Universität Stuttgart

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Satellite Navigation

Receivers

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Receivers

Decibel calculations

- Historical reason, easier calculations
- Using logarithm ($\log_{10}$)
- Multiplication => addition
- Division => subtraction

$$X[dB] = 10\log_{10}(X[.])$$
$$X[.] = 10^{X[dB]/10} \tag{5.1}$$

Examples:

$$X = 2 \;\Rightarrow X \approx 3dB$$
$$X = 10 \;\Rightarrow X = 10dB$$

Convenient for calculations. E.g. A signal of 100 W gets attenuated (damped) by factor 2 and then later amplified by a factor 40. Compute the power in dB of the signal at the end.

$$20dBW - 3dB + 16dB = 33dBW = 2000W$$

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Link budget

$$P_r = P_t + G_t + G_r - L_f - L_m - \dots \tag{5.2}$$

P_r	received power in decibels Watt [dBW]
P_t	transmitter output power [dBW]
G_t	transmitter antenna gain [dBi]
G_r	receiver antenna gain [dBi]
L_f	free space loss [dB]
L_m	miscellaneous losses (tranmitter loss, receiver loss, fading margin, body loss, polarization mismatch, other losses...) (dB)

Free space loss:

$$L_f = \left(\frac{4\pi \cdot d \cdot f}{c}\right)^2 \Rightarrow$$

$$L_f[\text{dB}] = 20\log_{10}(d) + 20\log_{10}(f) + \underbrace{20\log_{10}\left(\frac{4\pi}{c}\right)}_{-147.55} \tag{5.3}$$

d	distance between the transmitter and the receiver [m]
f	frequency [Hz]
c	speed of light (299 792 458 m/s)

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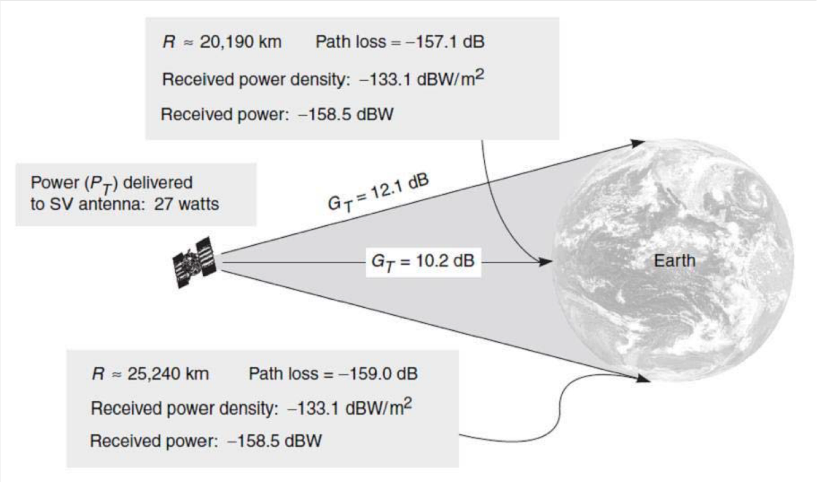
Equalizing the signal power



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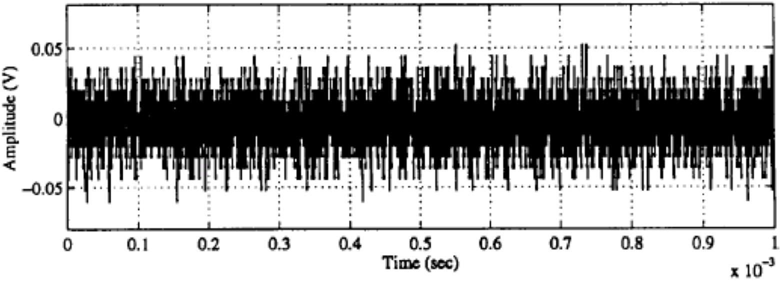
Where is the L1 C/A code in this 1ms sample data?



