GENEHMIGT

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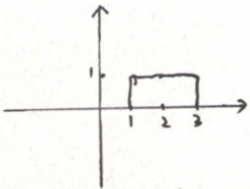
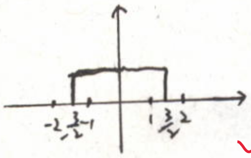
Signal Processing Lab 1

Graphical and Analytical Convolution

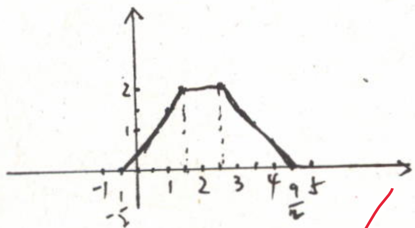
1.1

a) $u(t) = \mathrm{rect}\left(\frac{t-2}{2}\right)$ $h(t) = \mathrm{rect}\left(\frac{t}{3}\right)$





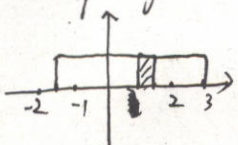
Hence, $g(t) = \begin{cases} 0, & t < -\frac{1}{2} \\ t+\frac{1}{2}, & -\frac{1}{2} \le t < \frac{3}{2} \\ 2, & \frac{3}{2} \le t < \frac{5}{2} \\ -t+\frac{9}{2}, & \frac{5}{2} \le t < \frac{9}{2} \\ 0, & t \ge \frac{9}{2} \end{cases}$

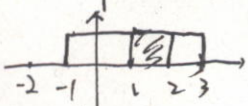


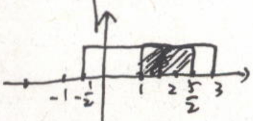
$u(t) \to u(\tau)$, $h(t) \to h(-\tau+t)$

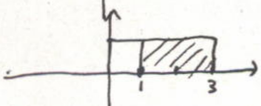
$t < -\frac{1}{2}$, no overlap $\Rightarrow g(t) = 0$

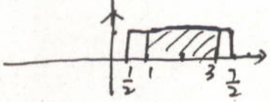
$t = -\frac{1}{2}$, overlap begins

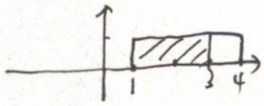
$t = 0$,  $\Rightarrow g(t) = 1 \cdot \frac{1}{2} = \frac{1}{2}$

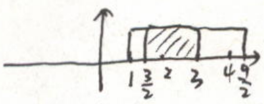
$t = \frac{1}{2}$,  $\Rightarrow g(t) = 1 \cdot 1 = 1$

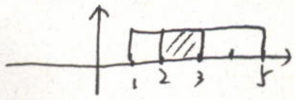
$t = 1$,  $\Rightarrow g(t) = 1 \cdot \frac{3}{2} = \frac{3}{2}$

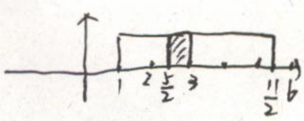
$t = \frac{3}{2}$,  $\Rightarrow g(t) = 1 \cdot 2 = 2$

$t = 2$,  $\Rightarrow g(t) = 1 \cdot 2 = 2$

$t = \frac{5}{2}$,  $\Rightarrow g(t) = 1 \cdot 2 = 2$

$t = 3$,  $\Rightarrow g(t) = 1 \cdot \frac{3}{2} = \frac{3}{2}$

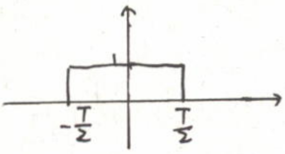
$t = \frac{7}{2}$,  $\Rightarrow g(t) = 1 \cdot 1 = 1$

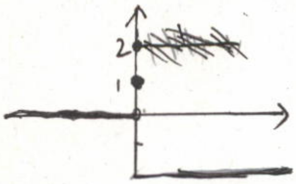
$t = 4$,  $\Rightarrow g(t) = 1 \cdot \frac{1}{2} = \frac{1}{2}$

$t = \frac{9}{2}$, overlap ends

$t > \frac{9}{2}$, no overlap $\Rightarrow g(t) = 0$.

