SOLUTION (Version 0)
Exercise on 18.06.2019

Task 1 (4 points)

Solution

Design matrix (order of columns: H_1,$\dot{H}_1$,H_2,$\dot{H}_2$,H_3,$\dot{H}_3$)

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Process noise matrix (IRW) (order of columns: H_1,$\dot{H}_1$,H_2,$\dot{H}_2$,H_3,$\dot{H}_3$)

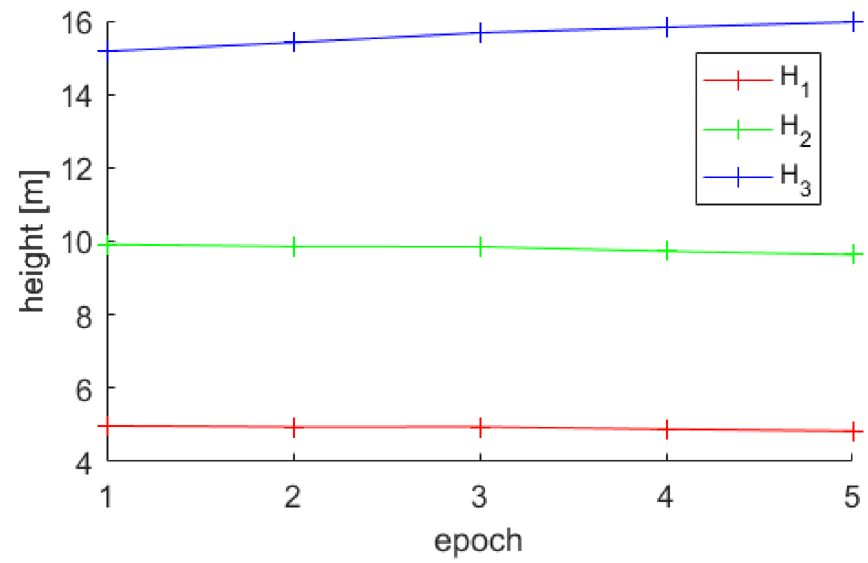
$$Q = \begin{bmatrix} s^2\frac{\Delta t^3}{3} & s^2\frac{\Delta t^2}{2} & 0 & 0 & 0 & 0 \\ s^2\frac{\Delta t^2}{2} & s^2\Delta t & 0 & 0 & 0 & 0 \\ 0 & 0 & s^2\frac{\Delta t^3}{3} & s^2\frac{\Delta t^2}{2} & 0 & 0 \\ 0 & 0 & s^2\frac{\Delta t^2}{2} & s^2\Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & s^2\frac{\Delta t^3}{3} & s^2\frac{\Delta t^2}{2} \\ 0 & 0 & 0 & 0 & s^2\frac{\Delta t^2}{2} & s^2\Delta t \end{bmatrix}$$

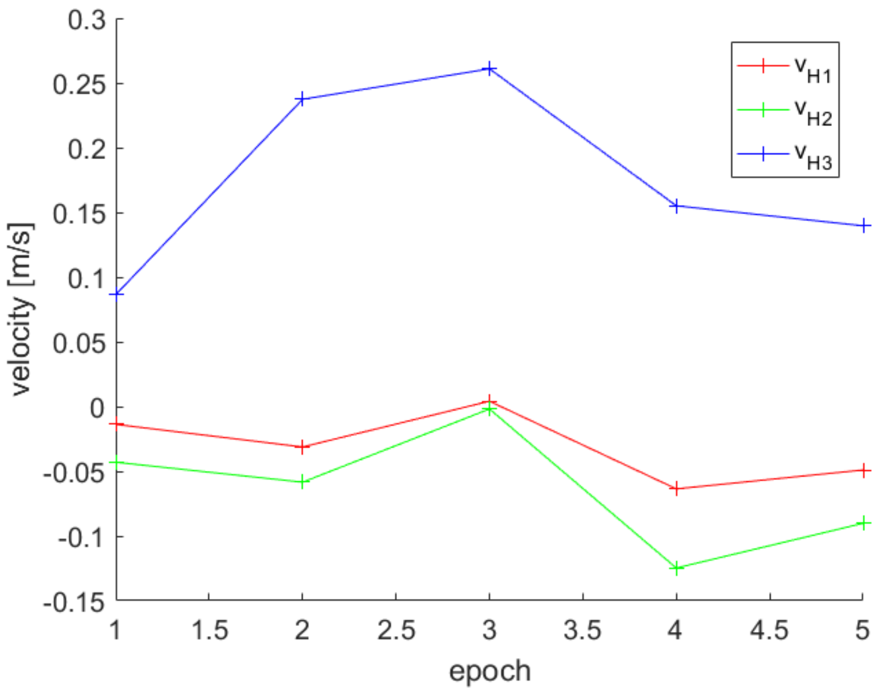
Transition matrix (order of columns: H_1,$\dot{H}_1$,H_2,$\dot{H}_2$,H_3,$\dot{H}_3$)

