Statistical Inference — Lab 6

Date of issue: 24. Januar 2019 Due date: none

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Weighted least-squares adjustment and Linearisation (A-model)

Task 1: To estimate the gravity acceleration g, 3 groups of student carry an experiment of free-fall. In this experiment, the position p of a mass falling in a vacuum chamber is measured at five different times (see files *Posgruppe1.mat*, *Posgruppe2.mat* und *Posgruppe3.mat* for the positions [m], *time.mat* for the time [s], available on Ilias). The constant g is then determined by adjusting to the measurements the function

$$p(t) = -\frac{1}{2}gt^2 + v_0 t + p_0$$

where t is the time, v_0 is the inital velocity of the mass and p_0 is the initial vertical position.

a) Compute the least-squares estimate $\hat{g}$ of g

b) Since the precision of the measurements decreases as the velocity of the mass increases, an estimated variance of the error degrading the measurements has been assigned for each measurement time: [1, 1, 2.25, 6.25, 16] $10^{-6}\,\mathrm{m}^2$. Compute now the weighted least-squares estimator $\hat{g_w}$ of g and the standard deviation of the estimator $\hat{g_w}$.

c) Compare the estimate $\hat{e}$ of the inconsistency and its quadratic sum $\|\hat{e}\|_2^2 := \hat{e}^\mathsf{T}\hat{e}$ of both cases.

Task 2: The **logistic function** is a common function that finds applications in a range of fields including demography, geosciences, chemistry and artificial neural network. In this exercise, we would like to fit a set of data (*X.mat* for the variable t and *Ynoisy.mat* for the observations) to the logistic function

$$f(t) = -\frac{K}{1 + ae^{-rt}}$$

where $K = 5$ and a and r are the unknown parameters.

a) Plot the data and give a first estimation a_0 and r_0 of the parameters a and r, respectively. To do this, you can for instance roughly estimate from the plot the values of $f(0)$ and $f'(0)$ and infer the parameters value.

b) Compute the least-squares estimators $\hat{a}$ and $\hat{r}$ of a and r. Please details more specifically the computation of the design matrix and the iterative process.

c) The function **logit** is defined as

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

calculate $\text{logit}(f)$ and $Y_l = \text{logit}(Ynoisy)$ and compute again the least-squares estimators $\hat{r}_l$ of a and r_l by adjusting this new function to Y_l.

d) Compare the results obtained with both approach in terms of estimate $\hat{e}$ of the inconsistency and its quadratic sum $\|\hat{e}\|_2^2 := \hat{e}^T\hat{e}$. Which one seems the best to you?