



Universität Stuttgart

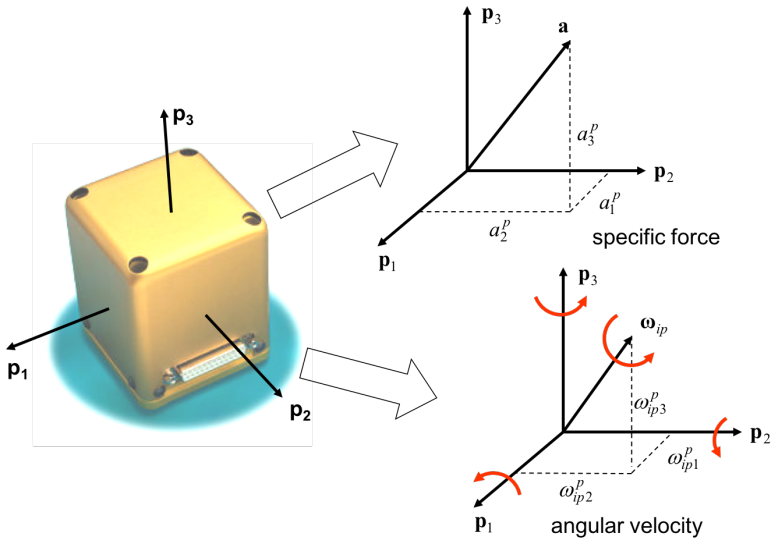
**Prof.Dr.
Thomas Hobiger**

Integrated Positioning and Navigation

Sensors

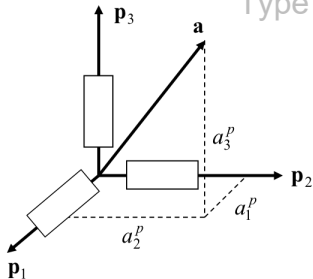
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Sensors



Sensors - cont'd

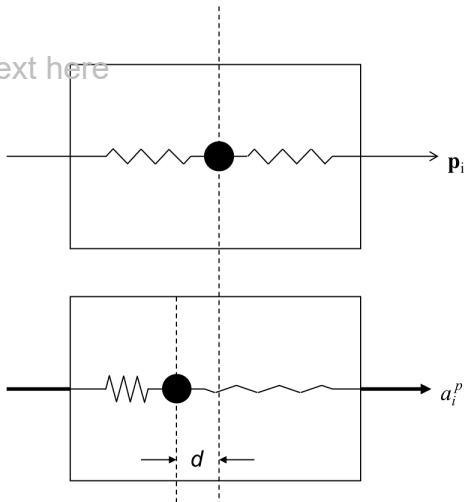
Accelerometers – Spring suspended mass



$$d = \frac{m}{k} \cdot a_i^p$$

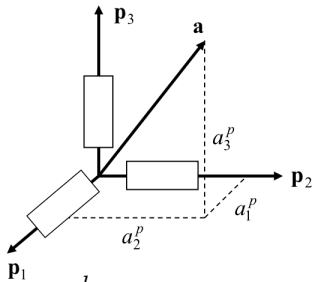
k : spring parameter [kg/s²]

m : mass [kg]



