rho h\boldsymbol{e}_z$.

i) Is this a conservative field? Is it a Laplace field?

ii) Suppose you are doing seaborne gravimetry on open ocean and you want to correct for the effect of bathymetry using the Bouguer plate. Given a depth of 4 km, calculate the attraction of the water layer.

The equation of rotational kinematics (see below) describes the several inertial accelerations in a moving reference frame. In shipborne gravimetry the so-called Eötvös-correction ($2\omega \sin\theta v_{\mathrm{E}}$) plays an important role.

iii) Describe all individual components and all variables in the equation of rotational kinematics.

iv) How is the Eötvös correction connected to this equation?

v) Calculate the Eötvös correction for a Westbound ship survey around the equator with a velocity of 20 knots.

In the formula below the geopotential number is converted into an orthometric height.

vi) Explain the approximations and assumptions behind this formula.

vii) Explain the concept of geopotential number. Is this an observable quantity? If yes, explain how it is observed. If no, explain why it is not observable.

During the seaborne gravity survey, the ship will obviously follow the ocean surface, which can be equated with the geoid. As such, her ellipsoidal height will be variable, because the geoid height varies roughly ± 100 m.

viii) Explain ellipsoidal, geoidal and orthometric heights graphically.

ix) What is the order of magnitude of the variation in ellipsoidal height during the survey? And how about the variation of the geopotential number?

Formulas and numbers

gravitational constant:	G	$=$	$6.672 \cdot 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$
	GM	$=$	$3.986\,005 \cdot 10^{14}\,\mathrm{m^3s^{-2}}$
Earth radius:	R	$=$	$6378\,137$ m
potential of sphere:	$V(r)$	$=$	$\frac{4}{3}\pi G\rho R^3 \frac{1}{r}$
Bouguer plate:	$a(z)$	$=$	$-2\pi G\rho h$
kinematics:	$\ddot{\boldsymbol{r}}_e$	$=$	$R\ddot{\boldsymbol{r}}_i - 2\boldsymbol{\omega}\times\dot{\boldsymbol{r}}_e - \dot{\boldsymbol{\omega}}\times\boldsymbol{r}_e - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}_e)$
orthometric height:	H_P	$=$	$\dfrac{C_P}{g_P + 0.0424 H_P}$
Laplace:	ΔV	$=$	0

closed book
pocket calculator only

GIS

Exam: 6.8.2012, 10:00–12:00, M24.12

Exam questions

i) The gravitational attraction of a point mass $\boldsymbol{a} = -\frac{GM}{r^3}\boldsymbol{r}$ is a *conservative* field. What does this property physically mean? Demonstrate that, mathematically, it follows that $\boldsymbol{a}$ can be written as a gradient field.

ii) The gravitational attraction of a Bouguer plate of thickness h and density ρ reads $\boldsymbol{a}(z) = -2\pi G\rho h\boldsymbol{e}_z$. Is this a conservative field? Does it fulfil Laplace's equation?

iii) Suppose you are doing seaborne gravimetry on open ocean and you want to correct for the effect of bathymetry using the Bouguer plate. Given a depth of 4 km, calculate the attraction of the water layer.

iv) The Eötvös correction corrects for the vertical component of the Coriolis acceleration. Starting from the equation of rotational kinematics below, explain the steps that are needed to arrive at the correction term $2\omega \sin\theta v_{\mathrm{E}}$. (Explain the steps; don't do the derivations.) Calculate its value for an Eastbound ship survey around the equator with a velocity of 20 knots.

v) **Bonus 10%:** In case the accuracy of seaborne gravimetry would be 1 mGal, calculate the required accuracy of the depth for the bathymetric correction and the required accuracy of the velocity for the Eötvös correction.

vi) Discuss, which of the three principles of gravimetry would be used for seaborne applications.

vii) Explain the concept of geopotential number. Is this an observable quantity? If yes, explain how it is observed. If no, explain why it is not observable.

viii) During the seaborne gravity survey, the ship will obviously follow the ocean surface, which can be equated with the geoid. As such, her ellipsoidal height will be variable, because the geoid height varies roughly ±100 m. Explain ellipsoidal, geoidal and orthometric heights graphically. What is the order of magnitude of the variation in geopotential number along the survey?

