Universität Stuttgart

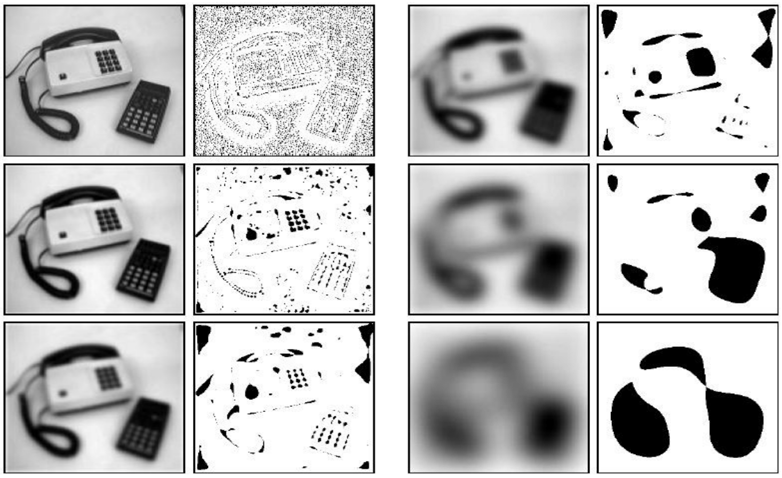
Pattern Recognition
Chapter 4: Scale Space

Prof. Dr.-Ing. Uwe Sörgel
soergel@ifp.uni-stuttgart.de

ifp

Scale space

- We can built a scale space by repeated smoothing of an image, for example, using Gaussian filters.
- By segmentation we detect a hierarchy of objects starting with fine and small structures and ending with coarse and large ones.



Contents

- Introduction
- Linear Scale Space
  - Remark: Other scale paces exists like non-linear or morphological ones, which are not discussed here.
- Scale Space Events
- Blob Detection
- Examples

Purpose of scale space

- The interpretation of the same image might be very different if we look at it from close distance or from far away.
- It is known that in the visual system of humans information is represented in different scales.
- In practice, it is more relevant that fine detail or small objects can not be recognized in imagery of coarse resolution.

T H I S
I S
N O T
WHAT IT
S E E M S
T O
B E

Scale and simplification

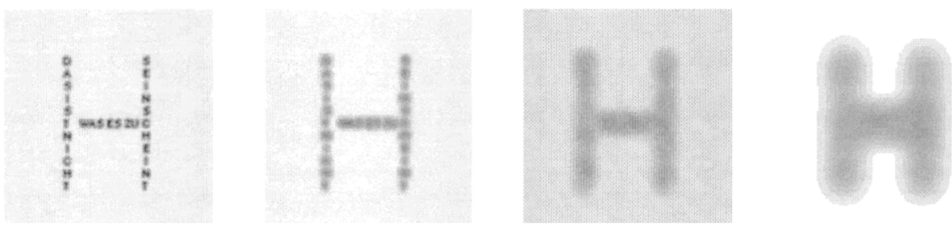
- The information content of an image is reduced at smaller scale by elimination of points, edges, and regions:
  - Both noise and real information are filtered out.

- Elimination of real information ≡ Elimination of substructures (object parts) ⇒ Simplification (in maps generalization)

- Simplification is very important: Implicit information becomes explicit – The „H“ can be recognized!

Scale space and scale parameter σ

- Scale space: Multi-level (scale) representation of an image $g(x,y)$ by a family of signals $L_\sigma(x,y)$, which are based on a certain parameter.
  - Systematical simplification of data and elimination of detail, i.e., information of high frequency (low-pass).
- The scale parameter $\sigma \in R+$ determines the scale.



$\sigma = 1$ $\sigma = 2$ $\sigma = 5$ $\sigma = 10$

Contents

Linear scale space I

- Combination of criteria:
  - Causality: Every feature (extremum) observed in a courser scale must be created by a "cause" from a finer scale, but not vice versa.
  - Continuous scale parameter σ
  - Homogeneity (invariance to location)
  - Isotropy (invariance to orientation)

各向同性

- It can be shown that the scale space signal "family" L is required to fulfill the diffusions equation:

$$\frac{\partial L}{\partial t} = \frac{1}{2}\nabla^2 L = \frac{1}{2}\left(\frac{\partial^2 L}{\partial x^2} + \frac{\partial^2 L}{\partial y^2}\right)$$

with $\sigma = \sqrt{2t}$.

Linear scale space II

- Solution of diffusion equation for infinite support of function: Convolution with Gaussian Kernel

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

→ Is the only function that fulfills desired properties

- Representation of a function $g(x,y)$ in scale space:

$$L_\sigma(x,y) = G_\sigma(x,y) * g(x,y)$$

with $g(x,y)$ … image in original resolution
$L_\sigma(x,y)$ … image in scale space of parameter σ

Boundary condition: $L_0(x,y) = g(x,y)$

Linear scale space III

- Semi-group structure:

$$L_{2\sigma}(x,y) = G_{2\sigma}(x,y) * g(x,y)$$
$$L_{2\sigma}(x,y) = G_\sigma(x,y) * G_\sigma(x,y) * g(x,y)$$

  - We may smooth the input image either n times with $G_\sigma(x,y)$ or once with $G_{n\sigma}(x,y)$
  - In case we need all images, the former variant is more efficient.

