Exercise on 11.12.2019

Task 1 *(2 Points)*

Show that $\Omega^e_{ie} \cdot \Omega^e_{ie} \cdot x^e$ corresponds to the centripetal acceleration

$$a_z = \omega_E^2 \cdot r$$

where r is the distance to the rotation axis and ω_E is the angular velocity of the earth.

Proposal for solution 1

$$\Omega^e_{ie} \cdot \Omega^e_{ie} \cdot x^e = \begin{bmatrix} 0 & -\omega_E & 0 \\ \omega_E & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -\omega_E & 0 \\ \omega_E & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = -\omega_E^2 \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}$$

$$\| -\omega_E^2 \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} \|_2 = \omega_E^2 \sqrt{x_1^2 + x_2^2} = \omega_E^2 \cdot r$$

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Task 2 *(3 Points)*

The pilot of a parking aircraft reads off the following values of the axis of the IMU:

$$\begin{aligned} a_1^p &= -0.5\,\mathrm{m\,s^{-2}} \\ a_2^p &= 0.6\,\mathrm{m\,s^{-2}} \\ \omega^p_{i,p1} &= 4.6035 \times 10^{-5}\,\mathrm{s^{-1}} \\ \omega^p_{i,p2} &= -8.1172 \times 10^{-6}\,\mathrm{s^{-1}} \end{aligned}$$

Calculate R, P, Y (Roll, Pitch and Yaw) of the platform. Additionally calculate the standard deviation of the heading angles under the assumption that the standard deviation of the IMU is $s_{a_1^p} = s_{a_2^p} = 0.003\,\mathrm{m\,s^{-2}}$ and $s_{\omega^p_{i,p1}} = s_{\omega^p_{i,p2}} = 3.0 \times 10^{-8}\,\mathrm{s^{-1}}$ and uncorrelated.

Proposal for solution 2

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\begin{aligned}
a_1^P &= -g \sin P \\
&\quad \rightarrow \underline{P} = \arcsin\left(-\frac{a_1^P}{g}\right) \approx 0.050\,99\,\mathrm{rad} \approx \underline{0.05\,\mathrm{rad}} \\
&\quad \rightarrow \underline{u_P} = \sqrt{\left(\frac{\partial P}{\partial a_1^P} u_{a_1^P}\right)^2} = ... \approx \underline{3.1 \times 10^{-4}\,\mathrm{rad}} \\
a_2^P &= g \sin R \cos P \\
&\quad \rightarrow \underline{R} = \arcsin\left(\frac{a_2^P}{g \cos P}\right) \approx 0.061\,28\,\mathrm{rad} \approx \underline{0.06\,\mathrm{rad}} \\
&\quad \rightarrow \underline{u_R} = \sqrt{\left(\frac{\partial R}{\partial a_2^P} u_{a_2^P}\right)^2 + \left(\frac{\partial R}{\partial P} u_P\right)^2} = ... \approx \underline{3.0 \times 10^{-4}\,\mathrm{rad}} \\
\omega_{ip_3}^p &= \sqrt{(7.2921 \times 10^{-5}\,\mathrm{rad\,s^{-1}})^2 - \omega_{ip_1}^{p\ 2} - \omega_{ip_2}^{p\ 2}} \approx 5.594\,77 \times 10^{-5}\,\mathrm{rad\,s^{-1}}
\end{aligned}
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C_b^n \cdot \omega_{ib}^b \overset{\text{here}}{=} C_p^n \cdot \omega_{ip}^p = ... \approx \begin{bmatrix} 1.152\,92 \times 10^{-5}\,\mathrm{s^{-1}} \cdot \sin Y + 4.879\,70 \times 10^{-5}\,\mathrm{s^{-1}} \cdot \cos Y \\ 4.879\,70 \times 10^{-5}\,\mathrm{s^{-1}} \cdot \sin Y - 1.152\,95 \times 10^{-5}\,\mathrm{s^{-1}} \cdot \cos Y \\ 5.294\,72 \times 10^{-5}\,\mathrm{s^{-1}} \end{bmatrix} \overset{!}{=} \begin{bmatrix} ? \\ 0 \\ ? \end{bmatrix} = \omega_{ib}^n
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\begin{aligned}
&\rightarrow \frac{\sin Y}{\cos Y} = \frac{1.152\,95 \times 10^{-5}\,\mathrm{s^{-1}}}{4.879\,70 \times 10^{-5}\,\mathrm{s^{-1}}} \\
&\rightarrow \underline{Y} = \arctan\left(\frac{1.152\,95 \times 10^{-5}\,\mathrm{s^{-1}}}{4.879\,70 \times 10^{-5}\,\mathrm{s^{-1}}}\right) \approx \underline{0.2320\,\mathrm{rad}} \\
&\rightarrow u_Y = ...
\end{aligned}
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Note: the eq. $\tan Y_0 = -\frac{w_{ip2}^p}{\omega_{ip1}^p}$ cannot be applied here, as therefore the IMU would have to be leveled.

Task 3 *(5 Points)*

Calculate the matrices Ω_{ie}^n for local level coordinate systems (**n**-systems), which are at the following positions:

i) Longitude $13°\ 17'\ 34.187''$
   Latitude $0°\ 0'\ 0.000''$
   Height $50.00\,\mathrm{m}$

ii) Longitude $17°\ 17'\ 24.356''$
   Latitude $47°\ 21'\ 26.483''$
   Height $125.13\,\mathrm{m}$

iii) Longitude $12°\ 13'\ 12.156''$
   Latitude $90°\ 0'\ 0.000''$
   Height $50.00\,\mathrm{m}$

Use $\omega_E = 7.292\,115\,816 \times 10^{-5}\,\mathrm{rad\,s^{-1}}$. Discuss the results.

Proposal for solution 3

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\begin{aligned}
\omega_{ie}^{n} &= \begin{bmatrix} \omega_E \sin\phi \\ 0 \\ -\omega_E \sin\phi \end{bmatrix} \\
&\rightarrow \Omega_{ie}^{n} \overset{(3.4)}{=} \begin{bmatrix} 0 & \omega_E \sin\phi & 0 \\ -\omega_E \sin\phi & 0 & -\omega_E \sin\phi \\ 0 & \omega_E \sin\phi & 0 \end{bmatrix} \\
&\rightarrow \text{Put in } \phi
\end{aligned}
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