Formulas and numbers

gravitational constant:	G	$=$	$6.672 \cdot 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$
	GM	$=$	$3.986\,005 \cdot 10^{14}\,\mathrm{m^3s^{-2}}$
Earth radius:	R	$=$	$6378\,137\,\mathrm{m}$
potential of sphere:	$V(r)$	$=$	$\frac{4}{3}\pi G\rho R^3 \frac{1}{r}$
Bouguer plate:	$a(z)$	$=$	$-2\pi G\rho h$
kinematics:	$\ddot{\boldsymbol{r}}_e$	$=$	$R\ddot{\boldsymbol{r}}_i - 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_e - \dot{\boldsymbol{\omega}} \times \boldsymbol{r}_e - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_e)$
orthometric height:	H_P	$=$	$\dfrac{C_P}{g_P + 0.0424H_P}$
Laplace:	ΔV	$=$	0

closed book
pocket calculator only

GIS

Exam: 16.3.2012, 11:00–13:00, M2.370

Exam questions

i) In your labs you have been making observations with a Scintrex gravimeter. Explain the principle behind this type of gravimeter. Your explanation must contain a drawing. Which factors determine the sensitivity of the instrument? And what is the benefit of a quartz spring?

ii) Your gravity measurements on the staircase yielded a vertical gravity gradient of approximately 0.3 mGal/m. Given the formula of the potential of a homogeneous sphere below, derive a formula for the radial gravitational gradient and confirm the observed gradient numerically.

iii) **Bonus 10%:** You measured the gravity on the staircase, using the Scintrex gravimeter, with a precision of, say, 10 μGal. How accurately do you have to know the height difference to be compatible with the gravimetry accuracy?

iv) The Laplace equation is fundamental to physical geodesy. Which role does it play in the Boundary Value Problems of geodesy?

v) Derive $P_{2,2}(t)$ from Ferrer's and Rodrigues's formulas and give its function value at the equator. Sketch accordingly the surface spherical harmonic $Y_{2,2}(\theta, \lambda)$. Is $Y_{2,2}$ sectorial, zonal or tesseral (and why)?

vi) Explain all quantities in the orthometric height formula below. How is C_P defined and measured? Where does the value 0.0424 come from?

vii) Discuss advantages and disadvantages of the system of orthometric heights.

Formulas and numbers

gravitational constant:	G	$=$	$6.672 \cdot 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$
	GM	$=$	$3.986\,005 \cdot 10^{14}\,\mathrm{m^3 s^{-2}}$
Earth radius:	R	$=$	$6378\,137\,\mathrm{m}$
potential of sphere:	$V(r)$	$=$	$\frac{4}{3}\pi G \rho R^3 \frac{1}{r}$
kinematics:	$\ddot{\boldsymbol{r}}_e$	$=$	$R\ddot{\boldsymbol{r}}_i - 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_e - \dot{\boldsymbol{\omega}} \times \boldsymbol{r}_e - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_e)$
Rodrigues:	$P_l(t)$	$=$	$\frac{1}{2^l\, l!} \frac{\mathrm{d}^l (t^2-1)^l}{\mathrm{d}t^l}$
Ferrer:	$P_{lm}(t)$	$=$	$(1-t^2)^{m/2} \frac{\mathrm{d}^m P_l(t)}{\mathrm{d}t^m}$
orthometric height:	H_P	$=$	$\frac{C_P}{g_P + 0.0424 H_P}$
Laplace:	ΔV	$=$	0
mathematical pendulum:	T	$=$	$2\pi \sqrt{\frac{l}{g}}$

closed book
pocket calculator only

GIS

Exam: 17.8.2011, 9:00–11:00, M24.01

Exam questions

i) In your labs you have been making observations with a Scintrex gravimeter. Explain the principle behind this type of gravimeter. Your explanation must contain a drawing. Which factors determine the sensitivity of the instrument?