Sensors - cont'd

Accelerometers – Pendulum

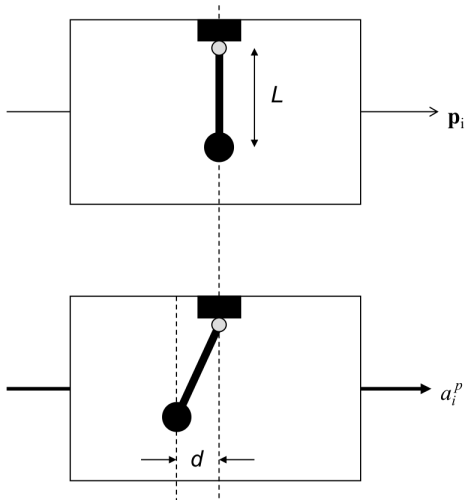


$$\frac{d}{L} \approx \alpha = \frac{m}{k} \cdot a_i^p$$

k : hinge parameter

L : pendulum length

m : mass



Sensors - cont'd

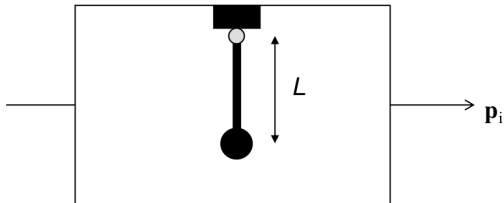
Accelerometers – Closed loop pendulum

$$T = m \cdot L \cdot a_i^p$$

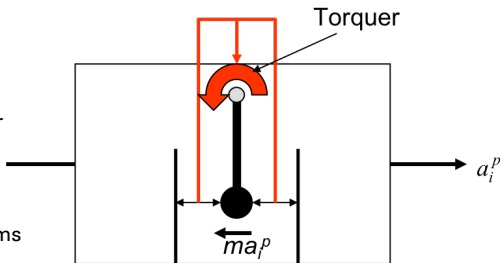
L : pendulum length

m : mass

T : torque to be supplied by
torquer to re-balance mass



- Closed-loop operation also for other types of accelerometers
- Closed-loop operation removes non-linearity problems



Sensors - cont'd

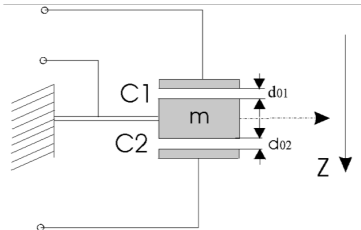
Accelerometers – Common Types

- **Capacitive:** metal beam or micro machined feature produces capacitance, change in capacitance related to acceleration
- **Piezoelectric:** piezoelectric crystal mounted to mass – voltage output converted to acceleration
- **Piezoresistive:** beam or micro machined feature whose resistance changes with acceleration
- **Hall Effect:** motion converted to electrical signal by sensing of changing magnetic fields
- **Magnetoresistive:** material resistivity changes in presence of magnetic field
- **Heat Transfer:** Location of heated mass tracked during acceleration by sensing temperature

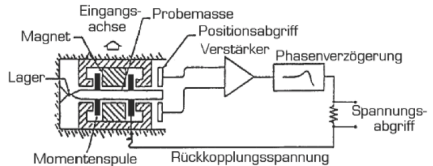
Sensors - cont'd

Accelerometers

Capacitive Measurement



Closed loop pendulum accelerometer



Sensors - cont'd

Accelerometers – Vibrating string

$$f = \frac{1}{2l} \sqrt{\frac{F}{\mu}}$$

F : force

m : mass

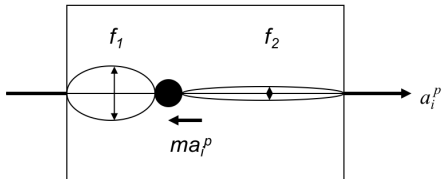
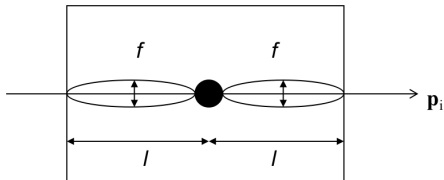
μ : string mass per unit length

For small displacements:

$$f_1 = \frac{1}{2l} \sqrt{\frac{F - ma_i^p}{\mu}}$$

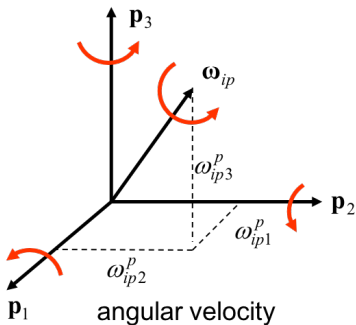
$$f_2 = \frac{1}{2l} \sqrt{\frac{F + ma_i^p}{\mu}}$$

$$f_1^2 - f_2^2 = \frac{m}{2\mu l^2} a_i^p$$



Sensors - cont'd

Gyroscopes (Rotation Rate Sensors)



