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Satellite Navigation

Atmospheric Effects

7

Atmospheric Effects

Maxwell Equations

$$\begin{array}{cc}
\nabla \cdot (\varepsilon_r \varepsilon_0 \vec{E}) = 0 & \nabla \cdot \vec{B} = 0 \\
\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & \nabla \times \vec{B} = \mu_r \mu_0 \varepsilon_r \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \\
\Downarrow & \Downarrow \\
\nabla \times (\nabla \times \vec{E}) & \nabla \times (\nabla \times \vec{B}) \\
\Downarrow & \Downarrow \\
\nabla^2 \vec{E} = \mu_r \mu_0 \varepsilon_r \varepsilon_0 \dfrac{\partial^2 \vec{E}}{\partial t^2} & \nabla^2 \vec{B} = \mu_r \mu_0 \varepsilon_r \varepsilon_0 \dfrac{\partial^2 \vec{B}}{\partial t^2}
\end{array} \tag{7.1}$$

Knowing that

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

we get

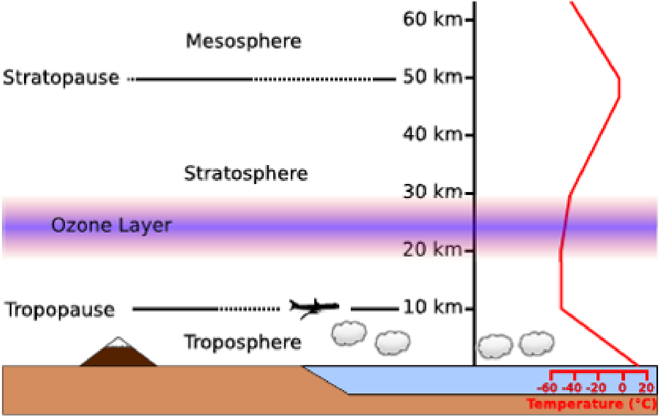
$$\boxed{\nabla^2 \vec{E} = \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{and} \quad \nabla^2 \vec{B} = \frac{n^2}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}} \tag{7.2}$$

where $n = \sqrt{\mu_r \varepsilon_r}$ is the so-called "refractive index".
n is a complex number

- Real parts relates to signal delay
- Imaginary part describes damping of the signal

Atmospheric Effects

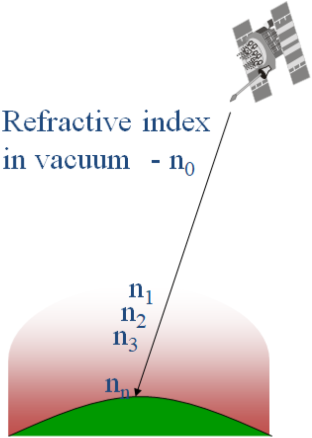
Basic atmospheric structure



Troposphere is where the temperature stops decreasing in the atmosphere. (ca. 10 km altitude)

Atmospheric Effects

Atmosphere and Observation Equation



$$P^k = \rho^k + c(d\tau - d\tau^k) + \Delta L_{atm} + \epsilon \tag{7.3}$$

Atmospheric Effects

Definition of the excess propagation path

- Assume a refractive index, n, in the atmosphere. The electrical path length L of a signal propagating along S is defined as

$$L = \int_s n \, ds \tag{7.4}$$

- The path S is determined from the index of refraction in the atmosphere using Fermat's Principle, to wit: the signal will propagate along the path that gives the minimum value of L.

- The geometrical straight line distance, G, through the atmosphere is always shorter than the path S of the propagated signal.

- The electrical path length of the signal propagating along G is longer than that for the signal propagating along S.

Atmospheric Effects

Definition of the excess propagation path The difference between the electrical path length and the geometrical straight line distance is called excess propagation path, path delay, or simply delay:

$$\Delta L = \int_s n\, ds - G \tag{7.5}$$

We may rewrite this expression as

$$\Delta L = \int_s (n-1) ds + S - G$$

where $S = \int_s ds$. The $(S - G)$ term is often referred to as the geometric delay or the delay due to bending, denoted as ΔL_g:

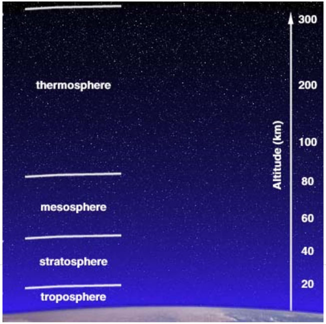
$$\Delta L_g \equiv S - G \tag{7.6}$$

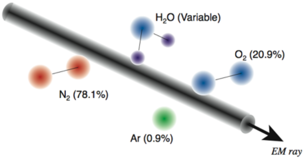
If the atmosphere is horizontally stratified, S and G are identical in the zenith direction and hence the geometric delay becomes equal to zero at this angle. (typically 3 cm at an elevation angle of 10°and 10 cm at 5°).

