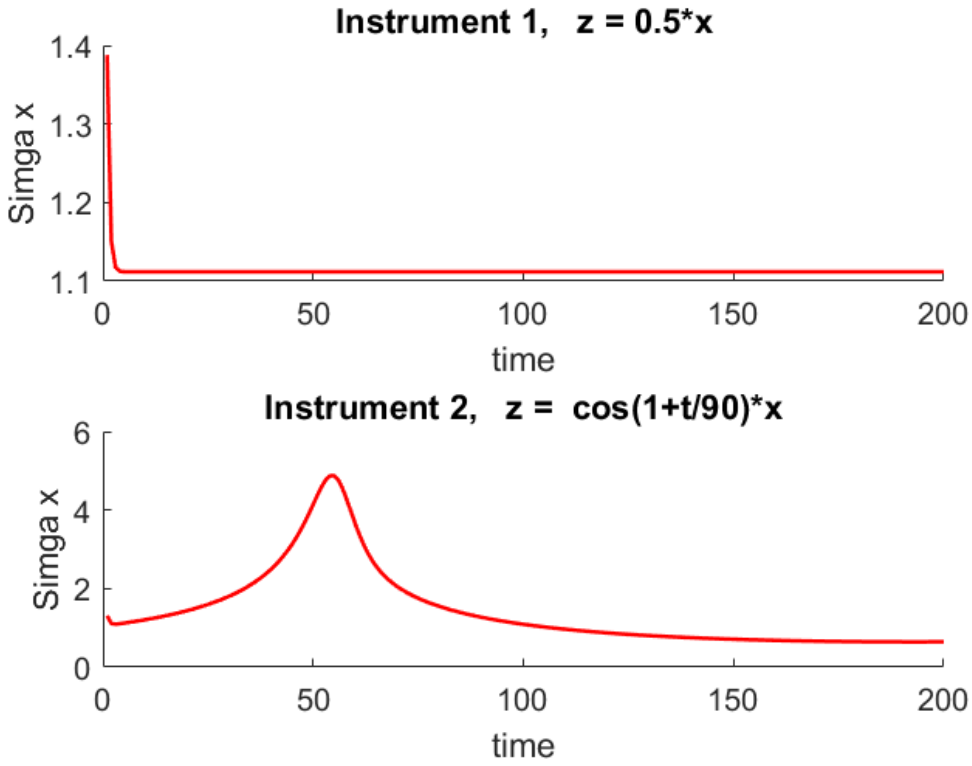
Exercise on 09.07.2019
SOLUTION (Version 0)

Task 1 (5 points)

Uncertainty plot for instrument 1 and 2:



The H-matrix has an influence on the uncertainty. For the case $t = 52$ the H-matrix is nearly zero.

Task 2 (5 points)

A damped mass-spring oscillator is defined by

$$m \cdot \ddot{x}(t) + b \cdot \dot{x}(t) + k \cdot x(t) = 0$$

where

- $x(t)$ denotes the position of the mass at time t
- m is the mass
- b is the damping coefficient
- k is the spring constant

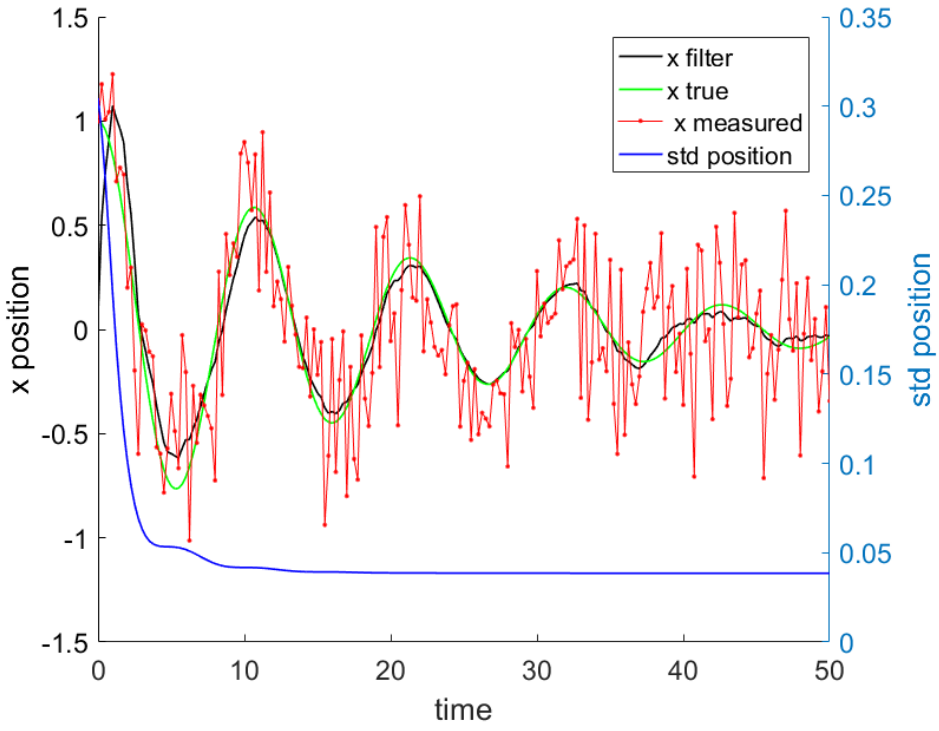
This equation can be rewritten to a first-order linear system