ii) Your gravity measurements on the staircase yielded a vertical gravity gradient of approximately 0.3 mGal/m. Given the formula of the potential of a homogeneous sphere below, derive a formula for the radial gravitational gradient and confirm the observed gradient numerically.

iii) The equation with the label *kinematics* below describes the accelerations in a rotating frame. Describe all individual variables and all individual terms.

iv) How big is the centrifugal acceleration at the equator and at the poles? How big is its radial gradient at the equator?

v) The Laplace equation is fundamental to physical geodesy. What does the equation express?

vi) Derive $P_{2,1}(t)$ from Ferrer's and Rodrigues's formulas and give its function value at the equator. Sketch accordingly the surface spherical harmonic $Y_{2,1}(\theta, \lambda)$. Is $Y_{2,1}$ sectorial, zonal or tesseral (and why)?

vii) Explain all quantities in the orthometric height formula below. How is C_P defined and measured? Where does the value 0.0424 come from?

viii) Explain graphically the difference between orthometric and ellipsoidal heights.

Formulas and numbers

$$
\begin{array}{lrcl}
\text{gravitational constant:} & G & = & 6.672 \cdot 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}} \\
 & GM & = & 3.986\,005 \cdot 10^{14}\,\mathrm{m^3s^{-2}} \\
\text{Earth radius:} & R & = & 6378\,137\,\mathrm{m} \\
\text{potential of sphere:} & V(r) & = & \frac{4}{3}\pi G\rho R^3 \frac{1}{r} \\
\text{kinematics:} & \ddot{\boldsymbol{r}}_e & = & R\ddot{\boldsymbol{r}}_i - 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_e - \dot{\boldsymbol{\omega}} \times \boldsymbol{r}_e - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_e) \\
\text{Rodrigues:} & P_l(t) & = & \frac{1}{2^l\, l!} \frac{\mathrm{d}^l (t^2-1)^l}{\mathrm{d}t^l} \\
\text{Ferrer:} & P_{lm}(t) & = & (1-t^2)^{m/2} \frac{\mathrm{d}^m P_l(t)}{\mathrm{d}t^m} \\
\text{orthometric height:} & H_P & = & \frac{C_P}{g_P + 0.0424 H_P} \\
\text{Laplace:} & \Delta V & = & 0 \\
\text{mathematical pendulum:} & T & = & 2\pi\sqrt{\frac{l}{g}}
\end{array}
$$

closed book
pocket calculator only

GIS

Exam: 18.3.2011, 10:00–12:00, M2.370

Exam questions

i) Explain the principle of an astatic spring gravimeter. Your explanation must contain a drawing. Formulas are optional.

ii) The equation with the label *kinematics* below describes the accelerations in a rotating frame. Describe all individual variables and all individual terms.

iii) Calculate the size of the Coriolis acceleration (in mGal) in case your traveling at the equator in East-West direction at a speed of 100 km/h and discuss its direction.

iv) Gravity is defined as the sum of gravitational attraction and centrifugal acceleration. How big is the radial gravity *gradient* at the equator?

v) In the spherical harmonic series of gravity $g(r, \theta, \lambda)$ below, what is the unit of the spherical harmonic coefficients $\bar{C}_{lm}$ and $\bar{S}_{lm}$?

vi) The Earth's flattening is expressed by the coefficient $\bar{C}_{2,0}$ with a numerical value of $-4.842 \cdot 10^{-4}$. What is the contribution of this coefficient to the gravity in Stuttgart ($\theta_{\mathrm{St}} = 41°15'$, $\lambda_{\mathrm{St}} = 9°10'$)?

vii) In the normal height formula below, explain all quantities and explain the steps necessary to arrive at this formula. (Don't do derivations, just explain the steps.)

viii) What are the advantages and disadvantages of the normal height system?

Formulas

gravitational constant: $G = 6.672 \cdot 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$

$GM = 3.986\,005 \cdot 10^{14}\,\mathrm{m^3 s^{-2}}$

Earth radius: $R = 6378\,137\,\mathrm{m}$

potential: $V(r) = \frac{4}{3}\pi G \rho R^3 \frac{1}{r}$

kinematics: $\ddot{\boldsymbol{r}}_e = R\ddot{\boldsymbol{r}}_i - 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_e - \dot{\boldsymbol{\omega}} \times \boldsymbol{r}_e - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_e)$

Rodrigues: $P_l(t) = \frac{1}{2^l\, l!} \frac{\mathrm{d}^l (t^2-1)^l}{\mathrm{d}t^l}$

Ferrer: $P_{lm}(t) = (1-t^2)^{m/2} \frac{\mathrm{d}^m P_l(t)}{\mathrm{d}t^m}$

normal height: $H_P^{\mathrm{n}} = \frac{C_P}{\gamma_{P_0} - 0.1543 H_P^{\mathrm{n}}}$

$$g(r,\theta,\lambda) = \frac{GM}{R^2} \sum_{l=0}^{L_{\max}} \sum_{m=0}^{l} \left(\frac{R}{r}\right)^{l+1} \bar{P}_{lm}(\cos\theta) \left[\bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda\right]$$

mathematical pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$

closed book
pocket calculator only

GIS

Exam: 6.8.2010, 10:00–12:00, M2.370

Exam questions

i) Explain the principle of pendulum gravimetry. Your explanation must contain a drawing. Formulas are optional.

ii) Given the equation of a mathematical pendulum below, discuss the timing accuracy required for a gravity error level of 1 mGal.

iii) The equation with the label *kinematics* below describes the accelerations in a rotating frame. Describe all individual variables and all individual terms.

iv) Calculate the size of the centrifugal acceleration at the equator in mGal and discuss its direction.

v) **Bonus 5%:** Gravity is defined as the sum of gravitational attraction and centrifugal acceleration. How big is the gravity *gradient*, due to the centrifugal acceleration, at the equator?

vi) In the spherical harmonic series of gravity $g(r,\theta,\lambda)$ below, what is the unit of the spherical harmonic coefficients $\bar{C}_{lm}$ and $\bar{S}_{lm}$?

vii) The Earth's flattening is expressed by the coefficient $\bar{C}_{2,0}$ with a numerical value of $-4.842 \cdot 10^{-4}$. What is the contribution of this coefficient to the gravity in Stuttgart ($\theta_{\text{St}} = 41°15'$, $\lambda_{\text{St}} = 9°10'$)?

viii) In the normal height formula below, explain all quantities and explain the steps necessary to arrive at this formula. (Don't do derivations, just explain the steps.)

ix) What are the advantages and disadvantages of the normal height system?

Formulas

gravitational constant: $G = 6.672 \cdot 10^{-11}\,\text{m}^3\text{kg}^{-1}\text{s}^{-2}$

$GM = 3.986\,005 \cdot 10^{14}\,\text{m}^3\text{s}^{-2}$

Earth radius: $R = 6378\,137\,\text{m}$

potential: $V(r) = \frac{4}{3}\pi G\rho R^3 \frac{1}{r}$

kinematics: $\ddot{\boldsymbol{r}}_e = R\ddot{\boldsymbol{r}}_i - 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_e - \dot{\boldsymbol{\omega}} \times \boldsymbol{r}_e - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_e)$

Rodrigues: $P_l(t) = \frac{1}{2^l\, l!}\frac{\mathrm{d}^l(t^2-1)^l}{\mathrm{d}t^l}$

