



Examination Spring 2011

March 31, 2011

Geomatics Methodology

Module 2

Module Section **Signal Processing**

Prof. Fritsch

Student ID

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Student's Surname

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Examination **Signal Processing** - Spring 2011

Question 1: (25%)

There is some similarity between least-squares adjustment using a Gauss-Markov model and random signal processing.

- (a) Write down the definition of the Gauss-Markov model and the random signal model in signal processing. Do you see already some similarities?
- (b) Which objective functions are used to estimate the unknown parameters in the Gauss-Markov model and the true random signal $y(m)$?
- (c) How can a Wiener filter be implemented?

Question 2: (25%)

The sampling theorem is an important issue when digitizing analog signals $x(t)$ to get $x(m)$.

- (a) How is the sampling theorem defined?
- (b) What has to be considered to get non-aliased sampling?
- (c) What is the ideal interpolator to reconstruct $x(t)$ using samples $x(m)$? (formula and sketch)
- (d) Which frequencies are aliased in the movement of a stage coach in a Western movie?

Question 3: (25%)

Given is the function $x(t) = e^{-|t|}$.

- (a) Make a sketch of the function $x(t)$.
- (b) Write down the equations for the continuous Fourier transformation (FT) and the inverse Fourier transformation (IFT).
- (c) Calculate the Fourier transformation of $x(t)$.

Question 4: (25%)

- (a) Using the accelerated computation scheme, compute the convolutions $f*x$ and $h*x$ of the following digital signal x and two filters f and h

$$x = [0 \ 4 \ 3 \ 4 \ 3 \ 4 \ 0]$$

$$f = 0.25 * [1 \ 1 \ -1 \ -1]$$

$$h = [-1 \ 1]$$



- (b) What is the effect of the filter h ?
- (c) Determine the filter kernel g such that $f=h*g$ holds. What is the effect of the filter g , what is the effect of the filter f ?

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Q1: a) $\mathbf{L} \mathbf{e} = \mathbf{A} \mathbf{x} \quad \mathbf{D}\{\mathbf{e}\} = \sigma \mathbf{P}^{-1}$

$x(m) = y(m) + r(m)$ "additive noise"

b) $\mathbf{e}^T \mathbf{e} \stackrel{!}{=} \min$

$E\{\sum \varepsilon(m) - E[\varepsilon(m)]\}^2 \stackrel{!}{=} \min$

c) see slides derivation 3.1.2 following

Q2: a) $f_{\text{sam}} > 2 f_{\text{max}}$

b) sample with a frequency 2 times larger than maximal signal frequency

c) $x(t) = \sum_{k=-\infty}^{\infty} x(k) \delta(t - k \Delta t)$

$\delta(t - k \Delta t) = \frac{\sin(\pi / \Delta t)(t - k \Delta t)}{(\pi / \Delta t)(t - k \Delta t)}$

$= \text{sinc} \quad (\text{Sketch})$

d) if the frequency of images taken is below 2 times the Nyquist frequency there will be artifacts

Q3 a) (Sketch)

b) FT: $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

IFT: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$

$$c) \Rightarrow \frac{2}{1+\omega^2} = X(j\omega)$$

Qu: a) $f * x = [0 \ 4 \ 7 \ 3 \ 0 \ 0 \ -3 \ -7 \ -4 \ 0] \cdot 0.25$
 $h * x = [0 \ -4 \ 1 \ -1 \ 1 \ -1 \ 4 \ 0]$

b) derivative

c) $-0.5 [1 \ 2 \ 1]$

negation,
smoothing