Types of Gyroscopes:

- Mechanical (rotating mass)
- Optical (Laser, Fibre Optics)
- MEMS (vibrating mass)

Sensors - cont'd

Gyroscopes (Rotation Rate Sensors)

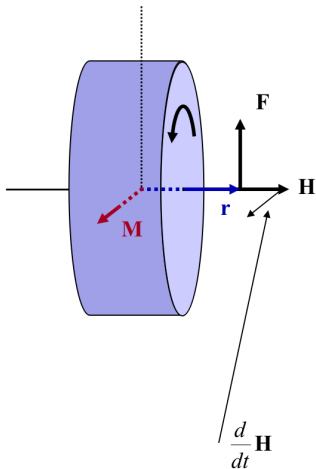
Type	mechanical	optical	MEMS
Drift [$^{\circ}/h$]	0.1 - 0.001	FOG: 0.1-100 RLG: 0.01 - 1 (0.00001)	1-500
Price [EUR]	> 10,000	1,000 - 50,000	10-100

Market segments

Market segment	Drift class [$^{\circ}/h$]	Size [cm ³]	Price [EUR]	Application
Mass market	5-100	10	10-500	automotive, games camera, guided projectiles, medical
Tactical navigation	0.1-1	1,000	2,000	guided munitions AHRS, missiles targeting
Precise navigation	<0.01	10,000	5,000 - 100,000	Military aircraft, land & marine nav., strategic aircraft, space

Sensors - cont'd

Mechanical Gyroscopes



Law of Conservation of Angular Momentum

$$\frac{d}{dt}H = M$$

$$M = r \times F$$

$$H = I\omega$$

M : Torque

H : Angular momentum

F : Force

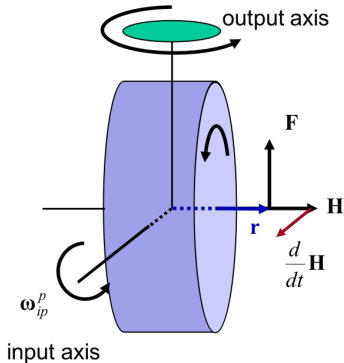
r : Lever Arm

I : Tensor of Inertial

ω : rotational velocity

Sensors - cont'd

Mechanical Gyroscopes



Rotational velocity about input axis generates force \mathbf{F}

Force \mathbf{F} generates torque \mathbf{M}

Torque is equal to change in angular momentum

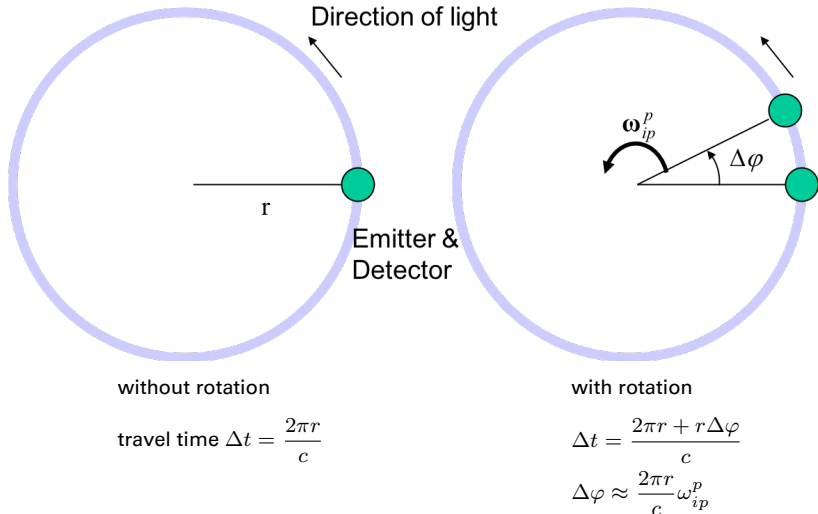
Torque generator on output axis keeps angular momentum constant

