**Exercise on 08.01.2020**

## Task 1 *(3 Points)*

In the lecture the relation between Geographic and Cartesian coordinates have been presented in eq. (6.19):

$$\begin{bmatrix} x_1^e \\ x_2^e \\ x_3^e \end{bmatrix} = \begin{bmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ [N(1-e^2)+h]\sin\phi \end{bmatrix}$$

Calculate the time deviations of $x_1^e$, $x_2^e$ and $x_3^e$ and show the correctness of eq. (6.20):

$$\begin{bmatrix} \dot{x}_1^e \\ \dot{x}_2^e \\ \dot{x}_3^e \end{bmatrix} = \begin{bmatrix} -\dot{\phi}(M+h)\sin\phi\cos\lambda - \dot{\lambda}(N+h)\cos\phi\sin\lambda + \dot{h}\cos\phi\cos\lambda \\ -\dot{\phi}(M+h)\sin\phi\sin\lambda + \dot{\lambda}(N+h)\cos\phi\cos\lambda + \dot{h}\cos\phi\sin\lambda \\ \dot{\phi}(M+h)\cos\phi + \dot{h}\sin\phi \end{bmatrix}$$

## Task 2 *(3 Points)*

The Coriolis acceleration is described by:

$$\mathbf{a}_{\text{cor}} = -2\,\Omega_{ie}^e \cdot \mathbf{v}^e$$

derive $||\mathbf{a}_{\text{cor}}||$ depending on the (North-)Azimuth, for a velocity of $1100\,\text{km}\,\text{h}^{-1}$ and located at

i) the equator

ii) a latitude of $39°$

and a height of $9.2\,\text{km}$ above WGS84. (Note: $\omega_E = 2\pi/86400\text{s}^{-1}$)

## Task 3 *(4 Points)*

Given is the following DCM:

$$C_p^e = \begin{bmatrix} -0.90680 & 0.41785 & -0.05585 \\ -0.34785 & -0.66680 & 0.65908 \\ 0.23815 & 0.61708 & 0.75000 \end{bmatrix}$$

Integrate the following differential equation over a time $n = 1 \ldots 200\,\text{s}$ with $\Delta t = 1\,\text{s}$ by deriving the start values for the quaternions $q_{p0}^e, q_{p1}^e, q_{p2}^e$ and $q_{p3}^e$ from the DCM above

$$\begin{bmatrix} \dot{q}_{p0}^e \\ \dot{q}_{p1}^e \\ \dot{q}_{p2}^e \\ \dot{q}_{p3}^e \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & \omega_{ip1}^p - \omega_{ie1}^p & \omega_{ip2}^p - \omega_{ie2}^p & \omega_{ip3}^p - \omega_{ie3}^p \\ -\omega_{ip1}^p + \omega_{ie1}^p & 0 & \omega_{ip3}^p - \omega_{ie3}^p & -\omega_{ip2}^p + \omega_{ie2}^p \\ -\omega_{ip2}^p + \omega_{ie2}^p & -\omega_{ip3}^p + \omega_{ie3}^p & 0 & \omega_{ip1}^p - \omega_{ie1}^p \\ -\omega_{ip3}^p + \omega_{ie3}^p & \omega_{ip2}^p - \omega_{ie2}^p & -\omega_{ip1}^p + \omega_{ie1}^p & 0 \end{bmatrix} \cdot \begin{bmatrix} q_{p0}^e \\ q_{p1}^e \\ q_{p2}^e \\ q_{p3}^e \end{bmatrix}$$

Also known are:

$$\begin{bmatrix} \omega_{ip1}^p \\ \omega_{ip2}^p \\ \omega_{ip3}^p \end{bmatrix} = \begin{bmatrix} 0.08 \\ 0.02 \\ -0.02 \end{bmatrix} \text{ and } \begin{bmatrix} \omega_{ie1}^p \\ \omega_{ie2}^p \\ \omega_{ie3}^p \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.02 \\ 0.01 \end{bmatrix}$$

Calculate the **euler angles** after each epoch and plot them.