- Example: Streets with vehicles

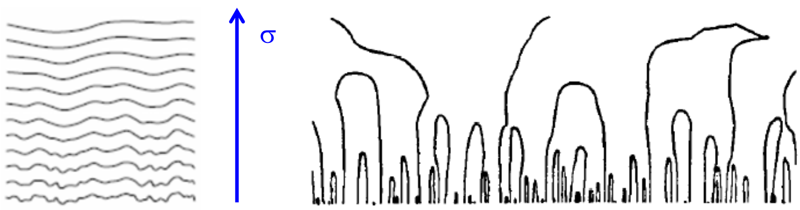
Input image (0.3 m) σ = 0.45 m σ = 1.45 m σ = 3.15 m

Contents

- Introduction
- Linear Scale Space
- Scale Space Events
- Blob Detection
- Examples

Scale space events

- Scale space events: Change of image structure in between two scale levels
- Example (1D): Numbers of edges (i.e., zero crossings of Laplace filter) declines with rising σ.



[Witkin86]

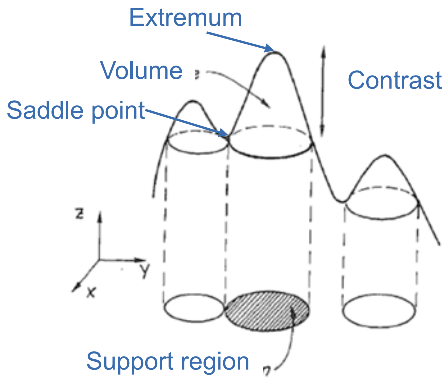
Position of edges (zero crossings of Laplace filter)

Scale space events: „Blobs“

- From an image $g(x,y)$ we establish a linear scale space: $L_\sigma(x,y)$
- Critical point: A point, at which the gradient of $L_\sigma(x,y)$ equals 0 (extremum or saddle point).
- Blob : Homogeneous region in image, which is either brighter or darker than background.
- Blobs coincide with two critical points, one extremum and one saddle point [Lindeberg, 1994].
- Blobs: Correspond with salient image structures

➔ SIFT features!

A Blob



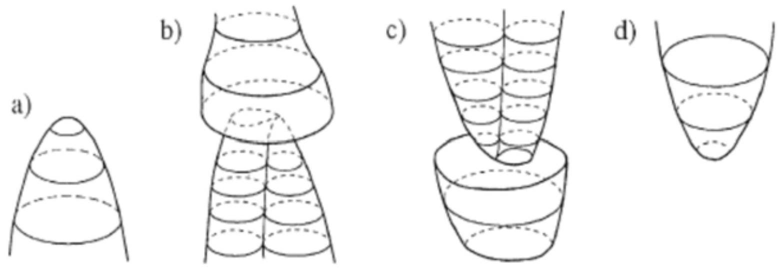
Scale space blob events after Lindeberg [1994]

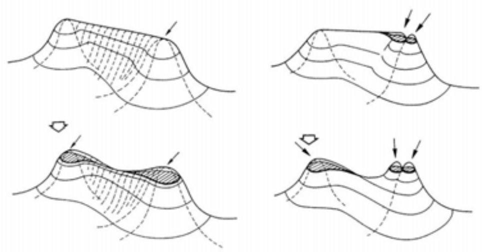
a) Blob vanishing („Annihilation“)

b) Blob merge

c) Blob split

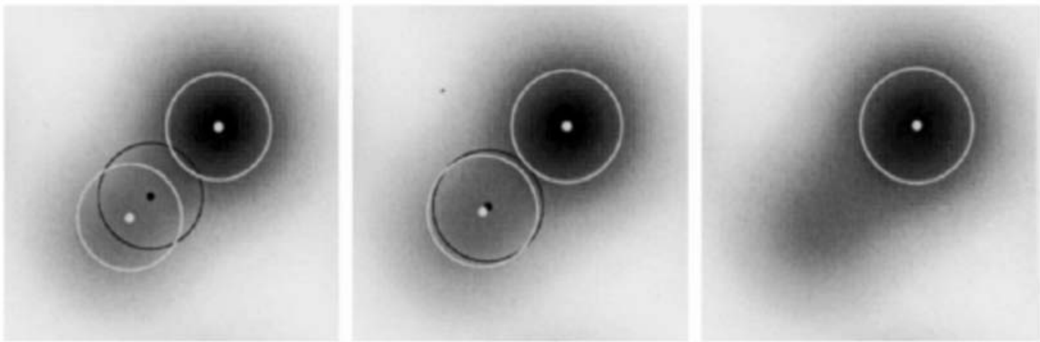
d) Blob creation





Scale space blob events: Example

- Example: Annihilation of one of two extremum-saddle point pairs with shared saddle point → blob merge

