I. Sequential Least Square Parameter Estimation

1. Basic LS adjustment.

$$y = A x + e \quad , \; P = P$$

$$\hat{x} = (A^T P A)^{-1} \cdot A^T P y$$

$$\hat{e} = y - A\hat{x}$$

$$\sigma_0^2 = \frac{e^T P e}{n-u}$$

$$\Sigma_{\hat{x}\hat{x}} = \sigma_0^2 \cdot (A^T P A)^{-1}$$

2. Sequential Least Square Adjustment.

$$y = \begin{bmatrix} y(t_1) \\ \vdots \\ y(t_n) \end{bmatrix} \quad A = \begin{bmatrix} A(t_1) \\ \vdots \\ A(t_n) \end{bmatrix} , \quad P = \begin{bmatrix} P(t_1) & & & 0 \\ & P(t_2) & & \\ & & \ddots & \\ 0 & & & P(t_n) \end{bmatrix}$$

- first epoch : Least Square
  further : update
- Real time application
- final result would be the same as LSE : no approximation

- uncorrelation between measurement epochs.
- assumption , $\hat{x}(t_{k+1}) \doteq \hat{x}(t_k) + \Delta x_{t_k}$

$$\Rightarrow \hat{x}_k = \hat{x}_{k-1} + \left(\hat{\sigma}_{0,k-1}^2 \cdot (\Sigma \hat{x}_{k-1})^{-1} + A_k^T P_k A_k \right)^{-1} A_k^T P_k \left(y_k - A_k \hat{x}_{k-1} \right)$$

$$\sigma_{0,k}^2 = \frac{\sigma_{0,k-1}^2 \cdot (n_{k-1} - u + \Delta\hat{x}^T (\Sigma \hat{x}_{k-1})^{-1} \Delta\hat{x}) + (y_k - A_k \hat{x}_k)^T P_k (y_k - A_k \hat{x}_k)}{n_k - u}$$

$$\Sigma \hat{x}_k = \sigma_{0,k}^2 \left(\sigma_{0,k-1}^2 (\Sigma \hat{x}_{k-1})^{-1} + A_k^T P_k A_k \right)^{-1}$$

Q: Would it be different if each epoch observations are different but the same unknowns?

II. Ordinary Differential Equations

1. mth-order Linear ODE $\Rightarrow$ system of m Linear ODE of 1st-order : $y^m(t) + a_1(t) y^{m-1}(t) + \cdots + a_m(t) y(t) = b(t)$

$$y_1(t) = y(t)$$
$$y_2(t) = y'(t)$$
$$\cdots$$
$$y_m(t) = y^{m-1}(t)$$

$$\Rightarrow \frac{d}{dt} \begin{bmatrix} y_1(t) \\ \vdots \\ y_m(t) \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 & \cdots & 0 \\ & & \vdots & & \\ a_m & a_{m-1} & \cdots & & a_1 \end{bmatrix} \begin{bmatrix} y_1(t) \\ \vdots \\ y_m(t) \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ b(t) \end{bmatrix}$$

i.e. $\underline{y}'(t) + \underline{A} \cdot \underline{y}(t) = \underline{b}(t)$

2. Runge-Kutta-Method : $y'(t) + a(t) y(t) = b(t) \Rightarrow y'(t) = f(t, y(t))$

2.1 theory : $y(t_{n+1}) = y(t_n) + y'(t_n) \cdot (t_{n+1} - t_n) + \frac{y''(t_n)}{2!} \cdot (t_{n+1} - t_n)^2 + \cdots$

with $y'(t_n) = f$

$$y''(t_n) = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial y} \cdot f \quad ?$$

2.2 Numerical integration — Runge-Kutta.

$$y(t_{n+1}) = y_{n+1} \qquad y'(t) = f(t, y(t)).$$
$$t_{n+1} - t_n = h$$

1st order : $y_{n+1} = y_n + h \cdot f(y_n, t_n)$.

2nd order : $y_{n+1} = y_n + \frac{h}{2} \cdot (k_1 + k_2)$

$k_1 = f(y_n, t_n)$

$k_2 = f(y_n + h \cdot k_1, t_n + h)$.

3rd order : $y_{n+1} = y_n + \frac{h}{6} \cdot (k_1 + 4k_2 + k_3)$*

$k_1 = f(y_n, t_n)$

$k_2 = f(y_n + \frac{h}{2} k_1, t_n + \frac{h}{2})$

$k_3 = f(y_n - h k_1 + 2h k_2, t_n + h)$

Q: $f(y(t), t)$ → $t_1 = t_2$; $y_1(t) = y_2(t)$?

relationship between $f(y, t)$

4th order : $y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = f(y_n, t_n)$

$k_2 = f(y_n + \frac{h}{2} k_1, t_n + \frac{h}{2})$

$k_3 = f(y_n + \frac{h}{2} k_2, t_n + \frac{h}{2})$

$k_4 = f(y_n + h k_3, t_n + h)$

III. Linear Dynamic Systems

$$\boxed{\dot{x}(t) = F(t)\, x(t) + G(t)\, u(t)} + L(t)\, s(t) \longrightarrow \text{solution has the form}$$

$$x(t) = \underbrace{\phi(t, t_0)\, x(t_0)}_{\text{general solution}} + \underbrace{\int_{t_0}^{t} \phi(t, t')\, G(t')\, u(t')\, dt'}_{\text{particular solution}}$$

$x(t)$: state vector

$u(t)$: random forcing function

$s(t)$: Deterministic control input

$F(t)$: square matrix

$G(t), L(t)$: matrices

→ method 1 : Taylor expansion for general solution (stationary system, i.e. $F(t)$ is constant).

$$x(t) = x(t_0) + \dot{x}(t_0)(t - t_0) + \frac{\ddot{x}(t_0)}{2!}(t - t_0)^2 + \cdots$$

$$\dot{x}(t_0) = F x(t_0), \quad \ddot{x}(t_0) = \dot{F} x(t_0) + F \cdot \dot{x}(t_0) = F^2 \cdot x(t_0), \cdots, x^{(n)}(t_0) = F^n x(t_0)$$

$$\Rightarrow x(t) = x(t_0) + F x(t_0)(t - t_0) + \frac{F^2}{2!} x(t_0)(t - t_0)^2 + \cdots$$

$$= \underline{[I + F(t - t_0) + \frac{F^2}{2!}(t - t_0)^2 + \cdots]} \cdot x(t_0) = \underline{e^{F(t - t_0)} \cdot x(t_0) = \overset{\text{state transition matrix}}{\phi(t, t_0)} \cdot x(t_0)}$$

method 2 : constant variation & integration.

→ ~~Concrete~~ Discrete form :

$$x(t_n) = \phi(t_n, t_{n-1}) \cdot x(t_{n-1}) + U(t_n)$$

with $\phi(t_n, t_{n-1}) = e^{F(t_n - t_{n-1})}$, $U(t_n) = \int_{t_{n-1}}^{t_n} \phi(t, t')\, G(t')\, u(t')\, dt'$

- $\phi = e^{F\Delta t}$ only stands for $\boxed{\text{stationary}}$ system $F(t) = F$.
- $x(t_n) = \phi(t_n, t_{n-1}) \cdot x(t_{n-1}) + U(t_n)$ is general.

IV. Random Processes (Stochastic Process)

Mean value: $\mu(t) = E(y(t)) = \int_{-\infty}^{\infty} y(t) \cdot \underline{p(y(t))} \cdot dy(t)$ ← probability density distribution

Auto-Covariance: $Cov(t_1,t_2) = E\left((y(t_1)-\mu(t_1)) \cdot (y(t_2)-\mu(t_2))^T\right)$

Auto-Correlation: $Cor(t_1,t_2) = E(y(t_1)\, y(t_2)^T)$

Cross-Covariance: $Cov_{xy}(t_1,t_2) = E\left((x(t_1)-\mu_x(t_1)) \cdot (y(t_2)-\mu_y(t_2))^T\right)$.

Stationarity: the probability density distribution is independent of time. (mean value also)

- ~~mean value,~~ auto-covariance, auto-correlation, cross-covariance depend only on time interval.
- Fourier transform of auto-correlation = power spectral density (PSD). $\phi(f) = \int_{-\infty}^{\infty} Cor(\tau) e^{-j2\pi f\tau} d\tau$

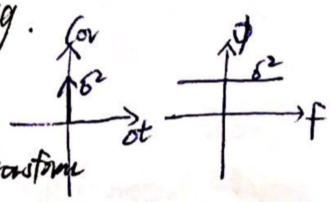
Ergodicity: the statistics of the RP can be derived from a single realisation of the RP in time domain.

e.g. $\mu(t) = E(y(t)) \Leftrightarrow \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} y(t)\, dt$.

