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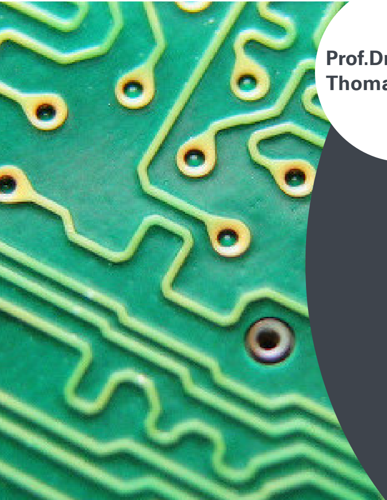

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Integrated Positioning and Navigation

10

Linearized error equations in the e-system

Linearized error equations in the e-system

- Initial conditions for integration have errors
- Measurements of accelerometers and gyroscopes have errors
- Gravity field is not perfectly known

$\Rightarrow$ Description of errors and error propagation needed

Start by taking the differential (denoted δ) of equation (9.1)

$$\begin{aligned} \delta\dot{\boldsymbol{x}}^e &= \delta\boldsymbol{v}^e \\ \delta\dot{\boldsymbol{v}}^e &= \delta\left[\boldsymbol{C}_p^e\boldsymbol{a}^p\right] - 2\delta\left[\boldsymbol{\Omega}_{ie}^e\boldsymbol{v}^e\right] - \delta\left[\boldsymbol{\Omega}_{ie}^e\boldsymbol{\Omega}_{ie}^e\boldsymbol{x}^e\right] + \delta\boldsymbol{g}^e \end{aligned} \tag{10.1}$$

Assuming the rotation rate of the e-system w.r.t. the i-system is known, the second equ. (10.1) can be expanded:

$$\delta\dot{\boldsymbol{v}}^e = \delta\boldsymbol{C}_p^e\boldsymbol{a}^p + \boldsymbol{C}_p^e\delta\boldsymbol{a}^p - 2\boldsymbol{\Omega}_{ie}^e\delta\dot{\boldsymbol{x}}^e - \boldsymbol{\Omega}_{ie}^e\boldsymbol{\Omega}_{ie}^e\delta\boldsymbol{x}^e + \boldsymbol{\Gamma}^e\delta\boldsymbol{x}^e + \delta\boldsymbol{g}^e \tag{10.2}$$

$\boldsymbol{\Gamma}^e$ denotes the gradient of the gravitational acceleration.

Linearized error equations in the e-system - cont'd

The differential of the DCM is obtained as the difference between the true DCM and a DCM accounting for additional small rotations about the three axes:

$$\delta \boldsymbol{C}_p^e \cdot \boldsymbol{a}^p = \left[(\boldsymbol{I} - \boldsymbol{\Psi}^e) \cdot \boldsymbol{C}_p^e - \boldsymbol{C}_p^e \right] \cdot \boldsymbol{a}^p = -\boldsymbol{\Psi}^e \cdot \boldsymbol{C}_p^e \cdot \boldsymbol{a}^p = -\boldsymbol{\Psi}^e \cdot \boldsymbol{a}^e \tag{10.3}$$

$\boldsymbol{\Psi}^e$ is the skew-symmetric form of the rotation angle vector $\boldsymbol{\psi}^e$ with the following properties for arbitrary vectors $\boldsymbol{k}$

$$\boldsymbol{\Psi}^e \cdot \boldsymbol{k} = \boldsymbol{\psi}^e \times \boldsymbol{k} = -\boldsymbol{k} \times \boldsymbol{\psi}^e \tag{10.4}$$

With (10.3) and (10.4), (10.2) can be re-written:

$$\delta \dot{\boldsymbol{v}}^e = \boldsymbol{a}^e \times \boldsymbol{\psi}^e + \boldsymbol{C}_p^e \delta \boldsymbol{a}^p - 2\boldsymbol{\Omega}_{ie}^e \delta \dot{\boldsymbol{x}}^e - \left[\boldsymbol{\Omega}_{ie}^e \boldsymbol{\Omega}_{ie}^e - \boldsymbol{\Gamma}^e \right] \delta \boldsymbol{x}^e + \delta \boldsymbol{g}^e \tag{10.5}$$

What is $\boldsymbol{\Psi}^e$? How is it related to the DCM and its derivative?

Linearized error equations in the e-system - cont'd

Time derivative of DCM (equ.(3.12))

$$\dot{\boldsymbol{C}}_p^e = \boldsymbol{C}_p^e \cdot \boldsymbol{\Omega}_{ep}^p \tag{10.6}$$

Take the differential

$$\delta \dot{\boldsymbol{C}}_p^e = \delta \boldsymbol{C}_p^e \cdot \boldsymbol{\Omega}_{ep}^p + \boldsymbol{C}_p^e \cdot \delta \boldsymbol{\Omega}_{ep}^p \tag{10.7}$$