See discussion on this Jupyter notebook

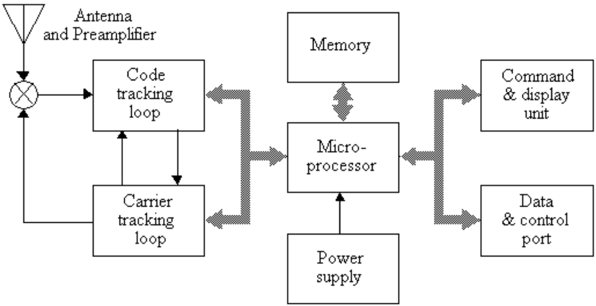
`https://github.com/spacegeodesy/SatNav/blob/master/example05.ipynb`

Receivers

Receiver design

A receiver for electromagnetic signals consists of:

- Antenna
- Preamplifier
- Measuring and processing unit (RF section, Signal trackers, Micro Processor)
- Interfaces/ data ports (in/out)



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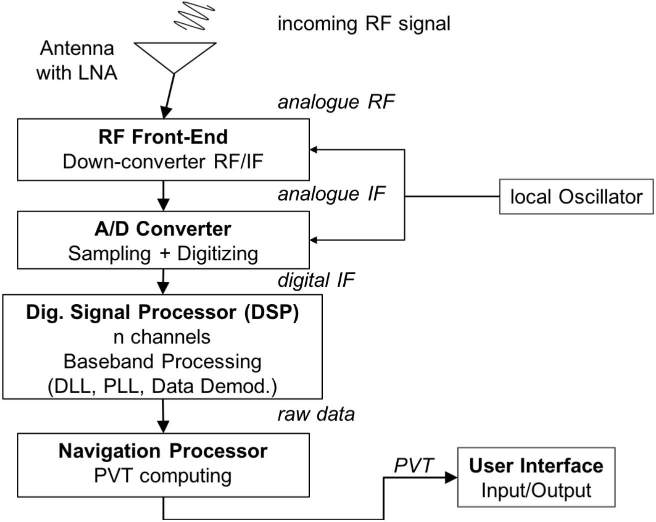
Receiver design



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Receiver design

Antenna

- The incoming electrical field at the antenna can be written as:

$$y = A\cos[\omega(t - \frac{\rho}{c})] \tag{5.4}$$

  ρ is the covered distance of the signal since its transmission by the satellite.
- The antenna is made up of conductive material. The incoming electrical field induces a current that is proportional to the electric field strength.

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{i\omega(t-\frac{\rho}{c})} \tag{5.5}$$

Preamplifier

- The analog primary signal is supplied to the tracking loops of the receiver via the preamplifier.

(Note: In reality there are many different electrical fields present at the antenna. The signal supplied to the receiver is the sum of all these signals.)

Receivers

Receiver design

Filtering and Down- conversion

- Frequency bands without wanted signal will be masked (by band pass filtering) before measurements on the signal are performed.
- high frequency signals are mixed with a reference signal and filtered (Low Pass Filter) afterwards for transformation in a lower frequency domain (directly down to ”baseband” or to an intermediate frequency and then down to baseband) ”Down-conversion”

$$
\begin{aligned}
y_{\text{mixed}} &= A\cos[\omega(t-\frac{\rho}{c})]\cdot\cos(\omega_0 t) \\
&= \frac{A}{2}\cos[\underbrace{(\omega-\omega_0)}_{\text{down conv. freq.)}}\ t-\omega\frac{\rho}{c}] + \frac{A}{2}\cos[\underbrace{(\omega+\omega_0)}_{\text{up conv. freq.}}\ t-\omega\frac{\rho}{c}]
\end{aligned}
$$

$$
y_{\text{mixed+filtered}} = A^*\cos[(\omega_{IF})t-\omega\frac{\rho}{c}]
$$

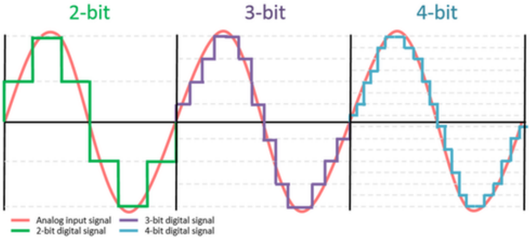
- This transformation is only for metrological and practical reasons and has no influence on the measurements (therefore it will be neglected further on)

