Exercise 1: Absolute and relative gravimetry

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Task 1: Absolute gravimetry

Absolute gravity experiment is to be conducted inside the staircase of the Siemens building (Geschwister-Scholl-Str. 24D 70174 Stuttgart) by the means of a pendulum. The pendulum consists of a mass attached to a thin rope. It is connected on a special construction on the fifth floor of the staircase. The pendulum starts to oscillate when the rope is deflected (approximately with 0.5 m). The main goal is to estimate the absolute gravity value at the center of mass of the pendulum. In doing so, the weight of the rope can be neglected and the following values must be first identified:

i. The length of the pendulum between the pivot point and the center of mass. The distance between the pivot point and the first floor is already measured using a distometer: 14.347 ± 0.001 m. Measure the distance between the center of mass and the ground using a ruler both before and after the time measurements.

ii. The period of the pendulum needs to be measured. To determine the period as accurately as possible, the duration of multiple oscillations (e.g., 30 oscillations) should be measured. Attention should be paid in counting the oscillations correctly. For instance, time measurements should start and end as the rope passes the initial (i.e., zero) position of rest.

Discuss the achievable accuracy of the pendulum by fulfilling the following tasks:

i. Estimate the accuracy of the gravity value by means of error propagation of the measurements of time and length.

ii. Determine the accuracy of the time measurements required to obtain an accuracy of 1 mGal for the absolute gravity value. In doing so, the length of the pendulum is assumed to be free of errors.

iii. Discuss whether or not the estimated gravity and its accuracy change if the experiment is done on the third floor.

Task 2: Relative gravimetry

In the staircase of the Siemens building (Geschwister-Scholl-Str. 24D 70174 Stuttgart) relative gravimetry measurements are to be conducted. The measurements are performed using a Scintrex gravimeter based on the step procedure in the following sequence: 1 OG – 3 OG – 1 OG – 3 OG – 6 OG – 3 OG – 6 OG. OG stands for *Obergeschoss*, which is the German word for *floor*.

From the gravity measurements, the gravity differences between the third and first floors, i.e., $\Delta g_{1.OG}^{3.OG}$, and that between the sixth and first floors, i.e., $\Delta g_{1.OG}^{6.OG}$, as well as a possible drift d are to be estimated. The following model for the measurements should be applied

$$y_n(t_k) = g_n + b + dt_k + \epsilon.$$

The model assumes that the observed gravity value of a single point contains not only the gravity itself g_n but also an unknown offset b, a linear drift d, and the measurement random noise ϵ.
Establish the linear system of equations, i.e., $\mathbf{l} = \mathbf{A}\mathbf{x}$, for observed gravity differences and solve (with a least-squares adjustment) for the unknown parameters, i.e., $\Delta g_{1.OG}^{3.OG}$, $\Delta g_{1.OG}^{6.OG}$, and d. Use the standard deviation of measurements of single points to obtain the covariance matrix of observed gravity differences. Use this matrix in your least-squares adjustment. The accuracy of the estimated unknown parameters are to be estimated using the law of error propagation.

i. Is the aimed precision of the relative gravity measurements of $\pm$10–50 μGal met?

ii. Can one achieve the same accuracy with Pendulum? Discuss.

iii. Compare the absolute gravity value obtained from the pendulum with the absolute gravity value 9.808929 m/s^2, which is obtained via gravity transfer of a nearby gravity point using the Scintrex relative gravimeter.

iv. Estimate the gravity vertical gradient, i.e., so-called free-air gradient, from the gravity differences $\Delta g_{1.OG}^{3.OG}$ and $\Delta g_{1.OG}^{6.OG}$.

v. How precise is the estimate of the gravity gradient? To that end, apply the law of error propagation.

vi. Compare the estimated gravity vertical gradient with the gravity gradient computed from a simple homogeneous spherical Earth model with the gravitational potential $V = GM/R$. Provide reasons for the difference between these two values.

Numerical values
Height difference between the third and first floor $\Delta h_{1.OG}^{3.OG}$=6.897 m.
Height difference between the sixth and first floor $\Delta h_{1.OG}^{6.OG}$=17.260 m.
Precision of the height differences: approximately $\sigma_{\Delta h} = \pm 0.03$ m.
Geocentric gravity constant: $GM = 3.986004415 \times 10^{14}$ m^3/s^2.
Radius of the Earth: $R = 6378\,136$ m.