Atmospheric Effects

Neutral Atmosphere

- Main components of dry air (N_2, O_2, Ar, CO_2)
- Composition constant up to about 100 km
- Above 100 km these gases appear as separate gases
- Other important constituents are Ozone and Water vapor
- Stratospheric Ozone important for absorption of UV-radiation and X-ray radiation from the sun (only 0.0001% of volume)
- Tropospheric Ozone is harmful





Atmospheric Effects

Neutral Atmosphere

- Lowest part of the Earth's Atmosphere
- Temperature decrease with an increase in altitude (0.6 - 0. 7°C/100 m)
- Almost all "weather" activity takes place here
- The thickness is about 9 km over the poles and exceeds 16 km over the equator (tropopause)
- Below 30 GHz not dispersive (e.g. GPS and GLONASS identically influenced)

The Stratosphere

- In the lower stratosphere the temperature is constant (10-30 km)
- Increasing temperature in upper part (30 - 50 km)
- Maximum temperature at stratopause at an altitude of about 50 km
- The increase in temperature is due to the natural presence of ozone
- Absorption of ultraviolet emission from the sun
- Small variation in temperature prevents from circulations → pollution only removed over a substantial period of time

Atmospheric Effects

The Upper Atmosphere

Mesophere

- From an altitude of about 50 km
- Temperature is decreasing with increasing altitude
- Minimum temperature at about 80-90 km (mesopause)

Thermosphere

- Increasing temperature with increasing altitude
- Maximum between 1500-3000 Kelvin at 500 km
- Very thin air and extremely low pressure
- Electromagnetic radiation and emission from the sun ionize gases

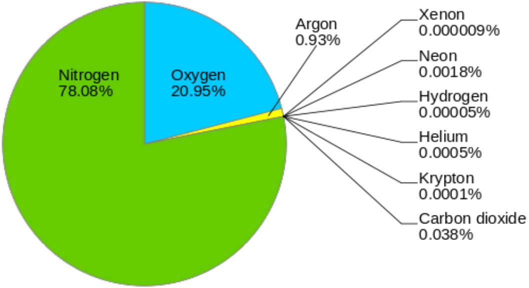
$\Rightarrow$ Ionosphere

- Dispersive medium for radio waves i.e. propagation velocity depends on frequency (e.g. GLONASS and GPS influenced differently)

Atmospheric Effects

Refractivity of air

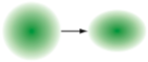
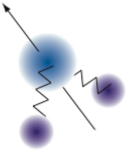
- Air is made up of specific combination of gases, the most important ones being oxygen and nitrogen.



- Each gas has its own refractive index that depends on pressure and temperature.
- For the main air constituents, the mixing ratio of the constituents is constant and so the refractivity of a packet of air at a specific pressure and temperature can be defined.
- The one exception to this is water vapor which has a very variable mixing ratio.
- Water vapor refractivity also depends on density/temperature due to dipole component.

Atmospheric Effects

Refractivity of air

Molecule Type	Atmospheric Examples	Modes of Interaction
	Ar, Ne, He, Kr, Xe	*Slight distortion of electron cloud*
	N_2, O_2	*Vibrational*
	H_2O, CO_2	*Vibrational plus rotational (dipole moment)*

Atmospheric Effects

Refractivity of air

The refractivity of moist air is given by:

$$N = k_1 \cdot \underbrace{\frac{P_d}{T} Z_d^{-1}}_{\text{density of dry air}} + k_2 \cdot \underbrace{\frac{P_w}{T} Z_d^{-1}}_{\text{density of water vapor}} + k_3 \cdot \underbrace{\frac{P_w}{T^2} Z_d^{-1}}_{\text{dipole component of water vapor}}$$

$$k_1 = 77.60 \pm 0.05 \text{ K/mbar} \tag{7.7}$$

$$k_2 = 70.4 \pm 2.2 \text{ K/mbar}$$

$$k_3 = (3.730 \pm 0.012) \times 10^5 \text{ K}^2\text{/mbar}$$

$$N = 10^6 (n - 1) \text{ where n is the refractive index}$$

For most constituents, refractivity depends on density (i.e., number of air molecules).
Water vapor dipole terms depends on temperature as well as density

Atmospheric Effects

Refractivity of air

We can write the refractivity in terms of density:

$$N = k_1 \frac{R}{M_d}\rho + \left(\frac{k_2'}{T} + \frac{k_3}{T^2}\right) P_w Z_w^{-1} \tag{7.8}$$

$$k_2' = k_2 - k_1 M_w / M_d = 22.1 \pm 2.2\text{K/mbar}$$

Density ρ is the density of the air parcel including water vapor.
R is universal gas constant, M_d and M_w are molecular weights. Z_w is compressibility (deviation from ideal gas law).

