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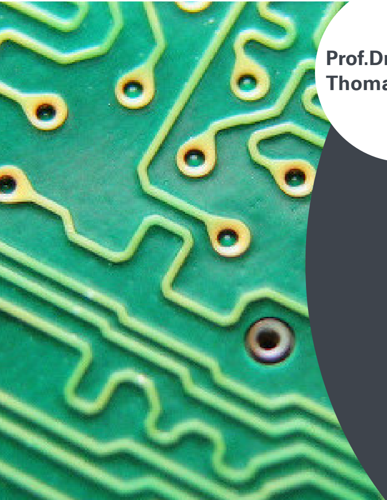


Prof.Dr.
Thomas Hobiger

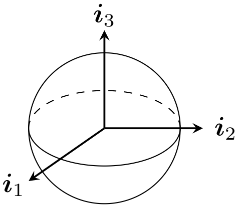
Integrated Positioning and Navigation

Coordinate Systems

4

Coordinate Systems

All coordinate systems used in these notes are right-handed orthogonal systems



Quasi-inertial coordinate system
(i-system; $\boldsymbol{i}_1$,$\boldsymbol{i}_2$, $\boldsymbol{i}_3$ axes)

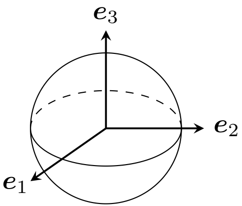
Inertial system: no translational acceleration of origin, no rotation of axes w.r.t. inertial space

Quasi inertial system: origin in geo-centre, no rotation of axes w.r.t. inertial space

$\boldsymbol{i}_1$ axis: towards Aries (γ) / "Vernal equinox"
$\boldsymbol{i}_3$ axis: rotation axis of the earth
$\boldsymbol{i}_2$ axis: complements right-handed orthogonal system

Newton's laws of motion are valid in an inertial system. Replacing the inertial system with a quasi-inertial system leads to mis-modelling effects of $10^{-7}\ g$ (g $\sim$9.81 m/s^2). These can be neglected here.

Coordinate Systems - cont'd



Earth-centered earth-fixed coordinate system (e-system; $\boldsymbol{e}_1$,$\boldsymbol{e}_2$, $\boldsymbol{e}_3$ axes)

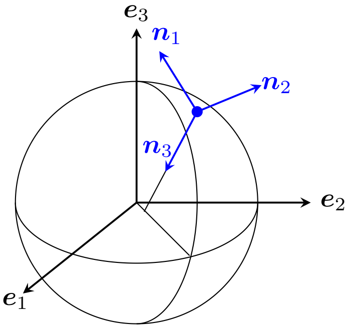
Rigidly attached to the earth; deformations of the earth are negligible (in the context of this course!)

Origin in geo-centre, no rotation of axes w.r.t. the earth's surface

$\boldsymbol{e}_1$ axis: in the meridian of Greenwich (zero longitude)
$\boldsymbol{e}_3$ axis: rotation axis of the earth (= $\boldsymbol{i}_3$ axis)
$\boldsymbol{e}_2$ axis: complements right-handed orthogonal system

The earth-centered earth-fixed system can be used to describe the position and orientation of objects in the vicinity of the earth's surface.

Coordinate Systems - cont'd



Local level coordinate system
(n-system; $\boldsymbol{n}_1$,$\boldsymbol{n}_2$, $\boldsymbol{n}_3$ axes)

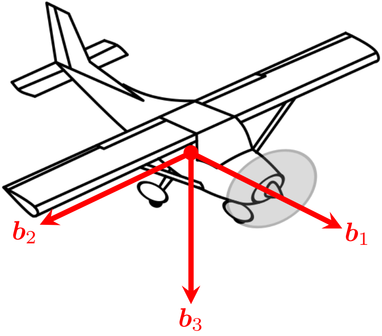
Origin in a point of interest near the earth surface (e.g. the IMU)

$\boldsymbol{n}_1$ axis: towards North (on the ellipsoid)
$\boldsymbol{n}_2$ axis: towards East (on the ellipsoid)
$\boldsymbol{n}_3$ axis: downwards ellipsoidal normal

($\boldsymbol{n}_1$ towards East, $\boldsymbol{n}_2$ towards North, $\boldsymbol{n}_3$ outward also in use!)

The local level coordinate system can be used to describe the orientation of objects in the vicinity of the earth's surface in terms of azimuth (yaw angle), roll angle, and pitch angle.

Coordinate Systems - cont'd



Body coordinate system
(b-system; b_1,b_2, b_3 axes)

Rigidly attached to the body of the vehicle to be positioned and oriented. Origin in the centre of gravity of the vehicle

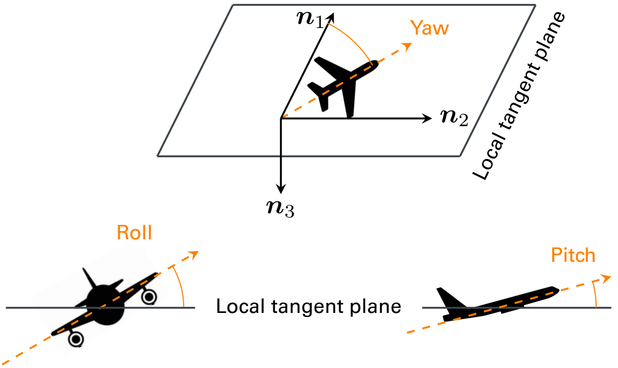
$\boldsymbol{b}_1$ axis: roll axis, pointing forward
$\boldsymbol{b}_2$ axis: pitch axis, to the right
$\boldsymbol{b}_3$ axis: downward
All directions w.r.t. the vehicle!

