Exercise on <u>28.05.2019</u>

Task 1 (3 points)

As discussed in the lectures, the so-called "integrated random walk" process is given by

$$\ddot{x}(t) = 0 + W(t).$$

The Gaussian white noise $W(t)$ is assumed to have a mean of 0 and a standard deviation of 1. A similar process can be set up with

$$\ddot{x}_c(t) = 0 + c(t),$$

where $c(t)$ is a correlated noise process defined by

$$\dot{c}(t) = -\beta c(t) + W(t).$$

Create a realization of the process with $\beta = 0.7$ for $t = 0{:}10$ with $\Delta t = 0.1$, assuming that $W(t)$ is Gaussian white noise (mean = 0 and standard deviation = 1). Compute the variance of $x(t)$ and $x_c(t)$. Discuss the influence of β on the result.

Task 2 (5 points)

A two-dimensional random noise process (random walk, RW) is given by

$$\dot{x}(t) = 0 + W_x(t) \qquad \text{and} \qquad \dot{y}(t) = 0 + W_y(t).$$

$W_x(t)$ corresponds to Gaussian white noise (mean = 0, standard deviation = 0.8) which is also the case for $W_y(t)$ (mean = 0, standard deviation = 0.2). The covariance matrix for x and y at $t_0 = 0$ is given by

$$\boldsymbol{\Sigma}_{xy}(t_0) = \begin{bmatrix} 1.2 & 0.3 \\ 0.3 & 1.0 \end{bmatrix}.$$

Compute the covariance matrix $\boldsymbol{\Sigma}_{xy}(\Delta t)$, where $\Delta t = 1$, assuming a random walk noise process.

After this, use your result and compute $\boldsymbol{\Sigma}_{xy}(2\Delta t)$, meaning you compute $t_0 \to t_0 + \Delta t$ with a RW first, and then, compute $t_0 + \Delta t \to t_0 + 2\Delta t$ assuming a two-dimensional integrated random noise process:

$$\ddot{x}(t) = 0 + W_x(t) \qquad \text{and} \qquad \ddot{y}(t) = 0 + W_y(t)$$

For this case the covariance matrix contains four parameters (position and velocity for x and y). $W_x(t)$ and $W_y(t)$ are the same as those used for the random walk.

Compute the parameter of the error ellipse (length of the axes and the orientation) for $\Delta t = 2$.

Task 3 (2 points)

Compute the numerical solution (i.e. use the "cook book"!) of the transition matrix Φ as well as the matrix of the process noise $\boldsymbol{Q}$ at $\Delta t = 1$. Assume that we have the following noise process:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -\sqrt{2}\omega_0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} W(t)$$

Use $\omega_0 = 0.2$ and $b^2 = 2\sqrt{2}\omega_0^3$ for the computation.