Universität Stuttgart

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Satellite Navigation

Real-time kinematic (RTK) positioning

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RTK positioning

"Standard positioning" (SPP, DGNSS) vs. "precise positioning" (RTK)

	Standard Positioning (code-based)	Precise Positioning (carrier-based)
Observables	pseudorange (code)	carrier phase (+ pseudorange)
Recv. noise	$\sim$ 30 cm	$\sim$ 3 mm
Multipath	30 cm - 30 m	1 - 3 cm
Discontinuities	none	cycle slips
Ambiguities	none	estimated / resolved
Accuracy	3 m / 5 m (H/V) SPP, 1 m / 2 m (H/V) DGNSS	1 cm / 2 cm (H/V) RTK
Applications	Basic navigation, timing, ...	surveying, prec. navigation, ...

Remember:

Measured carrier phase is denoted as

$$\phi_k^p = \phi_k - \phi^p + N_p^k \quad \text{[cycles]} \tag{10.1}$$

where N_p^k is the number of (integer cycles), which we will refer to from now as carrier phase ambiguity.

RTK positioning

Carrier phase

$$\begin{aligned}
\Phi_k^p &= \varphi_k(t_k) - \varphi^p(t^p) + N_p^k + \varepsilon_\Phi \qquad (\varphi_{k,0} = \varphi_k(t_0),\ \varphi_0^p = \varphi^p(t_0)) \\
&= (f(t_k + dt_k - t_0) + \varphi_{k,0}) - (f(t^p + dT^p - t_0) + \varphi_0^p) + N_p^k + \varepsilon_\Phi \\
&= \frac{c}{\lambda}(t_k - t^p) + \frac{c}{\lambda}(dt_k - dT^p) + (\varphi_{k,0} - \varphi_0^p + N_p^k) + \varepsilon_\Phi \quad \text{[cycles]}
\end{aligned} \tag{10.2}$$

$$\begin{aligned}
L_k^p &\equiv \lambda\Phi_k^p = c(t_k - t^p) + c(dt_k - dT^p) + \lambda(\varphi_{k,0} - \varphi_0^p + N_p^k) + \lambda\varepsilon_\Phi \\
&= \rho_k^p + c(dt_k - dT^p) - I_k^p + T_k^p + \lambda B_k^p + d_k^p + \varepsilon_L \quad \text{[m]}
\end{aligned} \tag{10.3}$$

where B_k^p is referred to as carrier-phase bias and d_r^p summarizes all other correction terms.

Compare to pseudorange

$$P_k^p = \rho_k^p + c(dt_k - dT^p) + I_k^p + T_k^p + \varepsilon_P \text{[m]} \tag{10.4}$$

RTK positioning

Carrier phase bias

$$B_k^p = \varphi_{k,0} - \varphi_0^p + N_p^k \tag{10.5}$$

N_p^k : **integer ambiguity**

$\varphi_{k,0}$: receiver initial phase

φ_0^p : satellite initial phase

Other correction terms (in simplified notation!)

$$d_k^p = \delta_{PCO,k} + \delta_{PCO}^p + \delta_{PCV,k} + \delta_{PCV}^p + \delta_{PWU} + \delta_{rel} \tag{10.6}$$

$\delta_{PCO,k}$: receiver antenna phase center offset effect

δ_{PCO}^p : satellite antenna phase center offset effect

$\delta_{PCV,k}$: receiver antenna phase center variation effect

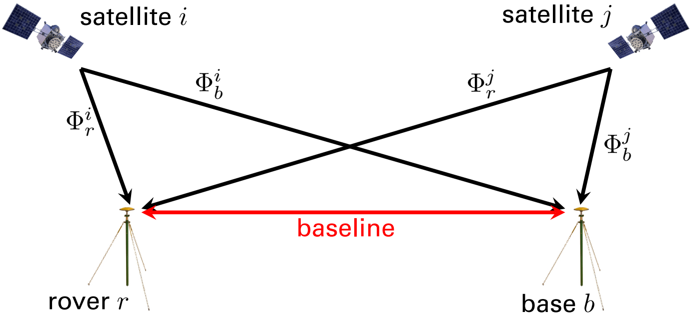
δ_{PCV}^p : satellite antenna phase center variation effect