Ferrer: $P_{lm}(t) = (1-t^2)^{m/2}\frac{\mathrm{d}^m P_l(t)}{\mathrm{d}t^m}$

normal height: $H_P^{\text{n}} = H_P^{\text{n}}\frac{C_P}{\gamma_{P_0} - 0.1543 H_P^{\text{n}}}$

$$g(r,\theta,\lambda) = \frac{GM}{R^2}\sum_{l=0}^{L_{\max}}\sum_{m=0}^{l}\left(\frac{R}{r}\right)^{l+1}\bar{P}_{lm}(\cos\theta)\left[\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\right]$$

mathematical pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$

closed book
pocket calculator only

Exam: 14.8.2009, 10:00–12:00, M2.370

GOCE gravity gradiometry

The satellite mission *Gravity field and steady-state Ocean Circulation Explorer* (GOCE) was launched on March 17$^{\text{th}}$ this year. It orbits the Earth on a near-circular and near-polar orbit at 283.5 km height over the Earth's surface. The goal of the mission is to determine the Earth's gravity field and geoid up till maximum spherical harmonic degree $L_{\max} = 250$.

i) After the launch, the popular press explained the concept of *geoid* as a *surface of constant gravitation*. Do you agree with that statement?

ii) The GOCE altitude is extremely low for a satellite. Discuss, using the spherical harmonic series below, why such a low orbital height is so important for a gravity field mission.

iii) What is the spatial resolution of GOCE? (or: Which spatial scale corresponds to $L_{\max} = 250$?)

GOCE does not measure the potential V itself. It measures the tensor (or Hesse matrix) of 2$^{\text{nd}}$ spatial derivatives in a local co-orbiting frame:

$$\mathbf{V} = \begin{pmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{yx} & V_{yy} & V_{yz} \\ V_{zx} & V_{zy} & V_{zz} \end{pmatrix} .$$

The notation V_{xx}, for instance, denotes the 2$^{\text{nd}}$ partial derivative of V towards x. The measurement of 2$^{\text{nd}}$ derivatives is known as *gravity gradiometry*.

iv) What can you say about the trace of this matrix?

The z-axis of the local frame is always pointing in radial direction. Thus $V_{zz} = V_{rr}$.

v) Comparing the spherical harmonic series of V and V_{rr} (also given below), discuss whether gravity gradiometry is a good observation approach for a gravity field mission.

vi) Calculate the order of magnitude of V_{rr} at GOCE orbit altitude.
Tip: the calculation of the central term ($l = m = 0$) is sufficient.

vii) Explain the fundamental height equation $h = H + N$ graphically.

viii) Discuss the practical relevance of knowing accurate geoid heights, e.g. from GOCE, in surveying.

ix) Explain in words the concept of orthometric height.

x) How is orthometric height H related to the geopotential number?

xi) In the orthometric height formula below, explain all quantities and explain the steps necessary to arrive at this formula. (Don't do derivations, just explain the steps.)

xii) What are the disadvantages of the orthometric height system?

xiii) **Bonus 5%:** Write down the gravity field determination from GOCE as a boundary value problem.

see reverse for formulas

closed book
pocket calculator only

Needed and/or useful

$$
\begin{aligned}
\text{gravitational constant:} \quad G &= 6.672 \cdot 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}} \\
\text{Earth radius:} \quad R_\mathrm{E} &= 6378\,137\,\mathrm{m} \\
\text{potential:} \quad V(r) &= \frac{4}{3}\pi G\rho R^3 \frac{1}{r} \\
\text{kinematics:} \quad \ddot{\vec{r}}_e &= R\ddot{\vec{r}}_i - 2\vec{\omega} \times \dot{\vec{r}}_e - \vec{\omega} \times (\vec{\omega} \times \vec{r}_e) - \dot{\vec{\omega}} \times \vec{r}_e \\
\text{Rodrigues:} \quad P_l(t) &= \frac{1}{2^l\, l!} \frac{\mathrm{d}^l (t^2-1)^l}{\mathrm{d}t^l} \\
\text{Ferrer:} \quad P_{lm}(t) &= (1-t^2)^{m/2} \frac{\mathrm{d}^m P_l(t)}{\mathrm{d}t^m} \\
\text{Stokes:} \quad N_P &= \frac{R_\mathrm{E}}{4\pi\gamma} \iint_\sigma St(\psi_{PQ}) \Delta g_Q \,\mathrm{d}\sigma \\
\text{orthometric height:} \quad H &= \frac{C}{g + 0.0424H}
\end{aligned}
$$

