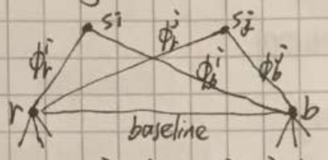
- Double differencing



$$L_{rb}^{ij} = \lambda\left((\phi_r^i-\phi_b^i)-(\phi_r^j-\phi_b^j)\right) = \rho_{rb}^{ij} + c\cdot(\underbrace{dt_{rb}^{ij}}_{\simeq 0} - \underbrace{dT_{rb}^{ij}}_{\simeq 0}) - \underbrace{I_{rb}^{ij} + T_{rb}^{ij}}_{\substack{\simeq 0 \\ \text{(short baseline)}}} + \lambda \underbrace{B_{rb}^{ij}}_{\substack{\parallel \\ N_{rb}^{ij}}} + \underbrace{d_{rb}^{ij}}_{\substack{\simeq 0 \\ \text{(same antenna)}}} + \varepsilon_L$$

$$\Rightarrow L_{rb}^{ij} \simeq \underbrace{\rho_{rb}^{ij}}_{f(\Delta x, \Delta y, \Delta z)} + \lambda N_{rb}^{ij} + \varepsilon_L$$

N_{rb}^{ij}:
- constant if no cycle slip
- integer → integer least square estimation (LAMBDA method)

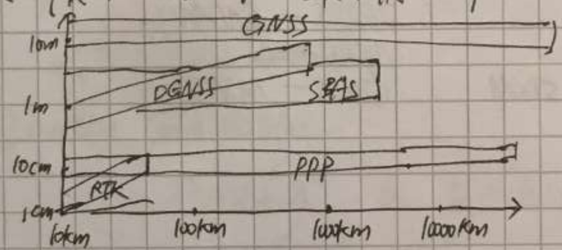
- accuracy : 1cm + 1ppm · baseline length

11. Precise point positioning (PPP)

{ precise orbit & clock information (precise ephemeris)
{ code + carrier phase ⇒ cm–dm accuracy
{ precise error modelling

$$L_k^p = \rho_k^p + c(dt_k - dT^p) - I_k^p + T_k^p + \lambda \overset{(\varphi_{k,0}-\varphi_0^p+N_k^p)}{B_k^p} + d_k^p + \varepsilon_L \quad (= \phi_k - \phi^p + N_k^p)$$

$$P_k^p = \rho_k^p + c(dt_k - dT^p) + I_k^p + T_k^p + \varepsilon_p$$



12. GNSS and outlook

- GPS modernization (24 satellites, 6 orbit planes)

1227.60 L2C = CM × CL → L2C higher accuracy / higher effective power & data structure / widely broadcast
1176.45 L5 = better multi-path suppression

BOC (Binary Offset Carrier) = applying a square-wave subcarrier to BPSK signal

BOC(n,m)
- n: subcarrier frequency n·1.023MHz
- m: code chiprate m·1.023Mcps

⇒ { auto-correlation function has narrower main peak
  { improve code-tracking & multi-path mitigating

- GLONASS (24 satellites, 3 orbit planes)

Frequency Division Multiple Access (FDMA) vs Code Division Multiple Access (CDMA)

GLONASS:
$f_{L1} = 1602 + m \cdot 0.5625$ (MHz)
$f_{L2} = 1246 + m \cdot 0.4375$ (MHz)
m=1,...,24 ↓ m=−7,...,+6.

other GNSS:
$f_{L1} = 1575.42$ MHz (GPS)

GLONASS modernization = satellite (clock stability, dynamic model)
receiving monitoring stations network extension
time keeping system
orbit determination / time synchronization
(reach GPS accuracy 2009)

- Galileo (27+ satellites, 3 frequencies, 3 orbit planes)

code (~~better~~ codes → better ~~f~~ accuracy
free service on 2 frequencies
phase (better phase observations (SNR)
3 carrier frequencies

- Beidou (30 MEO satellites, 5 GEO satellites, 6 orbit planes, 4 frequencies)

global + regional
(civil use 10-20m accuracy)