We can write Eq. (7.8) also as

$$N = \underbrace{k_1 \frac{R}{M_d}\rho}_{N_{\text{dry}}} + \underbrace{\left(\frac{k_2'}{T} + \frac{k_3}{T^2}\right) P_w Z_w^{-1}}_{N_{\text{wet}}} \tag{7.9}$$

$$= N_{\text{dry}} + N_{\text{wet}}$$

Atmospheric Effects

Integration of Refractivity

To model the atmospheric delay, we express the atmospheric delay as:

$$\Delta L = \int\limits_{atm} n(s)ds - \int\limits_{vac} ds \approx m(\varepsilon) \int\limits_{Z}^{\infty} (n(z) - 1)dz = m(\varepsilon) \int\limits_{Z}^{\infty} N(z) \times 10^{-6} dz \quad (7.10)$$

Where the atm path is along the curved propagation path; vac is straight vacuum path, z is height for station height Z and $m(e)$ is a mapping function. (Extended later for non-azimuthally symmetric atmosphere). The final integral is referred to as the "zenith delay".

$$\begin{aligned}
\Delta L &= \int\limits_{atm} n(s)ds - \int\limits_{vac} ds \approx m(\varepsilon) \int\limits_{Z}^{\infty} N(z) \times 10^{-6} dz \\
&= m_{dry}(\varepsilon) \int\limits_{Z}^{\infty} N_{dry}(z) \times 10^{-6} dz + m_{wet}(\varepsilon) \int\limits_{Z}^{\infty} N_{wet}(z) \times 10^{-6} dz \\
&= m_{dry}(\varepsilon)\Delta L_{dry} + m_{wet}(\varepsilon)\Delta L_{wet}
\end{aligned}$$

$$\Delta L_{dry} = ZHD$$

$$\Delta L_{wet} = ZWD$$

Atmospheric Effects

Zenith hydrostatic delay

The Zenith hydrostatic delay is given by:

$$ZHD = 10^{-6} k_1 \frac{R}{M_d} g_m^{-1} P_s \approx 0.00228\text{m/mbar} \tag{7.12}$$

where g_m is mean value of gravity in column of air (Davis et al. 1991)

g_m can be computed by

$$g_m = 9.8062(1 - 0.00265\cos(2\phi) - 3.1 \cdot 10^{-7}(0.9z + 7300)\text{ms}^{-2}$$

P_s is total surface pressure (again water vapor contribution included).
Since P_s is 1013 mbar at mean sea level a typical ZHD = 2.3 meters

Atmospheric Effects

Zenith wet delay (ZWD)

- In meteorology, the term "Precipitable water" (PW) is used. This is the integral of water vapor density with height and equals the depth of water if all the water vapor precipitated as rain (amount measured on rain gauge).
- If the mean temperature of atmosphere is known, PW can be related to Zenith Wet Delay (ZWD)

PW and ZWD

Relationship:

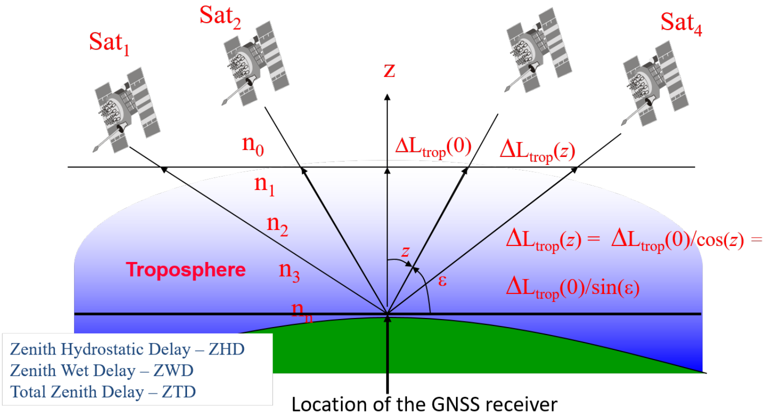
$$ZWD = 10^{-6}\frac{R}{M_w}(k_2' + k_3/T_m)PW \tag{7.13}$$

$$T_m = \frac{\int P_w/T dz}{\int P_w/T^2 dz} \tag{7.14}$$

- The factor for conversion is $\sim$ 6.7 mm delay/mm PW
- ZWD is usually between 0-40cm.