$$
\begin{aligned}
V(r,\theta,\lambda) &= \frac{GM}{R} \sum_{l=0}^{L_\mathrm{max}} \sum_{m=0}^{l} \left(\frac{R}{r}\right)^{l+1} \bar{P}_{lm}(\cos\theta) \left[\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\right] \\
V_{rr}(r,\theta,\lambda) = \frac{\partial^2 V(r,\theta,\lambda)}{\partial r^2} &= \frac{GM}{R^3} \sum_{l=0}^{L_\mathrm{max}} \sum_{m=0}^{l} \left(\frac{R}{r}\right)^{l+3} (l+1)(l+2)\bar{P}_{lm}(\cos\theta) \left[\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\right]
\end{aligned}
$$

GeoEngine: **Module 3, Submodule Physical Geodesy**

Exam: 6.8.2008, 10:00–11:20, M2.370

40 minutes — closed book, pocket calculator only

Gravity, gravitation, potential and the like

i) Below, the formula for gravitational *potential* of a homogeneous sphere is given. Given the fact that the gravitational *attraction* on the Earth's surface is about 980 Gal, and supposing that the Earth is a homogeneous sphere indeed, calculate the Earth's density.

ii) Now calculate the potential at a satellite altitude of 400 km.

iii) Determine the centrifugal acceleration at the equator and make a sketch of the centrifugal acceleration field in the equatorial plane.

iv) What does the Euler acceleration mean? Is it important in gravimetry?

v) Suppose you could take a Scintrex gravimeter with you in the Space Shuttle in Earth orbit. Does it measure gravity or gravitation or even something different? Explain.

vi) Starting from the formula for work $W = \int \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r}$, explain what a geopotential number is.

vii) How would you derive normal height H^{n} from a geopotential number? How can you turn a GPS-height into a normal height? Please use a drawing in your answer.

viii) Explain in words what the Laplace equation $\Delta V = 0$ means. What do Legendre functions have to do with the Laplace equation?

ix) *Bonus question, 10%:* Derive the Legendre function $P_{2,1}(t)$ from the analytical recipe below and make a sketch of it.

Needed and/or useful

gravitational constant:	G	$=$	$6.672 \cdot 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$
Earth radius:	R_{E}	$=$	$6378\,137\,\mathrm{m}$
potential:	$V(r)$	$=$	$\frac{4}{3}\pi G \rho R^3 \frac{1}{r}$
kinematics:	$\ddot{\boldsymbol{r}}_e$	$=$	$R\ddot{\boldsymbol{r}}_i - 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_e - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_e) - \dot{\boldsymbol{\omega}} \times \boldsymbol{r}_e$
Rodrigues:	$P_l(t)$	$=$	$\frac{1}{2^l\, l!} \frac{\mathrm{d}^l (t^2-1)^l}{\mathrm{d}t^l}$
Ferrer:	$P_{lm}(t)$	$=$	$(1-t^2)^{m/2} \frac{\mathrm{d}^m P_l(t)}{\mathrm{d}t^m}$
Stokes:	N_P	$=$	$\frac{R_{\mathrm{E}}}{4\pi\gamma} \iint_\sigma St(\psi_{PQ}) \Delta g_Q \,\mathrm{d}\sigma$

GeoEngine: **Module 3, Submodule Physical Geodesy**

Re-Exam: **8.2.2008**

40 minutes — closed book, pocket calculator

Saturn's moon Rhea and the satellite mission Cassini

Rhea is an icy moon of Saturn with a mean density of $1330\,\mathrm{kg/m^3}$. The low density indicates that it is composed of a rocky core and a layer of water-ice around it. The moon was discovered 1692 by Giovanni Domenic Cassini and was recently observed by the identically named satellite.

i) Calculate the gravitational potential *and* attraction on the surface of Rhea.

ii) Explain all the terms in the kinematic equation given below.

iii) Rhea revolves around Saturn in 4.5175 days. Knowing that Rhea has a so called bound rotation, which means that it always shows the same face to Saturn, determine its rotation rate.

iv) Determine the centrifugal acceleration at the equator and the pole.

