Statistical Inference — Lab 3

Date of issue: 22. November 2018 Due Date: 6. December 2018, 6:00 pm

Family Name: _____

First Name: _____

Student ID: _____

Free adjustment of a height network

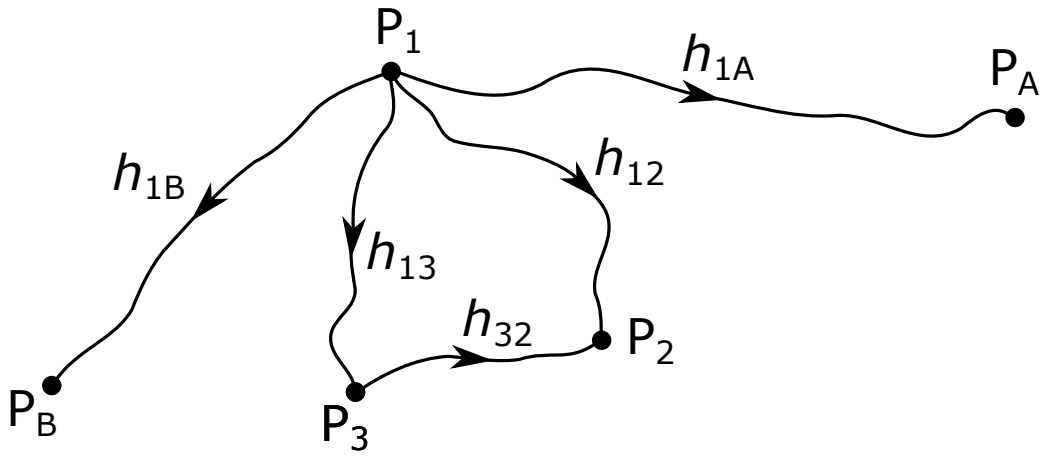
The scheme below illustrates the already-seen 5-point levelling network that will be constrained under the free network postulate, in order to determine the height of points P_A, P_1, P_2, P_3 and P_B, i.e.

$$H = [H_A, H_1, H_2, H_3, H_B]^\mathsf{T}.$$

The measured height differents are

$$h_{[m]} = [h_{1B}, h_{13}, h_{12}, h_{32}, h_{1A}]^\mathsf{T}$$

and can be found in lab 2.



1) Determine for this free adjustment :
   a) the observation vector h,
   b) the design matrix A, the constraint matrix D^T and the extended normal equation matrix N^*,
   c) the redundancy $r = m - (n - d)$ and verify that we have $\mathrm{rk}\left[A^\mathsf{T}|D\right] = n$.
2) Then, using the following approximative height values

$$H^0_{[m]} = [H^0_A, H^0_1, H^0_2, H^0_3, H^0_B]^\mathsf{T} = [100.96, 93.45, 107.76, 103.47, 100.72]^\mathsf{T}.$$

   compute :

a) the reduced observation vector $y_{[m]}$,
b) the least-squares estimate $\hat{x}_{[cm]} = \Delta\hat{H}$ (without Lagrange multipliers)) and $\hat{H}_{[m]}$,
c) the adjusted observation vector $\hat{h}_{[m]}$ and the estimated inconsistencies $\hat{e}_{[cm]}$,
d) the square sum of inconsistencies $\Omega_{[cm^2]} = \hat{e}^\mathsf{T}\hat{e}$,
e) the length of the vector $\hat{x}_{[cm]} = \Delta\hat{H}$, i.e. $\|\hat{x}\|_2 := \sqrt{\hat{x}^\mathsf{T}\hat{x}}$.

3) Repeat the instructions of point 2 with the modified approximative height values

$$H^0_{[m]} = [H^0_A,\, H^0_1,\, H^0_2,\, H^0_3,\, H^0_B]^\mathsf{T} = [101.00,\, 93.45 - 0.03\,k,\, 107.75,\, 103.47,\, 100.47]^\mathsf{T}\,.$$

The parameter k corresponds to the last two digits of your student ID number.

4) Comment on the differences between the results obtained at point 2 and 3 with respect to :
a) the adjusted observations $\hat{h}_{[mm]}$,
b) the estimated inconsistencies $\hat{e}_{[mm]}$,
c) the adjusted heights $\hat{H}_{[cm]}$,
d) the square sum of inconsistencies $\Omega_{[mm^2]}$,
e) the length of the vector $\hat{x}_{[cm]} = \Delta\hat{H}$.

5) Explain the apparently "curious" values of $\hat{e}$ in both cases.

Verify the correctness of your results with the help of the *orthogonality check*.