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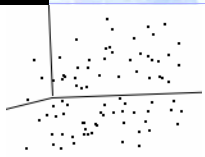
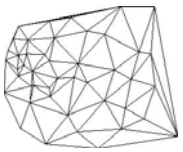
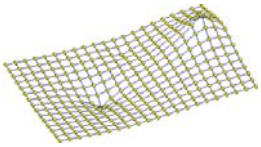
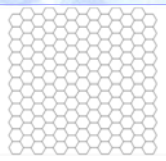
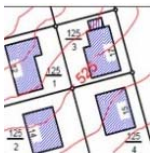
Data Modelling in 3D is much more complex ...


point clouds


2.5D TIN

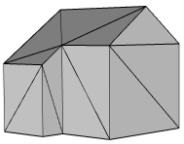

2.5D grid

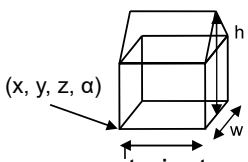

2.5D hexagon


2D + 1D


2.5D

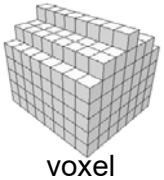

2.5D Voronoi


3D TIN

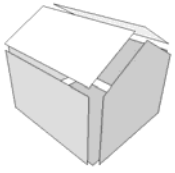


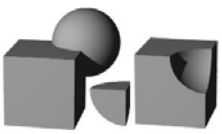
parametr. inst.

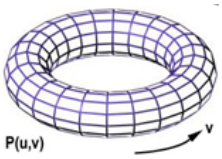

cell decomposition


voxel

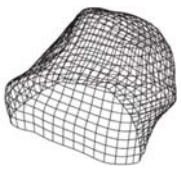

octree


BREP


CSG



sweep rep.


freeform

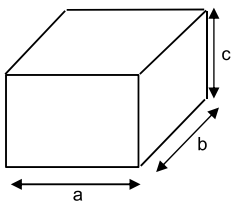
Geometric Modelling of 3D Objects

- Basic 3D Models:
  - Primitive Instancing
  - Cell Decomposition
  - Spatial Occupancy Enumeration
  - Boundary Representation (BREP)
  - Constructive Solid Geometry (CSG)
  - Sweep Representations
  - Freeform Shapes
  - …

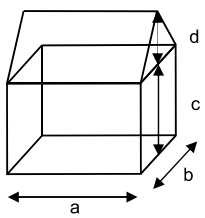
Primitive Instancing

- Define for each Object Type a complete List of Description Parameters
- We need Parameters to describe the **Form** of an Object:

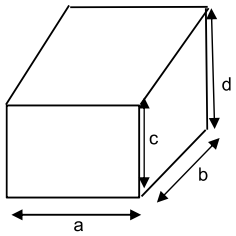
object type: flat roof



object type: sloped roof house

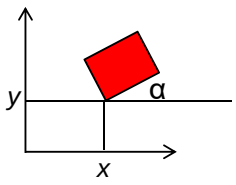


object type: pitch roof house

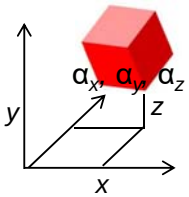


. . .

and the **Position**:



or



Primitive Instancing

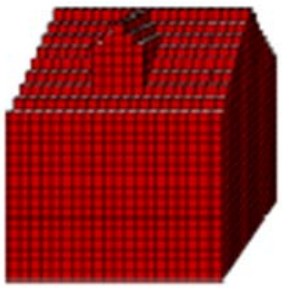
- **Pro**: Simple model, low Data Volume
- **Contra**: You can use only predefined Models -> it is not possible to combine Instances to create new more complex Structures

- In GIS it was used in the Past (15 – 20 years ago) for the Modelling of Houses because at this Time it was only possible to reconstruct simple Objects (Houses) from Point Clouds
  - 20 Models can describe more than 90% of all Houses (in rural Areas)
  - Complex Houses are difficult to describe (inner-city Areas)

- It is the basic Technique for Cell Decomposition and Constructive Solid Modelling (see later)

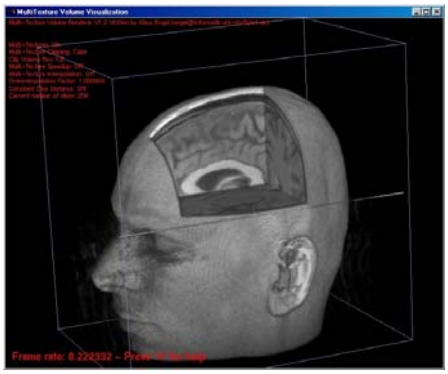
Spatial Occupancy Enumeration

- A 3D Object is modeled by a List of spatial Cells occupied by the Object
- The Cells (or Voxels) are Cubes of a fixed Size and are arranged in a fixed Spatial Grid.
- This Method is analog to raster Modelling in 2D
- **Pro**: Simple Structure (a list of occupied Cells). It is easy to determine if a Point lies inside or outside of an Object.
- **Contra**: Higher Model Accuracy requires higher Number of Cells -> huge Data Volumes



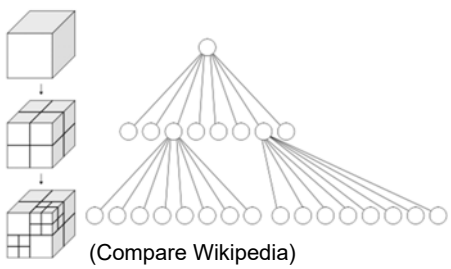
Spatial Occupancy Enumeration

- Representation used primarily for Volume Visualization
- Popular for medical Purposes such as
  - CAT scans
  - Magnetic Resonance Imaging (MRI)

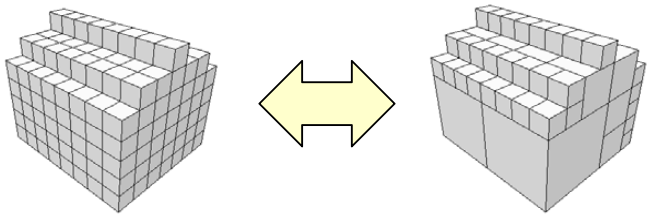


Octtree

- An Octree is a Tree Data Structure in which each (internal) Node has eight Children.
- The Leafs are analog to Voxels in Spatial Occupancy Enumeration
- Octrees are the three-dimensional analog to Quadtrees.
- **Pro**: less Data Volume as Spatial Occupancy Enumeration
- **Contra**: access to the Data is not so easy as in Spatial Occupancy Enumeration



(Compare Wikipedia)



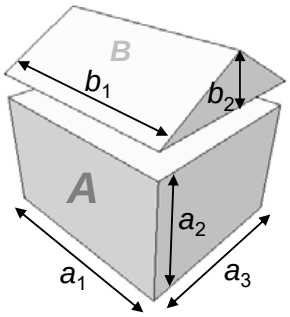
Spatial Occupancy Enumeration

Octree

Cell Decomposition

- *Cell Decomposition* is an Extension of *Spatial Occupancy Enumeration*
- The Cells are not only Cubes but also other Primitives like Prisms, Spheres, Cylinders, Cones, …
- The Cells can optionally be parameterized with *Primitive Instancing*

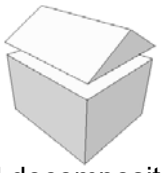
Cell Decomposition
without *Primitive Instancing*
= LEGO



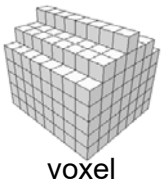
Cell Decomposition
with *Primitive Instancing*

Cell Decomposition

- **Pro**: better Representation of 3D Objects (no blocky Structures because of Voxels)

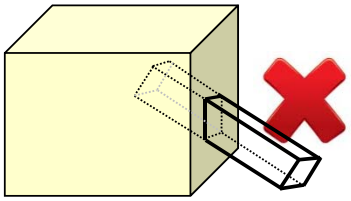


cell decomposition



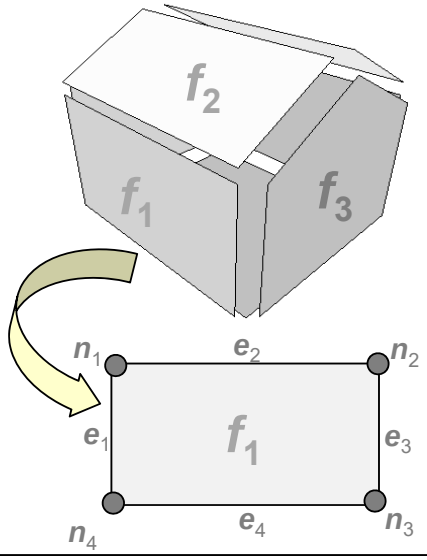
voxel

- **Contra**: more difficult to guarantee Correctness (Cells must fit together – it is not allowed that one Cell penetrates other Cells)



Boundary Representation (BREP)

- 3D Objects are defined by their enclosing Surfaces
- Topological Representation: an Object consist of *n* Faces – a Face consist of *n* Edges – an Edge consist of two Nodes – a Node has one Coordinate



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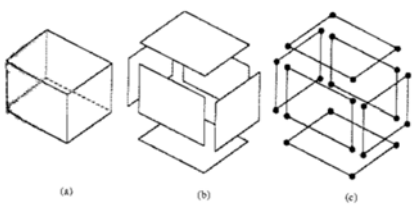
Boundary Representation (BREP)

- Commonly used in GIS (e.g. CityGML use BREP)
- Also often used in CAD Systems
- **Pro**:
  - very flexible
  - direct Extension of 2D Vector Data
  - it is possible to make local Changes without complete new Construction
  - Topology is completely stored -> Topological Analyses are possible
- **Contra**:
  - Correctness is difficult to prove
  - High Data Volumes



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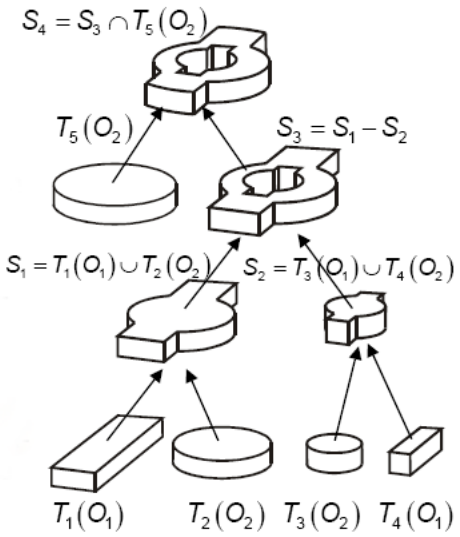
Constructive Solid Geometry (CSG)

- CSG uses Primitives (Prisms, Spheres, Cylinders, Cones, etc.) and Boolean Operations (Union, Subtraction, Intersection) to create 3D Objects
- The Primitives are modeled with Primitive Instancing

CSG-Tree:



Constructive Solid Geometry (CSG)

- Often used in CAD systems but also in GIS or in Game Engines
- **Pro:**
  - Easy to construct very complex Models with few Primitives
  - CSG Modelling need less Storage.
  - CSG can be converted to BREP
- **Contra:**
  - It is not unique (one 3D Object can have many different CSG Trees)
  - The Visualization of CSG Objects is CPU intensive

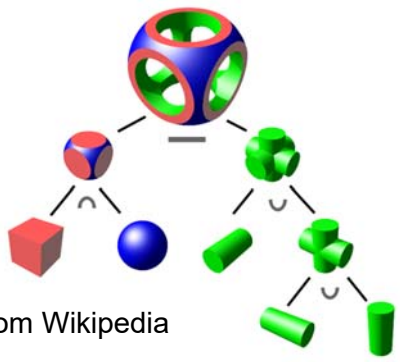
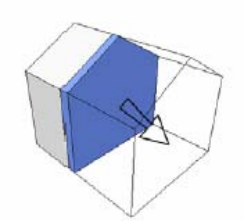


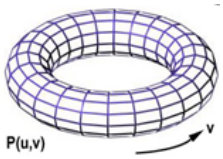
Image from Wikipedia

Sweep Representations

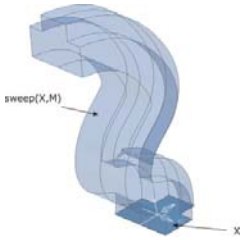
- Sweep Representations are used to construct 3D Objects that have some kind of Symmetry
- A Sweep Representation consists of a Shape and a Trajectory



Translational Sweep



Rotational Sweep

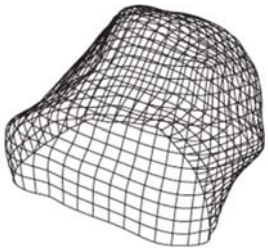


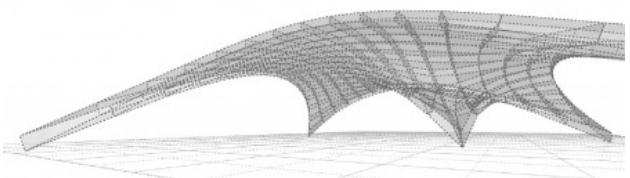
Complex Sweep

- Variations:
  - Vary the Shape along the Sweep Path
  - Vary the Orientation of the Shape relative to the Sweep Path.
- Sweep Representations allow the Modelling of very Complex Objects
- Used in many CAD Systems

Freeform Surfaces

- Freeform surfaces are used to describe the Skin of 3D objects
- Most Systems today use nonuniform rational B-Splines (NURBS) Mathematics to describe the Surface Forms
- Used in CAD Systems



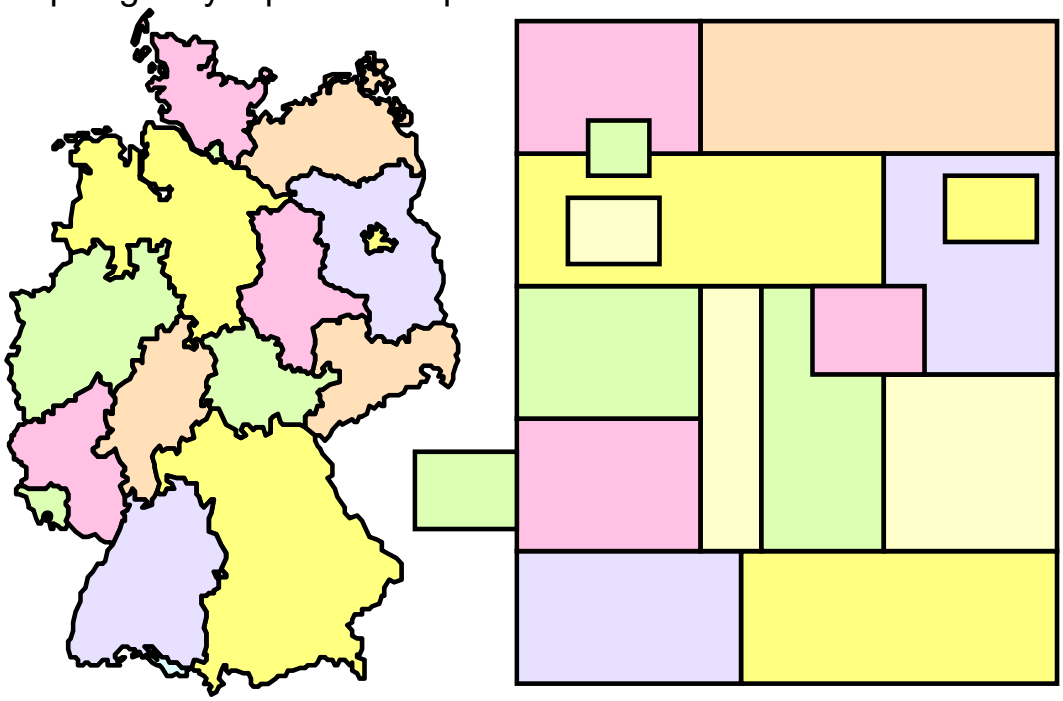


Topology

- describes spatial relations among geometric objects
- is based on Leibniz‘ analysis situs
- metric aspects are not relevant
- graph-theory
- spatial relations among the geometric primitives
  - node
  - edge
  - mesh

Topology

- topologically equivalent representations:



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Topology

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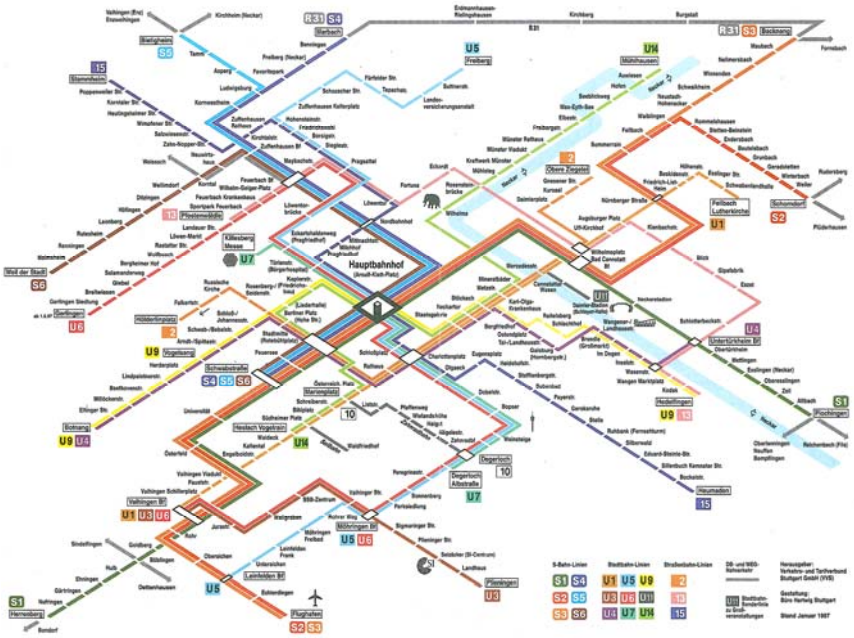

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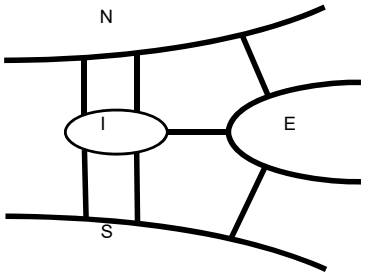
Das Königsberger Brückenproblem (The Königsberger Bridge-Problem)

The rivers „Alte Pregel“ and „Neue Pregel“ meet together in the city Königsberg. At that position is an isle with bridges connecting the isle with the city in the directions north, south and east.



Question to the mathematician L. Euler (1736):
„Is it possible to make a walk which uses every bridge exactly one time?“

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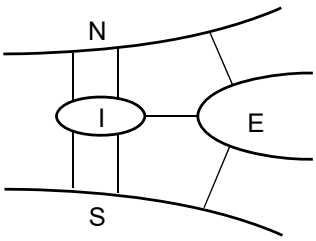
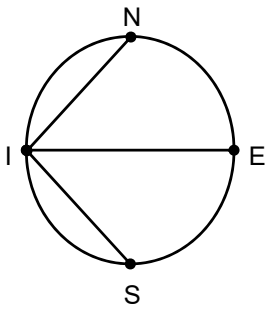
Das Königsberger Brückenproblem Solution 1

- Tip: the length, angle or position of the bridges is unimportant. The abstract representation results in the question: „Is it possible to draw the figure with one contiguous line?“
- Answer: „It is not possible to find a walk which uses every bridge exactly one time“

a) real situation



b) abstract representation



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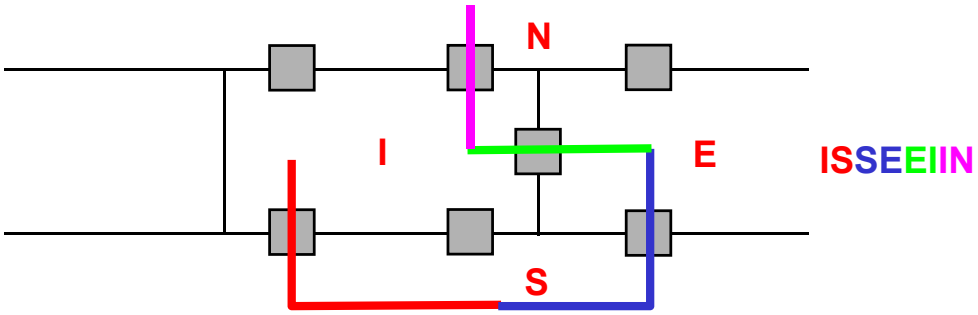
Königsberger Brückenproblem: Complete Solution

- In order to solve the problem we have to answer two questions:
- Question 1: when is a sequence of the letters N, I, E and S a path?



- Answer:
  - a sequence of letters is a sequence of bridges
  - a bridge ends in that area where the next bridge starts. Therefore the letter is used twice
- Conclusion 1: every letter NIES is represented even times - except for two letters (start and end)

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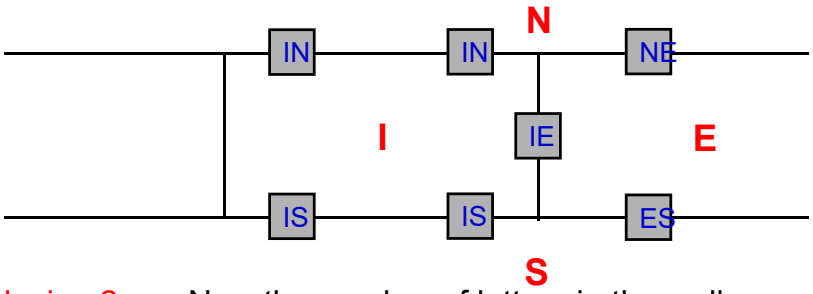
Königsberger Brückenproblem: Complete Solution

- Question 2: What is the characteristic of the walk?
- Answer: the walk uses every bridge once



- Conclusion 2: we Now the number of letters in the walk.
  I: E: S: N:
- Result: conclusion 1 and conclusion 2 are contradictory. Therefore no solution for the problem exist (indirect proof)

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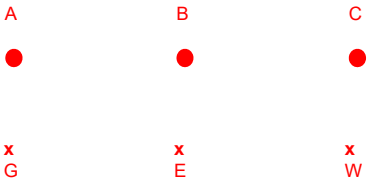
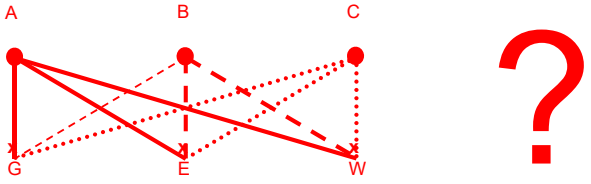
The Supply-Net-Problem

- Three houses A, B and C shall be connected with electricity, water and gas.



- Question: Is it possible to find a intersection-free configuration



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The Supply-Net-Problem

- Answer: „Problem is not solvable“

- These two problems have in common that the form and position of the figures is unimportant. Only neighborhoods (between points, lines or areas) play a role for the solution of the problem. That means that the problems will be unchanged if the figure are geometrically changed without changing the topology (for example with an affine transformation). These are „Topological Problems“

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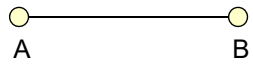
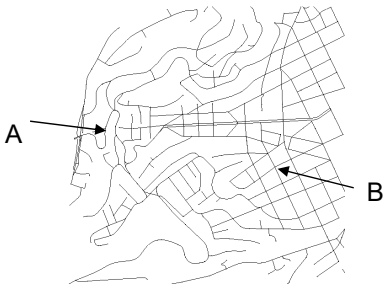
Further example for network analyses

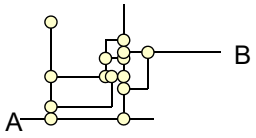
Search for the shortest (fastest, cheapest, ..) path from A to B



