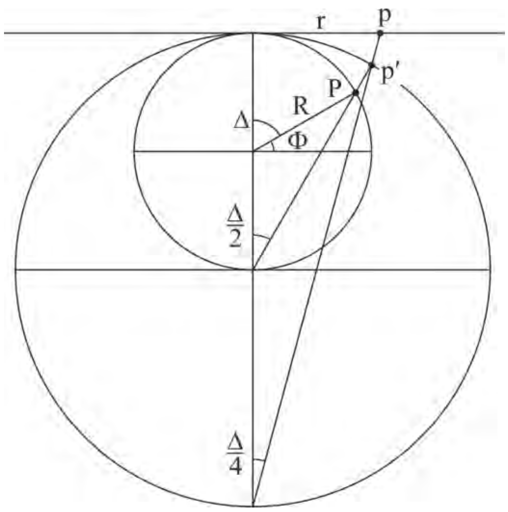
Exam Map Projections and Geodetic Coordinate Systems (SS 14)

Problem 1: (Map Projections)

The stereographic projection ("UPS") is a conformal azimuthal mapping for the polar regions. It complements the transverse Mercator projection ("UTM"), which is limited in latitude extension, e.g. $-80° \le \Phi \le 80°$. N. Solovyew has studied stereographic double projections on spheres with radii, which are a multiple of that of the reference sphere. The construction principle is e.g. so that the point $P(\Lambda, \Phi)$ of the reference sphere (radius R) is mapped stereographically onto point p' (on a sphere of radius 2R) and – in a second step – point p' is mapped stereographically onto point p of the plane, see figure.

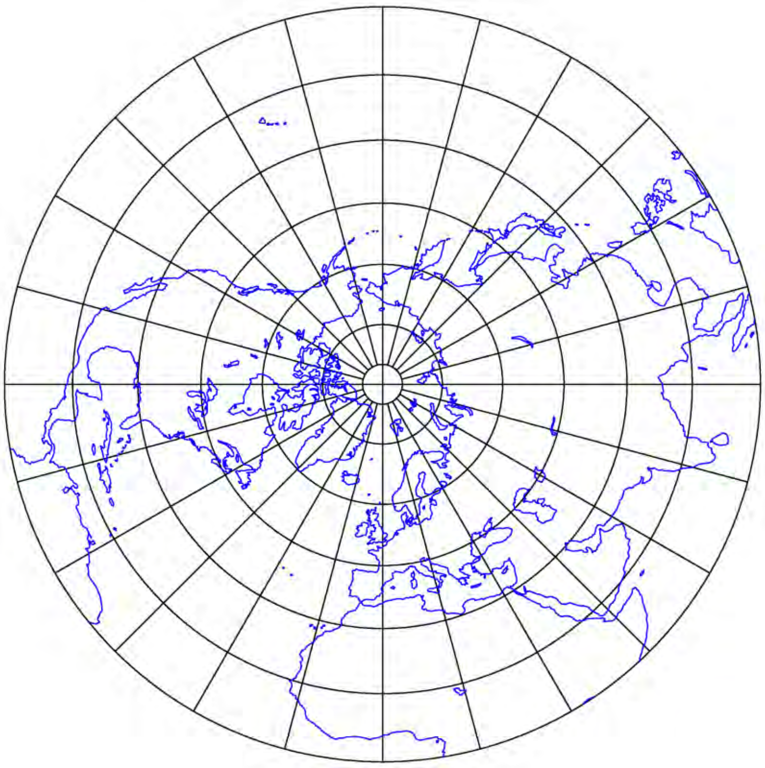


a) Derive the mapping equations for this kind of azimuthal stereographic double projection.

b) Compute the principle distortions Λ_1, Λ_2, and investigate if this mapping is also conformal.

c) Sketch the indicatrix (Tissot distortion ellipse) at selected locations Λ, Φ of the map below, so that the tendency of distortion becomes clear.

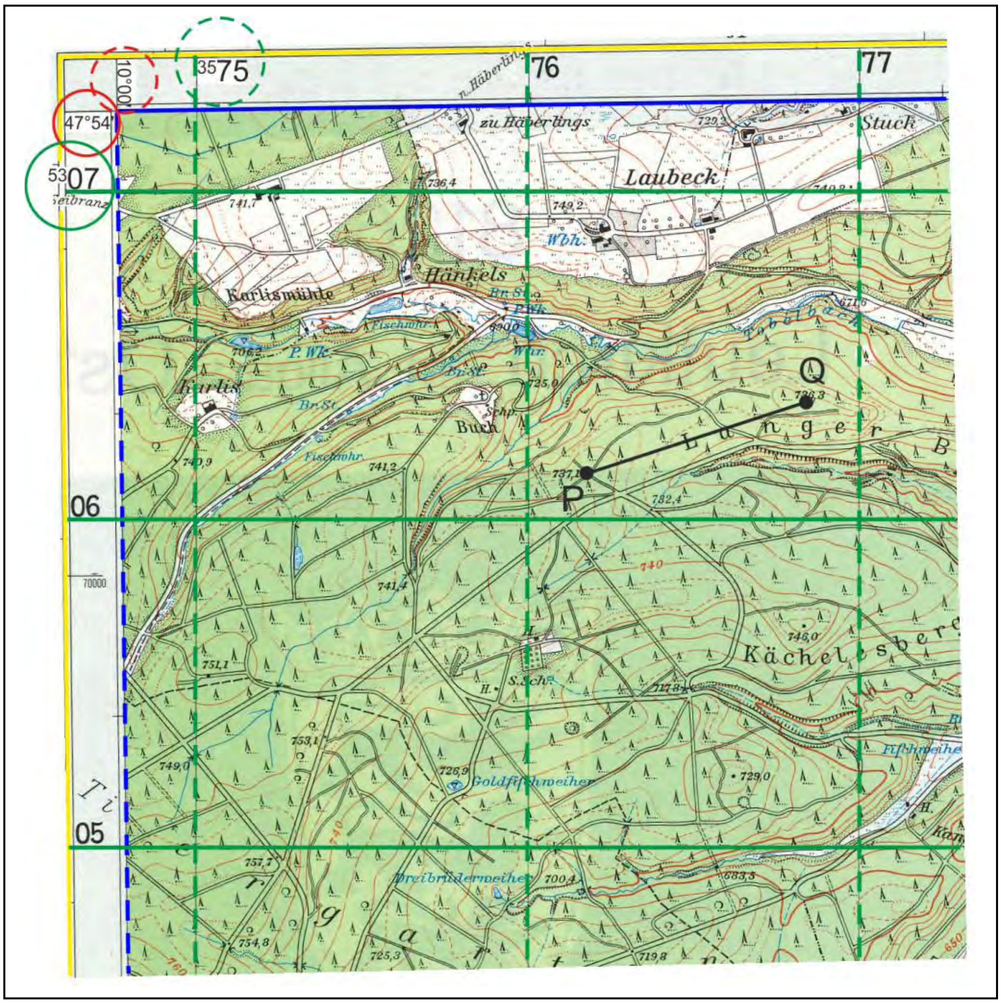
d) How must the mapping equations in a) be modified in order to map the parallel circle $\Phi_0 = \text{const} \ne 90°$ equidistantly ?



