Exercise on <u>08.01.2020</u>

Task 1 *(3 Points)*

In the lecture the relation between Geographic and Cartesian coordinates have been presented in eq. (6.19):

$$\begin{bmatrix} x_1^e \\ x_2^e \\ x_3^e \end{bmatrix} = \begin{bmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ [N(1-e^2)+h]\sin\phi \end{bmatrix}$$

Calculate the time deviations of x_1^e, x_2^e and x_3^e and show the correctness of eq. (6.20):

$$\begin{bmatrix} \dot{x}_1^e \\ \dot{x}_2^e \\ \dot{x}_3^e \end{bmatrix} = \begin{bmatrix} -\dot{\phi}(M+h)\sin\phi\cos\lambda - \dot{\lambda}(N+h)\cos\phi\sin\lambda + \dot{h}\cos\phi\cos\lambda \\ -\dot{\phi}(M+h)\sin\phi\sin\lambda + \dot{\lambda}(N+h)\cos\phi\cos\lambda + \dot{h}\cos\phi\sin\lambda \\ \dot{\phi}(M+h)\cos\phi + \dot{h}\sin\phi \end{bmatrix}$$

Proposal for solution 1

$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$
$$\frac{\partial N}{\partial \phi} = \frac{a \cdot e^2 \sin\phi\cos\phi}{(1-e^2\sin^2\phi)^{3/2}}$$

$$\frac{d}{dt}x_i^e = \frac{\partial x_i^e}{\partial h}\frac{dh}{dt} + \frac{\partial x_i^e}{\partial \phi}\frac{d\phi}{dt} + \frac{\partial x_i^e}{\partial \lambda}\frac{d\lambda}{dt} + \frac{\partial x_i^e}{\partial t} \tag{1}$$

First entry $\dot{x}_1^e$:
Use equation 1. All derivations trivial except:

$$\begin{aligned} \frac{\partial x_1^e}{\partial \phi} &= \frac{\partial}{\partial \phi}(N\cos\phi\cos\lambda) + \frac{\partial}{\partial \phi}(h\cos\phi\cos\lambda) = \\ &= ... = -\left(\frac{a\cdot(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}} + h\right)\sin\phi\cos\lambda = \\ &= -(M+h)\sin\phi\cos\lambda \end{aligned}$$

Second entry $\dot{x}_2^e$:

See $\dot{x}_1^e$

Third entry $\dot{x}_3^e$:
Use equation 1. Again all derivations trivial except:

$$\begin{aligned} \frac{\partial x_3^e}{\partial \phi} &= \frac{\partial}{\partial \phi}(N(1-e^2)\sin\phi) + \frac{\partial}{\partial \phi}(h\sin\phi) = \\ &= ... = \left(\frac{a\cdot(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}} + h\right)\cos\phi = \\ &= (M+h)\cos\phi \end{aligned}$$

Task 2 *(3 Points)*

The Coriolis acceleration is described by:

$$\mathbf{a}_{\text{cor}} = -2\,\Omega_{ie}^{e} \cdot \mathbf{v}^{e}$$

derive $||\mathbf{a}_{\text{cor}}||$ depending on the (North-)Azimuth, for a velocity of $1100\,\mathrm{km\,h^{-1}}$ and located at

i) the equator

ii) a latitude of $39°$

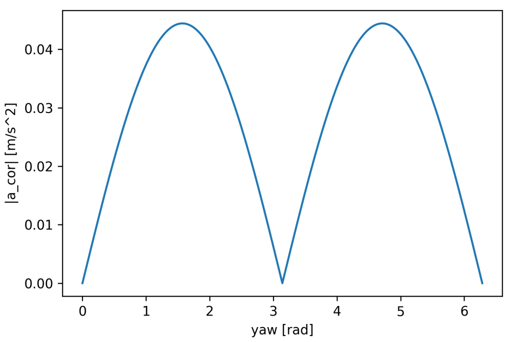
and a height of 9.2 km above WGS84. (Note: $\omega_E = 2\pi/86400\mathrm{s}^{-1}$)