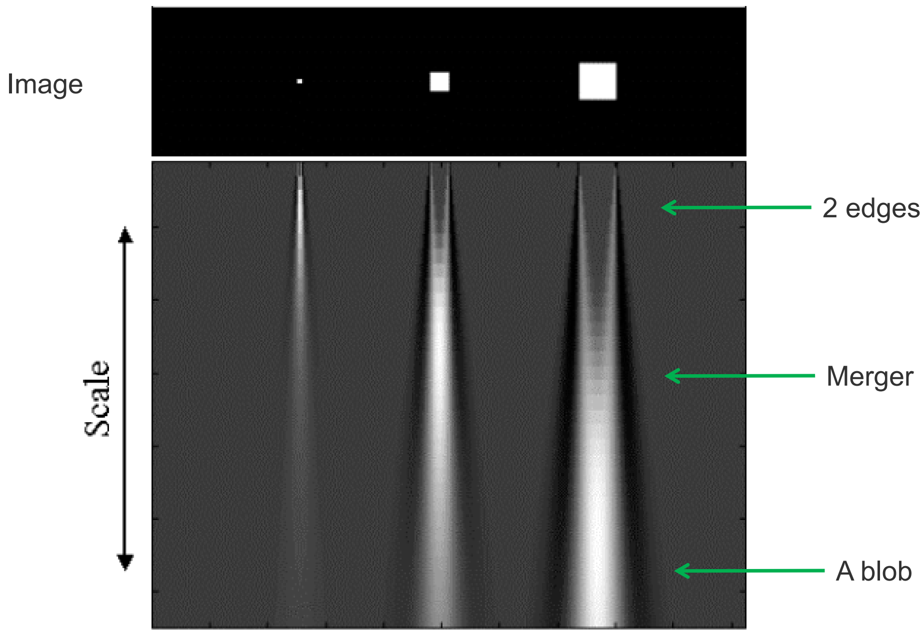


σ = 10.0 σ = 10.50 σ = 10.75

Two neighbored extrema (white), one saddle point (black)

[Florack&Kuijper00]

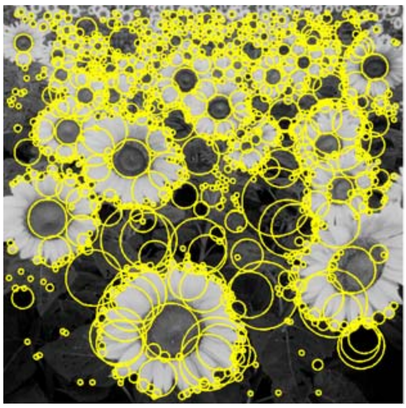
Another example



Contents

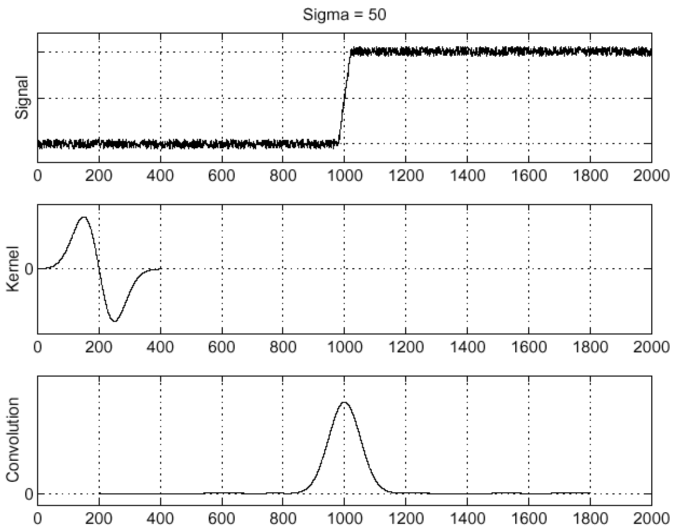
Search for blobs in scale space

- It turned out that blobs are very interesting features.
- Blobs are scale invariant (i.e., they stay blobs, only their size changes).
- We want to detect all blobs of an image at all scales.
- How can we do this?



