

SOLUTION (Version 0)
Exercise on 28.05.2019

Task 1 (3 points)

Solution

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{c} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\beta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} W(t)$$

$$A = \begin{bmatrix} -F & GWG^T \\ 0 & F^T \end{bmatrix} \Delta t$$

$$B = \expm(A) = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

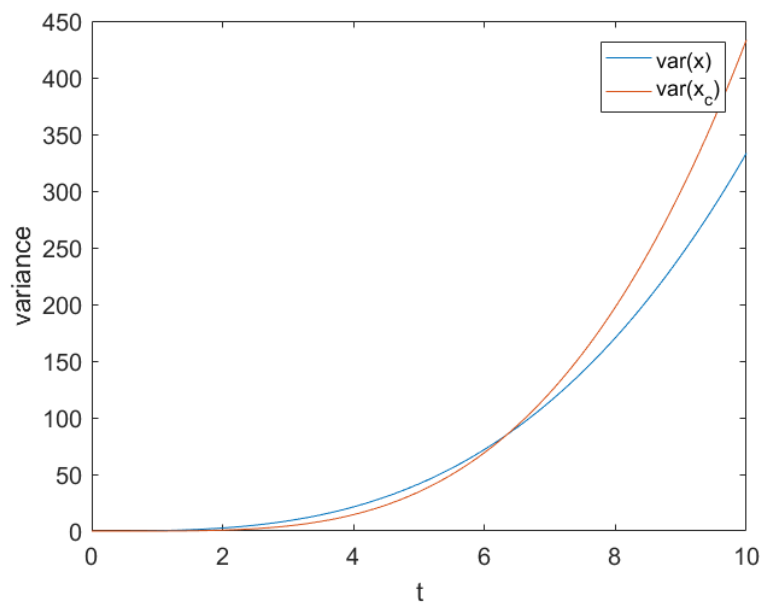
$$\Phi = B_{22}^T$$

$$Q = \Phi \cdot B_{12}$$

The variances (with $\beta = 0.7$) are extracted from

$$\sigma_{x_c}^2 = Q(1, 1)$$

$$\sigma_x^2 = \frac{\Delta t^3}{3}$$



Task 2 (5 points)**Solution**

Step 1 (Random Walk)

$$\Phi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Q_{process\ RW} = \begin{bmatrix} s_x^2 \Delta t & 0 \\ 0 & s_y^2 \Delta t \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} 1.2 & 0.3 \\ 0.3 & 1.0 \end{bmatrix}$$

$$Q(\Delta t = 1) = \Phi \cdot Q_0 \cdot \Phi^T + Q_{process\ RW}$$

Step 2 (Integrated Random Walk)

$$\Phi = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_{process\ IRW} = \begin{bmatrix} s_x^2 \frac{\Delta t^3}{3} & s_x^2 \frac{\Delta t^2}{2} & 0 & 0 \\ s_x^2 \frac{\Delta t^2}{2} & s_x^2 \Delta t & 0 & 0 \\ 0 & 0 & s_y^2 \frac{\Delta t^3}{3} & s_y^2 \frac{\Delta t^2}{2} \\ 0 & 0 & s_y^2 \frac{\Delta t^2}{2} & s_y^2 \Delta t \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} s_x^2 & 0 & s_{xy} & 0 \\ 0 & 0 & 0 & 0 \\ s_{xy} & 0 & s_y^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{extracted from } Q(\Delta t = 1), \text{ there is no variance for velocity! })$$

$$Q(\Delta t = 2) = \Phi \cdot Q_1 \cdot \Phi^T + Q_{process\ IRW} = \begin{bmatrix} 2.4533 & 0.9200 & 0.3000 & 0 \\ 0.9200 & 1.8400 & 0 & 0 \\ 0.3000 & 0 & 1.3867 & 0.5200 \\ 0 & 0 & 0.5200 & 1.0400 \end{bmatrix}$$

Parameter of error ellipse:

$$a = 1.5912 \quad b = 1.1437 \quad \Theta = 14.6789^\circ$$

Task 3 (2 points)

Solution

$$F = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -\sqrt{2}\omega_0 \end{bmatrix}$$

$$GWG^T = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix}$$

$$A = \begin{bmatrix} -F & GWG^T \\ 0 & F^T \end{bmatrix} \Delta t$$

$$B = \text{expm}(A) = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$\Phi = B_{22}^T = \begin{bmatrix} 0.9818 & 0.8652 \\ -0.0346 & 0.7371 \end{bmatrix}$$

$$Q = \Phi \cdot B_{12} = \begin{bmatrix} 0.0061 & 0.0085 \\ 0.0085 & 0.0171 \end{bmatrix}$$