($\boldsymbol{b}_2$ forward, $\boldsymbol{b}_1$ right and $\boldsymbol{b}_3$ upwards also in use!)

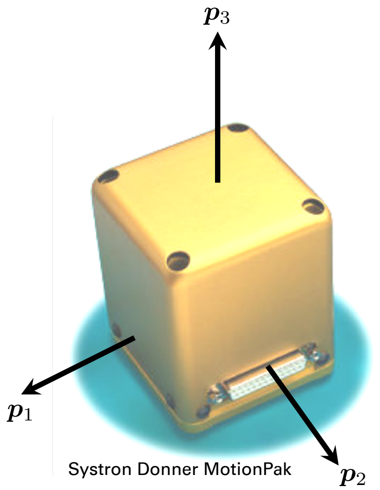
The directions of the body coordinate system describe the orientation of the vehicle in space.

Coordinate Systems - cont'd

Transformation: Body coordinate system to local level system



Coordinate Systems - cont'd



Systron Donner MotionPak

Platform coordinate system
(p-system; $\boldsymbol{p}_1$,$\boldsymbol{p}_2$, $\boldsymbol{p}_3$ axes)

Rigidly attached to the casing of the Inertial Measurement Unit (IMU).
Origin in the centre of the IMU assembly

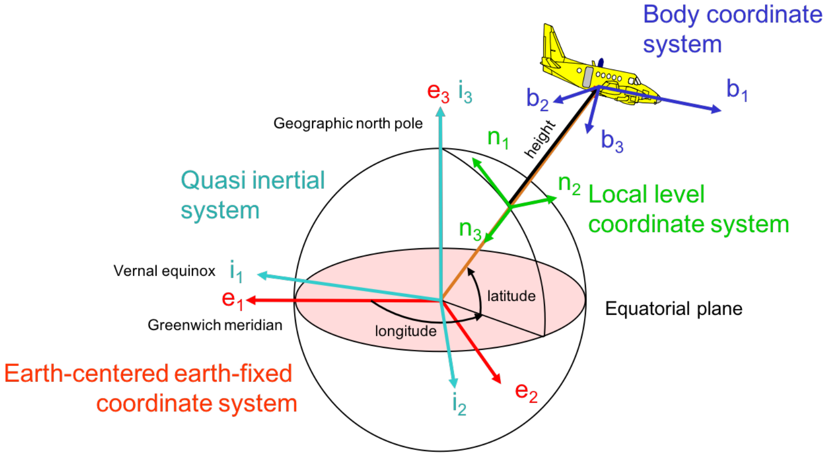
$\boldsymbol{p}_1$ axis: (x)-accelerometer/gyro
$\boldsymbol{p}_2$ axis: (y)-accelerometer/gyro
$\boldsymbol{p}_3$ axis: (z)-accelerometer/gyro

If the platform is rigidly attached to the vehicle, the orientation of the p-system is constant w.r.t. the b-system.

($\boldsymbol{b}_2$ forward, $\boldsymbol{b}_1$ right and $\boldsymbol{b}_3$ upwards also in use!)

Acceleration and rotation rate measurements of the IMU (after calibration) are along the platform coordinate system axes.

Coordinate Systems - cont'd



Coordinate Systems - cont'd

Transformation: Quasi-inertial to earth-centered earth-fixed system

Rotation of e-system w.r.t. i-system approximated by constant rotation rate about the $\boldsymbol{i}_1$ axis (deviations can be neglected here!)

$$\boldsymbol{\omega}_{ie}^{e} = \boldsymbol{\omega}_{ie}^{i} = [0 \;\; 0 \;\; \omega_E]^T \tag{4.1}$$

The resulting DCM can be written (see equ. (2.3)):

$$\boldsymbol{C}_i^e = \begin{bmatrix} \cos(\omega_E(t-t_0)) & \sin(\omega_E(t-t_0)) & 0 \\ -\sin(\omega_E(t-t_0)) & \cos(\omega_E(t-t_0)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.2}$$

Skew-symmetric matrix representation of angular velocity vector coordinates:

$$\boldsymbol{\Omega}_{ie}^{e} = \begin{bmatrix} 0 & -\omega_E & 0 \\ \omega_E & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.3}$$

Coordinate Systems - cont'd

Transformation: Local level to earth-centered earth-fixed system

1. Rotation about $\boldsymbol{n}_2$-axis with angle $(\phi + \pi/2)$, ϕ is latitude
2. Rotation about $\boldsymbol{n}_3$-axis with angle $(-\lambda)$, λ is longitude

