Universität Stuttgart

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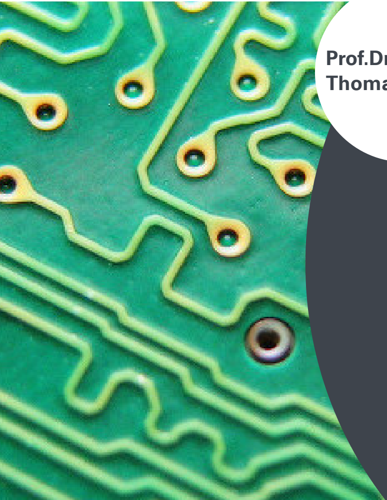


Integrated Positioning and Navigation

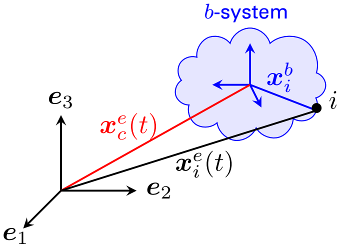
1

Vectors and Coordinates

Vectors and Coordinates

Objective of this course:
Describing the motion of objects on the Earths surface or close to it

- The motion of a body can be described by 6 parameters
  = 3 translations and 3 rotations
  = time series of 3 position parameters and 3 orientation states



$$\boldsymbol{x}_i^e(t) = \boldsymbol{x}_c^e(t) + \boldsymbol{C}_b^e(t) \cdot \boldsymbol{x}_i^b(t)$$

Time variable position component

Time variable rotation component

- Therefore a measurement system is required that can sense six independent quantities from which these parameters can be derived
  $\Rightarrow$ Inertial Navigation System

Vectors and Coordinates - cont'd

For the purpose of this course, a vector is a geometric object in (3D) space with a given length and orientation. The orientation of the vector can be described w.r.t. a 3D orthogonal coordinate system (Fig. 1.1). The axes of the coordinate system are shown as base (unit) vectors in the direction of the axes.

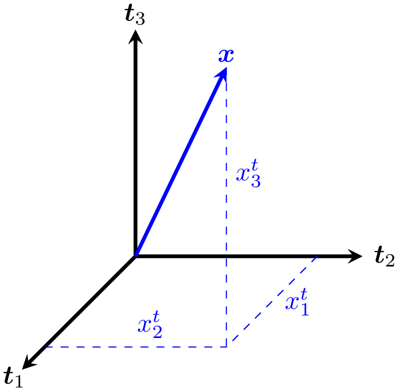


Figure 1.1: Coordinate systems and vectors

All coordinate systems used in this course are right-handed.

Vectors are denoted by bold letters ($\boldsymbol{t}_1$, $\boldsymbol{t}_2$, $\boldsymbol{t}_3$, and $\boldsymbol{x}$ in Fig 1.1 are vectors).

The coordinates of a vector are its projections onto the axes of the coordinate system.

Coordinate subscripts denote the axis, superscripts denote the coord. system.

Vectors and Coordinates - cont'd

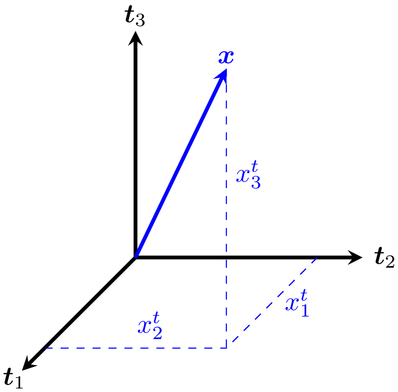


Figure 1.2: Same as Fig. 1.1

The coordinates of a vector may be assembled into a column matrix:

$$\boldsymbol{x}^t = \begin{bmatrix} x_1^t \\ x_2^t \\ x_3^t \end{bmatrix} \tag{1.1}$$

The base vectors of a coordinate system may be assembled in a row matrix:

$$\boldsymbol{t} = \begin{bmatrix} \boldsymbol{t}_1 & \boldsymbol{t}_2 & \boldsymbol{t}_3 \end{bmatrix} \tag{1.2}$$

Then the vector $\boldsymbol{x}$ with its coordinates in the t-system can be written as:

$$\boldsymbol{x} = \boldsymbol{t} \cdot \boldsymbol{x}^t = \begin{bmatrix} \boldsymbol{t}_1 & \boldsymbol{t}_2 & \boldsymbol{t}_3 \end{bmatrix} \cdot \begin{bmatrix} x_1^t \\ x_2^t \\ x_3^t \end{bmatrix} = x_1^t \boldsymbol{t}_1 + x_2^t \boldsymbol{t}_2 + x_3^t \boldsymbol{t}_3 \tag{1.3}$$

Vectors and Coordinates - cont'd

If a vector is attached to the origin of the coordinate system (c.f. Fig. 1.1), its coordinates can be used to describe a position in the 3D space with respect to the base vectors – position vector.

