Image alignment by estimation of a homography

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1. Homography

1) Basic Transformation Equation:

$$\mathbf{x}_i' = H \cdot \mathbf{x}_i$$

$$\mathbf{x}_i' \times H \cdot \mathbf{x}_i = 0$$

Where,

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \qquad \mathbf{x}_i' = \begin{pmatrix} x_i' \\ y_i' \\ w_i' \end{pmatrix} \qquad \underset{9\times 1}{\mathbf{h}} = \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = \begin{pmatrix} H(1,:) \\ H(2,:) \\ H(3,:) \end{pmatrix}$$

2) System of Linear Equations

Therefore,

$$H \cdot \mathbf{x}_i = \begin{pmatrix} \mathbf{h}^{1T}\mathbf{x}_i \\ \mathbf{h}^{2T}\mathbf{x}_i \\ \mathbf{h}^{3T}\mathbf{x}_i \end{pmatrix}$$

$$\mathbf{x}_i' \times H \cdot \mathbf{x}_i = \begin{pmatrix} y_i'\mathbf{h}^{3T}\mathbf{x}_i - w_i'\mathbf{h}^{2T}\mathbf{x}_i \\ w_i'\mathbf{h}^{1T}\mathbf{x}_i - x_i'\mathbf{h}^{3T}\mathbf{x}_i \\ x_i'\mathbf{h}^{2T}\mathbf{x}_i - y_i'\mathbf{h}^{1T}\mathbf{x}_i \end{pmatrix} = 0$$

Which results in system of linear equations,

$$\begin{bmatrix} \mathbf{0}^T & -w_i'\mathbf{x}_i^T & y_i'\mathbf{x}_i^T \\ w_i'\mathbf{x}_i^T & \mathbf{0}^T & -x_i'\mathbf{x}_i^T \\ -y_i'\mathbf{x}_i^T & x_i'\mathbf{x}_i^T & \mathbf{0}^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0$$

3) Solution of Linear Equations System

Row(3) is linear dependent of Row(1) and Row(2), thus we can eliminate that,

$$\begin{bmatrix} \mathbf{0}^T & -w_i'\mathbf{x}_i^T & y_i'\mathbf{x}_i^T \\ w_i'\mathbf{x}_i^T & \mathbf{0}^T & -x_i'\mathbf{x}_i^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0$$

Which can be solved by Singular Value Decomposition (SVD) with at least 4 pairs of identical points. By reshaping h1, h2, h3, we get the final H matrix.

2. Image Mapping

1) transformation

$$\mathbf{x}_i' = H \cdot \mathbf{x}_i$$

2) normalization

$$x_k = \frac{x'}{w'} = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{31}y + h_{33}}$$

$$y_k = \frac{y'}{w'} = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{31}y + h_{33}}$$

3. Results

1) Gebaeude

Fig1.1 Identical points

Fig1.2 Gebaeude4 in the geometry of Gebaeude5

Fig3. Panorama Image

Tabel1 Residuals of Gebaeude

	P1	P2	P3	P4	P5
x	-0.3506	0.4066	-0.5195	0.0996	0.2713
y	1.7821	-2.0090	-0.9313	4.5227	-3.3617

2) Geoengine

Fig2.1 Identical Points

Fig2.2 Geoengine1 in the geometry of Geoengine2

Fig2.3 Panorama Image

Tabel2 Residuals of Geoengine

	P1	P2	P3	P4	P5
x	-1.3633	0.7973	-0.9560	0.1307	1.4138

y	0.7552	1.0446	-4.6337	6.1964	-3.4509