For office use only	Team Control Number	For office use only
T1 ________________	**62513**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2017
MCM/ICM
Summary Sheet

Merge after toll

Summary

This paper studies the design of highway toll plazas, establishing several models to stimulate the process of the vehicles driving throughout the toll plazas and work out the best design plans.

Firstly, we build a 3-stage model to describe basic components of a toll plaza. We quote a simplified traffic hydromechanics model to describe the first arriving stage and an M/M/1 queuing model to simulate the second wait-for-toll stage.

We put focus on the third merging stage, designing a few initial merging patterns and use the simulation software VISSIM to pick out a best one---the improved VSL merging model. After totally discussing the detail of the VSL model, we consider the shape of the merging area and work out a series of expression for some typical portable shapes.

Secondly, we build a multi-objective optimization model to put the three main objectives (throughput, cost and accident prevention) as a whole. To solve this complicated problem, we use the weight methods to transform the multiple target into single target and build a comprehensive index (COP) to evaluate the overall performance of a toll plaza.

With the Genetic Algorithm, we figure out results and find a best solution: The merging pattern is the improved VSL pattern, and the shape of the zone edge is like a cubic curve

$$w_1(x) = 0.0010x^3 + 0.0015x^2 + 0.3050x + 28$$

or an arc tangent curve ($w_2(x) = -6600.2\arctan(x+150) + 10352$) (Fig. 15-17).

Thirdly, we take the proportion of different types of tollbooths into consideration and find those best proportions changing with the proportion of different vehicles (Fig. 20). Thus given a certain vehicle proportion, we can surely get the best tollbooth proportion.

Finally, we change the traffic flow and adjust the proportion of autonomous vehicles to analyze the sensitivity of our model, find strengths and weaknesses of our model and write a letter based on our ideas to the New Jersey Turnpike Authority.

Merge after toll

Team # 62513

Abstract

This paper studies the design of highway toll plazas, establishing several models to stimulate the process of the vehicles driving throughout the toll plazas and work out the best design plans.

Firstly, we build a 3-stage model to describe basic components of a toll plaza. We quote a simplified traffic hydromechanics model to describe the first arriving stage and an M/M/1 queuing model to simulate the second wait-for-toll stage.

We put focus on the third merging stage, designing a few initial merging patterns and use the simulation software VISSIM to pick out a best one---the improved VSL merging model. After totally discussing the detail of the VSL model, we consider the shape of the merging area and work out a series of expression for some typical portable shapes.

Secondly, we build a multi-objective optimization model to put the three main objectives (throughput, cost and accident prevention) as a whole. To solve this complicated problem, we use the weight methods to transform the multiple target into single target and build a comprehensive index (COP) to evaluate the overall performance of a toll plaza.

With the Genetic Algorithm, we figure out results and find a best solution: The merging pattern is the improved VSL pattern, and the shape of the zone edge is like a cubic curve

$$(w_1(x) = 0.0010x^3 + 0.0015x^2 + 0.3050x + 28)$$

or an arc tangent curve($w_2(x) = -6600.2\arctan(x+150)+10352$) (Fig. 15-17).

Thirdly, we take the proportion of different types of tollbooths into consideration and find those best proportions changing with the proportion of different vehicles (Fig. 20). Thus given a certain vehicle proportion, we can surely get the best tollbooth proportion.

Finally, we change the traffic flow and adjust the proportion of autonomous vehicles to analyze the sensitivity of our model, find strengths and weaknesses of our model and write a letter based on our ideas to the New Jersey Turnpike Authority.

Keywords: Toll Plaza, Merging Pattern, Multi-objective Optimization, VISSIM

Contents

1. Introduction... 4

1.1 Problem Restatement..4

1.2 Previous work..4

1.3 Our work..4

2. General Assumptions.. 5

3. Notations & symbols... 5

4. 3-stage toll plaza model...6

Ω1: Entering the tollbooths.. 6

Ω2: Queueing for toll.. 7

Ω3: Merging after toll.. 7

1) Merging pattern: the improved VSL model..8

2) Shape of merging area..9

5. A Multi-objective Optimization Model...12

5.1 The Multi-objective Optimization Model.. 12

5.2 Comprehensive Performance Index Model..12

5.3 The best toll plaza-building strategy... 13

6. Improved Model Considering Tollbooth-type.......................................14

7. Sensitivity Analysis...17

7.1 Light or Heavy Traffic.. 17

7.2 The proportions of different types of tollbooths.................................. 18

7.3 More autonomous vehicles...18

7.4 Change weights.. 18

8. Strengths and Weaknesses.. 18

8.1 Strengths..18

8.2 Weaknesses... 19

9. References... 19

Appendix.. 20

1. Introduction

1.1 Problem Restatement

In this paper, we have been asked to solve the following problems:

- Determine the shape, size, and merging pattern of the area following the toll barrier in which vehicles fan in from B tollbooth egress lanes down to L lanes of traffic, taking accident prevention, throughput and cost into consideration.
- Analyze the performance of different particular toll plaza designs that may already be implemented.
- Find a best solution (shape, size, and merging pattern).
- Determine the performance of the solution in light and heavy traffic.
- Consider the situation when more autonomous are added in the traffic mix.
- Consider the situation when the proportions of different types of tollbooths are changed.

1.2 Previous work

According to the traffic flow problem of freeway, researchers have proposed a number of solutions. Rongguo Ma and Libo Yang[1] introduced Fluid Mechanics Simulation Theory to study relationships between expressway traffic flow's parameters(velocity, flux, traffic density, the width of road, etc.). Minghui MA and Qingfang Yang used METANET[2] to describe the running state of traffic flow under variable speed limit in order to solve the problem of traffic congestion in expressway. It is also worth mentioning that nowadays, cellular automata theory is the most widely used analyze method, and it was first defined and used for single-lane traffic flow simulations by Nagel K and Scheckenberg M[3]. In this theory, the change rule of the motion of the vehicle on the road segment is expressed as the evolution rule of cellular automata. The researchers propose cellular automata rules which reflect vehicles' traffic behavior such as free-riding, follow-up driving and deceleration driving and then a variety of rules are described in detail.

1.3 Our work

Our work begins with a basic 3-stage model describing the whole process of the vehicles driving from the entrance to the exit of the toll plaza. We use a simplified traffic hydromechanics model to describe the first stage and a simplified queuing model to describe the second stage. We focus on the third stage, designing a few merging patterns and use

VISSIM to simulate the results of them. Thus we pick out a best merging pattern---- an improved VSL model, to make further discussions. Also, we discuss the shape of the merging area in detail.