Atmospheric Effects

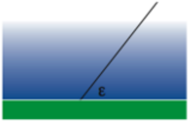
Tropospheric Model



Atmospheric Effects

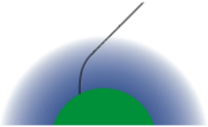
Mapping functions

- Zenith delays discussed so far; how to relate to measurements not at zenith
- Problem has been studied since 1970's.
- In simplest form, for a plain atmosphere, elevation angle dependence would behave as $1/\sin(\varepsilon)$. (At the horizon, $\varepsilon = 0$ and this form goes to infinity.)
- For a spherically symmetric atmosphere, the 1/sin(elev) term is "tempered" by curvature effects.
- Most complete form is "continued fraction representation" (Davis et al., 1991).



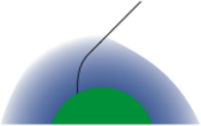
Bad: Flat Earth, no refraction

$\tau(\varepsilon) = \tau_z / \sin\varepsilon = \tau_z \csc\varepsilon = \tau_z\, m(\varepsilon)$



Better: Spherical layers, refraction included

$\tau(\varepsilon) = \tau_z / (\sin\varepsilon + a / (\sin\varepsilon + b / (\sin\varepsilon + c \ldots)))$
$= \tau_z\, m(\varepsilon)$



Best: Model complicated variations

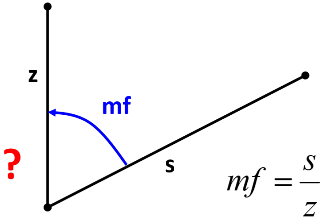
$\tau(\varepsilon, \alpha) = \tau_z \; m(\varepsilon, \alpha)$

τ_z: Zenith Delay
$m(\varepsilon)$ or $m(\varepsilon, \alpha)$: Mapping Function

Atmospheric Effects

Mapping functions

- Mapping functions (mf) relate slanted measurements (s) to equivalent zenith values (z) using a functional expression depending on the elevation angle ε;
- mf are approximated by analytical functions MF
- Mapping functions ensure that functional systems are over-determined.



Coefficients x_i characterize the behavior of the MF !

$$mf = \frac{s}{z} \approx MF(\varepsilon, \boxed{x_1, x_2, x_3 \ldots})$$

Atmospheric Effects

Mapping functions

$$MF(\varepsilon, a, b, c) = \frac{1 + \cfrac{a}{1 + \cfrac{b}{1 + c}}}{\sin\varepsilon + \cfrac{a}{\sin\varepsilon + \cfrac{b}{\sin\varepsilon + c}}} \tag{7.15}$$

Fitting of the 3 parameters a,b,c is carried out with ray-traced results from different elevation angles.

Basic form of mapping function was deduced by Marini (1972) and matches the behavior of the atmosphere at near-zenith and low elevation angles. Form is:

$$m(\varepsilon) = \frac{1}{\sin\varepsilon + \cfrac{a}{\sin\varepsilon + \cfrac{b}{\sin\varepsilon + \cfrac{c}{\sin\varepsilon + \ldots}}}} \tag{7.16}$$

Atmospheric Effects

Mapping functions

When the mapping function is truncated to the finite number of terms then the form is:

$$m(\varepsilon) = \frac{1 + \cfrac{a}{1 + \cfrac{b}{1 + c}}}{\sin\varepsilon + \cfrac{a}{\sin\varepsilon + \cfrac{b}{\sin\varepsilon + c}}} \tag{7.17}$$

when $\varepsilon = 90$, $m(e) = 1$

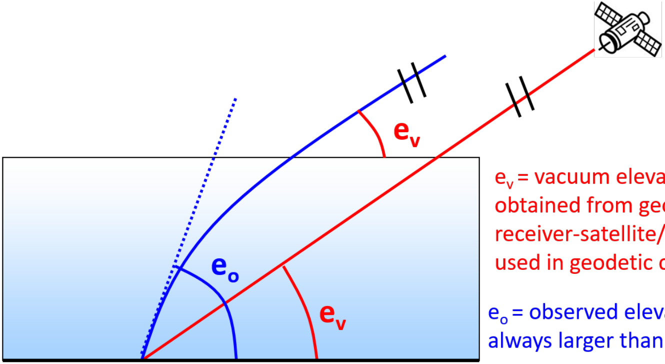
Basic problem with forming a mapping function is determining the coefficient a,b,c etc. for specific weather conditions. Different "solutions" available.