- QZSS (quasi-zenith satellite system)

4+7 quasi-zenith satellites in different orbit planes → the same ground track @Japan
~~or~~ increase availability in urban/mountain areas (one satellite >60° elevation always)

- IRNSS (Indian regional navigation satellite system)

4 quasi + 3 GEO
dual frequency in S & L5 band
<10m accuracy

- multi-GNSS & SBAS

GPS — WAAS
Galileo — EGNOS
Beidou & GLONASS — SDCM

IRNSS — GAGAN
~~MSAS~~
QZSS — MSAS

Satellite Navigation

1. Introduction

- GNSS segments: space / ground (control) / user
- GNSS orbit: 20 000 km GLONASS – GPS – Beidou – Galileo 12h / circle
  (66 ms) R G C E
- GNSS error sources:
  - satellite clock: < 1m
  - satellite orbit: cm (post-processing) – m (real time)
  - ionospheric delay: 10m – 100+m
  - troposphere delay: m → 30m (low elevation)
  - multi-path m
- GNSS signals: L-band (1-2 GHz). code + carrier

2. Coordinate and Time Systems

- Coordinate system
  - reference ellipsoid: semi-major axis a (6378), semi-minor axis b (6357), flattening $f = \frac{a-b}{a}$ $\frac{1}{298}$
  - geocentric reference frame

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (N+h)\cos\varphi\cos\lambda \\ (N+h)\cos\varphi\sin\lambda \\ ((1-e^2)N+h)\sin\varphi \end{pmatrix}$$

N: radius of curvature in N-S direction $N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}}$

e: eccentricity $e^2 = \frac{a^2-b^2}{a^2} = 2f - f^2$

  - reference system: procedures for creating reference frames. ITRS / ICRS
    reference frame: actual realization of a reference system.
    - ITRF: ~~ ~~ terrestrial
    - ICRF: celestial 天体的
    EOP (UT1)
  - Earth orientation parameters

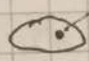
$$Q = \underset{\text{precession}}{P} \cdot \overset{\text{nutation}}{N} \cdot \underset{\text{earth's spin}}{U} \cdot \underbrace{X \cdot Y}_{\text{polar motion}}$$

precession: nutation: polar motion: reference pole → real pole

- Time system
  - universal time (UT1)
    - Greenwich mean sidereal time (GMST) 0h → midnight at Greenwich
    - zero meridian of ITRF → ICRF
    - length of day (LOD) $= -\frac{d(UT1-TAI)}{dt}$ (TAI: atomic time)
    - earth's rotation & solar second
    - not consider rotation angle
  - Coordinated Universal Time (UTC)
    - Based on atomic second & follow earth rotation
    - UTC – TAI = N (seconds) = leap seconds — always synchronize to the earth's rotation better than 1s
    - $|UTC - UT1| < 0.9s$
  - GNSS Time Systems
    - GPS: GPS Time = TAI – 19s (UTC on 1980.01.06 0:0:0)
      GPST 2019 = UTC + 18s
    - GLONASS: UTC
    - Beidou: UTC
    - Galileo: TAI – 19s

(time: counting circles)

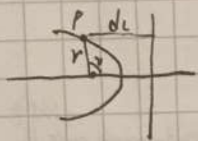
3. Orbits

- usually Medium Earth Orbit
- Spherically symmetric force field
  - gravitational potential $V = \frac{GM}{r}$
  - acceleration $a = -\frac{GM}{r^3} \cdot \vec{x}$
- Kepler Laws: ①  sun ②  $s_1 = s_2$ ③ $\frac{T^2}{a^3} = k$

- Keplerian Elements:
  - a semi-major axis] size & shape
  - e eccentricity
  - i inclination] orientation of orbital plane in space
  - Ω right ascension of the ascending node
  - ω argument of perigee = orientation of ~~orbital~~ ellipse w.r.t the nodal line
  - T_0 time of perigee passing = position of the satellite in its orbit

- shape of the orbit: Two-body Problem

  $r = \frac{p}{1+e\cdot\cos\nu}$ where $p = \frac{h^2}{GM}$, $e = \frac{A}{GM}$ "conic section"



elliptical:	$0<e<1$	$a>0$	$E<0$
parabolic:	$e=1$	$1/a=0$	$E=0$
hyperbolic:	$e>1$	$a<0$	$E>0$

  - energy law: $E = -GM\frac{m}{2a}$, $a = \frac{h^2}{GM(1-e^2)}$.