After that, we discuss the three objective of a good toll plaza, throughput, cost and security. Then a multi-objective optimization model is built to solve the model and a comprehensive index is designed to plan these three factors as a whole. Based on the index, we test a number of toll plazas.

Additionally, we add the proportion of different tollbooths into consideration, using an improved queuing model.

Finally, we analyze the impact of light or heavy flow and the increasing proportion of autonomous vehicles.

2. General Assumptions

- Vehicle types passing through the plaza area are all standard cars.
- B (the number of tollbooths in a barrier toll) and L (the number of lanes of travel in each direction) satisfy the equation of B = 2L
- All lanes have the same width, which is a constant number w_0.
- All drivers are completely rationally dependent on the length of stay to select lanes.
- No barrier. Barriers prevent drivers getting to lanes that they need to use.

3. Notations & symbols

Symbols	Descriptions	Units
v_0	Arriving velocity	
v_2	Velocity when vehicles exit the tollbooths	m/s
v_{limit}	Limited velocity when vehicles begin to merge	
q_0	Arriving flow	
q_1	Flow where vehicles exit the tollbooths	Pcu/h
q_2	Exiting flow	
$L_i(i = 1,2,3,4,5,tb)$	Length of each area	m
w_0	Width of a single traffic lane	
L	Number of lanes	/
B	Number of tollbooths	
μ	Average service rate	Pcu/h
ρ	Traffic density	Pcu/km

T_C	Time vehicles passing through the confluence zone	s
S_C	Area of fan-in sector	m^2
T_{AST}	Average sojourn time in tollbooths	s
$T_{tollbooth}$	Retention time in the tollbooths	s

4. 3-stage toll plaza model

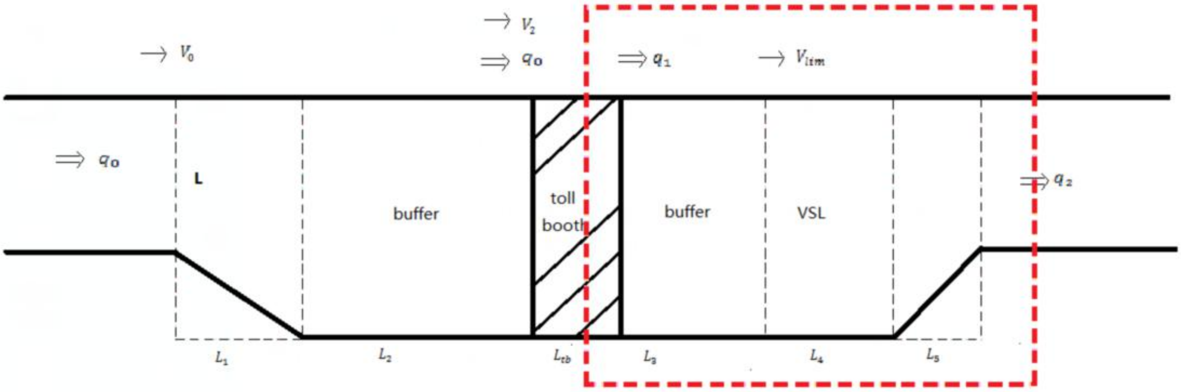


Fig. 1 3-Stage Toll Plaza Model

To begin with the “merge after toll” problem and build a basic theoretical framework for our solution, we expand the study area(i.e. include the area before exiting the tollbooths), building a 3-stage toll plaza model.

Ω1: Entering the tollbooths

In this part, we quote a simplified Traffic hydromechanics model[4] to simulate the entering process. This theory uses a basic principle of fluid mechanics to establish the continuity equation of the traffic flow, by simulating the continuity equation of the fluid. Assuming the traffic flow is free, the arriving flow q can be represented by the arriving velocity v_0 and traffic density ρ_0:

$$q = v_0 \cdot \rho_0$$

Similarly, we can get the flow $q(x)$ at any location:

$$q(x) = v_x \cdot \rho_x$$

Additionally, since the arriving flow is free, we can find the flow arriving at the tollbooths (Ω2) equal to that at the start point (q_0).

In a word, vehicles arrive at the toll plaza at a velocity v_0, a flow q_0, driving through the area Ω1, and enter the tollbooths still with a flow of q_0.

Ω2: Queueing for toll

In this part, we quote the queuing theory[5] to simulate the tolling process. A general single queue model is described as :

$$A/S/C$$

A is the time between arrivals to the queue, S is the size of jobs and C is the number of servers at the node[7].

According to statistical laws, we can assume that the arriving of vehicles obey negative exponential distribution and the service of the tollbooths obey Gaussian distribution[6], and each lane sets one tollbooth so the number of server is one. Thus we build an M/M/1 model for this tolling part.

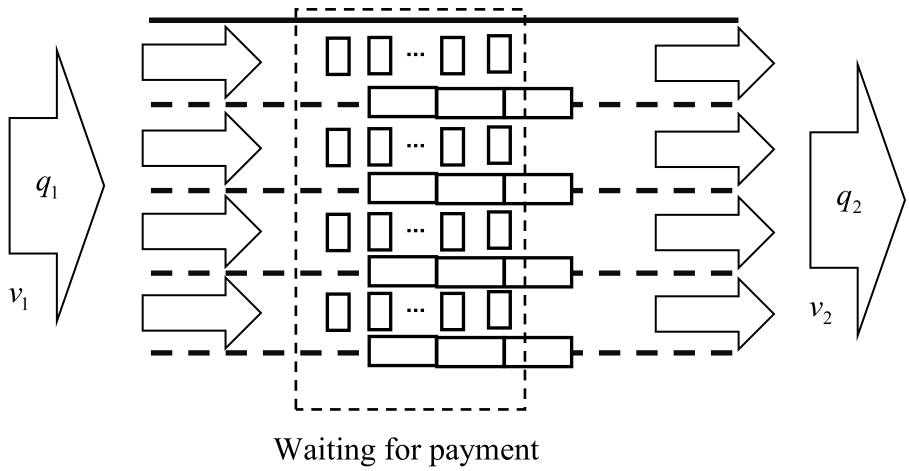


Fig. 2 M/M/1 Model for Tolling Part

By using the average arriving rate $\lambda(\lambda = q_0 / B)$ and the average service rate μ, we can deduce the average delay time in this part :

$$T_{tollbooth} = \frac{1}{\mu - \lambda} = \frac{B}{B \cdot \mu - q_0}$$

In a word, vehicles enter the area Ω2 with a flow of and exit with a flow of q_1, a velocity of v_2 (assumed), the time of which is $T_{tollbooth}$.