- The typical values for the coefficients are
- Hydrostatic: $a = 1.232 \cdot 10^{-3}$, $b = 3.16 \cdot 10^{-3}$; $c = 71.2 \cdot 10^{-3}$
- Wet delay: $a = 0.583 \cdot 10^{-3}$, $b = 1.402 \cdot 10^{-3}$; $c = 45.85 \cdot 10^{-3}$
- Since coefficients are smaller for wet delay, this mapping function increases more rapidly at low elevation angles.
- At 0 degrees, hydrostatic mapping function is ca. 36. Total delay ca. 82 meter

Atmospheric Effects

Caveats

Mapping functions are defined for vacuum delays!



e_v = vacuum elevation angle, obtained from geometry receiver-satellite/radio source, used in geodetic calculations

e_o = observed elevation angle, always larger than e_v

Thus: after ray-tracing the outgoing elevation angles (which are nearly identical to the vacuum delays) have to be used for MF fitting!

Atmospheric Effects

Tropospheric Mapping Functions

The total tropospheric delay, ΔL , at any elevation angle, ε, is often modeled as a function of the elevation angle

$$\Delta L(\varepsilon) = ZHD \cdot m_h(\varepsilon) + ZWD \cdot m_w(\varepsilon) \tag{7.18}$$

where m_h and m_w are the hydrostatic and wet mapping functions.
A simple example of such a mapping function could look like

$$m(\varepsilon) = 1/sin(\varepsilon) \tag{7.19}$$

For any elevation down to 15° we can often use the same mapping function for both the hydrostatic and the wet delay (better than 5 mm)

Atmospheric Effects

Gradients

- In recent years; more emphasis put on deviation of atmospheric delays from azimuthal symmetry.
- These effects are much smaller (usually <30mm) but do effect modern GNSS measurements.
- There is a mean NS gradient that is latitude dependent and probably due to equator to pole temperature gradient.
- Parameterized as $\cos(\alpha)$ and $\sin(\alpha)$ (α: azimuth) terms with a "tilted" atmosphere model.

Effects of atmospheric delays

Effects of the atmospheric delay can be approximately assessed using a simple model of the form:

$$y = \begin{bmatrix} 1 & \sin(\varepsilon) & m(e) \end{bmatrix} \cdot \begin{bmatrix} \Delta clk \\ \Delta H \\ \Delta ZWD \end{bmatrix} \tag{7.20}$$

Simulated data y (e.g. error in mapping function) can be used to see effects on clock estimate (Δclk), height (Δh), and atmospheric delay (Δatm).

If atmospheric zenith delay not estimated, then when data is used to 10° elevation angle, error in height is ca. 6 times zenith atmospheric delay error.

Atmospheric Effects

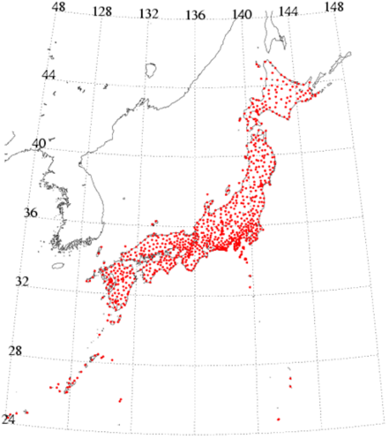
Parameterization of atmospheric delay

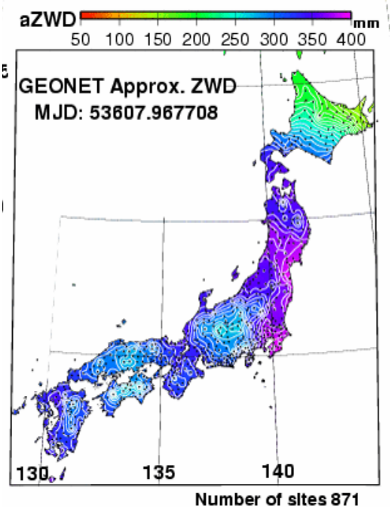
- Atmospheric delays are one the limiting error sources in GNSS;
- Parameterization is either Kalman filter or coefficients of piece-wise linear functions;
- Delays are almost always estimated:
  - At low elevation angles can be problems with mapping functions
  - Spatial inhomogenity of atmospheric delay still unsolved problem even with gradient estimates.
  - Estimated delays are being used for weather forecasting if latency <2 hrs.

