

Physical Geodesy Lab3: Gravity and Coriolis accelerations

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Task1: Gravity, gravitation, and centrifugal accelerations

1.

1.1 Gravitational potential V

$$V = \frac{GM}{R} = \frac{G \cdot 4/3\pi\rho R^3}{R} = \frac{4}{3}\pi G\rho R^2 = 6.256 \times 10^7 \text{ m}^2/\text{s}^2$$

1.2 Centrifugal potential V_c

$$V_c = \int \mathbf{a}_c = \frac{1}{2}\omega^2 R^2 (\cos\varphi)^2 = 9.530 \times 10^4 \text{ m}^2/\text{s}^2$$

1.3 Gravity potential W

$$W = V + V_c = \frac{4}{3}\pi G\rho R^2 + \frac{1}{2}\omega^2 R^2 (\cos\varphi)^2 = 6.266 \times 10^7 \text{ m}^2/\text{s}^2$$

1.4 Gravitational attraction \mathbf{a}

$$\mathbf{a} = -\frac{GM}{R^3} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{GM}{R^3} \cdot \begin{pmatrix} R\cos\varphi\cos\lambda \\ R\cos\varphi\sin\lambda \\ R\sin\varphi \end{pmatrix}$$
$$a = |\mathbf{a}| = \frac{GM}{R^2} = 9.8197 \text{ m/s}^2$$

1.5 Gravity attraction \mathbf{g}

Centrifugal attraction

$$\mathbf{a}_c = -\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_e) = \omega^2 \cdot \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \omega^2 \cdot \begin{pmatrix} R\cos\varphi\cos\lambda \\ R\cos\varphi\sin\lambda \\ 0 \end{pmatrix}$$

Gravity attraction \mathbf{g}

$$\mathbf{g} = \mathbf{a} + \mathbf{a}_c$$
$$g = |\mathbf{g}| = 9.7898 \text{ m/s}^2$$

1.6 Disturbance of the direction ξ

$$\xi = \arccos\left(\frac{\mathbf{a} \cdot \mathbf{g}}{|\mathbf{a}||\mathbf{g}|}\right) = 0.0633^\circ$$

1.7 Disturbance of the attraction δ_g

$$\delta_g = g - a = -0.0299 \text{ m/s}^2$$

2.