A blob

Recap: Edge extraction, 1. derivation



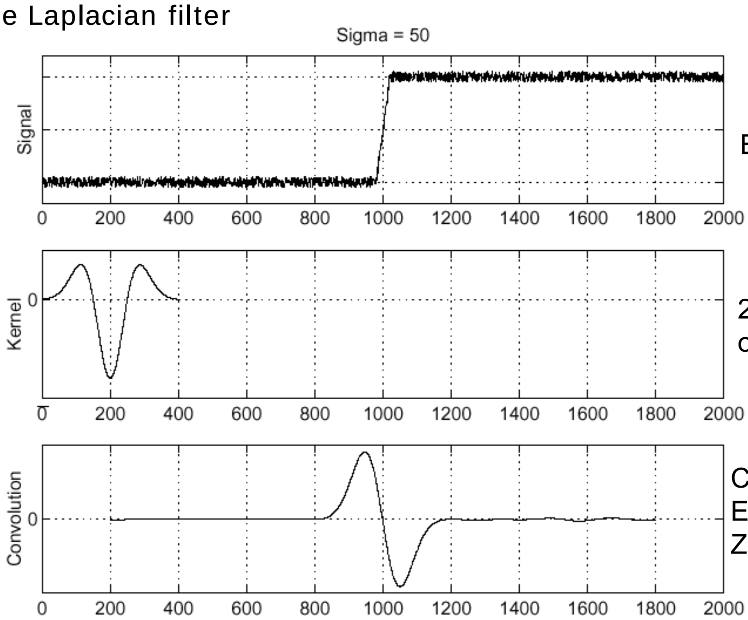
f — Edge

$\frac{d}{dx}g$ — 1. Derivation of Gaussian

$f * \frac{d}{dx}g$ — Convolution, Edge = Extremum

Recap: Edge extraction, 2. derivation

This is the Laplacian filter



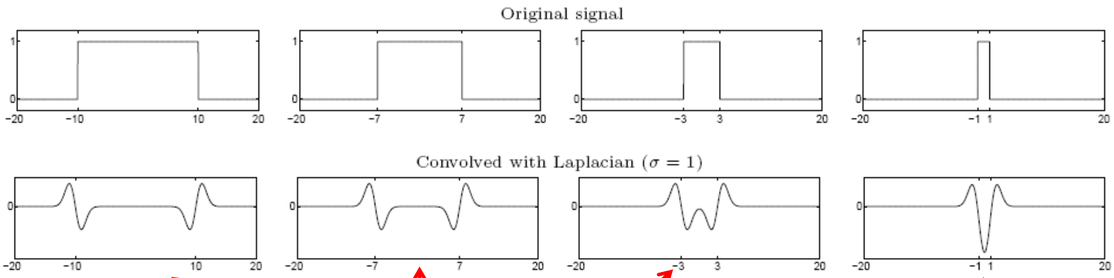
f — Edge

$\frac{d^2}{dx^2}g$ — 2. Derivation of Gaussian

$f * \frac{d^2}{dx^2}g$ — Convolution, Edge = Zero crossing

From edges to blobs (1D = Rectangle)

- Edge = Jump o grey value
- Blob = Two jumps up and down in a row



Here the filter mask is too small

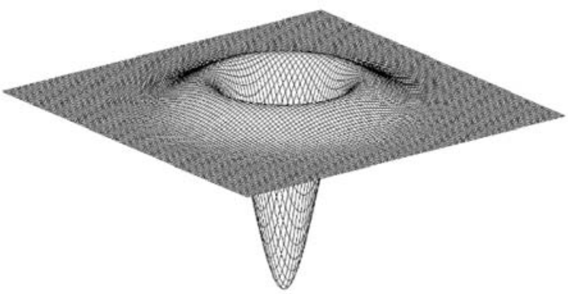
Extremum

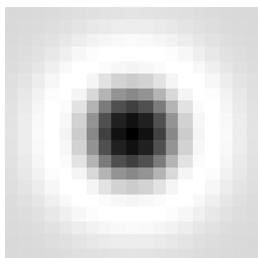
- Correct location: the maximum of the filter response coincides with blob center in case the right scale was chosen.
- This scale depends of course on σ of initial Gaussian.

Quelle: Lana Lazebnik

Let us turn to 2D now

- Laplacian of Gaussian (LoG): Rotationally symmetric blob detector

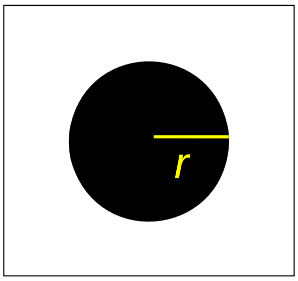




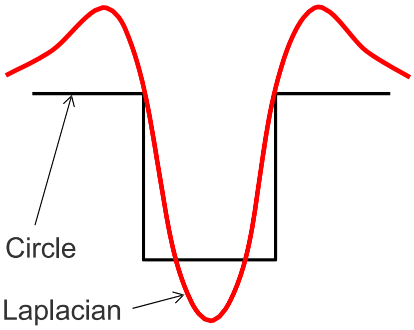
$$LoG(x,y)=\nabla^2 g(x,y)=\frac{\partial^2 g(x,y)}{\partial x^2}+\frac{\partial^2 g(x,y)}{\partial y^2}=\frac{x^2+y^2-2\sigma^2}{2\pi\sigma^6}e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Choice of scale I

- At which scale the answer of the filter becomes maximal?



Circle of radius r in an image



➔ Zero crossing of the filter should coincide with the edge of the circle:

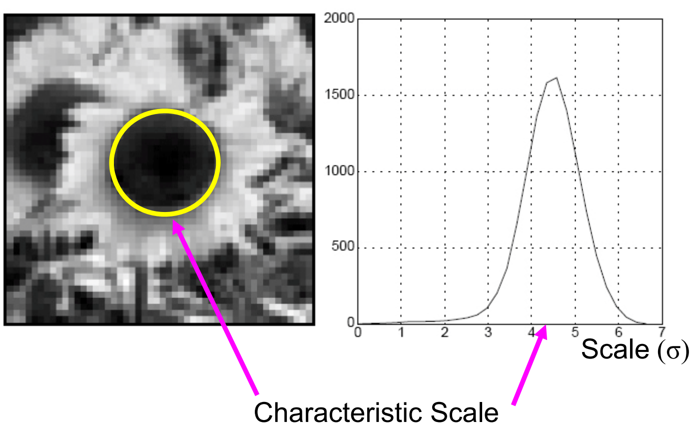
$$\frac{x^2+y^2-2\sigma^2}{2\pi\sigma^6} e^{-\frac{x^2+y^2}{2\sigma^2}} = 0$$

$$x^2+y^2=r^2$$

$$\Rightarrow \sigma = r/\sqrt{2}$$

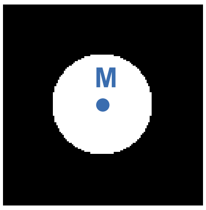
Choice of scale II

- Definition: The scale that coincides with maximum response of blob detector is called characteristic scale sig=r/1.414



T. Lindeberg (1998). "Feature detection with automatic scale selection." *International Journal of Computer Vision* **30** (2): pp 77--116.