By estimating troposphere delays GNSS can also be used to measure parameters of the atmosphere !!!

Atmospheric Effects

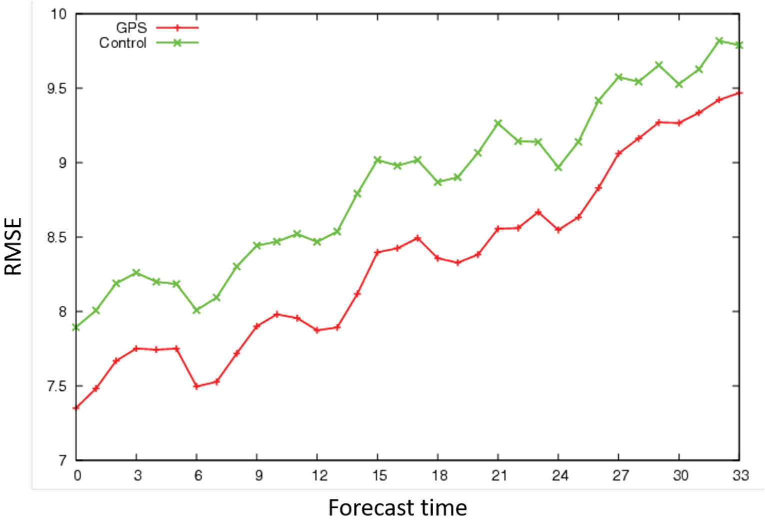
Outlook applications





Atmospheric Effects

Outlook applications

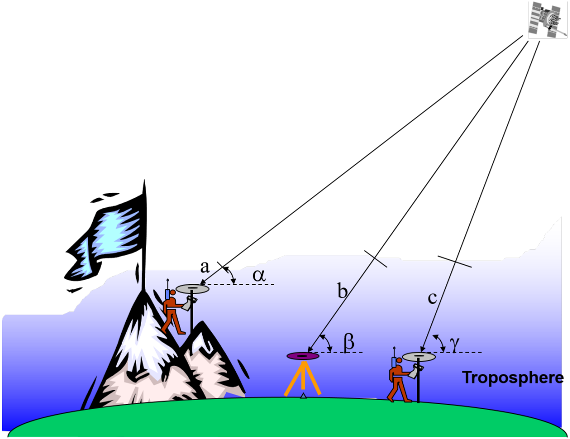


Atmospheric Effects

Influence of the Troposphere

Propagation path trough the troposphere varies in length (a, b, c) depending on:

- Elevation angle to the satellite (α, β, γ)
- Height difference, 8 m ⇔ 1 mbar in air pressure ⇔ 2 mm in signal delay in zenith
- Local variations in water vapor

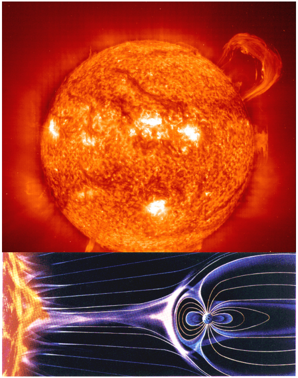


Atmospheric Effects

Propagation: Ionospheric delay

- Propagating waves and effects of low density plasma
- Additional effects
- Treatment of ionospheric delay in GPS processing
- Examples of some results

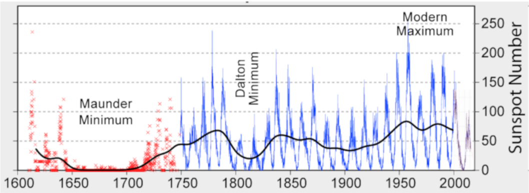
Solar activity

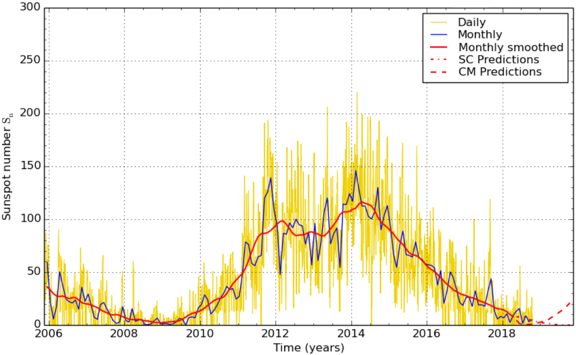


- The "solar cycle" is about 11 years.
- The sun has seasons
- Solar maximum occur with a periodicy of 11 years
- Outbursts at the surface of the sun result in radiation towards the Earth. The particles are spread by the "solar wind" ("wind speed" up to 800 km/s) consisting of protons and electrons
- For real-time "space weather reports" see e.g. `http://www.sec.noaa.gov`