Receivers

Receiver design

Preprocessing of the signal

- Depending on the particular receiver several measurements on the signal (amplitudes, phases, frequencies) will be done
- The signal is converted from analogue form in discrete numerical values. After the conversion each operation on the signal is done by software (before by hardware). In modern receivers the analog/digital converter is closer to the signals input port compared to early receiver designs.



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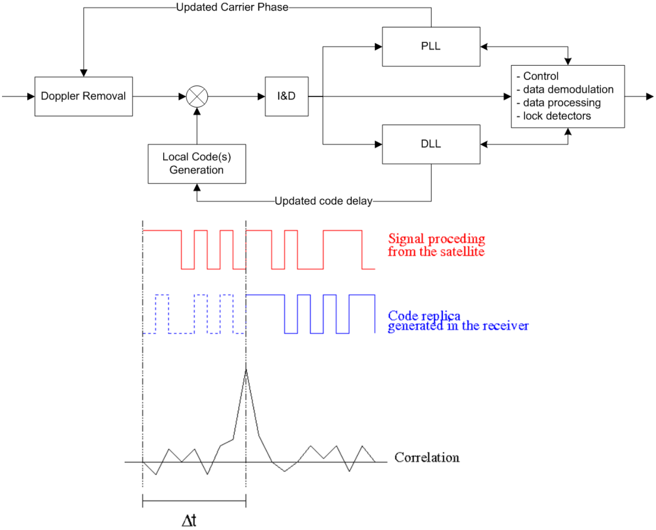
Baseband processing - Overview

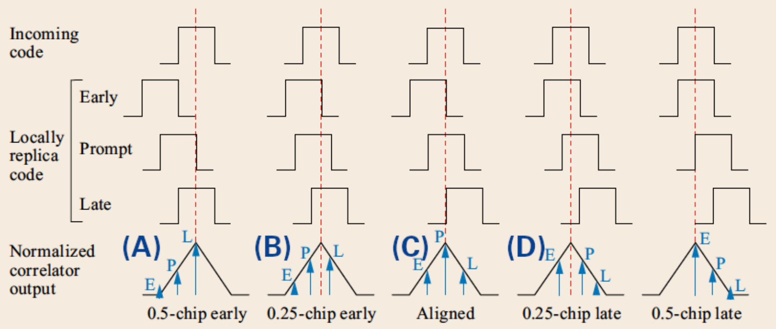


(source: Navipedia, ESA)

Receivers

Delay lock loop (DLL)

E(arly)/P(rompt)/L(ate) channels are usually half-chip spaced

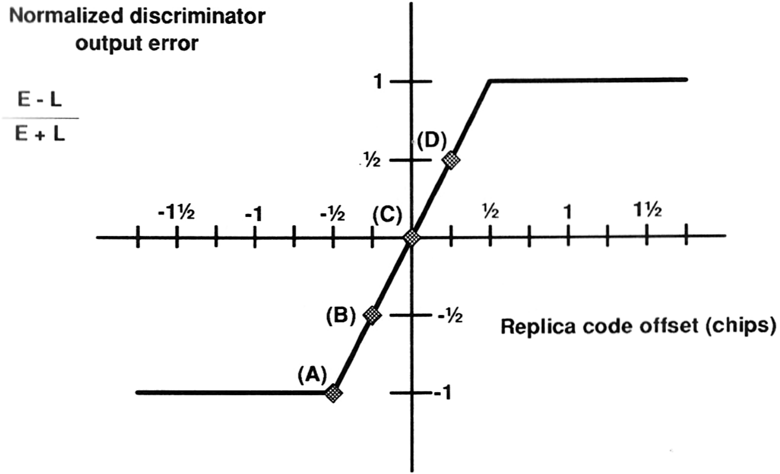


(modified after: Springer Handbook of Global Navigation Satellite Systems, 2017

Question: How can we determine if the replica code is aligned with the incoming signal?

