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Dynamic System Estimation

State vector augmentation

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State vector augmentation

We start with a very basic example assuming a Wiener (Brownian motion) process. Such a process is defined as integrated Gaussian white noise with the additional constraint that the initial value is zero. The continuous linear dynamic system with the usual differential equation

$$\dot{x}(t) = F(t)x(t) + G(t)w(t)$$

simplifies to

$$\dot{x}(t) = w(t) \tag{6.1}$$

as $F(t) = 0$ and $G(t) = 1$, if we assume unity white noise. The state transition matrix degrades to a scalar and simply becomes

$$\Phi = e^{F(t)\Delta t} = e^0 = 1.$$

Also the process noise covariance degrades and we are only left with the variance, which can be computed as

$$Q = \sigma^2 \int_0^{\Delta t} d\tau = \sigma^2 \Delta t.$$

Assuming now that we predict for steps of $\Delta t = 1$ s, we find $Q = \sigma^2$.
Describe/sketch such a process!

State vector augmentation - cont'd

In several cases the simple white Gaussian noise model does not describe the system dynamic noise adequately and one would like to have a model that represents the empirical auto-covariance or power spectral density of a system. Thus we need some kind of trick. Starting with a system in the form

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(t)\boldsymbol{x}(t) + \boldsymbol{G}(t)\boldsymbol{w}(t) + \boldsymbol{B}(t)\boldsymbol{c}(t) \tag{6.2}$$

where $\boldsymbol{x}$ describes the state of the system, $\boldsymbol{w}$ is white noise and $\boldsymbol{F}$ and $\boldsymbol{G}$ are the usual matrices, which we have encountered so far. $\boldsymbol{c}$ is generated from white noise $\boldsymbol{w}_c$ (uncorrelated in time) and due to the mathematical nature of the expression has a temporal correlations (random walk, random constant, Gauß-Markov processes, etc.) expressed by the matrix $\boldsymbol{F}_c$ as described below

$$\dot{\boldsymbol{c}}(t) = \boldsymbol{F}_c(t)\boldsymbol{c}(t) + \boldsymbol{G}_c(t)\boldsymbol{w}_c(t) \tag{6.3}$$

Hereby $\boldsymbol{w}_c(t)$ represent uncorrelated white noise.

State vector augmentation - cont'd

Combining (6.2) with the equ. (6.3) in a new system of equations:

$$\begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{c}}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}(t) & \boldsymbol{B}(t) \\ \boldsymbol{0} & \boldsymbol{F}_c(t) \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{c}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{G}(t) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{G}_c(t) \end{bmatrix} \begin{bmatrix} \boldsymbol{w}(t) \\ \boldsymbol{w}_c(t) \end{bmatrix} \tag{6.4}$$

This equation differs from equ. (6.2) in that respect, that the RPs in the second term on the right hand side are all white noise processes; the correlated RPs have augmented the state vector $\boldsymbol{x}$ that preserves the form of (6.2).

Assume now a similar process as in Eq. (6.1) which is not driven by Gaussian white noise but for example by a Gauss-Markov process. In this case we have to augment the vector differential equation by an additional state and obtain

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\beta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \sqrt{2\sigma^2\beta} \end{bmatrix} w(t) \tag{6.5}$$