Atmospheric Effects

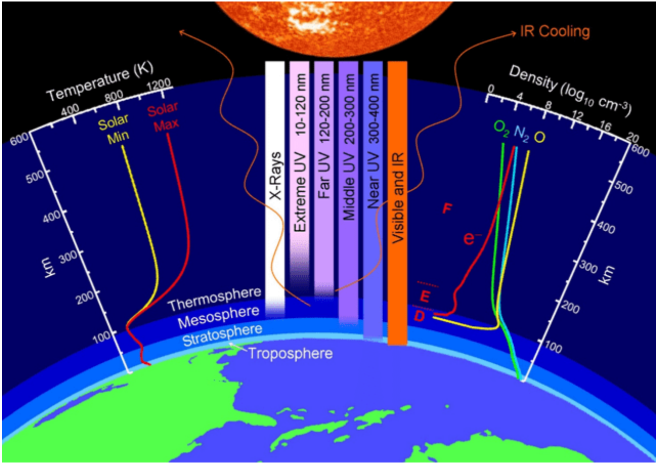
The Solar Cycle





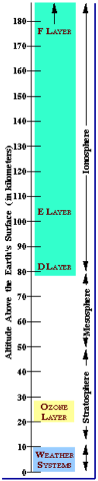
Atmospheric Effects

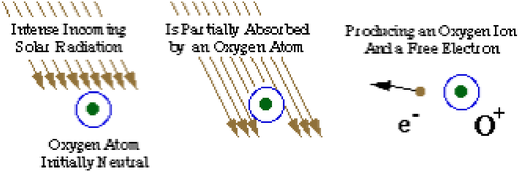
The Ionosphere



Atmospheric Effects

Where is the Ionosphere and what happens there?





Atmospheric Effects

The Ionosphere

The ionosphere lies above 99.9% of the mass of the Earth's atmosphere and at an altitude of about between 50-1000 kilometers.

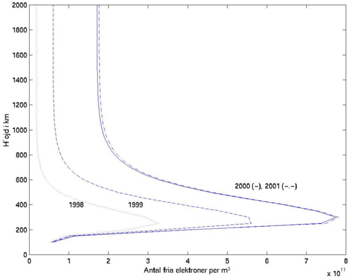
Because the Sun is the primary source of free electron production, the ionosphere varies to a large extent with the rotation of the Earth and with solar activity. The ionosphere is generally classified by the D, E, F_1, and F_2 regions.

Region	Altitude	Electron Density	Remark
D	below 90 km	$10^8 el/m^3$	
E	90 - 120 km	$10^{11} el/m^3$	
F_1	120 - 180 km	$10^{12} el/m^3$	refraction, delay
F_2	180 - 1000 km	$10^{12} el/m^3$	polarization, absorption

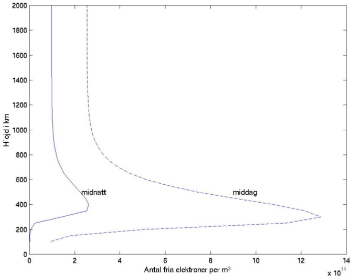
Atmospheric Effects

Ionospheric free electron content

Annual differences



Differences between night and day

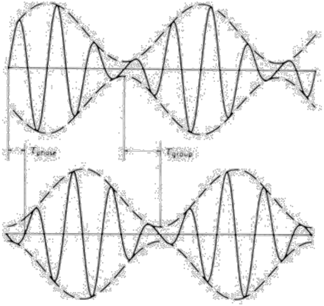


Atmospheric Effects

The Ionosphere and radio propagation

- When radio waves propagate through the ionosphere several effects occur. These are proportional to the total electron content (TEC).
- Retardation of the modulation carrier wave also known as the ionospheric group delay.
- Advance of the carrier, known as the ionospheric phase advance.
- TEC in the ionosphere varies in time and space. TEC is a function of solar activity, zenith angle, season, time of day and magnetic latitude.