White noise: A stationary RP whose auto-covariance is zero for non-zero lag.

$Cov(\Delta t) = \sigma^2 \cdot \underline{\delta(t-\Delta t)}$. ? ← $\delta(\Delta t)$? $Cor(\tau) = \sigma^2 \cdot \delta(\tau)$

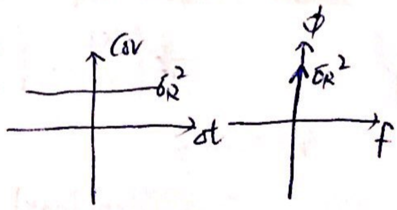
PSD: $\phi(f) = \int_{-\infty}^{\infty} \sigma^2 \cdot \delta(\tau) e^{-j2\pi\tau} d\tau = \sigma^2$ ... constant. ← Fourier transform



Random Constant: A stationary RP with random constant value for all times

$\dot{R}(t) = 0$, $R(t_0) = R_0$

$\underline{Cov_{RR}(\Delta t) = \sigma_R^2}$? . $\phi_R(f) = \underline{\sigma_R^2\, \delta(f)}$. (no ergodic)



Random Walk: $\dot{R}(t) = w(t)$, $R(t_0) = 0 \Rightarrow R(t) = \int_{t_0}^{t} w(t)\, dt$. (no stationary).

$Cov(t_1,t_2) = \begin{cases} \sigma^2(t_2-t_1), & t_2 > t_1 \\ \sigma^2(t_1-t_2), & t_2 < t_1 \end{cases}$. σ is the amplitude of the psd of $w(t)$

Discrete random process: Q

$\dot{x}(t) = -\beta \cdot x(t) + \alpha \cdot \underline{w(t)}$ (→ zero-mean Gaussian white noise.) , $\alpha, \beta \geq 0$. $\Leftrightarrow$ $x_{n+1} = b_n x_n + a_{n+1} w_{n+1}$

$\alpha = \beta = 0 \Rightarrow$ random constant $\dot{x} = 0$ — $b_n = 1$, $a_n = 0$ — $\sigma_{x,n+1}^2 = \sigma_{x,n}^2$

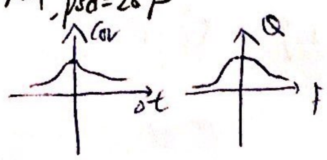
$\alpha = 1, \beta = 0 \Rightarrow$ random walk. $\dot{x} = w(t)$ (Wiener Process) — $b_n = a_n = 1$ — $\sigma_{x,n+1}^2 = \sigma_{x,n}^2 + \sigma_{w,n+1}^2$

$\alpha = 1, \beta \neq 0 \Rightarrow$ Gauss-Markov process 1^{st} order — $b_n \neq 0$, $a_n = 1$. — $\sigma_{x,n+1}^2 = b_n^2 \sigma_{x,n}^2 + a_{n+1}^2 \cdot \sigma_{w,n+1}^2$

$\dot{x} = -\beta \cdot x + w(t)$

$Cov(t_1,t_2) = Cov(\Delta t) = \sigma^2 e^{-\beta|\Delta t|}$, psd $= 2\sigma^2\beta$

$\phi(f) = \frac{2\sigma^2\beta}{4\pi^2 f^2 + \beta^2}$.



V. State and Covariance Prediction

$$\dot{x}(t) = F(t)\,x(t) + G(t)\,u(t)$$

$$\Rightarrow x_n = \phi(t_n, t_{n-1}) \cdot x_{n-1} + \underbrace{\int_{t_{n-1}}^{t_n} \phi(t_n, t')\,G(t')\,u(t')\,dt'}_{u_n} \quad , \quad \phi(t_n, t_{n-1}) = e^{F(t_n - t_{n-1})}$$

1. Analytically

for assumption " $u(t)$ is white noise" (uncorrelation w.r.t. time).

spectral density matrix

$$\Rightarrow Cov_{x_n, x_n} = \phi(t_n, t_{n-1}) \cdot Cov_{x_{n-1}, x_{n-1}} \cdot \phi(t_n, t_{n-1})^T + \underbrace{\int_{t_{n-1}}^{t_n} \phi(t_n, \tau)\,G(\tau)\,\Lambda(\tau)\,G(\tau)^T \phi(t_n, \tau)^T d\tau}_{Q_n}$$