Proposal for solution 2

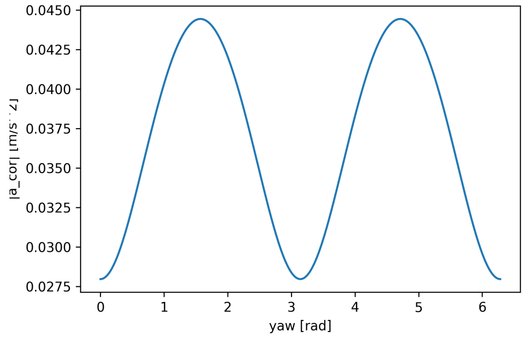
$$\dot{h} = 0 \qquad v_E = v \cdot \sin Y \qquad v_N = v \cdot \cos Y \tag{2}$$

$$\begin{aligned}
\mathbf{a}_{\text{cor}} &= -2\,\Omega_{ie}^{e} \cdot \mathbf{v}^{e} = -2 \begin{bmatrix} 0 & -\omega_E & 0 \\ \omega_E & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1^e \\ \dot{x}_2^e \\ \dot{x}_3^e \end{bmatrix} = 2\omega_E \begin{bmatrix} \dot{x}_2^e \\ -\dot{x}_1^e \\ 0 \end{bmatrix} = \\
&= 2\omega_E \begin{bmatrix} -\dot{\phi}(M+h)\sin\phi\sin\lambda + \dot{\lambda}(N+h)\cos\phi\cos\lambda + \dot{h}\cos\phi\sin\lambda \\ \dot{\phi}(M+h)\sin\phi\cos\lambda + \dot{\lambda}(N+h)\cos\phi\sin\lambda - \dot{h}\cos\phi\cos\lambda \\ 0 \end{bmatrix} = \\
&= 2\omega_E \begin{bmatrix} -v_N\sin\phi\sin\lambda + v_E\cos\phi\cos\lambda + \dot{h}\cos\phi\sin\lambda \\ v_N\sin\phi\cos\lambda + v_E\cos\phi\sin\lambda - \dot{h}\cos\phi\cos\lambda \\ 0 \end{bmatrix} = \\
&\overset{\text{use (2)}}{=} 2\omega_E \begin{bmatrix} -v\cos Y\sin\phi\sin\lambda + v\sin Y\cos\phi\cos\lambda \\ v\cos Y\sin\phi\cos\lambda + v\sin Y\cos\phi\sin\lambda \\ 0 \end{bmatrix} \\
\rightarrow ||\mathbf{a}_{\text{cor}}|| &= \cdots = 2 \cdot \omega_E \cdot v\sqrt{\sin^2\phi\cos^2 Y + \sin^2 Y}
\end{aligned}$$

i) Graph at the equator:



ii) Graph at latitude of 39°



Task 3 *(4 Points)*

Given is the following DCM:

$$C_p^e = \begin{bmatrix} -0.90680 & 0.41785 & -0.05585 \\ -0.34785 & -0.66680 & 0.65908 \\ 0.23815 & 0.61708 & 0.75000 \end{bmatrix}$$

Integrate the following differential equation over a time $n = 1 \ldots 200\,\mathrm{s}$ with $\Delta t = 1\,\mathrm{s}$ by deriving the start values for the quaternions $q^e_{p0}, q^e_{p1}, q^e_{p2}$ and q^e_{p3} from the DCM above

$$\begin{bmatrix} \dot{q}^e_{p0} \\ \dot{q}^e_{p1} \\ \dot{q}^e_{p2} \\ \dot{q}^e_{p3} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & \omega^p_{ip1} - \omega^p_{ie1} & \omega^p_{ip2} - \omega^p_{ie2} & \omega^p_{ip3} - \omega^p_{ie3} \\ -\omega^p_{ip1} + \omega^p_{ie1} & 0 & \omega^p_{ip3} - \omega^p_{ie3} & -\omega^p_{ip2} + \omega^p_{ie2} \\ -\omega^p_{ip2} + \omega^p_{ie2} & -\omega^p_{ip3} + \omega^p_{ie3} & 0 & \omega^p_{ip1} - \omega^p_{ie1} \\ -\omega^p_{ip3} + \omega^p_{ie3} & \omega^p_{ip2} - \omega^p_{ie2} & -\omega^p_{ip1} + \omega^p_{ie1} & 0 \end{bmatrix} \cdot \begin{bmatrix} q^e_{p0} \\ q^e_{p1} \\ q^e_{p2} \\ q^e_{p3} \end{bmatrix}$$

Also known are:

$$\begin{bmatrix} \omega^p_{ip1} \\ \omega^p_{ip2} \\ \omega^p_{ip3} \end{bmatrix} = \begin{bmatrix} 0.08 \\ 0.02 \\ -0.02 \end{bmatrix} \text{ and } \begin{bmatrix} \omega^p_{ie1} \\ \omega^p_{ie2} \\ \omega^p_{ie3} \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.02 \\ 0.01 \end{bmatrix}$$

Calculate the **euler angles** after each epoch and plot them.

Proposal for solution 3