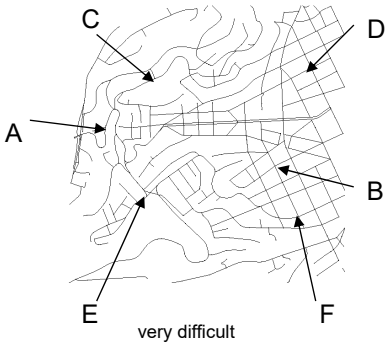
simple



difficult



more difficult



and visit also C,D,E,F

very difficult

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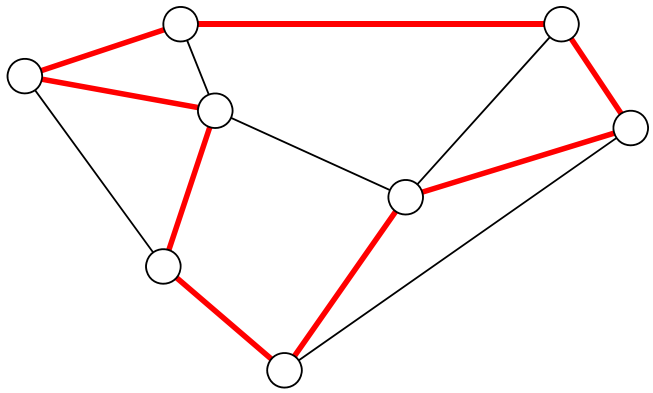
Further example for network analyses

- **Travelling Salesman Problem:** visit n cities and come back to the starting point in shortest time. Example n=8:



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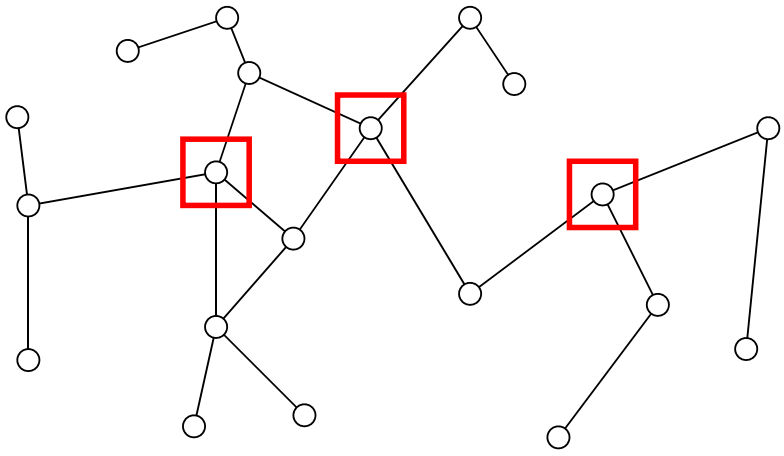
Further example for network analyses

- **Center problem:** k firehouses shall be build in n cities in such a way that every city is reachable as fast as possible.
  Example n = 20, k = 3



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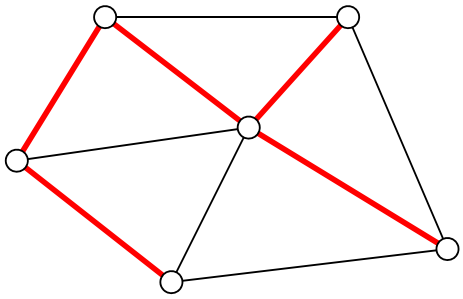
Further example for network analyses

- **Minimal spanning tree:** connect n cities in such a way that the resulting tree is minimal. Example n = 6



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- Graphs
  - A Graph G(N, E) consists of a set of Nodes N and Edges E
  - A node is on the position where an edge starts or ends or several edges meet
  - Every edge is a connection between two nodes. Every edge has a start and an end node

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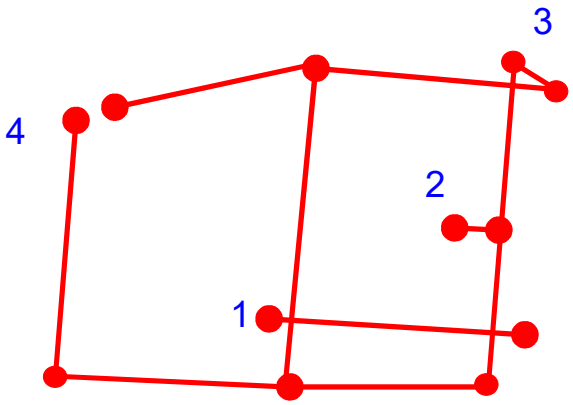
Importance of Topology

- description of relations between elementary objects (node, edge, area)
- important for consistency checks:
  - under- / overshoots (1)
  - dead ends (2)
  - weird polygons (3)
  - closed polygons (4), ...



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