Problem 2: (Geodetic Coordinate Systems, Datum transformations, see WS 09/10, Problem 2)

Problem 3: (Geodetic Coordinate Systems, Datum transformations)

The picture below shows a small part of a topographic map, which was made by conformally mapping the ellipsoid-of-revolution onto a transverse cylinder using Gauß-Krüger coordinates. Apart from the frame lines (indicated in yellow), which are not important here, there is a set of (solid and dashed) blue and a set of (solid and dashed) green lines. Furthermore, two points P and Q are displayed together with their connecting line (in black). Answer the following questions and – if the answer is only 'yes' or 'no' – justify your answer.



a) What do the blue lines (solid, dashed) represent ?

b) If you would look at the entire map, are the blue lines straight ? If no, when are they ?

c) Are the blue lines orthogonal to each other ?

d) What do the green lines (solid, dashed) represent ?

e) If you would look at the entire map, are the green lines straight ?

f) Are the green lines orthogonal to each other ?

g) Is the set of blue lines orthogonal to the set of green lines ?

Problem 3 (continued):

h) Which lines (solid, dashed, blue, green) point north ?

i) What do the numbers in the red circles (solid, dashed) indicate ?

j) What do the numbers in the green circles (solid, dashed) indicate ?

k) How can the distance of the straight line PQ be found/computed ? (The distance is not to be measured by a ruler/set square !)

l) Does the distance of the straight line PQ coincide with the true (shortest) distance ("geodesic") PQ on the ellipsoid-of-revolution ?

m) How can the bearing T of the line PQ be found/computed ?

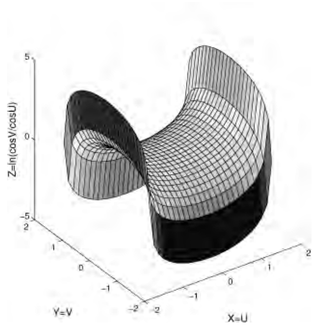
n) How can the north direction (north azimuth A) of the line PQ be found/computed ?

o) Where is north in point P ? Indicate it in the map by an arrow. Also plot the quantities from m) and n) which are required to find the north direction.

Exam Map Projections and Geodetic Coordinate Systems (SS 13)

Problem 1: (Map Projections)

The minimal surface by Scherk is defined by the equation



$$\mathbf{X}(U,V) = U\mathbf{E}_1 + V\mathbf{E}_2 + \ln\frac{\cos V}{\cos U}\mathbf{E}_3$$

$$-\frac{\pi}{2} + \varepsilon \le U,V \le \frac{\pi}{2} - \varepsilon\ ,\ 0 < \varepsilon \ll 1$$

a) Compute the Gauß' tangent vectors $\mathbf{G}_1$ and $\mathbf{G}_2$.

b) Determine the metric matrix $\underline{G}$.

c) Decide whether or not the manifold is orthogonally parameterized. Justify your decision.

Problem 2: (Map Projections)

Let us assume that the map projection used in Google Earth is a spherical perspective projection in the polar aspect, which maps a spherical cap onto a tangential plane attached to the North Pole. Here we want to treat the case of a secant plane, instead. The secant plane is parallel to the equatorial plane and intersects the sphere at a latitude $0 \ll \Phi_0 < 90°$.

a) Derive the corresponding mapping equations for a sphere of radius R with the perspective centre at height H above the North Pole.

b) Specify the principal distortions Λ_1 and Λ_2.
Hint: Initially, the radius r (in the chart) should be taken as a function of polar distances (co-latitudes) $\Delta = 90° - \Phi, \Delta_0 = 90° - \Phi_0$.

c) Which parallel circles are mapped equidistantly ?

d) Which points/lines are mapped isometrically, i.e. free from any distortion ?

Problem 3: (Geodetic Coordinate Systems, Datum transformations)

a) Set up the model of the 7-parameter transformation in order to transform 3D Cartesian co-ordinates X,Y,Z referring to a set of base vectors $\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3$ into 3D Cartesian coordinates x,y,z referring to base vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$.

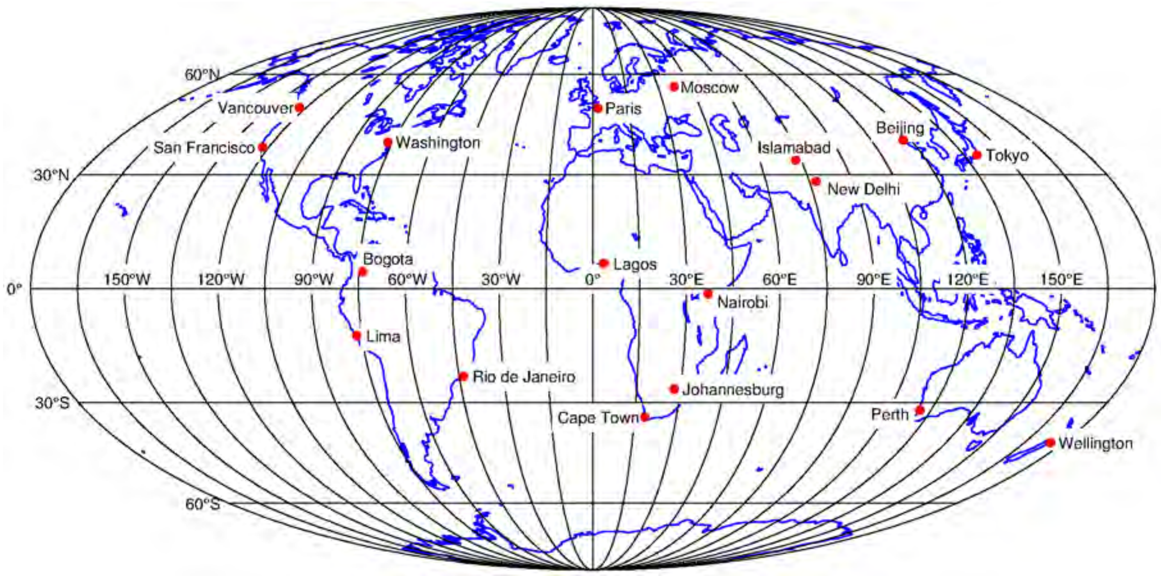
b) Explain accurately the elements of the model, both graphically and in words.

c) What is meant by "transformation close to the identity" ?