Blob extraction: Example



Original image radius of circle: 32 pixel

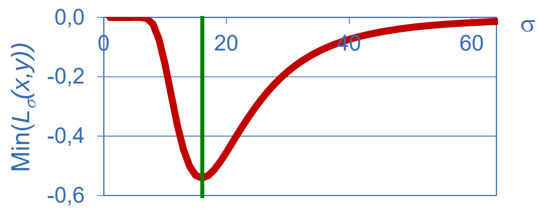
$L_\sigma(x,y) = \Delta L_\sigma(x,y)$ for five values of σ:

$\sigma = 1$ $\sigma = 4$ $\sigma = 8$ $\sigma = 16$ $\sigma = 32$

Values of $L_\sigma(x,y)$ in **M**:

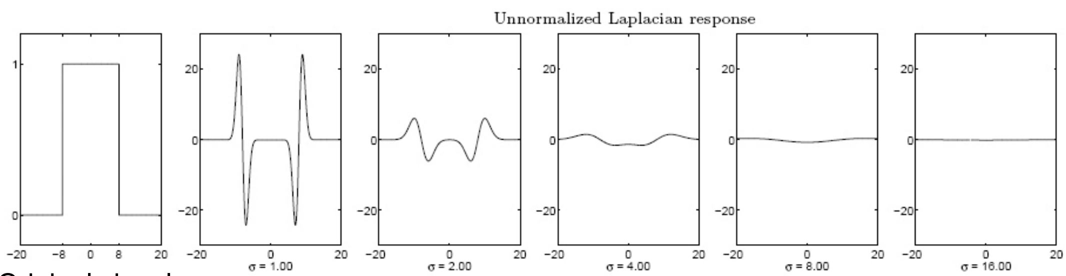
Absolute minimum at $\sigma = 16$

Results in $r = 22.6$ instead 32!?



What is the problem?

- If we repeatedly convolve the image with LoG functions of rising σ, we encounter a surprising effect:
  - The stronger the smoothing, the flatter the resulting curve!



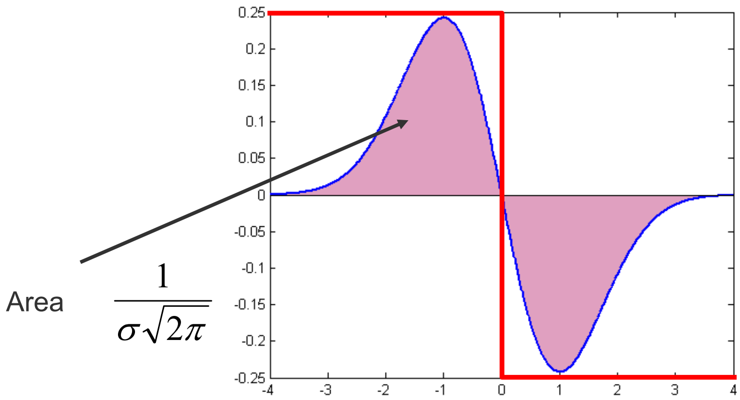
Original signal (Radius=8)

rising σ

"Extremum" small

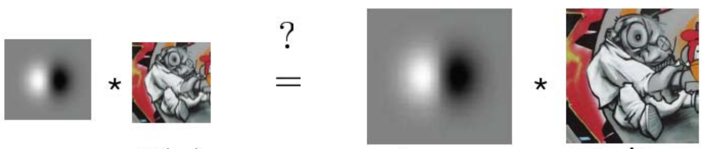
Normalization of scale I

- Below we see a perfect step edge together with the 1. derivation of a Gaussian curve.
- For rising σ the integral of the area under curve becomes smaller.
- This effect needs to be compensated.



Normalization of scale II

- In the example shown we enlarge scale σ and image x stepwise by factor s.
- This enlargement leads to stronger smoothing as discussed.
- However, we want that the derivation leads to same effect over all scales.