b) $u(t) = \mathrm{rect}\left(\frac{t}{T}\right)$ $\quad$ $h(t) = -2\gamma(t)$



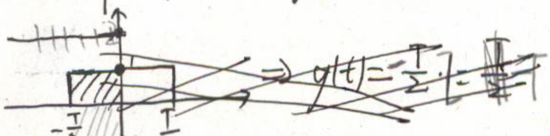


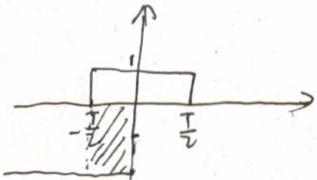
$u(t) \to u(\tau)$
$h(t) \to h(t-\tau)$

$t < -\frac{T}{2}$, no overlap $\Rightarrow g(t) = 0$

$t = -\frac{T}{2}$, overlap begins

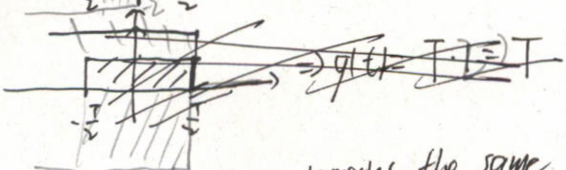
$t = 0$,

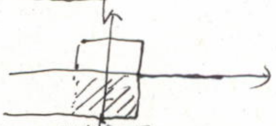




$\Rightarrow g(t) = \frac{T}{2}\cdot(-2) = -T$

$t = \frac{T}{2}$,

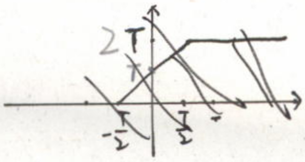


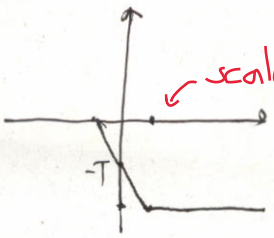


$\Rightarrow g(t) = \frac{T}{T}\cdot(-2) = -2T$

$t \geq \frac{T}{2}$, the overlap area remains the same $\Rightarrow g(t) = -2T$.

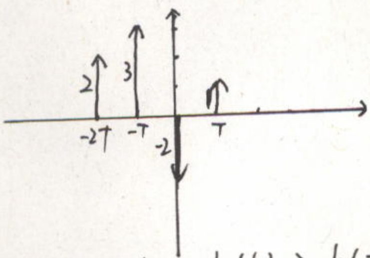
Hence $g(t) = \begin{cases} 0, & t < -\frac{T}{2} \\ -2t-T, & -\frac{T}{2} \leq t < \frac{T}{2} \\ -2T, & t \geq \frac{T}{2} \end{cases}$

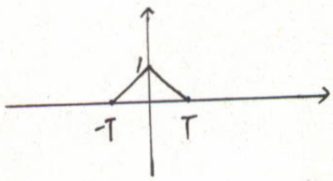




scale?

c) $u(t) = 2\delta(t+2T) + 3\delta(t+T) - 2\delta(t) + \delta(t-T)$ $\quad$ $h(t) = \mathrm{tri}\left(\frac{t}{T}\right)$





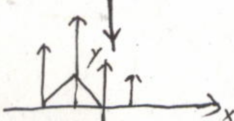
$u(t) \to u(\tau)$, $h(t) \to h(t-\tau)$

$t < -3T$, no overlap $\Rightarrow g(t) = 0$

$t = -2T$,

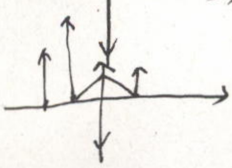
$\Rightarrow g(t) = 2$.

$t = -T$,



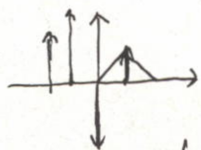
$\Rightarrow g(t) = 3$

$t = 0$,



$\Rightarrow g(t) = -2$

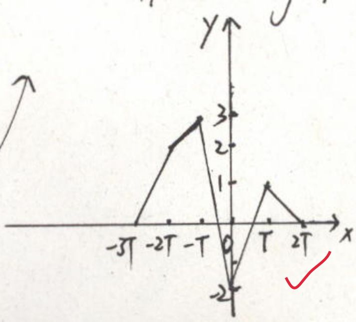
$t = T$,



$\Rightarrow g(t) = 1$.

$t \geq 2T$, no overlap $\Rightarrow g(t) = 0$

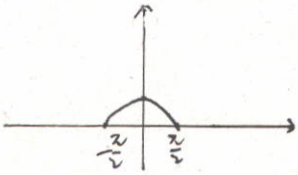
Hence $g(t) = \begin{cases} 0, & t < -3T \\ \frac{2}{T}t + 6, & -3T \leq t < -2T \\ \frac{1}{T}t + 4, & -2T \leq t < -T \\ -\frac{5}{T}t - 2, & -T \leq t < 0 \\ \frac{3}{T}t - 2, & 0 \leq t < T \\ -\frac{1}{T}t + 2, & T \leq t < 2T \\ 0, & t \geq 2T \end{cases}$



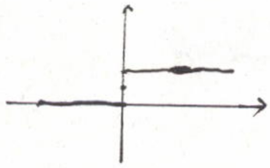
1.2 $\left(\cos(t)\cdot \mathrm{rect}\left(\frac{t}{\pi}\right)\right) * \gamma(t)$

Graphically:

$u(t) = \cos(t)\cdot \mathrm{rect}\left(\frac{t}{\pi}\right)$.