- Keplerian Elements to Cartesian coordinates

  ① K.E. to orbital plane coord.

$$n = \sqrt{\frac{GM}{a^3}} \Rightarrow M(t) = n(t-t_0)$$

$$M(t) = E(t) - e\sin E(t) \Rightarrow E_{i+1}(t) = M(t) + e\cdot\sin E_i(t), \quad E_0(t) = M(t)$$

$$\Rightarrow \begin{cases} x(t) = a(\cos E(t) - e) \\ y(t) = a\sqrt{1-e^2}\cdot\sin E(t) \\ z(t) = 0 \end{cases}$$

$$\Rightarrow r(t) = a(1 - e\cos E(t)) \Rightarrow$$

$$\begin{cases} \dot{x}(t) = -n\cdot\frac{a^2}{r(t)}\sin E(t) \\ \dot{y}(t) = n\frac{a^2}{r(t)}\sqrt{1-e^2}\cdot\cos E(t) \\ \dot{z}(t) = 0 \end{cases}$$

  ② orbital plane to XYZ

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R_3(-\Omega)\cdot R_1(-i)\cdot R_3(-\omega)\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- Cartesian coordinates to Keplerian Elements

$$\vec{h} = \vec{X} \times \dot{\vec{X}} \Rightarrow \tan\Omega = \frac{h_X}{-h_Y}, \quad \tan i = \frac{\sqrt{h_X^2 + h_Y^2}}{h_Z}$$

$$\left.\begin{aligned} r &= \sqrt{X^2+Y^2+Z^2} \\ v &= \sqrt{\dot{X}^2+\dot{Y}^2+\dot{Z}^2} \end{aligned}\right] \Rightarrow a = \left(\frac{2}{r} - \frac{v^2}{GM}\right)^{-1} \Rightarrow e = \sqrt{1 - \frac{h^2}{a\cdot GM}}$$

$$n = \sqrt{\frac{GM}{a^3}} \Rightarrow \tan E = \frac{\vec{X}\cdot\dot{\vec{X}}/(a^2 n)}{1 - r/a} \Rightarrow \tan\frac{\nu(t)}{2} = \sqrt{\frac{1+e}{1-e}}\cdot\tan\frac{E(t)}{2}$$

$$\cos u = \frac{1}{r}(X\cos\Omega + Y\sin\Omega), \quad \text{if } Z<0,\ u = 2\pi - u$$

$$\Rightarrow \omega = u - \nu$$

$$T_0 = t - \frac{M}{n}$$

non-spherically symmetric force field ⇒ time varying K.E.
"Osculating K.E."

- The disturbed keplerian motion

$$\ddot{X}(t) = -\frac{GM}{(r(t))^3} X(t) + F_{as}(t) + F_S(t) + F_M(t) + F_R(t) + F_A(t)$$

non-symmetric (max) km — sun — moon (10^{-1} km) — pressure from solar radiation — resistence of atmosphere

(3 hour)

- GNSS satellite orbit

GPS constellation : $6 \times 4 = 24$ sats. $i = 55°$ $a = 26500$ km

GLONASS : $3\times$

Galileo : $3\times$

Broadcast Ephemeris = update each hour accuracy < 5m . 3 uploads/day ; 10m . 1 upload/day

Almanac : (ID/t_0, week, $\sqrt{a}$ e, M, w_0. i_0, Ω_0, $\dot{\Omega}$; $\underline{A_{f0}, A_{f1}}$) clock offset / drift

for planning, satellite tracking

update once a week (6 days).

