Exercise on <u>20.11.2019</u>

Task 1 *(5 points)*

Given are the following combinations of three consecutive rotations around the three axes:

$$\alpha_1 = 0.3°,\ \beta_1 = 0.2°,\ \gamma_1 = 0.05° \tag{1}$$

$$\alpha_2 = -30°,\ \beta_2 = 35°,\ \gamma_2 = -20° \tag{2}$$

i) Calculate the DCMs following equation (2.4) and (2.6) from the lecture (different rotation conventions)

ii) Derive the corresponding Euler Symmetric Parameters from the DCMs

Proposal for solution 1

i) • Angles from (1)

- using equation (2.4)

$$C_t^s = \begin{bmatrix} 9.99981e-01 & 6.10861e-03 & 3.04617e-06 \\ -6.10858e-03 & 9.99975e-01 & 3.49065e-03 \\ 1.82769e-05 & -3.49060e-03 & 9.99994e-01 \end{bmatrix}$$

as comparison first order Taylor ($\sin(x) = x,\ \cos(x) = 1.0$)):

$$C_t^s = \begin{bmatrix} 9.99127e-01 & 6.10865e-03 & 3.04617e-06 \\ -6.10865e-03 & 9.99995e-01 & 3.49066e-03 \\ 1.82770e-05 & -3.49066e-03 & 1.00000e+00 \end{bmatrix}$$

- using equation (2.6)

$$C_t^s = \begin{bmatrix} 9.99994e-01 & 8.72659e-04 & -3.49065e-03 \\ -8.54376e-04 & 9.99986e-01 & 5.23593e-03 \\ 3.49517e-03 & -5.23292e-03 & 9.99980e-01 \end{bmatrix}$$

• Angles from (2)

- using equation (2.4)

$$C_t^s = \begin{bmatrix} 0.67371 & -0.71248 & -0.19617 \\ 0.68107 & 0.49561 & 0.53899 \\ -0.28679 & -0.49673 & 0.81915 \end{bmatrix}$$

as comparison first order Taylor ($\sin(x) = x,\ \cos(x) = 1.0$)):

$$C_t^s = \begin{bmatrix} 1.34907 & -0.87266 & -0.21323 \\ 0.87266 & 0.81723 & 0.61087 \\ -0.31985 & -0.61087 & 1. \end{bmatrix}$$

- using equation (2.6)

$$C_t^s = \begin{bmatrix} 0.76975 & -0.28017 & -0.57358 \\ 0.02671 & 0.91189 & -0.40958 \\ 0.63779 & 0.29995 & 0.70941 \end{bmatrix}$$

ii)
- Angles from (1)
  - using equation (2.4)

$$q_0 = 9.99994e-01 \qquad q_1 = 1.74532e-03$$
$$q_2 = 3.80771e-06 \qquad q_3 = 3.05432e-03$$

  - using equation (2.6)

$$q_0 = 9.99995e-01 \qquad q_1 = 2.61723e-03$$
$$q_2 = 1.74646e-03 \qquad q_3 = 4.31761e-04$$

- Angles from (2)
  - using equation (2.4)

$$q_0 = 0.86436 \qquad q_1 = 0.29956$$
$$q_2 = -0.02621 \qquad q_3 = -0.40306$$

  - using equation (2.6)

$$q_0 = 0.92074 \qquad q_1 = -0.19265$$
$$q_2 = 0.32891 \qquad q_3 = -0.08332$$

Task 2 *(5 points)*

In the lecture has been shown (equation (3.12)), that the time derivative of the transformation-matrix $\dot{\boldsymbol{C}}_t^s$ can be expressed using the equation

$$\dot{\boldsymbol{C}}_t^s = \boldsymbol{C}_t^s \cdot \boldsymbol{\Omega}_{st}^t,$$

where $\boldsymbol{\Omega}_{st}^t$ is the matrix representation of the angular velocity vector $\boldsymbol{\omega}_{st}^t$. In the derivation the linearisation of small Euler Angles was utilized by the use of the limit $\Delta t \to 0$. This equation could also have been achieved by strict differentiation of $\boldsymbol{C}_t^s = \boldsymbol{C}(1,\alpha) \cdot \boldsymbol{C}(2,\beta) \cdot \boldsymbol{C}(3,\gamma)$.
Show analytically, that eq. (3.12) holds, using the simplification that only rotations around the first axis are taken into account:

$$\boldsymbol{C}_t^s = \boldsymbol{C}(1,\alpha), \qquad \alpha = \omega_1 \cdot t, \qquad \boldsymbol{\omega}_{ts}^t = \begin{bmatrix} \omega_1 \\ 0 \\ 0 \end{bmatrix}$$

Proposal for solution 2

l.h.s.:

$$\dot{C}_t^s(2,\beta) = \frac{\partial}{\partial t} \begin{bmatrix} 1 & 0 & \\ 0 & \cos(\omega_1 t) & \sin(\omega_1 t) \\ 0 & -\sin(\omega_1 t) & \cos(\omega_1 t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & \\ 0 & -\omega_1 \sin(\omega_1 t) & \omega_1 \cos(\omega_1 t) \\ 0 & -\omega_1 \cos(\omega_1 t) & -\omega_1 \sin(\omega_1 t) \end{bmatrix}$$

r.h.s.:

$$\omega_{ts}^t = \begin{bmatrix} \omega_1 \\ 0 \\ 0 \end{bmatrix} \to \omega_{st}^t = \begin{bmatrix} -\omega_1 \\ 0 \\ 0 \end{bmatrix} \to \Omega_{st}^t = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \omega_1 \\ 0 & -\omega_1 & 0 \end{bmatrix}$$

$$C_t^s \Omega_{st}^t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1 t) & \sin(\omega_1 t) \\ 0 & -\sin(\omega_1 t) & \cos(\omega_1 t) \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \omega_1 \\ 0 & -\omega_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\omega_1 \sin(\omega_1 t) & \omega_1 \cos(\omega_1 t) \\ 0 & -\omega_1 \cos(\omega_1 t) & -\omega_1 \sin(\omega_1 t) \end{bmatrix}$$

$\blacksquare$