## Lab 3: Equivalent water height derived from GRACE data and estimation of trends in the water cycle

The Gravity Recovery and Climate Experiment (GRACE, and now its following mission GRACE-FO) was a joint mission of NASA and the German Aerospace Centre dedicated to the mapping of the Earth's gravity field and in particular of its time-variable part. GRACE was launched in March 2002 and ended its science mission in October 2017. It consisted of 2 twin satellites orbiting the Earth closed to each other with a near-polar inclination at a typical altitude of 485 km. The measurement principle was based on the precise estimation of the orbit and inter-range dynamics which were eventually related to the Earth's gravitational field and mass distribution. In particular, GRACE has revolutionized the way scientists monitor the mass change associated to the global water cycle. In practice, the time variable gravity field is often expressed in terms of equivalent water height (EWH), which is computed as follows

$$EWH(\theta,\lambda)[t] = \frac{a_{e}\rho_{ave}}{3\rho_{w}} \sum_{l=0}^{L} \frac{2l+1}{1+k_{l}} \sum_{m=0}^{l} \bar{P}_{lm}(\cos\theta) \left(\Delta C_{l,m}[t] \cos m\lambda + \Delta S_{l,m}[t] \sin m\lambda\right), \quad (1)$$

where  $\rho_{\rm ave}$  is the mean density of the Earth (5514 kg/m³),  $\rho_{\rm w}$  is the mean density of water(1000 kg/m³),  $a_{\rm e}$  refers to the Earth's semi-major axis (6378.1363 km),  $k_l$  is the *load Love number* for the spherical harmonic function of degree l (see the MATLAB function *lovenr.m*),  $\bar{P}_{lm}(\cos\theta)$  is the normalized Legendre function of degree l and order m and  $\Delta C_{lm}[t]$ ,  $\Delta S_{lm}[t]$  are the time-varying spherical harmonic coefficients of the Earth's gravitational potential obtained after subtracting a long-term mean. The GRACE data (Level-2) from April 2002 to February 2015 can be found in the file *GRACE\_SH.mat*. The data are organized in 10 columns as follows

where the coefficients  $C_{lm}$  and  $S_{lm}$  located in the  $9^{th}$  column are ordered for every month in a cell array as follows (L is the maximum degree):

$$\begin{bmatrix} C_{00} & S_{11} & S_{21} & \cdots & S_{L1} \\ C_{10} & C_{11} & S_{22} & \cdots & S_{L2} \\ C_{20} & C_{21} & C_{22} & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & S_{LL} \\ C_{L0} & C_{L1} & C_{L2} & \cdots & C_{LL} \end{bmatrix}$$

1. Compute the mean coefficients  $\bar{C}_{lm}$ ,  $\bar{S}_{lm}$  for the time 2002–2015 in order to obtain  $\Delta C_{lm}[t]$  and  $\Delta S_{lm}[t]$ , where

$$\Delta C_{lm}[t] = C_{lm}[t] - \bar{C}_{lm}, \ \Delta S_{lm}[t] = S_{lm}[t] - \bar{S}_{lm}$$



2. According to the rule of thumb of Kaula, the gravity potential signal decreases continuously with increasing degrees and thus, higher spatial resolution. At the same time, the error degrading the spherical surface function coefficients increases. As a consequence, it seems appropriate to filter out higher degrees in order to maximize the signal to noise ratio. Filter the coefficients  $\Delta C_{lm}$  and  $\Delta S_{lm}$  with the Gaussian filter of radius 350 km by multiplying the Stokes' coefficients as follows

$$\Delta C_{lm} = w_l \Delta C_{lm}, \ \Delta S_{lm} = w_l \Delta S_{lm}$$

where the filter coefficients  $w_l$  are computed with the help of the function *gaussian.m.* 

- 3. Compute the equivalent water height with the formula given hereinabove for every month.
- 4. Plot a map of the equivalent water height for both February and July 2006 and compare them. What do you observe?
- 5. Compute and plot the long-term trend in equivalent water height and comment the result. In particular, try to interpret the origin of the large amplitude of the trend in some regions.
- 6. How can you explain the positive trend over Canada and northern Europe?
- 7. Estimate the trend in ice mass loss over Greenland in gigaton per year.
- 8. Is it possible with GRACE to detect the gravitational signal associated to the long-lasting drought in California? Suggest potential reasons for this drought.

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