Precise Ephemeris = $X(t)$. $\dot{X}(t)$.

measured data , post-mission , less accurate — ultra-rapid / rapid — for real time

orbit / sat&sta clock — 2.5cm | update every Thursday | sample 15min / 30s & 5min

- Compute XYZ satellite position from broadcast ephemeris parameters

$$\underset{\text{mean motion}}{n_0} \rightarrow \underset{\text{anomalies}}{\Delta n + n_0} \rightarrow u(t), r(t), i(t) \rightarrow \Omega(t), \lambda(t) \rightarrow \begin{cases} r(t), 0, 0 \\ x(t), y(t), 0 \end{cases} \rightarrow \vec{X}(t) = R_3(-\lambda) \cdot R_1(-i) \cdot R_3(-u) \cdot \vec{x}$$

4. Signals

- Atmospheric electromagnetic opacity

$\underbrace{\gamma\text{-ray. X-ray. ultra-violet}}_{\times}$. $\underbrace{\text{visible, near infrared}}_{\checkmark}$. $\underset{\times}{\text{infrared}}$. $\underset{/\checkmark}{\text{radio wave}}$ $\underset{\times}{\text{long-wavelength radio}}$

GNSS = UHF L-band 1-2 GHz (30-15cm)

- Radio signals
  - structure

$$\begin{cases} y(t) = A \cdot \cos(2\pi f_c t) & \text{temporal} \\ y(x) = A \cdot \cos(\frac{2\pi}{\lambda} x) & \text{spatial} \end{cases}$$

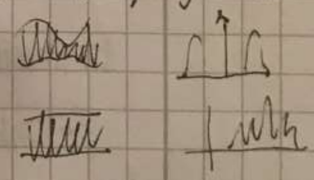
A. amplitude f_c. frequency . $w_c = 2\pi f_c$ angular frequency

$2\pi f_c t$ - phase

  - Modulation
    - amplitude = carrier wave frequency remains the same, amplitude changes

$$y(t) = \underbrace{(A_0 + \delta A \cos(2\pi f_m t))}_{A(t)} \underbrace{\cos(2\pi f_c t)}_{\text{carrier}}$$

Q? space, time & frequency domain?



    - frequency = amplitude the same, frequency changes

$$y(t) = A \cos(2\pi (\underbrace{f_0 + \delta f \cos(2\pi f_m t)}_{\text{carrier}}) t)$$

    - phase = (GPS ✓)

$$y(t) = A \cos(2\pi (f_c t + \underline{\varphi(t)}))$$

"Binary Phase Shift Keyed (BPSK)" — $y(t) = \begin{cases} 0 & \text{, unchanged} \\ \pi & \text{, reversed} \end{cases}$

- GPS signals
  - carrier signals : fundamental frequency $f_0 = 10.23$ MHz

carrier wave $f_{L1} = 154 \cdot f_0 = 1575.42$ MHz

$f_{L2} = 120 \cdot f_0 = 1227.60$ MHz

$f_{L5} = 115 \cdot f_0 = 1176.45$ MHz

• GPS Pseudo Random Noise Codes (PRN)

L_1 – C/A-code (coarse/acquisition) $f_{C/A} = 1.023$ MHz $\Delta t_{C/A} = 1\mu s$ $\Delta S_{C/A} \approx 300m$

L_1&L_2 – P-code (precise) $f_P = 10.23$ MHz $\Delta t_P \approx 0.1\mu s$ $\Delta S_P \approx 30m$

$L_2C(M)$-code (civil-moderate) $f_{CM} = f_{CL} = 0.5115$ MHz $\Delta t \approx 2\mu s$ $\Delta S \approx 600m$

$L_2C(L)$-code (civil-long) "IIR satellite"

• C/A-code: 10-stage shift register ⊗ ⟹ $2^{10}-1 = 1023$ chips for each period

$1023/1.023$ MHz $= 1ms$.