2. Numerically — "cookbook" method.

$$A = \begin{bmatrix} -F & GWG^T \\ 0 & F^T \end{bmatrix} \delta t$$

$$B = \exp(A) = \begin{bmatrix} \cdots & \phi^{-1} Q \\ 0 & \phi^T \end{bmatrix}$$

$$\Rightarrow \phi = B_{22}^T \qquad Q = \phi \cdot B_{12}$$

Example:

Integrated Random Walk" : $\ddot{x}(t) = 0 + \sigma u(t)$.

$$\frac{d}{dt}\begin{pmatrix} x \\ \dot{x} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ \dot{x} \end{pmatrix} + \begin{pmatrix} 0 \\ \sigma \end{pmatrix} u(t)$$

① Analytically

$$\phi = \exp(F \cdot \delta t) = I + F\delta t + \frac{F^2 \delta t^2}{2!} + \cdots = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & \delta t \\ 0 & 0 \end{bmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \cdots = \begin{pmatrix} 1 & \delta t \\ 0 & 1 \end{pmatrix}$$

$$\phi \cdot G = \begin{pmatrix} 1 & \delta t \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ \sigma \end{pmatrix} = \begin{pmatrix} \delta t \\ 1 \end{pmatrix} \cdot \sigma \quad , \quad G^T \phi^T = \sigma \cdot (\delta t, 1)$$

$$Q = \int_{t_{n-1}}^{t_n} \phi G \Lambda G^T \phi^T d\tau = \int_{t_{n-1}}^{t_n} \begin{pmatrix} t_n - \tau \\ 1 \end{pmatrix} (t_n - \tau, 1)\, d\tau \cdot \sigma^2 = \int_{t_{n-1}}^{t_n} \begin{pmatrix} (t_n - \tau)^2 & t_n - \tau \\ t_n - \tau & 1 \end{pmatrix} d\tau \cdot \sigma^2$$

$$= \sigma^2 \cdot \begin{pmatrix} t_n^2 \cdot \tau - t_n \cdot \tau^2 + \frac{\tau^3}{3} & t_n \cdot \tau - \frac{\tau^2}{2} \\ t_n \cdot \tau - \frac{\tau^2}{2} & \tau \end{pmatrix} \Bigg|_{t_{n-1}}^{t_n} = \sigma^2 \cdot \begin{pmatrix} \frac{\delta t^3}{3} & \frac{\delta t^2}{2} \\ \frac{\delta t^2}{2} & \delta t \end{pmatrix} \qquad Q: \sigma?$$

② numerically

$$A = \begin{pmatrix} -F & GWG^T \\ 0 & F^T \end{pmatrix} \delta t \qquad B = \exp(A) = \begin{pmatrix} \cdots & \phi^{-1} Q \\ 0 & \phi^T \end{pmatrix}$$

$$\phi = B_{22}^T = \begin{pmatrix} 1 & \delta t \\ 0 & 1 \end{pmatrix} . \quad Q = \phi B_{12} = \sigma^2 \cdot \begin{pmatrix} \frac{\delta t^3}{3} & \frac{\delta t^2}{2} \\ \frac{\delta t^2}{2} & \delta t \end{pmatrix} .$$

VI. State Vector Augmentation ?

In reality the simple white Gaussian noise is not enough, we need empirical autocovariance model of a system. $\Rightarrow B(t)\, c(t)$

$$\dot{x}(t) = F(t)\,x(t) + G(t)\,\underbrace{u(t)}_{\text{white noise}} + B(t)\,\underline{c(t)}$$

$$\dot{c}(t) = F_c(t)\,c(t) + G_c(t)\,\underbrace{w_c(t)}_{\text{uncorrelated white noise}} \quad ?$$

- In reality, the random process may be correlated over time, thus here it is divided into uncorrelated part $u(t)$ and correlated part $c(t)$.

$$\Rightarrow \begin{pmatrix} \dot{x}(t) \\ \dot{c}(t) \end{pmatrix} = \begin{pmatrix} F(t) & B(t) \\ 0 & F_c(t) \end{pmatrix} \begin{pmatrix} x(t) \\ c(t) \end{pmatrix} + \begin{pmatrix} G(t) & 0 \\ 0 & G_c(t) \end{pmatrix} \cdot \begin{pmatrix} u(t) \\ w_c(t) \end{pmatrix}$$

