University of Stuttgart
Institute of Geodesy

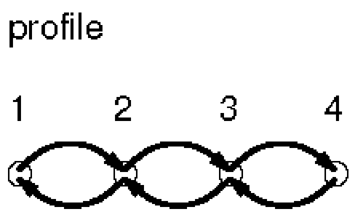
Physical Geodesy

Gravimetry network

GIS

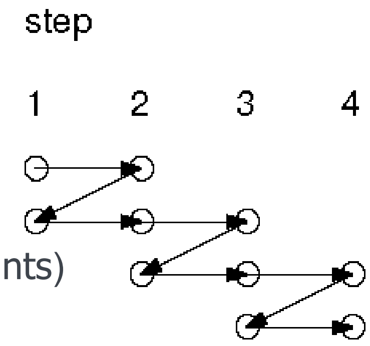
Gravimetry

Relative gravity measurement procedure



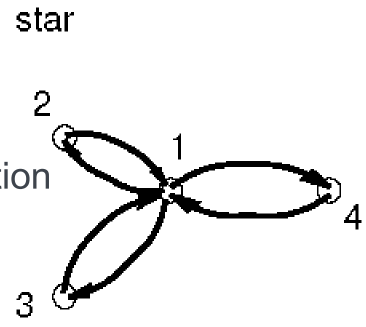
i. Profile

- ✓ Each point is measured twice (except for the end points)
- ✓ Provides various time difference between measurements of the same point
- ✓ Good for drift estimation
- ✓ Recommended for precise networks



ii. Step

- ✓ Each point is measured three times (except for the end points)
- ✓ Revisit time differences are short
- ✓ Ideal when drift is non-linear



iii. Star

- ✓ Measurements of the central point are used for drift estimation
- ✓ Other points are unreliable
- ✓ Prone to hidden systematic errors

Gravimetry

Relative gravity observation equation

- Gravity measurement of point n at time t_k

$$y_n(t_k) = g_n + b + dt_k + \epsilon$$

  - ✓ g_n: gravity at point n
  - ✓ b: bias
  - ✓ d: drift (linear)
  - ✓ ϵ: noise

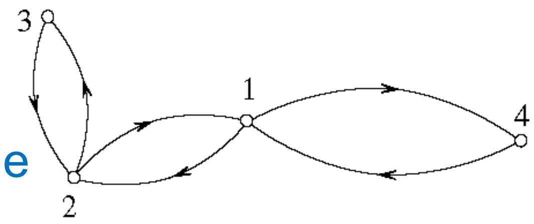
- Observation equation for measurements of points n & m at times t_k & t_l

$$\begin{cases} y_n(t_k) = g_n + b + dt_k + \epsilon \\ y_m(t_l) = g_m + b + dt_l + \epsilon \end{cases} \Rightarrow \Delta y_{nm}(t_{kl}) = \Delta g_{nm} + d\Delta t_{kl} + \epsilon$$

  - ✓ $\Delta g_{nm} = g_m - g_n$
  - ✓ $\Delta t_{kl} = t_l - t_k$

Gravimetry

Relative gravity observation equation: Example



- Sequence

$$1_{t_1}^{y_1} \longrightarrow 2_{t_2}^{y_2} \longrightarrow 3_{t_3}^{y_3} \longrightarrow 2_{t_4}^{y_4} \longrightarrow 1_{t_5}^{y_5} \longrightarrow 4_{t_6}^{y_6} \longrightarrow 1_{t_7}^{y_7}$$

- Equations

$$\begin{cases} \Delta y_{12}(t_{12}) = \Delta g_{12} + d\Delta t_{12} \\ \Delta y_{23}(t_{23}) = \Delta g_{23} + d\Delta t_{23} \\ \Delta y_{34}(t_{34}) = \Delta g_{32} + d\Delta t_{34} \\ \Delta y_{45}(t_{45}) = \Delta g_{21} + d\Delta t_{45} \\ \Delta y_{56}(t_{56}) = \Delta g_{14} + d\Delta t_{56} \\ \Delta y_{67}(t_{67}) = \Delta g_{41} + d\Delta t_{67} \end{cases} \xRightarrow{\begin{cases} \Delta g_{32} = -\Delta g_{23} \\ \Delta g_{21} = -\Delta g_{12} \\ \Delta g_{41} = -\Delta g_{14} \end{cases}} \begin{cases} \Delta y_{12}(t_{12}) = \Delta g_{12} + d\Delta t_{12} \\ \Delta y_{23}(t_{23}) = \Delta g_{23} + d\Delta t_{23} \\ \Delta y_{34}(t_{34}) = -\Delta g_{23} + d\Delta t_{34} \\ \Delta y_{45}(t_{45}) = -\Delta g_{12} + d\Delta t_{45} \\ \Delta y_{56}(t_{56}) = \Delta g_{14} + d\Delta t_{56} \\ \Delta y_{67}(t_{67}) = -\Delta g_{14} + d\Delta t_{67} \end{cases}$$