G1: $1+x^3+x^{10}$ (e.g.) → x^{10} initial → "1"

⊕ "modulo-2 add" ⊕ even 0, odd 1

G2 $= 1+x^2+x^3+x^6+x^8+x^9+x^{10}$ → $x^2 \oplus x^6$

each satellite has a different C/A code

• P-code: 12-stage shift register ×4 $\xrightarrow{\text{W-code}}$ Y-code

• CL & CM-code: 27-stage shift register

different satellites — different initial states

{ CM reset 20ms
  CL reset 1.5s

• Nav Data (Message)

1 frame = 5 subframes = 50 words = 1500 bits (=30s)

1 message = 25 frames (=12.5 min)

1	2	3	4	5
6s	6s	6s	6s	6s

1–3: positioning 30s; 4–5: sat. geometry 12.5 min

subframe 1: flags (week number, accuracy, ...) + clock correction coeff.] repeate 30s

2&3: satellite orbit para. (broadcast ephemeris)

4&5: almanac for GPS constellation + health status — repete 12.5 min

$\Delta \varphi = \frac{\pi}{2}$

• message is modulo-2 added to C/A-code on the quadrature component of L1 carrier signal

• IIR-M satellites: additional L2 component

→ message modulo-2 added to CM-code] $\xrightarrow[\text{multiplication}]{\text{chip by chip}}$ L_2 carrier

CL-code 1.023 GHz

BPSK (binary phase shift key)

nav message

$$S_{1i}(t) = P_i(t) \oplus W_i(t) \oplus D_i(t) \otimes A_1\cos(2\pi f_1 t) + C/A_i(t) \oplus D_i(t) \otimes A_1\sin(2\pi f_1 t)$$

$$S_{2i}(t) = P_i(t) \oplus W_i(t) \oplus D_i(t) \otimes A_2\cos(2\pi f_2 t) + [CM_i(t) \oplus D_i(t)] \cdot CL_i(t) \otimes A_2\sin(2\pi f_2 t)$$

5. Receivers

• Link budget

$$P_r = P_t + G_t + G_r - L_f - L_m - \cdots$$

P_r: received power; P_t: transmitter output; G_t: antenna gain transmitter; G_r: receiver antenna gain; L_f: free space loss; L_m: miscellaneous losses

$x\,dB = 10\log_{10} X$

3dB → 2

10dB → 10

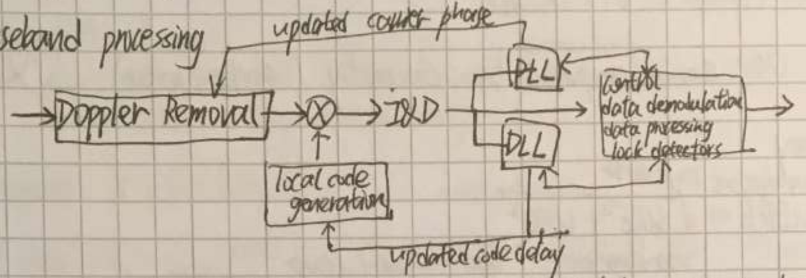
• Receiver design

components: antenna, preamplifier, measuring & processing unit, interface/data ports

(RF section, signal tracker, micro processor)

antenna (conductive material) $y = A\cos[w(t-t_c)]$
preamplifier (supply the primary signal to the tracking loop)
filtering & down-conversion : [low pass] down to baseband
preprocessing : A/D conversion

- Baseband processing



DLL : Delay lock loop (early / prompt / late) — usually half-chip

$\begin{cases} E-L \\ E^2-L^2 & \text{for 0.5 chip correlator spacing} \\ (E-L)/(E+L) & \text{good error} \le 1.5 \text{ input error} \end{cases}$

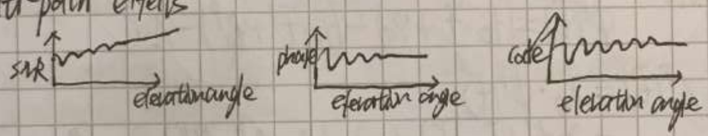
consider < GNSS signal structures / multi-path suppression

PLL : Phase lock loop — "costas loop" ?
- extract navigation message as well (phase jump by 180° every 20ms)

frequency tracking (doppler shift, satellite clock drift, etc.)

