Computer Vision Exercise 2

Spatial Intersection and Resection

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I. Processing Steps

1.

Compute projection matrix:

```
$ImageID___________________________________________________(ORI_Ver_1.0)
            8919
$IntOri_FocalLength___________________________________________[mm]
             5.98760000
$IntOri_PixelSize_____(x|y)___________________________________[mm]
               0.001530         0.001530
$IntOri_SensorSize____(x|y)_______________________________[pixel]
                   3000              4000
$IntOri_PrincipalPoint_(x|y)__________________________________[pixel]
           1499.50000000                  1999.50000000
$IntOri_CameraMatrix______________________________(ImageCoordinateSystem)
          -3913.4640522      -0.00000000                 1499.50000000
            -0.00000000   3913.46405229                  1999.50000000
             0.00000000      0.00000000                     1.00000000
$ExtOri_RotationMatrix____________________(World->ImageCoordinateSystem)
          0.540702141171              -0.840413805693              -0.036685552114
          0.833792484528               0.541202069167              -0.109043170690
          0.111495682777               0.028371738229               0.993359832685
$ExtOri_TranslationVector_________________________(WorldCoordinateSystem)
           513028.292610              5427687.148050                  490.883330
```

$$\frac{1}{m_x},\frac{1}{m_y}$$

$$\boldsymbol{K} = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} c & & p_x \\ & c & p_y \\ & & 1 \end{bmatrix}$$

$$\mathbf{R}$$

$$\tilde{\mathbf{X}}_0$$

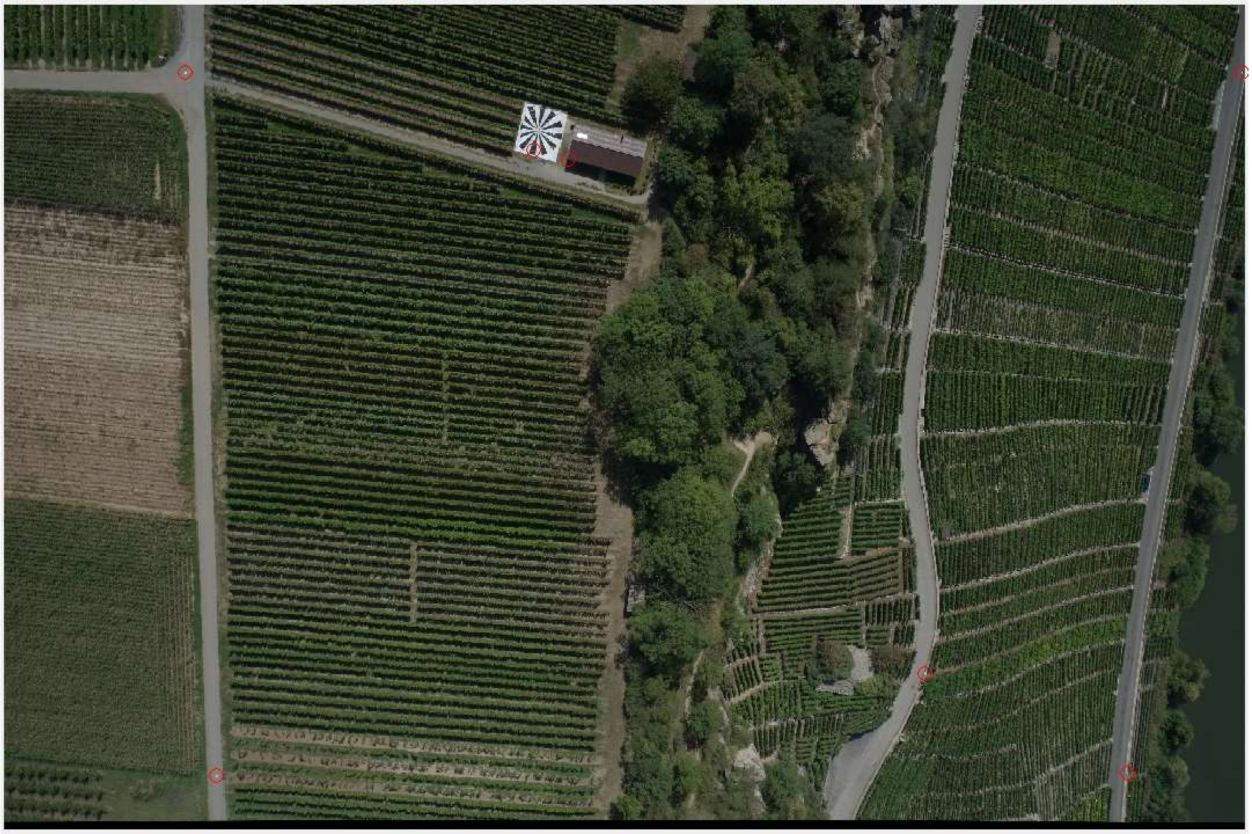
$$[\mathbf{R} \mid \mathbf{t}] = \begin{bmatrix} \mathbf{R} & -\mathbf{R} \cdot \tilde{\mathbf{X}}_0 \end{bmatrix}$$