Problem 4: (Geodetic Coordinate Systems, Datum Transformations)

a) Which cities from the map below have the following UTM-coordinates ?

	Point 1	Point 2
False Easting	631 535.88 m	318 473.70 m
Northing	485 137.03 m	3 772 859.59 m
Zone	18	43



b) Specify the structure of the two equations which are used in the transformation of isometric coordinates L and Q (referring to a local origin with isometric coordinates L_0 and Q_0) of the ellipsoid into conformal coordinates x and y of the map.

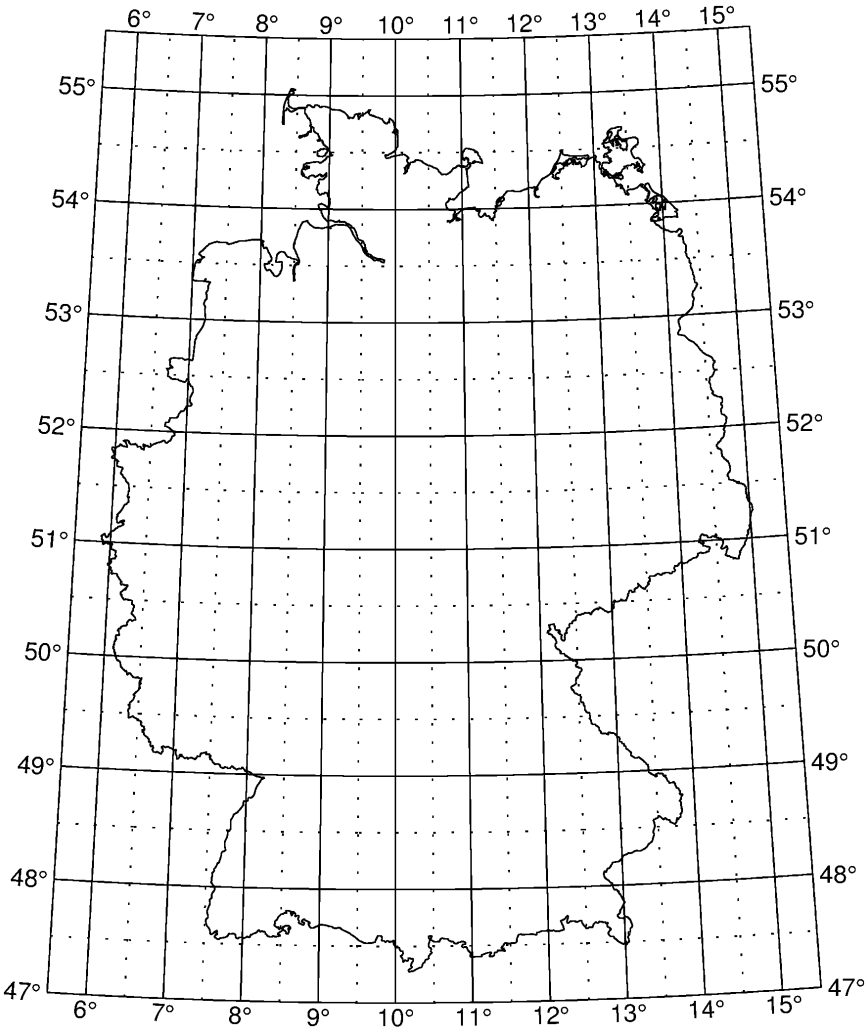
c) As a result from bivariate or univariate conformal series, conformal coordinates x, y are given. How are UTM-coordinates (False Easting E, Northing N) established from x, y ? Which additional information is needed in order to uniquely georeference a point ?

d) Explain the meaning/usage of meridian convergence c (e.g. in connection with UTM-coordinates) both in an equation and in a picture.

e) Which of the points with given Gauß-Krüger coordinates are located inside Germany ?

Plot them (together with their numbers) in the map

Point	False Easting [m] (Rechtswert)	Northing [m] (Hochwert)
1	1492682.821	5429632.451
2	2492984.006	5654871.695
3	3603587.143	5736592.266
4	4516687.181	5902471.701
5	4558097.419	5096359.461
6	5465090.211	5680017.185
7	6480563.795	6041612.280



Exam Map Projections and Geodetic Coordinate Systems (SS 12)

Problem 1: (Map Projections)

The Austrian geodesist Kurt Bretterbauer (1929-2009) has done quite a lot contributions to world map projections. As an example he has invented an exponential map projection (see figure) the mapping equations of which are

$$x = R\left(e^{\Lambda} - e^{-\Lambda}\right)\cos\Theta$$

$$y = R\left(e^{\Lambda} + e^{-\Lambda}\right)\sin\Theta$$

where $\Theta = \ln\tan\left(\frac{\pi}{4} + \frac{\Phi}{2}\right)$ is the well-known Mercator latitude. For the reason that $\lim_{|\Phi|\to 90°} \Theta = \infty$ the extension in latitude is limited, cp. figure

Prove whether or not the mapping is conformal.