$$\Delta f = \frac{\phi_2-\phi_1}{t_2-t_1} \quad \text{(phase jump treated properly)}$$

- multi-path effects



- Receiver outputs :
  - observables : code phase P (from DLL)
    carrier phase ϕ (from PLL) $-2N\pi$.
    doppler shift f_D (PLL/FLL)
    signal-to-noise ratio (PLL)
  - navigation message bits → satellite position
  - receiver position (based on code phase of >4 satellites)
  - (switch satellites in/out view)

6. Observables and error budget

- Pseudorange equation : satellite clock offset

$$P_k^p = \rho_k^p + (\tau_k - \tau^p)\cdot c + I_k^p + A_k^p$$

true range, receiver clock offset, ionosphere delay, troposphere delay

- observation equation :

$$\rho^p(t,t^p) = \sqrt{(X^p(t^p)-x(t))^2 + (Y^p(t^p)-y(t))^2 + (Z^p(t^p)-z(t))^2}$$

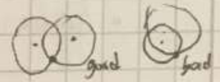
$$p^i = \sqrt{(x^i-x)^2 + (y^i-y)^2 + (z^i-z)^2} + c\tau - c\tau^i + \varepsilon_i \qquad \text{4 unknowns } (x,y,z,\tau)$$

$$\xrightarrow{\text{linearization}} \Delta p^i = \left(\frac{\partial p^i}{\partial x}\ \frac{\partial p^i}{\partial y}\ \frac{\partial p^i}{\partial z}\ \frac{\partial p^i}{\partial \tau}\right)\cdot\begin{pmatrix}\Delta x\\ \Delta y\\ \Delta z\\ \Delta \tau\end{pmatrix} + \varepsilon_i$$

i.e. $\Delta P = A\cdot\Delta x + \varepsilon \Rightarrow \Delta x = \underbrace{(A^TA)^{-1}}_{Q_{xx}} A^T\Delta P$, $\Sigma_{xx} = \sigma_P^2 Q_{xx}$ (σ_P: measurement accuracy of pseudorange)

- Dilution of Precision (DOP)
  - PDOP (overall): $\sigma_P = \sigma_r \sqrt{q_{xx}^2 + q_{yy}^2 + q_{zz}^2}$
  - HDOP (horizontal): $\sigma_H = \sigma_r \sqrt{q_{nn} + q_{ee}}$ — (transform to local coordinate system)
  - VDOP (vertical): $\sigma_V = \sigma_r \sqrt{q_{uu}}$
  - TDOP (time): $\sigma_T = \sigma_r \sqrt{q_{\tau\tau}}$

  DOP can be computed from Geometry (smaller → better)



- Accuracy
  user equivalent range error
  - $UERE = \sqrt{\underbrace{URE}_{\text{user range error}}^2 + \underbrace{UEE}_{\text{user equipment error}}^2}$
  - typical single frequency error budget [m]:
    satellite clock: 1
    ionosphere: 7 $\xrightarrow[HDOP=1.0]{VDOP=1.7}$ $\begin{cases} \sigma_V = 12.1 \\ \sigma_H = 7.1 \end{cases}$
  - pseudorange noise:
    - correlation function width 1% < C/A: 3m ; P: 0.3m
    - thermal noise: C/A < P
    - multi-path
  - Receiver noise = 10cm ~ 1m (pseudorange observation)

- $\boxed{\text{Carrier Phase}}$ Measurement
  - observation:

$$\phi^P(\tilde{t}) = (\underbrace{f_0 \cdot \tilde{t} + \varphi_0}_{\varphi_k(\tilde{t}_k)}) - (\underbrace{f_0 \cdot \tilde{t}^P + \varphi_0^P}_{\varphi^P(\tilde{t}^P)}) - N^P$$

$$\Rightarrow L^P(\tilde{t}) = \lambda_0 \cdot \phi^P(\tilde{t})$$

$$= \lambda_0 f_0 (\tilde{t} - \tilde{t}^P) + \lambda_0 (\varphi_0 - \varphi_0^P - N^P)$$

$$= c(\tilde{t} - \tilde{t}^P) + \underbrace{\lambda_0 (\varphi_0 - \varphi_0^P - N^P)}_{\to B^P}$$

$$= \rho^P(t, t^P) + c\tau - c\tau^P + A^P - \underbrace{I^P}_{\text{"phase advance"}} + B^P$$