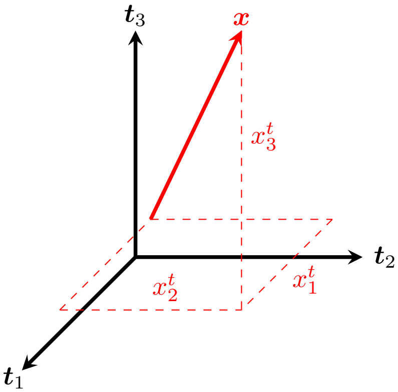


Figure 1.3: Vector in 3D space

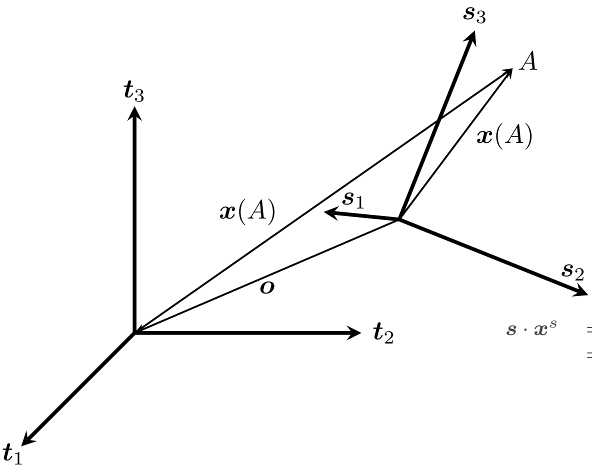
In general, vectors will not be position vectors; only their length and orientation with respect to 3D space is defined like shown in Fig. 1.3.

The coordinates of vectors and position vectors are different in different coordinate systems.

Between coordinate systems, the coordinates of vectors and position vectors transform differently.

Vectors and Coordinates - cont'd

Transformation of a position vector describing point A in space.



$$\begin{aligned} \boldsymbol{x}(A) &= \boldsymbol{t} \cdot \boldsymbol{x}^t \\ \boldsymbol{x}(A) &= \boldsymbol{s} \cdot \boldsymbol{x}^s \end{aligned} \tag{1.4}$$

$$\begin{aligned} \boldsymbol{s} \cdot \boldsymbol{x}^s &= \boldsymbol{s} \cdot \boldsymbol{o}^s + \boldsymbol{t} \cdot \boldsymbol{x}^t \\ &= \boldsymbol{s} \cdot \boldsymbol{o}^s + \boldsymbol{s} \cdot \boldsymbol{C}_t^s \boldsymbol{x}^t \end{aligned} \tag{1.5}$$

Figure 1.4: Position vector transformation

Vectors and Coordinates - cont'd

Transformation of a vector

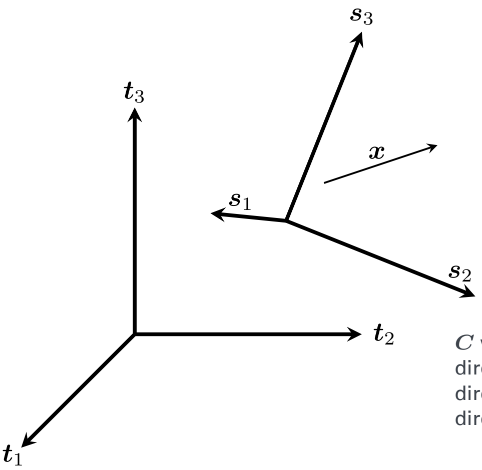


Figure 1.5: Vector transformation

$$\begin{aligned} \boldsymbol{x} &= \boldsymbol{t} \cdot \boldsymbol{x}^t \\ \boldsymbol{x} &= \boldsymbol{s} \cdot \boldsymbol{x}^s \end{aligned} \tag{1.6}$$

$$\boldsymbol{s} \cdot \boldsymbol{x}^s = \boldsymbol{t} \cdot \boldsymbol{x}^t = \boldsymbol{s} \cdot \boldsymbol{C}_t^s \boldsymbol{x}^t \tag{1.7}$$

$\boldsymbol{C}$ with subscript t and superscript s is the direction cosine matrix (DCM) relating the directions of the t-system axes to the directions of the s-system axes.

Vectors and Coordinates - cont'd

The direction cosine matrix

$$\boldsymbol{s} \cdot \boldsymbol{x}^s = \boldsymbol{t} \cdot \boldsymbol{x}^t = \boldsymbol{s} \cdot \boldsymbol{C}_t^s \boldsymbol{x}^t \tag{1.8}$$

From equ. (1.8)

$$\boldsymbol{x}^s = \boldsymbol{C}_t^s \cdot \boldsymbol{x}^t$$

$$\text{and} \tag{1.9}$$

$$\boldsymbol{t} = \boldsymbol{s} \cdot \boldsymbol{C}_t^s$$

For orthogonal right-handed coordinate systems, the DCM is an orthonormal matrix:

$$[\boldsymbol{C}_t^s] \cdot [\boldsymbol{C}_t^s]^T = \boldsymbol{I}, \quad [\boldsymbol{C}_t^s]^T = [\boldsymbol{C}_t^s]^{-1} \tag{1.10}$$

From equ. (1.9)

$$\begin{aligned} \left[\boldsymbol{C}_t^s\right]^{-1} \cdot \boldsymbol{x}^s = \left[\boldsymbol{C}_t^s\right]^{-1} \cdot \left[\boldsymbol{C}_t^s\right] \cdot \boldsymbol{x}^t \;\Rightarrow\; \boldsymbol{x}^t = \left[\boldsymbol{C}_t^s\right]^{-1} \cdot \boldsymbol{x}^s \\ \Rightarrow\; \left[\boldsymbol{C}_t^s\right]^{-1} = \left[\boldsymbol{C}_s^t\right] \end{aligned} \tag{1.11}$$