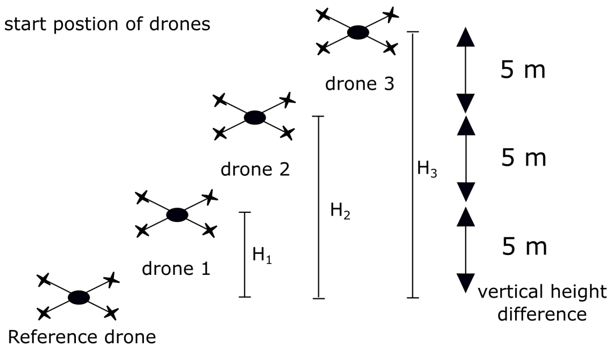
Exercise on 23.04.2019

Task 1 (4 points)

Sequential adjustment

A company for drone development has to verify the flying performance from a new drone series. In order to determine a possible vertical drift during the flight, three drones of the same model are investigated. In addition, a fourth drone is used, which is assumed to be free of drifts and will serve as a reference. When the drones start the formation flight, they are positioned with vertical differences of exactly 5 m for adjacent drones (see figure). All drones are supposed to fly an equal route while the vertical distance of the three drones under investigation to the reference drone should stay the same. During the flight, several height differences between drones are measured and have to be processed by a sequential adjustment. The observations have a standard deviation of 2 cm.



Following measurements are provided:

Epoch	$\Delta h_{R,1}$	$\Delta h_{1,2}$	$\Delta h_{2,3}$	$\Delta h_{3,2}$	$\Delta h_{2,1}$	$\Delta h_{1,R}$
1	4.96	4.93	5.25	-5.26	-4.96	-4.99
2	4.96	4.94	5.56	-5.55	-4.89	-4.92
3	4.97	4.91	5.84	-5.81	-4.91	-4.92
4	4.86	4.83	6.09	-6.09	-4.86	-4.89
5	4.85	4.81	6.33	-6.34	-4.80	-4.82

where

$$\Delta h_{i,j} = H_j - H_i$$

is the height difference from drone i to j. Compute the heights H_1, H_2, and H_3 of the three drones above the reference drohne height. In addition, calculate the formal errors of parameters in each epoch. Give a statement about a possible height drift of drones. Plot your results.

Task 3 (1 point)

Is it possible to compute a separat adjustment of each epoch? If yes, why would it be possible? Would it have an influence on the formal errors of the heights?

Task 2 (1 point)

Is there a possibility which allows to take the temporal changes into account when doing a sequential adjustment? If yes, describe or test your approach with the data from task 1.

Task 4 (2 points)

Runge-Kutta method

Following differential equation is given

$$y' = \frac{-2ty}{y^2 - t^2} \qquad y(0) = 4$$

Compute the function value $y(t)$ at $t = 4$ with a step size of $h = 1$ by using the 3^{rd} order Runge-Kutta method. In Addition, determine the function value of $y(t = 4)$ with a step size of $h = 1$ by using the Runge-Kutta of 4^{th} order. Compare both results and explain the difference.

Task 5 (2 points)

The first order of the Runge-Kutta method (Eqn. (2.13) in lecture slides) is defined as:

$$y_{n+1} = y_n + hf(y_n, t_n).$$

Compute the numeric solution for following differential equation

$$y' = c$$

where c is an (arbitrary) constant parameter. Find an analytic expression for the uncertainty (variance) of y_{n+m} by applying the law of error propagation. Note: you need m steps to go from y_n to y_{n+m}. The $\text{Var}(y_n)$ and $\text{Var}(c)$ is supposed to be known and is uncorrelated.