Receivers

Delay lock loop (DLL)



(Source: D. Kaplan, Understanding GPS - Principles and Applications, 1996

Receivers

Delay lock loop (DLL) - Discriminators

$E - L$	Early minus late envelope; for 0.5 chip correlator spacing; good tracking error with $\pm$ 0.5 chip of input error
$(E^2 - L^2)$	Early minus late power; similar performance as $(E - L)$ but less computational load (not square root)
$(E - L)/(E + L)$	Early minus late envelope normalized by the early plus late envelope; removes amplitude sensitivity; for 0.5 chip correlator spacing; good tracking error with $\pm$ 1.5 chip of input error; high computational load; unstable (division by zero) at $\pm$ 1.5 chip input error

Nowadays computational load is no longer a real criteria for DLL design but there are other considerations for selecting a discriminator:

- GNSS signal structure (modulations and codes)
- multi-path suppression

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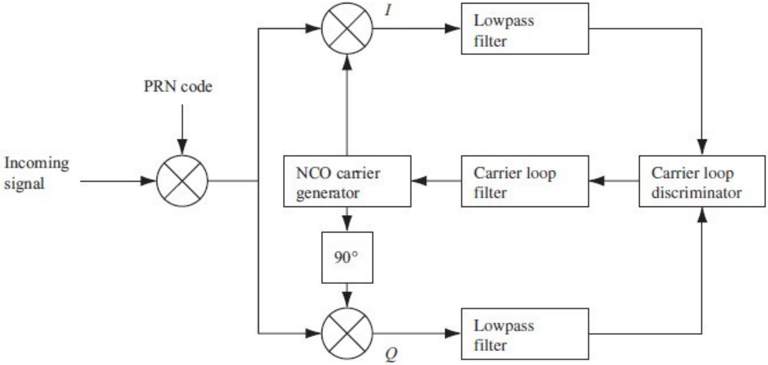
Phase lock loop (PLL)



A so-called Costas loop is usually used to the track the carrier wave. The PRN code of the prompt channels is thereby utilized.

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Costas loop discriminators

Discriminator algorithm	Output phase error.	Characteristics
$\text{sign}(I) \cdot Q$	$\sin\phi$	near optimal at high signal to noise ratio (SNR); slope proportional to signal amplitude
$I \cdot Q$	$\sin 2\phi$	near optimal at low SNR; slope proportional to signal amplitude squared
Q/I	$\tan\phi$	suboptimal, but good at low and high SNR; slope not depending on amplitude; division by zero possible
$\arctan(Q, I)$	ϕ	requires two quadrant arctangent; optimal at low and high SNR; slope not depending on amplitude;

Important: The phase can jump by $\pm$ 180°every 20 ms (in case of GPS) $\Rightarrow$ we can extract the navigation bit message from the PLL as well!

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Frequency tracking (in a simplified explanation)

- The replica code does not only need to be aligned with the proper delay, but also with the proper frequency
- The received signal differs from the the nominal frequency f_0 due to Doppler shift, satellite clock drifts, etc.
- If we use the basic relation between phase ϕ and time t

$$\phi = 2\pi f t$$

  we can use determine the change of frequency $\Delta f = f_2 - f_1$ by measuring the phase states at two consecutive epochs t_1 and $_2$ as

$$\Delta f = \frac{\phi_2 - \phi_1}{t_2 - t_1} \tag{5.6}$$

- The approach denoted by (5.6) requires that phase jumps (by $\pm$ 180°) due to navigation bit changes are treated properly.
- consecutive updating of the local replica frequency allows to keep the coherency between the received signal and the replica code

