Examination **Dynamic System Estimation** - Spring 2009

Qu. 1 10%

Explain all the differences and similarities between an ordinary Least Squares Parameter Estimation and the Sequential Least Squares Parameter Estimation.

Qu. 2 20%

The Runge-Kutta-Methods (RKM) are a family of numerical integrators for systems of first order Linear Differential Equations. Explain

a) the characteristic property of the RKM compared to other methods

b) the differences between the RKM of first, second, and third order

Qu. 2 25%

The following equation (4.9) is taken from chapter 4 of the course material:

$$\mathbf{\Phi}(t, t_0) = e^{\mathbf{F}\,(t-t_0)}$$

It describes the computation of the Transition Matrix $\mathbf{\Phi}$ in stationary linear systems through the exponential function. Explain step by step how you would compute $\mathbf{\Phi}$ for a given square matrix **F**.

Qu. 3 20%

Describe with the help of graphical sketches the covariance functions and the power spectral density functions of the two random processes

a) Random Constant
b) Gauß-Markov-Process of first order

Qu. 4 25%

The following equations describing the prediction step of the Kalman Filter and the computation of the gain matrix have been taken from chapter 8 of the course material:

$$\delta\mathbf{x}_n(-) = \mathbf{F}\,\delta\mathbf{x}_{n-1}(+)$$

$$\mathbf{C}x_n(-) = \mathbf{F}\mathbf{C}x_{n-1}(+)\mathbf{F}^T + \mathbf{G}\mathbf{C}w_n\mathbf{G}^T$$

$$\mathbf{K}_n = \mathbf{C}x_n(-)\mathbf{H}_n^T\left(\mathbf{H}_n\mathbf{C}x_n(-)\mathbf{H}_n^T + \mathbf{C}v_n\right)^{-1}$$

Give brief explanations of all quantities appearing in these equations.

Examination **Dynamic System Estimation** - Spring 2010

Qu. 1 15%

Explain the relation between Least Squares Parameter Estimation and **Sequential** Least Squares Parameter Estimation.

Qu. 2 25%

Explain how an Ordinary Linear Differential Equation of m-th order is transformed into a system of m Ordinary Linear Differential Equations of first order. What is the purpose of this transformation?

Qu. 3 20%

The following equation describes the computation of the transition matrix $\mathbf{\Phi}$.

$$\mathbf{\Phi}(t,t_0) = e^{\mathbf{F}\,(t-t_0)}$$

Write explicitly the matrix $\mathbf{\Phi}$ for the case

$$t - t_0 = 4, \ \ \mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Qu. 4 20%

Describe with the help of graphical sketches the auto-covariance functions and the power spectral density functions of the two random processes “White Noise” and “Gauss-Markov process of first order”. How can the power spectral density function be computed from the auto-covariance function?

Qu. 5 20%

In a Kalman Smoother the result of a forward Kalman Filter is optimally combined with a backward Kalman Filter using the following equations:

$$\mathbf{C}x_n = (\mathbf{C}x_{nb}^{-1}(+) + \mathbf{C}x_n^{-1}(+))^{-1}$$

$$\delta\mathbf{x}_n = \mathbf{C}x_n(\mathbf{C}x_n^{-1}(+)\delta\mathbf{x}_n(+) + \mathbf{C}x_{nb}^{-1}(+)\delta\mathbf{x}_{nb}(+))$$

- Explain the meaning of all symbols in these equations.
- Explain the purpose of the Kalman Smoother.

Examination **Dynamic System Estimation** - Spring 2011

Qu. 1 25%

The following equations for the second order Runge-Kutta integration algorithm are taken from the lecture notes.

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$
$$k_1 = f(y_n, t_n)$$
$$k_2 = f(y_n + h\,k_1, t_n + h)$$

Use these equations to integrate the first order Differential Equation $y'(t) - \frac{2t}{1+t^2}\,y(t) = t^3$

with initial value $y(t_0) = y(0) = 1$

to obtain a solution for ***t = 0.01***; use the step size ***h = 0.01*** (i.e. only one integration step is required!). Write down all intermediate calculations. Retain the accuracy of the result with four decimals. Remark: first transform the Differential Equation into the form

$$y'(t) = f(y(t), t)$$

Qu. 2 25%

Chapter 7 of the lecture notes is entitled “State vector augmentation”. It is related to a particular treatment of correlated Random Processes in the context of Linear Models. Explain in your own words, what this chapter is all about.