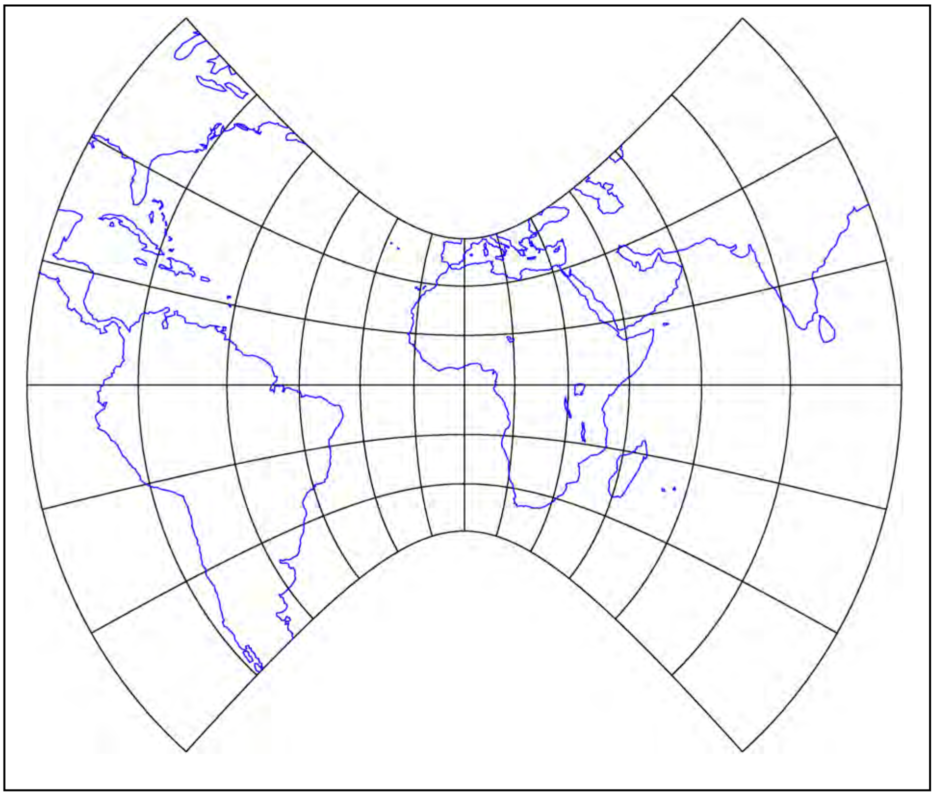


Figure: Exponentialabbildung nach K. Bretterbauer (2002): Die runde Erde eben dargestellt. Abbildungslehre und sphärische Kartennetzentwürfe. Geowissenschaftliche Mitteilungen 59, S. 82, Studienrichtung Vermessung und Geoinformation, TU Wien

Exam Map Projections and Geodetic Coordinate Systems (SS 12)

Problem 2: (Map Projections)

a) Explain the meaning of the distortion measures (i) $\Lambda_1 = \sqrt{C_{11}/G_{11}}$, $\Lambda_2 = \sqrt{C_{22}/G_{22}}$, (ii) $\Lambda_1 = 1, 0 \le \Lambda_2 < \infty$, (iii) $0 \le \Lambda_1 < \infty, \Lambda_2 = 1$ und (iv) $\Lambda_1 = \Lambda_2 = 1$?

b) How are the general mapping equations for (i) azimuthal mappings, (ii) cylindrical mappings and (iii) conical mappings ?

c) Explain the mathematical concept of how oblique spherical map projections may be established. Draw a figure on the surface of a sphere, where all given and unknown quantities are plotted and named. (It is not required to write down the detailed transformation equation, which might be derived from spherical trigonometry)

Problem 3: (Geodetic Coordinate Systems, Conformal Coordinates)

a) Explain the meaning/usage of meridian convergence both in words and in a picture.

b) For two points P_1,P_2 Gauß-Krüger-coordinates, for a third point P_3 UTM-coordinates are given.

	False Easting [m]	Northing [m]	Zone
P_1:	4 420 000,00	5 301 912,12	-
P_2:	3 512 129,04	5 500 000,00	-
P_3:	534 012,45	4 989 991,34	33

Describe the position of the points on the GRS80-ellipsoid.

c) Conformal map coordinates x, y of three points have been determined by evaluating the bivariate conformal polynomials $x(\ell, q)$ and $y(\ell, q)$. Compute Gauss-Krüger coordinates (H, R) for P_1 and UTM coordinates (N, E) for P_2 and P_3 under the condition that the corresponding reference point P_0 is located on the meridian L_0 and 5400 km north of the equator.

	y [m]	x [m]	L_0
P_1:	229 741,67	128 844,77	6°E
P_2:	-128 844,77	58 123,55	15°E
P_3:	0,00	0,00	3°E

Problem 4: (Geodetic Coordinate Systems, Datum Transformations)

A Cartesian planar coordinate transformation is defined through the equations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \omega & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

Show that this is not a conformal mapping.

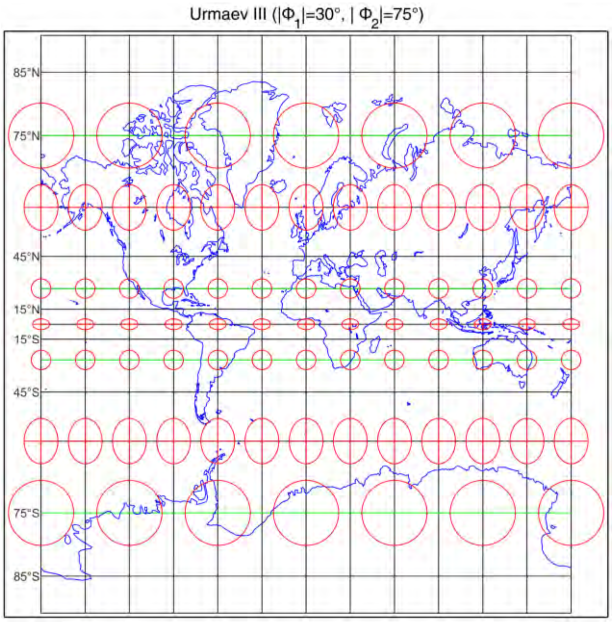
Hint: The general expression for the extremal distortions of map projections is given by $\Lambda_{1,2}^2 = \frac{1}{2}\left\{ \mathrm{tr}(\underline{C}\underline{G}^{-1}) \pm \sqrt{[\mathrm{tr}(\underline{C}\underline{G}^{-1})]^2 - 4\det(\underline{C}\underline{G}^{-1})} \right\}$.

Exam Map Projections and Geodetic Coordinate Systems (SS 11)

Problem 1: (Map Projections)

Given the Urmaev-III general mapping equations

$$\begin{bmatrix} x \\ y \end{bmatrix} = R \begin{bmatrix} \Lambda \\ a_0\Phi + \frac{a_2}{3}\Phi^3 \end{bmatrix} \quad , \quad a_0, a_2 = \text{const.}$$



a) determine the numerical values of the constants a_0, a_2 so as to make the parallel circle images $\Phi_1 = \pm 30°$ and $\Phi_2 = \pm 75°$ free from any angular distortion (conformality on $\Phi_1 = \pm 30°$ and $\Phi_2 = \pm 75°$).

b) Sketch the Tissot distortion ellipses at $\Phi = \pm 85°$.

c) formulate the constants a_0, a_2 as functions of latitudes Φ_1 and Φ_2 for an equivalent mapping (equal area mapping).