Amount of torque generated on output axis (voltage) is a measure for rotational velocity about input axis

The torque generated is proportional to rotational velocity

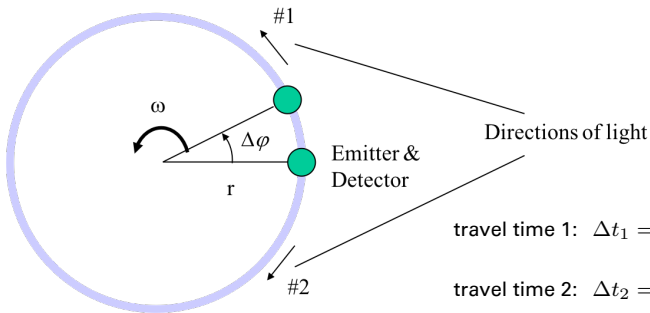
Sensors - cont'd

Optical Gyroscopes



Sensors - cont'd

Optical Gyroscopes



$$\text{travel time 1: } \Delta t_1 = \frac{2\pi r + r\Delta\varphi}{c}$$

$$\text{travel time 2: } \Delta t_2 = \frac{2\pi r - r\Delta\varphi}{c}$$

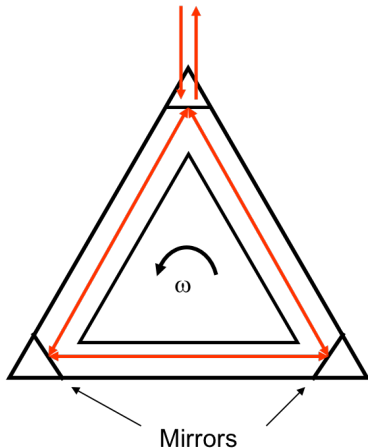
$$\text{from the difference: } \omega_{ip}^p = \frac{c^2(\Delta t_1 - \Delta t_2)}{4\pi r^2}$$

Measured time difference is proportional to

- rotational velocity
- area enclosed by light path

Sensors - cont'd

Optical Gyroscopes - Lasergyro

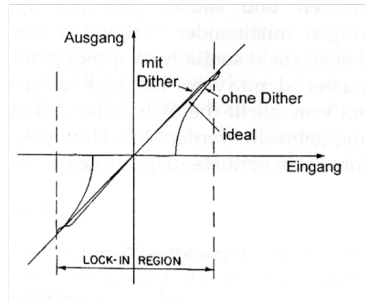


Advantages:

- Insensitive to accelerations
- Large working range

Disadvantage

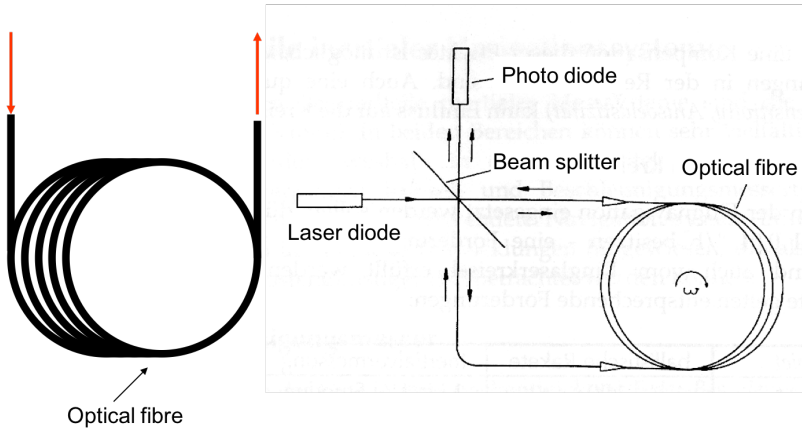
- lock in effect (minimized by small high frequency movements)