Example: Gauss-Markov process

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & -\beta \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \sqrt{2\sigma^2\beta} \end{pmatrix} \cdot u(t) \quad ?$$

VII. Discrete Kalman Filter

- precondition: $x_n = \phi_{n,n-1} \cdot x_{n-1} + \underline{u_n} \rightarrow$ zero-mean Gaussian white noise $GRN(0,\sigma)$.

$$P_n = \phi_{n,n-1} \cdot P_{n-1} \cdot \phi_{n,n-1}^T + Q.$$

observation $\leftarrow z_n = H_n \cdot x_n + \underline{v_n} \rightarrow GRN(0,\sigma)$.

$$\Rightarrow \left.\begin{aligned} \hat{x}_{n|n-1} &= \phi_{n-1|n-1} \cdot \hat{x}_{n-1|n-1} \\ \hat{P}_{n|n-1} &= \phi_{n-1|n-1} \cdot P_{n-1|n-1} \cdot \phi_{n-1|n-1}^T + Q. \end{aligned}\right\} \text{prediction}$$

$$\left.\begin{aligned} \hat{x}_{n|n} &= \hat{x}_{n|n-1} + \underbrace{K_n}_{\text{Kalman Gain.}} \cdot (z_n - H_n \cdot \hat{x}_{n|n-1}) \\ P_{n|n} &= (I - K_n H_n) \cdot P_{n|n-1} \end{aligned}\right\} \rightarrow \text{correction}$$

with $K_n = P_{n|n-1} \cdot H_n^T (H_n P_{n|n-1} H_n^T + \underline{R_n})^{-1}$ (R_n: covariance of measurement errors)

[Appendix] process of the calculation of K_n:

$$P_{n|n} = E\left((x_n - \hat{x}_{n|n})(x_n - \hat{x}_{n|n})^T\right) = E\left(\left(x_n - \hat{x}_{n|n-1} - K_n(z_n - H_n \cdot \hat{x}_{n|n-1})\right)\left(x_n - \hat{x}_{n|n-1} - K_n(z_n - H_n \cdot \hat{x}_{n|n-1})\right)^T\right)$$

$$= (I - K_n \cdot H_n) \cdot P_{n|n-1} \cdot (I - K_n \cdot H_n)^T + K_n \cdot R_n \cdot K_n^T$$

Basic idea of Kalman filter: minimize the mean-square error of the posteriori state estimation

i.e. $J_n = tr(P_{n|n}) \overset{!}{=} \min \Leftrightarrow \frac{\partial J_n}{\partial K_n} \overset{!}{=} 0$

$$\Rightarrow \frac{\partial\, tr(P_{n|n})}{\partial K_n} = -2(I - K_n H_n) \cdot P_{n|n-1} \cdot H_n^T + 2 K_n R_n \overset{!}{=} 0$$

$$\Rightarrow K_n = P_{n|n-1} \cdot H_n^T \cdot (H_n P_{n|n-1} H_n^T + R_n)^{-1}$$

[Note]
- residuals don't affect uncertainty
- don't trust uncertainty (unless manually tell it) ?

VIII. Backward filter and smoothing

Backward : $t_0 \leftrightarrow t_n$. $\Delta t \leftrightarrow -\Delta t$.

Smoothing : $\hat{x}_n = A \cdot \hat{x}_{n|n} + (I-A) \cdot \hat{x}^b_{n|n}$ where $A = P^b_{n|n} \cdot (P_{n|n} + P^b_{n|n})^{-1}$

$P_n = A P_{n|n} A^T + (I-A) P^b_{n|n} (I-A)^T$

*

Calculation of A: minimize the trace of final covariance matrix

$$tr(P_n) \overset{!}{=} min \Rightarrow \frac{\partial tr(P_n)}{\partial A} \overset{!}{=} 0 \Rightarrow A = P^b_{n|n} \cdot (P_{n|n} + P^b_{n|n})^{-1}$$

Smoothed result :

$$P_n = \left((P^b_{n|n})^{-1} + (P_{n|n})^{-1}\right)^{-1}$$

$$\hat{x}_n = P_n \cdot \left((P_{n|n})^{-1} \cdot \hat{x}_{n|n} + (P^b_{n|n})^{-1} \cdot x^b_{n|n}\right)$$

$\dot{x} = F \cdot x + G(t) \cdot u(t)$.