  - accuracy:
    - 1% of wavelength < L1: 2mm ; L2: 2.4mm
    - ionospheric delay: 30m
    - troposphere delay: 3-30m

- GNSS data format
  - NMEA (national marine electronics association)
  - RINEX (receiver independent exchange) 2.11 & 3.XX
    only raw data = pseudorange, carrier phase, doppler, SNR.

- single point positioning (SPP)
  - ionosphere 10-100m, phase advance
  - troposphere 3-30m depends on elevation angle

PRN: <1ms time error in 1m satellite position — satellite moves 1km/s
phase: 1μs time error

7. Atmospheric Effects

- Atmospheric structure:

  troposphere —(ozone layer)— stratosphere (同温层) — mesosphere (中间层) — thermosphere (热电离层)
  10km 25km 50km 90km (ionosphere) >1000km
  "temperature": ↘ ↗ ↘ D-E-F₁-F₂ → radio wave propagation velocity $\propto f$

• Excess propagation path (path delay):

$$\Delta L = \int s(n-1)ds + \underbrace{S - G}_{\text{geometric delay (bending) } \Delta L_g \text{ — 0 in zenith direction}}$$

G — geometric direct path

• Refractivity of air:

moist air — constant, depends on < temperature / pressure (density)

water vapor — temperature & density

• Integration of refractivity:

$$\text{"zenith delay"}\ \Delta L = \int_{atm} n(s)ds - \int_{vac} ds = m(\varepsilon)\int_{z}^{\infty} N(z)\cdot 10^{-6}dz = m_{dry}\,\Delta L_{dry} + m_{wet}\,\Delta L_{wet}$$

$\int_{atm} n(s)ds$ — curved propagation; $\int_{vac} ds$ — straight vacuum; $m(\varepsilon)$ — mapping function; z — station height

ΔL_{dry} — ZHD (zenith hydrostatic delay) ~2.3m; ΔL_{wet} — ZWD (zenith wet delay) 0~40cm

$$\Delta L_{trop}(\varepsilon) = \Delta L_{trop}(0)/\sin(\varepsilon) \quad (\varepsilon\text{: elevation angle})$$

• mapping function (MF): relate slant measurements(s) to equivalent zenith values(z)

$$mf = \frac{s}{z} \approx MF(\varepsilon, x_1, x_2, x_3, \ldots) \quad (\text{simplest}: m(\varepsilon) = \frac{1}{\sin\varepsilon})$$

$$m(\varepsilon) = \frac{1 + \frac{a}{1+\frac{b}{1+c}}}{\sin\varepsilon + \frac{a}{\sin\varepsilon + \frac{b}{\sin\varepsilon + c}}}, \quad \varepsilon = 90° \rightarrow m(\varepsilon) = 1.$$

hydrostatic / wet: variant parameters can be estimated with simulated data — parameterization

• Ionospheric Delay (solar activity — free electron)

ionospheric group delay — TEC (total electron content) < solar activity / zenith angle / season / time / magnetic latitude

ionospheric phase advance

• $\Delta L = \frac{40.3}{f^2}\int n_e(s)ds = \pm\frac{40.3}{f^2}\cdot TEC \quad (1\,TECU = 10^{16}/m^2)$ ~ 15mm/km

• "dual-frequency":

$$L_3 = \frac{f_{L1}^2}{f_{L1}^2 - f_{L2}^2}\cdot L_1 - \frac{f_{L2}^2}{f_{L1}^2 - f_{L2}^2}\cdot L_2$$

the closer the frequencies, the larger multiply factor

$\sigma_{L3} > \sigma_{L1}$ or σ_{L2}

8. Parameter estimation & Code phase positioning

• ionospheric delay (from nav message file) — Klobuchar model ($3\alpha + 3\beta$)

nequick model (3α)

• zenith hydrostatic delay (ZHD) — ellipsoidal latitude, height, pressure

— can be estimated by adding parameters in zenith and get mapped down to observation elevation

9. Differential GNSS and augmentation systems

• Positioning models:

3m/5m SPP (single point positioning): absolute positioning, single frequency code observation, broadcast ephemeris

dm-cm PPP (precise point positioning): absolute positioning, carrier phase observation, precise ephemeris + ...

