Exercise on 18.06.2019

Task 1 (4 points)

In the first assignment (on 23.04.2019) you computed the altitude of the three drones H_1, H_2, and H_3 above the reference drone height for several epochs by using sequential adjustment. You found out that the heights of the drones above the reference drone are not stable over time, but show a time-dependent drift.

Now your task is to take the drift into account by Kalman fitlering instead of sequential adjustment. Assume that the three heights can be represented by an integrated random walk, i.e.

$$\begin{aligned}\ddot{H}_1 &= 0 + w(t)\\ \ddot{H}_2 &= 0 + w(t)\\ \ddot{H}_3 &= 0 + w(t).\end{aligned}$$

The following measurements (identical to assignment 1) are provided:

Epoch	$\Delta h_{R,1}$	$\Delta h_{1,2}$	$\Delta h_{2,3}$	$\Delta h_{3,2}$	$\Delta h_{2,1}$	$\Delta h_{1,R}$
1	4.96	4.93	5.25	-5.26	-4.96	-4.99
2	4.96	4.94	5.56	-5.55	-4.89	-4.92
3	4.97	4.91	5.84	-5.81	-4.91	-4.92
4	4.86	4.83	6.09	-6.09	-4.86	-4.89
5	4.85	4.81	6.33	-6.34	-4.80	-4.82

where

$$\Delta h_{i,j} = H_j - H_i$$

is the height difference between drone i and j. The covariance matrix of the state for the first epoch is given by

$$\Sigma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The process noise variance is $\sigma^2 = 0.01$ and the standard deviation of the measurement noise $\sigma = 0.02$. For the first epoch the state parameters

$$\begin{bmatrix} H_1 \\ \dot{H}_1 \\ H_2 \\ \dot{H}_2 \\ H_3 \\ \dot{H}_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 10 \\ 0 \\ 15 \\ 0 \end{bmatrix}$$

can be used.

In addition, compute the standard deviation of the parameters and plot your results.

Task 2 (6 points)

A satellite flying in the gravity field of the Earth follows Newton's law of universal gravitation i.e.

$$\frac{d^2}{dt^2}\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{-GM}{(x^2+y^2+z^2)^{3/2}}\begin{bmatrix}x\\y\\z\end{bmatrix} + \begin{bmatrix}n_x\\n_y\\n_z\end{bmatrix}$$

which can be rearranged to

$$\frac{d}{dt}\begin{bmatrix}x\\y\\z\\v_x\\v_y\\v_z\end{bmatrix} = \begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\a&0&0&0&0&0\\0&a&0&0&0&0\\0&0&a&0&0&0\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\\v_x\\v_y\\v_z\end{bmatrix} + \begin{bmatrix}0\\0\\0\\n_x\\n_y\\n_z\end{bmatrix}$$

where

$$a = \frac{-GM}{(x^2+y^2+z^2)^{3/2}} \quad \text{and} \quad \begin{bmatrix}v_x\\v_y\\v_z\end{bmatrix} = \begin{bmatrix}\dot{x}\\\dot{y}\\\dot{z}\end{bmatrix}, \quad GM = 3.986005\cdot 10^{14}.$$

Compute the position and velocity of the satellite by Kalman filtering. Observations of the satellite position (x,y,z) are provided in the file `exampleKF2.txt` for each epoch recorded at 1 Hz and having a measurement noise of 50 m. You can obtain the matrix of the process noise Q together with the state transition matrix by applying the ”cook book recipe”. The velocity noise n_i is assumed to be 30 m/sec. The state vector and the covariance matrix of the state for the first epoch are given as

$$\boldsymbol{x}_0 = \begin{bmatrix}-12823317\\-11933101\\20070042\\2000\\-1000\\1000\end{bmatrix} \quad \text{and} \quad \boldsymbol{\Sigma}_0 = \begin{bmatrix}10&0&0&0&0&0\\0&10&0&0&0&0\\0&0&10&0&0&0\\0&0&0&100&0&0\\0&0&0&0&100&0\\0&0&0&0&0&100\end{bmatrix}$$

Compute the forward, backward and smoothed Kalman filter solution and plot your results.

Hint 1: In order to compute the elements a in the $\mathbf{F}$ matrix, you need the position of the satellite, i.e. its coordinates x, y, z. The position at the previous epoch is a good enough approximation for that, since you are updating your parameters every second!

Hint 2: The $\mathbf{G}$ vector is of the form $\mathbf{G} = \sigma_n \cdot [0\,0\,0\,1\,1\,1]^T$ where σ_n is the velocity noise that drives the uncertainty of your prediction.