Qu. 3 15%

Describe with the help of graphical sketches the covariance functions and the power spectral density functions of the random processes

c) White Noise
d) Random Constant

Qu. 4 25%

The following equations describing the prediction step of the Kalman Filter and the computation of the gain matrix have been taken from chapter 8 of the course material:

$$\delta\mathbf{x}_n(-) = \mathbf{F}\,\delta\mathbf{x}_{n-1}(+)$$
$$\mathbf{C}x_n(-) = \mathbf{F}\mathbf{C}x_{n-1}(+)\mathbf{F}^T + \mathbf{G}\mathbf{C}w_n\mathbf{G}^T$$
$$\mathbf{K}_n = \mathbf{C}x_n(-)\mathbf{H}_n^T\left(\mathbf{H}_n\mathbf{C}x_n(-)\mathbf{H}_n^T + \mathbf{C}v_n\right)^{-1}$$

Give brief explanations of all quantities appearing in these equations.

Qu. 5 10%

Let **s** be an n-dimensional random variable with covariance matrix **Σ**(**s**). The m-dimensional random variable **r** is computed according to **r** = **As** with **A** being a deterministic matrix.

a) What is the dimension of the matrix **A**?
b) What is the equation to compute the covariance matrix **Σ**(**r**) of the random variable **r**?

Examination **Dynamic System Estimation** - Spring 2012

Qu. 1 25%

The following equations for the second order Runge-Kutta integration algorithm are taken from the lecture notes.

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$
$$k_1 = f(y_n, t_n)$$
$$k_2 = f(y_n + h\,k_1, t_n + h)$$

Use these equations to integrate the first order Differential Equation $y'(t) - \frac{3t}{1+t^3} y(t) = t$ with initial value $y(t_0) = y(2) = 3$

to obtain a solution for ***t = 2.01***; use the step size ***h = 0.01*** (i.e. only one integration step is required!). Write down all intermediate calculations. Retain the accuracy of the result with four decimals. Remark: first transform the Differential Equation into the form $y'(t) = f(y(t), t)$

Qu. 2 15%

Numerical integration methods (example: the Runge-Kutta methods) will always produce errors in the integration results. Explain the causes of these integration errors and which measures can be taken to reduce the errors.

Qu. 3 15%

Describe with the help of graphical sketches the covariance functions and the power spectral density functions of the random processes

a) White Noise
b) 1st Order Gauss-Markov process

Qu. 4 25%

The following equation for the computation of the matrix exponential function is taken from chapter 4 of the course material:

$$e^{\mathbf{A}} = \mathbf{I} + \mathbf{A} + \frac{\mathbf{A}^2}{2!} + \frac{\mathbf{A}^3}{3!} + ..$$

Use this equation to compute the matrix exponential for the following matrix A: $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

Remark: The powers of this matrix A have a very simple structure; use this!
Remember: For a scalar x the exponential function is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ..$$

Qu. 5 20%

Explain under which condition a **sequential** Least Squares estimation can be performed. Explain differences and similarities between Least Squares estimation and **sequential** Least Squares estimation.

Total: 100%

Examination **Dynamic System Estimation** - Spring 2013

Qu. 1 25%

The following equations for the second order Runge-Kutta integration algorithm are taken from the lecture notes.

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$
$$k_1 = f(y_n, t_n)$$
$$k_2 = f(y_n + h\,k_1, t_n + h)$$

$$y'(t) - \frac{2t}{1+t^3}\,y(t) = t$$

Use these equations to integrate the first order Differential Equation with initial value $y(t_0) = y(1.0) = 2$

to obtain a solution for ***t = 1.02***; use the step size ***h = 0.02*** (i.e. only one integration step is required!). Write down all intermediate calculations. Retain the accuracy of the result with four decimals. Show explicitly all calculation steps. Remark: first transform the Differential Equation into the form $y'(t) = f(y(t), t)$

Qu. 2 15%

Attached is one page from chapter 8 of the course material with several equations. Give brief explanations of all quantities appearing in these equations.

Qu. 3 15%

Describe with the help of graphical sketches the covariance functions and the power spectral density functions of the random processes

a) White Noise
b) Random Constant

Qu. 4 25%

The following equation for the computation of the matrix exponential function is taken from chapter 4 of the course material: $e^{\mathbf{A}} = \mathbf{I} + \mathbf{A} + \frac{\mathbf{A}^2}{2!} + \frac{\mathbf{A}^3}{3!} + ..$

Use this equation to compute (numerically, with 4 decimals) the matrix exponential for the following matrix A:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

Remark: The powers of this matrix A have a very simple structure; use this!

Remember: For a scalar x the exponential function is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ..$$

Qu. 5 20%

Describe (in your own words) the purpose of a Kalman filter (which quantities must be defined in order to get the filter running; which quantities are estimated with the filter; etc).