Ω3: Merging after toll

In this part, we build a merging model to control the merging area Ω3 in which vehicles fan in from B tollbooth egress lanes down to L lanes of traffic to exit the toll plaza system. We analyze two main questions, one the merging pattern (or strategy), the other the shape of the area.

1) Merging pattern: the improved VSL model

In order to determine a best merging pattern, we set a few initial merging strategies, control other variable parameters, and use the simulation software VISSIM to simulate each strategy's result (PS: Actually the result mainly reflects in the output flow).

Table. 1 Exiting Flow of Different Merging Strategies

Merging type	M1(none)	M2	M3	M4	M5(VSL+)
	1071	998	942	919	1120

From the table we can pick out that best merging strategy: the improved VSL model.

Based on the simulation results, we select the improved VSL (Variable Speed Limits)[1] [2] model to best stimulate the merging pattern, dividing the area Ω3 into 3 parts: the buffer zone, the VSL zone and the fan-in zone. (Fig.3)

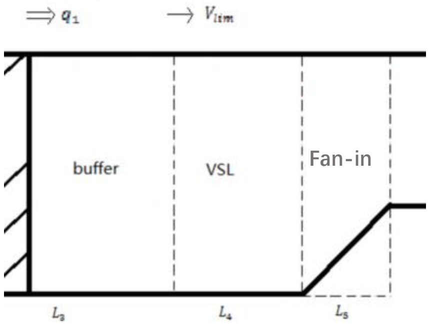


Fig.3 Merging area

- The buffer zone is set for vehicles to prepare for the VSL control, the velocity of which remains .
- The VSL zone is set for speed control, vehicles change velocity to a limited value v_{limit}

$$v_{limit} = \frac{q_{2max}}{\rho_{limit}} = \frac{q_{2max}}{2q_0 - q_{2max}} \cdot v_2, \quad (\rho_{limit} = \frac{2q_0 - q_{2max}}{v_2})$$

(PS: In this part we assume that, thus the tollbooths wouldn't overload.)

$$q_{1max} = \mu \cdot B, q_{2max} = \mu \cdot L$$

Where q_{1max}describes the ultimate service capacity of the tollbooths; q_{2max} describes the ultimate capacity of the merging pattern; ρ_{limit} describes the ultimate density of vehicles considering security and traffic capacity.[7]

Through the limited velocity v_{limit} we can deduce the average delay time T_3:

$$T_3 = \frac{L_4 + L_5}{v_{limit}} - \frac{L_4 + L_5}{v_2}$$

Where L_4 and L_5 are determined by the width of the tollbooth area considering safety[6]. Also, we can deduce the length of the buffer zone L_3:

$$L_3 = T_3 \cdot v_2$$

- The fan-in zone is set for vehicles merging from the VSL area to the outside traffic lanes. At the left side of the area, we set traffic lights to control the number of vehicles in the merging area (N) to maximize the portable output flow q_2[7].

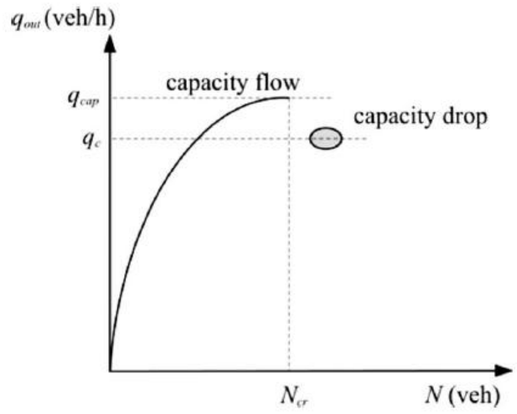


Fig. 4 Change of q_2 with Number of Vehicles in the Fan-in Zone[8]

The specific driving situation in the area is simulated by the software VISSIM as follows:

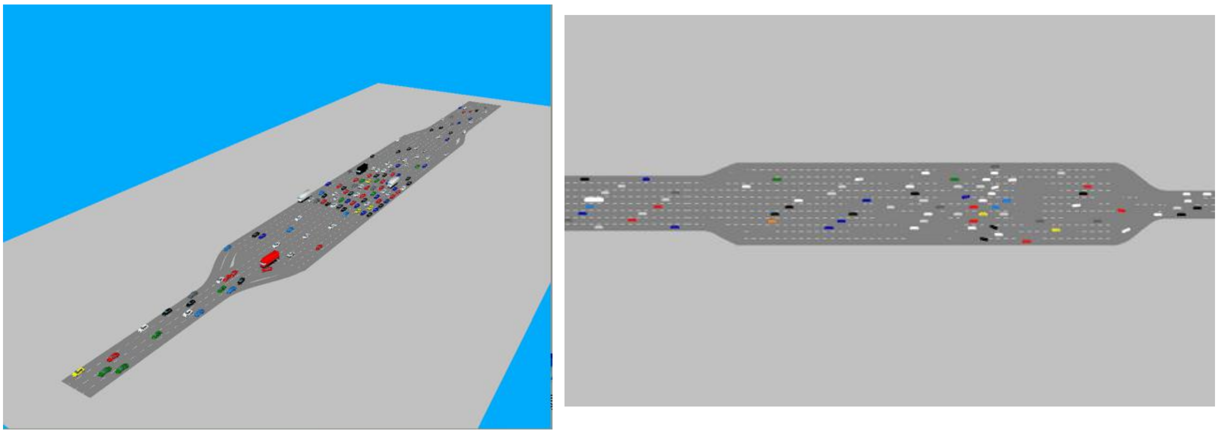


Fig. 5 VISSIM Simulation

Additionally, we put the analysis of the specific shape of the area in the next part.

2) Shape of merging area

While considering the shape of the fan-in zone, we simplify the shape variable into the edge of the fan-in zone and set several typical shapes (Fig. 7).