$$\mathrm{P} = \mathrm{K}[\mathrm{R} \mid \mathrm{t}] = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix}$$

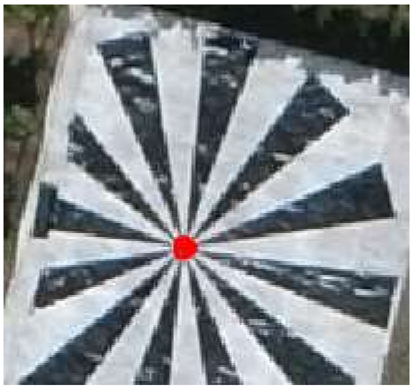
Compute pixel coordinates:

$$\mathbf{x} = \mathbf{P} \cdot \mathbf{X}$$

Plot points:



2. Measure one object



3. Spatial intersection

For unknown object coordinate $\boldsymbol{X}$ at least two pixel measures $\boldsymbol{x}$ and $\boldsymbol{x}'$ from two cameras with known projection matrix $\boldsymbol{P}$ and $\boldsymbol{P}'$are available:

$$\mathbf{x} = \mathbf{P} \cdot \mathbf{X} \qquad \mathbf{x}' = \mathbf{P}' \cdot \mathbf{X}$$

From which we can build identity equation:

$$\mathbf{x} \times (\mathbf{PX}) = \mathbf{0} \qquad \mathbf{x}' \times (\mathbf{P}'\mathbf{X}) = \mathbf{0}$$

With

$$\mathrm{P} = \mathrm{K}\left[\mathrm{R} \mid \mathrm{t}\right] = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix}$$

Then we get

$$\begin{aligned} x(\mathbf{p}^{3\mathrm{T}}\mathbf{X}) - (\mathbf{p}^{1\mathrm{T}}\mathbf{X}) &= 0 \\ y(\mathbf{p}^{3\mathrm{T}}\mathbf{X}) - (\mathbf{p}^{2\mathrm{T}}\mathbf{X}) &= 0 \\ x(\mathbf{p}^{2\mathrm{T}}\mathbf{X}) - y(\mathbf{p}^{1\mathrm{T}}\mathbf{X}) &= 0 \end{aligned} \qquad \begin{aligned} x'(\mathbf{p}'^{3\mathrm{T}}\mathbf{X}) - (\mathbf{p}'^{1\mathrm{T}}\mathbf{X}) &= 0 \\ y'(\mathbf{p}'^{3\mathrm{T}}\mathbf{X}) - (\mathbf{p}'^{2\mathrm{T}}\mathbf{X}) &= 0 \\ x'(\mathbf{p}'^{2\mathrm{T}}\mathbf{X}) - y'(\mathbf{p}'^{1\mathrm{T}}\mathbf{X}) &= 0 \end{aligned}$$

For both x and x’, the third equation is linearly dependent on the other two, therefore we eliminate it and get:

$$\mathbf{AX} = \mathbf{0}$$

where

$$\mathbf{A} = \begin{pmatrix} x\mathbf{p}^{3\mathrm{T}} - \mathbf{p}^{\mathrm{IT}} \\ y\mathbf{p}^{3\mathrm{T}} - \mathbf{p}^{2\mathrm{T}} \\ x'\mathbf{p}'^{3\mathrm{T}} - \mathbf{p}'^{\mathrm{T}} \\ y'\mathbf{p}'^{3\mathrm{T}} - \mathbf{p}'^{2\mathrm{T}} \end{pmatrix} = \begin{pmatrix} x\mathbf{p}(3;\mathrm{i}) - \mathbf{p}(1,:) \\ y\mathbf{p}(3,:) - \mathbf{p}(2,:) \\ x'\mathbf{p}'(3,\vdots) - \mathbf{p}'(1;) \\ y'\mathbf{p}'(3,:) - \mathbf{p}'(2,\vdots) \end{pmatrix}$$