δ_{PWU} : phase wind-up effect

δ_{rel} : relativistic effects

RTK positioning

Double differencing



$$
\begin{aligned}
L_{rb}^{ij} &= \lambda((\Phi_r^i - \Phi_b^i) - (\Phi_r^j - \Phi_b^j)) \\
&= \rho_{rb}^{ij} + c(dt_{rb}^{ij} - dT_{rb}^{ij}) - I_{rb}^{ij} + T_{rb}^{ij} + \lambda B_{rb}^{ij} + d_{rb}^{ij} + \varepsilon_L
\end{aligned}
\tag{10.7}
$$

RTK positioning

Double differencing

Clock effects:

$$dt_{rb}^{ij} = dt_r^{ij} - dt_b^{ij} = 0$$
$$dT_{rb}^{ij} = dt_{rb}^{i} - dt_{rb}^{j} \approx 0$$

(Differenced) phase bias:

$$\begin{aligned} B_{rb}^{ij} &= (\varphi_{r,0} - \varphi_0^i + N_r^i) - (\varphi_{b,0} - \varphi_0^i + N_b^i) - (\varphi_{r,0} - \varphi_0^j + N_r^j) + (\varphi_{b,0} - \varphi_0^j + N_b^j) \\ &= N_{rb}^{ij} \end{aligned}$$

Thus Eq. (10.7) becomes

$$L_{rb}^{ij} = \rho_{rb}^{ij} - I_{rb}^{ij} + T_{rb}^{ij} + \lambda N_{rb}^{ij} + d_{rb}^{ij} + \varepsilon_L \qquad (10.8)$$

In case of **short baselines** differential troposphere (T_{rb}^{ij}) and ionosphere (I_{rb}^{ij}) contributions are negligible. If rover r and base station b are equipped with the same antennas also $d_{rb}^{ij} \approx 0$ which leaves the very simple double-difference equation

$$\boxed{L_{rb}^{ij} \approx \rho_{rb}^{ij} + \lambda N_{rb}^{ij} + \varepsilon_L} \qquad (10.9)$$

RTK positioning

Integer ambiguity resolution

- Eq. (10.9) contains the position of the rover w.r.t. the base, i.e.

$$\rho_{rb}^{ij} = f(\Delta x, \Delta y, \Delta z)$$

  as well as the unknown number of ambiguities N_{rb}^{ij}.
- N_{rb}^{ij} is constant as long as no cycle slip happens
- We have usually an overdetermined problem if we track enough satellites and form double differences. Thus we could solve the problem by means of simple weighted least-squares
- However, we know that solutions for N_{rb}^{ij} should belong to the space of integer numbers

Mathematically we face the minimization problem

$$\boxed{\hat{\boldsymbol{x}} = \underset{\boldsymbol{a} \in \mathbb{Z}^n,\ \boldsymbol{b} \in \mathbb{R}^m}{\arg\min} \left\{ (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})^T \boldsymbol{Q}_y^{-1} (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}) \right\}} \tag{10.10}$$

where the state vector $\boldsymbol{x}$ has length $n + m$ and contains n unknowns $\boldsymbol{a}$ which should lie in the space of integer numbers $\mathbb{Z}^n$ while the other m parameters $\boldsymbol{b}$ can take arbitrary (float) values und thus lie in the space of real numbers $\mathbb{R}^m$.

RTK positioning

Integer least squares (ILS) estimation

- different approaches (e.g. simple rounding)
- most common is the so-called LAMBDA method (Teunissen, 1995, `https://link.springer.com/content/pdf/10.1007%2FBF00863419.pdf`).
- ILS provides us also with a statistical criteria that tells us if we could successfully find ("fix") to an integer ambiguity.
  - Numbers close to 100% indicate good RTK solutions
  - fix ratios get worse with baseline length!

RTK summary

- Real-time positioning of the rover
- requires a data-link so that carrier phase observations are available in real-time at the rover
- On-the-fly (OTF) integer ambiguity resolution
- Typical (horizontal) accuracies: 1 cm + 1 ppm $\times$ baseline length
- Extension of (network-) RTK: virtual reference station, FKP, ... (not discussed in this lecture series!, contact us if you need more information on such concepts)