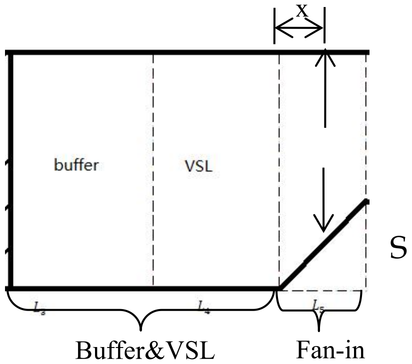


Fig. 6 Merging area

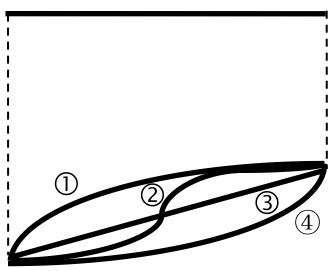


Fig. 7 Several Typical Shapes

To determine the specific shape, we set the width as $w(x)$ (Fig. 6). In order to enable the vehicle to run smoothly, it is suggested that the gradient rate S/L_5 should be less than 1/3 ("Japanese highway design essentials" (1987)). So we set S/L_5 as 1/5 according to recommendations. If S is determined, L_5 is determined, and the shape of fan-in zone is merely defined by $w(x)$.

According to fluid mechanics mentioned in Previous work, T_C and S_C can be represented as follows:

$$T_c = \frac{B \cdot w_0}{v_{limit}} \int_0^{L_c} \frac{1}{w(x)} dx$$

$$S_c = \int_0^{L_c} w(x) dx$$

As for the ability of preventing accidents, we select a few values of the $w'(x)$ (PS: $w'(x)$ is the derivative of $w(x)$) at equal intervals and use the variance of these values as COS, our parameter of the ability of preventing accidents.

$$COS = Var[w'(x_1), w'(x_2) \dots w'(x_n)]$$

$Var[]$ is the function of variance.

- **For type ① ③ ④ in Fig. 7:**

We assume $w(x) = ax^3 + bx^2 + cx + d$. Then limitation factors can be expressed as $w(0) = w_0 \cdot B$ and $w(L_5) = w_0 \cdot L$. Because in this fan-in area, width decreases as x increases, so another limitation factor is $w'(x) \le 0, (0 < x < L_5)$. With these three limitation factors, we can get the conclusion that T_C and S_C are both decided by a and b, while a and b must satisfy the following condition:

$$3ax^2 + 2bx + \frac{w_0(L-B)}{L_5} - aL_5^{\ 2} - bL_5 \le 0, 0 < x < L_5$$

We introduce an algorithm similar to EM algorithm. Firstly, we set b as a constant number and respectively find the value of a which minimize T_C or minimize S_C. Secondly, we set a as the value found before and respectively find the value of b which minimize T_C or minimize S_C. After a finite number of iterations, the optimal solutions of a and b can be obtained from this, which means that the shape of confluence zone can be obtained.

- **For type ② in Fig. 7:**

We assume $w(x)=k\arctan(x+x_0)+b_0$. By considering the same limitation factors of $w(0)$ and $w(L_C)$, $w(x)$ can be expressed as the following expression and then with integral action, T_C and S_C can be expressed merely as the function of x_0.

$$w(x)=\frac{(w_0B-w_0L)\arctan(x+x_0)}{\arctan x_0-\arctan(L_5+x_0)}+w_0B-\frac{(w_0B-w_0L)\arctan x_0}{\arctan x_0-\arctan(L_5+x_0)}$$

In our experiment, we set $w_0=3.5m, B=8, L=4, L_5=70m, v_{\lim it}=8.3m/s$, and then get the relationship between T_C and x_0 and relationship between S_C and x_0.