Group and Phase Delay



For an animated explanations watch `https://www.youtube.com/watch?v=tlM9vq-bepA`

Atmospheric Effects

Calculating the Total Electron Content using GNSS

- The impact on GNSS radio propagation is quite accurately modeled and measured using dual-frequency (e.g. GPS L1 $\sim$ 1575 MHz and L2 $\sim$ 1227 Mhz) observations

Below elevation of 15°, other propagation errors such as multipath, tropospheric delay add to effects from ionospheric propagation significantly

The refractive index in the ionosphere

The refractive index depends on frequency, number of electrons, and the electron charge and mass.

$$n \approx 1 \pm \left(\frac{n_e e^2}{\pi m_e}\right)\left(\frac{1}{f^2}\right) + \frac{k}{f^3} + \dots \tag{7.21}$$

Refractivity

$$N = (n-1) \cdot 10^6 \approx \pm \frac{40.3 \times 10^6 n_e}{f^2} \tag{7.22}$$

Atmospheric Effects

Calculating TEC

Total Electron Content (TEC) in number of electrons/m^2 (1 TECU = 10^{16} electrons/m^2)

$$TEC = \int n_e(s)ds \tag{7.23}$$

Propagation path delay in the Ionosphere

$$\boxed{\begin{aligned} \Delta L = \int n(s)ds - \int ds \approx \int N \times 10^{-6} ds = \pm \frac{40.3}{f^2} \int n_e(s)ds \\ \Delta L = \pm \frac{40.3}{f^2} TEC \end{aligned}} \tag{7.24}$$

Linear Combinations

$$\begin{aligned} L_1 = L_t + \Delta L_1 \qquad L_2 = L_t + \Delta L_2 \\ f_{F1}^2 \cdot L_1 - f_{L2}^2 \cdot L_2 = (f_{L1}^2 - f_{L2}^2) \cdot L_t \end{aligned} \tag{7.25}$$

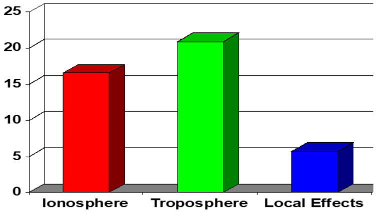
$$\boxed{L_3 = \frac{f_{L1}^2}{f_{L1}^2 - f_{L2}^2} L_1 - \frac{f_{L2}^2}{f_{L1}^2 - f_{L2}^2} L_2} \tag{7.26}$$

Notice that the closer the frequencies, the larger multiplying factors.
For GPS frequencies: 2.546 and 1.546

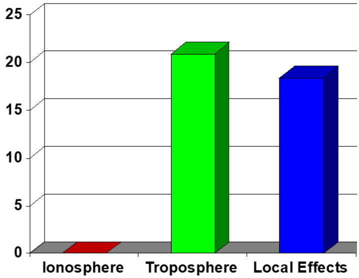
Atmospheric Effects

Magnitudes

- The factors 2.546 and 1.546 which multiple the L1 and L2 range measurements, mean that the noise in the ionospheric free linear combination is larger than for L1 and L2 separately.
- If the range noise at L1 and L2 is the same, then the R_c range noise is 3-times larger.
- For GNSS receivers separated by small distances, the differential position estimates may be worse when dual frequency processing is done
- As a rough rule of thumb; the ionospheric delay is 1-10 parts per million (i.e. 1-10 mm over 1 km)



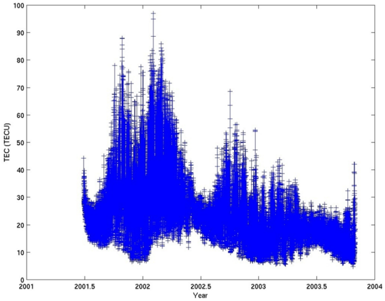
L1

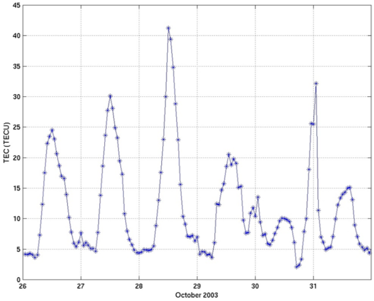


L3

Atmospheric Effects

Real-time TEC values





Atmospheric Effects

Summary

- Effects of ionospheric delay are large on GPS (10s of meters in point positioning); 1-10ppm for differential positioning
- Largely eliminated with a dual frequency correction at the expense of additional noise (and multipath)
- Residual errors due to neglected terms are small but can reach a few centimeters when ionospheric delay is large.