$h(t) = \gamma(t)$



$u(t) \Rightarrow u(\tau)$

$h(t) \Rightarrow h(t-\tau)$

$t < -\frac{\pi}{2}$, no overlap $\Rightarrow g(t) = 0$

$t = -\frac{\pi}{2}$, overlap begins

$t = 0$,

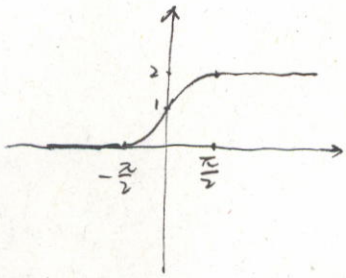
$\Rightarrow y(t) = \int_{-\frac{\pi}{2}}^{0} \cos t \, dt = 1$

$t = \frac{\pi}{2}$,

$\Rightarrow g(t) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t \, dt = 2$

$t > \frac{\pi}{2}$, overlap remains the same $\Rightarrow g(t) = 2$

$$\Rightarrow g(t) = \begin{cases} 0, & t < -\frac{\pi}{2} \\ \sin t + 1, & -\frac{\pi}{2} \le t < \frac{\pi}{2} \\ 2, & t \ge \frac{\pi}{2} \end{cases}$$



Analytically:

$u(t) * h(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau$

$\tau < -\frac{\pi}{2}$ or $\tau > \frac{\pi}{2}$, $u(\tau) = 0$, $g(t) = u(t) * h(t) = 0$.

$-\frac{\pi}{2} \le \tau \le \frac{\pi}{2}$, $u(\tau) = \cos\tau$.

$t < -\frac{\pi}{2}$, $h(t-\tau) = 0$, $g(t) = u(t) * h(t) = 0$

$-\frac{\pi}{2} \le t < \frac{\pi}{2}$, $h(t-\tau) = \begin{cases} 1, & \tau < t \\ 0, & \tau > t \end{cases}$ $g(t) = \begin{cases} \int_{-\frac{\pi}{2}}^{t} u(\tau)\cdot 1\, d\tau = \int_{-\frac{\pi}{2}}^{t} \cos\tau\, d\tau = \sin t + 1 \\ \int_{t}^{\frac{\pi}{2}} u(\tau)\cdot 0\, d\tau = 0. \end{cases}$

$t \ge \frac{\pi}{2}$, $h(t-\tau) = 1$, $g(t) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} u(\tau)\, d\tau = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\tau\, d\tau = 2$

$$\Rightarrow g(t) = \begin{cases} 0, & t < -\frac{\pi}{2} \\ \sin t + 1, & -\frac{\pi}{2} \le t < \frac{\pi}{2} \\ 2, & t \ge \frac{\pi}{2} \end{cases}$$

Discrete Convolution of two signals

2.1

a)

```
     1 3 1        1 3 1        1 3 1
 1 3 1          1 3 1          1 3 1
 -------      ---------      -----------
 1×1=1        1×3+3×1=6      1×1+3×3+1×1=11

 1 3 1          1 3 1
     1 3 1          1 3 1
 ---------      ---------
 3×1+1×3=6      1×1=1
```

$\Rightarrow$ [..., 0, 0, 0, 1, 6, 11, 6, 1, 0, 0, 0, ...]

b) [...1 3 1...] × [...1 3 1...]

```
1×1  1×3  1×1
     3×1  3×3  3×1
          1×1  1×3  1×1
---------------------------
1    6    11   6    1
```

✓

$\Rightarrow$ [..., 0, 0, 0, 1, 6, 11, 6, 1, 0, 0, 0, ...].

2.2 ..., 0, 1, 0, 1, 0, 1, ...
..., 0, 0.5, 0.5, 0, 0, ...

$\Rightarrow$

```
... 0   0.5  0   0.5  0   0.5 ...
... 0   0.5  0   0.5  0   0.5 ...
-----------------------------------
... ~~0.5~~ 0.5 0.5 0.5 0.5 0.5 0.5 ...
```

(✓)

$\Rightarrow$ [..., 0.5, 0.5, 0.5, ...].

2.3 ..., 0, 1, 0, 1, 0, 1, ...
-1, 0, 1,

$\Rightarrow$

```
... 0, -1, 0, -1, 0, -1 ...
... 0  0  0  0  0  0 ...
... 0  1  0  1  0  1 ...
---------------------------
... 0  0  0  0  0  0  0  0  0 ...
```

✗

$\Rightarrow$ [..., 0, 0, 0, ...].