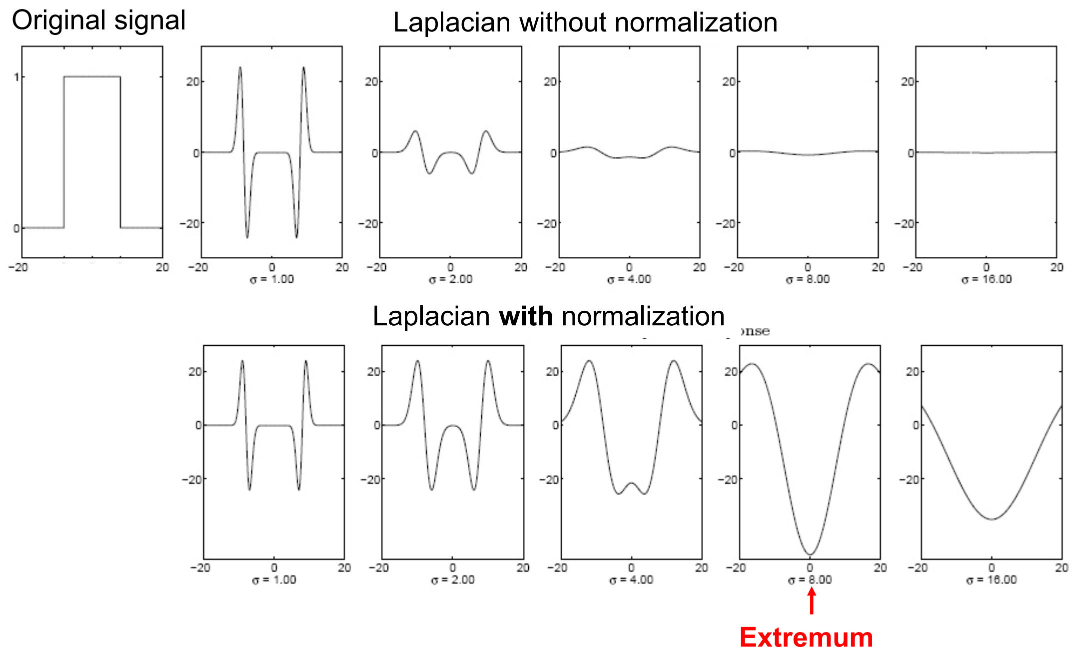
$$x' = sx \qquad \sigma * I(x) \overset{?}{=} \sigma' = s\sigma * I(x') = I(sx)$$

- then: $$\frac{\partial}{\partial x} I(x') = \frac{\partial}{\partial x} I(sx) = \boxed{s} \frac{\partial}{\partial x} I(x)$$

➔ Normalization necessary: For each derivation we have to multiply with s, in case of Laplacian therefore with s^2.

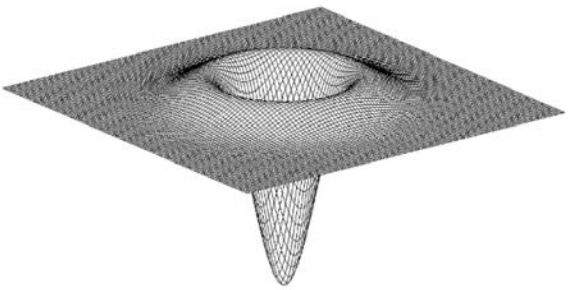
Quelle: Stefan Roth, TU DA

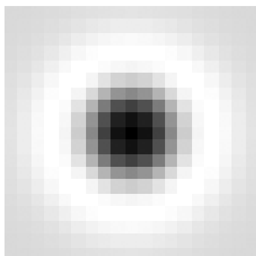
Effect of normalization



Let us turn to 2D again

- Laplacian of Gaussian (LoG): Rotationally symmetric blob detector

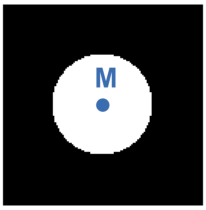




Normalized scale

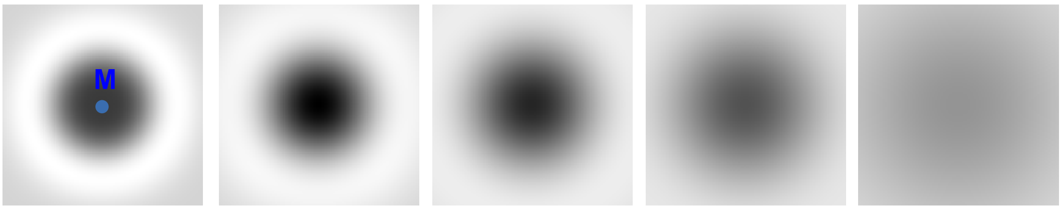
$$\nabla^2_{norm} g = \sigma^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$$

Blob extraction: Example revisited



Original image radius of circle: 32 pixel

$\Delta_{norm}\, L_\sigma(x,y) = \sigma^2 \cdot \Delta\, L_\sigma(x,y)$ for five values of σ:

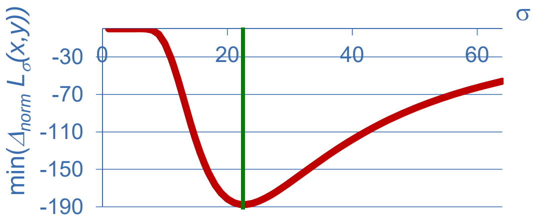


$\sigma = 16$ $\sigma = 23$ $\sigma = 32$ $\sigma = 41$ $\sigma = 64$

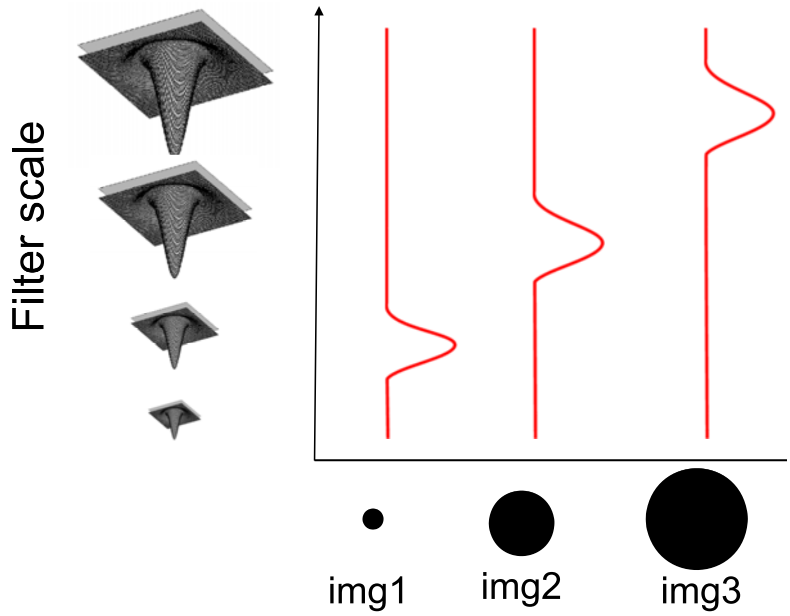
$\Delta_{norm}\, L_\sigma(x,y)$ at point **M**

Absolute minimum at $\sigma = 22.6$

$$r = \sqrt{2} \cdot \sigma = 32$$

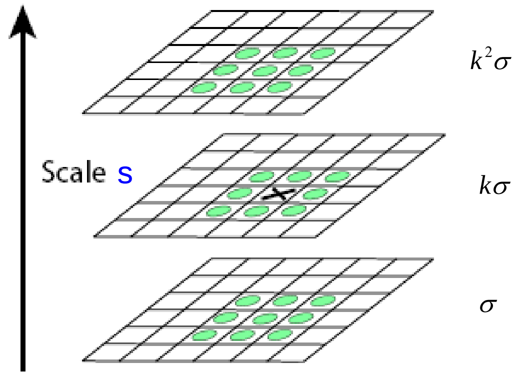


Choice of scale III



Search for characteristic scale (SIFT)

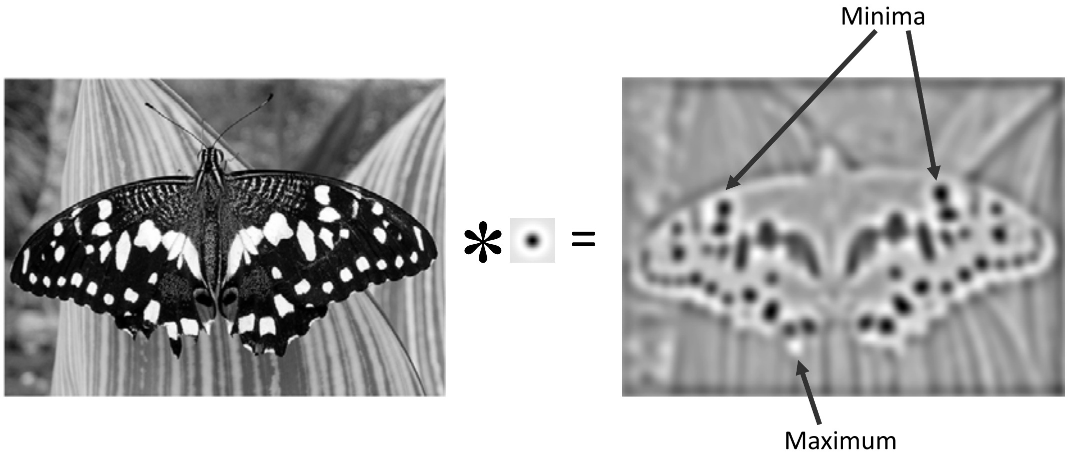
- We built a LoG or DoG scale space.
- Extrema in DoG stack are hints to salient points of respective characteristic scale s.
  (Detail of SIFT in Computer Vision)



Example: An image of a butterfly

Example: We filter the image with LoG

- “Blob” detector



- We search *maxima and minima* of LoG response with respect to scale and location.

Example: The scale space (LoG images)