$$w(x)=ax^3+bx^2+cx+d$$

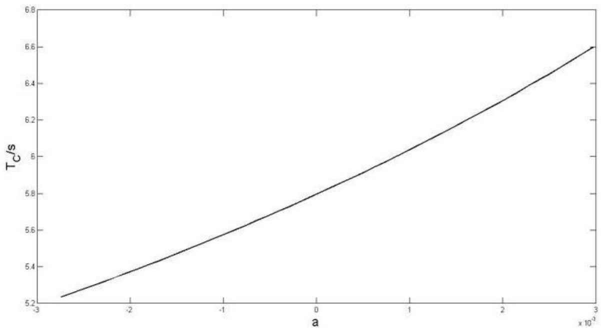


Fig. 8 Relationship between T_C and a

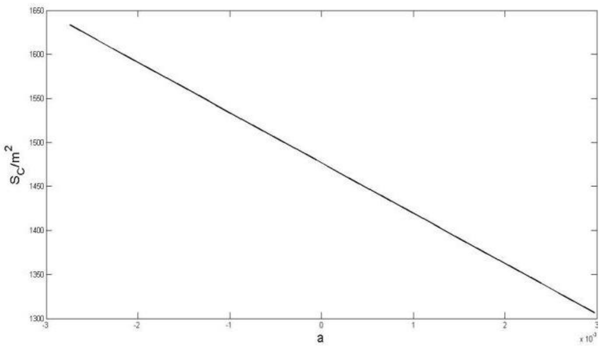


Fig. 10 Relationship between S_C and a

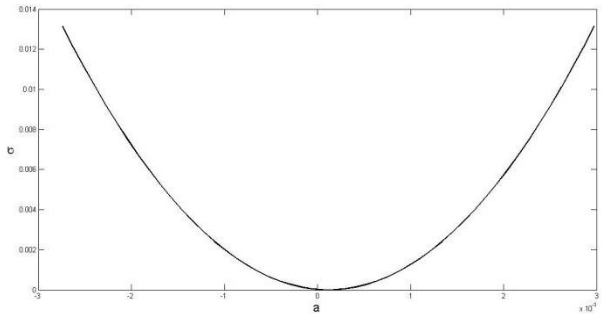


Fig. 12 Relationship between σ and x_0

$$w(x)=k\arctan(x+x_0)+b_0$$

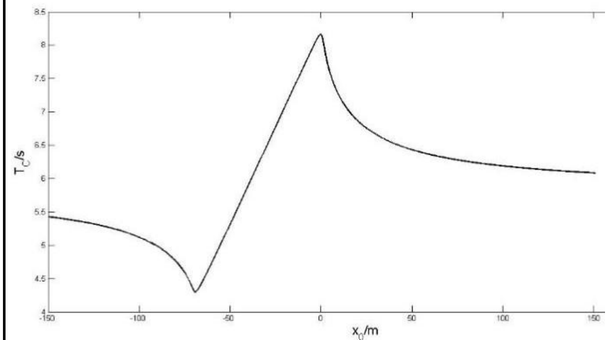


Fig. 9 Relationship between T_C and x_0

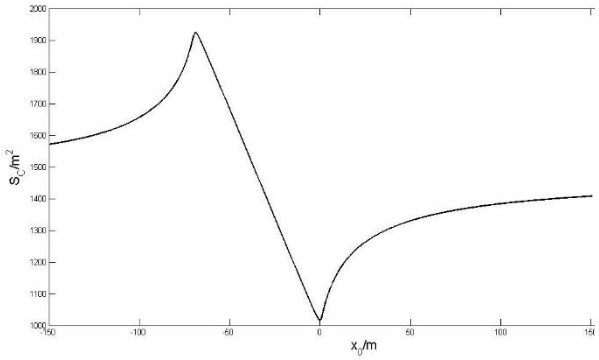


Fig. 11 Relationship between S_C and x_0

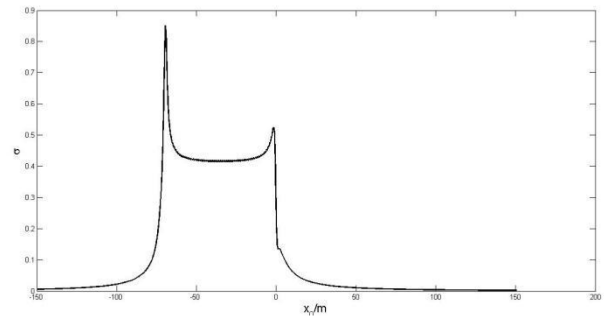


Fig. 13 Relationship between σ and x_0

5. A Multi-objective Optimization Model

When it comes to the comprehensive performance of a toll plaza, we build a multi-objective optimization model, taking accident prevention, throughput and cost into consideration, designing an index to describe all these three objectives.

5.1 The Multi-objective Optimization Model

- The throughput is evaluated by the delay time (the throughput decreases with T_{delay} increasing). For example, T_{delay} in our improved VSL model is T_3 . Other merging models' T_{delay} can also be deduced or simulated by VISSIM.
- The cost is evaluated by the shape and size of the merging area and the tollbooths:

$$\text{Cost} = S_c \cdot cost_0 + cost_{tollbooths}$$

Where S_c describes the area of the merging area, $cost_0$ is the cost per unit area, $cost_{tollbooths}$ is the cost of those tollbooths and is seen as constant.

- The accident prevention is evaluated mainly by the shape:

$$COS = Var(w'(x_1), w'(x_2), w'(x_3) \dots w'(x_n))$$

Based on this expression, the security index COS declines with the variance increasing. In order to integrate the three factors, we normalize them into a range [0, 1].

$$\begin{cases} X = \dfrac{T_{delay}}{T_{ideal}} \\ Y = \dfrac{S_c}{S_{max}} \\ Z = COS(the\ \ security\ \ coefficient) \end{cases}$$

X describes throughput, Y describes cost, and Z describes accident prevention.

Then we get the final multi-objective optimization model:

find shape, size and merging pattern

min X,Y

max Z

$$\text{st.}\begin{cases} shape \\ size \\ merging\ \ pattern \end{cases}$$

5.2 Comprehensive Performance Index Model

To solve the model as well as to evaluate a toll plaza's comprehensive performance, we assign a weight to each factor and make linear combination to obtain the final comprehensive index COP (coefficient of performance).

$$\mathrm{COP} = -\omega_1 \cdot X - \omega_2 \cdot Y + \omega_3 \cdot Z$$

Thus the multi-objective optimization model changes to a single objective optimization model:

$$\text{find} \quad \text{shape, size and merging pattern}$$

$$\max \quad \mathrm{COP}$$

$$\text{st.} \begin{cases} shape \\ size \\ merging\ \ pattern \end{cases}$$

We use the Genetic Algorithm to determine the weights, the values of the parameters in which are shown as follows:

Table. 2 Values of Genetic Algorithm Parameters

Parameter	Evolutionary times	Crossover probability	Mutation probability	Sample size	ω_1	ω_2	ω_3
Value	10	0.5	0.1	100	0.3	0.3	0.4

5.3 The best toll plaza-building strategy

Using the above-mentioned method, we test the performance of a number of different toll plazas and get a best one. The specific expression, variables and the values of those constant parameters are as follows:

$$\mathrm{COP} = -\omega_1 \cdot \frac{T_{delay}}{T_{delay} \cdot v_2 + L_4 + L_5} - \omega_2 \cdot \frac{M_s \cdot \int_0^{L_5} w_1(a,x)dx + (1 - M_s) \cdot \int_0^{L_5} w_2(x_0,x)dx}{L_5 \cdot w_0 \cdot B}$$
$$-\omega_3 \cdot (M_s \cdot Var\{w_1'(a,x)\} + (1 - M_s) \cdot Var\{w_2'(x_0,x)\}$$

Table. 3 Actual Specific Variables

Direct factor	Shape & Size		Merging pattern	Others
Proxy variable	M_s (0 or 1)	$w_i(x) = \begin{cases} w_1(a,x) \\ w_2(x_0,x) \end{cases}$	T_{delay}	$\omega_i (i = 1,2,3)$
Value range	{0,1}	$w_1(a,x) = ax^3 + bx^2 + cx + d$ $w_2(x_0,x) = k \arctan(x + x_0) + b_0$	{11.3,10.5,9.4,9.3,8.2}	{0.3,0.4,0.3}

Table. 4 Values of the Constant Parameters

L	B	w_0	v_2	L_4	L_5	q_0	μ
4	8	3.5	8.3	20	70	3000	400

Results:

$$w_1(x) = ax^3 + bx^2 + cx + d$$

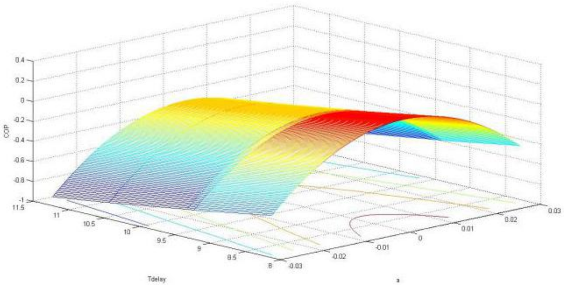


Fig. 14 Relationship between COP, T_{delay} and a

Relatively optimized solution:

$$a = 0.0010$$

$$w_1(x) = 0.0010x^3 + 0.0015x^2 + 0.3050x + 28$$

Merging pattern：5

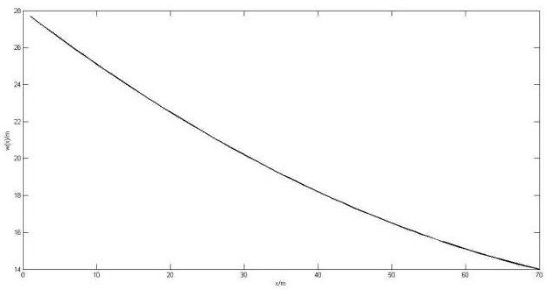


Fig. 16 Shape of $w_1(x)$

$$w_2(x) = k_0 \arctan(x - x_0) + b_0$$

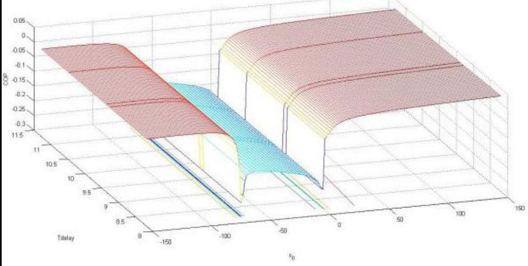


Fig. 15 Relationship between COP, T_{delay} and x_0

Relatively optimized solution:

$$x_0 = 150$$

$$w_2(x) = \text{-}6600.2\arctan(\mathrm{x} + 150) + 10352$$

Merging pattern：5

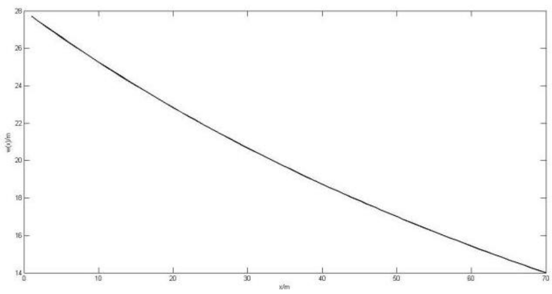


Fig. 17 Shape of $w_2(x)$

6. Improved Model Considering Tollbooth-type

In theory, the use of ETC technology can increase the service capacity of toll stations. However, if the ETC lane utilization rate is not high enough, the human-staffed toll lane reduction makes the human-staffed toll lane delays increase. On the contrary, if the usage of ETC lane is too high, and lane configuration is not adjusted, it may lead to the situation where ETC lane’s delay is higher than human-staffed lane.So in our improved model, we take the proportions of different tollbooth types into consideration.

If a vehicle passes the ETC lane, it must have an OBU (On - Board Unit). So we divide vehicles arriving at the toll plaza into two categories. Vehicles of category1 don’t have OBUs, and those of cateory2 do. The corresponding relationship between the vehicle and the toll lane they selected can be determined and then we can build vehicles' queuing system in the toll station (Fig. 18).

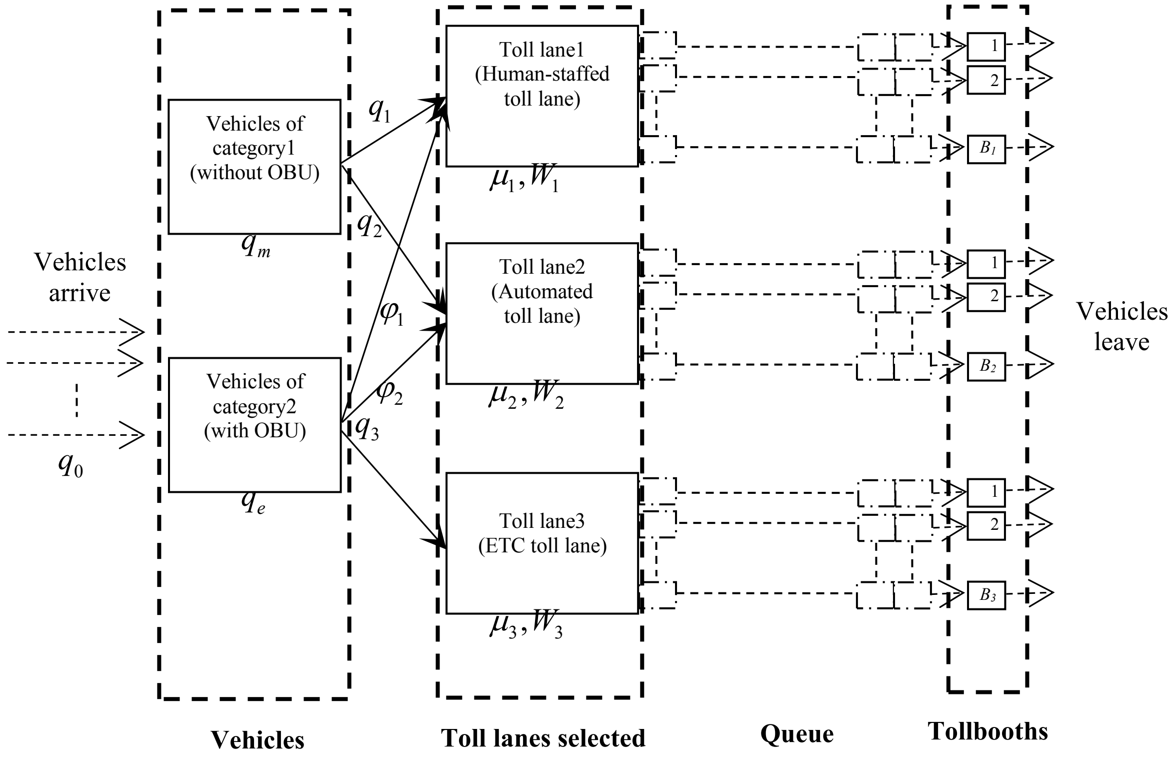


Fig. 18 Vehicles' Queuing System in the Toll Station

Table. 5 Symbols Used in This Part and Meaning of Them

Symbols	Meaning
B_1, B_2, B_3	Number of three types of tollbooths
,, μ_3	Service rate of three types of tollbooths
W_1 W_2 W_3	Sojourn time of three types of tollbooths
q_0	flow of vehicles arriving
q_m, q_e	flow of vehicles without OBU and with OBU
q_1, q_2	flow of vehicles of category1 selecting toll lane1 and toll lane2
q_3	flow of vehicles with OBU
φ_1, ϕ_2	probability of vehicles of category2 selecting toll lane1 and toll lane2

As OBU's utilization rate is ϕ, we can get:

$$q_m = q_0(1-\varphi), q_e = q_0 \cdot \varphi$$

Then, the general equivalent relationships are listed as follow:

$$\begin{cases} q_1 + q_2 = q_m \\ \dfrac{q_1 \cdot \dfrac{1}{\mu_1}}{q_2 \cdot \dfrac{1}{\mu_2}} = \dfrac{B_1}{B_2} \\ q_3 = q_e \end{cases}$$

Based on the principle of queuing theory, three queuing models of toll station vehicles are analyzed, and three types of tollbooths allocation models are established with W_{AST} as the objective function. We have general constraints and use them to express W_{AST} as the function of B_1 and B_2.

$$\begin{cases} B_1 + B_2 + B_3 = B \\ \dfrac{(q_1 + q_3 \cdot \varphi_1)\dfrac{1}{\mu_1}}{(q_3 - q_3 \cdot \varphi_1)\dfrac{1}{\mu_3}} = \dfrac{B_1}{B_3} \\ \dfrac{(q_2 + q_3 \cdot \varphi_2)\dfrac{1}{\mu_2}}{(q_3 - q_3 \cdot \varphi_2)\dfrac{1}{\mu_3}} = \dfrac{B_2}{B_3} \end{cases}$$

$$W_{AST} = W_1 + W_2 + W_3 = \frac{B_1}{B_1 \cdot \mu_1 - (q_1 + q_3 \cdot \varphi_1)} + \frac{B_2}{B_2 \cdot \mu_2 - (q_2 + q_3 \cdot \varphi_2)} + \frac{B_3}{B_3 \cdot \mu_3 - q_3(1 - \varphi_1 - \varphi_2)}$$

We fix B, μ_1, μ_2, μ_3 and q_0, and get Fig. 19, which shows the condition when ϕ is 0.3:

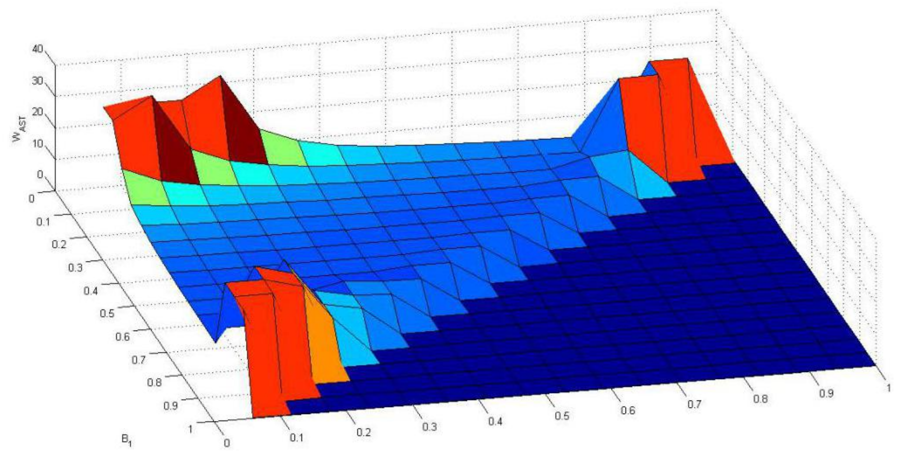


Fig. 19 3-D Plot W_{AST} of Different Proportions of B_1 and B_2

With ϕ varying from 0 to 1, the best set solution of proportions of three types of tollbooths is shown in Fig. 20 and the minimum W_{AST} is shown in Fig. 21.

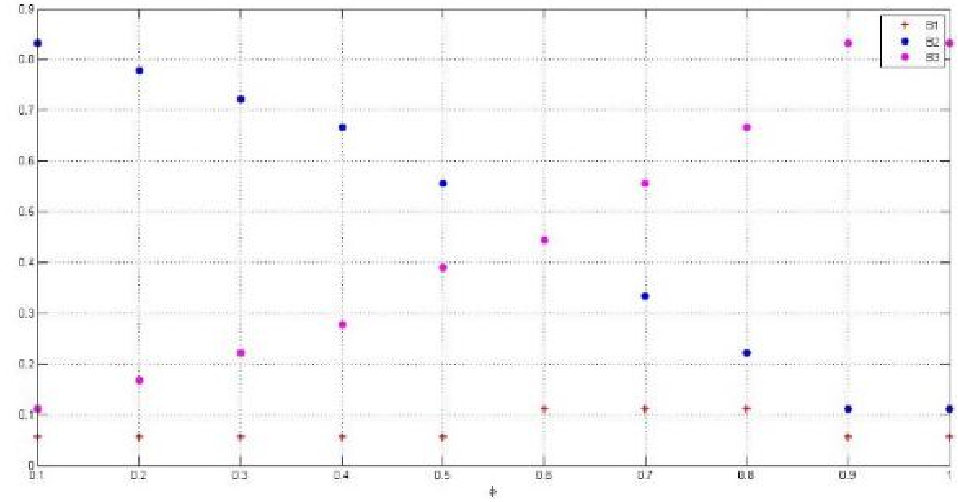


Fig. 20 Best Solution of Proportions with Different ϕ

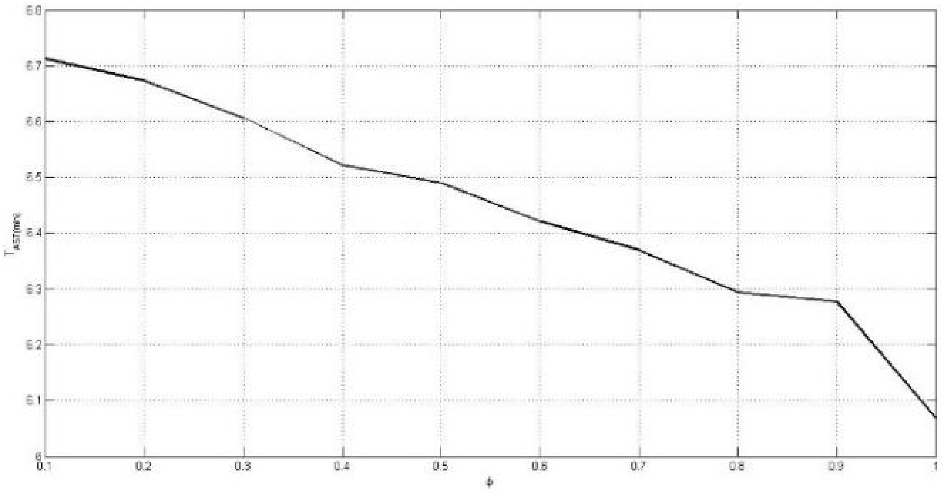


Fig. 21 Minimum W_{AST} of Different ϕ

7. Sensitivity Analysis

7.1 Light or Heavy Traffic

The magnitude of traffic flow affects the throughput of our model.

- When the traffic is light, the vehicles need not wait at the tollbooths, the throughout is the same as the entering flow.
- When the traffic is heavy, the model would change according to (the flow exiting the tollbooths and entering the merging area Ω3):

$$v_{limit} = \frac{q_{2max}}{\rho_{limit}} = \frac{q_{2max}}{2q_1 - q_{2max}} \cdot v_2, \quad (\rho_{limit} = \frac{2q_1 - q_{2max}}{v_2})$$

$$q_1 = \begin{cases} q_0, & when\ q_0 < q_{1max} \\ q_{1max}, & when\ q_0 > q_{1max} \end{cases}$$

$$q_{1max} = \mu \cdot B, q_{2max} = \mu \cdot L$$

As is shown above, the limited velocity v_{limit} in the VSL area changed according to the value of q_1. When $q_0 < q_{1max}, q_1$ depends on q_0, and the whole model is just like that we discuss above; when $q_0 > q_{1max}$, q_1 is limited by the maximal service capacity of the tollbooths, and the model should be adjusted.

7.2 The proportions of different types of tollbooths

In our improved model, we give the best proportion of three types of tollbooths with OBU's utilization rate ϕ as variable, in which condition, the average sojourn time is the shortest.

However, in the actual construction, the proportion may be different from the best solution. In this condition, the average sojourn will be longer. As a result, the throughput will get smaller.

7.3 More autonomous vehicles

We take autonomous vehicles as vehicles equipped with OBU. On the one hand, as more autonomous vehicles are added, ϕ will increase. When the proportion stays the same, the average sojourn will increase, which means that the throughput will decrease. On the other hand, considering from the perspective of accident prevention, because our model introduce the buffer area, where drivers will choose lanes sensibly. So each the flow of each lane will be roughly equal. It comes to the conclusion that accident prevention ability will not deteriorate from higher proportion of autonomous vehicles.

7.4 Change weights

Meanwhile, because of the consideration of the weights ω_1, ω_2 and ω_3 ,which denote the demand of construction units, we can design different kinds of toll plazas which will fulfill their requirements.

8. Strengths and Weaknesses

8.1 Strengths

- Our model detailedly describes the whole process of the vehicles passing through the toll plaza, each stage with a most suitable model.
- We get explicit formulas for both the whole system and each single model.

- We combine rigorous mathematical derivation with stochastic simulation.

8.2 Weaknesses

- Before solving the model, we use the fluid mechanics to simulation traffic flow. So the result will not be exactly the same as the real situation.
- We don't consider the effect of the continuous change of vehicles on accident prevention ability, so our index of accident prevention ability can be improved.

9. References

[1]. Rongguo M, Libo Y. Theory and Method of Traffic Engineering Design: China Communications Press; 2002.

[2]. Minghui M, Qingfang Y, Shidong L, Wei D. Coordinated Control Method of Freeway Main Line and Ramp Road Confluence Area [J]. Journal of Harbin Engineering University. 2015,36(12):1603-8.

[3]. Nagel K, Scheckenberg M. A Cellular Automaton model for freeway traffic [J]. J Phys, 1992 (I-2): 2221 - 2229.

[4]. Chunlei W, Yulin C. Research on toll lane allocation of toll plaza in expressway [J]. Road construction. 2008,33(5):172-5.

[5]. Chonghua Z, Kyushu Z, Zhizhe S. Model configuration based on queuing theory and the incremental cost-effectiveness ratio to maximize ETC lane [J]. Journal of Transportation Systems Engineering and Information Technology. 2009,9(5):77-84.

[6]. Li S. Study on Design of Toll Station of Expressway [master]: Changan University; 2012.

[7]. Tijms, H.C, Algorithmic Analysis of Queues", Chapter 9 in A First Course in Stochastic Models, Wiley, Chichester, 2003.

[8]. Spiliopoulou AD, Papamichail I, Papageorgiou M. Toll plaza merging traffic control for throughput maximization [J]. Journal of Transportation Engineering. 2009,136(1):67-76.

Appendix

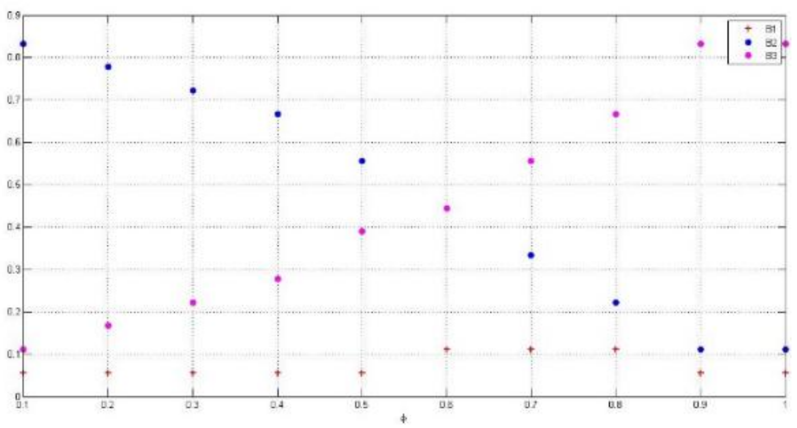
Dear Sir or Madam:

With the booming traffic vehicles and road network, how to design those essential road facilities has become a more and more important problem. It shows that due to 2015, the New Jersey's daily VMT have reached to 206,557,313. To improve the traffic condition on toll highways, we have tried to determine a feasible toll plaza design, considering objectives like throughput, cost and security.

With a basic 3-stage model describing the whole toll plaza, we put our focus on the third stage to design the best shape, size and merging pattern of the toll plaza. Using the simulation software Vissim, we analyze a number of toll plaza designs that may already be implemented and pick out a best one---the improved VSL merging model. Based on this model, we suggest you divide the fan-in area after the tollbooths into buffer, VSL and merging area. Additionally, it is better to set some traffic lights to reduce the risk of accidents as well as make the traffic flow well-organized.

As to the shape and size of the toll plaza, we build a multi-objective optimization model to figure out the best values of these two variables. By analyzing the statistics of the New Jersey traffic and some general premises, we work out that the best shape of the merging area is like a cubic curve ($w_1(x) = 0.0010x^3 + 0.0015x^2 + 0.3050x + 28$) or an arc tangent curve($w_2(x) = \text{-}6600.2\arctan(x + 150) + 10352$) , and the length of this area is better determined by the width of the main lanes at a certain formula.

Additionally, we have also considered the proportion of different types of tollbooths, since more and more autonomous vehicles are added into the traffic mix. The most suitable proportion is shown as follows:



We have a strong belief that our model is of high feasibility. Hope you can think about our idea.

Your sincerely