Lab 2: Kepler-Cartesian transformations and orbit computation

Aufgabe 1: Keplertransformation

A satellite evolving in a force field deriving from the earth gravitational potential $V(r) = GM/r$ follows a Keplerian orbit. Such orbits can be fully parameterized into 6 Keplerian elements $a, e, I, \Omega, \omega, M(t)$ and the law of motion thereof easily integrated. The objective of this lab is to compute the orbit of satellites whose initial position and velocity are given, with the help of Keplerian elements.

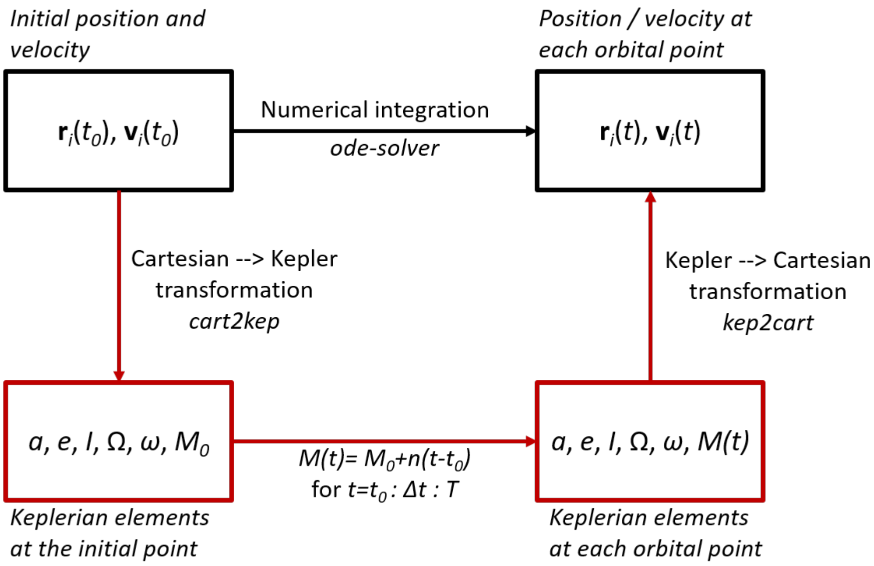

Abbildung 1: Overview of the numerical integration of non-perturbed Keplerian orbit with Keplerian elements.

- Select for the initial time $t_0 = 0\,\mathrm{s}$ an initial satellite position $\mathbf{r_i}(t_0)$ and velocity $\mathbf{v_i}(t_0)$ between (CHAMP or MOLNIJA) and determine the corresponding Keplerian elements. For that, write a MATLAB function *cart2kep*.
- Compute the Keplerian elements of the orbiting satellite for 1 day with a sampling period of $\Delta t = 10\,\mathrm{s}$. To compute the mean anomaly $M(t)$ from its initial value M_0 use the equation $M(t) = M_0 + n(t - t_0)$ where n is the mean angular velocity of the satellite.
- Finally, transform the Keplerian elements of each orbital point back to position vectors $\mathbf{r_i}(t)$ and velocity vectors $\mathbf{v_i}(t)$. To do so, write a new MATLAB function *kep2cart* that computes the Keplerian to Cartesian transformation. Keep in mind that the computation of the eccentric anomaly E from the mean anomaly M follow an iterative algorithm.
- Plot the numerically integrated satellite orbit with the help of the MATLAB function **plot3**. For the sake of clarity, plot also a point or a sphere at the location of the earth's centre.
- To validate the quality and precision of the numerical integration of the orbit carried out in lab 1, compare the Keplerian elements of the latter with the Keplerian elements you obtained in this lab. In particular, quantify the variations of Keplerian elements that are theoretically constant over time.

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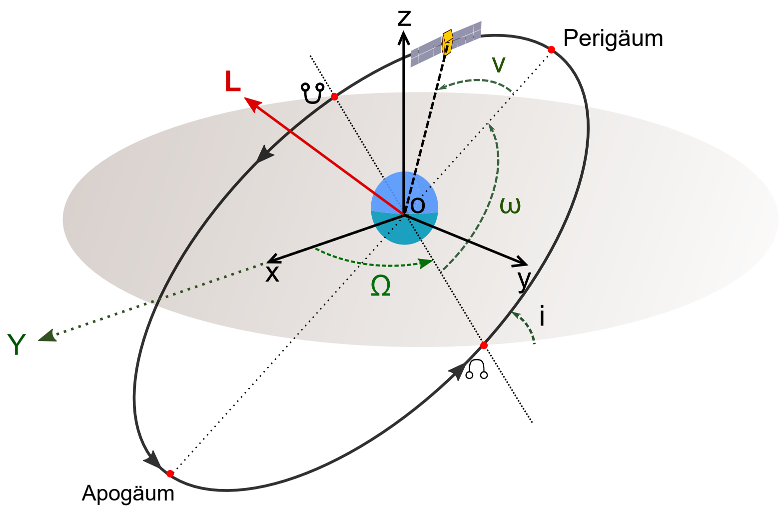
Parameter

$$GM = 3.986\,005 \cdot 10^{14}\,\mathrm{m}^3\mathrm{s}^{-2}$$



i	Inklination
Ω	Rektaszension des aufsteigenden Knoten
ω	Argument der Periapsis
v	wahre Anomalie
e	Exzentrizität
a	große Halbachse

☊	aufsteigender Knoten
☋	absteigender Knoten
☋☊	Knotenlinie
Y	Frühlingspunkt
L	Drehimpulsvektor

Abbildung 2: Keplerian elements

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