Sensors - cont'd

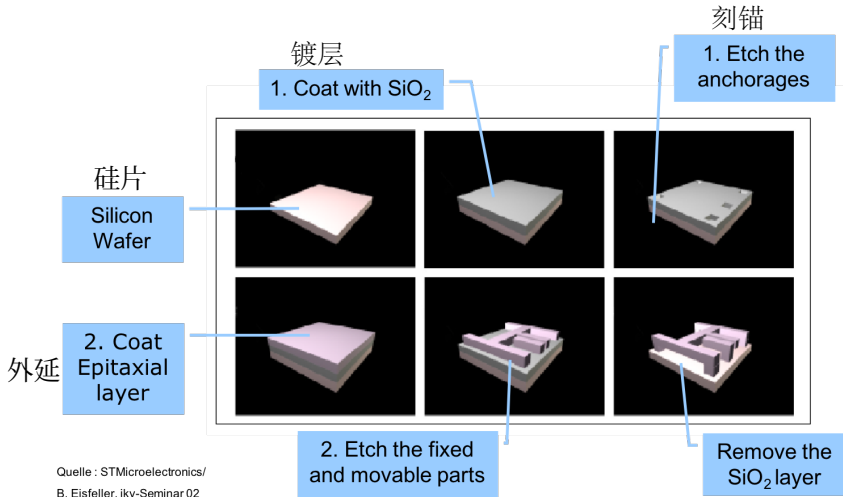
Optical Gyroscopes - Fibre-optics Gyro

For details see guest lecture next week!



Sensors - cont'd

Vibrating Mass Gyroscopes (MEMS) - Production

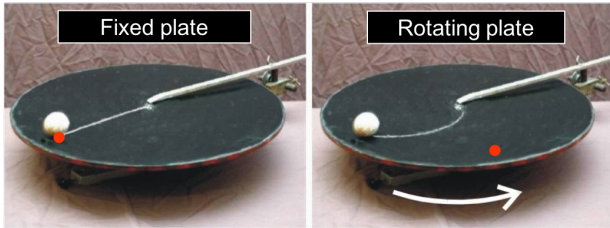


Quelle : STMicroelectronics/
B. Eisfeller, ikv-Seminar 02

Sensors - cont'd

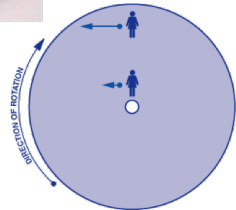
Vibrating Mass Gyroscopes – Principle

Work on the basis of the Coriolis force: $F_c = 2 \cdot M \cdot (v \times \omega)$



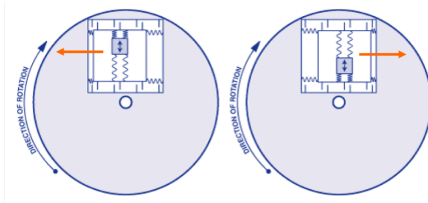
from: FH Merseburg

Person on a moving plate walking outwards

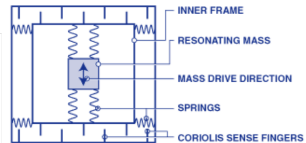


Sensors - cont'd

Vibrating Mass Gyroscopes – Principle



$$F_c = 2 \cdot M \cdot (v \times \omega)$$



from: Analog Devices

Principle:

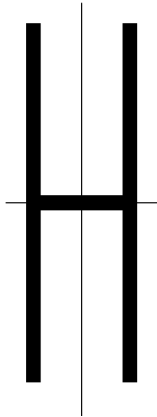
- Capacitive sensing element attached to the resonator

Advantage

- Immunity to shocks and vibrations

Sensors - cont'd

Vibrating Mass Gyroscopes (Tuning fork)



Conservation of Linear Momentum

$$\frac{d}{dt}(m\dot{\mathbf{x}}) = \mathbf{F}$$

Constant mass, force-free:

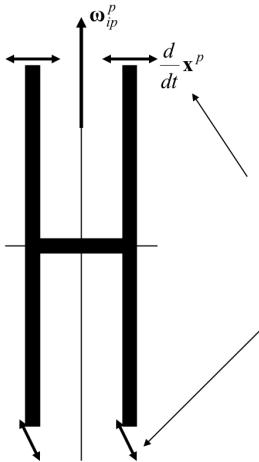
$$m \frac{d^2}{dt^2} \mathbf{x} = \mathbf{0}$$

Transform from i -system to p -system
(c.f. Module 5)

$$\frac{d^2}{dt^2} \mathbf{x}^p + 2\boldsymbol{\Omega}_{ip}^p \cdot \frac{d}{dt} \mathbf{x}^p + \boldsymbol{\Omega}_{ip}^p \cdot \boldsymbol{\Omega}_{ip}^p \cdot \mathbf{x}^p = \mathbf{0}$$

Sensors - cont'd

Vibrating Mass Gyroscopes (Tuning fork)



Re-arrange equation

$$\frac{d^2}{dt^2}x^p = -2\Omega_{ip}^p \cdot \frac{d}{dt}x^p - \Omega_{ip}^p \cdot \Omega_{ip}^p \cdot x^p$$

Induce high linear velocity (vibrations), then first term on r.h.s. dominates

$$\frac{d^2}{dt^2}x^p \approx -2\Omega_{ip}^p \cdot \frac{d}{dt}x^p = -2\omega_{ip}^p \times \frac{d}{dt}x^p$$

Vibrating mass reacts with accelerations proportional and orthogonal to

- linear velocity
- rotational velocity

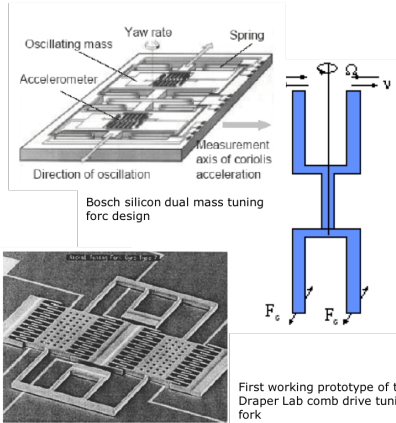
Sensors - cont'd

音叉

Vibrating Mass Gyroscopes (Tuning fork)

Principle:

- Two masses driven to oscillate with equal amplitude in opposite directions
- Rotation generates a Coriolis force \rightarrow orthogonal vibration results
- Capacity is a measure for the angular rate (Amplitude & $\Delta\varphi$)



Disadvantage:

- Very sensible to shocks and vibrations

Watch: <https://www.youtube.com/watch?v=W12KARSKNhQ>

Sensors - cont'd

共鸣器

Vibrating Mass Gyroscopes (Wine glass resonator / Hemispherical Resonator Gyros)

Principle:

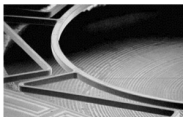
- A vibrating ring is driven to resonance 共振
- Rotation displaces nodal points → measure of angular rate
- E.g.: Gyro of Sumitomo and British Aerospace (29x29x18 mm³)

Advantage:

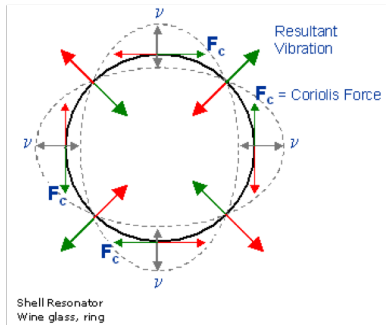
- non-sensitive against shocks and vibrations



from: Silicon Sensing Systems



Part of a vibrating ring
from: sensormag.com



Sensors - cont'd

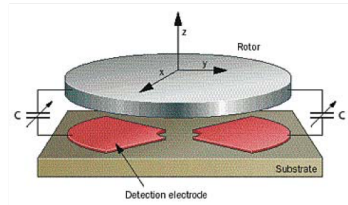
Vibrating Mass Gyroscopes (Vibrating Wheel)