Problem 2: (Map Projections)

In order to map countries with distinct East-West extension normal cylindrical or normal conical mappings with two equidistant parallel circles (secant cylinder/secant cone) are well suited. Given the general mapping equations of a cylinder secant at $\Phi = \pm\Phi_0$, i.e.

$$x = R\Lambda\cos\Phi_0 \; , \; y = f(\Phi)$$

find unknown function $f(\Phi)$ for a mapping with equidistant meridians.

Exam Map Projections and Geodetic Coordinate Systems (SS 11)

Problem 3: (Geodetic Coordinate Systems, Datum Transformations)

a) Set up the 2D datum transformation model for a similarity transformation (4 parameter transformation).
b) Comment on the geometric meaning of the transformation parameters, on the number of control points necessary to estimate them and on the procedure how they will be estimated from the model equations.
c) Set up the 2D datum transformation model for an affine transformation (6 parameter transformation)
d) Comment on the geometric meaning of the transformation parameters, on the number of control points necessary to estimate them and on the procedure how they will be estimated from the model equations.

Problem 4: (Geodetic Coordinate Systems, Conformal Coordinates)

Give a detailed description of Gauß-Krüger-/UTM-coordinates:

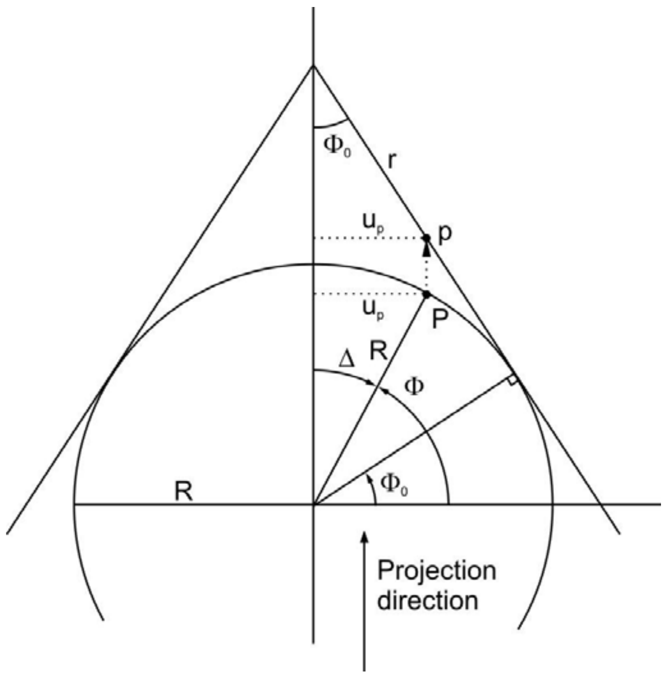
a) What is the famous relevance of using Gauß-Krüger-/UTM-coordinates ?
b) Describe detailed how Gauß-Krüger coordinates of a point P are generated from a set of conformal coordinates x,y which refer to a certain "Taylor point" P_0
c) Explain the difference between Gauß-Krüger-coordinates and UTM-coordinates.

Exam Map Projections and Geodetic Coordinate Systems (SS 10)

Problem 1: (Map Projections)

a) Specify the **general mapping equations** $x = x(\Phi, \Lambda), y = y(\Phi, \Lambda)$ for a conical mapping in the normal aspect which touches the sphere of radius R at $\Phi = \Phi_0$ (circle of contact).

b) Develop the mapping equations $x = x(\Phi, \Lambda), y = y(\Phi, \Lambda)$ for the special case of a projective mapping, where the projection centre is located at infinity in direction of the Earth rotation axis (cp. figure)



c) Compute analytically the distortions of the mapping in form of the semi axes of Tissot distortion ellipses.

d) Prepare a picture of the chart with selected parameter lines of the northern hemisphere $(0 \le \Phi \le \pi/2, -\pi \le \Lambda \le \pi)$ and properly graph a collection of distortion ellipses so as to show realistically the distortion characteristics. In addition, describe the distortion characteristics in words.

Problem 2: (Map Projections)

Assume you would like to map a surface S_1 in 3D-space onto a surface S_2 in 3D-space, and you are interested in the resulting deformations/distortions in terms of Tissot distortion ellipse elements.

a) Which steps are necessary to do both, mapping and distortion analysis ? Which quantities have to be computed in order to judge the quality of the mapping ?

b) How would you mathematically/analytically describe the map characteristics "Conformality", "Equivalence" and "Equidistance" ?

c) Find out whether or not the parameter lines U and V of the "Elliptic Cone: $\mathbf{X}(U,V) = AV\cos U\mathbf{E}_1 + BV\sin U\mathbf{E}_2 + CV\mathbf{E}_3 \quad (A \neq B, C = \text{const})$ " intersect orthogonally.

Exam Map Projections and Geodetic Coordinate Systems (SS 10)

Problem 3: (Geodetic Coordinate Systems, Datum transformations)

a) Set up the model of the 7-parameter transformation in order to transform 3D Cartesian coordinates X,Y,Z referring to a set of base vectors $\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3$ into 3D Cartesian coordinates x,y,z referring to base vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$.

b) Draw a meaningful picture and explain accurately the elements of the model

c) What is meant by "transformation close to the identity" ?

Problem 4: (Geodetic Coordinate Systems, Conformal Coordinates)

Give a detailed description of Gauß-Krüger-/UTM-coordinates:

a) What is the famous relevance of using Gauß-Krüger-/UTM-coordinates ?

b) Describe detailed how Gauß-Krüger coordinates of a point P are generated from a set of conformal coordinates x,y which refer to a certain "Taylor point" P_0

c) Explain the difference between Gauß-Krüger-coordinates and UTM-coordinates.

Exam Map Projections and Geodetic Coordinate Systems (WS 09/10)

Problem 1: (Map Projections)

The stereographic Gall-Projection of the sphere of radius R as shown below is generated by the mapping equations $x = \frac{R\Lambda}{\sqrt{2}}, \quad y = R\left(1 + \frac{1}{\sqrt{2}}\right)\tan\frac{\Phi}{2}$.

a) How is the metric matrix $\underline{G}$ of the sphere of radius R ?

b) How is the metric matrix $\underline{g}$ of the map ?