- Linear system

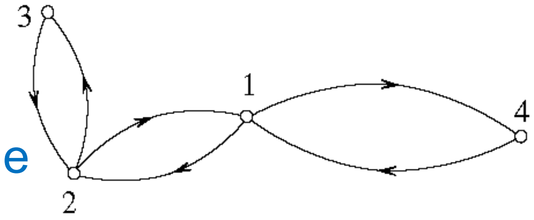
$$\underbrace{\begin{bmatrix} \Delta y_{12}(t_{12}) \\ \Delta y_{23}(t_{23}) \\ \Delta y_{34}(t_{34}) \\ \Delta y_{45}(t_{45}) \\ \Delta y_{56}(t_{56}) \\ \Delta y_{67}(t_{67}) \end{bmatrix}}_{\boldsymbol{l}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \Delta t_{12} \\ 0 & 0 & 1 & \Delta t_{23} \\ 0 & 0 & -1 & \Delta t_{34} \\ -1 & 0 & 0 & \Delta t_{45} \\ 0 & 1 & 0 & \Delta t_{56} \\ 0 & -1 & 0 & \Delta t_{67} \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} \Delta g_{12} \\ \Delta g_{14} \\ \Delta g_{23} \\ d \end{bmatrix}}_{\boldsymbol{x}}$$

- Solution

$$\boldsymbol{x} = \left(\boldsymbol{A}^T \boldsymbol{A}\right)^{-1} \boldsymbol{A}^T \boldsymbol{l}$$

Gravimetry

Relative gravity observation equation: Example



- Sequence

$$1_{t_1}^{y_1} \longrightarrow 2_{t_2}^{y_2} \longrightarrow 3_{t_3}^{y_3} \longrightarrow 2_{t_4}^{y_4} \longrightarrow 1_{t_5}^{y_5} \longrightarrow 4_{t_6}^{y_6} \longrightarrow 1_{t_7}^{y_7}$$

- Linear system

$$\underbrace{\begin{bmatrix} \Delta y_{12}(t_{12}) \\ \Delta y_{23}(t_{23}) \\ \Delta y_{34}(t_{34}) \\ \Delta y_{45}(t_{45}) \\ \Delta y_{56}(t_{56}) \\ \Delta y_{67}(t_{67}) \end{bmatrix}}_{\boldsymbol{l}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \Delta t_{12} \\ 0 & 0 & 1 & \Delta t_{23} \\ 0 & 0 & -1 & \Delta t_{34} \\ -1 & 0 & 0 & \Delta t_{45} \\ 0 & 1 & 0 & \Delta t_{56} \\ 0 & -1 & 0 & \Delta t_{67} \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} \Delta g_{12} \\ \Delta g_{14} \\ \Delta g_{23} \\ d \end{bmatrix}}_{\boldsymbol{x}}$$

- Covariance matrix

$$\boldsymbol{C} = \operatorname{diag}\left(\left[\sigma^2_{\Delta y_{12}(t_{12})} \quad \sigma^2_{\Delta y_{23}(t_{23})} \quad \sigma^2_{\Delta y_{34}(t_{34})} \quad \sigma^2_{\Delta y_{45}(t_{45})} \quad \sigma^2_{\Delta y_{56}(t_{56})} \quad \sigma^2_{\Delta y_{67}(t_{67})}\right]\right)$$

$$\sigma^2_{\Delta y_{nm}(t_{kl})} = \sigma^2_{y_n(t_k)} + \sigma^2_{\Delta y_m(t_l)}$$

- Solution

$$\boldsymbol{x} = \left(\boldsymbol{A}^T \boldsymbol{C}^{-1} \boldsymbol{A}\right)^{-1} \boldsymbol{A}^T \boldsymbol{C}^{-1} \boldsymbol{l}$$

- Solution uncertainties

$$\boldsymbol{C}_{\boldsymbol{x}} = \left(\boldsymbol{A}^T \boldsymbol{C}^{-1} \boldsymbol{A}\right)^{-1}$$