Principle:

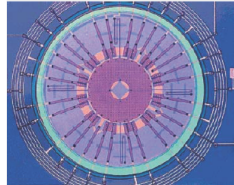
- Wheel is driven to vibrate around its axis of symmetry
- Rotation about either in plane axis results in the wheels tilting
- Changes detected with capacitive electrodes under the wheel
- Degree of angular rate

Advantage:

- Sensitive in 2 directions



Schematic design concept for Robert Bosch vibrating wheel



Polysilicon surface-micromachined vibrating wheel from Berkley Sensors and Actuators Center

Sensors - cont'd

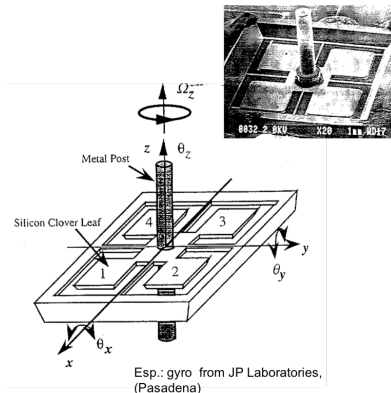
Vibrating Mass Gyroscopes (Foucault pendulum gyroscope)

Principle:

- Vibrating rod
- Rotation around the rod leads to an orthogonal oscillation
- Amplitude and Phase are a measure for the rate of rotation

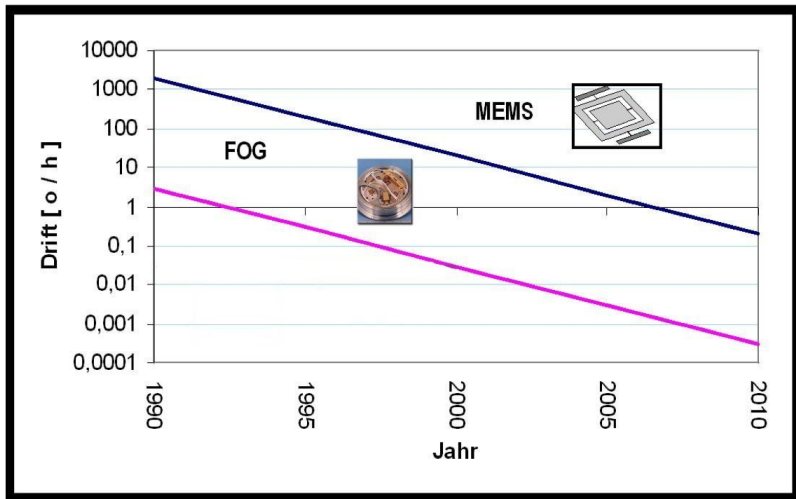
Disadvantage:

- Relative large metal rod necessary
- No planar realization possible



Sensors - cont'd

Preview



Quelle: Vortrag Prof. Eissfeller, ikv-Seminar 02

Sensors - cont'd

Accelerometer and Gyroscope Measurements (cf. module 04)

Most modern Inertial Measurement Units (IMU) provide digital readouts at regular time intervals

Readouts (measurements) at time t_k are integrals (sums) of the sensed specific force and rotational velocity since the last readout at time t_{k-1}

Velocity increment:
$$\Delta \mathbf{v}^p(t_k) = \int_{t_{k-1}}^{t_k} \mathbf{a}^p(\tau) d\tau$$

Angular increment:
$$\Delta \boldsymbol{\alpha}_{i^p}^p(t_k) = \int_{t_{k-1}}^{t_k} \boldsymbol{\omega}_{i^p}^p(\tau) d\tau$$

- Measurement output rates are 50Hz – 200Hz
- Internal sampling rate can be in kHz-range
- Any non-linear platform motion within readout interval cannot be recovered from readout data