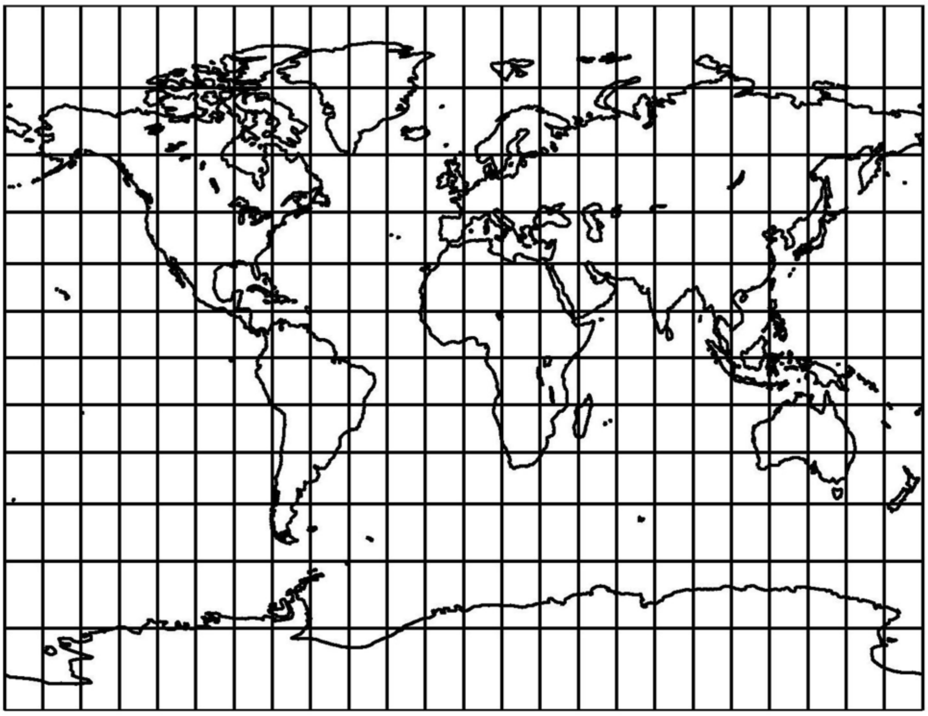
c) Derive the Cauchy-Green deformation matrix $\underline{C}$ from the mapping equations.

d) Do the principal distortions (principal stretches) show up in direction of the parameter line images ? Justify your answer !

e) Do the mapping equations generate an area preserving mapping (equal area or equivalent mapping) ?

f) Which parameter line images are free from any distortion ?

Stereographic Gall-Projection



Exam Map Projections and Geodetic Coordinate Systems (WS 09/10)

Problem 2: (Geodetic Coordinate Systems, Datum transformations)

In order to perform a simple planar coordinate transformation the following model equations have been set up

$$u = U\, m_u \cos\beta - V\, m_v \sin\alpha + t_u$$
$$v = V\, m_v \cos\alpha + U\, m_u \sin\beta + t_v$$

a) Specify the explicit equations so as to determine the unknown transformation parameters m_u, m_v, α, β, t_u and t_v from the coordinates of control points P_1-P_3.

b) Compute the unknown parameters using the given data.

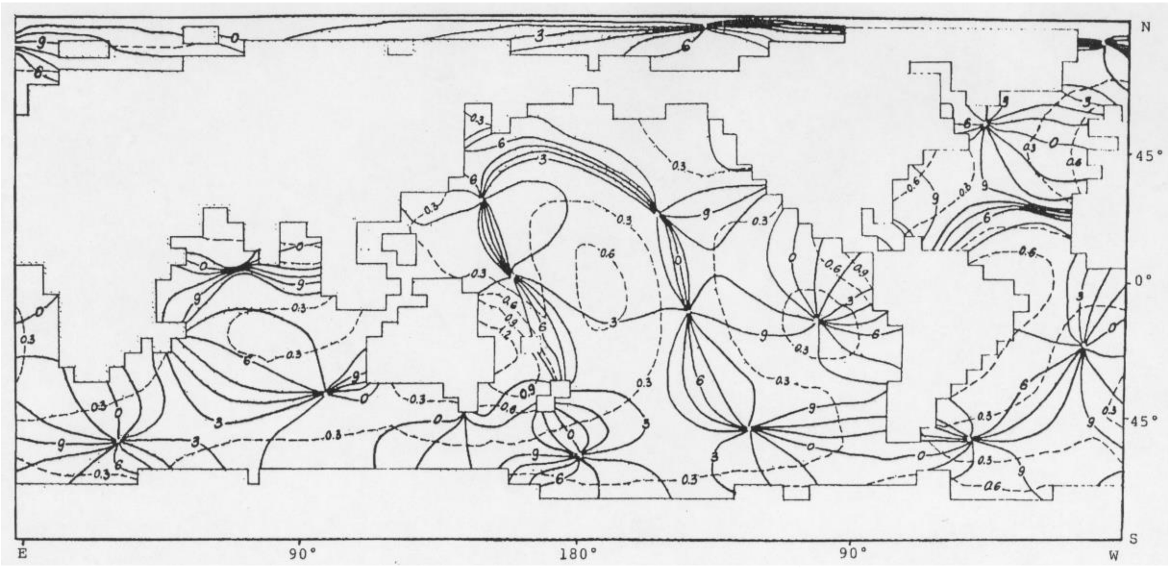
c) Determine the coordinates u_4,v_4 for given U_4,V_4

	U	V	u	v
P_1	7	5	7	-1
P_2	5	7	5	1
P_3	4	4	6	-2
P_4	6	6	?	?

