Statistical Inference — Lab 2

Date of issue: 08. November 2018 Due Date: 22. November 2018, 18 Uhr

Family Name: _____

First Name: _____

Student ID: _____

Adjustment of height networks

The 3 following schemes outline simple levelling networks consisting of 5 points (the arrows indicate the direction of track of levels).The objective of this exercice is to adjust the height network by using a linear model $y = Ax + e$ and the given measured height differences (the parameter k corresponds to the last two digits of your student ID number). The measured height differences are:

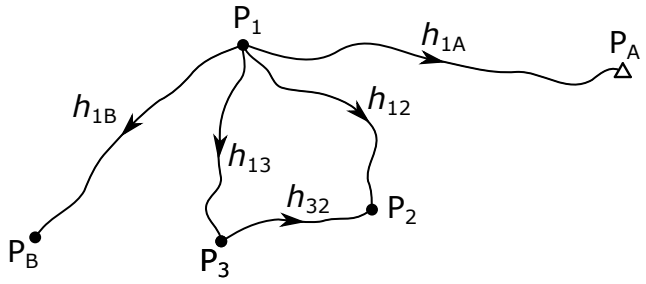
$$h_{[m]} = [h_{1B},\, h_{13},\, h_{12},\, h_{32},\, h_{1A}]^T$$

$$= \left[6.9803 + \tfrac{k}{1000},\, 10.0213 - \tfrac{k}{1000},\, 14.3030,\, 4.2871,\, 7.5001 + \tfrac{k}{1000}\right]^T$$

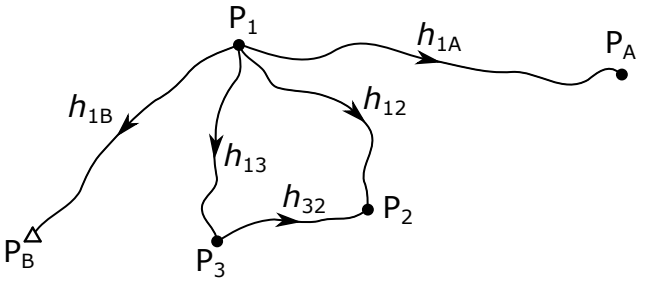
and the unknown absolute heights are:

- for case 1: $x = [H_1,\, H_2,\, H_3,\, H_B]^T$
- for case 2: $x = [H_A,\, H_1,\, H_2,\, H_3]^T$
- for case 3: $x = [H_1,\, H_2,\, H_3]^T$

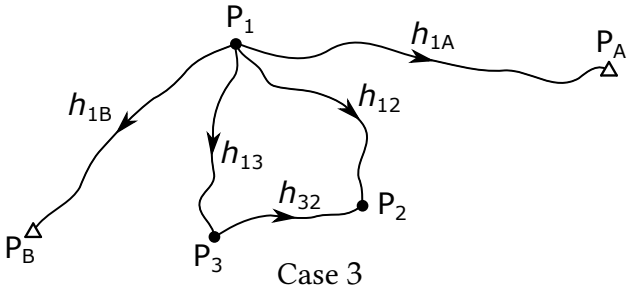
In case 1 (resp. case 2) only the height of P_A is known exactly (resp. P_B) while in case 3 both levelling points are supposed to be known exactly ($H_A = 100.956\,\text{m}$, $H_B = 100.459\,\text{m}$).



Case 1



Case 2



Case 3

For each of the 3 cases, follow the step-by-step approach to adjust the height network by determining

- the set of equations describing height network,
- the reduced observation vector y,
- the design matrix A, the number of rows (observations) m, columns (unknowns) n and the rank of A $\mathrm{rk}\, A$,
- $\mathrm{rk}\,[A|y]$,
- the Least-squares estimate of the vector $\hat{H}$,
- the corresponding adjusted reduced observation vector $\hat{y}$,
- the adjusted observations $\hat{h} = \left[\hat{h}_{1B}, \hat{h}_{13}, \hat{h}_{12}, \hat{h}_{32}, \hat{h}_{1A}\right]^{\mathsf{T}}$,
- the estimate $\hat{e}$ of the inconsistency parameter and its quadratic sum $\|\hat{e}\|_2^2 := \hat{e}^{\mathsf{T}}\hat{e}$,
- the redundancy r and the redundancy-normalized inconsistency quadratic sum $\frac{\hat{e}^{\mathsf{T}}\hat{e}}{r}$.

Comment on and interpret the differences in the results between the 3 cases, in particular in what concerns the estimated inconsistencies and the corresponding quadratic sum.

Verify the correctness of your results with the help of the *orthogonality check* and the *main check*.