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

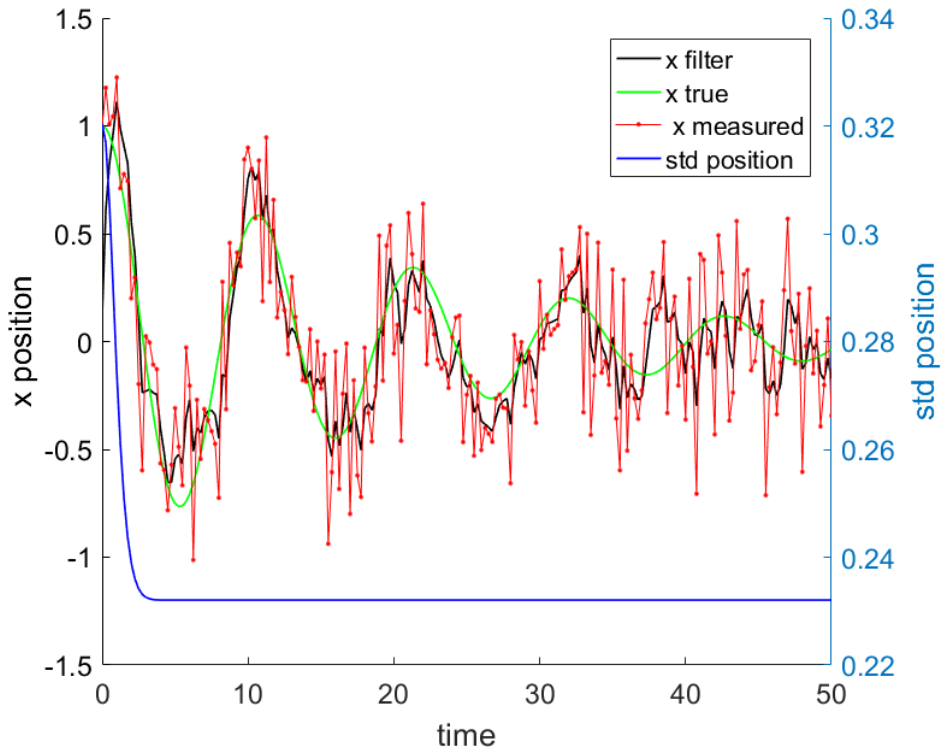
where x_1 is the position x and x_2 is the velocity $\dot{x}$. The transition matrix is derived from the coefficient matrix:

$$\Phi = \text{expm}(F \cdot \Delta t)$$

a) Kalman filter result for the given R and Q:

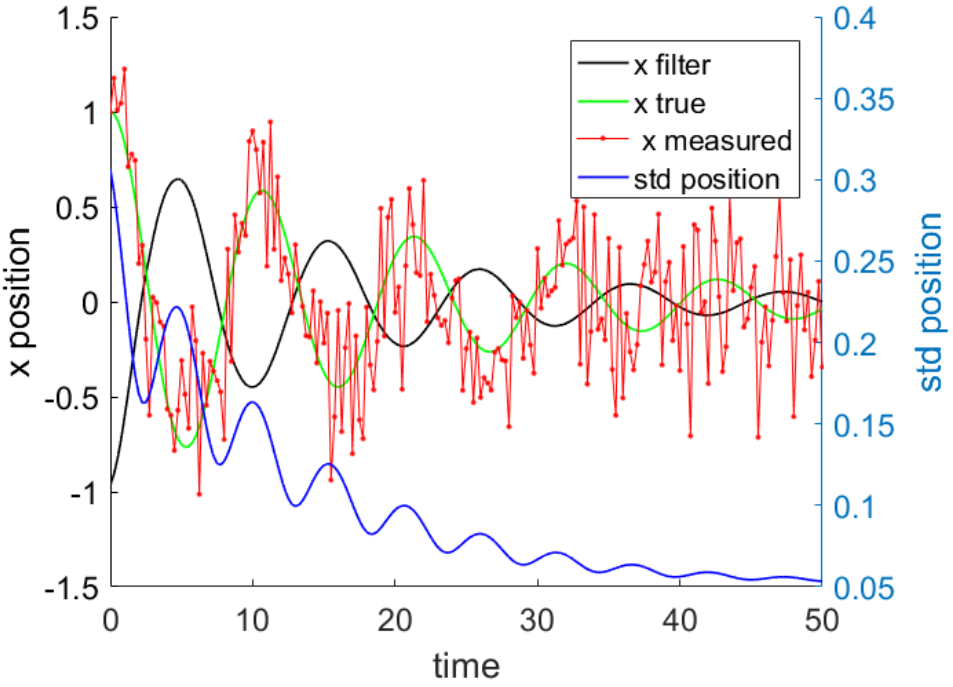


b) Kalman filter result when Q is multiplied with factor 100:



→The filter follows the observations.

c) Kalman filter result when R is multiplied with factor 100:



→The filter follows the process model. Since the initial value was assumed to be $x = -1$ the filtering results are shifted/mirrored to the expected results.