① $\phi = \exp(F \cdot \Delta t)$

$\hat{x}_{n|n-1} = \phi_{n-1} \cdot \hat{x}_{n-1|n-1}$

$P_{n|n-1} = \phi_{n-1} \cdot P_{n-1|n-1} \cdot \phi_{n-1}^T + Q$

② $z_n = H_n \cdot x_n + v_n$

③ $\hat{x}_{n|n} = \hat{x}_{n|n-1} + K_n \cdot (z_n - H_n \cdot \hat{x}_{n|n-1})$.

$P_{n|n} = (I - K_n \cdot H_n) P_{n|n-1}$.

$K_n = P_{n|n-1} \cdot H_n^T \cdot (H_n \cdot P_{n|n-1} \cdot H_n^T + R_n)^{-1}$

④ Backward.

⑤ $P_n = \left((P_{n|n})^{-1} + (P^b_{n|n})^{-1}\right)^{-1}$

$\hat{x}_n = P_n \cdot \left((P_{n|n})^{-1} \cdot x_{n|n} + (P^b_{n|n})^{-1} \cdot x^b_{n|n}\right)$

IX. Nonlinear Version of Kalman Filter.

$$x_n = f(x_{n-1}, \underline{s_k}) + u_n \qquad s_k \text{ — control vector, usually zero.}$$
$$z_n = h(x_n) + v_n$$

Extended Kalman Filter (EKF):

$$\hat{x}_{n|n-1} = f(\hat{x}_{n-1|n-1}, s_k)$$

$$P_{n|n-1} = F_{n-1} P_{n-1|n-1} F_{n-1}^T + Q_n, \quad \text{with } F_{n-1} = \frac{\partial f}{\partial x}\Big|_{\hat{x}_{n-1|n-1}, s_k}.$$

$$H_n = \frac{\partial h}{\partial x}\Big|_{\hat{x}_{n|n-1}}$$

$$K_n = P_{n|n-1} H_n^T (H_n P_{n|n-1} H_n^T + R_n)^{-1} \quad \text{— near-optimal.}$$

$$P_{n|n} = (I - K_n H_n) P_{n|n-1}$$

$$\hat{x}_{n|n} = \hat{x}_{n|n-1} + K_n \cdot (z_n - h(\hat{x}_{n|n-1})).$$

Unscented Kalman Filter (UKF):

① unscented transform (UT):

select "sigma points" → transform through non-linear function → assign weights → reconstruct $\bar{x}_f$, P_f.

② Kalman filter:

$$\hat{x}_{n|n-1} = \sum_{i=0}^{2n} w_m^{[i]} f(\chi^{[i]})$$

$$P_{n|n-1} = \sum_{i=0}^{2n} w_c^{[i]} \left[f(\chi^{[i]}) - \hat{x}_{n|n-1} \right] \left[f(\chi^{[i]}) - \hat{x}_{n|n-1} \right]^T + Q.$$

$$K_n = \underbrace{P_{n|n-1} H_n^T}_{P^{x,z}_{n|n-1}} \big(\underbrace{H_n P_{n|n-1} H_n^T + R_n}_{S_n} \big)^{-1}$$

$P^{x,z}_{n|n-1}$: covariance between observation & state

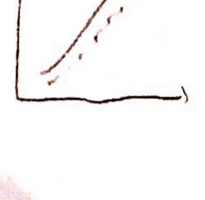
S_n: innovation matrix

$$\hat{x}_{n|n} = \hat{x}_{n|n-1} + K_n (z_n - z_n^*)$$

$$P_{n|n} = (I - K_n H_n) P_{n|n-1} = P_{n|n-1} - K_n S_n K_n^T.$$

Q: Linearized KF?

Monte Carlo simulations?



$x = x^* + \Delta x$

X. Comparison between KF & SLSE (sequential least squares estimation)

Kalman filter $\equiv$ SLSE in case of time-invariant parameters

- State transition matrix $\Phi_{k|k-1}$ is identity matrix
- process noise covariance matrix Q is zero.

] prediction is invalid
⇓
all lie on measurements.