Computer Vision Exercise 3

Epipolar Lines

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1. Fundamental matrix

$$(x_1^{cam} \quad y_1^{cam} \quad -c) \cdot \mathbf{E} \cdot \begin{pmatrix} x_2^{cam} \\ y_2^{cam} \\ -c \end{pmatrix} = 0$$

$$\mathbf{E} = \left(\begin{pmatrix} 0 & -b_z & b_y \\ b_z & 0 & -b_x \\ -b_y & b_x & 0 \end{pmatrix} \mathbf{R}'_z \right) = [\mathbf{B}] \cdot \mathbf{R}_\mathbf{i}^z = [-\mathbf{R_z}(\mathbf{X_z^o} - \mathbf{X_i^o})] \cdot (\mathbf{R_z} \cdot \mathbf{R}_\mathbf{i}^r)$$

$$\begin{pmatrix} x_1^{pix} & y_1^{pix} & 1 \end{pmatrix} \cdot \mathbf{F} \cdot \begin{pmatrix} x_2^{pix} \\ y_2^{pix} \\ 1 \end{pmatrix} = \mathbf{x'}_{pix}{}^T \cdot \mathbf{F} \cdot \mathbf{x}_{pix} = 0$$

$$\mathbf{F} = \left(\mathbf{K}'^{-1}\right)^T \cdot \mathbf{E} \cdot \mathbf{K}^{-1} = \left(\mathbf{K}'^{-1}\right)^T \cdot \left[-\mathbf{R}_2\left(\mathbf{X}_2^0 - \mathbf{X}_1^0\right)\right]_\times \cdot \left(\mathbf{R}_2 \cdot \mathbf{R}_1^T\right) \cdot \mathbf{K}^{-1}$$

Where E is essential matrix (mapping of camera coordinates) and F is fundamental matrix (mapping of pixel coordinates)

2. Epipolar lines

The formula

$$\mathbf{I}' = \mathbf{F}\mathbf{x}$$

provides all potential positions of corresponding point x', then homogenous coordinates x' of points on line l' are computed by normal form

$$\mathbf{I}' \cdot \mathbf{x}' = \begin{pmatrix} l'_x \\ l'_y \\ l'_z \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = l'_x \cdot x' + l'_y \cdot y' + l'_z = 0$$

From which we can get the epipolar line:

$$y'(x') = \frac{-l'_x \cdot x' - l'_z}{l'_y}$$

3. Result

As is shown in Fig1, we select 3 points in 20774.

Fig1. Image 20774

Then we get the fundamental matrix:

$$\begin{pmatrix} -2.26e-07 & 6.20e-08 & -0.0069 \\ -5.79e-08 & 5.16e-09 & 0.00034 \\ 0.0078 & -0.00091 & -3.7016 \end{pmatrix}$$

With which we get the epipolar lines: