Universität Stuttgart

Prof.Dr.
Thomas Hobiger

Satellite Navigation

Parameter estimation and code phase positioning

8

Parameter estimation and code phase positioning

As discussed in module 6, pseudorange (i.e. code phase observations) can be expressed as

$$\boxed{P_k^p = \rho_k^p + (\Delta t_k - \Delta t^p) \cdot c + \underbrace{I_k^p}_{\text{ionosphere delay}} + \underbrace{A_k^p}_{\text{troposphere delay}}} \tag{8.1}$$

where ρ_k^p is the true (geometrical) range between receiver k and satellite p. This range is equal to the Euclidian distance computed from the position of the satellite at transmission time t_p and the position of the receiver at the time t the signal was captured.

Dropping now the index k, i.e. focusing on a single receiver, we obtain

$$\rho^p(t, t^p) = \sqrt{[X^p(t^p) - x(t)]^2 + [Y^p(t^p) - y(t)]^2 + [Z^p(t^p) - z(t)]^2} \tag{8.2}$$

which implies that we have to deal with

- three position components of the receiver
- the receiver clock offset
- the troposphere delay
- the ionosphere delay

Parameter estimation and code phase positioning

Besides the unknowns mentioned before we also need to know

- the satellite position at transmission time
- the satellite clock offset

This information is basically available from the broadcast ephemeris (see module 2 and others)

Example RINEX Nav-Format (see `ftp://igs.org/pub/data/format/rinex210.txt`):

```
     2              NAVIGATION DATA                         RINEX VERSION / TYPE
CCRINEXN V1.6.0 UX  CDDIS               04-AUG-18 17:31     PGM / RUN BY / DATE
IGS BROADCAST EPHEMERIS FILE                                COMMENT
    0.4657D-08  0.1490D-07 -0.5960D-07 -0.5960D-07          ION ALPHA
    0.7782D+05  0.4915D+05 -0.6554D+05 -0.3277D+06          ION BETA
    0.186264514923D-08 0.888178419700D-14    61440     2013 DELTA-UTC: A0,A1,T,W
    18                                                      LEAP SECONDS
                                                            END OF HEADER
 1 18  8  3  0  0  0.0-0.722697004676D-04-0.397903932026D-11 0.000000000000D+00
    0.660000000000D+02 0.843750000000D+00 0.461090634847D-08-0.107690595368D+01
    0.188127160072D-06 0.805073522497D-02 0.469572842121D-05 0.515366210747D+04
    0.432000000000D+06 0.203028321266D-06 0.240613246279D+01 0.223517417908D-07
    0.972082336160D+00 0.292250000000D+03 0.672681593740D+00-0.825677249915D-08
    0.288583449230D-09 0.100000000000D+01 0.201200000000D+04 0.000000000000D+00
    0.200000000000D+01 0.000000000000D+00 0.558793544769D-08 0.660000000000D+02
    0.424818000000D+06 0.400000000000D+01 0.000000000000D+00 0.000000000000D+00
 2 18  8  3  0  0  0.0 0.395588576794D-04-0.112549969344D-10 0.000000000000D+00
    0.910000000000D+02 0.790625000000D+01 0.516057196975D-08-0.759203844230D+00
    :
```

clock af_0, af_1, af_2

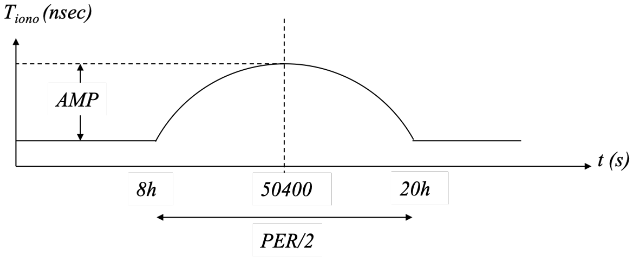
satellite orbit (Kepler elements)

Parameter estimation and code phase positioning

(Simple) Ionosphere corrections are also transmitted via the navigation message

```
     2              NAVIGATION DATA                          RINEX VERSION / TYPE
CCRINEXN V1.6.0 UX  CDDIS               04-AUG-18 17:31      PGM / RUN BY / DATE
IGS BROADCAST EPHEMERIS FILE                                 COMMENT
    0.4657D-08  0.1490D-07 -0.5960D-07 -0.5960D-07           ION ALPHA
    0.7782D+05  0.4915D+05 -0.6554D+05 -0.3277D+06           ION BETA
    0.186264514923D-08 0.888178419700D-14    61440     2013 DELTA-UTC: A0,A1,T,W
    18                                                       LEAP SECONDS
    :
```

The coefficients provide the so-called Klobuchar model which can be used to calculate a reasonable value for ionosphere delay T_{iono} in zenith direction.



Parameter estimation and code phase positioning

We can relate T_{iono} to TEC by

$$T_{\text{iono}} \cdot c = 40.28 \frac{TEC}{f_1^2} \tag{8.3}$$

where f_1 is the carrier frequency of GPS L1.

The actual computation follows a cookbook scheme

1. Calculate the earth-centered angle (elevation E in semicircles)

$$\psi = \frac{0.0137}{E + 0.11} - 0.022 \quad \text{[semicircles]}$$

2. Compute the latitude of the Ionospheric Pierce Point (IPP)

$$\phi_i = \phi + \psi \cos A \quad \text{[semicircles]}$$

   If $\phi_i > 0.416$ then $\phi_i = 0.416$. If $\phi_i < -0.416$ then $\phi_i = -0.416$.

3. Compute the longitude of the IPP

$$\lambda_i = \lambda + \frac{\psi \sin A}{\cos \phi_i} \quad \text{[semicircles]}$$

Parameter estimation and code phase positioning

4. Find the geomagnetic latitude of the IPP

$$\phi_m = \phi_i + 0.064\cos(\lambda_i - 1.617) \quad \text{[semicircles]}$$

5. Find the local time at the IPP

$$t = 43200\lambda_i + t_{\text{GPS}} \quad \text{[s]}$$

If $t > 86400$ s then subtract 86400 s. If $t < 0$ s then add 86400 s.

6. Compute the amplitude of ionospheric delay (use the broadcasted coeff.)

$$A_I = \sum_{n=0}^{3} \alpha_n \phi_n^m \quad \text{[s]}$$

If $A_I < 0$ then $A_I = 0$.

7. Compute the period of ionospheric delay (use the broadcasted coeff.)

$$P_I = \sum_{n=0}^{3} \beta_n \phi_m^n \quad \text{[s]}$$

If $P_I < 72000$ then $P_I = 72000$.

Parameter estimation and code phase positioning

8. Compute the phase of ionospheric delay

$$X_I = \frac{2\pi(t - 50400)}{P_I} \quad \text{[rad]}$$

9. Compute the slant factor (elevation E in semicircles).

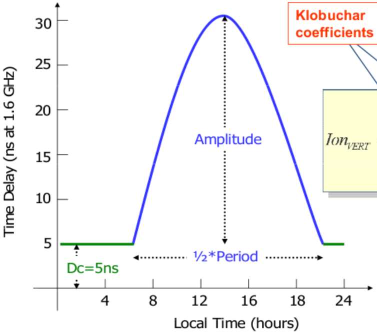
$$F = 1.0 + 1.0(0.53 - E)^3$$

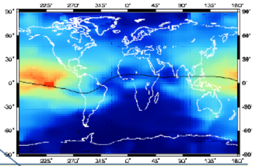
10. Compute the ionospheric time delay

$$T_{\text{iono}} = \begin{cases} \left[5 \cdot 10^{-9} + A_I \cdot (1 - \frac{X_I^2}{2} + \frac{X_I^4}{24})\right] \cdot F\,; & |X_I| \leq 1.57 \\ 5 \cdot 10^{-9} \cdot F\,; & |X_I| \geq 1.57 \end{cases}$$

Parameter estimation and code phase positioning

Klobuchar model





Klobuchar coefficients

$$Ion_{VERT} = \begin{cases} DC + A\cos\left[\dfrac{2\pi(t-\Phi)}{P}\right] & (day) \\ DC \quad ; \ if \left[\dfrac{2\pi(t-\Phi)}{P}\right] > \dfrac{\pi}{2} & (night) \end{cases}$$

Being :

$$A = \sum_{n=0}^{3} \alpha_n \varphi^n \quad ; \quad P = \sum_{n=0}^{3} \beta_n \varphi^n$$

φ = Geomagnetic Latitude

$$Ion_{SLANT} = Ion_{VERT} \; m(elev)$$

$$m(elev) = 1 + 16\left(0.53 - elev/\pi\right)^3$$

Where:
DC= 5ns
Φ= 14 (ctt. phase offset)
t = Local Time

Parameter estimation and code phase positioning

The Saastamoinen Hydrostatic Delay Model

The zenith hydrostatic delay (ZHD) (see module 7) after Saastamoinen is given by:

$$ZHD = \frac{0.0022767 \cdot p}{1 - 0.00266 \cos(2\phi) - 0.00028 \cdot h} \tag{8.4}$$

where ϕ is the ellipsoidal latitude, h is the height (in km) above the ellipsoid and p is the total pressure.

From where do we get surface pressure p?

- barometer measurement at the station
- a very rough model, like the one from Berg (1948)

$$p(h) = 1013.25(1 - 0.0226 \cdot h)^{5.225} \quad h \text{ in km}$$

- from empirical models which provide us mean pressure for a certain location (latitude, longitude and height) on a given epoch (hour and day of the year), e.g. see `https://link.springer.com/article/10.1007/s00190-007-0135-3`

Parameter estimation and code phase positioning

Assuming tropospheric and ionospheric delays are either taken care of or ignored, we can set up a non-linear equation system for N pseudorange observations collected at epoch t:

$$\begin{aligned}
P^1 &= \sqrt{(X^1 - x)^2 + (Y^1 - y^2)^2 + (Z^1 - z)^2} + c\tau - c\tau^1 + \epsilon_1 \\
P^2 &= \sqrt{(X^2 - x)^2 + (Y^2 - y^2)^2 + (Z^2 - z)^2} + c\tau - c\tau^2 + \epsilon_2 \\
&\vdots \\
P^N &= \sqrt{(X^N - x)^2 + (Y^N - y^N)^2 + (Z^N - z)^2} + c\tau - c\tau^N + \epsilon_N
\end{aligned} \tag{8.5}$$

This implies that we have four unknowns (three position components + the receiver clock).
Eq. (8.5) is non-linear in position coordinates (x, y, z) and the terms ϵ_k denote the (random) noise contribution to a pseudorange observation. Linearization of (8.2) of the pseudorange equations yields,

$$\begin{aligned}
P^k(x, y, z, \tau) \quad = \quad & \underbrace{P^k(x_0, y_0, z_0, \tau_0)}_{P^k_{\text{computed}}} + \underbrace{(x - x_0)}_{\Delta x} \frac{\partial P^k}{\partial x} + \underbrace{(y - y_0)}_{\Delta y} \frac{\partial P^k}{\partial y} + \\
& + \underbrace{(z - z_0)}_{\Delta z} \frac{\partial P^k}{\partial z} + \underbrace{(\tau - \tau_0)}_{\Delta \tau} \frac{\partial P^k}{\partial \tau}
\end{aligned} \tag{8.6}$$

Parameter estimation and code phase positioning

$$
\begin{aligned}
\Delta P^k &= P^k_{\text{observed}} - P^k_{\text{computed}} \\
&= \frac{\partial P^k}{\partial x}\Delta x + \frac{\partial P^k}{\partial y}\Delta y + \frac{\partial P^k}{\partial z}\Delta z + \frac{\partial P^k}{\partial \tau}\Delta \tau + \epsilon
\end{aligned}
\tag{8.7}
$$

In matrix notation

$$
\Delta P^k = \begin{pmatrix} \frac{\partial P^k}{\partial x} & \frac{\partial P^k}{\partial y} & \frac{\partial P^k}{\partial z} & \frac{\partial P^k}{\partial \tau} \end{pmatrix} \cdot \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta \tau \end{pmatrix} + \epsilon \tag{8.8}
$$

Thus, the equation array (8.5) can be written as

$$
\boldsymbol{\Delta P} = \underbrace{\begin{pmatrix} \frac{x_0 - x^1}{\rho^1} & \frac{y_0 - y^1}{\rho^1} & \frac{z_0 - z^1}{\rho^1} & c \\ \frac{x_0 - x^2}{\rho^2} & \frac{y_0 - y^2}{\rho^2} & \frac{z_0 - z^2}{\rho^2} & c \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_0 - x^N}{\rho^N} & \frac{y_0 - y^N}{\rho^N} & \frac{z_0 - z^N}{\rho^N} & c \end{pmatrix}}_{\boldsymbol{A}} \cdot \underbrace{\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta \tau \end{pmatrix}}_{\boldsymbol{\Delta x}} + \underbrace{\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_N \end{pmatrix}}_{\boldsymbol{\epsilon}} \tag{8.9}
$$

Parameter estimation and code phase positioning

Eq. (8.9) can be used for Single Point Positioning (SPP) or referred to as code phase positioning), i.e.

- an epoch-wise least-squares adjustment, i.e. computing (x, y, z, t) independently if we have $N >= 4$ GNSS pseudorange observations
- a Kalman filter where we model (x, y, z, t) as continuous stochastic processes which evolves over time and is updated by GNSS measurements

As for the ionosphere we can

- ignore its effect $\rightarrow$ our results will be biased
- significantly reduce its effect with empirical models
- remove its effect by dual-frequency observations (see last lesson)

As for the troposphere we can

- ignore its effect $\rightarrow$ our results will be slightly biased
- significantly reduce its effect with empirical models
- estimate its effect, by adding an additional unknown parameter that describes troposphere delay in zenith and which gets mapped down to the observation elevation (see last lecture)