The resulting DCM can be written (see equ. (2.2) and (2.3)):

$$\boldsymbol{C}_n^e = \boldsymbol{C}(3, -\lambda) \cdot \boldsymbol{C}(2, \phi + \pi/2) = \begin{bmatrix} -\sin\phi\cos\lambda & -\sin\lambda & -\cos\phi\cos\lambda \\ -\sin\phi\sin\lambda & \cos\lambda & -\cos\phi\sin\lambda \\ \cos\phi & 0 & -\sin\phi \end{bmatrix} \tag{4.4}$$

Skew-symmetric matrix representation of angular velocity vector coordinates using equ. (3.12):

$$\dot{\boldsymbol{C}}_n^e = \boldsymbol{C}_n^e \cdot \boldsymbol{\Omega}_{en}^n \Rightarrow \boldsymbol{C}_e^n \cdot \dot{\boldsymbol{C}}_n^e = \boldsymbol{C}_e^n \cdot \boldsymbol{C}_n^e \cdot \boldsymbol{\Omega}_{en}^n \Rightarrow \boldsymbol{\Omega}_{en}^n = \boldsymbol{C}_e^n \cdot \dot{\boldsymbol{C}}_n^e \tag{4.5}$$

$$\boldsymbol{\Omega}_{en}^n = \begin{bmatrix} 0 & \sin\phi \cdot \dot{\lambda} & -\dot{\phi} \\ -\sin\phi \cdot \dot{\lambda} & 0 & -\cos\phi \cdot \dot{\lambda} \\ \dot{\phi} & \cos\phi \cdot \dot{\lambda} & 0 \end{bmatrix} \tag{4.6}$$

Coordinate Systems - cont'd

Transformation: Body coordinate system to local level system

1. Rotation about $\boldsymbol{b}_1$-axis with angle $(-R)$, R is roll angle
2. Rotation about $\boldsymbol{b}_2$-axis with angle $(-P)$, P is pitch angle
3. Rotation about $\boldsymbol{b}_3$-axis with angle $(-Y)$, Y is yaw angle

The resulting DCM can be written:

$$\boldsymbol{C}_b^n = \boldsymbol{C}(3, -Y) \cdot \boldsymbol{C}(2, -P) \cdot \boldsymbol{C}(1, -R) \tag{4.7}$$

$$\boldsymbol{C}_b^n = \begin{bmatrix} \cos Y \cos P & \cos Y \sin P \sin R - \sin Y \cos R & \cos Y \sin P \cos R + \sin Y \sin R \\ \sin Y \cos P & \sin Y \sin P \sin R + \cos Y \cos R & \sin Y \sin P \cos R - \cos Y \sin R \\ -\sin P & \cos P \sin R & \cos P \cos R \end{bmatrix} \tag{4.8}$$

The skew-symmetric matrix representation of angular velocity vector coordinates can be obtained applying equ. (3.12):

$$\dot{\boldsymbol{C}}_b^n = \boldsymbol{C}_b^n \cdot \boldsymbol{\Omega}_{nb}^b \Rightarrow \boldsymbol{C}_n^b \cdot \dot{\boldsymbol{C}}_b^n = \boldsymbol{C}_n^b \cdot \boldsymbol{C}_b^n \cdot \boldsymbol{\Omega}_{nb}^b \Rightarrow \boldsymbol{\Omega}_{nb}^b = \boldsymbol{C}_n^b \cdot \dot{\boldsymbol{C}}_b^n \tag{4.9}$$

Coordinate Systems - cont'd

Transformation: Platform coordinate system to body coordinate system

In general: Three-axes rotation accounting for the differences in orientation

$\Rightarrow$ See DCM from equ. (2.6)

If the IMU platform is (approximately) aligned to the body coordinate system, the DCM may be approximated by equ. (2.8) for small angle rotations:

$$\boldsymbol{C}_b^n = \begin{bmatrix} 1 & \alpha_3 & -\alpha_2 \\ -\alpha_3 & 1 & \alpha_1 \\ \alpha_2 & -\alpha_1 & 1 \end{bmatrix} \tag{4.10}$$

There is no angular velocity between the p-system and the b-system.

Coordinate Systems - cont'd

Accelerometer and Gyroscope Measurements

Most modern Inertial Measurement Units (IMU) provide digital readouts at regular time intervals

Readouts (measurements) at time t_k are integrals (sums) of the sensed specific force and rotational velocity since the last readout at time t_{k-1}

Velocity increment: $$\Delta \boldsymbol{v}^p(t_k) = \int_{t_{k-1}}^{t_k} \boldsymbol{a}^p(\tau) d\tau$$

Angular increment: $$\Delta \boldsymbol{\alpha}_{ip}^p(t_k) = \int_{t_{k-1}}^{t_k} \boldsymbol{\omega}_{ip}^p(\tau) d\tau$$

- Measurement output rates are 50Hz – 200Hz
- Internal sampling rate can be in kHz-range
- Any non-linear platform motion within readout interval cannot be recovered from readout data