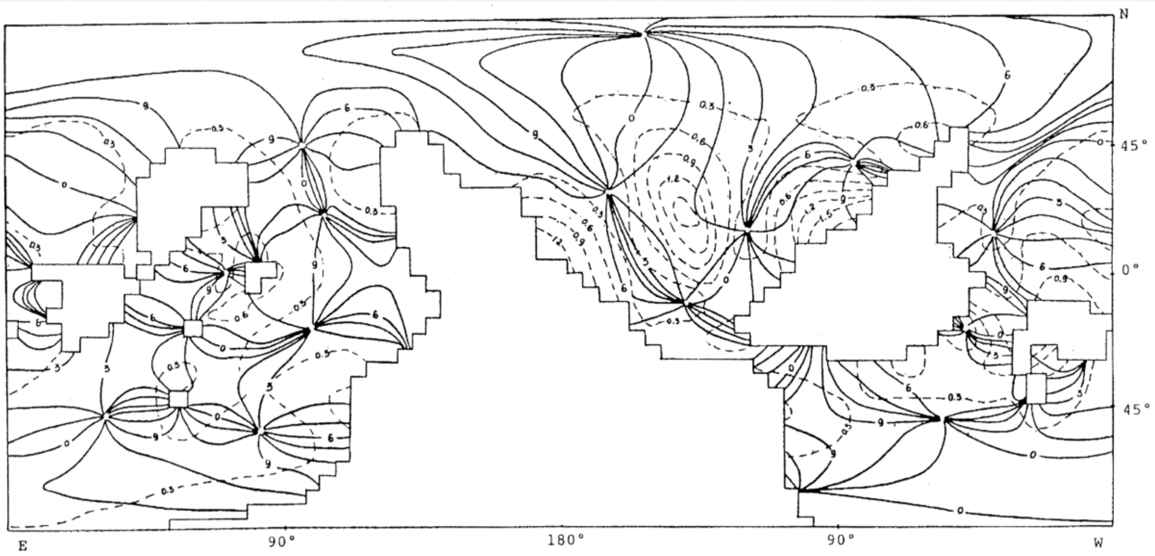
Exam Map Projections and Geodetic Coordinate Systems (SS09)

Problem 1: (Map Projections)

In order to determine the continental-to-ocean area ratio and to investigate the changes of tectonic activities on a geological time scale, paleontologists often use paleogeographical maps of different geological epochs (see examples below).



Present



Early Cambrian
(~ 566 Ma before present)

a) It is often assumed that above maps show the Earth in a Plate Carrée projection (or isoparametric mapping). Prove that this is not correct.

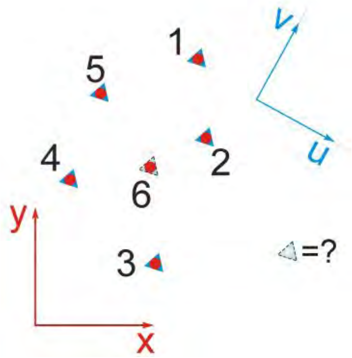
b) Explain why – for the purpose of the determination of the continental-to-ocean area ratio – the Plate Carrée projection would be the wrong choice. Prove your answer !

c) If a Plate-Carrée projection is not proper, but you would still like to use a normal cylindrical mapping for the above scientific questions, which mapping (equations) should be applied ? Derive the equations !

Exam Map Projections and Geodetic Coordinate Systems (SS09)

Problem 2: (Geodetic Coordinate Systems, Datum Transformations)

a) Assume a two dimensional geodetic network consisting of a set of points the coordinates of which are specified with respect to two different coordinate systems x,y="red" and u,v="blue" (different origins, different scales, different axes orientation). Assume further that an additional point is equipped with coordinates in the "red" x-y-system, only.



b) Make two different proposals (specifying two different datum transformation models) how to transform the additional point from the x-y-system into the u-v-system thus computing its "blue" coordinates.

c) In three dimensional datum transformation models three different rotation matrices are involved. Specify the matrices and explain the term "mathematically positive rotation".

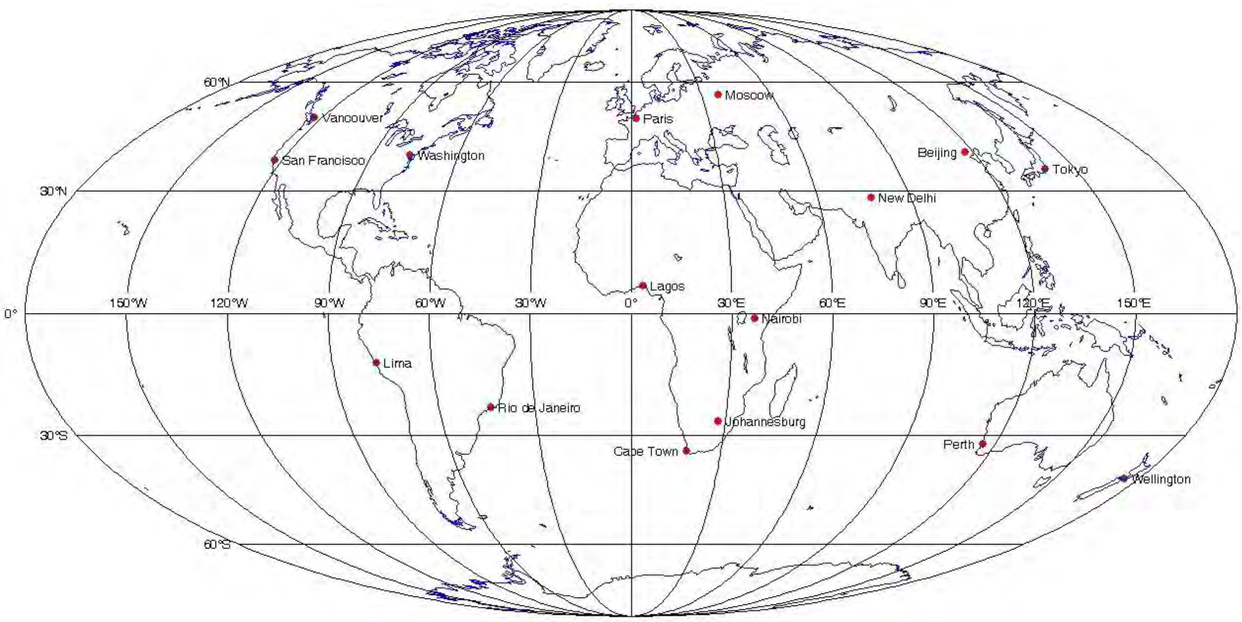
d) Which of the points as shown on the map (see below) has UTM-coordinates

"False Easting" E = 729 741.67 m

"False Northing" N = 3 128 844.77 m

Zone = 43.

Explain the numbers and give reasons for your choice!



Exam Map Projections and Geodetic Coordinate Systems (SS08)

Problem 1: (Map Projections)

Most likely in 1862 P. Foucaut published for the first time a map projection which was composed of the weighted arithmetical average of a Lambert cylindrical and a sinusoidal mapping of the sphere (radius R). Following the usual classification of map projections it is therefore called a pseudo cylindrical mapping. The mapping equations are

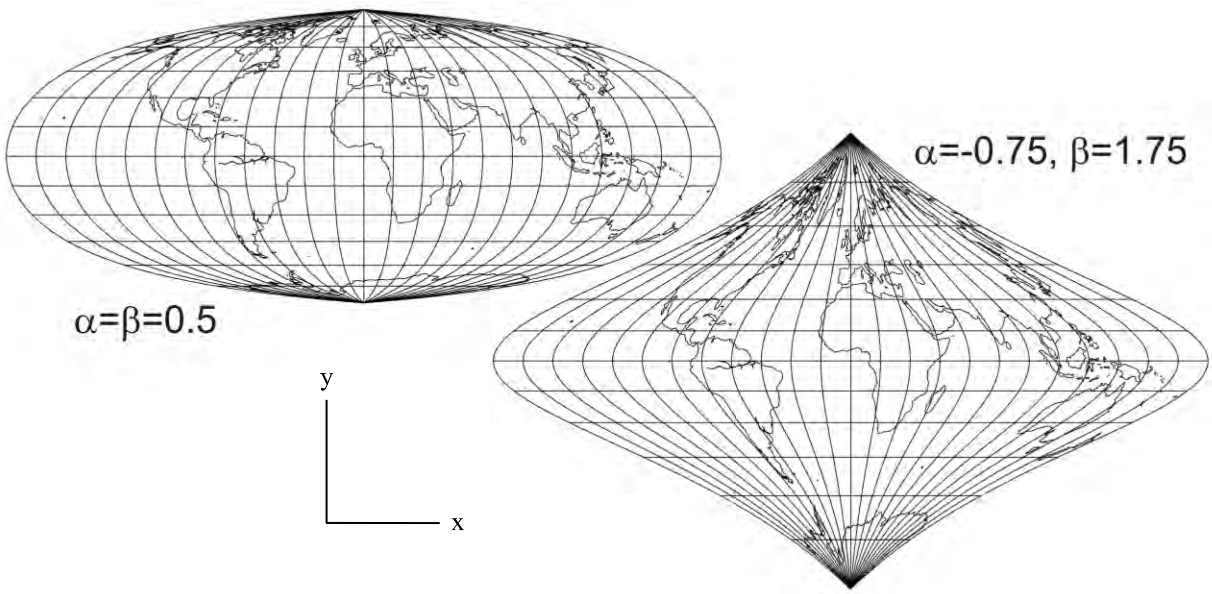
$$x = \frac{R\Lambda\cos\Phi}{\beta + \alpha\cos\Phi}, \quad y = R(\beta\Phi + \alpha\sin\Phi) \qquad \text{with weights } \alpha \text{ and } \beta.$$

a) Do the extremal distortions appear along the parameter lines ? If not, for which weights α and β is this the case ? Prove your answer !

b) Do the mapping equations guarantee an equal-area mapping ? Prove your answer !

c) Show for two different cases (c_1) $\alpha = 1-\beta$ and (c_2) $\alpha = 0$, $\beta = 1$, along which typical parameter lines conformality is achieved.

Hint: Avoid using the time-consuming explicit computation of the extremal distortions Λ_1, Λ_2. Use instead well-known rules for the computation of determinants.



Exam Map Projections and Geodetic Coordinate Systems (SS08)

Problem 2: (Geodetic Coordinate Systems, Datum Transformations)

a) Describe how conformal coordinates (Gauß-Krüger / UTM-coordinates) are generated from ellipsoidal longitude L and ellipsoidal latitude B.

b) Explain how the location of the point P on the ellipsoid-of-revolution can be deduced from UTM-coordinates

"False Easting" E = 534 012,45 m

"False Northing" N = 4 989 991,34 m

Zone = 33.

c) Sketch out a commutative diagram for datum transformations between six different sets: two-dimensional Cartesian chart-coordinates (x_1,y_1; x_2,y_2), three-dimensional Cartesian coordinates (X_1,Y_1,Z_1; X_2,Y_2,Z_2) and three-dimensional curvilinear coordinates (L_1,B_1,H_1; L_2,B_2,H_2). Describe succinctly the relations between the knots.

d) Model the datum transformation between two sets of three-dimensional Cartesian coordinates (X_1,Y_1,Z_1; X_2,Y_2,Z_2) using 3 translation parameters, 3 rotation parameters and 1 scale parameter.

Exam Map Projections and Geodetic Coordinate Systems (SS07)

Problem 1: (Map Projections)

The conformal projection of the sphere $\mathbb{S}^2_R$ onto a tangential plane $\mathbb{P}^2_O$ at the North Pole (Stereographic projection in the polar aspect) is parameterized by the mapping equations

$$\begin{bmatrix} x \\ y \end{bmatrix} = 2R\tan\left(\frac{\pi}{4} - \frac{\Phi}{2}\right)\begin{bmatrix} \cos\Lambda \\ \sin\ \Lambda \end{bmatrix}$$

with principal distortions (principal stretches)

$$\Lambda_1 = \Lambda_2 = \frac{1}{\cos^2\left(\frac{\pi}{4} - \frac{\Phi}{2}\right)}.$$

a) Determine the function $f(\Phi_0)$ by which both mapping equations have to be multiplied in order to map the parallel circle $\Phi = \Phi_0$ equidistantly, i.e. preserve its length in the chart.

b) Where is the position of the plane ?

c) Compute the radius of the image of the parallel circle, mapped equidistantly in this way.

Problem 2: (Geodetic Coordinate Systems, Datum Transformations)

In order to perform a simple planar coordinate transformation the following model equations have been set up

$$x = X\ m_x \cos\mu - Y\ m_y \sin\omega + c_1$$
$$y = Y\ m_y \cos\omega + X\ m_x \sin\mu + c_2$$

a) Specify the equations so as to determine the unknown transformation parameters m_x, m_y, μ, ω, c_1 and c_2 from the coordinates of control points P_1-P_3.

b) Compute the unknown parameters using the given data.

c) Determine the coordinates X_4,Y_4 given x_4,y_4

	X	Y	x	y
P_1	-1	7	5	7
P_2	1	5	7	5
P_3	-2	6	4	4
P_4	?	?	6	6