Take time derivative of (10.3)

$$\delta \dot{\boldsymbol{C}}_p^e = -\dot{\boldsymbol{\Psi}}^e \cdot \boldsymbol{C}_p^e - \boldsymbol{\Psi}^e \cdot \dot{\boldsymbol{C}}_p^e = -\dot{\boldsymbol{\Psi}}^e \cdot \boldsymbol{C}_p^e - \boldsymbol{\Psi}^e \cdot \boldsymbol{C}_p^e \cdot \boldsymbol{\Omega}_{ep}^p \tag{10.8}$$

Set r.h.s. of equ. (10.7) equal to r.h.s. of equ. (10.8): :

$$\delta \boldsymbol{C}_p^e \cdot \boldsymbol{\Omega}_{ep}^p + \boldsymbol{C}_p^e \cdot \delta \boldsymbol{\Omega}_{ep}^p = -\dot{\boldsymbol{\Psi}}^e \cdot \boldsymbol{C}_p^e - \boldsymbol{\Psi}^e \cdot \boldsymbol{C}_p^e \cdot \boldsymbol{\Omega}_{ep}^p \tag{10.9}$$

Use equ. (10.3) to replace $\delta \boldsymbol{C}$

$$\dot{\boldsymbol{\Psi}}^e = -\boldsymbol{C}_p^e \cdot \delta \boldsymbol{\Omega}_{ep}^p \cdot \boldsymbol{C}_e^p \tag{10.10}$$

Linearized error equations in the e-system - cont'd

In terms of vectors, this is equivalent to (see equ. (3.3), (3.6)):

$$\dot{\boldsymbol{\psi}}^e = -\boldsymbol{C}_p^e \cdot \delta\boldsymbol{\omega}_{ep}^p \tag{10.11}$$

Replacing

$$\boldsymbol{\omega}_{ep}^p = \boldsymbol{\omega}_{ip}^p - \boldsymbol{\omega}_{ie}^p = \boldsymbol{\omega}_{ip}^p - \boldsymbol{C}_e^p \cdot \boldsymbol{\omega}_{ie}^e \tag{10.12}$$

and taking differentials on both sides:

$$\delta\boldsymbol{\omega}_{ep}^p = \delta\boldsymbol{\omega}_{ip}^p - \delta\boldsymbol{C}_e^p \cdot \boldsymbol{\omega}_{ie}^e - \boldsymbol{C}_e^p \cdot \delta\boldsymbol{\omega}_{ie}^e \tag{10.13}$$

The differential of the rotation rate of the e-system w.r.t. the i-system is zero, the differential of a DCM is the transpose of the differential of the inverse DCM.

$$\begin{aligned} &\delta\boldsymbol{C}_e^p = \left(\delta\boldsymbol{C}_p^e\right)^T \Rightarrow \text{(10.3)} \Rightarrow \delta\boldsymbol{C}_e^p = \left(-\boldsymbol{\Psi}^e \cdot \boldsymbol{C}_p^e\right)^T = \boldsymbol{C}_e^p \cdot (-\boldsymbol{\Psi}^e)^T \\ &\Rightarrow \delta\boldsymbol{C}_e^p = \boldsymbol{C}_e^p \cdot \boldsymbol{\Psi}^e \end{aligned} \tag{10.14}$$

Linearized error equations in the e-system - cont'd

Combining equ. (10.11), (10.13), (10.14), (10.3) and (10.4)

$$\dot{\boldsymbol{\psi}}^e = -\boldsymbol{C}_p^e \cdot \delta\boldsymbol{\omega}_{ip}^p - \boldsymbol{\omega}_{ie}^e \times \boldsymbol{\psi}_e \tag{10.15}$$

or

$$\dot{\boldsymbol{\psi}}^e = -\boldsymbol{C}_p^e \cdot \delta\boldsymbol{\omega}_{ip}^p - \boldsymbol{\Omega}_{ie}^e \cdot \boldsymbol{\psi}_e \tag{10.16}$$

Equations(10.11), (10.5) and (10.16) are combined to form a linear system describing the error propagation:

$$\boxed{\begin{aligned} \frac{d}{dt}\begin{bmatrix} \boldsymbol{\psi}^e \\ \delta\dot{\boldsymbol{x}}^e \\ \delta\boldsymbol{x}^e \end{bmatrix} = & \begin{bmatrix} -\boldsymbol{\Omega}_{ie}^e & \mathbf{0} & \mathbf{0} \\ \boldsymbol{a}^e \times & -2\boldsymbol{\Omega}_{ie}^e & -(\boldsymbol{\Omega}_{ie}^e \cdot \boldsymbol{\Omega}_{ie}^e - \boldsymbol{\Gamma}^e) \\ \mathbf{0} & \boldsymbol{I} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\psi}^e \\ \delta\dot{\boldsymbol{x}}^e \\ \delta\boldsymbol{x}^e \end{bmatrix} \\ & + \begin{bmatrix} -\boldsymbol{C}_p^e & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{C}_p^e & \boldsymbol{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \delta\boldsymbol{\omega}_{ip}^p \\ \delta\boldsymbol{a}^p \\ \delta\boldsymbol{g}^e \end{bmatrix} \end{aligned}} \tag{10.17}$$