2.1 Disturbance of the direction ξ

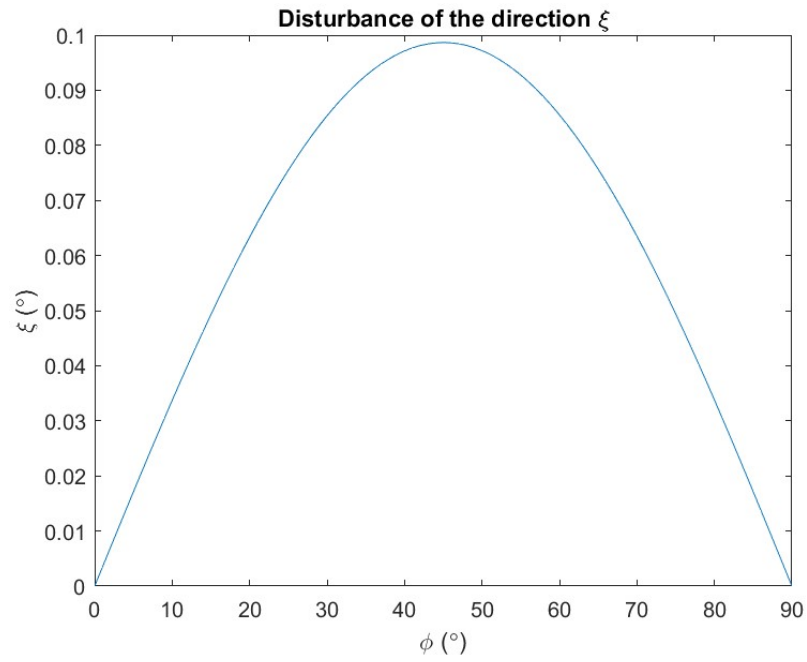


Fig 2.1

2.2 Disturbance of the attraction δ_g

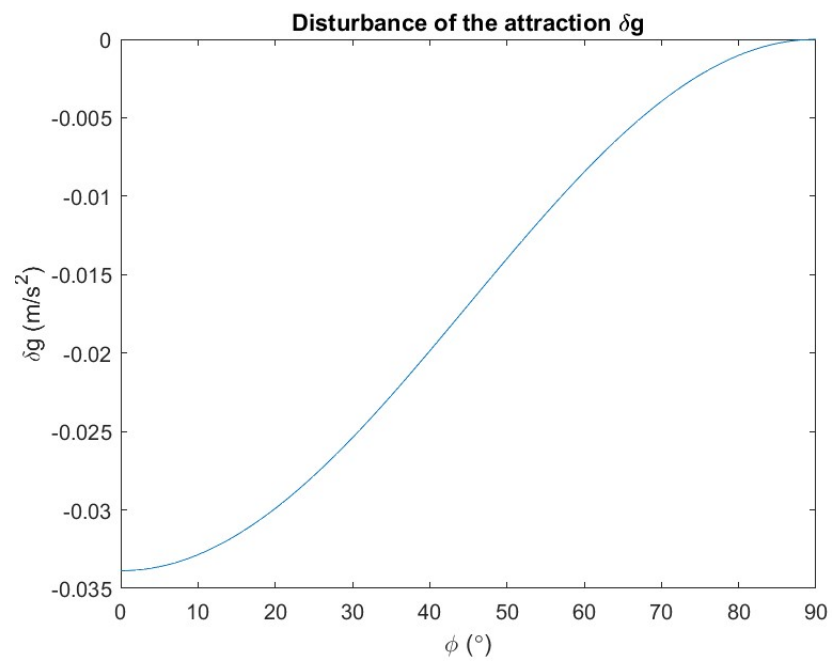


Fig 2.2

2.3 Centrifugal potential V_c

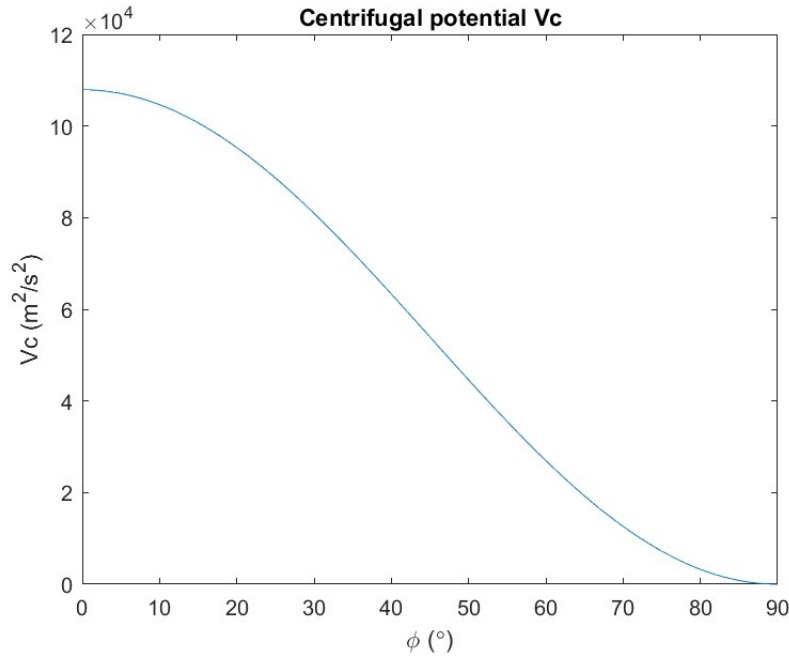


Fig 2.3

Task 2: Eotvos correction

3.1 Coriolis acceleration

$$\mathbf{a}_{cor,e} = -2 \cdot \boldsymbol{\omega} \times \mathbf{r}_e = 2\omega \cdot \begin{pmatrix} \dot{y} \\ -\dot{x} \\ 0 \end{pmatrix} = 2\omega \cdot \begin{pmatrix} -\sin\phi \cos\lambda \cdot v_N - \sin\lambda \cdot v_E \\ -\sin\phi \sin\lambda \cdot v_N + \cos\lambda \cdot v_E \\ \cos\phi \cdot v_N \end{pmatrix}$$

East-West:

$$\mathbf{a}_{cor,e} = 2\omega \cdot \begin{pmatrix} -\sin\lambda \cdot v_E \\ \cos\lambda \cdot v_E \\ 0 \end{pmatrix} = \begin{pmatrix} -0.0028 \\ 0.0160 \\ 0 \end{pmatrix}$$

$$a_{cor,e} = 0.0162 \text{ m/s}^2$$

North-South:

$$\mathbf{a}_{cor,e} = 2\omega \cdot \begin{pmatrix} -\sin\phi \cos\lambda \cdot v_N \\ -\sin\phi \sin\lambda \cdot v_N \\ \cos\phi \cdot v_N \end{pmatrix} = \begin{pmatrix} -0.0107 \\ -0.0019 \\ 0.0120 \end{pmatrix}$$

$$a_{cor,e} = 0.0162 \text{ m/s}^2$$

3.2 Eotvos correction

$$\mathbf{a}_{cor,t} = S_1 \cdot R(\varphi) \cdot R(\lambda) \cdot \mathbf{a}_{cor,e} = 2\omega \cdot \begin{pmatrix} -\sin\varphi \cdot v_E \\ \sin\varphi \cdot v_N \\ \cos\varphi \cdot v_E \end{pmatrix}$$

$$a_{cor,t} = 2\omega \cdot \sqrt{v_E^2 + (\sin\varphi)^2 \cdot v_N^2}$$

East-West:

$$\mathbf{a}_{cor,t} = 2\omega \cdot \begin{pmatrix} -\sin\varphi \cdot v_E \\ 0 \\ \cos\varphi \cdot v_E \end{pmatrix} = \begin{pmatrix} -0.0108 \\ 0 \\ 0.0120 \end{pmatrix}$$

$$a_{cor,t} = 0.0162 \text{ m/s}^2$$

$$da = 2\omega \cdot dv_E$$

$$\sigma_v = \frac{\sigma_a}{2\omega} = 0.0686 \text{ m/s}$$

North-South:

$$\mathbf{a}_{cor,t} = 2\omega \cdot \begin{pmatrix} 0 \\ \sin\varphi \cdot v_N \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.0108 \\ 0 \end{pmatrix}$$

$$a_{cor,t} = 0.0108 \text{ m/s}^2$$

$$da = 2\omega \cdot \sin\varphi \cdot dv_N$$

$$\sigma_v = \frac{\sigma_a}{2\omega \cdot \sin\varphi} = 0.1025 \text{ m/s}$$