$$\Phi = \begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Epoch	H_1	$\dot{H}_1$	H_2	$\dot{H}_2$	H_3	$\dot{H}_3$
1	4.9725	-0.0138	9.9142	-0.0430	15.1723	0.0864
2	4.9414	-0.0312	9.8560	-0.0584	15.4094	0.2373
3	4.9439	0.0041	9.8519	-0.0019	15.6754	0.2609
4	4.8810	-0.0636	9.7293	-0.1247	15.8250	0.1551
5	4.8308	-0.0490	9.6346	-0.0902	15.9691	0.1396

Epoch	s_{H_1}	$s_{\dot{H}_1}$	s_{H_2}	$s_{\dot{H}_2}$	s_{H_3}	$s_{\dot{H}_3}$
1	0.0141	0.7113	0.0200	0.7113	0.0245	0.7113
2	0.0141	0.0611	0.0200	0.0643	0.0245	0.0673
3	0.0138	0.0591	0.0193	0.0624	0.0236	0.0659
4	0.0138	0.0590	0.0192	0.0622	0.0235	0.0657
5	0.0138	0.0590	0.0192	0.0622	0.0235	0.0657





Task 2 (6 points)

Solution

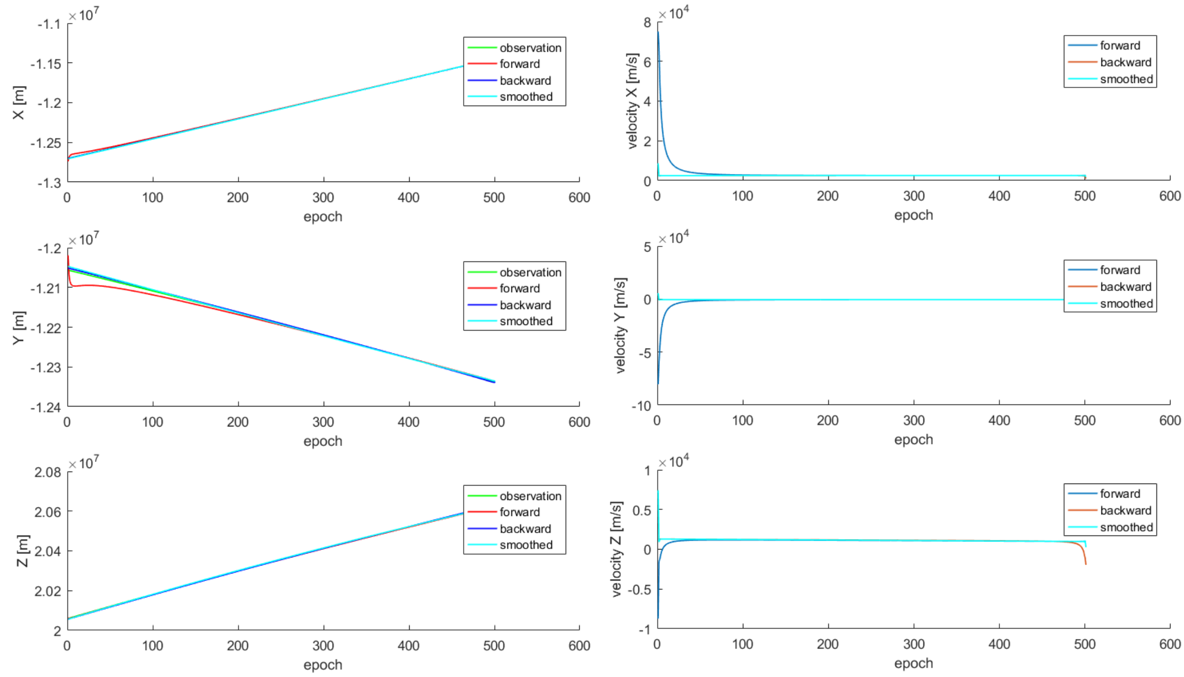
Φ and Q can be derived from the "cook book recipe" (see lecture slides V0 05) but have to be computed in every epoch since a depends on the position of the satellite.

Design matrix (order of columns: X, Y, Z v_X, v_Y, v_Z)

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

For the backward filtering one can use the state vector of the last epoch from the forward filtering as an initial state vector and the initial covariance matrix as written in the task.

Position and velocity of satellite:



Standard deviation (position of satellite):