Exercise on 02.07.2019

Task 1 (4 points)

From the top of a tower a car is tracked. The car moves away horizontally from the bottom of the tower as shown in the figure below. At time t_0 the horizontal distance between the tower and the car is x_0=13 m. Since then the slant distance from the top of the tower (height = 21.86 m) to the car is measured in irregular time intervals. The horizontal distance x from the tower to the car shall be computed by extended Kalman filtering. x is assumed to be modelled as a random walk, i.e.

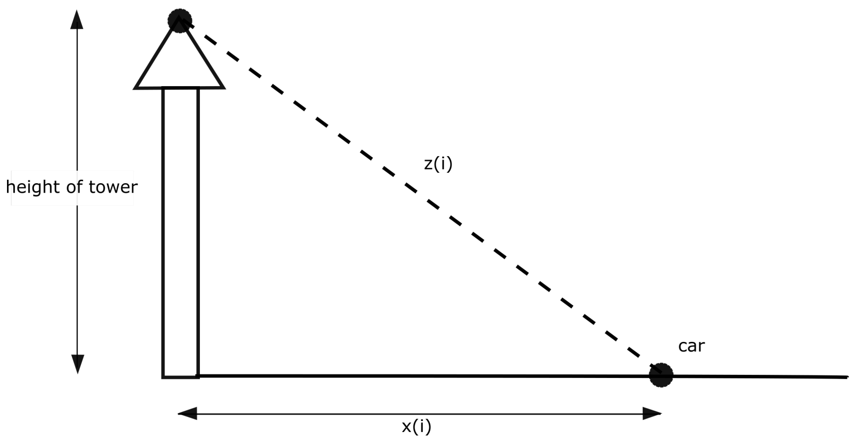
$$\dot{x} = 0 + w(t)$$

The variance of the process noise is given with $\sigma^2 = 4$ and the variance of the measurement noise is $\sigma^2 = 0.01\,\mathrm{m}^2$. For the first epoch $\Sigma_0 = 1$ can be used.

The following measurements are provided:

epoch i	time t [sec]	measurement z [m]
1	3.2	36.25
2	7.5	55.06
3	12.1	76.66
4	17.8	104.42

Compute x and $\sigma_x(t)$ and plot your results.



Task 2 (6 points)

A vehicle performs a test drive on a specific road which has multiple turns. During the drive the horizontal distances from two control points P_1 and P_2 to the vehicle are measured. These measurements are available in the file `EKF_task2.txt`. The data shall be processed by means of extended Kalman filtering, assuming an integrated random walk, i.e.:

$$\ddot{x} = 0 + w(t)$$

$$\ddot{y} = 0 + w(t)$$

The variance of the process noise is $\sigma^2 = 4$ and the standard deviation of the measurement noise is $\sigma = 0.01$. For the first epoch the following covariance matrix of the state can be used:

$$\Sigma_0 = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

The state vector of the first epoch is

$$x = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and the coordinates (x,y) of the control points are

$$P_1 = (-3.5, -3.5)$$

$$P_2 = (3.5, -3.5)$$

Plot the positions, velocities and their standard deviations. Compare your results with the true positions which is given by

$$\begin{aligned} t &= 0 \ldots 1 \text{ with } \Delta t = 0.0025 \\ r &= 2 + \sin(20\pi \cdot t) \\ x_{true} &= r \cdot \sin(2\pi \cdot t) \\ y_{true} &= r \cdot \cos(2\pi \cdot t) \end{aligned}$$

The true velocity can be obtained by the computing $\dot{x}_{true}$ and $\dot{y}_{true}$.