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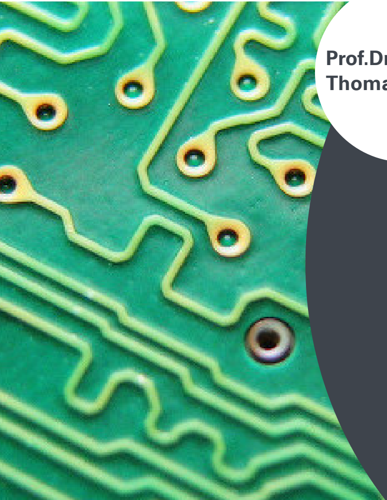


Prof.Dr.
Thomas Hobiger

Integrated Positioning and Navigation

What do Inertial Sensors measure?

5

What do Inertial Sensors measure?

Note: In this module the b-system is assumed to be aligned with the p-system

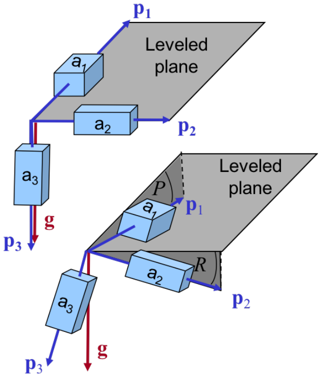
Theoretical Measurements of a stationary Accelerometer triad:

The gravity of the Earth ($\boldsymbol{g}$) is the only signal sensed
Stationary Leveled Accelerometer Triad
Nominal measurements:

$$a_1^p = 0, \quad a_2^p = 0, \quad a_3^p = g$$

Stationary Mis-Leveled Accelerometer Triad
$\boldsymbol{a}^n = [0, 0, g]^T$, $\boldsymbol{a}^p = \boldsymbol{a}^b = (\boldsymbol{C}_b^n)^T \boldsymbol{a}^n$,
for $\boldsymbol{C}_b^n$ see Equ. (4.8)

$$a_1^p = -g \sin P$$
$$a_2^p = g \sin R \cos P$$
$$a_3^p = g \cos R \cos P$$



Actual measurements = Nominal measurements + Errors

What do Inertial Sensors measure? - cont'd

Theoretical Measurements of a stationary Gyro triad:

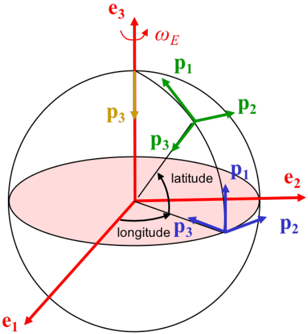
The only rotation acting, is the Earths rotation rate ω_E

Under the Assumption that: $Y = 0°$, $P = 0°$, $R = 0°$

At the pole
$$\omega^p_{ip1} = 0$$
$$\omega^p_{ip2} = 0$$
$$\omega^p_{ip3} = -\omega_E$$

At the equator
$$\omega^p_{ip1} = \omega_E$$
$$\omega^p_{ip2} = 0$$
$$\omega^p_{ip3} = 0$$

At an arbitrary position
$$\omega^p_{ip1} = \omega_E \cdot ?$$
$$\omega^p_{ip2} = 0$$
$$\omega^p_{ip3} = \omega_E \cdot ?$$



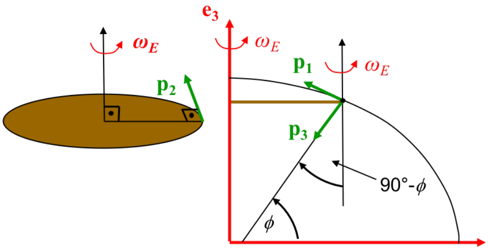
Gyro measurements depend on the latitude!

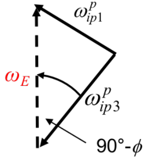
What do Inertial Sensors measure? - cont'd

Theoretical Measurements of a stationary Gyro triad:

Under the Assumption that:
$Y = 0°, P = 0°, R = 0°$

- Because the 2-gyro is located in a plane perpendicular to the Earth's rotation axis, it will not sense any part of ω_E
- The 1- and 3-gyros are in the same plane as the Earth rotation axis, so each of them will sense a part of ω_E





$$\omega_E = \sqrt{(\omega^p_{ip1})^2 + (\omega^p_{ip3})^2}$$

$$\omega^p_{ip1} = \omega_E \sin(90° - \phi) = \omega_E \cos\phi$$

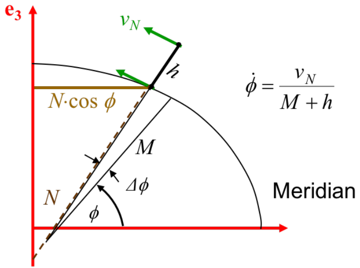
$$\omega^p_{ip3} = -\omega_E \cos(90° - \phi) = -\omega_E \sin\phi$$