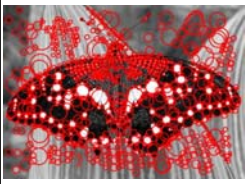
sigma = 2

sigma = 6.126

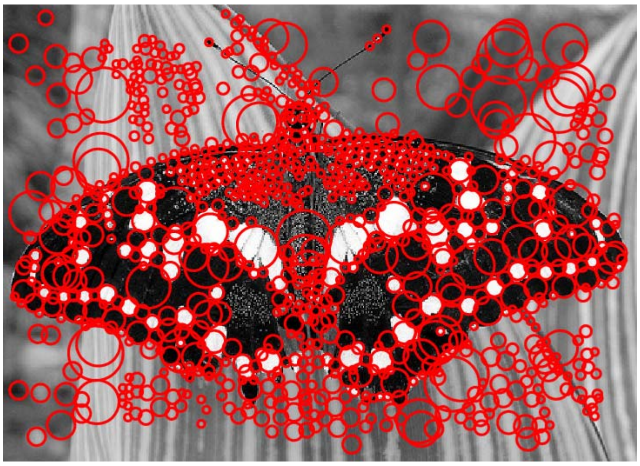
sigma = 9.5859

sigma = 15

Final result



$\Delta\sigma$



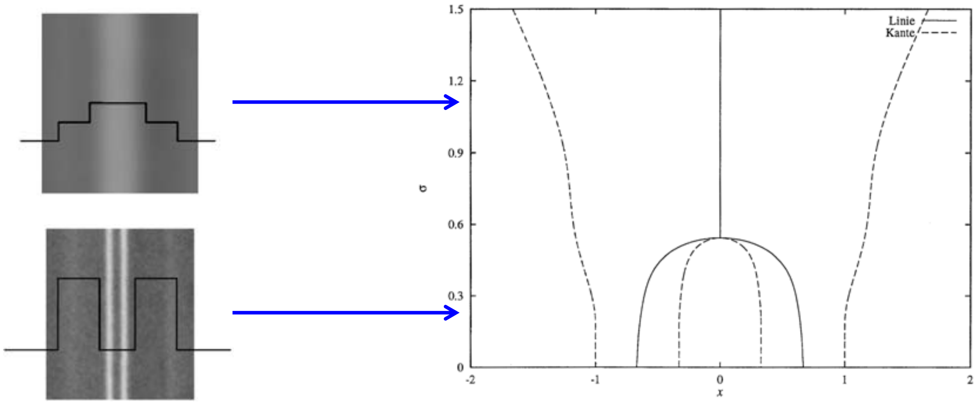
- In smaller scale $\Delta\sigma$ the corresponding features show-up shifted in scale space also by $\Delta\sigma$!

Contents

- Introduction
- Linear Scale Space
- Scale Space Events
- Blob Detection
- Examples

Example: Roads in different scales, simulation

- In across track direction roads show an interesting scale space behavior.
- Scale change comes along with abstraction
- Analytical example: Two-lane road (parallel lines)



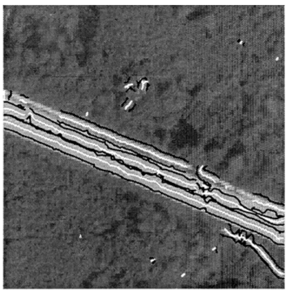
Line and edge position

- Scale change: The stripes of two-lane road merges to one line

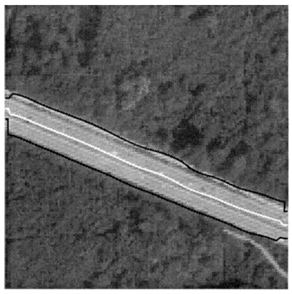
Example: Roads in different scales, real data

- This scale behavior requires to tailor the model for object extraction (i.e., which objects do we see, how do they look like) as a function of scale
  - Large scale→ Details visible → 2 lanes
  - Small scale → Details vanish → 1 lane

GSD = 4 m



σ = 5.6 m

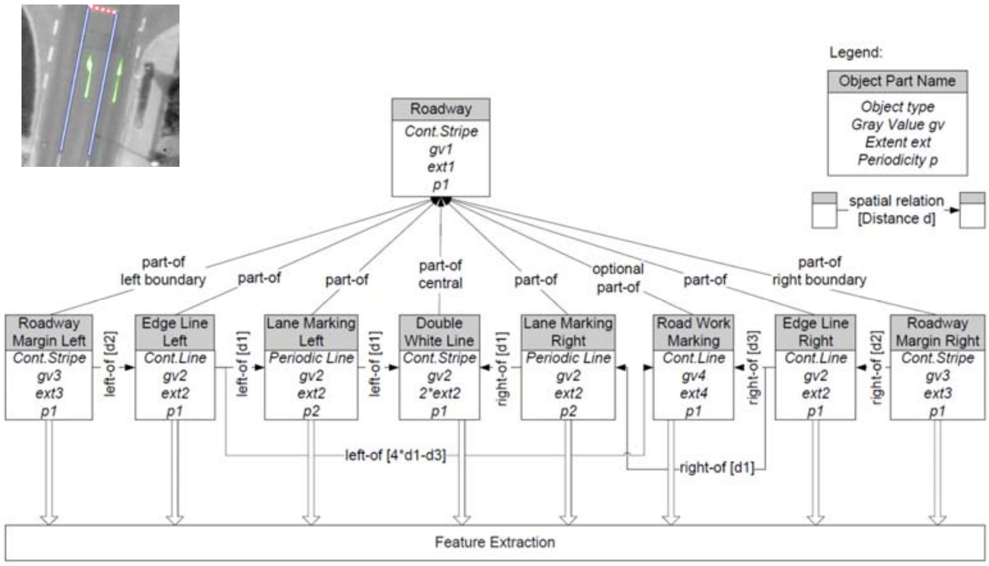


σ = 18 m

white: road axis; black: road margins

Example: Adaption of model to scale I

- Semantic net for large scale



Example: Adaption of model to scale II

Semantic net for medium scale

Semantic net for small scale