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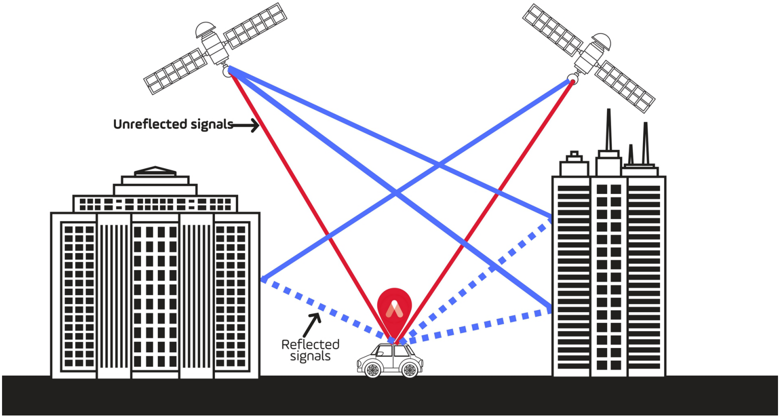
Multipath



(Source: Argus Tracking, 2018)

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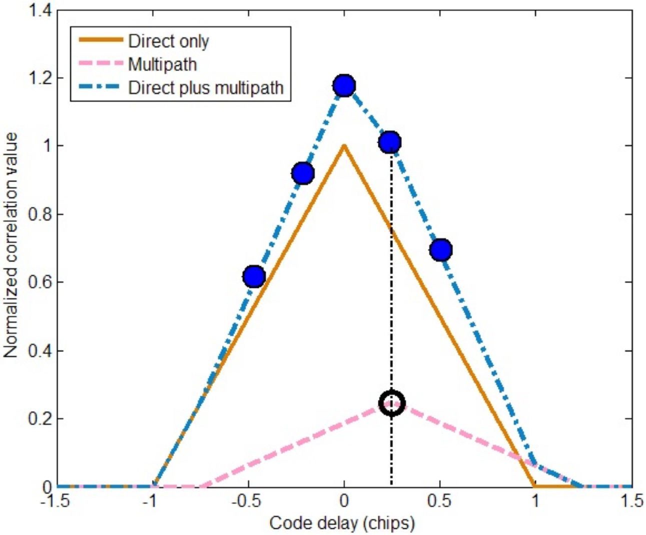
Multipath



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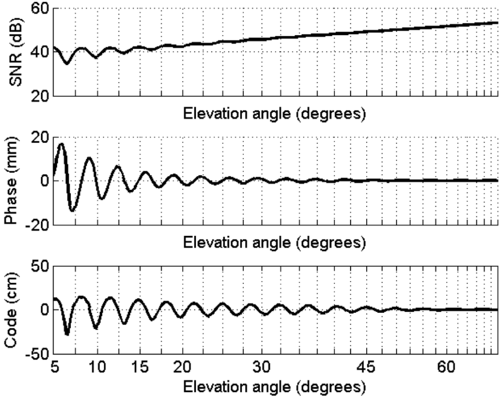
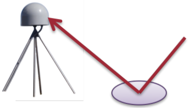
(Simulated) multi-path effects





Signal-to-noise Ratio (SNR)

Carrier phase

Code phase

Receivers

Receiver - Summary

A GNSS receiver

- produces the following observables
  - Code phase P (from the DLL)
  - Carrier phase ϕ which is ambiguous by $2\pi N$ (from the PLL)
  - Doppler shift f_D (from the PLL/FLL)
  - Signal to noise ratio (SNR) (from the PLL)

  and outputs them to internal or external storages or transmit these data over network for differential positioning applications
- extracts the navigation message bits and computes the position of the i-th satellite (X_i, Y_i, Z_i) at transmission time T_i
- computes the (static or dynamic) position (x, y, z, t) based on code phase observations to at least 4 satellites
- continuously switches satellites that come in/go out of view (computed from the almanac which is transmitted in the navigation message)