What do Inertial Sensors measure? - cont'd

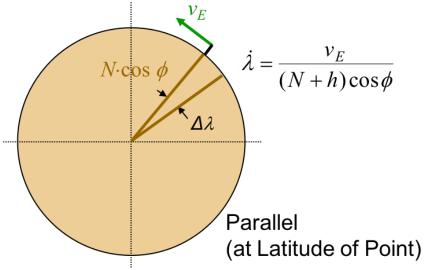
Theoretical Measurements of a moving Gyro triad: Changes in Latitude and Longitude under the assumption, that:

- The b-system (and p-system) is aligned with the n-system and
- The vehicle is moving with velocity v_E and v_N along the east and north directions respectively

Side view



Top view

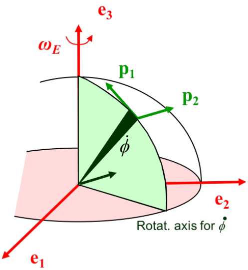


What do Inertial Sensors measure? - cont'd

Theoretical Measurements of a moving Gyro triad:

Angular velocity of the local level system

		Stationary component	non stationary component
$\omega^p_{ip1} =$	$\omega_E \cos\phi + \dot{\lambda}\cos\phi =$	$\omega_E \cos\phi$	$+\dfrac{v_E}{N+h}$
$\omega^p_{ip2} =$	$-\dot{\phi} =$		$-\dfrac{v_N}{M+h}$
$\omega^p_{ip3} =$	$-\omega_E \sin\phi - \dot{\lambda}\sin\phi =$	$-\omega_E \sin\phi$	$-\dfrac{v_E}{N+h}\tan\phi$
$\Uparrow$		$\Uparrow$	$\Uparrow$
$\boldsymbol{\omega}^n_{in}$		$\boldsymbol{\omega}^n_{ie}$	$\boldsymbol{\omega}^n_{en}$



Stationary component will be used for sensor calibration and alignment

What do Inertial Sensors measure? - cont'd

What do we need?

In the previous example the b-system was aligned with the local level system (n-system)!

But, in reality the b-system can take any arbitrary orientation, with respect to the local level system

Thus, at the beginning of the survey, flight, etc.:

- An alignment process is needed to establish the relationship between the body system and the local level system $\Rightarrow$ initial attitude angles R_0, P_0, Y_0 have to be estimated
- Attitude angles are used in generating the initial DCM between the b-system and the n-system

What do Inertial Sensors measure? - cont'd

Initialization - Purpose

Finding initial values for the following integration procedure

$\downarrow$	$\downarrow$	$\downarrow$
position ($\boldsymbol{x}_0$)	velocity ($\boldsymbol{v}_0$)	attitude (Y_0, P_0, R_0)
$\downarrow$	$\downarrow$	$\downarrow$
mostly from external sensors (like GNSS)		alignment
or known values (e.g. zero velocity)		

Alignment:

- Establish the connection between the body system and the local level system
- Consists of two consecutive steps:
  - Accelerometer leveling
  - Gyro compassing

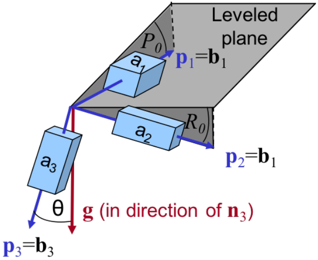
What do Inertial Sensors measure? - cont'd

Initalization - Accelerometer leveling

- Aligns the 3 axes of the accelerometer triad to the 3 axes of the local level system
- Therefore driving the output of the „horizontal" accelerometers to zero

$$\sin P_0 = -\frac{a_1^p}{g}$$

$$\sin R_0 = \frac{a_2^p}{g \cos P_0}$$



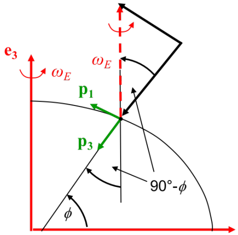
- Assumptions: error free accelerometers, knowledge of g
  $\Rightarrow$ coarse alignment only

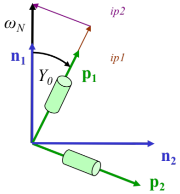
What do Inertial Sensors measure? - cont'd

Initalization - Gyro compassing

- A stationary gyro senses only a component of the Earth rotation rate
- If the gyro is within a leveled plane this component has its maximum when the gyro points towards North (zero when it points East)
- Assumption: Accelerometer leveling is completed

Side view $\omega_N = \omega_E \cos\phi$ Top view





$$
\begin{aligned}
\omega^p_{ip2} &= -\omega_N \cdot \sin Y_0 \\
&= -\omega_E \cdot \cos\phi \cdot \sin Y_0 \\
\omega^p_{ip1} &= \omega_N \cdot \cos Y_0 \\
&= \omega_E \cdot \cos\phi \cdot \cos Y_0 \\
\tan Y_0 &= -\frac{\omega^p_{ip2}}{\omega^p_{ip1}}
\end{aligned}
$$