And

$$\mathbf{X} = \begin{pmatrix} X & Y & Z & W \end{pmatrix}^T$$

Solve this equation using singular vector decomposition, we can get the final object coordinates.

4. Back transformation and errors

We apply back transformation with

$$\mathbf{x} = \mathbf{P} \cdot \mathbf{X} \qquad \mathbf{x}' = \mathbf{P}' \cdot \mathbf{X}$$

And calculate the error with

$$\mathbf{v'v} = \sum\left(\mathbf{x}_{meas} - \mathbf{x}_{trafo}\right)^2 \qquad \sigma_0 = \sqrt{\frac{\mathbf{v'v}}{2 \cdot n_{images} - 3}}$$

5. Direct Linear Transformation

For the direct linear transformation we use the following equation, where P matrix is what we need.

$$\mathbf{x}_i \times \mathbf{P}\cdot\mathbf{X}_i = \begin{pmatrix} y_i\mathbf{p}^{3T}\mathbf{X}_i - w_i\mathbf{p}^{2T}\mathbf{X}_i \\ w_i\mathbf{p}^{1T}\mathbf{X}_i - x_i\mathbf{p}^{3T}\mathbf{X}_i \\ x_i\mathbf{p}^{3T}\mathbf{X}_i - y_i\mathbf{p}^{2T}\mathbf{X}_i \end{pmatrix} = 0$$

This can be rewritten into

$$\begin{pmatrix} \mathbf{0}^T & -w_i\mathbf{X}_i^T & y_i\mathbf{X}_i^T \\ w_i\mathbf{X}_i^T & \mathbf{0}^T & -x_i\mathbf{X}_i^T \\ -y_i\mathbf{X}_i^T & x_i\mathbf{X}_i^T & \mathbf{0}^T \end{pmatrix}\begin{pmatrix} \mathbf{p}^1 \\ \mathbf{p}^2 \\ \mathbf{p}^3 \end{pmatrix} = 0$$

The third row is linear dependent on the first two rows, therefore we can eliminate it:

$$\begin{pmatrix} \mathbf{0}^T & -w_i\mathbf{X}_i^T & y_i\mathbf{X}_i^T \\ w_i\mathbf{X}_i^T & \mathbf{0}^T & -x_i\mathbf{X}_i^T \end{pmatrix}\begin{pmatrix} \mathbf{p}^1 \\ \mathbf{p}^2 \\ \mathbf{p}^3 \end{pmatrix} = A_i\mathbf{p} = 0$$

To solve this equation we need more than 6 pairs of points:

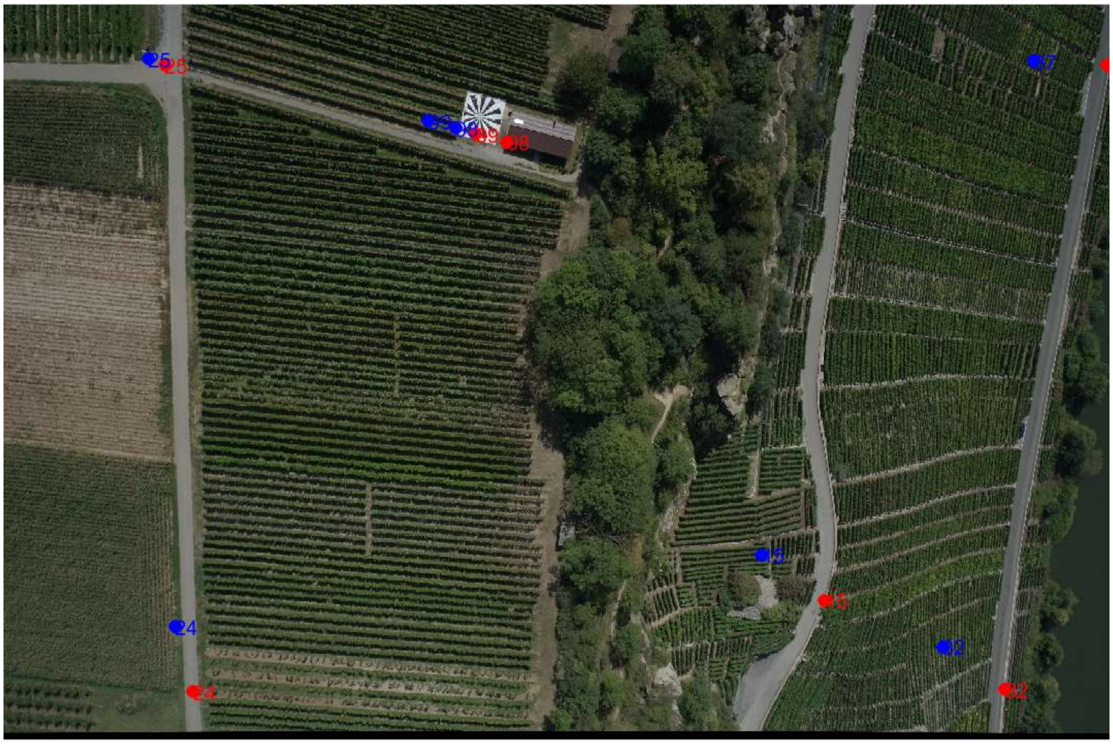
$$\underset{2n\times 12}{\mathbf{A}} \cdot \mathbf{p} = 0$$

Similar to before, we use singular vector decomposition to calculate the P matrix.

$$[\cup, \mathrm{D}, \mathrm{V}] = \mathrm{svd}(\mathrm{A}, 0)$$
$$\%\ \text{Extract homography } P = \text{reshape } (V(:, 12), 4, 3)'$$

6. Re-mapping and comparison

Similar to task 1, $\mathbf{x} = \mathbf{P}\cdot\mathbf{X}$ is used to re-compute the mapping



7. Reconstruct the camera parameters

a) translation vector X0

$\boldsymbol{X}0$ can be computed from Singular Value Decomposition (SVD) of P:

$$[U, D, V] = svd(P, 0)$$

Where X0 is the last column of V

b) camera matrix K and rotation matrix R

$$\boldsymbol{P} = \boldsymbol{K}\left[\boldsymbol{R} \mid \boldsymbol{t}\right] = \boldsymbol{K}\left[\boldsymbol{R} \mid -\boldsymbol{R}\tilde{\boldsymbol{X}}_0\right] = \boldsymbol{K}\boldsymbol{R}\left[\boldsymbol{I}_3 \mid -\tilde{\boldsymbol{X}}_0\right] = \boldsymbol{M}\left[\boldsymbol{I}_3 \mid -\tilde{\boldsymbol{X}}_0\right]$$

Where $\boldsymbol{M} = \boldsymbol{KR}$ is the left 3x3-Sub-Matrix of P, With $M^{-1} = R^T K^{-1}$ matrix $\boldsymbol{M}$ can be decomposed into QR decomposition:

$$[q, r] = qr(M^{-1})$$

And

$$R = q^{-1}$$

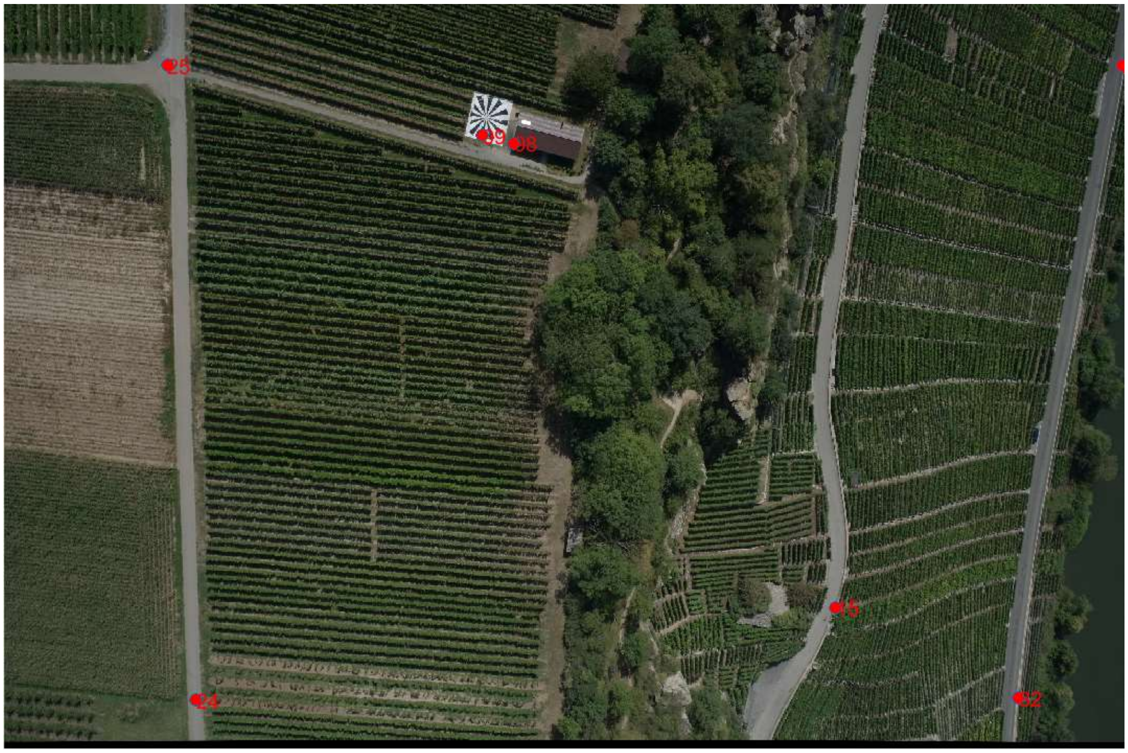
$$K = r^{-1}$$

PS: we have to normalize the K and X0 by the scale factor.

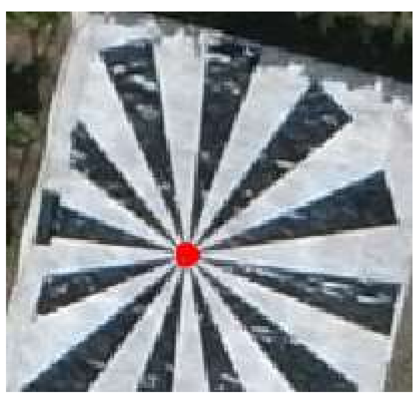
II. Results

1. Fundamental matrix & Pixel coordinates

3.4975e+03	2.0830e+03	1.8694e+03	-1.3101e+10
2.3237e+03	-3.3321e+03	997.5443	1.6893e+10
0.1131	-0.0513	0.9923	2.1995e+05



2. Measure an object



3. Object Coordinates (m)

5.1300e+05
5.4277e+06
327.2105
1

4. Back transformation errors (pixel)

```
sigma0_x =

    4.5173


sigma0_y =

    3.6680
```

5. Direct Linear Transformation

```
P_20851_B =

    0.0000    0.0000    0.0000   -0.6128
    0.0000   -0.0000    0.0000    0.7902
    0.0000   -0.0000    0.0000    0.0000
```

6. Remapping Difference (pixel) & Error (pixel)

243.8114	69.7921	64.1058	242.0076	282.3202	197.3427	182.8119
176.9169	252.3815	24.1761	165.4313	15.5587	54.0080	50.1713

```
sigma0_T6_x =

  159.2771


sigma0_T6_y =

  108.1346
```

7. Reconstruct Camera Parameters

X0=

5.1297e+05
5.4277e+06
536.5390
1

K=

-3.9334e+03	-0.0087	2.1435e+03
0	3.9334e+03	1.4235e+03
0	0	1

R=

```
   -0.8276   -0.5575    0.0655
    0.5499   -0.8286   -0.1055
    0.1131   -0.0513    0.9923
```

Differences:

```
diff_X0 =

  -10.9532
   -1.1280
   21.7447
```

```
diff_K =

   -0.0086   -0.0087    0.0023
         0    0.0017    0.0180
         0         0         0
```

```
diff_R =

   1.0e-05 *

    0.0193   -0.0254    0.0272
   -0.0264    0.0388   -0.4424
    0.2694   -0.3509   -0.0488
```