Suppose the Cassini mission dropped off an autonomous skatebot (skating robot) onto the surface of Rhea. It is equipped with a LaCoste-Romberg type gravimeter.

v) Explain the principles of spring-type gravimeters like LaCoste-Romberg.

vi) Once in stationary mode, does the gravimeter measure gravity or gravitation? Explain.

vii) Your skatebot is now skating westward on the equator of Rhea with a speed of $100\,\mathrm{km/h}$. Calculate the magnitude of the Coriolis acceleration it experiences? What can you say about the direction?

viii) Explain the Stokes equation below in detail and explain the necessary steps to determine a geoid of Rhea?

ix) *Bonus question, 10%:* Derive the Legendre function $P_{2,1}(t)$ from the analytical recipe below and make a sketch of it.

Needed and/or useful

$\rho_{\mathrm{ice}} = 917\,\mathrm{kg/m^3}$, $\rho_{\mathrm{core}} = 3500\,\mathrm{kg/m^3}$, $R_{\mathrm{Rhea}} = 765\,\mathrm{km}$, $M_{\mathrm{Saturn}} = 5.7 \cdot 10^{26}\,\mathrm{kg}$,
$G = 6.672 \cdot 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$, $d_{\mathrm{Rhea-Saturn}} = 527\,040\,\mathrm{km}$

$$V(r) = \frac{4}{3}\pi G \rho R^3 \frac{1}{r}$$

$$\ddot{\boldsymbol{r}}_e = R\ddot{\boldsymbol{r}}_i - 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_e - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_e) - \dot{\boldsymbol{\omega}} \times \boldsymbol{r}_e$$

$$P_l(t) = \frac{1}{2^l\, l!} \frac{\mathrm{d}^l (t^2-1)^l}{\mathrm{d}t^l}$$

$$P_{lm}(t) = (1-t^2)^{m/2} \frac{\mathrm{d}^m P_l(t)}{\mathrm{d}t^m}$$

Stokes:

$$N_P = \frac{R_{\mathrm{E}}}{4\pi\gamma} \iint_\sigma St(\psi_{PQ}) \Delta g_Q \, \mathrm{d}\sigma$$

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Examination Physical Geodesy — Summer 2007

Short questions—short answers

i. Explain the principle of free-fall gravimetry.
ii. How do modern absolute gravimeters achieve their very high accuracy (a few μGal)?
iii. What is the difference between gravity and gravitation? Answer verbally and graphically.
iv. Suppose that point P in the figure below is Stuttgart. Draw the (approximate) centrifugal acceleration vector.
v. How are orthometric heights defined geometrically? Use the figure below to explain graphically.
vi. Are surfaces of equal orthometric height equipotential surfaces? Explain.
vii. Given geoid undulation N, ellipsoidal height h and orthometric height H. How are they related? Also explain graphically in the figure.
viii. What are the 3 components of a boundary value problem?
ix. Why are spherical harmonic functions used in physical geodesy?
x. Given the analytical formulae below, determine $P_{3,1}(t)$.
xi. Explain the concept of orthogonality of Legendre functions. How would you go about to calculate the normalization factor $N_{3,1}$ in order to normalize $P_{3,1}(t)$?

Formulae

$$P_l(t) = \frac{1}{l!2^l}\frac{d^l(t^2-1)^l}{dt^l}$$

$$P_{lm}(t) = (1-t^2)^{m/2}\frac{d^m P_l(t)}{dt^m}$$

Figure