1m/2m DGNSS (differential GNSS): relative positioning, code observation, reference station / service

1cm/2cm RTK (Real time kinematic): relative positioning, carrier phase observation, reference station / service

• DGNSS — types of corrections:

DGNSS with correction data < correction of position / correction of measurements

raw data

- Correction of measurements

$$P_R^p = \rho_R^p + (\delta t_R - \delta t^p)\cdot c + I_R^p + A_R^p + \varepsilon_R^p$$

$$\Rightarrow P_R^p = \underbrace{(\rho_R^p)^0 + \delta\rho_R^p}_{\text{geometric distance}} + (\underbrace{(\delta t_R)^0 + \delta\delta t_R}_{\text{receiver clock error}} - \delta t^p)c + I_R^p + A_R^p + \varepsilon_R^p$$

$$\Rightarrow \underbrace{P_R^p - (\rho_R^p)^0 - c\cdot(\delta t_R)^0}_{-PRC_R^p} = \delta\rho_R^p + (\delta\delta t_R - \delta t^p)\cdot c + I_R^p + A_R^p + \varepsilon_R^p$$

pseudo range correction

$$\Rightarrow \overline{P}_{RU}^p = P_U^p + PRC_R^p$$

$$= (\rho_U^p - \delta\rho_R^p) + (\delta t_U - \delta\delta t_R)\cdot c + \underbrace{(I_U^p - I_R^p)}_{\approx 0} + \underbrace{(A_U^p - A_R^p)}_{\approx 0} + (\varepsilon_U^p - \varepsilon_R^p)$$

removed errors: ephemeris, satellite clock, ionosphere, troposphere   accuracy 0.5m~3m

- additional transmission of rates of change of corrections
  - to solve problem: calculation & transmission need time ← correction only valid for a single t_0

$\Rightarrow$ Rate Range Correction (RRC):

$$RRC_R^p = \frac{\delta PRC_R^p}{\delta t} \Rightarrow RC(t) = PRC_R^p(t_0) + RRC_R^p(t - t_0)$$

- transmission of correction data
  - up link: reference $\xrightarrow{\text{correction}}$ user
  - down link: user $\xrightarrow{\text{~~coord.~~}}$ reference, calculate

correction data
- proprietary formats
- RTCM
  - GBAS + LAAS — terrestrial stations — local [SAPOS in Germany]
  - WADGPS — GEO satellites — regional (OmniSTAR)
- Raw Data
- RTCA — (satellite based augmentation service) SBAS — GEO satellites (reference station → GEO satellite → user) — regional
  - WAAS (US)
  - EGNOS (EU)
  - MSAS (Japan)
  - • geostationary satellites
  - • airborne application
  - • L1/L5
- Ntrip Protocol RTCM — internet

10. Real-time Kinematic Positioning (RTK)

- Carrier Phase Observation

$$\phi_k^p = \phi_k - \phi^p + N_p^k \quad \text{(cycles)}$$

number of integer cycles = "carrier phase ambiguity"

$$L_k^p = \rho_k^p + c(dt_k - dT^p) - I_k^p + T_k^p + \lambda B_k^p + d_k^p + \varepsilon_L \quad (m)$$

carrier phase bias: $\varphi_{k,0} - \varphi_0^p + N_p^k$

other corrections (antenna offset, rotation, ...)

(compare to pseudorange)

$$P_k^p = \rho_k^p + c(dt_k - dT^p) + I_k^p + T_k^p + \varepsilon \quad (m)$$